Tecnologías de la Información, Comunicaciones y Matemática Computacional

Universidad de Valencia

Non-convex Distributed Power Allocation Games in Cognitive Radio Networks

Xiaoge Huang

Advisors

Prof. Baltasar Beferull Lozano
Dr. Carmen Botella Mascarell

Thesis submitted for the Degree of Philosophiae Doctor (Ph.D.)

Valencia, May 2013

To my family

Resumen

Motivación y objetivos

En las últimas décadas, las comunicaciones inalámbricas han experimentado un desarrollo espectacular, con el objetivo de proporcionar una experiencia de usuario similar a los sistemas cableados, pero bajo la filosofía de poder comunicarse "en cualquier lugar y en cualquier momento". El desarrollo de los sistemas comerciales ha sido posible gracias a la aparición de nuevas tecnologías, que, en general, requieren cada vez de canales con un mayor ancho de banda. Sin embargo, el espectro electromagnético es un recurso limitado, cuyo uso viene regulado por el gobierno de cada país. En el caso particular de Estados Unidos, los últimos resultados publicados por la Comisión Federal de Comunicaciones (FCC, *Federal Communications Commission*) muestran que el espectro electromagnético está infrautilizado actualmente; algunas bandas de frecuencias son muy utilizadas, mientras que otras están sólo parcialmente ocupadas. De hecho, la ocupación varía en espacio y tiempo, es decir, estas bandas están desocupadas en diferente regiones del espacio durante ciertos intervalos de tiempo. La tecnología de radios cognitivas se propone recientemente como una solución para promover un uso eficaz del espectro, ya que la base de su funcionamiento es la detección en un determinado lugar y momento de "agujeros en el espectro" o bandas de frecuencias desocupadas por el usuario primario o principal con la correspondiente licencia. Gracias a esta detección, un usuario secundario o sin licencia para ocupar esa banda de frecuencias o canal, puede transmitir/recibir temporalmente en esas frecuencias utilizando una radio cognitiva sin perturbar las comunicaciones entre usuarios primarios. El hecho de compartir el espectro es beneficioso para los usuarios secundarios o radios cognitivas, y aumenta la eficiencia en el uso del espectro. Sin embargo, desde el punto de vista práctico, esta tecnología presenta muchos retos que o bien siguen sin resolverse, o bien todavía no se han resuelto de una forma eficiente. Por un lado, uno de los principales retos es la implementación de un método de detección fiable para encontrar agujeros en el espectro, identificando así las oportunidades de transmisión del usuario secundario sin comprometer la integridad de las comunicaciones entre usuarios primarios. Hay que tener en cuenta que si un usuario secundario detecta erróneamente que un canal de frecuencias está libre y puede transmitir, cuando no lo está en realidad, producirá una interferencia que degradará la experiencia del usuario primario. Por tanto, es imprescindible mantener una probabilidad de detección errónea lo suficientemente pequeña. Por otra parte, otro criterio de diseño es disminuir la probabilidad de falsa alarma tanto como sea posible, ya que ésta refleja el porcentaje de espectro vacante que está clasificado erróneamente como ocupado, aumentando así el uso oportunista del espectro de los usuarios secundarios de radio cognitiva. Por otro lado, con el fin de limitar la probabilidad de interferir con los usuarios primarios, es deseable mantener la probabilidad de fallo de detección tan baja como sea necesario para cumplir las restricciones exigidas para proteger a los usuarios primarios. El umbral de detección es el parámetro que determina el equilibrio entre la probabilidad de falsa alarma y la probabilidad de fallo de detección: un umbral bajo aumenta la probabilidad de falsa alarma, disminuyendo la probabilidad de fallo (aumentando la probabilidad de detección), y viceversa.

Otro parámetro que influye en el proceso de detección es el tiempo de detección, es decir, el tiempo que el usuario secundario emplea en el proceso de detección. La elección de este tiempo ofrece una solución de compromiso entre la calidad y la velocidad de detección: aumentar el tiempo de detección disminuye tanto la probabilidad de falsa alarma como la probabilidad de detección, pero reduce también el tiempo disponible para las transmisiones secundarias, lo que reduce el rendimiento en transmisión de la radio cognitiva. Esta dependencia entre los parámetros es la que justifica el diseño conjunto de los parámetros de detección y transmisión de la radio cognitiva, siempre suponiendo que estas radios presentan una naturaleza egoísta y por lo tanto, no están dispuestas a cooperar entre ellas.

Esta tesis se centra en las tecnologías de comunicaciones relacionadas con la radio cognitiva, tomando como base un sistema de comunicaciones de tipo OFDM (*Orthogonal Frequency Division Multiplexing*), como los propuestos en los futuros sistemas de comunicaciones de cuarta generación (LTE, *Long Term Evolution*). Concretamente, a lo largo de la tesis, se investigan sistemas de comunicaciones con un solo usuario secundario o con varios usuarios secundarios (caso multiusuario). En el caso de los sistemas con un solo usuario secundario, el objetivo es resolver el problema de asignación de potencias de transmisión sobre diferentes canales en base a la información de detección disponible bajo el modelo de acceso oportunista al espectro. En el caso de los sistemas con múltiples usuarios secundarios, el comportamiento no cooperativo entre las radios cognitivas se modela utilizando teoría de juegos. El principal objetivo de este segundo caso es modelar y analizar el problema de optimización multiusuario competitivo, teniendo en cuenta la incertidumbre asociada con el proceso de detección.

Metodología

En esta tesis, se combinan los métodos de análisis teórico y la simulación por ordenador.

En el análisis teórico de los casos de un usuario y de múltiples usuarios secundarios, se ha seguido la metodología clásica de construir el nuevo concepto teórico como un sistema lógico con definiciones y operaciones para demostrar los distintos teoremas. En primer lugar, el problema se modela como un problema de optimización matemática. Después, se aplican métodos de optimización para analizar y resolver este problema. Especialmente, la tesis se centra en el problema de optimización de recursos, tanto para sistemas de comunicaciones con una sola radio cognitiva como para el caso multiusuario. Para el caso de un solo usuario, la función objetivo, así como las restricciones para el problema de optimización de recursos son no convexas, lo que da lugar a un problema complejo de resolver que motiva la utilización del método alternante para resolver el problema. Para los sistemas multiusuario, el problema de optimización consiste en un juego no cooperativo, donde se utiliza el concepto nuevo de cuasi-equilibrio de Nash.

Tras el análisis teórico, los algoritmos propuestos para los dos casos considerados (un único usuario y múltiples usuarios), se simulan por ordenador mediante el programa Matlab. Es importante destacar que tanto para el caso de un usuario como para el caso multiusuario, los escenarios simulados han tenido en cuenta parámetros y modelos de simulación estándar, propuestos por organismos de estandarización o por la comunidad científica, con el objetivo de facilitar la reproducción de los resultados obtenidos en esta tesis. Del mismo modo, los algoritmos propuestos se han comparado en todos los casos con los mejores algoritmos de optimización propuestos en la literatura de radios cognitivas. Aunque las simulaciones se han realizado con el programa Matlab, sería posible utilizar otras plataformas de simulación y lenguajes de programación.

Conclusiones y Contribuciones

En esta tesis, se considera un sistema de comunicaciones de tipo *interweave* con radios cognitivas donde el objetivo general es maximizar la tasa de cada usuario secundario mediante la optimización conjunta de la operación de detección y la asignación de potencias de transmisión en diferentes canales, teniendo en cuenta la influencia de la incertidumbre en el proceso de detección y el hecho de que la potencia de interferencia que puede aceptar un usuario primario está limitada. Este problema se plantea tanto para sistemas de comunicaciones con un único usuario secundario como para sistemas con múltiples usuarios secundarios. Además, en el caso multiusuario, también se contempla el escenario donde tanto el usuario primario como los usuarios secundarios disponen de múltiples antenas (canal MIMO, *Multi-Input Multi-Output*).

En primer lugar, se estudia el problema de optimización de la asignación de recursos para sistemas de comunicaciones con un único usuario secundario, donde tanto el usuario primario como el usuario secundario disponen de una única antena (canal SISO, *Single-Input Single-Output*). El usuario primario dispone de varios canales para transmitir. El objetivo del usuario secundario es maximizar su tasa mediante la optimización de forma conjunta de la información de detección (el resultado del proceso de detección es común a todos los canales) y de la asignación de potencia para cada canal.

En este escenario, se considera que el sistema de comunicaciones es de tipo *interweave*, con un acceso oportunista al espectro, donde la radio cognitiva detecta si hay transmisión del usuario primario en todos los canales, y decide transmitir si los resultados de la detección indican que el usuario primario está inactivo en ese canal. Sin embargo, debido a los errores de detección, la radio cognitiva podría acceder a un canal cuando todavía está ocupado por el usuario primario, provocando interferencias perjudiciales tanto para los usuarios secundarios como para los usuarios primarios. Por este motivo, en el algoritmo de asignación de potencia se propone una restricción en la potencia de interferencia introducida por el usuario secundario, llamada *rate-loss gap*, que asegura que la degradación experimentada por el usuario primario está acotada. El problema de optimización resultante es no convexo. Para resolverlo, se proponen un algoritmo de optimización exhaustivo y un algoritmo de optimización de dirección alternante. El análisis de la complejidad del algoritmo de optimización de dirección alternante, junto con los resultados de las simulaciones, prueban que este algoritmo resuelve el problema de forma eficaz.

En segundo lugar, la tesis se centra en el problema de la asignación de recursos en sistemas de comunicaciones con múltiples usuarios primarios y secundarios, pero con una única antena por usuario. En este escenario, se asume que cada usuario primario dispone de un canal distinto en el que transmitir. En este caso, el esquema de acceso al espectro es de tipo espectro compartido, y el problema de asignación de recursos se plantea como un juego de estrategia no cooperativa, donde cada radio cognitiva es egoísta y se esfuerza por utilizar tantos canales como sea posible con el fin de maximizar su propia tasa, considerando también el impacto de disponer de información de detección imperfecta. En el esquema de espectro compartido, las radios cognitivas pueden coexistir con los usuarios primarios y ajustar la potencia de transmisión en cada canal en función del resultado de la detección.

Cuando se aplica la teoría de juegos a este escenario, el juego resultante pertenece a la clase de juegos no convexos. La no convexidad se debe tanto a las funciones objetivo como al conjunto de resultados posibles resultantes de los problemas de optimización individual de cada radio cognitiva. En un primer paso, se propone utilizar un esquema distribuido de detección cooperativa, basado en un algoritmo de consenso, donde las radios cognitivas comparten su información de detección únicamente a nivel local. Tras la detección, para resolver el problema de asignación de recursos, se propone el algoritmo de optimización de dirección alternante, demostrando que es posible alcanzar un equilibrio local de Nash. A continuación, se utiliza el nuevo concepto de equilibrio relajado o cuasi-equilibrio de Nash. Se realiza el análisis de las condiciones suficientes para demostrar la existencia del cuasi-equilibrio de Nash para el juego bajo consideración. Tras este análisis, se propone un algoritmo iterativo de punto interior primal-dual que converge al cuasi-equilibrio de Nash del juego considerado. A partir de los resultados de las simulaciones, se comprueba que el método propuesto mejora considerablemente la tasa de las radios cognitivas con respecto a distintos métodos alternativos propuestos en la literatura.

Finalmente, se investiga un escenario con múltiples usuarios primarios y secundarios, que además disponen de múltiples antenas. El esquema de acceso al espectro considerado es el de acceso oportunista. En este último caso, el problema a resolver sigue siendo la asignación de recursos de las radios cognitivas cuando cada usuario primario dispone de una canal distinto en el que transmitir. El problema de optimización se analiza como un juego no cooperativo estratégico, donde la matriz de covarianza de transmisión, el tiempo de detección y el umbral de detección son las variables a optimizar conjuntamente. El juego resultante es no convexo, por lo tanto, se utiliza nuevamente el concepto de cuasi-equilibrio de Nash, y se demuestra analíticamente que el juego propuesto puede lograr un cuasi-equilibrio de Nash bajo ciertas condiciones, mediante la utilización del método de *Variational Inequality* (VI). En particular, se demuestra teóricamente la condición suficiente de la existencia y la unicidad del cuasi-equilibrio de Nash para el juego propuesto. Por otra parte, se presenta una posible extensión de este trabajo teniendo en cuenta el tiempo de detección para las radios cognitivas. A partir de los resultados de las simulaciones, se demuestra que el algoritmo iterativo de punto interior primal-dual converge de forma eficiente al cuasi-equilibrio de Nash.

Como trabajo futuro, se contempla el desarrollo de la introducción del factor de precio en el escenario con múltiples usuarios primarios y secundarios. Otra extensión futura es la introducción de un método de detección robusto basado en detección cooperativa. Una alternativa a la detección, es la utilización de mapas de interferencia, que actuarían como *soft info*. Por último, todos los algoritmos y esquemas presentados en esta tesis se han analizado mediante simulaciones. Su implementación en un testbed sería de gran utilidad de cara a su posible aplicación práctica.

Acknowledgement

This dissertation could never have been written without help and support from many people around me.

First of all, I would like to thank my supervisor Prof. Baltasar Beferull Lozano for hiring me as a PhD student in GSIC and providing me such a good working environment. Your work as my advisor not only shaped the student I am today, but the professor I aspire to become. You managed to provide me the freedom to think and explore, while simultaneously giving me the guidance I needed to stay focused. Specially, I would like to thank your for not assigning me heavy tasks in the funding projects so that I could finish my thesis writing on time.

I would also like to thank my other advisor and friend Prof. Carmen Botella Mascarell, who not only constantly fueled me with strength, hope and support to take on challenges and go through frustrations, but also helped me a lot during all the period of my research with guidance and stimulating suggestions. Thank you for believing in me and for your advice and friendship. Specially, I would like to thank your mother and your family for their thoughtful kindness to me, which warmed my heart and greatly eased my homesickness.

To my colleagues and friends in the Group of Information and Communication Systems: Eugenio Celada, César Asensio, Santosh Shah, Ibra Khalife, Fernando Camaro Nogues, Daniel Alonso, Vinay Prasad, and Gustavo Hernandez. We had many valuable discussions and glad cooperations. Special thanks goes to Eugenio Celada, who is not just a friend, but also a free driver, Spanish teacher, and the guy who is always helpful. It is truly a pleasure to get to know each of you. Thank you for your help during my life in Spain and I will treasure this wonderful memory during my whole life.

To my friends: Yan Duan, Gang Qin, Jie Lu, Xuehua, Kaiyang, Yixuan, Qinhui, Jiuxing, Zhibin, En Wang, Teng Xu, and all the friends who helped me. I consider myself lucky that I could meet you guys in Spain. Thanks for your support, encouragement and friendship. I will never forget those happy days we spent together. Looking forward to meeting you guys again in the world, someday, somewhere, so stay in touch and I hope our paths will cross again soon.

Finally, I owe an enormous debt of gratitude to my parents Chao Huang, Junxuan Jiang, Aiping Tang, Xuemei Peng. Your unconditional love, understanding, and unwavering support are the origin of the power for me to keep on moving in my research. I am extremely indebted to my husband Weifan Tang who gives me deep love and the continuous encouragement. Thank you for listening to my complaints and frustrations about study and research, and for believing in me. Without all of this, I would not have completed this thesis. A special thanks to my forthcoming baby for coming to my life at this remarkable moment, and making everyday so beautiful.

Publications

Journal Articles

- Xiaoge Huang, Baltasar Beferull-Lozano, and Carmen Botella, "Quasi-nash equilibria for non-convex distributed power allocation games in cognitive radios," Wireless Communications, IEEE Transactions on. To be published. Accepted May, 2013.
- Xiaoge Huang, Baltasar Beferull-Lozano, and Carmen Botella, "Joint sensing and power allocation in non-convex power allocation games in MIMO cognitive radio networks," Wireless Communications, IEEE Transactions on. To be submitted, 2013.

Conference Proceedings

- Xiaoge Huang and Baltasar Beferull-Lozano, "Joint optimization of detection and power allocation for OFDM-based cognitive radios," In IEEE Global Telecommunications Conference (GLOBECOM 2010), pages 1-5, December, 2010.
- Xiaoge Huang and Baltasar Beferull-Lozano, "Power allocation optimization in OFDM-based cognitive radios based on sensing information," In IEEE International Conference on Acoustics, Speech and Signal Process (ICASSP 2011), pages 3192-3195, May, 2011.
- Xiaoge Huang and Baltasar Beferull-Lozano, "Non-cooperative power allocation game with imperfect sensing information for cognitive radio," In IEEE International Conference on Communications (ICC 2012), pages 1666-1671, June, 2012.

- Xiaoge Huang, Baltasar Beferull-Lozano, and Carmen Botella, "Non-convex power allocation games in MIMO cognitive radio networks," In IEEE International Workshop on Signal Processing Advances for Wireless Communications (SPAWC 2013), June, 2013.

Abstract

In this thesis, we explore interweave communication systems in cognitive radio networks where the overall objective is to maximize the sum-rate of each cognitive radio user by optimizing jointly both the detection operation based on sensing and the power allocation across channels, taking into account the influence of the sensing accuracy and the interference limitation to the primary users. The optimization problem is addressed in single and multiuser cognitive radio networks for both single-input single-output and multi-input multi-output channels.

Firstly, we study the resource allocation optimization problem for single-input single-output single user cognitive radio networks, wherein the cognitive radio aims at maximizing its own sum-rate by jointly optimizing the sensing information and power allocation over all the channels. In this framework, we consider an opportunistic spectrum access model under interweave systems, where a cognitive radio user detects active primary user transmissions over all the channels, and decides to transmit if the sensing results indicate that the primary user is inactive at this channel. However, due to the sensing errors, the cognitive users might access the channel when it is still occupied by active primary users, which causes harmful interference to both cognitive radio users and primary users. This motivates the introduction of a novel interference constraint, denoted as rate-loss gap constraint, which is proposed to design the power allocation, ensuring that the performance degradation of the primary user is bounded. The resulting problem is non-convex, thus, an exhaustive optimization algorithm and an alternating direction optimization algorithm are proposed to solve the problem efficiently.

Secondly, the resource allocation problem for a single-input single-output multiuser cognitive radio network under a sensing-based spectrum sharing scheme is analyzed as a strategic non-cooperative game, where each

cognitive radio user is selfish and strives to use the available spectrum in order to maximize its own sum-rate by considering the effect of imperfect sensing information. The resulting game-theoretical formulations belong to the class of non-convex games, where the non-convexity occurs at both the objective functions and feasible constraint sets of the cognitive radio users' optimization problems. A distributed cooperative sensing scheme based on a consensus algorithm is considered in the proposed game, where all the cognitive radio users can share their sensing information locally. We start with the alternating direction optimization algorithm, and prove that the local Nash equilibrium is achieved by the alternating direction optimization algorithm. In the next step, we use a new relaxed equilibrium concept, namely, quasi-Nash equilibrium for the non-convex game instead of the traditional Nash equilibrium for the convex game. The analysis of the sufficient conditions for the existence of the quasi-Nash equilibrium for the proposed game is provided. Furthermore, an iterative primal-dual interior point algorithm that converges to a quasi-Nash equilibrium of the proposed game is also proposed. From the simulation results, the proposed algorithm is shown to yield a considerable performance improvement in terms of the sum-rate of each cognitive radio user, with respect to previous state-of-the-art algorithms.

Finally, we investigate a multiple-input multiple-output multiuser cognitive radio network under the opportunistic spectrum access scheme. We focus on the throughput of each cognitive radio user under correct sensing information, and exclude the throughput due to the erroneous decision of the cognitive radio users to transmit over occupied channels. The optimization problem is analyzed as a strategic non-cooperative game, where the transmit covariance matrix, sensing time, and detection threshold are considered as multidimensional variables to be optimized jointly. The resulting game is non-convex, hence, we also use the new relaxed equilibrium concept quasi-Nash equilibrium and prove that the proposed game can achieve a quasi-Nash equilibrium under certain conditions, by making use of the variational inequality method. In particular, we prove theoretically the sufficient condition of the existence and the uniqueness of the quasi-Nash equilibrium for this game. Furthermore, a possible extension of this work considering equal sensing time is also discussed. Simulation results show that the iterative primal-dual interior point algorithm is an efficient solution that converges to the quasi-Nash equilibrium of the proposed game.

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List of Abbreviations

ADO	Alternating Direction Optimization
ADOS	ADO for SISO CRNs
APD	Alternating Optimization of Power Allocation and Detection
CR	Cognitive Radio
CRNs	Cognitive Radio Networks
CCC	Common Control Channel
CSI	Channel State Information
DG	Deterministic Game
DSA	Dynamic Spectrum Access
ECC	Electronic Communications Committee
EPD	Exhaustive Optimization of Power Allocation and Detection
FCC	Federal Communications Commission
ISM	Industrial, Scientific and Medical
KKT	Karush-Kuhn-Tucker
LICQ	Linear Independent Constraint Qualification
MIMO	Multiple-Input Multiple-Output
MUI	Multiuser Interference
NCG	Non-Cooperative Game
NE	Nash Equilibrium
NNPG	Non-Convex Non-Cooperative Power Allocation Game
OFDM	Orthogonal Frequency Division Multiplexing
OSA	Opportunistic Spectrum Access

PDIP Primal-Dual Interior Point

PDIPM Primal-Dual Interior Point Optimization in MIMO CRNs

PDIPS Primal-Dual Interior Point Optimization in SISO CRNs

PLR Power Budget Limited Regime

PU Primary User

QNE Quasi-Nash Equilibrium

Rx Receiver

RLR Rate-Loss Limited Regime

SNR Signal to Noise Radio

SISO Single-Input Single-Output

SSS Sensing-Based Spectrum Sharing

Tx Transmitter

VI Variational Inequality

WF Water-Filling

WLANs Wireless Local Area Networks

WPANs Wireless Personal Area Networks

WRANs Wireless Regional Area Networks

Chapter 1

Introduction

The science is nothing more than a refinement of everyday thinking.
——Albert Einstein(1879-1955)

During the last decade, wireless communication networks have been greatly developed including third generation 3G, fourth generation cellular networks, IEEE 802.11 Wireless Local Area Networks (WLANs), IEEE 802.15.4 WPANs, Bluetooth, etc. The radio spectrum ranging from 3KHz to 300GHz is the basic resource to carry data in wireless networks. In each region, spectrum is regulated by its radio regulatory agency, such as Federal Communications Commission (FCC) in USA [6], Electronic Communications Committee (ECC) in Europe [7], and Ofcom in UK [8]. Spectrum is traditionally assigned via a fixed frequency allocation policy. For example, the spectrum allocation table by FCC is shown in Figure 1.1, where each portion of spectrum is exclusively allocated to a specific wireless system, and all subscribers to a wireless system should be granted to access the exclusive spectrum. Following this traditional approach, the spectrum resource is in danger of being exhausted. Obtaining a license on a spectrum band is becoming more and more difficult and expensive.
The Industrial, Scientific and Medical (ISM) spectrum band, which is mostly located around 2.4 GHz and 5 GHz, is the only spectrum that can be shared by different networks. WLANs, WPANs, cordless phones, and even microwave ovens are operating simultaneously in the ISM spectrum band, and thus experiencing interference from each other. Therefore, the

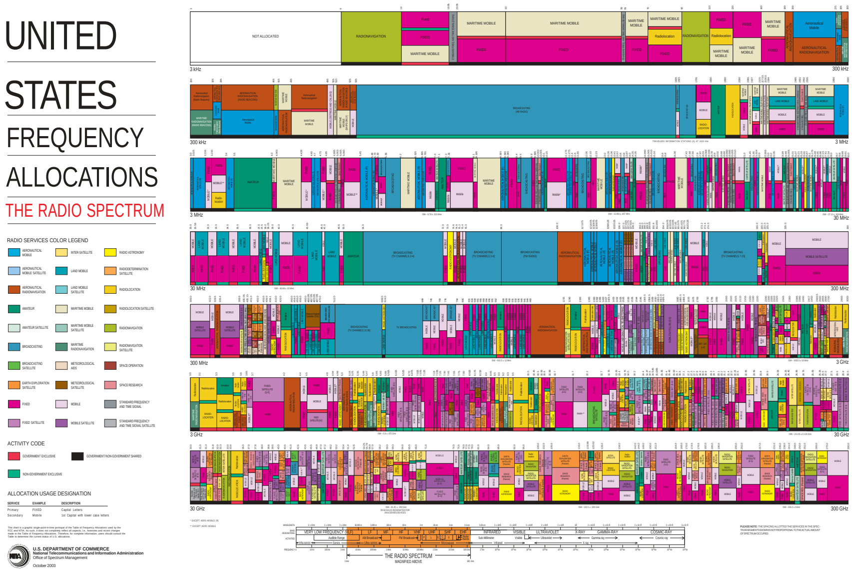

Figure 1.1: Spectrum allocation table from FCC [1]

performance of wireless networks working in the ISM spectrum band is highly limited by the coexistence of other nearby wireless networks. In addition, the licensed spectrum utilization is highly dependent on the location and time. For instance, during some time periods in a certain geographic area, the allocated spectrum bands may be seldom used.
In November 2002, the FCC published a report to indicate that during 90% of the time, many licensed frequency bands remain unused [1]. Furthermore, the Shared Spectrum Company (SCC) has published a bunch of spectrum measurement results of USA and some European Countries since 2004 [9]. From their spectrum reports [10, 11], the average utilization of many licensed frequency bands in many cities is less than 25%. This means that it is not an actual spectrum scarcity what is worrisome, but rather the inefficient spectrum usage. As a result, since 2004, FCC has recommended to consider authorizing new devices in the TV broadcast spectrum at locations where TV channels are not being used for authorized services, including broadcast television, broadcast auxiliary services such as wireless microphones, and private land mobile radio [12]. The IEEE 802.22 Working Group on Wireless Regional Area

Networks (WRANs) was formed in October 2004, and has been working on the standardization for the rural broadband wireless access using the TV broadcast spectrum by Cognitive Radio (CR) technologies [13].
The basic idea behind IEEE 802.22 is to exploit the unused or not fully utilized licensed spectrum, which is called "spectrum hole". Actually, this idea was proposed in the light of the concept of CR by Joseph Mitola III at Royal Institute of Technology (KTH), Sweden, in 1999 [14]. In essence, CR technology differs from conventional radio devices in that a CR can equip users with cognitive capability and reconfigurability (e.g., frequency, power, modulation), allowing for Dynamic Spectrum Access (DSA). Following this concept, many national regulatory bodies (i.e., the FCC in the USA) have recently proposed expanding the unlicensed spectral bands to obtain more flexible utilization of the available spectrum through the use of CR technology. As such, CR is foreseen as one of the most viable technical paradigms to improve the spectrum utilization significantly, and contribute to solving the problem of spectrum shortage.

1.1 Motivation

Although spectrum sharing brings opportunities for CR users to access the licensed channels*, many new challenges come up when deploying CR in practice. On the one hand, the challenge for a reliable sensing method to find the "spectrum hole" is to identify suitable transmission opportunities without compromising the integrity of the Primary User (PU). One of the design criteria is to make the probability of false alarm as low as possible, since it measures the percentage of vacant spectrum that is misclassified as busy, increasing thus the opportunistic usage of the spectrum from the CR users. On the other hand, in order to limit the probability of having CR users interfering with PUs, it is desirable to keep the missed detection probability as low as possible. The detection thresholds are the trade-off factor between the false alarm and the missed detection probabilities: generally speaking, low thresholds will result in high false alarm rates in favor of low missed detection probability and vice versa. Alternatively, the choice of the sensing time offers a trade-off between the quality and speed of sensing: increasing

*In this thesis, a channel means a frequency subband or an aggregation of frequency subbands (frequency bands) for spectrum sensing and transmitting.

the sensing time permits to decrease both false alarm and miss detection probability values, however, it reduces the time available for secondary transmissions, which decreases CR throughput. The above trade-off calls naturally for a joint determination of the sensing and transmission parameters of the CR users, assuming a paradigm of selfish behavior among these CR users, where the CR users have no willing to cooperate with each other.
In this thesis, for the application of CR technology, we investigate applications for both single user and multiuser Cognitive Radio Networks (CRNs). In the case of single user CRNs, we study the problem of power allocation making use of the detection information under an opportunistic spectrum access model. For multiuser CRNs, we analyze the noncooperative behavior of the CR users based on game theory. The modeling and analysis of the competitive multiuser optimization, taken into consideration the sensing accuracy, is the main overall subject of this thesis.

1.2 Contributions

In [15–19], we have explored a sensing-based access scheme in CRNs where the overall objective is to maximize the sum-rate (sum-throughput) of each CR user by optimizing jointly both the detection operation based on sensing and the power allocation, taking into account the influence of the sensing accuracy and the interference limitation to the PUs. The optimization problem is addressed in single and multiuser CRNs for both Single-Input Single-Output (SISO) and Multiple-Input Multiple-Output (MIMO) channel. In the following, we enumerate the main topics where this thesis provides contributions.

1.2.1 Joint optimization of detection and power allocation in single user CRNs

We start with the resource allocation and optimization problem for single user CRNs [15,16], where joint power allocation and spectrum detection are key issues. In single user CRNs, the CR-Transmitter (Tx) has to perform the spectrum sensing before accessing the channel. We consider the Opportunistic Spectrum Access (OSA) model under the opportunistic spectrum access scheme. In the OSA model, CR users are allowed to transmit over the channel of interest when all the PUs are detected as not transmitting at this channel. One essential enabling technique for OSA-based CRNs is spectrum sensing, where the CR users individually or collaboratively detect active PU transmissions over the channel, and decide to transmit if the sensing results indicate that all the PUs are inactive in this channel. The main contributions of Chapter 3 are the following:

- We consider an Orthogonal Frequency Division Multiplexing (OFDM) based communication system and present efficient algorithms to maximize the sum-rate of the CR by optimizing jointly both the detection operation and the power allocation. The problem is non-convex, and can be formulated as a two-variable problem and solved by the alternating direction optimization method operating sequentially over the power allocation and the detection threshold.
- We show that the algorithm operates basically in two regimes depending on the constraints involved. As compared to previous work, a novel interference constraint is proposed to design the power allocation scheme, ensuring that the performance degradation of the PU is bounded.

1.2.2 Joint optimization of detection and power allocation in multiuser CRNs

In [17], we analyze the resource allocation problem among CR users for the Sensing-Based Spectrum Sharing (SSS) scheme as a strategic Non-Cooperative Game (NCG), where each CR user is selfish and strives to use the available spectrum in order to maximize its own sum-rate by considering the effect of imperfect sensing information. The resulting game-theoretical formulations belong to the class of non-convex games, where the non-convexity occurs at both the objective functions and the feasible sets of the CR users' optimization problems. Therefore, traditional mathematical tools from [20] are not applicable to show the existence of an equilibrium for this game. We analyze our Non-Convex Non-Cooperative Power Allocation Game (NNPG) based on the new relaxed mathematical equilibria concept introduced in [21], namely, the Quasi-Nash Equilibrium (QNE). The main contributions of Chapter 4 are the following:

- We propose a NNPG, where each CR user aims at maximizing its own sum-rate by jointly optimizing the sensing operation as well as the transmit power over all channels, which differs from the disjoint case, called deterministic game where the sensing parameters are not considered as a part of the optimization, as shown in [22–26].

- Deviating from the constraints considered in previous work [22–33] (such as interference temperature and outage probability constraints), we introduce a rate-loss constraint in order to effectively protect the PU from harmful interference caused due to the imperfect sensing information. We analyze the optimization problem in two different limited regimes, namely, power budget limited regime and rate-loss limited regime. The performance of the CR users in these regimes are evaluated extensively through simulation.

- In addition, a distributed cooperative sensing scheme based on a consensus algorithm is considered in the proposed game for a SSS scenario. Compared with the OSA scenario discussed in [31–33], in our scenario, the CR users can coexist with PUs, and adjust the transmit power on each channel based on the sensing result.

- The fourth major contribution of this chapter is to prove that the proposed NNPG can achieve a QNE under certain conditions, by making use of the Variational Inequality (VI) method. Meanwhile, we show that, under the so-called linear independent constraint qualification, the achieved QNE coincides with the Nash Equilibrium (NE).

- Finally, an iterative Primal-Dual Interior Point (PDIP) algorithm that converges to a QNE of the proposed game is provided here. The PDIP algorithm can run at each node in parallel, since it requires only the local information of each CR user (e.g. its own transmit power and the channel gain), and hence, it can be regarded as a distributed solution. Simulation results show that the PDIP algorithm yields a considerable performance improvement, in terms of the sum-rate of each CR user, with respect to previous state-of-the-art algorithms, such as alternating direction optimization algorithm [16] and the deterministic game proposed in [26].

In Chapter 5, we move a step ahead from Chapter 4, and consider an OSA scenario in multiuser MIMO CRNs [19]. The optimization problem is analyzed as a strategic NCG, where the transmit covariance matrix, sensing time, and detection threshold are considered as variables to be optimized. The resulting game is non-convex, hence, we also use the new relaxed equilibria concept QNE, and prove that the proposed game can achieve a QNE under certain conditions, by making use of the VI method. Simulations show that the proposed game can achieve a considerable performance improvement with respect to the deterministic game in [34].

1.3 Thesis Organization

This thesis is organized as follows. Chapter 2 introduces the background of CRNs, and summarizes the related works in optimization of power allocation and game theory in CRNs. Chapter 3 describes the proposed joint optimization of detection and power allocation schemes for single user CRNs. The proposed joint optimization of detection and power allocation schemes for multiuser SISO CRNs is given in Chapter 4. Chapter 5 provides the proposed joint optimization of detection and power allocation schemes for multiuser MIMO CRNs. Chapter 6 concludes our study in this thesis and points out several future directions in the research on CRNs. The relationship between the major chapters from Chapter 3 to Chapter 5 can be seen from Table 1.1, where we summarize the scenarios addressed in the different chapters.

Table 1.1: **Scenarios addressed in each chapter**

Chapter	Number of CR	Spectrum Share Mode	Channel Mode
3	single user	OSA	SISO
4	multiuser	SSS	SISO
5	multiuser	OSA	MIMO

Chapter 2

State of the Art

Cognitive radio (CR) is viewed as a novel approach for improving the utilization of a precious natural resource: the radio electromagnetic spectrum. The ultimate goal for the CR is to accommodate the increasing demand for wireless data transmission by using the radio spectrum more efficiently.

In this chapter, we introduce the background of CR technologies, a brief description of game theory and present the related work. We first introduce in Section 2.1 and Section 2.2 the background of CR technologies including the definition, key technologies, and main topic in CRNs, respectively. The main concepts of game theory that are used in our optimization problem, are presented in Section 2.3. Finally, the related work in Section 2.4 is organized around two main themes of our research in CRNs: (i). Resource allocation in single user CRNs; (ii). Resource allocation in multiuser CRNs. In Section 2.5, we provide the conclusion.

2.1 Cognitive Radio, a Brief Introduction

2.1.1 Definition of CR

The ever-increasing demand for higher data rates in wireless communications in the face of under utilization of the electromagnetic spectrum motivated the detection and exploitation of spectrum holes, which are defined as [35]: "a spectrum hole is a frequency band assigned to a PU,

at a particular time and specific geographic location, where the band is not being utilized by that user." Spectrum utilization can be improved significantly by making it possible for an unlicensed secondary user (who is not being served) to access a spectrum hole unoccupied by the PU at the right location and time in question. The concept of CR, which is based on the software-defined radio, has been proposed as the means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes.
The term "CR" was firstly introduced by Joseph Mitola in his paper in 1999, where he defined CR as: "A radio that employs model based reasoning to achieve a specified level of competence in radio related domains [14]." In 2005, Professor Simon Haykin defined CR as [36]: "an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding by-building to learn from the environment and adapt its internal states to statistical variations in the incoming radio frequency (RF) stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- Highly reliable communications whenever and wherever needed;
- Efficient utilization of the radio spectrum.

Six key words stand out in this definition: awareness, intelligence, learning, adaptivity, reliability, and efficiency." Implementation of this far-reaching combination of capabilities is indeed feasible today, thanks to the spectacular advances in digital signal processing, networking, machine learning, computer software, and computer hardware [36].
On the other hand, the regulator FCC defined CR as: "A radio that can change its transmitter parameters based on interaction with the environment in which it operates [2]." There will be a lot of benefits from the new radio regulations, such as obtaining more capacity, decreasing the cost of communications, improving reliability, and reaching longer distances with wireless equipments.

2.1.2 Main tasks and key technologies

The CR technology makes use of tools from signal-processing and machine-learning for its implementation. The cognitive process starts with the passive sensing of RF stimuli and culminates with action. There are three main tasks for CR [2]:

- Radio-scene analysis, which encompasses the following:
  - Estimation of interference temperature of the radio environment. The interference temperature is defined as a maximum amount of tolerable interference for a given channel in a particular location. Any unlicensed transmitter (CR-Tx) utilizing this channel must guarantee that its transmissions added to the existing interference must not exceed the interference temperature limit at a licensed receiver (PU-Rx) [13];
  - Detection of spectrum holes.
- Channel identification, which encompasses the following:
  - Estimation of Channel State Information (CSI) for secondary users;
  - Prediction of available channel capacity to be used by the transmitter.
- Transmit-power control and dynamic spectrum management.

The first task and the second task are carried out at the receiver, and the third task is carried out at the CR-Tx. Through interaction with the RF environment, these three tasks form a cognitive cycle, which is pictured in its most basic form in Figure 2.1, wherein the receiver is required to perform spectrum sensing, analysis, and estimation before transmission in order to protect PUs. The transmitter will then select an appropriate channel and control the transmit power to guarantee that the interference to PUs is not harmful. Three basic approaches have been considered to allow concurrent communications for CR: spectrum overlay, underlay, and interweave.

- In overlay systems, as proposed in [37], CR users allocate part of their power for secondary transmissions, while the remaining power is used to assist (relay) primary transmissions. By exploiting sophisticated coding techniques, such as dirty paper coding, based on the knowledge of the PUs' message or codebook at the

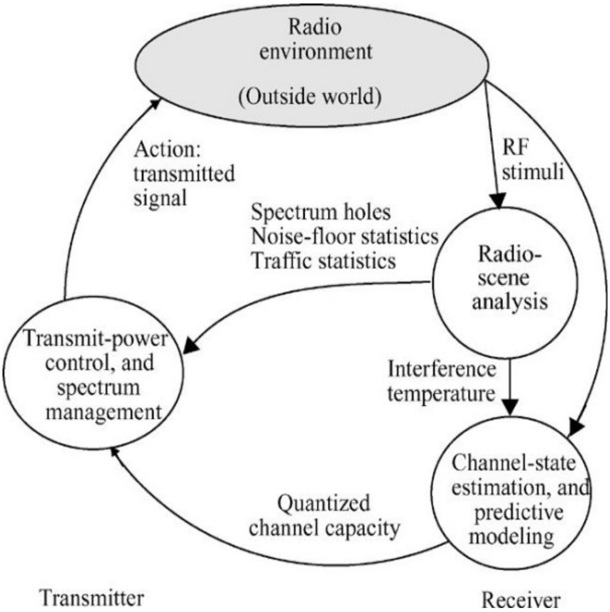

Figure 2.1: Cognitive radio operation cycle [2]

CR-Tx, these systems offer the possibility of concurrent transmissions without capacity penalties. However, although these technology is interesting from an information theoretic perspective, these techniques are difficult to implement as they require noncausal knowledge of the primary signals at the CR-Tx.

- In underlay systems, CR users are allowed to share resources with the PUs, but without any knowledge about the PUs' signals and under the strict constraint that the spectral density of their transmitted signals falls below the noise floor at the primary receivers. This interference constraint can be met using spread spectrum or ultra-wide band communications from the CR users. This transmission technique does not require the estimation of the electromagnetic environment from CR users, but it is mostly appropriate for short distance communications, due to the strong constraints imposed on the maximum power radiated by the CR users.

- Conversely, interweave communications, initially envisioned in [38], are based on an opportunistic or adaptive usage of the spectrum, as a function of its real utilization. CR users are allowed to adapt their power allocation as a function of time and frequency, depending on what they are able to sense and learn from the environment, in a non-intrusive manner. Rather than imposing a severe constraint on their transmit power spectral density, in interweave systems, the CR users have to figure out when and where to transmit. As opposed to underlay systems, this opportunistic spectrum access requires an opportunity identification phase, through spectrum sensing, followed by an opportunity exploitation mode [39]. In this thesis, we focus on an interweave communications model, as it seems to be the most suitable for the current spectrum management policies and legacy wireless systems [39].

Spectrum sensing

The main tasks of radio-scene analysis are based on spectrum sensing, which is one of the most important procedures in CRNs. The essential problem of spectrum sensing is to decide whether a particular slice of the spectrum is available or not for transmission. Thus, a spectrum sensor is required in order to detect spectrum holes. This should provide high spectral-resolution capability, estimate the average power in each channel of the spectrum, and identify the unknown directions of interfering signals [40].

In the literature, there are three major methods for spectrum sensing, i.e., matched filter detection [40], cyclostationary detection [41] and energy detection [3,42]. Each of them has its advantages and disadvantages in different scenarios. Matched filter-based detection is considered to be an optimal signal-detection method when the signal format of the PU is known, e.g., modulation type, pulse shaping, and synchronization of timing and carrier. Moreover, in case of PUs belonging to different types of networks, the CR will need a dedicated receiver for each type of PU, which makes it difficult for practical implementation. Cyclostationary detection needs to know the periodicity of the cyclic prefix of the primary signal, which may not be available to the secondary users in practice. In addition, it requires a substantial computational complexity.

On the contrast, energy detection requires no information of the primary signal and it is robust to unknown channels. This makes it a very desirable spectrum sensing technique for CR. Among these methods, energy detection has been widely used in CRNs because of its computational and implementation simplicities, even though its robustness comes

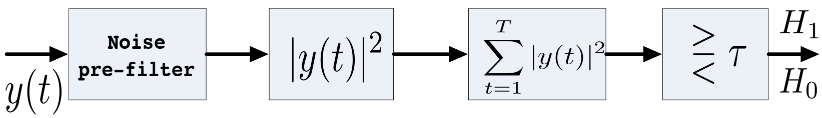

Figure 2.2: Energy detection [3]

together with some decrease in detection performance as compared to previous methods. The energy detection model is shown in Figure 2.2, which consists of a noise prefilter that serves to limit the noise bandwidth, and a square law device followed by a finite time integrator. The output of the integrator at any time is the energy of the input to the squaring device over the interval T in the past. The output of the integrator is finally compared with a predefined threshold τ. The detection is a binary hypothesis test with the following hypothesis:

- H_0: $y(t)$ is noise alone;
- H_1: $y(t)$ is signal plus noise.

In practice, the reliability of the PU detection at the CR-Tx is limited by several factors, such as the attenuation due to path loss, as well as shadowing and fading. Therefore, cooperative sensing [43, 44], which allows several nodes to sense jointly the spectrum environment and make the decision in a cooperative manner combining their sufficient statistics, can be see as an efficient way to solve such problems and ensure robustness. The concept of cooperative sensing is to use multiple sensors and combine their measurements into a common decision. There are two ways for this approach, soft combining and hard combining which are described in [43, 44].

Resource allocation

Spectrum sensing results are used as the basis for optimizing resource allocation. Several dynamic spectrum access schemes such as [45–47] have been proposed using the sensing-based opportunistic spectrum access approaches.

There are currently two main approaches for interweave cognitive communications:

- Opportunistic spectrum access (OSA) [48]: In the OSA model, the CR users are allowed to transmit over channel of interest when all the PUs are not transmitting there. One essential enabling technique for OSA-based CRNs is spectrum sensing, where the CR users individually or collaboratively detect active PU transmissions over the channel, and decide to transmit if the sensing results indicate that all the PUs are inactive at this channel.

- Sensing-based spectrum sharing (SSS) [49]: In the SSS model, the CR users are allowed to transmit simultaneously with the PUs in the same channel even if they are active, thus each CR user coexists with the PU and adapts its transmit power based on the detector decision from the spectrum sensing, ensuring that the performance degradation of each active PU link is within a tolerable margin.

As a crucial part of the resource allocation process, CR users should decide the transmit power on the CR-Tx access the available degree of freedom. Different from traditional spectrum assignment, the CR paradigm enables CR users to transmit on channel overlapping with PUs, provided that the degradation induced on the PUs' performance is tolerable. The way about how to measure the interference on PUs in an efficient way is a complex and open regulatory issue [36]. Restrictive constraints may marginalize the potential gains offered by the dynamic resource assignment mechanism, whereas loose constraints may affect the compatibility with legacy systems [50].
Several works have considered the interference constraints for CR users, e.g., both deterministic and probabilistic interference constraints have been suggested in the literature [14,39], namely: the Multiuser Interference (MUI) power level perceived by any active PU (the so-called interference temperature limit) [36], and the maximum probability that the MUI interference level at each PU's receiver may exceed a prescribed threshold [39, 51]. In the presence of sensing errors, the access to channels identified as idle should also depend on the goodness of the channel estimation. As shown in [52], in this case the optimal strategy is probabilistic, with a probability depending on both the false alarm and miss detection probabilities.

2.1.3 The main challenges

There are many challenges in making CR to become a reality, including hardware, spectrum sensing techniques, resource allocation, and the common control channel. Many improvements from various perspectives are necessary, such as:

- **Spectrum sensing issue**

  Without efficient sensing capabilities, the cognition requirement (for whites spaces, geographical information, etc.) is simply impossible. In fact, spectrum sensing is not always perfect, thus it gives rise to non-zero false alarm and miss detection. False alarm happens when the spectrum sensing results report activity of PUs, which actually do not exist. Following the sensing result, CR users may stop the current transmission and decide to switch to another channel. This causes additional channel access delay and reduction of throughput. In contrast to false alarm, miss detection happens when CR users fail to detect the active PUs, and continue working on that channel. Thus, it can cause uncontrolled interference to PUs. It is not only harmful to PUs but also harmful to CR users. In terms of cooperative spectrum sensing, the main issues are related to how to fuse individual CR users' decisions, and how to perform distributed spectrum sensing with limited feedback from each detector.

- **Resource allocation issue**

  A channel is said to be available for CR users when it is not occupied by any PUs or the interference from CR users to PU is under a tolerable threshold. The channel availability of CR users on different locations may be distinct from each other because of different PU activities. The CR users may have different available channels because of hardware limitations such as sensing constraints and transmission constraints. This phenomenon would result in the problem of channel heterogeneity where CR users have different available channels at a certain time [53]. In this heterogeneous situation, neighbor CR users should negotiate a common channel to communicate with each other before data transmission. However, if the CR users do not have the willing to share the information with each other, they will decide to access the available channels based on their own local information and their behavior will become selfish.

- **Hardware issue**

  One of the basic features of CR technologies is the wide spectrum working capability. The frequent dynamic variations of the carrier frequency and the communication bandwidth require either wideband or narrowband tunable hardware devices (e.g., amplifiers). From a hardware perspective, frequency tunable elements are difficult to design and costly, while wideband elements are inherently less efficient due to the higher noise floor.

- **Common control channel issue**

  Before establishing communication, the CR users do not know which channel can be used by each other, so they need to exchange messages to know the available channels globally. A common channel can be chosen based on their agreement, and generally, this requires exchanging messages through a Common Control Channel (CCC) [54].
  In order to achieve the "social welfare", (i,e,. maximize the utility of the CRNs) and avoid a harmful interference to the PU due to the transmission from CR users, a simple solution is to have a dedicated CCC. This channel is a dedicated licensed channel to CR users for the exchange of control messages, thus it will not be interrupted by any PU. In the literature, many contributions are based on this assumption such as [55, 56]. Another solution is to choose a control channel among the available channels, such as in [57]. However, there are several challenges related to this latter case. Firstly, CR users should vacate the channels or reduce the transmit power in certain channels when PUs are detected. Therefore, the control channel should be the most reliable channel at each moment, so that it can not be interrupted frequently. On the other hand, if the CR users can not or do not want to exchange their informations with each other, the CCC is redundant.

2.2 Game Theory in CRNs

Similar to other types of communication networks, the deployment of CRNs can be justified in financial terms if and only if the network is utilized by multiple users [58].
Currently, mobile wireless communication networks, such as cellular systems, are centralized. These systems require an infrastructure of base stations to route calls from one user to another. In contrast, for both civilian and military applications, it is desirable for CRNs to be decentralized, allowing also the existence of device to device (D2D) communications, as it is being considered in the latest standards (LTE Release II). In other words, the network is configured in a self organized manner [59], which makes it possible to dispense with the need for a costly pre-established infrastructure. Self organization builds on two basic mechanisms: cooperation and competition; these two mechanisms operate in a complementary manner so as to “bring order in the network out of disorder [58]” :

- Cooperation is used to facilitate communication across the nodes of the network without any fixed infrastructure.
- Competition is used to provide control over the power transmitted from each individual node of the network to maintain the interference temperature at a receiving node below a prescribed limit.

In this thesis, we focus on the problem of distributed power allocation, thus, the goal is to design an efficient and effective transmit power allocation policy. Most importantly, this policy does not require synchronization nor centralization among the multiple users, thereby simplifying the design of the network. In this scenario, there is limited or no information exchange among CR users. The common control channel and any fusion center are then not required in this model. Each CR user will follow the criterion of competitive optimality for maximizing its own total achievable throughput based on their own information, subject to certain constraints. We focus on the transmit power allocation problem in non-cooperative multiuser CRNs, where the overall objective is to maximize the sum-rate (throughput) of each CR user by optimizing jointly both the detection operation based on sensing and the power allocation across the channels, which can be formulated, as we show, in terms of an equivalent non-cooperative power game.

2.2.1 Basic concepts of game theory

Game theory is widely used in the study of economics [60]; it has also been applied in other areas such as machine learning [61] and neuroscience [62]. Recently, game theory has been used in CRNs [63], involving the following ingredients:

- Multiple players who, by virtue of their responsibilities as decision-makers, are required to take specific actions.
- The actions may lead to consequences, which could be of mutual conflict to the players themselves.

The formulation of a mathematical framework for a non-cooperative game is based on three key elements:

- State space, which is the product of the individual players' states.
- State transitions, which are functions of joint actions taken by individual players.
- Payoffs to individual players that depend on joint actions as well.

2.2.2 Nash equilibrium

In [64, 65], John Nash focused his study of game theory on a class of games described as non-cooperative, simultaneous-move, one-shot, and finite games with complete information, where:

- "Simultaneous move" means that each player picks an action without knowledge of the other players' actions.
- "One-shot" implies that the game is played only once.
- "Finite game" refers to the fact that the game involves a finite number of players, with each player taking only a finite number of possible actions.

The concept of Nash equilibrium of a game is defined as follows [64]:

Definition 1. *A NE is defined as an action profile (i.e., vector of players' actions) in which the action of each player is a best response to the actions of all the other players.*

The NE is a solution of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibria actions of the other players, and no player has anything to gain by changing only his own action unilaterally [64]. If each player has chosen an action and no player can benefit by changing actions while the other players keep theirs unchanged, then the current set of action choices and the corresponding payoffs constitute a NE. The NE features prominently in the study of game theory. This concept works perfectly well provided two assumptions are satisfied:

- The players engaged in a game are all rational.
- The underlying structure of the game is of common knowledge to all the players.

Under these two assumptions, the NE offers an intuitively satisfying approach that predicts the equilibrium outcome of the game as follows: any player, being "rational", will play a "best-response" action (i.e., the point at which each player in a game has selected the best response to the other players' strategies). Moreover, under the "common knowledge" assumption, this action is known to all the other players and, being rational, they will therefore play their own "best-response" actions, leading the game to a NE [58].

2.3 The Main Challenges in This Thesis

2.3.1 Spectrum sensing

In practice, the reliability of the PU detection at the CR-Tx is limited by several factors, such as the attenuation due to path loss, as well as shadowing and fading. Therefore, decisions made by independent CR users with local sensing capability about transmission parameters (e.g., power, etc.) generate harmful interference to the PU system or will use very conservative allocation policies involving unnecessary transmission

back-off and generating a throughput lower than the one that can be achieved. As a consequence, a certain degree of performance degradation of the PU is usually unavoidable. In this case, the influence of the sensing accuracy on the throughput of the CR user should be taken into account in order to perform an appropriate power allocation.
In this thesis, the detection results are based on the performance of the energy detector in terms of its receiver operating characteristics curve, which gives certain probability of detection and certain probability of false alarm. The fundamental problem of this detector is to set the optimal detection threshold, as well as the optimal sensing time, to achieve the desired detection performance, which is optimized depending on the particular network utility to be maximized. In order to reduce the interference from the CR to the PU due to the non-zero probability of miss detection and increase the probability for CR to access the available channel, we optimize both the detection threshold and the sensing time of the energy detector in order to obtain a better sensing accuracy. For multiuser SISO CRNs, we consider a cooperative sensing scheme, which can be implemented by a distributed consensus algorithm with limited interaction among nearby CR users.

2.3.2 Power allocation

Power allocation in CRNs is substantially more complex than in traditional wireless networks. In CRNs, CR users control transmit power not only to achieve the "best-response" actions, but also to protect PUs. The interference generated by CR users to any PU should be carefully considered, and should not exceed a tolerable threshold.
In this thesis, we consider the power allocation problem in single user and multiuser CRNs for both SISO and MIMO channels based on the sensing information. In single user CRNs, we focus on the optimization of the power allocation for the CR-Txs jointly with the sensing, while keeping the performance degradation of the PUs due to the transmission of CR when a miss detection occurs. This can be enforced through a constraint that limits the rate-loss by the PU. As a explained in Chapter 3, the result optimization problem is non-convex.
In the multiuser case, we assume that the CR users are not willing to exchange any information, thus, the optimization problem can be reformulated as a non-cooperative game. The joint optimization of detection and power allocation result in a non-convex game, which presents a new

challenge when analyzing the equilibrium of this game. In Chapter 4, we focus on this non-convex property and find the equilibrium for the proposed game.

2.4 Related Work

The general resource allocation problem in CRNs includes both the channel assignment, and the power allocation schemes. The various schemes depend on the number of PU channels, the number of CR users, the particular spectrum access schemes that are used, and on what kind of type of sensing they used for their decision. In the following, we will discuss the previous works according to different number of CR users.

2.4.1 Power allocation in single user CRNs

The problem of maximizing the throughput of the CR user without sensing information (or under perfect sensing information, e.g., the probability of miss detection and false alarm are zero) has been widely studied in the literature [66–70].
Some previous works have focused on the combination of the sensing information together with the throughput of simplified CRNs with one CR user and one PU [28–30, 49, 71–73]. The problem of designing the optimal sensing time and power allocation strategy that maximizes the average throughput for SSS schemes was studied in [29]. The work in [29] was extended in [30], where the problem of finding the optimal sensing time and power allocation was studied based on the outage capacity constraint and the truncated channel inversion constraint, namely, a sensing-enhanced spectrum sharing CR system.
In the literature [28, 71, 72], the authors considered the optimization problem considering only the sensing parameters as optimization variables. In [71], the authors proposed alternative centralized schemes that optimize the detection thresholds for a bank of energy detectors, in order to maximize the so-called opportunistic throughput, while keeping the sensing time and the transmission parameters of the CR fixed and given a priori. The optimization of the sensing time and detection thresholds for a given miss detection probability and target transmission rate of one CR in the presence of one PU was addressed in [28, 72], respectively.

Joint optimization of sensing information and power allocation is discussed in [49]. In [49], the sensing time and the transmit power of one CR were jointly optimized while keeping the detection probability (and thus the decision threshold) fixed to a target value. In [73], the authors focused on the joint optimization of the power allocation and the equi-false alarm of one CR over all the channels, for a fixed sensing time.

2.4.2 Power allocation in multiuser CRNs

In the case of SISO system model

All the aforementioned schemes are applicable for single user CRNs, and the schemes are applicable only to CR scenarios composed by one pair of PU Tx-Receiver (Rx) and one pair of CR Tx-Rx.
The work in [74–80] addresses the optimization of the CR users' transmit power in a multiuser OFDM SISO CR scenario, where [74, 75] focuses only on centralized schemes. In a decentralized multiuser scenario, CR users can self-enforce the negotiated agreements on the usage of the available spectrum. Every CR user aims at the transmission strategy that maximizes its own utility function, usually the average throughput. This inherently competitive nature of the decentralized multiuser scenario leads to a non-cooperative game (NCG) [20], where the solution of the game is the well-known concept of Nash equilibrium. The NCG theoretical model for power allocation in the SISO interference channels has been addressed in [76–80], while the equilibrium model based on pricing has been discussed in [81, 82]. However, the power allocation schemes proposed in the mentioned papers are not applicable to CRNs, since they do not provide any mechanism to limit the performance degradation caused to PUs.
NCG theory has been successfully applied to the power allocation problem in CRNs [22–26]. The finite-dimensional variational inequality (VI) method [83] has been used in [22–25] to analyze the existence and uniqueness of the solution for the NCG in the CRNs. Those works are extended in [26] for a more practical scenario with imperfect CSI. However, in [22–26], no sensing is performed by CR users.
Recently, the sensing information is considered in [31] for a multiuser scenario. The resulting problem is non-convex due to the information from the sensing information. We provide an alternating direction method to obtain the sub-optimal solution of the non-convex game. The OSA

model is considered in [31] and the analysis of the equilibria of this game in [31] is based on a new concept called quasi-Nash equilibrium (QNE) [21]. QNE is a solution of a VI problem obtained under the first-order optimality conditions of each player's optimization problem while retaining the convex constraints in the defining set of the VI problem. The prefix quasi is intended to signify that a NE (if it exists) must be a QNE under certain conditions to be satisfied by the constraints (constraint qualifications) [21].

In the case of MIMO system model

The incorporation of MIMO techniques into CRNs can improve the channel capacity by sending independent data streams simultaneously over different antennas. There are some works that attempt to protect PUs in MIMO CRNs while maximizing the CRNs' throughput [34, 84–89]. In [86], the authors consider the optimization over the set of precoding matrices for each CR and PU, allocating power over both space and frequency dimensions and yielding radiation patterns that induce minimum interference, so as to maximize the network throughput. However, due to the challenges associated with power and spectrum optimization, all the existing works on MIMO CRNs do not consider the joint optimization including also the sensing information. In Chapter 5, we consider an OSA scenario in MIMO CRNs where the overall objective is to maximize the total throughput of each CR by jointly optimizing both the detection operation and the power allocation over all the channels, under an interference constraint bound to PUs. The optimization problem is analyzed as a strategic NCG, and the resulting game is non-convex, hence, the analysis of the equilibria of this game is based on the new concept QNE.

2.5 Conclusions

In this chapter, we first introduced the main concepts, the main tasks and key concepts in CRNs. The current state-of-the art CR technologies, which are certainly not able to satisfy all the technical requirements, and the existing challenges to make CR a reality are presented in this chapter. In addition, we discussed the multiuser CRNs that can be formulated using a non-cooperative game theoretic approach, as well as the main challenges of the work in this thesis. Finally, we introduced the related work in the same area. In the following chapter, we start with the resource allocation problem in single user CRNs, where the spectrum sensing problem and the optimal transmit power allocation are the main issues we focus on.

Chapter 3

Joint Optimization of Detection and Power Allocation in Single User CRNs*

The first scenario considered in this thesis is the resource allocation optimization problem in single user CRNs, where joint power allocation and spectrum detection is one of the most important issues. In the single user CRNs, one pair of CR Tx-Rx performs the spectrum sensing before accessing the channel. We assume an interweave system, where the two main approaches for CR users regarding the way secondary users access the licensed spectrum are opportunistic spectrum access (OSA) and sensing-based spectrum sharing (SSS).

There exists currently a debate about which operation model, OSA or SSS, is better to deploy CR users in practical systems. Generally speaking, SSS utilizes the spectrum more efficiently than OSA, since the former supports concurrent PU and CR transmissions over the same channel, while the latter only allows orthogonal transmissions between them. In this chapter, we assume an OSA model, while the SSS model is considered in Chapter 4.

*The publications associated to this chapter are [15, 16]

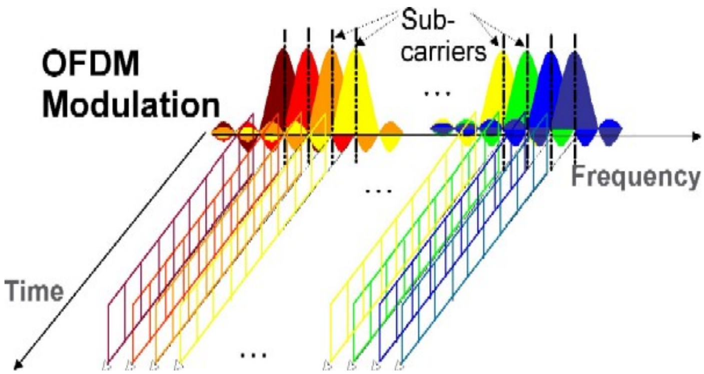

Figure 3.1: OFDM modulation [4]

The reliability of the PU detection at the CR-Rx is limited by attenuation due to shadowing, fading, as well as the hidden node problem, that is, a CR-Rx may be interfered from the PU-Tx, but without being blocked from the PU-Rx, an effect that is known as the hidden terminal problem. As a result, the PU' action is not detected and the CR transmission could significantly interfere to the PU-Rx. A CR-Tx usually needs to deal with a performance tradeoff between maximizing the rate and minimizing the performance degradation caused to the PU transmission.
OFDM is a modulation technique, depicted in the Figure 3.1, which uses many sub-carriers, or tones, to carry a signal, which has developed into a popular scheme for wideband digital communications, such as digital television and audio broadcasting, wireless networks, and 4G mobile communications. The primary advantage of OFDM over single-carrier schemes is its ability to cope with severe channel conditions (for example, attenuation of high frequencies in a long copper wire, narrowband interference and frequency-selective fading due to multipath) without complex equalization filters. OFDM can be viewed as using many slowly modulated narrowband signals rather than one rapidly modulated wideband signal. The low symbol rate makes the use of a guard interval between symbols affordable, making it possible to eliminate intersymbol interference (ISI) and utilize echoes and time-spreading to achieve a diversity gain, i.e. a signal-to-noise ratio improvement [4].
In this chapter, we consider an OFDM based communication system and present efficient algorithms to maximize the sum-rate of the CR by optimizing jointly both the detection operation and the power al-

Table 3.1: **Notation for single user SISO CRNs**

Symbol	Meaning
Rx	Receiver
Tx	Transmitter
N	Number of channels
τ	Detection threshold
$\mathcal{P}_d$	Probability of detection
$\mathcal{P}_{fa}$	Probability of false alarm
γ_k^p	SNR of PU at PU-Rx in channel k
γ_k^i	SNR of PU at CR-Rx i in channel k
P_k	Transmit power of CR in channel k
$H_{0,k}$	Channel k is detected to be idle
$H_{1,k}$	Channel k is detected to be occupied
$P_{\max}$	Maximum total transmit power of the CR-Tx
Γ_k	Maximum acceptable rate-loss gap for the PU
$I_{k,cp}$	Total interference experienced by the PU-Rx in channel k
$\lvert h_{k,cp}\rvert^2$	Channel gain in channel k between CR-Tx and PU-Rx
$\lvert h_{k,pc}\rvert^2$	Channel gain in channel k between PU Tx and CR-Rx
$\lvert h_{k,cr}\rvert^2$	Channel gain in channel k between CR-Tx and CR-Rx

location, taking into account the influence of the probabilities of miss detection and probabilities of false alarm, namely, the sensing accuracy. This problem can be formulated as a two-variable problem and it is solved here by the alternating direction optimization method, operating sequentially over the power allocation and the detection threshold. This algorithm operates basically in two regimes depending on which constraints become active. In addition, a novel criterion is proposed to design the power allocation, ensuring that the performance degradation of the PU is bounded.

In the following, we introduce the system model and formulate the optimization problem in Section 3.1 and Section 3.2, respectively. Power allocation with optimal spectrum sensing and the solution based on the alternating method is presented in Section 3.3. Finally, performance evaluation results are presented in Section 3.4. Section 3.5 states the conclusion. Table 3.1 lists the notation used in this chapter. Matrices and vectors are indicated in boldface.

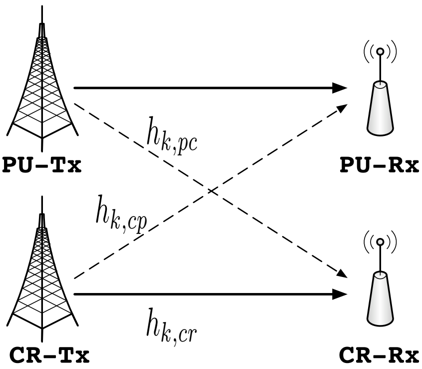

Figure 3.2: System Model: one pair of PU Tx-Rx and one pair of CR Tx-Rx

3.1 System Model

In this chapter, we consider the OSA model, where the CR user can access the channel only if the PU is detected to be absent, and the CR-Tx deals with a performance tradeoff between maximizing its sum-rate and minimizing the performance degradation caused to the PU.
Consider simplified OFDM based CRNs with one pair of single antenna CR Tx-Rx and N channels belonging to one PU that are available for the CR user, as given in Figure 3.2. We assume that the local CSI, i.e., the channel gain between the CR-Tx and its target Rx and the PU, is known by the CR-Tx. In practice, CSI on the CR user's own channel can be obtained via the classical channel training and feedback methods, while the CSI from the CR user to the PU can be obtained by the CR-Tx via estimating the reversed channel from the PU-Rx, under the assumption of channel reciprocity.
In the assumed system model, the possibility that the PU's transmit power is a function of the received interference power from the CR-Tx has been deliberately excluded. Otherwise, we would also need to take into account any predictable reaction of the PU upon receiving the interference from the CR-Tx, e.g., changes in the PU transmit power will result in a change of the interference power level at the CR-Rx. Before accessing the channel, each CR-Tx must first perform spectrum sensing to determine the status of each channel. We assume that simultaneous spectrum sensing of all the N channels is performed by the CR-Rx using an energy detection scheme. The received signal at the CR-Rx is given

by:

$$y_k(l) = h_{k,pc} S_k(l) + n_k(l) \tag{3.1}$$

where y_k is the CR received signal corresponding to the kth channel, S_k is the PU transmitted signal, $h_{k,pc}$ is the CSI from PU-Tx to CR-Rx for the kth channel, and n_k is the background noise at the CR-Rx in channel k, which is assumed to be independent and identically distributed (i.i.d.) additive complex Gaussian with zero mean and variance σ_k^2, and l is the index of the discrete sample. The statistic is computed as the sum of the received energy over an interval of L_s samples, and the decision is based on:

$$Y_k = \sum_{l=1}^{L_s} |y_k(l)|^2 \gtrless_{H_{0,k}}^{H_{1,k}} \tau_k, \quad k = 1, 2, \ldots, N. \tag{3.2}$$

where τ_k is the threshold of channel k, the hypothesis $H_{0,k}$ represents the absence of a PU in channel k, and the alternative hypothesis $H_{1,k}$ represents the presence of a PU in channel k, which can be reformulated to the following two hypotheses [90]:

$$H_{0,k} : y_k(l) = n_k(l) \tag{3.3}$$

$$H_{1,k} : y_k(l) = h_{k,pc} S_k(l) + n_k(l) \tag{3.4}$$

According to the central limit theorem, for large L_s, $y_k(l)$ are approximately normally distributed. The probabilities of detection $\mathcal{P}_{k,d}$ and false alarm $\mathcal{P}_{k,fa}$ for the kth channel for the CR-Tx, under the energy detection scheme are given, respectively, by [90]:

$$\mathcal{P}_{k,d}(\tau_k, t) = Q\left((\frac{\tau_k}{\sigma_k^2} - \gamma_k - 1)\sqrt{\frac{tf_s}{(2\gamma_k + 1)}} \right) \tag{3.5}$$

$$\mathcal{P}_{k,fa}(\tau_k, t) = Q\left((\frac{\tau_k}{\sigma_k^2} - 1)\sqrt{tf_s} \right) \tag{3.6}$$

where t and γ_k denote, respectively, the sensing time and the received Signal to Noise Radio (SNR) from the PU-Tx to CR-Rx i on the channel k. Let $L_s = tf_s$, where f_s and σ_k^2 represent the sampling frequency and the noise power, respectively. The function $\mathcal{Q}(x)$ is defined as follows:

$$\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{+\infty} e^{-\frac{x^2}{2}} dx \tag{3.7}$$

The choice of the detection threshold τ_k leads to a tradeoff between probability of false alarm $\mathcal{P}_{k,fa}$ and probability of detection $\mathcal{P}_{k,d}$. In an OSA model, a low probability of false alarm $\mathcal{P}_{k,fa}$ is necessary to maintain high spectral utilization in CR systems, since it would prevent the unused spectrum from being accessed by CR users. Furthermore, $\mathcal{P}_{k,d}$ measures the interference of CR users to the PU, which should be limited in order to protect the PU.

3.2 Problem Formulation

Let P_k denote the CR transmit power over the channel k. Since spectral efficiency is the main overall goal of the CR users, the objective function chosen to be maximized is the sum-rate. In this section, we analyze the problem of optimizing the power allocation for the CR user in order to maximize the sum-rate, taking into account the detection result. We start from the perfect sensing case, where the probability of false alarm $\mathcal{P}_{k,fa} = 0$ and the probability of detection $\mathcal{P}_{k,d} = 1$, indicating that there is no interference to the PU. Hence, the total achievable sum-rate for CR user, denoted as $R_{wf}(P_k)$, is given by:

$$R_{wf}(P_k) = \sum_{k=1}^{N} \log_2 \left(1 + \frac{P_k |h_{k,cr}|^2}{\sigma_k^2}\right) \tag{3.8}$$

The total transmit power of the CR-Tx over all channels should not exceed its maximum allowed power. Thus, a power budget constraint can be formulated as:

$$\sum_{k=1}^{N} P_k \leq P_{\max} \tag{3.9}$$

where $P_{\max}$ denotes the maximum total transmit power of the CR-Tx over the N channels. The optimization problem for maximizing the sum-rate of the CR user can be formulated as $P3.1$:

$$\begin{aligned} \max_{\mathbf{P}} \quad & \sum_{k=1}^{N} \log_2(1+\frac{P_k|h_{k,cr}|^2}{\sigma_k^2}) \\ \text{s.t.} \quad & \sum_{k=1}^{N} P_k \le P_{\max}, \\ & P_k \ge 0, \quad k=1,2,\dots,N. \end{aligned} \tag{3.10}$$

where $\mathbf{P} = [P_k]_{k=1}^N$, $|h_{k,cr}|^2$ is the channel gain in channel k between the CR-Tx and the CR-Rx. The optimization problem $P3.1$ is a convex problem, thus we can establish the Karush-Kuhn-Tucker (KKT) conditions and solve it efficiently. The solution is the well-known model of water-filling, given by:

$$P_{k,wf}^{\star} = \left[\frac{1}{\beta} - \frac{\sigma_k^2}{|h_{k,cr}|^2}\right]^+ \tag{3.11}$$

where $[x]^+ = \max(0,x)$, $\frac{\sigma_k^2}{|h_{k,cr}|^2}$ is the water level, and β is a non-negative dual variable associated with the power budget constraint (3.9).
Considering the fact that the spectrum sensing information is not always reliable, the accuracy of detection is limited by attenuation due to shadowing, fading, as well as the hidden node problem, leading to probabilities of detection $\mathcal{P}_{k,d} < 1$ and probabilities of false alarm $\mathcal{P}_{k,fa} > 0$. As a consequence, in the OSA model, four different rates at the CR-Rx in channel k can be defined as shown in the following, where the first index number describes the actual status of the PU ("0" for idle and "1" for busy), and the second index number indicates the sensing result obtained by energy detection.

- If channel k is idle and estimated to be idle, the rate is given by $r_{k,00} = \log_2\left(1+\frac{P_k|h_{k,cr}|^2}{\sigma_k^2}\right)$.

- If channel k is idle and estimated to be occupied, the rate is given by 0.

- If channel k is busy and estimated to be idle, the rate is given by $r_{k,10} = \log_2\left(1+\frac{P_k|h_{k,cr}|^2}{\sigma_k^2\gamma_k+\sigma_k^2}\right)$.

- If channel k is busy and estimated to be occupied, the rate is given by 0.

We can formulate the objective function as follows:

$$R(P_k, \tau_k) = \sum_{k=1}^{N} \Big((1 - \mathcal{P}_{k,fa}(\tau_k)) r_{k,00} + (1 - \mathcal{P}_{k,d}(\tau_k)) r_{k,10} \Big) \tag{3.12}$$

The most important constraint of CRNs involves protecting the PU from harmful performance degradation. In this thesis, a rate-loss gap constraint to design the power allocation is proposed, ensuring that the performance degradation of the PU due to imperfect sensing in each channel is bounded. On the one hand, the maximum achievable rate of the PU in channel k without the interference from CR user is given as:

$$U_{k,\max} = \log_2(1 + \gamma_k^p) \tag{3.13}$$

On the other hand, the maximum achievable average rate of the PU in channel k with the interference from the CR user is given by:

$$U_k(\tau_k) = \mathcal{P}_{k,d}(\tau_k) \log_2(1 + \gamma_k^p) + (1 - \mathcal{P}_{k,d}(\tau_k)) \log_2(1 + \frac{\gamma_k^p \sigma_k^2}{I_{k,cp}}) \tag{3.14}$$

where γ_k^p is the SNR of the PU in channel k, $I_{k,cp}$ denotes the interference experienced by the PU-Rx due to the transmission of the CR-Tx in the same channel k and the noise, that is:

$$I_{k,cp} = |h_{k,cp}|^2 P_k + \sigma_k^2 \tag{3.15}$$

Given this, the rate-loss gap constraint can be written as follows:

$$U_{k,\max} - U_k(P_k, \tau_k) \leq \Gamma_k U_{k,\max} \tag{3.16}$$

where Γ_k is the maximum acceptable rate-loss gap for the PU in channel k. Furthermore, the power budget constraint (3.9) is also considered here. Specifically, in a real system, a high $\mathcal{P}_{k,d}$ and a low $\mathcal{P}_{k,fa}$ are typically required. Thus in this work, we restrict the target detection probability and false alarm to the ranges $\mathcal{P}_{k,d} \geq \frac{1}{2}$ and $\mathcal{P}_{k,fa} \leq \frac{1}{2}$, respectively. This constraint can ensure that the minimum opportunistic spectral utilization to be achieved is $\frac{1}{2}$. According to the monotonicity of the $\mathcal{Q}$-function, taking into account expressions (3.5) and (3.6), the constraints are equivalent to:

$$\tau_{k,\min} \leq \tau_k \leq \tau_{k,\max} \tag{3.17}$$

The optimization problem for maximizing the sum-rate of the CR user, $R(\mathbf{P}, \boldsymbol{\tau})$, $\mathbf{P} = [P_k]_{k=1}^N, \boldsymbol{\tau} = [\tau_k]_{k=1}^N$, can be formulated as the following problem $P3.2$:

$$\begin{aligned} \max_{\mathbf{P},\boldsymbol{\tau}} \quad & \sum_{k=1}^{N} \Big((1 - \mathcal{P}_{k,fa}(\tau_k)) r_{k,00} + (1 - \mathcal{P}_{k,d}(\tau_k)) r_{k,10} \Big) \\ \text{s. t.} \quad & \sum_{k=1}^{N} P_k \leq P_{\max}, \\ & U_{k,\max} - U_k(P_k, \tau_k) \leq \Gamma_k U_{k,\max}, \\ & \tau_{k,\min} \leq \tau_k \leq \tau_{k,\max}, \\ & P_k \geq 0, \quad k = 1, 2, \ldots, N. \end{aligned} \tag{3.18}$$

3.3 Joint Optimization of Detection and Power Allocation

The objective function in $P3.2$ is a non-convex function of variables: $\mathbf{P}$ and $\boldsymbol{\tau}$, thus, finding the exact optimal solution for the above problem entails a high complexity. In the following, we present two iterative algorithms:

- Exhaustive Optimization of Power Allocation and Detection (EPD) algorithm, which is based on an exhaustive search of the detection threshold $\boldsymbol{\tau}$;

- Alternating Optimization of Power Allocation and Detection (APD) algorithm, which solves only convex problems in each iteration and updates the variables in an alternating fashion.

Moreover, we prove analytically that both algorithms can converge to a fixed point.

3.3.1 Exhaustive optimization of power allocation and detection

The global optimal solution to problem $P3.2$ can be found by computing the optimal $\mathbf{P}^{\star}$, for all $\frac{N(\boldsymbol{\tau}_{\max}-\boldsymbol{\tau}_{\min})}{\delta}$ possible choices for $\boldsymbol{\tau}$, where δ is the step size for $\boldsymbol{\tau}$, and finding the $(\mathbf{P}^{\star}, \boldsymbol{\tau}^{\star})$ that yield the maximum sum-rate $R(\mathbf{P}^{\star}, \boldsymbol{\tau}^{\star})$ for the CR user. For any given $\tilde{\tau}_k$, the optimization problem $P3.2$ can be reformulated as the following problem $P3.3$:

$$\begin{aligned}
\max_{\mathbf{P}} \quad & \sum_{k=1}^{N} \Big((1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) r_{k,00} + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k)) r_{k,10} \Big) \\
\text{s.t.} \quad & \sum_{k=1}^{N} P_k \leq P_{\max}, && (3.19)\\
& U_{k,\max} - U_k(\tilde{\tau}_k) \leq \Gamma_k U_{k,\max}, && (3.20)\\
& P_k \geq 0, \quad k = 1, 2, \ldots, N. && (3.21)
\end{aligned}$$

In order to analyze the problem in a more convenient form, constraint (3.20) can be rewritten as an equivalent convex form:

$$P_k \leq C_k(\tilde{\tau}_k) \tag{3.22}$$

where:

$$C_k(\tilde{\tau}_k) = \frac{\gamma_k^p \sigma_k^2}{\left(2^{(1-\frac{\Gamma_k}{1-\mathcal{P}_{k,d}(\tilde{\tau}_k)})U_{k,\max}} - 1\right)|h_{k,cp}|^2} - \frac{\sigma_k^2}{|h_{k,cp}|^2} \tag{3.23}$$

In order to ensure there is feasible solution of P_k, thus the set of P_k defined in problem $P3.3$ is nonempty, we need the following two necessary conditions per channel k:

$$2^{(1-\frac{\Gamma_k}{1-\mathcal{P}_{k,d}(\tilde{\tau}_k)})U_{k,\max}} - 1 < \gamma_k^p \tag{3.24}$$

$$\mathcal{P}_{k,d}(\tilde{\tau}_k) < 1 - \Gamma_k \tag{3.25}$$

The conditions above are based on the constraint (3.20), which shows the relationship between the sensing performance and system parameters. Regarding the first two constraint inequalities (3.19) and (3.20) in problem $P3.3$, the optimization problem is working in two possible regimes:

- Power Budget Limited Regime (PLR), where $P_{\max} \leq \sum_{k=1}^{N} C_k(\tilde{\tau}_k)$, implying that the power allocation is bounded by the total power budget $P_{\max}$, which leads to the worst case interference condition as considered in [36]. In this case, the first two constraints are equivalent to $\sum_{k=1}^{N} P_k = P_{\max}$ and $P_k \leq C_k(\tilde{\tau}_k)$;

- Rate-Loss Limited Regime (RLR), where $P_{\max} > \sum_{k=1}^{N} C_k(\tilde{\tau}_k)$, implying that the power allocation is bounded by the rate-loss gap constraint. Increasing the total power budget $P_{\max}$ will not lead to an increase in the sum-rate of the CR user. In this case, the optimization solution is achieved when $P_k^\star = C_k(\tilde{\tau}_k)$.

The Lagrangian of problem $P3.3$ is denoted as $L(\mathbf{P}, \boldsymbol{\alpha}, \beta)$, and is given by:

$$\begin{aligned} L(\mathbf{P}, \boldsymbol{\alpha}, \beta) = & \sum_{k=1}^{N} \Big((1 - \mathcal{P}_{k,fa}(\tilde{\tau}_k)) r_{k,00} + (1 - \mathcal{P}_{k,d}(\tilde{\tau}_k)) r_{k,10} \Big) \\ & - \beta\Big(\sum_{k=1}^{N} P_k - P_{\max}\Big) - \sum_{k=1}^{N} \alpha_k (P_k - C_k(\tilde{\tau}_k)) \end{aligned} \quad (3.26)$$

where $\boldsymbol{\alpha} = [\alpha_k]_{k=1}^{N}$ and β are nonnegative multipliers corresponding to the dual variables associated with the power budget constraint and rate-loss gap constraint, respectively. The Lagrange dual optimization problem becomes:

$$\min_{\boldsymbol{\alpha} \geq 0, \beta \geq 0} g(\boldsymbol{\alpha}, \beta) \quad (3.27)$$

where $g(\boldsymbol{\alpha}, \beta) = \max_{\mathbf{P}} L(\mathbf{P}, \boldsymbol{\alpha}, \beta)$. Furthermore, $P3.3$ is a convex problem in both of those regimes. Thus, we can establish the KKT conditions and solve $P3.3$ efficiently:

$$\frac{(\mathcal{P}_{k,fa}(\tilde{\tau}_k) - 1)|h_{k,cr}|^2}{P_k |h_{k,cr}|^2 + \sigma_k^2} + \frac{(\mathcal{P}_{k,d}(\tilde{\tau}_k) - 1)|h_{k,cr}|^2}{P_k |h_{k,cr}|^2 + \sigma_k^2 + \gamma_k \sigma_k^2} + \alpha_k + \beta = 0, \quad (3.28)$$

$$\alpha_k (P_k - C_k(\tilde{\tau}_k)) = 0, \quad (3.29)$$

$$\beta(\sum_{k=1}^{N} P_k - P_{\max}) = 0, \tag{3.30}$$

$$\alpha_k \geq 0, \quad \beta \geq 0. \tag{3.31}$$

In the RLR, the optimal solution can be obtained similar to the water-filling (WF) algorithm:

$$P_k^\star(RLR) \begin{cases} \left[\dfrac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{\alpha_k \ln 2 - g(P_k)} - \dfrac{1}{w_k}\right]^+ & \text{if } \dfrac{1}{w_k} \geq W \\ C_k(\tilde{\tau}_k) & \text{if } \dfrac{1}{w_k} < W \end{cases} \tag{3.32}$$

where $w_k = \frac{|h_{k,cr}|^2}{\sigma_k^2(\gamma_k+1)}$, and $g(P_k)$ and W are given, respectively, by:

$$g(P_k) = \frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k))\gamma_k\sigma_k^2|h_{k,cr}|^2}{(P_k|h_{k,cr}|^2+\sigma_k^2)(P_k|h_{k,cr}|^2+\gamma_k\sigma_k^2+\sigma_k^2)} \tag{3.33}$$

$$W = \frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{\alpha_k - g(C_k(\tilde{\tau}_k))} - C_k(\tilde{\tau}_k) \tag{3.34}$$

In addition, we define the function $f(P_k)$ as:

$$f(P_k) = \frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{\alpha_k \ln 2 - g(P_k)} \tag{3.35}$$

$$f_{\min}(P_k) = \frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{\alpha_k \ln 2 - g(C_k(\tilde{\tau}_k))} \tag{3.36}$$

$$f_{\max}(P_k) = \frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{\alpha_k \ln 2 - g(0)} \tag{3.37}$$

Notice that $f(P_k)$ is a decreasing function of P_k. Assuming $\frac{1}{w_k} \geq W$, Figure 3.3 illustrates the solution of $P_k^\star$, which is obtained as the intersection between a 45-degree line starting from the point $(0, \frac{1}{w_k})$, and the curve of the function $f(P_k)$. In this case, the multi-level water-filling level $f(0)$ is related to $\mathcal{P}_{k,fa}(\tilde{\tau}_k)$, $\mathcal{P}_{k,d}(\tilde{\tau}_k)$ and channel gain $|h_{k,cr}|^2$, the interference $\frac{1}{w_k}$ is related to channel gain $|h_{k,cr}|^2$ and γ_k.
Interestingly, the extreme case of $\mathcal{P}_{k,fa}(\tilde{\tau}_k) = 0$, $\mathcal{P}_{k,d}(\tilde{\tau}_k) = 1$ is given by the perfect detection information of each channel. Then, the optimal solution is a modified version of the standard WF algorithm. Compared

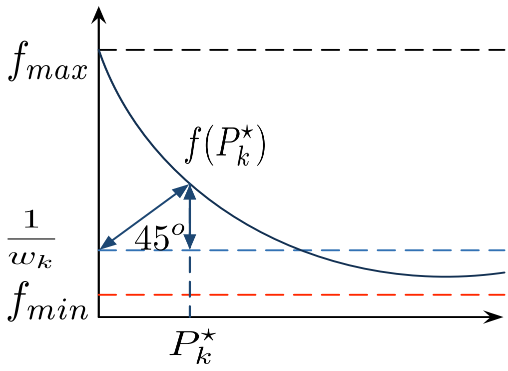

Figure 3.3: The optimal solution $P_k^\star$

with the standard WF algorithm, (3.32) differs in that the water-level is no longer a constant, but it is instead a function of $g(P_k)$. If the function $g(P_k) = 0$, (3.32) becomes the standard WF policy with a constant water-level $\frac{1}{w_k}$, since in this case the CR transmission does not interfere with the PU. On the other hand, if $g(P_k) = \infty$, from (3.32) it follows that the water-level becomes zero and thus $P_k = 0$, regardless of the interference $\frac{1}{w_k}$, suggesting that in this case no CR transmission is allowed since any finite CR transmit power will result in an infinite interference power at the PU-Rx.

In the PLR, we can obtain the global optimal solution in a similar way, which is given as:

$$P_k^\star(PLR) = \left[\frac{(1-\mathcal{P}_{k,fa}(\tilde{\tau}_k)) + (1-\mathcal{P}_{k,d}(\tilde{\tau}_k))}{(\alpha_k+\beta)\ln 2 - g(P_k)} - \frac{1}{w_k}\right]^+ \tag{3.38}$$

The ellipsoid method can be used here to find the optimal solutions of $\boldsymbol{\alpha}$, β and $\mathbf{P}$ [36], which require the subgradient form of the dual function. The subgradient of the dual function $h(\boldsymbol{\alpha}, \beta)$ is given by $(Z_0, \mathbf{Z}_1)$, where $Z_0 = P_{\max} - \sum_{k=1}^{N} P_k^\star$, and $\mathbf{Z}_1 = [Z_{k,1}]_{k=1}^N$, $Z_{k,1} = C_k(\tau_k) - P_k^\star$, where $P_k^\star$ is the optimal power allocation for any fixed $\boldsymbol{\alpha}$, β. The proposed EPD algorithm is summarized in **Algorithm 1**, where v_1 and v_2 are the iteration values, θ is the step size, and ε is the tolerance which is fixed and dependent on the system accuracy.

Algorithm 1 EPD Algorithm

for $\boldsymbol{\tau} = \boldsymbol{\tau}_{\min}:\boldsymbol{\tau}_{\max}$ **do**
  repeat
    Initialize $\beta(v_1)$, $v_1 := 0$
    repeat
      Initialize $\alpha_k(v_2)$, $v_2 := 0$
      repeat
        Find $\mathbf{P}^\star$, solve $P2$;
        Update $\alpha_k(v_2+1) = \alpha_k(v_2) + \theta(P_k^\star - C_k)$;
      until If $\alpha_k(v_2+1) < 0$, set $\alpha_k(v_2+1) = 0$, Stop;
            Or, when $|\alpha_k(v_2+1) - \alpha_k(v_2)| \le \varepsilon$, Stop.
      Update $\beta(v_1+1) = \beta(v_1) + \theta(\sum_{k=1}^{N} P_k^\star - P_{\max})$;
    until $\beta(v_1+1) < 0$, set $\beta(v_1+1) = 0$, Stop;
          Or, when $|\beta(v_1+1) - \beta(v_1)| \le \varepsilon$, Stop.
  until $|\mathbf{P}^\star(v_1) - \mathbf{P}^\star(v_1-1)| \le \varepsilon$
  Update the optimal $\mathbf{P}^\star$ and the maximum sum-rate $R(\mathbf{P}^\star)$
end for

3.3.2 Complexity analysis of the EPD algorithm

In the case of the EPD algorithm, the complexity is related with the possible region of the detection threshold $\boldsymbol{\tau}$, the step size θ, as well as the number of iterations needed to achieve the optimal Lagrange multipliers $\boldsymbol{\alpha}^\star$ and $\beta^\star$. The time complexity to find the multipliers $\boldsymbol{\alpha}$ and β is associated with the step size θ of the ellipsoid method and the number of constraints. The ellipsoid method shows polynomial complexity [20] which is given by $O(\frac{N}{\theta^2})$. Hence, the total complexity of the EPD algorithm is $O\Bigg(\ln \frac{1}{\varepsilon} \frac{N^2(\boldsymbol{\tau}_{\max} - \boldsymbol{\tau}_{\min})}{\theta^2 \delta} \Bigg)$.

3.3.3 Alternating direction optimization of power allocation and detection

The complexity of finding the global optimal solution based on the exhaustive search approach is prohibitively high. Instead of directly solving the non-convex optimization problem by the EPD algorithm, in the following, we propose the APD algorithm, which is based on the Alternating Direction Optimization (ADO) method in [91], and finds ef-

ficiently a suboptimal solution for the non-convex optimization problem $P3.2$. The ADO method is a simple but powerful method that is well suited to convex optimization. It takes the form of a decomposition-coordination procedure, in which the solutions to small local subproblems are coordinated to find a solution to a large global problem. The ADO method can be viewed as an attempt to blend the benefits of dual decomposition and augmented Lagrangian methods for constrained optimization [91]. In ADO, the variables are updated in an alternating or sequential fashion, which accounts for the term alternating direction. However, for our non-convex problem, the ADO may not converge to the global optimal points, therefore, it must be considered just a local optimization method.
We divide the original problem $P3.2$ into two stages, referred as optimal power allocation and local threshold optimization, respectively. In this case, the APD finds the optimal transmit power $\mathbf{P}$ and detection threshold $\boldsymbol{\tau}$ alternately.

- In the first optimal power allocation step, we maximize the sum-rate of the CR based on the given detection threshold $\tilde{\tau}_k$. Notice that $\mathcal{P}_{k,fa}(\tilde{\tau}_k)$ and $\mathcal{P}_{k,d}(\tilde{\tau}_k)$ become constants in this case. Then, $P3.2$ can be reformulated to the optimization problem $P3.3$, in a similar way as for the EPD algorithm.

- Substituting the $P_k^\star$ obtained from the optimal power allocation step, we optimize the local threshold $\boldsymbol{\tau}$ to get the maximum sum-rate of the CR in the local threshold optimization step, which is given as the following $P3.4$:

$$
\begin{aligned}
\max_{\boldsymbol{\tau}} \quad & \sum_{k=1}^{N} \Big((1-\mathcal{P}_{k,fa}(\tau_k))r_{k,00}(P_k^\star) + (1-\mathcal{P}_{k,d}(\tau_k))r_{k,10}(P_k^\star) \Big) \\
\text{s. t.} \quad & A_k(P_k^\star) - \mathcal{P}_{k,d}(\tau_k) \le 0, && (3.39)\\
& \tau_{\min,k} \le \tau_k \le \tau_{\max,k}, \quad k = 1,2,\ldots,N. && (3.40)
\end{aligned}
$$

where:

$$
A_k(P_k^\star) = 1 - \frac{\Gamma_k U_{k,\max}}{U_{k,\max} - \log_2\left(1 + \frac{\gamma_k \sigma_k^2}{P_k^\star |h_{k,cp}|^2 + \sigma_k^2}\right)} \tag{3.41}
$$

The objective and the constraint functions in $P3.4$ are generally non-convex. However, this seemingly non-convex problem can be solved by exploiting the convexity properties. For this purpose, we derive the following proposition.

Proposition 1. *The optimal solution of $P3.4$ is achieved when the detection threshold $\boldsymbol{\tau}$ is equal to the upper bound.*

Proof. From (3.5) and (3.6), we can observe that $\mathcal{P}_d(\boldsymbol{\tau})$ and $\mathcal{P}_{fa}(\boldsymbol{\tau})$ are decreasing functions of the detection threshold $\boldsymbol{\tau}$. Moreover, $\boldsymbol{\tau}$ is bounded by the target probability of false alarm and detection, as well as the rate-loss constraint (3.39). Consequently, the objective function in $P3.4$ is an increasing function of $\boldsymbol{\tau}$, and $P3.4$ achieves its maximal sum-rate when the detection threshold $\boldsymbol{\tau}$ reaches its upper bound. □

In addition, the upper bound of the detection threshold $\boldsymbol{\tau}$ for channel k is given by:

$$\tau_k^\star = \max\left[\tau_{\max,k}, \sigma_k^2(Q^{-1}(A_k(P_k^\star))\left(\frac{tf_s}{2\gamma_k+1}\right)^{-\frac{1}{2}} + \gamma_k + 1)\right] \tag{3.42}$$

Combining the two steps, the proposed APD algorithm is summarized in **Algorithm** 2. Noting that the objective function is nondecreasing at each iteration, we can obtain:

$$R(P_k(p), \tau_k(p)) \leq R(P_k(p+1), \tau_k(p)) \leq R(P_k(p+1), \tau_k(p+1)) \tag{3.43}$$

where p is the iteration number. Combined with the fact that both P_k and τ_k are upper bounded, the convergence follows. However, in general, the convergence point is not always the global optimal. Therefore, the solution of the APD algorithm should be considered as a sub-optimal solution.

3.3.4 Complexity analysis of the APD algorithm

For the APD algorithm, the complexity is related with the tolerance ε and the number of iterations in the local threshold optimization step.

Algorithm 2 APD Algorithm

Initialize $v_0 := 0$, $\boldsymbol{\tau}(v_0)$, $\mathbf{P}$, $\forall k = 1, 2, \ldots$
repeat
  Initialize $\beta(v_1)$, $v_1 := 0$
  repeat
    Initialize $\alpha_k(v_2)$, $v_2 := 0$
    repeat
      Given $\boldsymbol{\tau}(v_0)$, find $\mathbf{P}^{\star}(v_0)$, solve $P3.3$;
      Update $\alpha_k(v_2+1) = \alpha_k(v_2) + \theta(P_k^{\star} - C_k)$;
    until If $\alpha_k(v_2+1) < 0$, set $\alpha_k(v_2+1) = 0$, Stop;
        Or, when $|\alpha_k(v_2+1) - \alpha_k(v_2)| \leq \varepsilon$, Stop.
    Update $\beta(v_1+1) = \beta(v_1) + \theta(\sum_{k=1}^{N} P_k^{\star} - P_{\max})$;
  until $\beta(v_1+1) < 0$, set $\beta(v_1+1) = 0$, Stop;
      Or, when $|\beta(v_1+1) - \beta(v_1)| \leq \varepsilon$, Stop.
  Update $\mathbf{P}(v+1) = \mathbf{P}^{\star}$;
  Given $\mathbf{P}(v_0+1)$ find the optimal value $\boldsymbol{\tau}^{\star}$ that solve $P3.4$;
  Update $\boldsymbol{\tau}(v_0+1) := \boldsymbol{\tau}^{i,\star}$;
until $|\mathbf{P}(v_0) - \mathbf{P}(v_0-1)| \leq \varepsilon$, and $|\boldsymbol{\tau}(v_0) - \boldsymbol{\tau}(v_0-1)| \leq \varepsilon$, Stop.

The optimization problem $P3.4$ is a linear programming problem, which requires $O(N)$ operations in each round. Furthermore, for the optimal power allocation step, the complexity is $O(\frac{N}{\theta^2})$. Hence, the total complexity of the APD algorithm is $O(\ln\frac{1}{\varepsilon}(\frac{N}{\theta^2} + N))$. More specifically, with a small value of the tolerance ε and a large number of PUs, the APD algorithm can achieve a considerable performance improvement in convergence speed with respect to the EPD algorithm.

3.4 Simulation Results

In this section, we present some numerical results to illustrate the performance of the proposed algorithms. First of all, the performance of the proposed EPD algorithm and APD algorithm with different values of $P_{\max}$ and Γ, are evaluated respectively. Then, we show the performance comparison of EPD algorithm, APD algorithm, determined power constraint (DPC) algorithm [66,67] and interference power constraint (IPC) algorithm in [92].

Table 3.2: **Simulation parameters**

Parameter	Value
Primary network	
Primary transmit power	4W
Number of channels, N	8
Cognitive radio network	
Power budget $P_{\max}$	1W - 20W
Number of samples tf_s	100
Rate-loss gap Γ_k	$0.1\% - 1.5\%$
Noise σ^2	1
$\lvert h_{k,cp}\rvert^2$	0.1
$\lvert h_{k,pc}\rvert^2$	0.1
$\lvert h_{k,cr}\rvert^2$	0.4
$\lvert h_{k,pu}\rvert^2$	0.4

3.4.1 Scenario description

We consider a CRN with one pair of CR Tx-Rx and one pair of PU Tx-Rx. The number of available channels for the CR is $N = 8$. We assume that the sensing environment is stable in the optimization process. The noise from PU transmissions is treated as floor noise that together with the thermal noise is normalized to a unit variance. All the parameters used in the simulation are given in Table 3.2. Note that the proposed algorithms can also be applied to more realistic scenarios with multiple CR pairs and multiple PU pairs. In the following chapter, we will focus on a multiuser CRN, where there are multiple CR pairs, multiple PU pairs, and the multiuser interference is taken into account.

3.4.2 Simulation results analysis

Figure 3.4 and Figure 3.5 show the sum-rate of the CR versus the rate-loss gap Γ_k and the power budget $P^i_{\max}$, respectively. The results show that an optimal detection threshold $\boldsymbol{\tau}$ that maximizes the sum-rate of the CR user exists for the EPD algorithm. In order to reduce the computational complexity in the EPD algorithm, we initialize the detection threshold with the same value for all channels. Specifically, a small value of detection threshold leads to a higher probability of false alarm and

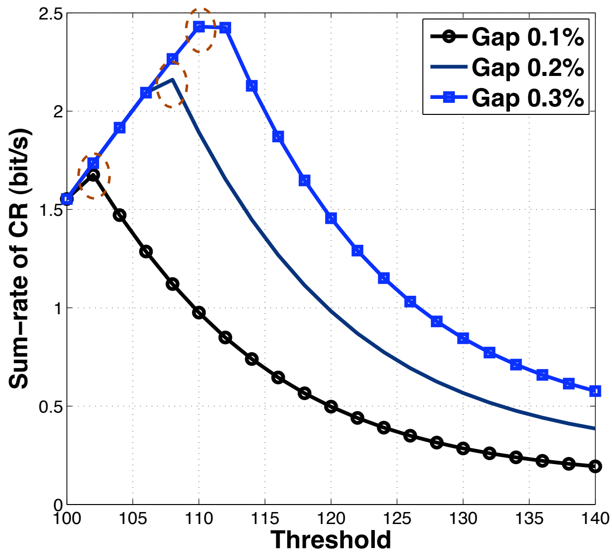

Figure 3.4: EPD algorithm: sum-rate versus threshold for different values of rate-loss gap Γ_k, with $P_{\max}$=$6W$

a lower probability of miss detection, and consequently the available chance for the CR user to reuse the channel is less. On the other hand, when the detection threshold is higher, there is more chance for the CR user to access the channel, leading to an increase in the sum-rate.

In Figure 3.4, we show the achievable sum-rate for different detection thresholds $\boldsymbol{\tau}$ under different values of rate-loss gap $\Gamma_k = 0.1\%, 0.2\%, 0.3\%$, respectively, for a power budget $P_{\max} = 6W$. From the picture, we can see that there is an optimal detection threshold $\boldsymbol{\tau}$ for each value of rate-loss gap, where the CR user can achieve the maximum sum-rate. Specifically, when the value of the rate-loss gap is less stringent, the CR user can achieve a better performance than in the stringent one.

More specifically, the optimal threshold decreases when the Γ_k is more stringent. This is because for the lower rate-loss gap case, we need a more precise detection information to avoid harmful interference to PU, thus a lower probability of miss detection is required, leading to a reduction of detection threshold.

Figure 3.5 shows the sum-rate of the CR for different detection thresholds under different value of the power budget $P_{\max} = 3W, 6W, 9W$,

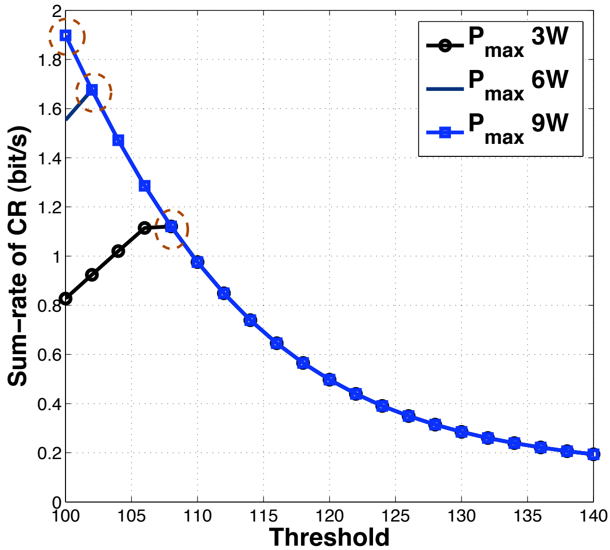

Figure 3.5: EPD algorithm: sum-rate versus threshold for different values of power budget $P_{\max}$, with Γ_k=0.1%

respectively, when $\Gamma_k = 0.1\%$. From the picture, we can see that there is an optimal detection threshold $\boldsymbol{\tau}$ for each value of power budget $P_{\max}$, where the CR user can achieve the maximum sum-rate. Specifically, with a larger value of power budget, the CR user can achieve a better performance than in the lower case.

As we mentioned in Section 3.3, both EPD and APD algorithms work in two different limited regimes, namely, power budget limited regime (PLR), implying that the power allocation is bounded by the total power budget $P_{\max}$; rate-loss limited regime (RLR), implying that the power allocation is bounded by the rate-loss constraint. Increasing the total power budget $P_{\max}$ will not lead to an increase in the sum-rate of the CR user. Based on the results, the CR user in Figure 3.4 works in the RLR, and in Figure 3.5 the result is more related to the power budget, thus the CR user is bounded by the PLR.

Figure 3.6 shows the sum-rate versus power budget $P_{\max}$ for different values of rate-loss gap Γ_k=$0.1\%, 0.2\%, 0.3\%$ for the APD algorithm. With the increasing power budget of the CR, the optimization problem will be bounded by different constraints. When $\Gamma_k = 0.1\%$, the CR user

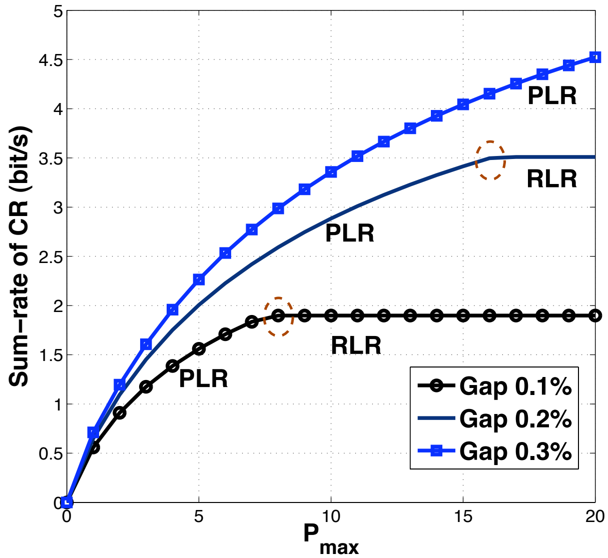

Figure 3.6: APD algorithm: sum-rate versus power budget $P_{\max}$ for different values of rate-loss gap Γ_k=0.1%, 0.2%, 0.3%

works in the PLR between $P_{\max} = 1W$ to $P_{\max} = 8W$, when the power budget becomes larger the sum-rate is not increased. Then, the CR user is bounded by the RLR. The CR user changes from PLR to RLR when the power budget is larger than $16W$, when $\Gamma_k = 0.1\%$. Moreover, the CR always works in the PLR when we relax the rate-loss gap to 0.3%. Clearly from the picture, the sum-rate of the CR increases with the increase of Γ_k, which shows the tradeoff between improving the spectral utilization and reducing the interference.

Figure 3.7 plots the sum-rate versus Γ_k for different values of $P_{\max}$ for the APD algorithm. From the picture, the sum-rate of the CR user can not be improved in the PLR even when we increase the rate-loss gap Γ_k. The rate-loss gap constraint becomes insignificant in the PLR.

In Figure 3.8, we compare the sum-rate for EPD, APD, the interference power constraint (IPC) algorithm in the literature [92], and the deterministic power constraint (DPC) algorithm, where the sensing information is not considered as a part of optimization. Let Γ_0 be the maximum total interference power that PU can tolerate. The IPC can

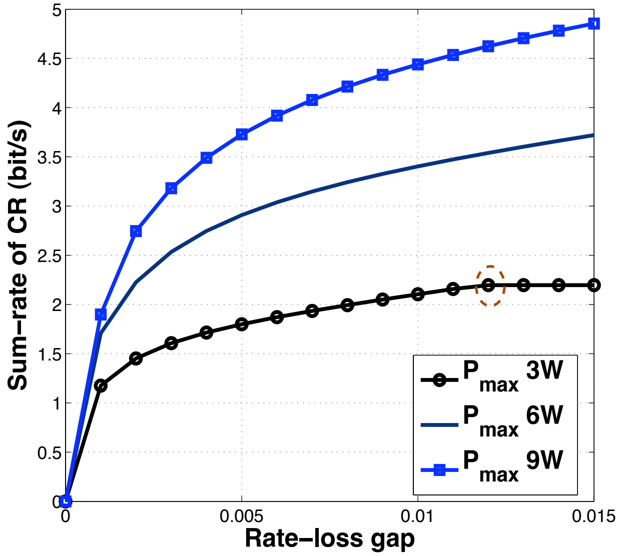

Figure 3.7: APD algorithm: sum-rate versus rate-loss gap Γ_k for different values of power budget $P_{\max}$=$3W, 6W, 9W$

be written as:

$$\sum_{k=1}^{N}(1-\mathcal{P}_{k,d}(\tau_k))P_k|h_{k,cp}|^2 \leq \Gamma_0 \tag{3.44}$$

Then, the maximum rate-loss of the PU is upper-bounded as follows:

$$\begin{aligned}
&\frac{1}{N}\sum_{k=1}^{N}(U_{k,\max}-U_k)\\
&=\frac{1}{N}\sum_{k=1}^{N}\left[(1-\mathcal{P}_{k,d}(\tau_k))U_{k,\max}-(1-\mathcal{P}_{k,d}(\tau_k))\log_2\left(1+\frac{P_k|h_{k,cp}|^2}{I_{k,cp}}\right)\right]\\
&\leq\frac{1}{N}\sum_{k=1}^{N}(1-\mathcal{P}_{k,d}(\tau_k))\log_2\left(1+\frac{P_k|h_{k,cp}|^2}{\sigma_k^2}\right)\\
&\leq\frac{1}{N}\sum_{k=1}^{N}\frac{(1-\mathcal{P}_{k,d}(\tau_k))P_k|h_{k,cp}|^2}{\sigma_k^2\ln 2}
\end{aligned}$$

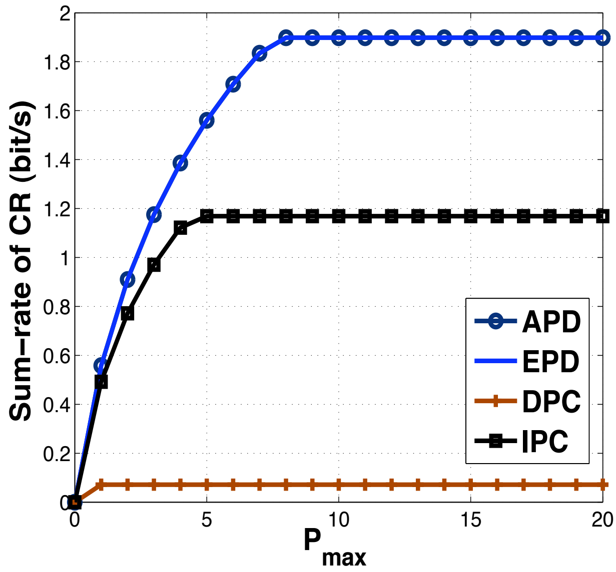

Figure 3.8: Sum-rate versus power budget $P_{\max}$ for different algorithms, with Γ_k=0.1%

$$\leq \frac{\Gamma_0}{N\sigma_k^2 \ln 2} \tag{3.45}$$

which follows from the fact that $x \log(e) \geq log(1 + x)$. From (3.44)-(3.45), it can be seen that the IPC provides an upper bound on the average rate-loss of the PU for $P3.2$. From Figure 3.8, the EPD algorithm and the APD algorithm can achieve a higher sum-rate than the IPC algorithm and the DPC algorithm, thus, making a better use of the available channels. All the algorithms are bounded by the PLR when the value of the power budget $P_{\max}$ is small. With the increasing power budget, the EPD algorithm and the APD algorithm turn to the RLR at the same point $P_{\max} = 8$, while the IPC algorithm turns to the RLR after $P_{\max} = 5$. This is because the rate-loss gap is more stringent for the IPC algorithm than for the other two algorithms. More specifically, the DPC algorithm is always bounded by the RLR, and shows the worst performance. This is because a stringent rate-loss gap constraint imposes lower transmit power for the DPC algorithm, while higher transmit power is allowed for other algorithms, due to the accurate sensing information. Furthermore, the proposed APD algorithm can achieve the same performance as the global optimal solution obtained by the EPD algorithm with much lower computation complexity.

3.5 Conclusions

In this chapter, we consider a CRN with one PU Tx-Rx pair and one CR Tx-Rx pair, under OSA model. We analyze the optimal power allocation for the CR user to maximize the sum-rate by considering the effect of the non-zero probabilities of miss detection and false alarm of active PU transmission. Our problem can be formulated as a two-variable problem and solved by the EPD algorithm and the APD algorithm. A novel criterion is proposed to design the power allocation, ensuring that the performance degradation of the PU is bounded. In addition, we discuss the two algorithms in two different constraint regimes, namely, PLR and RLR. Simulation results demonstrate that the proposed algorithms can considerably improve system performance with respect to the state of the art IPC and DPC algorithms. In the following chapter, we will focus on multiuser CRNs, where the multiuser interference is taken into account. The resulting optimization problems can be formulated as a non-cooperative game, wherein the selfish players seek to optimize their individual objectives in the face of competition from their rivals under appropriate constraints.

Chapter 4

Joint Optimization of Detection and Power Allocation in Multiuser SISO CRNs*

In the previous chapter, we have considered a sensing-based spectrum sharing scenario for single user CRNs, where the overall objective is to maximize the sum-rate of the CR user by optimizing jointly both the detection operation based on sensing and the power allocation, taking into account the influence of the sensing accuracy and the interference limitation to PUs. We now focus on multiuser CRNs, where the multiuser interference is taken into account. The resulting optimization problems can be formulated to a non-cooperative game, wherein the selfish players seek to optimize their individual objectives in the face of competition from their rivals under the constraints. A fundamental concept for the resolution of such a game is the Nash equilibrium (NE). The classical case where a NE exists is when the players' objective functions are convex with their rivals' strategies fixed and the players' constraint sets are convex. However, our game is a non-convex game, which presents a new challenge when analyzing the equilibria of

*The publications associated to this chapter are [17, 18] this game where each cognitive user represents a player. In order to deal with the non-convexity of the game, we start with the ADO algorithm mentioned in the previous chapter, and we proof that the local NE is achieved by the ADO for SISO CRNs (ADOS) algorithm. In the second step, we use a new relaxed equilibrium concept, namely, quasi-Nash equilibrium (QNE) [31, 32] instead of the traditional NE for the convex game. We show the sufficient conditions for the existence and the uniqueness of the QNE for the proposed game. Moreover, an iterative Primal-Dual Interior Point Optimization in SISO CRNs (PDIPS) that converges to a QNE of the proposed game is provided here. The PDIPS algorithm can run at each node in parallel, since it requires only the local information of each CR user (e.g. its own transmit power and the channel gain), and hence, it can be regarded as a distributed solution. Simulation results show that the PDIPS algorithm yields a considerable performance improvement, in terms of the sum-rate of each CR user, with respect to previous state-of-the-art algorithms, such as ADOS algorithm [16] and the deterministic game proposed in [26].

The rest of the chapter is organized as follows. Section 4.1 presents our system model. The analysis of the optimization problem is presented in Section 4.2. Section 4.3 provides a detailed analysis of the ADOS algorithm. The concept and the existence of a QNE, a as well as the detailed analysis of the PDIPS algorithm are discussed in Section 4.4. Extensive performance evaluation results are presented in Section 4.5. Section 4.6 states the conclusions. Table 4.1 presents the notation used in this chapter. Matrices and vectors are indicated in boldface, and CR i denotes CR Tx-Rx pair i.

Table 4.1: **Notation for multiuser SISO CRNs**

Symbol	Meaning
Rx	Receiver
Tx	Transmitter
$[x]^+$	$\max(0, x)$
t	Sensing time
CR i	ith CR pair
N	Number of PUs
M	Number of CR pairs
τ_k^i	Detection threshold of CR i in channel k
$P_{k,0}^i$	Transmit power of CR i for $H_{k,0}$ in channel k
$P_{k,1}^i$	Transmit power of CR i for $H_{k,1}$ in channel k
$\mathcal{P}_d$	Probability of detection
$\mathcal{P}_{fa}$	Probability of false alarm
γ_k^p	SNR of PU at PU-Rx in channel k
γ_k^i	SNR of PU at CR-Rx i in channel k
$\mathbb{R}_+^n$	Nonnegative n-dimensional space
$H_{0,k}$	Channel k detected to be idle
$H_{1,k}$	Channel k detected to be occupied
Γ_k	Maximum acceptable rate-loss gap for the PU
$I_{k,cr}^i$	Total interference observed by CR-Rx i in channel k
$I_{k,cp}$	Total interference experienced by the PU in channel k
$\lvert h_{k,cp}^i\rvert^2$	Channel gain in channel k between CR-Tx i and PU
$\lvert h_{k,cp}^i\rvert^2$	Channel gain in channel k between PU and CR-Rx i
$\lvert h_{k,cr}^{ii}\rvert^2$	Channel gain in channel k between CR-Tx i and CR-Rx i
$\lvert h_{k,cr}^{ji}\rvert^2$	Channel gain in channel k between CR-Tx j and CR-Rx i

4.1 System Model

In this section, we introduce the system model for the multiuser CRNs, the coexistence condition of PUs and CR users, the interference from CR-Tx to PUs, the multiuser interference between CR users, and the total achievable rate of each CR user.

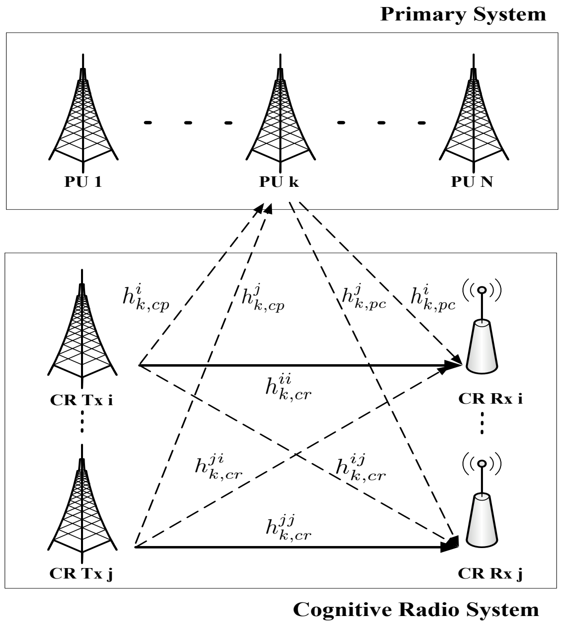

Figure 4.1: System model: N PUs and M CR Tx-Rx pairs. PU k uses channel k, $k = 1, ..., N$.

4.1.1 System model for multiuser CRNs

We consider the interweave communication, where the CR user senses the status of the channel and adapts its transmit power based on the decision made by spectrum sensing, and the CR-Tx deals with a performance tradeoff between maximizing its rate and minimizing the performance degradation caused to the PU.

Consider the multiuser SISO OFDM CRNs with M CR Tx-Rx pairs and N PUs in a certain area, each PU uses a different channel (i.e., PU k uses channel k, $k = 1, ..., N$). We focus on block transmission over SISO OFDM channels. CR users are allowed to access the N channels simultaneously, thus, multiuser interference (MUI) from different CR users in the same channel (see Figure 4.1) must be taken into account. We assume that no interference cancellation is performed and the MUI is treaded as additive colored noise at each CR-Rx.

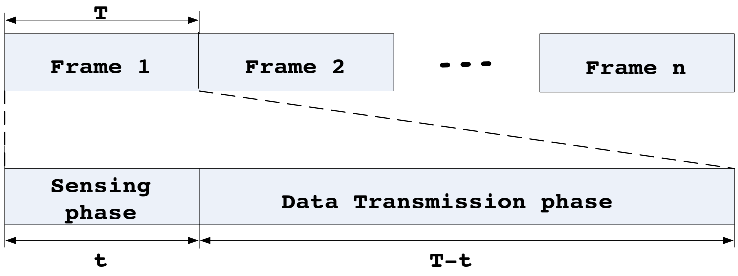

Figure 4.2: Frame structure of conventional sensing-based spectrum sharing.

4.1.2 Spectrum sensing

Before accessing the channel, each CR-Tx must first perform spectrum sensing to determine the status of each channel. We assume that simultaneous spectrum sensing of all the N channels is performed by each CR-Rx using an energy detection scheme. Specifically, for channel k, at the discrete sample l, the received signal y_k^i at the CR-Rx i, $i = 1, 2, ..., M$, is given by [90]:

$$H_{0,k}: \quad y_k^i(l) = n_k(l) \tag{4.1}$$

$$H_{1,k}: \quad y_k^i(l) = S_k^i(l) + n_k(l) \tag{4.2}$$

where $n_k(l)$ denotes additive background noise on the k-th channel, which is assumed to be independent and identically distributed additive complex Gaussian with zero mean and variance $(\sigma_{k,n}^i)^2$, and $S_k^i(l)$ stands for the PU transmit signal in channel k. Let $P_{k,pc}^i = |S_k|^2 |h_{k,pc}^i|^2$ denote the received power by CR-Rx i from the PU in channel k, and $L_s = tf_s$ denote the number of samples, where t is the sensing time and f_s represents the sampling frequency. Under an energy detection scheme, for each channel k, the statistic is computed as the sum of the received energy over an interval of L_s samples over each channel, and the decision is based on:

$$Y_k = \sum_{l=1}^{L_s} |y_k^i(l)|^2 \gtrless_{H_{0,k}}^{H_{1,k}} \tau_k^i, \quad k = 1, 2, \ldots, N. \tag{4.3}$$

Note that the longer the sensing time t, the better the energy estimation accuracy. However, for a fixed frame length, with a longer sensing time t, the transmission time has to be reduced (see Figure 4.2). In order to improve the sensing accuracy without increasing sensing time t, a distributed cooperative scheme is adopted here. We assume that the nearby CR-Rxs have the possibility to exchange their local measurements if their distance is less than a chosen coverage radius. Hence, the cooperative sensing can be implemented by the distributed consensus algorithm from [93], which requires only the interaction among nearby CR-Rxs. Let us denote by M the number of cooperative CR-Rxs. State update occurs at discrete time for each CR-Rx locally, and the final average consensus result is asymptotically reached for all nodes [93] as:

$$y^i_{k,c}(l) = \frac{1}{M}\sum_{i=1}^{M} y^i_k(l) \tag{4.4}$$

The final sensing decision at each CR-Rx is made by comparing the consensus result with a primary detection threshold τ^i_k as follows:

$$Y_{k,c} = \sum_{l=1}^{L_s} |y^i_{k,c}(l)|^2 \gtrless^{H_{1,k}}_{H_{0,k}} \tau^i_k, \quad k = 1, 2, \ldots, N. \tag{4.5}$$

According to the Central Limit Theorem, for large L_s, $y^i_{k,c}(l)$ are approximately normally distributed: $Y_{k,c} \sim \mathcal{N}(\mu^i_{k,0}, (\sigma^i_{k,0})^2)$ for $H_{0,k}$, and $Y_{k,c} \sim \mathcal{N}(\mu^i_{k,1}, (\sigma^i_{k,1})^2)$ for $H_{1,k}$, where:

$$\mathcal{N}(\mu^i_{k,0}, (\sigma^i_{k,0})^2)\begin{cases} \mu^i_{k,0} = \frac{L_s}{M}\sum_{i=1}^{M}(\sigma^i_{k,n})^2 \\ (\sigma^i_{k,0})^2 = \frac{L_s}{M^2}\sum_{i=1}^{M}(\sigma^i_{k,n})^4 \end{cases} \tag{4.6}$$

$$\mathcal{N}(\mu^i_{k,1}, (\sigma^i_{k,1})^2)\begin{cases} \mu^i_{k,1} = \frac{L_s}{M}\sum_{i=1}^{M}((\sigma^i_{k,n})^2 + P^i_{k,pc}) \\ (\sigma^i_{k,1})^2 = \frac{L_s}{M^2}\sum_{i=1}^{M}((\sigma^i_{k,n})^2 + P^i_{k,pc})^2 \end{cases} \tag{4.7}$$

The probabilities of detection $\mathcal{P}^i_{k,d}$ and false alarm $\mathcal{P}^i_{k,fa}$ for the kth channel for CR-Rx i, $i = 1, 2, ..., M$, are given by:

$$\mathcal{P}^i_{k,fa}(\tau^i_k, t) = \mathcal{Q}\left(\frac{\tau^i_k - \mu^i_{k,0}}{\sigma^i_{k,0}}\right) \tag{4.8}$$

Table 4.2: **Four instantaneous rates at CR-Rx i**

Actual status	Detection result	Actual rate at CR-Rx i
$H_{0,k}$	$H_{0,k}$	$r^i_{k,00} = \log_2\left(1 + \frac{P^i_{k,0}\lvert h^{ii}_{k,cr}\rvert^2}{I^i_{k,0}}\right)$
$H_{0,k}$	$H_{1,k}$	$r^i_{k,01} = \log_2\left(1 + \frac{P^i_{k,1}\lvert h^{ii}_{k,cr}\rvert^2}{I^i_{k,1}}\right)$
$H_{1,k}$	$H_{0,k}$	$r^i_{k,10} = \log_2\left(1 + \frac{P^i_{k,0}\lvert h^{ii}_{k,cr}\rvert^2}{I^i_{k,0}+P^i_{k,pc}}\right)$
$H_{1,k}$	$H_{1,k}$	$r^i_{k,11} = \log_2\left(1 + \frac{P^i_{k,1}\lvert h^{ii}_{k,cr}\rvert^2}{I^i_{k,1}+P^i_{k,pc}}\right)$

$$\mathcal{P}^i_{k,d}(\tau^i_k, t) = \mathcal{Q}\left(\frac{\tau^i_k - \mu^i_{k,1}}{\sigma^i_{k,1}}\right) \tag{4.9}$$

In our chapter, we consider a sensing-based spectrum sharing (SSS) scheme, where CR user can coexist with the PU, and CR-Txs transmit simultaneously on the N channels and adapt their transmit power on each channel based on the sensing information. If channel k is detected to be idle ($H_{0,k}$), CR-Tx i transmits using power $P^i_{k,0}$, whereas if channel k is sensed to be active ($H_{1,k}$), then each CR-Tx i transmits using a relatively lower power $P^i_{k,1}$, in order to reduce the interference caused to the PU. This scheme can be seen as a hybrid approach between protecting the PU and improving the spectrum utilization.

4.2 Problem Formulation

Since spectral efficiency is the main overall goal of the CR users, the objective function chosen by each user to be maximized is the sum-rate (average) over all the channels. In this section, we analyze the problem of optimizing the power allocation for the CR users in order to maximize their own sum-rate, taking into account the detection result.

4.2.1 Total achievable rate of the CR users

Considering the fact that the spectrum sensing information is not always reliable, which implies having probabilities of detection $\mathcal{P}^i_{k,d} < 1$

and probabilities of false alarm $\mathcal{P}^i_{k,fa} > 0$, we have four different instantaneous rates at CR-Rx i in channel k, as shown in Table 4.2. In this table, the first subindex number of r^i_k (the third column of Table 4.2) describes the actual status of the PU ("0" for idle and "1" for active), and the second subindex number indicates the sensing result obtained by energy detection. $I^i_{k,0}$ and $I^i_{k,1}$, presenting the noise and the interference observed by CR-Rx i from other CR-Txs in channel k, under sensing results $H_{0,k}$ and $H_{1,k}$ are given by, respectively:

$$I^i_{k,0} = (\sigma^i_{k,n})^2 + \sum_{j=1,j\neq i}^{M} P^i_{k,0}|h^{ji}_{k,cr}|^2 \tag{4.10}$$

$$I^i_{k,1} = (\sigma^i_{k,n})^2 + \sum_{j=1,j\neq i}^{M} P^i_{k,1}|h^{ji}_{k,cr}|^2 \tag{4.11}$$

Note that the instantaneous rates $r^i_{k,00}$ and $r^i_{k,11}$ are based on the correct sensing information,whereas the instantaneous rates $r^i_{k,01}$ and $r^i_{k,10}$ are due to the incorrect sensing information. In some literature [74–80], the instantaneous rates due to the incorrect sensing information are ignored from the sum-rate. In our system model, we consider the complete model, thus all the four possible rates are take in to consideration.
Let $\mathcal{P}(H_{0,k})$ denote the prior probability that the k-th channel is idle, and $\mathcal{P}(H_{1,k})$ denote the prior probability that the k-th channel is active. The total achievable rate at CR-Rx i based on a given sensing time t, denoted as $f^i(\mathbf{P}^i_1, \mathbf{P}^i_0, \boldsymbol{\tau}^i)$, $\mathbf{P}^i_1 = [P^i_{k,1}]^N_{k=1}$, $\mathbf{P}^i_0 = [P^i_{k,0}]^N_{k=1}$, $\boldsymbol{\tau}^i = [\tau^i_k]^N_{k=1}$, can be formulated as follows:

$$\begin{aligned} f^i(\mathbf{P}^i_1, \mathbf{P}^i_0, \boldsymbol{\tau}^i) = \sum_{k=1}^{N} \Bigg(&\mathcal{P}(H_{0,k})\Big((1 - \mathcal{P}^i_{k,fa}(\tau^i_k))r^i_{k,00} + \mathcal{P}^i_{k,fa}(\tau^i_k)r^i_{k,01}\Big) \\ &+ \mathcal{P}(H_{1,k})\Big((1 - \mathcal{P}^i_{k,d}(\tau^i_k))r^i_{k,10} + \mathcal{P}^i_{k,d}(\tau^i_k)r^i_{k,11}\Big)\Bigg) \end{aligned} \tag{4.12}$$

Due to the inherent trade-off between $\mathcal{P}^i_{k,d}$ and $\mathcal{P}^i_{k,fa}$, maximizing the total achievable rate of CR i will result in low $\mathcal{P}^i_{k,fa}$ and a high $\mathcal{P}^i_{k,d}$.

4.2.2 Constraints for the CR users

The most important constraint of the CRNs involves protecting the PU from harmful performance degradation. In a multiuser scenario, this constraint can be imposed on an "individual" or "global level". The individual constraint requires the transmit power of each CR user in channel k to be always less than a given threshold. Instead of specifying individual constraints on the transmit power of each CR user per channel, the global constraint adapts the transmit power of each CR-Tx depending on the actions from other CR users that share the same channel, so that the accumulated interference from all the CR users at a PU does not exceed a given threshold. Though the global constraint may result in higher network rate (with price mechanism), it requires a large information exchange and coordination among CR users [32, 33]. In our scenario, we assume that the CR users are not willing to exchange information in the transmission stage. Therefore, we use an individual constraint, namely, rate-loss constraint, to design the power allocation, ensuring that the performance degradation experienced by each PU is bounded. This individual constraint leads to a distributed scenario. Note that the only local information exchange among nearby CR-Rxs is needed in the cooperative sensing stage. On the one hand, the maximum achievable rate of the PU in channel k without the interference from CR-Tx i is denoted as:

$$R^i_{k,\max} = \mathcal{P}(H_{1,k}) \log_2 \left(1 + \frac{|S_k|^2}{(\sigma^i_{k,n})^2} \right) \tag{4.13}$$

On the other hand, the maximum achievable rate of the PU in channel k with the interference from CR-Tx i is denoted as:

$$\begin{aligned} R^i_k =& \mathcal{P}(H_{1,k}) \mathcal{P}^i_{k,d}(\tau^i_k) \log_2 \left(1 + \frac{|S_k|^2}{I^{i,p}_{k,1}} \right) \\ &+ \mathcal{P}(H_{1,k})(1 - \mathcal{P}^i_{k,d}(\tau^i_k)) \log_2 \left(1 + \frac{|S_k|^2}{I^{i,p}_{k,0}} \right) \end{aligned} \tag{4.14}$$

where $I^{i,p}_{k,0}$ and $I^{i,p}_{k,0}$ are the interference contributions from CR-Tx i to the PU in channel k under sensing results $H_{0,k}$ and $H_{1,k}$, respectively:

$$I^{i,p}_{k,0} = (\sigma^i_{k,n})^2 + P^i_{k,0} |h^i_{k,cp}|^2 \tag{4.15}$$

$$I^{i,p}_{k,1} = (\sigma^i_{k,n})^2 + P^i_{k,1} |h^i_{k,cp}|^2 \tag{4.16}$$

Let Γ_k denote the maximum acceptable rate-loss gap of the PU in channel k, $k = 1, ..., N$, then, the rate-loss constraint for CR-Tx i can be written as follows:

$$R^i_{k,\max} - R^i_k \leq \Gamma_k R^i_{k,\max} \tag{4.17}$$

In order to simplify the development of (4.17), we use $x \log_2(e)$ instead of $\log_2(1 + x)$, which amounts to rewrite this constraint as:

$$\begin{aligned}(1 - \Gamma_k)\mathcal{P}(H_{1,k})\frac{|S_k|^2 \log_2 e}{(\sigma^i_{k,n})^2} - \mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k)\frac{|S_k|^2 \log_2 e}{I^{i,p}_{k,1}} \\ - \mathcal{P}(H_{1,k})(1 - \mathcal{P}^i_{k,d}(\tau^i_k))\frac{|S_k|^2 \log_2 e}{I^{i,p}_{k,0}} \leq 0\end{aligned} \tag{4.18}$$

Given this, the new rate-loss constraint results in:

$$\Gamma^i_{k,c} I^{i,p}_{k,1} I^{i,p}_{k,0} - \mathcal{P}^i_{k,d}(\tau^i_k) I^{i,p}_{k,0} - (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) I^{i,p}_{k,1} \leq 0 \tag{4.19}$$

where $\Gamma^i_{k,c} = (1 - \Gamma_k)/(\sigma^i_{k,n})^2$. In fact, since $x \log_2 e \geq \log_2(1 + x)$, the actual rate-loss gap resulting from the constraint (4.19) is not the same as in the original constraint (4.17). The modified constraint (4.19) is more restrictive than (4.17), as will be shown in simulation results. However, the resulting solution is valid and satisfactory, which is equal to the results from the original constraint (4.17) with a smaller rate-loss gap. The further detailed discussion is given in Appendix C. Furthermore, the total transmit power of each CR-Tx i over all channels should not exceed its maximum allowed power. The power budget constraint for each CR-Tx i can be formulated as:

$$\begin{aligned}\sum_{k=1}^{N} \Bigg(& \mathcal{P}(H_{0,k})((1 - \mathcal{P}^i_{k,fa}(\tau^i_k))P^i_{k,0} + \mathcal{P}^i_{k,fa}(\tau^i_k)P^i_{k,1}) \\ & + \mathcal{P}(H_{1,k})(\mathcal{P}^i_{k,d}(\tau^i_k)P^i_{k,1} + (1 - \mathcal{P}^i_{k,d}(\tau^i_k))P^i_{k,0}) \Bigg) \leq P^i_{\max}\end{aligned} \tag{4.20}$$

where $P^i_{\max}$ denotes the maximum total transmit power of the CR-Tx i over all the N channels. Finally, we restrict the target detection probability and false alarm to the ranges $\mathcal{P}^i_{k,d} \geq \frac{1}{2}$ and $\mathcal{P}^i_{k,fa} \leq \frac{1}{2}$, respectively. According to the monotonicity of the $\mathcal{Q}$ function, and taking into account

(5.7), constraints in $\mathcal{P}^i_{k,d}$ and $\mathcal{P}^i_{k,fa}$ are equivalent to the inequalities:

$$\tau^i_{k,min} \leq \tau^i_k \leq \tau^i_{k,max} \tag{4.21}$$

where $\tau^i_{k,min} = \mu^i_{k,0}$, $\tau^i_{k,max} = \mu^i_{k,1}$. Note that the ranges of target detection probability and false alarm do not represent real loss of generality, because practical CRNs are required to satisfy even stronger constraints from the standard [94].

4.2.3 Optimization problem

Under the above assumption, the optimization problem for maximizing the sum-rate of CR i can be formulated as the following problem $P4.1$, where $\mathbf{P}^i_0 = [\mathbf{P}^i_{k,0}]^N_{k=1}, \mathbf{P}^i_1 = [\mathbf{P}^i_{k,1}]^N_{k=1}, \boldsymbol{\tau}^i = [\tau^i_k]^N_{k=1}$:

$$\begin{aligned}
\max_{\mathbf{P}^i_1,\mathbf{P}^i_0,\boldsymbol{\tau}^i} \;& f^i(\mathbf{P}^i_1, \mathbf{P}^i_0, \boldsymbol{\tau}^i) \\
\text{s. t. } & (a1)\; \Gamma^i_{k,c} I^{i,p}_{k,1} I^{i,p}_{k,0} - \mathcal{P}^i_{k,d}(\tau^i_k) I^{i,p}_{k,0} - (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) I^{i,p}_{k,1} \leq 0, \\
& (a2)\; \sum_{k=1}^{N} \Big(\mathcal{P}(H_{0,k})((1 - \mathcal{P}^i_{k,fa}(\tau^i_k)) P^i_{k,0} + \mathcal{P}^i_{k,fa}(\tau^i_k) P^i_{k,1}) \\
& \qquad + \mathcal{P}(H_{1,k})(\mathcal{P}^i_{k,d}(\tau^i_k) P^i_{k,1} + (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) P^i_{k,0}) \Big) \leq P^i_{\max}, \\
& (b1)\; \tau^i_{k,min} \leq \tau^i_k \leq \tau^i_{k,max}, \\
& (b2)\; 0 \leq P^i_{k,1}, 0 \leq P^i_{k,0}, \;\; 1 \leq i \leq M, \;\; 1 \leq k \leq N.
\end{aligned} \tag{4.22}$$

where $(a1), (a2)$ are the non-convex constraint sets, and $(b1), (b2)$ are the convex constraint sets. Each CR i aims at maximizing its own rate under the power budget constraint and the rate-loss constraint. Both the power budget constraint and the rate-loss constraint are individual constraints, meaning that the CR users are allowed to choose their power allocation individually. Therefore, there is no information exchange between CR users.

For the multiuser scenario, all the CR users are selfish and strive to maximize their own sum-rate under several constraints. The presence of concurrent CR users competing over the same resources adds dynamics to the system, as every CR user will dynamically react to the strategies adopted by the other CR users sharing the same resources.

The main question is then to establish under what conditions the overall system can eventually converge to an equilibrium from which every CR user is not willing to unilaterally move. This form of equilibrium coincides with the well-known concept of NE in game theory. Hence, game theory is addressed here for our distributed scenario, which allows the CR users to find out their best response to any given channel and interference scenario and to derive the conditions for the existence and uniqueness of NE.

4.3 Local Nash Equilibrium and Alternating Optimization

The resulting optimization problem $P4.1$ is non-convex; even if the constraints $(b1), (b2)$ are linear, the objective function and the constraints $(a1), (a2)$ are non-convex due to the presence of the $\mathcal{Q}$ function in the false alarm and detection probability. In order to simplify the game, the sensing time t is considered as a constant.

4.3.1 Alternating direction optimization in SISO CRNs

The optimization problem $P4.1$ for CR i is convex with respect to the transmit powers $\mathbf{P}_1^i$ and $\mathbf{P}_0^i$, but not with respect to the detection threshold $\boldsymbol{\tau}^i$, thus the optimal solution can not be obtained using conventional convex optimization techniques. The global optimal solution to problem $P4.1$ can be found by computing the optimal $\mathbf{P}_0^{i,\star}$, $\mathbf{P}_1^{i,\star}$ for all $(N\frac{\boldsymbol{\tau}^i_{max}-\boldsymbol{\tau}^i_{min}}{\delta})^M$ possible choices for $\boldsymbol{\tau}^i$, and find the $(\mathbf{P}_0^{i,\star}, \mathbf{P}_1^{i,\star}, \boldsymbol{\tau}^{i,\star})$ that yield the maximum rate $f^i(\mathbf{P}_0^{i,\star}, \mathbf{P}_1^{i,\star}, \boldsymbol{\tau}^{i,\star})$ for CR i, where θ is the step size. However, the complexity of finding the global optimal solution based on the exhaustive search approach is prohibitively high. As a consequence, a suboptimal algorithm is proposed in this section to find the local NE of the non-convex game $P4.1$.

In this section, we propose an ADO algorithm for SISO CRNs (ADOS) to solve the non-convex optimization problem $P4.1$. However, for non-convex problems, due to the non-convexity of the problem $P4.1$, the ADOS algorithm may not converge to the global optimal points, it must be considered just another local optimization method.

We divide the original problem $P4.1$ into two stages, referred as local threshold optimization and optimal power allocation, respectively. In the local power allocation optimization step, we find the optimal transmit powers $\mathbf{P}_1^i$ and $\mathbf{P}_0^i$ for any given fixed threshold $\tilde{\boldsymbol{\tau}}^i$. Next, in the detection threshold optimization step, substituting the power $\mathbf{P}_1^{i,\star}, \mathbf{P}_0^{i,\star}$ obtained from the previous optimal power allocation stage, we find the optimal detection threshold $\boldsymbol{\tau}^{i,\star}$.

4.3.2 Local power allocation optimization

For every fixed action from all the CR users except CR i, denoted as CR $-i$, the optimization problem is convex with respect to the transmit powers $\mathbf{P}_1^i$ and $\mathbf{P}_0^i$ for any given fixed threshold $\tilde{\boldsymbol{\tau}}^i$. In order to initialize the ADOS algorithm, we first maximize the sum-rate of CR i based on an initial detection threshold $\boldsymbol{\tau}^i(0)$, corresponding to a target probability false alarm $\mathcal{P}_{k,fa}^i(\tau_k^i)$. In the following, we focus on finding the optimal power allocation for any given detection threshold $\tilde{\boldsymbol{\tau}}^i$. The optimization problem can be formulated as $P4.2$:

$$\begin{aligned}
\max_{\mathbf{P}_1^i,\mathbf{P}_0^i} \quad & \sum_{k=1}^{N}\left(a_{k,0}^i(\tilde{\tau}_k^i)r_{k,00}^i + a_{k,1}^i(\tilde{\tau}_k^i)r_{k,01}^i + b_{k,0}^i(\tilde{\tau}_k^i)r_{k,10}^i + b_{k,1}^i(\tilde{\tau}_k^i)r_{k,11}^i\right)\\
\text{s.t.} \quad & \Gamma_{k,c}^i I_{k,1}^{i,p} I_{k,0}^{i,p} - \mathcal{P}_{k,d}^i(\tau_k^i) I_{k,0}^{i,p} - (1-\mathcal{P}_{k,d}^i(\tau_k^i)) I_{k,1}^{i,p} \le 0, \qquad (4.23)\\
& \sum_{k=1}^{N}\left((a_{k,0}^i(\tilde{\tau}_k^i) + b_{k,0}^i(\tilde{\tau}_k^i))P_{k,0}^i + (a_{k,1}^i(\tilde{\tau}_k^i) + b_{k,1}^i(\tilde{\tau}_k^i))P_{k,1}^i\right) \le P_{\max}^i. \qquad (4.24)
\end{aligned}$$

where:

$$a_{k,0}^i(\tau_k^i) = \mathcal{P}(H_{0,k})(1-\mathcal{P}_{k,fa}^i(\tau_k^i)), \tag{4.25}$$

$$a_{k,1}^i(\tau_k^i) = \mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i), \tag{4.26}$$

$$b_{k,0}^i(\tau_k^i) = \mathcal{P}(H_{1,k})(1-\mathcal{P}_{k,d}^i(\tau_k^i)), \tag{4.27}$$

$$b_{k,1}^i(\tau_k^i) = \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i). \tag{4.28}$$

$P4.2$ is a convex problem with respect to the transmit power $\mathbf{P}_1^i, \mathbf{P}_0^i$. Therefore, the optimal duality gap is zero, and we can establish the KKT conditions and solve the problem efficiently. The Lagrangian with respect to the transmit powers $\mathbf{P}_1^i$ and $\mathbf{P}_0^i$ is defined as $L(\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\alpha}^i, \beta^i)$, and given by:

$$\sum_{k=1}^{N}\Bigg(a^i_{k,0}(\tilde{\tau}^i_k)r^i_{k,00}+a^i_{k,1}(\tilde{\tau}^i_k)r^i_{k,01}+b^i_{k,0}(\tilde{\tau}^i_k)r^i_{k,10}+b^i_{k,1}(\tilde{\tau}^i_k)r^i_{k,11}\Bigg)$$
$$-\sum_{k=1}^{N}\alpha^i_k\Bigg(\Gamma^i_{k,c}I^{i,p}_{k,1}I^{i,p}_{k,0}-\mathcal{P}^i_{k,d}(\tau^i_k)I^{i,p}_{k,0}-(1-\mathcal{P}^i_{k,d}(\tau^i_k))I^{i,p}_{k,1}\Bigg)$$
$$-\beta^i\Bigg(\sum_{k=1}^{N}((a^i_{k,0}(\tilde{\tau}^i_k)+b^i_{k,0}(\tilde{\tau}^i_k))P^i_{k,0}+(a^i_{k,1}(\tilde{\tau}^i_k)+b^i_{k,1}(\tilde{\tau}^i_k))P^i_{k,1})-P^i_{\max}\Bigg) \tag{4.29}$$

where $\boldsymbol{\alpha}^i=[\alpha^i_k]^N_{k=1}$ and β^i are nonnegative multipliers corresponding to the dual variables associated with the power budget constraint and rate-loss constraint respectively. The Lagrange dual optimization problem becomes:

$$\max_{\mathbf{P}^i_1,\mathbf{P}^i_0} L(\mathbf{P}^i_1,\mathbf{P}^i_0,\boldsymbol{\alpha}^i,\beta^i) \tag{4.30}$$

In order to find the solution for the dual problem (4.30), we decompose the joint optimization problem into two optimization subproblems, as follows:

SP1:

$$\max_{\mathbf{P}^i_0}\quad \sum_{k=1}^{N}(a^i_{k,0}(\tilde{\tau}^i_k)r^i_{k,00}+b^i_{k,0}(\tilde{\tau}^i_k)r^i_{k,10})-\beta^i\sum_{k=1}^{N}(a^i_{k,0}(\tilde{\tau}^i_k)+b^i_{k,0}(\tilde{\tau}^i_k))P^i_{k,0}$$
$$-\sum_{k=1}^{N}\alpha^i_k\mathcal{P}(H_{1,k})(1-\mathcal{P}^i_{k,d}(\tilde{\tau}^i_k))\frac{|S_k|^2}{I^{i,p}_{k,0}\ln 2} \tag{4.31}$$

SP2:

$$\max_{\mathbf{P}^i_1}\quad \sum_{k=1}^{N}(a^i_{k,1}(\tilde{\tau}^i_k)r^i_{k,01}+b^i_{k,1}(\tilde{\tau}^i_k)r^i_{k,11})-\beta^i\sum_{k=1}^{N}(a^i_{k,1}(\tilde{\tau}^i_k)+b^i_{k,1}(\tilde{\tau}^i_k))P^i_{k,1}$$
$$-\sum_{k=1}^{N}\alpha^i_k\mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tilde{\tau}^i_k))\frac{|S_k|^2}{I^{i,p}_{k,1}\ln 2} \tag{4.32}$$

The optimal powers $\mathbf{P}_0^i$ and $\mathbf{P}_1^i$ for the given $\boldsymbol{\alpha}^i$, β^i are presented as follows:

$$P_{k,0}^{i,\star} = \left[\frac{1}{2}(A_{k,0}^i + \sqrt{B_{k,0}^i}) - I_{k,0}^i\right]^+ \tag{4.33}$$

$$P_{k,1}^{i,\star} = \left[\frac{1}{2}(A_{k,1}^i + \sqrt{B_{k,1}^i}) - I_{k,1}^i\right]^+ \tag{4.34}$$

where:

$$A_{k,0}^i = A_{k,1}^i = \frac{|h_{k,cr}^{ii}|^2}{\beta^i} \tag{4.35}$$

$$B_{k,0}^i = \left(\frac{|h_{k,cr}^{ii}|^2}{\beta^i}\right)^2 - \frac{4\alpha_k^i b_{k,0}^i(\tilde{\tau}_k^i)|S_k|^2}{\beta^i(a_{k,0}^i(\tilde{\tau}_k^i) + b_{k,0}^i(\tilde{\tau}_k^i))\ln 2} \tag{4.36}$$

$$B_{k,1}^i = \left(\frac{|h_{k,cr}^{ii}|^2}{\beta^i}\right)^2 - \frac{4\alpha_k^i b_{k,1}^i(\tilde{\tau}_k^i)|S_k|^2}{\beta^i(a_{k,1}^i(\tilde{\tau}_k^i) + b_{k,1}^i(\tilde{\tau}_k^i))\ln 2} \tag{4.37}$$

The ellipsoid method can be used here to find the optimal solutions of $\boldsymbol{\alpha}^i$, β^i and $\mathbf{P}_0^i$, $\mathbf{P}_1^i$ [95], which requires the subgradient form of the dual function.

Proposition 2. *The subgradient of the dual function $L(\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\alpha}^i, \beta^i)$ is given by $[Z_0^i, \mathbf{Z}_1^i]$, where Z_0^i and $\mathbf{Z}_1^i = [Z_{k,1}^i]_{k=1}^N$ are given by:*

$$Z_0^i = P_{\max}^i - \sum_{k=1}^N (a_{k,0}^i(\tau_k^i)P_{k,0}^{i,\star} + a_{k,1}^i(\tau_k^i)P_{k,1}^{i,\star} + b_{k,0}^i(\tau_k^i)P_{k,0}^{i,\star} + b_{k,1}^i(\tau_k^i)P_{k,1}^{i,\star})$$

$$Z_{k,1}^i = \Gamma_{k,c}^i I_{k,1}^{i,p} I_{k,0}^{i,p} - \mathcal{P}_{k,d}^i(\tau_k^i) I_{k,0}^{i,p} - (1 - \mathcal{P}_{k,d}^i(\tau_k^i)) I_{k,1}^{i,p}$$

$\mathbf{P}_0^{i,\star}$ and $\mathbf{P}_1^{i,\star}$ are the optimal power allocation for any fixed $\boldsymbol{\alpha}^i$, β^i.

4.3.3 Local threshold optimization

Substituting the optimal powers $\mathbf{P}_1^{i,\star}$ and $\mathbf{P}_0^{i,\star}$ obtained from the previous optimal power allocation stage, the terms $r_{k,00}^{i,\star}, r_{k,01}^{i,\star}, r_{k,10}^{i,\star}$ and $r_{k,11}^{i,\star}$ become constants. In this step, we optimize the detection threshold $\boldsymbol{\tau}^i$ based on the obtained powers, and the problem $P4.1$ can be rewritten as the problem $P4.3$:

$$
\begin{aligned}
\max_{\boldsymbol{\tau}^i} \quad & \sum_{k=1}^{N} \Big(\mathcal{P}(H_{0,k}) \mathcal{P}^i_{k,fa}(\tau^i_k)(r^{i,\star}_{k,01} - r^{i,\star}_{k,00}) + \mathcal{P}(H_{1,k}) \mathcal{P}^i_{k,d}(\tau^i_k)(r^{i,\star}_{k,11} - r^{i,\star}_{k,10}) \\
& + \mathcal{P}(H_{0,k}) r^{i,\star}_{k,00} + \mathcal{P}(H_{1,k}) r^{i,\star}_{k,10} \Big) \\
\text{s.t.} \quad & \Gamma^i_{k,c} I^{i,p}_{k,1} I^{i,p}_{k,0} - \mathcal{P}^i_{k,d}(\tau^i_k) I^{i,p}_{k,0} + (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) I^{i,p}_{k,1} \leq 0 \\
& \sum_{k=1}^{N} \Big((a^i_{k,0}(\tau^i_k) + b^i_{k,0}(\tau^i_k)) P^{i,\star}_{k,0} + (a^i_{k,1}(\tau^i_k) + b^i_{k,1}(\tau^i_k)) P^{i,\star}_{k,1} \Big) \leq P^i_{\max} \\
& \tau^i_{k,min} \leq \tau^i_k \leq \tau^i_{k,max}
\end{aligned} \tag{4.38}
$$

The objective and the constraint function in $P4.3$ are generally non-convex, however, we can solve this seemingly non-convex problem by exploiting the convexity properties. We have the following proposition.

Proposition 3. *The optimal solution of $P4.3$ is achieved when the detection threshold $\boldsymbol{\tau}^i$ is equal to the upper bound.*

Proof. From (5.7) and (5.8), we can observe that $\mathcal{P}^i_d(\boldsymbol{\tau}^i)$ and $\mathcal{P}^i_{fa}(\boldsymbol{\tau}^i)$ are decreasing functions of detection threshold $\boldsymbol{\tau}^i$. Furthermore, the main priority of the CRNs are the protection of the PU. In order to reduce the interference caused to the PU, CR users attempt to allocate less power when the channel is considered to be busy. When the target probability of detection and the target probability of false alarm are satisfied, the optimal transmit power $\mathbf{P}^{i,\star}_1$ is relatively lower than the optimal transmit power $\mathbf{P}^{i,\star}_0$. Thus, from the formulations of $r^i_{k,00}, r^i_{k,01}, r^i_{k,10}, r^i_{k,11}$ we can have $r^i_{k,01} < r^i_{k,00}$ and $r^i_{k,11} < r^i_{k,10}$. Consequently, the objective function in $P4.3$ is an increasing function of $\boldsymbol{\tau}^i$, and $P4.3$ achieves its maximal rate when the detection threshold $\boldsymbol{\tau}^i$ reach its upper bound. □

4.3.4 Local Nash equilibrium

Combining the power allocation optimization stage discussed in Section 4.3.2 and the detection threshold optimization stage discussed in Section 4.3.3, the proposed ADOS algorithm is summarized in **Algorithm** 3, where v_0, v_1 and v_2 are the iteration values, θ is the step size, and ε is the tolerance which is small enough, in our algorithm $\varepsilon = 10^{-6}$. Since the objective function is nondecreasing at each iteration, we have:

$$\begin{aligned} &f^i(\mathbf{P}_0^i(v_0), \mathbf{P}_1^i(v_0), \boldsymbol{\tau}^i(v_0)) \leq f^i(\mathbf{P}_0^i(v_0+1), \mathbf{P}_1^i(v_0+1), \boldsymbol{\tau}^i(v_0)) \\ &\leq f^i(\mathbf{P}_0^i(v_0+1), \mathbf{P}_1^i(v_0+1), \boldsymbol{\tau}^i(v_0+1)) \end{aligned} \tag{4.39}$$

and considering the fact that $\mathbf{P}_0^i, \mathbf{P}_1^i$, and $\boldsymbol{\tau}^i$ are upper bounded, convergence follows.
It can be concluded that there exists a NE of the proposed game $P4.1$ if there exists a NE of $P4.2$. Note that $P4.2$ always admits a NE for any value of $\boldsymbol{\tau}^i$, since it is convex game, and hence, the NE of the non-convex game is achieved. Furthermore, since ADOS may not converge to a globe optimal point, thus we call this NE as the local NE (LNE) of the game.

4.3.5 Complexity analysis of the ADOS algorithm

The complexity of the ADOS algorithm is dominated by the procedure of finding the multipliers $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in the optimal power allocation step, the iteration in the local optimization step, and the size of the CRNs. For our problem, the time complexity to find the multipliers $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is associated with the step size θ of the ellipsoid method and the number of the constraint. The ellipsoid method have polynomial complexity [20], which is given by $O(\frac{N}{\theta^2})$. In addition, the optimization problem $P4.3$ is a linear programming problem, which requires $O(N)$ operations in each iteration. Hence, for the CRNs with M CR users and a accuracy ε, the total complexity of the ADOS algorithm is $O\left(\ln \frac{1}{\varepsilon} M(\frac{N}{\theta^2} + N)\right)$.

Algorithm 3 ADO in SISO CRNs (ADOS)

Initialize $v_0 := 0$, $\boldsymbol{\tau}^i(v_0)$, $\mathbf{P}_1^i(v_0)$, $\mathbf{P}_0^i(v_0)$, $k = 1, 2, \ldots, N, i = 1, 2, \ldots, M$
for i=1:M **do**
  repeat
    Initialize $\beta^i(v_1)$, $v_1 := 0$
    repeat
      Initialize $\alpha_k^i(v_2)$, $v_2 := 0$
      repeat
        Given $\boldsymbol{\tau}^i(v_0)$, find $\mathbf{P}_1^{i,\star}(v_0)$, $\mathbf{P}_0^{i,\star}(v_0)$ solve (4.33)-(4.37);
        Update $\alpha_k^i(v_2+1) = \alpha_k^i(v_2) + \theta\Big(\Gamma_{k,c}^i I_{k,1}^{i,p} I_{k,0}^{i,p} - \mathcal{P}_{k,d}^i(\tau_k^i) I_{k,0}^{i,p} + (1 - \mathcal{P}_{k,d}^i(\tau_k^i)) I_{k,1}^{i,p}\Big)$;
      until If $\alpha_k^i(v_2 + 1) < 0$, set $\alpha_k^i(v_2 + 1) = 0$, Stop;
        Or, when $|\alpha_k^i(v_2 + 1) - \alpha_k^i(v_2)| \le \varepsilon$, Stop.
      Update $\beta^i(v_1 + 1) = \beta^i(v_1) + \theta\Big(\sum_{k=1}^{N} (a_{k,0}^i(\tau_k^i(v_0)) P_{k,0}^{i,\star} + a_{k,1}^i(\tau_k^i(v_0)) P_{k,1}^{i,\star} + b_{k,0}^i(\tau_k^i(v_0)) P_{k,0}^{i,\star} + b_{k,1}^i(\tau_k^i(v_0)) P_{k,1}^{i,\star}) - P_{\max}^i\Big)$;
    until $\beta^i(v_1 + 1) < 0$, set $\beta^i(v_1 + 1) = 0$, Stop;
      Or, when $|\beta^i(v_1 + 1) - \beta^i(v_1)| \le \varepsilon$, Stop.
    Update $\mathbf{P}_1^i(v_0 + 1) = \mathbf{P}_1^{i,\star}$, $\mathbf{P}_0^i(v + 1) = \mathbf{P}_0^{i,\star}$;
    Given $\mathbf{P}_1^i(v_0 + 1)$, $\mathbf{P}_0^i(v_0 + 1)$, find the optimal value $\boldsymbol{\tau}^{i,\star}$ that solve $P4.3$;
    Update $\boldsymbol{\tau}^i(v_0 + 1) := \boldsymbol{\tau}^{i,\star}$;
  until $|\mathbf{P}_1^i(v_0) - \mathbf{P}_1^i(v_0 - 1)| \le \varepsilon$, $|\mathbf{P}_0^i(v_0) - \mathbf{P}_0^i(v_0 - 1)| \le \varepsilon$ and $|\boldsymbol{\tau}^i(v_0) - \boldsymbol{\tau}^i(v_0 - 1)| \le \varepsilon$, Stop.
end for

4.4 QNE for Non-Convex Game in SISO CRNs

The fundamental concept for the noncooperative game is the NE, which exists when the objective function and the player's constraint are convex. However, the proposed noncooperative game is non-convex, an NE may not exist without the convexity of the problem. In Section 4.3, we found the local solution for the proposed non-convex game by ADOS algorithm. In order to find the globe solution for the proposed game, in this section we use a relaxed equilibrium concept proposed from [21], namely, the QNE. The QNE is by definition a tuple that satisfies the KKT conditions of all the players' optimization problems; the prefix "quasi" is intended to signify that a NE (if it exists) must be a QNE under certain constraint qualifications, as explained in [21]. In this section, we provide sufficient conditions to ensure the existence of a QNE for the non-convex game $P4.1$ by VI method. Finally, we propose a primal-dual interior point optimization for SISO CRNs (PDIPS), which converges to a QNE.

Table 4.3: **Notation of quasi-Nash equilibrium**

Symbol	Meaning
$g_k^i(\mathbf{x}^i)$	Non-convex individual constraint $(a1)$ of $P4.1$
$h^i(\mathbf{x}^i)$	Non-convex individual constraint $(a2)$ of $P4.1$
$\mathbf{J}_{g_k^i(\mathbf{x}^i)}$	Jacobian matrix of the vector function $g_k^i(\mathbf{x}^i)$
$\mathbf{J}_{h^i(\mathbf{x}^i)}$	Jacobian matrix of the vector function $h^i(\mathbf{x}^i)$
$\nabla^2_{\mathbf{x}^i} g_k^i(\mathbf{x}^i)$	Hessian matrix of the vector function $g_k^i(\mathbf{x}^i)$
$\nabla^2_{\mathbf{x}^i} h^i(\mathbf{x}^i)$	Hessian matrix of the vector function $h^i(\mathbf{x}^i)$
$T(\mathcal{X}^i; \mathbf{x}^i)$	Tangent cone of the set $\mathcal{X}^i$ at $\mathbf{x}^i \in \mathcal{X}^i$
$\mathcal{X}^i$	Convex individual constraints $(b1), (b2)$ of $P4.1$
$\mathcal{Y}^i$	Feasible set of CR i

4.4.1 Equivalent reformulation of game theory

Consider that there are M players, corresponding to the M CR-Txs, each one controlling the variables $\mathbf{x}^i = (\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\tau}^i)$. We denote by $\mathbf{x}$ the overall vector of all variables: $\mathbf{x} = [\mathbf{x}^i]_{i=1}^M$, while $\mathbf{x}^{-i}$ denotes the vector of the variables associated to all CR users except CR i. The main definitions and symbols used in this section are given in Table 4.3.
The non-convex individual constraints $(a1)$ and $(a2)$ are denoted as $g_k^i(\mathbf{x}^i)$. We define the function vectors $\mathbf{G}(\mathbf{x}) = [(g_k^i(\mathbf{x}^i))_{k=1}^N]_{i=1}^M$, and $h^i(\mathbf{x}^i)$, $\mathbf{H}(\mathbf{x}) = [h^i(\mathbf{x}^i)]_{i=1}^M$, respectively, whereas the convex individual constraints $(b1), (b2)$ are embedded in the defining set of $\mathbf{x}^i$, denoted as $\mathcal{X}^i$. We denote the non-cooperative power allocation game $\mathcal{G}(\mathbf{H}, \mathbf{G})$, given as problem $P4.4$:

$$\max_{\mathbf{x}^i} \qquad f^i(\mathbf{x}^i, \mathbf{x}^{-i})$$

$$\text{s. t.} \qquad g_k^i(\mathbf{x}^i) \leq 0, \;\; h^i(\mathbf{x}^i) \leq 0, \;\; \mathbf{x}^i \in \mathcal{X}^i. \tag{4.40}$$

The resulting game $P4.4$ is non-convex; the objective function and the constraints are non-convex. As a consequence, traditional mathematical tools are not applicable to prove the existence of a NE for the game. In this section, we analyze the proposed non-convex game based on a relaxed equilibrium concept that has been recently introduced by Pang and Scutari [31, 32], namely, the QNE.

4.4.2 VI method and KKT conditions

The NE is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally [60, 96].

Definition 2. *A Nash equilibrium of the game* $\mathcal{G}(\mathbf{H}, \mathbf{G})$ *is defined and formed by solution tuple* $\mathbf{x}^\star$*, such that:*

$$f^i(\mathbf{x}^{i,\star}, \mathbf{x}^{-i,\star}) \leq f^i(\mathbf{x}^i, \mathbf{x}^{-i,\star}), \quad \mathbf{x}^i \in \mathcal{Y}^i. \tag{4.41}$$

where $\mathcal{Y}^i$ is the feasible set of $\mathbf{x}^i$. The NE always exists for the convex game, where the constraint sets are compact and convex and the objective function are convex or concave for the variables [96]. For the non-cooperative game $P4.4$, none of those properties it is satisfied. Both the objective function and the constraint functions are non-convex. Hence, the existence of a NE for the non-convex game $P4.4$ is uncertain.
To overcome this problem for the non-cooperative game $P4.4$, we use the new (relaxed) equilibrium concept QNE from the literature [21, 31], where the QNE is by definition a tuple that satisfies the KKT conditions of all the players' optimization problems; the prefix *quasi* is intended to signify that a NE (if it exists) must be a QNE under a certain constraint qualification (CQ), as explained in [31, 32]. Notice that for a nonlinear program constrained by finite equations and inequalities and a differentiable objective function, KKT conditions are not always necessary conditions for a given point to be a solution to the problem. When an appropriate CQ holds, the solutions of the KKT conditions are equal to stationary solutions of the associated problem [33]. In the following, the

KKT conditions of the problem $P4.4$ are rewritten to a proper variational inequality (VI) problem [83]. Let $\mathcal{Y}^i$ denote the feasible strategy set of each CR i, which can be written as:

$$\mathcal{Y}^i = \{\mathbf{x}^i \in \mathcal{X}^i \mid g_k^i(\mathbf{x}^i) \le 0, h^i(\mathbf{x}^i) \le 0\}, \;\; 1 \le k \le N. \tag{4.42}$$

Instead of explicitly accounting all the multipliers as variables of the KKT conditions for each player's optimization problem, we introduce multipliers only for the non-convex constraints $h^i(\mathbf{x}^i) \le 0$ and $g_k^i(\mathbf{x}^i) \le 0$, and the convex constraints are embedded in the defining set $\mathcal{X}^i$. Denoting by α_k^i and β^i the multipliers associated with the non-convex constraints $g_k^i(\mathbf{x}^i) \le 0$ and $h^i(\mathbf{x}^i) \le 0$ of player CR-Tx i, respectively, the Lagrangian function of player CR-Tx i is given by:

$$L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) = -f^i(\mathbf{x}^i) + \sum_{k=1}^{N} \alpha_k^i g_k^i(\mathbf{x}^i) + \beta^i h^i(\mathbf{x}^i) \tag{4.43}$$

The KKT conditions based on Lagrangian function (4.43) are given by:

$$\begin{aligned} &0 \le \mathbf{x}^i \perp \nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) \ge 0 \\ &0 \le \alpha_k^i \perp -g_k^i(\mathbf{x}^i) \ge 0 \\ &0 \le \beta^i \perp -h^i(\mathbf{x}^i) \ge 0 \end{aligned} \tag{4.44}$$

where $0 \le a \perp b \ge 0$ implies $a \ge 0$, $b \ge 0$, $a \cdot b = 0$, and $\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i)$ is defined as:

$$\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) = -\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) + \sum_{k=1}^{N} \alpha_k^i \mathbf{J}_{g_k^i(\mathbf{x}^i)} + \beta^i \mathbf{J}_{h^i(\mathbf{x}^i)} \tag{4.45}$$

The components of the gradient $\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) = (\nabla_{P_{k,0}^i} f^i(\mathbf{x}^i), \nabla_{P_{k,1}^i} f^i(\mathbf{x}^i), \nabla_{\tau_k^i} f^i(\mathbf{x}^i))$ are given, respectively, by:

$$\nabla_{P_{k,0}^i} f^i(\mathbf{x}^i) = \frac{a_{k,0}^i(\boldsymbol{\tau})|h_{k,cr}^{ii}|^2}{I_{k,0}^i + P_{k,0}^i|h_{k,cr}^{ii}|^2} + \frac{b_{k,0}^i(\boldsymbol{\tau})|h_{k,cr}^{ii}|^2}{P_{k,pc}^i + I_{k,0}^i + P_{k,0}^i|h_{k,cr}^{ii}|^2} \tag{4.46}$$

$$\nabla_{P_{k,1}^i} f^i(\mathbf{x}^i) = \frac{a_{k,1}^i(\boldsymbol{\tau})|h_{k,cr}^{ii}|^2}{I_{k,1}^i + P_{k,1}^i|h_{k,cr}^{ii}|^2} + \frac{b_{k,1}^i(\boldsymbol{\tau})|h_{k,cr}^{ii}|^2}{P_{k,pc}^i + I_{k,1}^i + P_{k,1}^i|h_{k,cr}^{ii}|^2} \tag{4.47}$$

$$\nabla_{\tau_k^i} f^i(\mathbf{x}^i) = \mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i)'(r_{k,01}^i - r_{k,00}^i) \\ + \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i)'(r_{k,11}^i - r_{k,10}^i) \tag{4.48}$$

where: $a_{k,0}^i(\boldsymbol{\tau}), a_{k,1}^i(\boldsymbol{\tau}), b_{k,0}^i(\boldsymbol{\tau}), b_{k,1}^i(\boldsymbol{\tau})$ are given in (4.25)-(4.28). The components $\mathbf{J}_{g_k^i(\mathbf{x}^i)}$ and $\mathbf{J}_{h^i(\mathbf{x}^i)}$ denote the Jacobian matrix of the vector function $g_k^i(\mathbf{x}^i)$ and $h^i(\mathbf{x}^i)$, respectively:

$$\mathbf{J}_{g_k^i(\mathbf{x}^i)} = \begin{pmatrix} \Gamma_{k,c}^i I_{k,1}^{i,p}|h_{k,cp}^i|^2 - \mathcal{P}_{k,d}^i(\tau_k^i)|h_{k,cp}^i|^2 \\ \Gamma_{k,c}^i I_{k,0}^{i,p}|h_{k,cp}^i|^2 - (1 - \mathcal{P}_{k,d}^i(\tau_k^i))|h_{k,cp}^i|^2 \\ \mathcal{P}_{k,d}^i(\tau_k^i)'(I_{k,1}^{i,p} - I_{k,0}^{i,p}) \end{pmatrix} \tag{4.49}$$

$$\mathbf{J}_{h^i(\mathbf{x}^i)} = \begin{pmatrix} \sum\limits_{k=1}^{N}(1 - \mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i) - \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i)) \\ \sum\limits_{k=1}^{N}(\mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i) + \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i)) \\ \sum\limits_{k=1}^{N}(\mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i)' + \mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i)')(P_{k,1}^i - P_{k,0}^i) \end{pmatrix} \tag{4.50}$$

More specifically, if $\mathbf{x}^\star$ are the stationary solutions of game $\mathcal{G}(\mathbf{H}, \mathbf{G})$, and some CQ holds at $\mathbf{x}^\star$, the KKT conditions (4.44) can be reformulated to the equivalent form:

$$\begin{pmatrix} \mathbf{x} - \mathbf{x}^\star \\ \alpha_k - \alpha_k^\star \\ \beta - \beta^\star \end{pmatrix}^T \underbrace{\begin{pmatrix} \nabla_{\mathbf{x}^i} L(\mathbf{x}^{i,\star}, \boldsymbol{\alpha}^{i,\star}, \boldsymbol{\beta}^{i,\star}) \\ -g_k^i(\mathbf{x}^{i,\star}) \\ -h^i(\mathbf{x}^{i,\star}) \end{pmatrix}_{i=1}^{M}}_{\boldsymbol{\Theta}(\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star)} \geq 0,$$

$$(\mathbf{x}^i, \alpha_k^i, \beta^i) \in \underbrace{\prod_{i=1}^{M} \mathcal{X}^i \times \mathbb{R}_+^r}_{\mathbf{Q}} \tag{4.51}$$

The above system of inequalities defines a VI problem with variables $(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$, denoted as $VI(\mathbf{Q}, \boldsymbol{\Theta})$, where the vector function $\boldsymbol{\Theta}$ and feasible set $\mathbf{Q}$ are defined in (4.51). This $VI(\mathbf{Q}, \boldsymbol{\Theta})$ is an equivalent reformulation of the KKT conditions (4.44), where the convex constraints are embedded in the feasible set $\mathbf{Q}$, and r is the total number of multipliers $\boldsymbol{\alpha}, \boldsymbol{\beta}$. The $VI(\mathbf{Q}, \boldsymbol{\Theta})$ problem is to find a point $\mathbf{z}^\star = (\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star) \in \mathbf{Q}$, such that $(\mathbf{z} - \mathbf{z}^\star)^T \mathbf{\Theta}(\mathbf{z}^\star) \geq 0$. In addition, if $(\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star)$ is the solution of the $VI(\mathbf{Q}, \mathbf{\Theta})$, there exists $\boldsymbol{\gamma}^\star$ such that $(\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star, \boldsymbol{\gamma}^\star)$ is a solution of the game, where $\boldsymbol{\gamma}^\star$ are the multipliers associated with the players' convex constraints $(b1), (b2)$ [32].

4.4.3 Definition and basic concepts of QNE

Definition 3. *A quasi-Nash equilibrium of the game $\mathcal{G}(\mathbf{H}, \mathbf{G})$ is defined and formed by the solution tuple $(\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star)$ of the equivalent $VI(\mathbf{Q}, \mathbf{\Theta})$, which is obtained under the first-order optimality conditions of each player's problems, while retaining the convex constraints in the defined set $\mathbf{Q}$. A QNE is said to be trivial, if $\mathbf{P}_0^\star, \mathbf{P}_1^\star = 0$ for all $i = 1, ..., M$ [31, 32].*

The concept of QNE is conceptually similar but formally different from other forms of local equilibrium introduced in the literature [97]. The QNE is a stationary solution of the game, which has the following equivalent interpretation: the $\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star$ is a QNE of the game $P4.4$ if and only if it is an optimal solution of the players' optimization problems satisfied the following condition:

(A) The optimal solution $\mathbf{P}_1^\star, \mathbf{P}_0^\star$ is the NE of the game $\mathcal{G}(\mathbf{H}, \mathbf{G})$ when $\boldsymbol{\tau} = \boldsymbol{\tau}^\star$ for $i = 1, ..., M$.

(B) The $\boldsymbol{\tau}^\star$ is the optimal solution of the $P4.4$ when $\mathbf{P}_1^\star, \mathbf{P}_0^\star$ for $i = 1, ..., M$.

(C) There exist optimal multipliers $\boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star$ associated with the non-convex constraints $h^i(\mathbf{x}^{i,\star}) \leq 0$ and $g_k^i(\mathbf{x}^{i,\star}) \leq 0$ at $\mathbf{x}^{i,\star}$.

In a words, at a QNE$(\mathbf{x}^\star, \boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star)$ we have that:

- Each CR i maximizes his own function with respect to each of his own strategies $\mathbf{x}^i$, while keeping the rivals strategies fixed at the optimal value $\mathbf{x}^{-i,\star}$.
- $\boldsymbol{\alpha}^\star, \boldsymbol{\beta}^\star$ are the optimal multipliers associated with the non-convex constraints $h^i(\mathbf{x}^{i,\star}) \leq 0$ and $g_k^i(\mathbf{x}^{i,\star}) \leq 0$ at $\mathbf{x}^{i,\star}$.

Note that under condition (A), the problem $P4.4$ can be reformulated to a linearly constrained concave maximization problem, similar to the optimization problem $P4.2$; thus multipliers exist for this problem. Under condition (B), the problem $P4.4$ can be reformulated to a monotonous maximization problem with convex constraints, similar to the optimization problem $P4.3$. Thus they also have constraint multipliers. Moreover, the constraint sets under condition (A) and condition (B) are bounded. In addition, if a certain CQ holds at the optimal solution $\mathbf{x}^{i,\star}$, we can conclude that the KKT conditions are valid necessary conditions for an optimal solution of problem $P4.4$, and condition (C) is automatically guaranteed.

4.4.4 The existence of the QNE

Note that a matrix $\mathbf{A}$ is copositive when $\mathbf{x}^T\mathbf{A}\mathbf{x} \geq 0$ for all $\mathbf{x} \geq 0$. $\mathbf{T}(\mathcal{X}^i;\mathbf{x}^i)$ denotes the tangent cone of the set $\mathcal{X}^i$ at $\mathbf{x}^i \in \mathcal{X}^i$ [5], i.e.,

$$\mathbf{T}(\mathcal{X}^i;\mathbf{x}^i) = \Big\{ \lim_{q\to\infty} \frac{\mathbf{x}_q^i - \mathbf{x}^i}{y_q} \mid \mathbf{x}_q^i \in \mathcal{X}^i,\ y_q \in \mathbb{R}_+ \text{ with } \lim_{q\to\infty} \mathbf{x}_q^i = \mathbf{x}^i,\ \lim_{q\to\infty} y_q = 0 \Big\} \tag{4.52}$$

Details of the tangent cone are given in Appendix A.

Theorem 1. *The $VI(\mathbf{Q},\mathbf{\Theta})$ has a solution, and equivalently the game $\mathcal{G}(\mathbf{H},\mathbf{G})$ has a QNE, if the following conditions are satisfied [21]:*

(A) Set $\mathcal{X}^i$ is convex, $i = 1, ..., M$.

(B) The function $\mathbf{F}(\mathbf{x}) = [-\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i)]_{i=1}^M$ is continuously differentiable on its domain, and each $\mathbf{H}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are twice continuously differentiable on their domains.

(C) There exists a vector $\mathbf{x}^{ref} = [\mathbf{x}^{i,ref}]_{i=1}^M \in \mathcal{X}, \mathcal{X} = [\mathcal{X}^i]_{i=1}^M$, such that

(C1) $\Psi^i(\mathbf{x}^{i,ref}) < 0$, where $\Psi^i(\mathbf{x}^{i,ref}) = \big(g_k^i(\mathbf{x}^{i,ref}),\ h^i(\mathbf{x}^{i,ref})\big)$.

(C2) The Hessian matrix $\nabla^2_{\mathbf{x}^i} g_k^i(\mathbf{x}^i)$ is copositive on $\mathbf{T}(\mathcal{X}^i;\mathbf{x}^{i,ref})$ for $\mathbf{x}^i \in \mathcal{X}^i$.

(C3) The Hessian matrix $\nabla^2_{\mathbf{x}^i} h^i(\mathbf{x}^i)$ *is copositive on* $\mathbf{T}(\mathcal{X}^i; \mathbf{x}^{i,ref})$ *for* $\mathbf{x}^i \in \mathcal{X}^i$.

(C4) The set $\left\{\mathbf{x}^i \in \mathcal{X}^i | (\mathbf{x}^i - \mathbf{x}^{i,ref})\mathbf{F}^i(\mathbf{x}^i) \leq 0\right\}$ *is bounded (possibly empty).*

Theorem 2. *The* $VI(\mathbf{Q}, \boldsymbol{\Theta})$ *has a solution, thus the game* $\mathcal{G}(\mathbf{H}, \mathbf{G})$ *has a QNE, which is nontrivial.*

Proof. The non-convex problem $P4.4$ satisfies the hypotheses (A) and (B), and the proof for the hypotheses in (C1- C4) is given in Appendix C. □

An interiority condition (C1) is needed for the non-convex constraints. Conditions (C2) and (C3) highlight the significance of distinguishing the non-convex constraints $\Psi^i(\mathbf{x}^{i,ref}) < 0$ from the convex constraints contained in each set $\mathcal{X}^i$. The condition (C4) is an assumption imposed for the existence of solutions of the $VI(\mathcal{X}, \mathbf{F})$.
In order to show that the KKT conditions are valid necessary conditions for an optimal solution of problem $P4.4$, we need to verify that an appropriate CQ holds, as shown in [98]. In this thesis, we use the Linear Independent Constraint Qualification (LICQ). If the gradients of the constraints are linearly independent at $\mathbf{x}^i$, we can prove that the LICQ holds at $\mathbf{x}^i$ [98].

Lemma 1. *The LICQ holds at every feasible solution of the problem* $P4.4$.

Proof. Let the rank of $A^{m\times n}$ denote as $\mathcal{R}(A^{m\times n})$. If $\mathcal{R}(A^{m\times n}) = \min(m, n)$, the matrix $A^{m\times n}$ is full rank and nonsingular. According to **Theorem 1**, problem $P2$ admits a solution $\mathbf{x}^{i,\star} = (\mathbf{P}_1^{i,\star}, \mathbf{P}_0^{i,\star}, \boldsymbol{\tau}^{i,\star})$, which is nontrivial. Define the Jacobian matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})} = \left(\mathbf{J}_{g_k^i(\mathbf{x}^{i,\star})}, \mathbf{J}_{h^i(\mathbf{x}^{i,\star})}\right)$, where $\mathbf{J}_{g_k^i(\mathbf{x}^i)}, \mathbf{J}_{h^i(\mathbf{x}^i)}$ are given by (4.49), (4.50), respectively. We can observe that in the first row of matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, the first item contains the variables $\mathbf{P}_1^i$ and $\boldsymbol{\tau}^i$, while the second item just contains the variable $\boldsymbol{\tau}^i$. Moreover, in the second row of matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, the variables in the first item are not equal to the ones in the second item. Hence, the first column $\mathbf{J}_{g_k^i(\mathbf{x}^{i,\star})}$ and the second column $\mathbf{J}_{h^i(\mathbf{x}^{i,\star})}$ are linear independent at $\mathbf{x}^{i,\star}$, if $|h_{k,cp}^i|^2 \neq 0$. The rank of $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, defined as $\mathcal{R}(\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})})$, is 2. Therefore, we can state that the Jacobian matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$ is nonsingular for any given set of non-zero channels gain, and hence, the LICQ holds at every feasible solution of the problem $P2$. $\square$

Based on **Lemma 1**, we conclude that the KKT conditions are valid necessary conditions for an optimal solution of the problem $P4.4$, namely, the achieved QNE coincides with the NE.

4.5 Primal-Dual Interior Point Optimization in SISO CRNs

In Section 4.3, we used the ADOS algorithm for solving our problem $P4.1$, and find the LNE of the non-convex noncooperative game. In this section, we analyze the iterative primal-dual interior point optimization algorithm for SISO CRNs (PDIPS) based on the IP method from [99,100] for solving constrained equations (CE), which are reformulated from the $VI(\mathbf{Q}, \mathbf{\Theta})$. In addition, this PDIPS algorithm requires no information exchange between CR users, and hence, it can be regarded as a distributed solution.

There has been much research in using interior point algorithms for nonlinear programming; most of it concerns line search methods. The special case when the problem is a convex program can be handled by line search methods that are direct extensions of interior point algorithms for linear programming [101]. In the convex case, the step generated by the solution of the primal-dual equations can be shown to be a descent direction for several merit functions, and this allows one to establish global convergence results. Other research [102, 103] has focused on the local behavior of interior point line search methods for nonlinear programming. Conditions have been given that guarantee superlinear and quadratic rates of convergence. These algorithms can also be viewed as a direct extension of linear programming methods, in that they do not make provisions for the case when the problems is non-convex.

Several line search algorithms designed for non-convex problems have

Table 4.4: **Notation of PDIPS**

Symbol	Value
$\|\|\mathbf{x}\|\|_2$	Euclidean norm of vector $\mathbf{x}$
$\|\|\mathbf{x}\|\|_\infty$	Maximum norm of vector $\mathbf{x}$
$\mathbf{z}^i$	$(\mathbf{x}^i, \mathbf{s}^i)$
$\mathbf{u}^i$	$(\boldsymbol{\alpha}^i, \boldsymbol{\beta}^i, \boldsymbol{\gamma}^i)$
$\boldsymbol{\Lambda}^i$	$\mathrm{Diag}(\mathbf{u}^i)$
$\mathcal{S}^i$	$\mathrm{Diag}(\mathbf{s}^i)$
Υ^i	Trust region radius of CR i
$\mathbf{s}^i$	$(s^i_{k,0}, s^i_1, s^i_{k,2})^N_{k=1}$, Slack variables
$\mathbf{v}^i$	(v^i_0, v^i_1, v^i_2), Barrier parameters
$M_{c^i}(\mathbf{z}^i)$	Merit function
$D_{M_{c^i}(\mathbf{z}^i;\mathbf{d}_{\mathbf{z}^i})}$	Directional derivative of $M_{c^i}(\mathbf{z}^i)$

been proposed [104, 105]. An important feature of many of these methods is a strategy for modifying the KKT system used in the computation of the search direction. This modification ensures that the search direction is a descent direction for the merit function. The trust region strategies in interior point algorithms for linear and nonlinear problems is discussed in [106, 107].

Due to the non-convexity of problem $P4.1$, we propose a PDIPS algorithm based on the IP method from literature [99,100]. The typical iteration computes a primary step by solving the primal-dual equations and performs a line search to ensure decrease in a merit function. However, in order to obtain global convergence in the presence of non-convexity, the primary step is replaced by trust region step, under certain situation. The iterative PDIPS algorithm can use exact second derivatives of the objective function and constraints, ensuring that the search direction is a descent direction for the merit function, and converges to a solution of $VI(\mathbf{Q}, \boldsymbol{\Theta})$, thus to a QNE of our game. The iterative PDIPS algorithm combines a line search step that computes iterative steps by factoring the primal-dual equations, and a trust region step. We first compute the steps using line search whenever the conditions of these steps can be guaranteed, and turning to the trust region step otherwise. A merit function is a function that measures the agreement between data and the fitting model for a particular choice of the parameters [108]. For problem $P4.1$, we consider the merit function as an objective function component and a component comprising constraints of the problem. The main definitions and symbols are given in Table 4.4.

4.5.1 Line search iterations

The PDIPS reformulates problem $P4.1$ as a sequence of barrier problem $P4.5$:

$$\min_{\mathbf{z}^i} \quad \varphi_{\mathbf{v}^i}(\mathbf{z}^i) = -f^i(\mathbf{x}^i) - v_0^i \sum_{k=1}^{N} \ln s_{k,0}^i - v_1^i \ln s_1^i - v_2^i \sum_{k=1}^{N} \ln s_{k,2}^i \tag{4.53}$$

$$\text{s.t.} \quad g_k^i(\mathbf{x}^i) + s_{k,0}^i = 0 \tag{4.54}$$

$$h^i(\mathbf{x}^i) + s_1^i = 0 \tag{4.55}$$

$$\tilde{g}_k^i(\mathbf{x}^i) + s_{k,2}^i = 0 \tag{4.56}$$

where $\tilde{g}_k^i(\mathbf{x}^i)$ denotes the convex constraints $(b1), (b2)$, $s_{k,0}^i, s_1^i, s_{k,2}^i > 0$ are slack variables, denoted as $\mathbf{s}^i = (s_{k,0}^i, s_1^i, s_{k,2}^i)_{k=1}^N$. $v_0^i, v_1^i, v_2^i > 0$ are the barrier parameters, denoted as $\mathbf{v}^i = (v_0^i, v_1^i, v_2^i)$. To simplify the problem, we denote $\mathbf{z}^i = (\mathbf{x}^i, \mathbf{s}^i)$, and $\mathbf{u}^i = (\boldsymbol{\alpha}^i, \boldsymbol{\beta}^i, \boldsymbol{\gamma}^i)$. The Lagrangian function associated with the problem $P4.5$ is given by:

$$\begin{aligned} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \varphi_{\mathbf{v}^i}(\mathbf{z}^i) + \sum_{k=1}^{N} \alpha_k^i (g_k^i(\mathbf{x}^i) + s_{k,0}^i) + \beta^i (h^i(\mathbf{x}^i) + s_1^i) \\ + \sum_{k=1}^{N} \gamma_k^i (g_k^i(\tilde{\mathbf{x}}^i) + s_{k,2}^i) \end{aligned} \tag{4.57}$$

where $\varphi_{\mathbf{v}^i}(\mathbf{z}^i)$ is given in (4.53). Let $\boldsymbol{\Lambda}^i = \mathrm{Diag}(\mathbf{u}^i)$, and $\mathcal{S}^i = \mathrm{Diag}(\mathbf{s}^i)$, $\mathbf{e}$ is the all-ones column vector. The first order optimality conditions of the problem $P4.5$ can be written as:

$$\nabla_{\mathbf{z}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \begin{pmatrix} \nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) \\ \mathcal{S}^i \boldsymbol{\Lambda}^i \mathbf{e} - \mathbf{v}^i \mathbf{e} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4.58}$$

where $\nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is given by:

$$\nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = -\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) + \sum_{k=1}^{N} \alpha_k^i \mathbf{J}_{g_k^i(\mathbf{x}^i)} + \beta^i \mathbf{J}_{h^i(\mathbf{x}^i)} + \sum_{k=1}^{N} \gamma_k^i \mathbf{J}_{\tilde{g}_k^i(\mathbf{x}^i)} \tag{4.59}$$

$\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i), \mathbf{J}_{g_k^i(\mathbf{x}^i)}, \mathbf{J}_{h^i(\mathbf{x}^i)}$ are given by (4.46)-(4.48), (4.49) and (4.50), respectively. The $\mathbf{J}_{\tilde{g}_k^i(\mathbf{x}^i)}$ is the Jacobian matrix of the convex constraints $\tilde{g}_k^i(\mathbf{x}^i)$. Applying Newton's method to problem $P4.5$, we obtain the following primal-dual system:

$$\begin{pmatrix} \mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) & \mathbf{J}(\mathbf{x}^i) \\ \mathbf{J}^T(\mathbf{x}^i) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{d}_{\mathbf{z}^i} \\ \mathbf{d}_{\mathbf{u}^i} \end{pmatrix} = \begin{pmatrix} \nabla_{\mathbf{z}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) \\ \mathbf{B}(\mathbf{z}^i) \end{pmatrix} \tag{4.60}$$

$\mathbf{B}(\mathbf{z}^i)$ is defined as:

$$\mathbf{B}(\mathbf{z}^i) = \begin{pmatrix} g_k^i(\mathbf{x}^i) + s_{k,0}^i \\ h^i(\mathbf{x}^i) + s_1^i \\ \tilde{g}_k^i(\mathbf{x}^i) + s_{k,2}^i \end{pmatrix}_{k=1}^{N} \tag{4.61}$$

and $\mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is defined as:

$$\mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \begin{pmatrix} \nabla^2_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) & 0 \\ 0 & (\mathcal{S}^i)^{-1}\mathbf{\Lambda}^i \end{pmatrix} \tag{4.62}$$

where $\nabla^2_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is the Hessian matrix of $L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$, and $\mathbf{J}(\mathbf{x}^i)$ is given by:

$$\mathbf{J}(\mathbf{x}^i) = \begin{pmatrix} \mathbf{J}_{g_k^i(\mathbf{x}^i)} & \mathbf{J}_{h^i(\mathbf{x}^i)} & \mathbf{J}_{\tilde{g}_k^i(\mathbf{x}^i)} & \mathbf{I} \end{pmatrix}_{k=1}^{N} \tag{4.63}$$

We define the search directions $\mathbf{d}_{\mathbf{z}^i}$ and $\mathbf{d}_{\mathbf{u}^i}$ as:

$$\mathbf{d}_{\mathbf{z}^i}^T = \begin{pmatrix} d_{P_{k,0}^i}, & d_{P_{k,1}^i}, & d_{\tau_k^i}, & d_{s_{k,0}^i}, & d_{s_1^i}, & d_{s_{k,2}^i} \end{pmatrix}_{k=1}^{N} \tag{4.64}$$

$$\mathbf{d}_{\mathbf{u}^i}^T = \begin{pmatrix} d_{\alpha_k^i}, & d_{\beta^i}, & d_{\gamma_k^i} \end{pmatrix}_{k=1}^{N} \tag{4.65}$$

The objective function component and the component comprising constraints of the problem $P4.5$ are used as the merit function, which can be defined by:

$$M_{c^i}(\mathbf{z}^i) = \varphi_{\mathbf{v}^i}(\mathbf{z}^i) + c^i \|\mathbf{B}(\mathbf{z}^i)\|_2 \tag{4.66}$$

where $c^i > 0$ is the penalty parameter, which is updated at each iteration so that the search direction $\mathbf{d}_{\mathbf{z}^i}$ is a descent direction for $M_{c^i}(\mathbf{z}^i)$. The iterations are given by:

$$\mathbf{z}^i(p+1) = \mathbf{z}^i(p) + \boldsymbol{\rho}^i_{\mathbf{z}^i}\mathbf{d}_{\mathbf{z}^i}(p) \tag{4.67}$$

$$\mathbf{u}^i(p+1) = \mathbf{u}^i(p) + \boldsymbol{\rho}^i_{\mathbf{u}^i}\mathbf{d}_{\mathbf{u}^i}(p) \tag{4.68}$$

where p is the number of the inner iteration loop, $\boldsymbol{\rho}^i_{\mathbf{z}^i}$ and $\boldsymbol{\rho}^i_{\mathbf{u}^i}$ are the step-lengths. We then perform a backtracking line search that computes the step-lengths which provide a sufficient decrease in the merit function. The step-lengths $\boldsymbol{\rho}^i_{\mathbf{z}^i}, \boldsymbol{\rho}^i_{\mathbf{u}^i} \in (0, 1]$ are given by:

$$\boldsymbol{\rho}^i_{\mathbf{z}^i} = \{\mathbf{s}^i + \boldsymbol{\rho}^i_{\mathbf{z}^i}\mathbf{d}_{\mathbf{s}^i} \geq \xi_0 \mathbf{s}^i\}, \tag{4.69}$$

$$\boldsymbol{\rho}^i_{\mathbf{u}^i} = \{\mathbf{u}^i + \boldsymbol{\rho}^i_{\mathbf{u}^i}\mathbf{d}_{\mathbf{u}^i} \geq \xi_0 \mathbf{u}^i\} \tag{4.70}$$

where $\xi_0 \in (0, 1]$ is a constant. Moreover, the directional derivative of $M_{c^i}(\mathbf{z}^i)$ is given by:

$$D_{M_{c^i}(\mathbf{z}^i;\mathbf{d}_{\mathbf{z}^i})} = \nabla\varphi_{\mathbf{v}^i}(\mathbf{z}^i)\mathbf{d}_{\mathbf{z}^i} - c^i\|\mathbf{B}(\mathbf{z}^i)\|_2 \tag{4.71}$$

4.5.2 Trust region iterations

The expressions (4.60)-(4.70) provide the basis for the line search steps in the interior point algorithm. However, due to the non-convexity of the problem $P4.1$, the line search iterations may converge to non-stationary points. If the steplengths $\rho^i_{\mathbf{z}^i}, \rho^i_{\mathbf{u}^i}$ converge to zero, we turn to the trust region iterations, which provide a sufficient reduction in the chosen merit function for both feasibility and optimality at every iteration and guarantee progress towards stationary [100].

The trust region method treats convex and non-convex problems uniformly, and permits the direct use of second derivative information. It is desirable to provide a trust region radius Υ^i that reflects current problem informations. In addition to preserving the global convergence properties of the trust region method, the size of Υ^i affects the backtracking line search iterations. Note that if a trust region iteration is rejected, the following iterations are still computed by the trust region method until a successful step is obtained.

In the trust region step, a step $\mathbf{d}$ is acceptable if the ratio of actual reduction (ared($\mathbf{d}$)) to predicted reduction (pred($\mathbf{d}$)) of the merit function is greater than a given constant $\eta > 0$, denoted as:

$$\eta < \frac{\text{ared}(\mathbf{d})}{\text{pred}(\mathbf{d})} \tag{4.72}$$

where $ared(\mathbf{d})$ and $pred(\mathbf{d})$ are, respectively, given as,

$$ared(\mathbf{d}) = M_{c^i}(\mathbf{z}^i) - M_{c^i}(\mathbf{z}^i + \mathbf{d}_{\mathbf{z}^i}) \tag{4.73}$$

$$\begin{aligned} pred(\mathbf{d}) &= -\nabla\varphi_{\mathbf{v}^i}(\mathbf{z}^i)\mathbf{d}_{\mathbf{z}^i} - \frac{1}{2}\mathbf{d}_{\mathbf{z}^i}^T W \mathbf{d}_{\mathbf{z}^i} \\ &+ c^i(\|\mathbf{B}(\mathbf{z}^i)\|_2 - \|\mathbf{B}(\mathbf{z}^i) + J(\mathbf{x}^i)\mathbf{d}_{\mathbf{z}^i}\|_2) \end{aligned} \tag{4.74}$$

and $\mathbf{W}$ is defined in (4.62), $\mathbf{d}_{\mathbf{z}^i}^T W \mathbf{d}_{\mathbf{z}^i} > 0$. Instead of requiring only that the directional derivative of the merit function be negative, the value of c^i is based on the decrease in a quadratic/linear model of the merit function achieved by the step $\mathbf{d}$. Following [100], the update rule for the penalty parameter c^i is:

$$c^i = \begin{cases} c_0^i & \text{if} \quad c^i > c_0^i \\ c_0^i + 1 & \text{if} \quad c^i \leq c_0^i \end{cases} \tag{4.75}$$

where

$$c_0^i = \frac{\nabla\varphi_{\mathbf{v}^i}(\mathbf{z}^i) d_{\mathbf{z}^i} + \frac{1}{2} d_{\mathbf{z}^i}^T W d_{\mathbf{z}^i}}{(1 - \xi_1)\|B(\mathbf{z}^i)\|_2} \tag{4.76}$$

and $\xi_1 \in (0, 1)$ is a constant. The trust region step, which is described in [100], starts by constructing a quadratic model of the Lagrangian function. The search direction $\mathbf{d}$ is computed by minimizing the quadratic model, subject to the constraints and the trust region in this step, which provides sufficient reduction in the merit function.

We outline the iterative PDIPS algorithm in **Algorithm** 4, where N_e^i is the number of negative eigenvalues of the matrix in (4.60), and N_b is the maximum number of backtracking search steps. For our problem, if $N_e^i > 4N$, then $\mathbf{d}_{\mathbf{z}^i}$ can not be guaranteed to be the descent direction [109]. In this case, we turn to the trust region steps. We choose $\eta = 10^{-8}$, $\varepsilon = 10^{-6}$, and $N_b = 4$. The resulting algorithm is ensured to have global convergence, thus achieving a QNE of the $VI(\mathbf{Q}, \mathbf{\Theta})$. For more details of the trust region iterations and the global convergence analysis, refer to [99, 100].

4.5.3 Complexity analysis of the PDIPS algorithm

The complexity of the iterative PDIPS algorithm is dominated by the procedure of line search iteration steps and trust region iteration steps, as well as the size of the CRNs. Generally, for the inner loop, the time complexity of line search is based on the Newton iteration, which requires at most $O((2NM+M)^3)$ computations. For the ε-accurate iteration, the computation of Newton iterations reduce to $O(\ln(\frac{1}{\varepsilon})\sqrt{2NM+M})$ [110], and according to [111], the complexity for the logarithmic barrier function is the best one given by $O(\sqrt{2NM+M})$. For our problem, the maximum number of backtracking search steps is given by N_b, thus the time complexity of the line search is $O(\sqrt{2NM+M}) \sim O(N_b\sqrt{2NM+M})$. In addition, the trust region iterations step is based on the sequential quadratic programming techniques [112, 113], and the worst-case complexity of reaching a scaled stationary point is $O(2NM+M+\sqrt{2NM+M})$ [114]. The outer loop for a CRNs with M CR users is a linear problem with the accuracy ε, thus the total complexity of the PDIP algorithm is given by $O_{PDIP} = O(\ln(\frac{1}{\varepsilon})M\sqrt{2NM+M}) \sim O(\ln(\frac{1}{\varepsilon})M((N_b+1)\sqrt{2NM+M}+2NM+M))$. Notice that here we did not consider the time complexity of the convergence of the consensus algorithm in the cooperative sensing step.

Algorithm 4 PDIP Optimization for SISO CRNs (PDIPS)

Initialize $\mathbf{x}^i(0) = (\mathbf{P}_0^i(0), \mathbf{P}_1^i(0), \boldsymbol{\tau}^i(0))$, $\mathbf{z}^i(0) = (\mathbf{x}^i(0), \mathbf{s}^i(0))$. Compute initial values for the multipliers $\mathbf{u}^i(0) = (\boldsymbol{\alpha}^i(0), \boldsymbol{\beta}^i(0), \boldsymbol{\mu}^i(0))$, the trust-region radius $\Upsilon^i(0) > 0$ and the barrier parameter $\mathbf{v}^i(0) > 0$.
repeat
  for i =1: M
    repeat
      repeat
      Compute the number N_e^i from (4.60), set LS = 0
        if $N_e^i \leq 3N$
        Get the search direction $\mathbf{d}(p) = (d_{\mathbf{z}^i}(p), d_{\mathbf{u}^i}(p))$ from (4.60)
        Compute $\rho^i_{\mathbf{z}^i}$, $\rho^i_{\mathbf{u}^i}$
          if $\min\{\rho^i_{\mathbf{z}^i}, \rho^i_{\mathbf{u}^i}\} > \varepsilon$
          Set $j = 0, \rho^i_T = 1$
            repeat
              if $M_{c^i}(\mathbf{z}^i(p) + \rho^i_T \rho^i_{\mathbf{z}^i} d_{\mathbf{z}^i}(p))$
              $\leq M_{c^i}(\mathbf{z}^i(p)) + \eta \rho^i_T \rho^i_{\mathbf{z}^i} D_{M_{c^i}(\mathbf{z}^i(p))}$
              Update $\rho^i_{\mathbf{z}^i} = \rho^i_T \rho^i_{\mathbf{z}^i}$, $\rho^i_{\mathbf{u}^i} = \rho^i_T \rho^i_{\mathbf{u}^i}$
              Update $\mathbf{z}^i(p+1), \mathbf{u}^i(p+1)$ using (4.67), (4.68)
              Update $\Upsilon^i(p+1)$
              Set LS = 1
              else
              Update $j = j+1$, choose a smaller value of ρ^i_T
              endif
            until $j > N_b$ Or $\rho^i_T < \varepsilon$ Or LS == 1
          endif
        endif
        if LS == 0
        Compute $\mathbf{z}^i(p+1), \mathbf{u}^i(p+1)$ using the trust region method
        Compute $\Upsilon^i(p+1)$
        endif
        Set $\mathbf{v}^i(p+1) = \mathbf{v}^i(p)$, $p = p+1$
      until $\|\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \mathbf{u}^i)\|_\infty \leq \varepsilon$ and $\|\mathbf{S}^i e \Lambda^i - \mathbf{v}^i e\|_\infty \leq \varepsilon$
      Reset the barrier parameters, so that $\mathbf{v}^i(p+1) < \mathbf{v}^i(p)$
    until $\|\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \mathbf{u}^i)\|_\infty \leq \varepsilon$ and $\|\mathbf{S}^i \Lambda^i\|_\infty \leq \varepsilon$
    Update $\mathbf{x}^i(p_0) = \mathbf{x}^i(p)$
  endfor
until $\|\mathbf{x}^i(p_0) - \mathbf{x}^i(p_0 - 1)\|_2 \leq \varepsilon$

Algorithm 5 Trust region iterations method

Compute the step $\mathbf{d}(p) = (d_{\mathbf{z}^i}(p), d_{\mathbf{u}^i}(p))$.
Compute Lagrange multiplier $\mathbf{u}^i(p+1)$
Update the penalty parameter c^i by (4.75).
Compute $ared(d)$ by (4.73), and $pred(d)$ by (4.74)
if $ared(d) \geq \eta pred(d)$
Set $\mathbf{z}^i(p+1) = \mathbf{z}^i(p) + \mathbf{d}_{\mathbf{z}^i}(p)$
Enlarge the trust region radius $\Upsilon^i(p+1)$
else
Set $\mathbf{z}^i(p+1) = \mathbf{z}^i(p)$
Shrink the trust region $\Upsilon^i(p+1)$
endif

4.6 Simulation Results

4.6.1 Scenario description

Table 4.5: **Simulation parameters**

Symbol	Value
Sensing time t	1ms
Sampling frequency, f_s	$2MHz$
Probability of channel k idle, $\mathcal{P}(H_{0,k})$	$0.1, 0.5$
Probability of channel k occupied, $\mathcal{P}(H_{1,k})$	$0.9, 0.5$
Transmit power budget of CR i, $P^i_{\max}$	$0 \sim 10W$
Transmit power of PU in channel k, $\lvert S_k\rvert^2$	$10W$
Rate-loss gap of channel k, Γ_k	0.1%,0.3%,1%

We consider a CRNs with $M = 3$ CR Tx-Rx pairs and $N = 2$ PU channels. All PUs and CR users are randomly placed in a 50 meter $\times$ 50 meter square. The radio environment map is shown in Figure 4.3, where the color-bar shows the received power from PUs in Watt. We use the channel model from the 3rd Generation Partnership Project (3GPP) Indoor scenario for Long Term Evolution (LTE) [115]. The distance-depended path loss is given by $PL_{dB} = 7 + 56\log_{10}(d)$; $d = d_{ji}/d_{ii}$ (m) is the relative distance between CR-Tx j and CR-Rx i, where d_{ii}

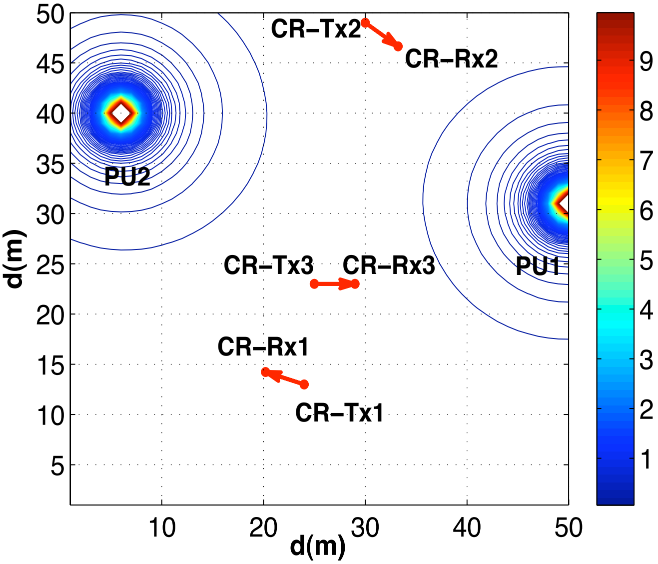

Figure 4.3: Network topology: location of two PUs and three CR Tx-Rx pairs.

and d_{ji} are the distances between CR-Tx i and CR-Rx i, CR-Tx j and CR-Rx i, respectively. A lognormal shadowing variable with variance 10 dBs is considered here, and $(\sigma_{k,n}^{i})^2 = 1$. Assume that the sensing environment is stable in the optimization process, and the local channel state information, i.e., the channel gain between CR-Tx and its target Rx and each PU, is known by each CR-Tx. The main simulation parameters are given in Table 4.5.

4.6.2 Simulation results analysis

In this section, we first compare the performance of the proposed game, in terms of the sum-rate achieved at the QNE for one CR user by the PDIPS algorithm, with those achieved by the ADOS algorithm and the Deterministic Game (DG) proposed in [26]. The sensing information is not considered as a part of optimization for the DG. Then, we investigate the influence of the activity of the PUs and compare the sum-rate achieved by different constraints, respectively. Finally, we show the actual rate-loss of the PUs under constraints (4.17) and (4.19).

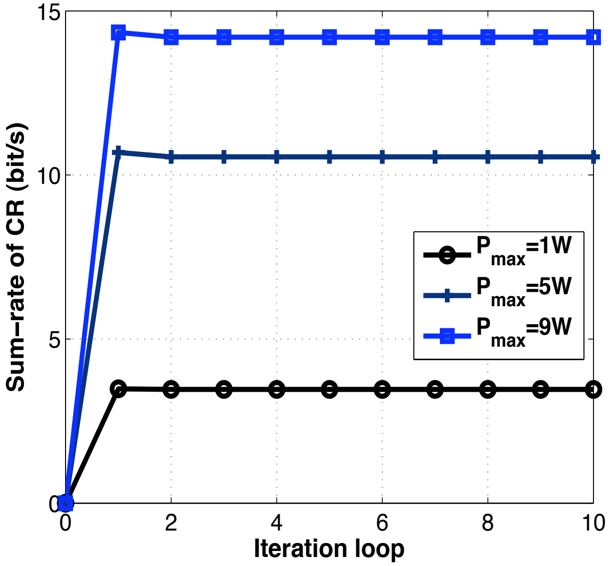

Figure 4.4: Sum-rate achieved at the QNE with different $P^i_{\max}$

Note that, except the results in Figure 4.6, all the other results are for $\mathcal{P}(H_{0,k}) = \mathcal{P}(H_{1,k}) = 0.5$.

In Figure 4.4, we compare the sum-rate of CR i achieved at a QNE with different $P^i_{\max}$. Our results show that CR i is able to achieve a QNE within a few iterations under any value of power budget. Moreover, CR users can achieve higher sum-rate with bigger $P^i_{\max}$.

In Figure 4.5, we compare the sum-rate achieved at the QNE by the PDIPS algorithm with those achieved by the ADOS algorithm and the DG. For the ADOS algorithm, the first step is to maximize the sum-rate of each CR i based on an initial detection threshold, and then optimize the threshold based on the optimal power obtained in the first step. Regarding the constraint inequalities given in (4.19), (4.20), we have the same limited regime as well as the single user CRNs. The optimization problem works in two possible regimes, namely, power budget limited regime (PLR) and rate-loss limited regime (RLR).

Our results shows that when the CR users work in PLR, when $\Gamma_k = 1\%$, the performances of these three algorithms are almost the same, while the proposed game which joint optimization of the sensing information and transmit power by PDIPS algorithm yields a considerable perfor-

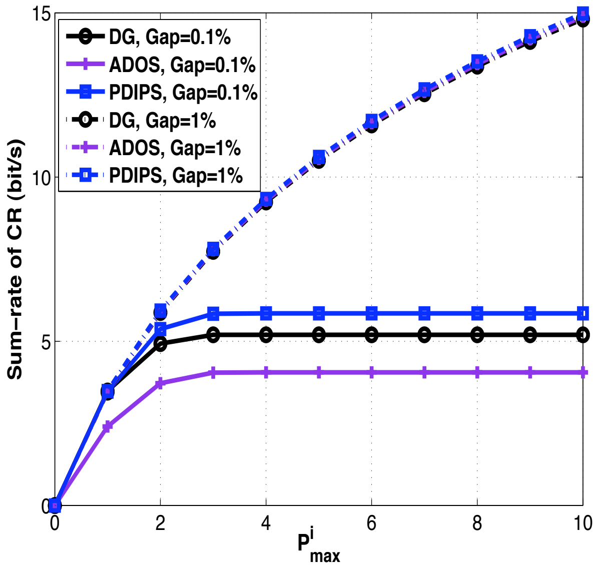

Figure 4.5: Sum-rate achieved at the QNE versus $P^i_{\max}$; comparison between DG, ADOS and PDIPS algorithms.

mance improvement in RLR, when $\Gamma_k = 0.1\%$, with respect to the ADOS algorithm and the disjoint case of the DG. In fact, the DG can be considered as the perfect sensing information case (i.e. $\mathcal{P}^i_{k,fa} = 0$ and $\mathcal{P}^i_{k,d} = 1$) with a deterministic interference constraint. Specifically, in RLR, a higher transmit power is allowed due to the accurate sensing information in the proposed game compared to the DG with a deterministic interference constraint, thus the performance can be improved. In addition, when $\Gamma_k = 0.1\%$, the sum-rate of CR users does not change after $P^i_{\max} > 1W$, indicating that the transmit power changes from PLR to RLR.

Figure 4.6 presents the sum-rate achieved at the QNE versus the power budget $P^i_{\max}$ for different average fractions of the PU's activity, $\mathcal{P}(H_{1,k}) = 0.5, 0.9$, which are directly related to the traffic load of the PU. It can be observed that in RLR, when $\Gamma_k = 0.1\%$, the traffic load of the PU affects the sum-rate of the CR users. The CR users suffer a decrease in sum-rate when the traffic load of the PU increases from 0.5 to 0.9, in other words, when there is more activity of the PU, there is less chance for the CR users to use the channel. Additionally, in PLR, the perfor-

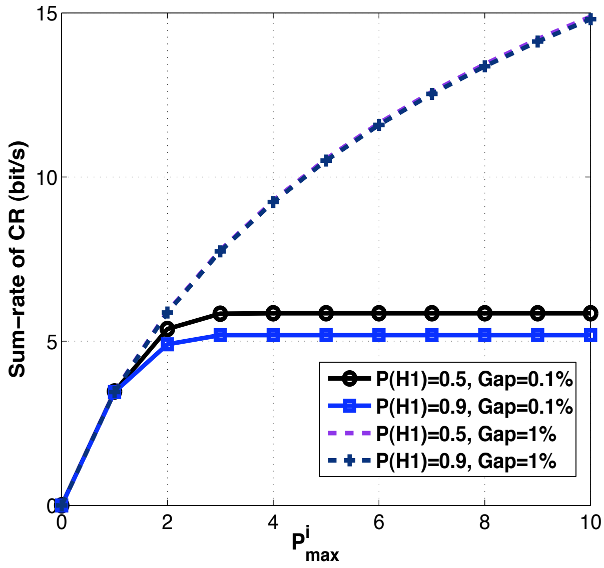

Figure 4.6: Sum-rate achieved at the QNE versus $P^i_{\max}$; comparison between $\mathcal{P}(H_{k,1}) = 0.5$ and $\mathcal{P}(H_{k,1}) = 0.9$.

mance of the CR users is not sensitive to the traffic load of the PU.
In Figure 4.7, we compare the performance achieved by the global constraint with the individual constraint (4.19), respectively. In order to have the same total interference to the PU, we use a rate-loss gap $\Gamma_{k,g} = \Gamma_k \times M$ for the global constraint. Based on the individual constraint (4.19), the global constraint can be written as:

$$(1 - \Gamma_{k,g}) I^{i,pt}_{k,1} I^{i,pt}_{k,0} - \mathcal{P}^i_{k,d}(\tau^i_k) I^{i,pt}_{k,0} - (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) I^{i,pt}_{k,1} \leq 0 \qquad (4.77)$$

where $I^{i,pt}_{k,0}, I^{i,pt}_{k,1}$ stand for the total interference from all the CR users. It is rather interesting to notice that when the rate-loss constraint is active, the performance of the CR users under the individual constraint is better than those achieved by the global constraint. However, this is due to the unfairness among the CR users in the global constraint. Each iteration of the game follows a sequential order, indicating that the CR users having the priority to choose their action can have the preference to maximize their own benefit in the global constraint case, and the CR users at the bottom of the iteration loop have to be switched off in RLR.

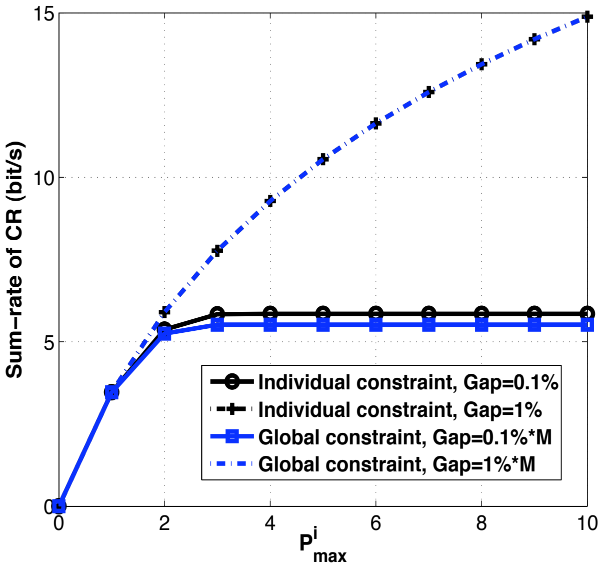

Figure 4.7: Sum-rate achieved at the QNE versus $P^i_{\max}$; comparison between global constraint and individual constraint.

These inherently unfairness for the global constraint leads to a lower utilization of the channel, yielding a worst performance of the CR users. Actually, the global constraint can result in a better performance than the individual constraint by pricing mechanism, which uses a penalty in the objective function and encourages the CR users to work in a cooperative manner to achieve a higher social welfare [32, 33, 116].
Finally, in Figure 4.8, we evaluate the interference experienced by the PU under constraint (4.17) and the modified constraint (4.19). The rate-loss gap is defined as $(R^i_{k,\max} - R^i_k)/R^i_{k,\max}$, and $R^i_{k,\max}$, R^i_k are given by (4.13), (4.14), respectively. It can be observed that in RLR, the constraint (4.17) imposes a less strict condition on the transmit power of the CR users than the one imposed by the modified constraint (4.19). This leads to a higher interference and a larger rate-loss gap experienced by the PUs, and a increasing of the sum-rate of the CR users. In other words, the modified constraint (4.19) can be seen as the constraint (4.17) with a smaller rate-loss gap.

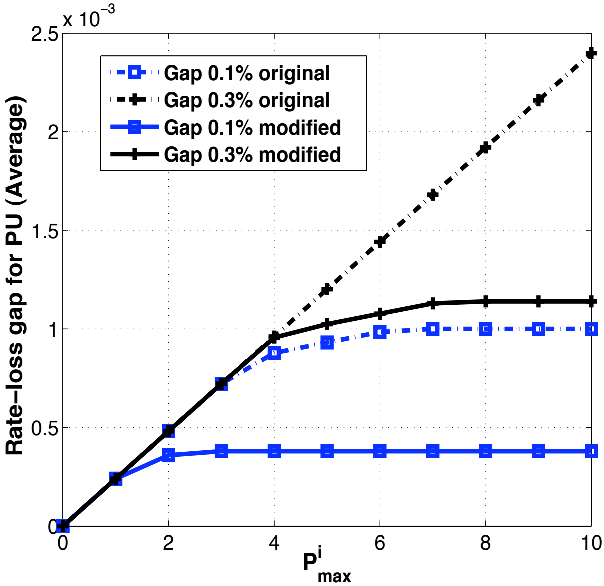

Figure 4.8: Average-rate gap for PU achieved at the QNE versus $P^i_{\max}$; comparison between constraints (4.17) and (4.19).

4.7 Conclusions

In this chapter, extending the single user case from Chapter 3, we consider the sensing-based spectrum sharing scenario for multiuser SISO CRNs. The overall objective is to maximize the sum-rate of each CR user by optimizing jointly both the detection operation and the power allocation. In order to deal with the non-convexity of the game, we use a relaxed equilibrium concept, the QNE. We present the sufficient conditions for the existence of a QNE based on VI theory, and prove that the LICQ holds at every feasible solution of the proposed game, thus the achieved QNE coincides with the NE. Finally, a distributed iterative PDIPS algorithm is stated and shown to converge to a QNE of the proposed game. Simulation results show that the iterative PDIPS yields a considerable performance improvement with respect to the ADOS algorithm and the DG. In the following, we are going to study the optimization problems in MIMO multiuser CRNs under OSA model.

Chapter 5

Joint Optimization of Detection and Power Allocation in Multiuser MIMO CRNs*

In Chapter 3 and 4, we have investigated the optimization problems in SISO CRNs under both the OSA and SSS models, respectively. In this chapter, we increase the number of antennas in CRNs and consider a MIMO scenario under the OSA model.

MIMO is a powerful wireless technology that uses multiple antennas at the Tx and Rx to enable a variety of signal paths to carry the data. One of the core ideas in MIMO systems is the use of the spatial dimension inherent in the use of multiple spatially distributed antennas. By increasing the number of receive and transmit antennas, it is possible to linearly increase the throughput of the channel [117]. MIMO wireless technology is actually one of the most important wireless techniques that have been incorporated in recent standards such as LTE.

In the context of this thesis, the incorporation of MIMO techniques into CRNs can improve the channel capacity by sending independent data streams simultaneously over different antennas. There are some

*The publication associated to this chapter is [19] works that attempt to protect PUs in MIMO CRNs while maximizing the CRNs' throughput [34, 84–86]. However, due to the challenges associated to power allocation and spectrum optimization, all the existing works on MIMO CRNs do not consider the joint optimization over the sensing information.

In this chapter, we move a step ahead from current approaches, and consider the optimization problem in MIMO CRNs where the overall objective is to maximize the total throughput of each CR by jointly optimizing both the detection operation and the power allocation over all channels, under an interference constraint to the PUs. In order to reduce the complexity of the non-convex optimization problem*, we only consider the throughput of CR users under the correct sensing information, and exclude the throughput due to the erroneous decision of CR users to transmit over occupied channels. The optimization problem is analyzed as a strategic non-cooperative game, where the transmit covariance matrix, sensing time, and detection threshold are considered as multidimensional variables to be optimized. The resulting game is non-convex, hence, we use the new relaxed equilibrium concept QNE introduced in [31], and prove that the proposed game can achieve the unique QNE under certain conditions, by making use of the VI method. Furthermore, a Primal-Dual Interior Point Optimization in MIMO CRNs (PDIPM) that converges to the QNE is discussed in this chapter [19].

In the following, we present the system model in Section 5.1. The non-cooperative game is discussed in Section 5.2. The proof for the existence and uniqueness of the QNE, as well as the connection between optimal sensing time and equi-sensing time is shown in Section 5.3. Then, we outline the PDIPM in Section 5.4. The simulation results are presented in Section 5.5. Section 5.6 states the conclusions. For the sake of readability, Table 5.1 presents the main notations used in this chapter. Matrices and vectors are indicated in boldface, and CR i denotes CR Tx-Rx pair i.

*For multiuser MIMO CRNs, theoretical analysis of multidimensional variables in multiple terms of the objective function is quite complicated, in this chapter we only consider one term (the throughput obtained from the correct sensing information) in the objective function.

Table 5.1: **Notation for multiuser MIMO CRNs**

Symbol	Meaning
Rx	Receiver
Tx	Transmitter
det(.)	Determinant
Tr(.)	Trace
$(.)^{-1}$	Inverse
t	Sensing time
L	Number of antennas
T	Length of the frame
$P_{\max}^{i}$	Power budget of CR i
N	Number of PU
M	Number of CR pairs
τ	Detection threshold
Diag(.)	Diagonal matrix
$\mathcal{P}_{d}$	Probability of detection
$\mathcal{P}_{fa}$	Probability of false alarm
$P_{mask,k}$	Interference mask for the PU
γ_{k}^{i}	SNR from the PU to CR-Rx i in channel k
$\|\|\mathbf{x}\|\|_{2}$	Euclidean norm of vector $\mathbf{x}$
$\|\|\mathbf{x}\|\|_{\infty}$	Maximum norm of vector $\mathbf{x}$
$\mathbb{R}_{+}^{n}$	Nonnegative n-dimensional space
$(.)^{H}$	Hermitian matrix transpose
$T(X^{i};x^{i})$	Tangent cone of the set X^{i} at x^{i}
$H_{0,k}$	Channel k is detected to be idle
$H_{1,k}$	Channel k is detected to be occupied
$\nabla_{\mathbf{x}}^{2}R(\mathbf{x})$	Hessian matrix of function $R(\mathbf{x})$
$\nabla_{\mathbf{x}}R(\mathbf{x})$	Gradient of function $R(\mathbf{x})$ at point $\mathbf{x}$
$J_{R(\mathbf{x})}$	Jacobian matrix of the vector function $R(\mathbf{x})$
$\mathbf{Q}_{k}^{i}$	Transmission covariance matrix of CR i in channel k
$\mathbf{C}_{k}^{i}$	Total interference observed by CR-Rx i in channel k
$\mathbf{H}_{k}^{ii}$	Channel matrix in channel k between CR-Tx i and Rx i
$\mathbf{H}_{k}^{ji}$	Channel matrix in channel k between CR-Tx j and Rx i
$\mathbf{G}_{k}^{i}$	Channel matrix in channel k between CR-Tx i and PU

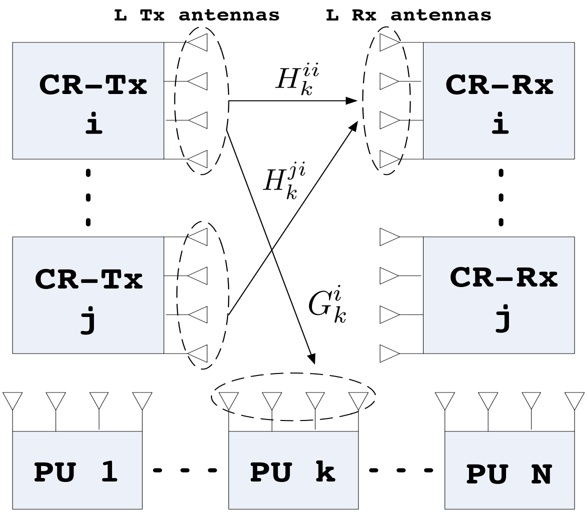

Figure 5.1: System model for MIMO CRNs.

5.1 System Model

5.1.1 System model for multiuser MIMO CRNs

We consider a multiuser environment of M CR Tx-Rx pairs and N PUs, where each PU uses a different channel (PU k uses channel k, $k = 1, ..., N$). The various systems coexisting in the network (the primary system and the CR system) do not cooperate with each other, and the CR users compete against each other to maximize their own performance for the same resources. The spectrum to be allocated is comprised of N OFDM channels, and each node is equipped with L antennas, as shown in Figure 5.1. Each CR can simultaneously communicate over multiple channels, thus, multiuser interference (MUI) from different CR users in the same channel must be taken into account. We impose a half-duplex constraint on all transmissions, meaning that a CR can not transmit and receive the data at the same time. Each CR-Tx can send up to L independent data streams on its L antennas over a given channel. A node controls the emitted antenna pattern and the power allocation for these streams through its precoding matrices. We assume that each CR-Rx is able to estimate the CSI from its intended Tx and the overall

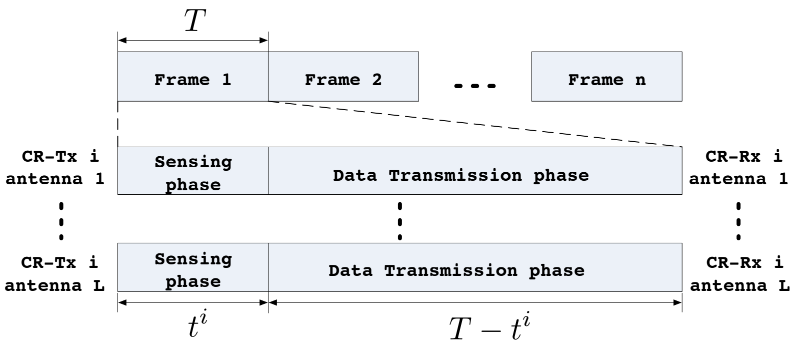

Figure 5.2: Frame structure of conventional sensing-based spectrum sharing with L antennas for MIMO CR i.

MUI covariance matrix. Furthermore, each CR-Tx is able to estimate the CSI from all the PUs.

5.1.2 Spectrum sensing

Consider the interweave communication under the OSA model, where CR users are able to adapt their power allocation depending on the sensing information and decide to transmit if the channel is detected to be idle. The frame structure of CR i consists of a sensing slot of duration t^i and a data transmission slot of duration $T - t^i$ over the L spatial sub-channels. If no PU is detected to be present during the sensing slot, the CR-Tx sends data in the transmission slot (the frame structure is shown in Figure 5.2). In the context of dynamic spectrum sharing, multiple antenna CR users can be used for a reliable signal transmission as well as spectrum sensing. In fact, using multiple antenna techniques in CR users is an important approach for spectrum sensing by exploiting available spatial domain observations [118–122].
In [118], the energy detector has been proposed for spectrum sensing by using multiple antennas. The PU signal has been treated as an unknown deterministic signal and based on this model the performance of the energy detector has been evaluated in Rayleigh fading channels. In [119], a blind energy detector based on SNR maximization has been proposed and its performance has been evaluated in different cases. The author of [120, 121] proposed a Generalized Likelihood Ratio detector, in which all the parameters are unknown, this is a blind and invariant detector

with a low computational complexity. A summary of multiple antenna spectrum sensing techniques is given in [122].

In this chapter, we consider first the spectrum sensing problem by using multiple antennas when the PU signal is assumed to be modeled as a complex Gaussian random signal in the presence of an Additive White Gaussian Noise (AWGN). We assume that simultaneous spectrum sensing of each frequency channel is performed by multiple antennas at each CR-Rx using an energy detection scheme. The detection problem on each frequency channel k is modeled as a hypothesis test, given by:

$$H_{0,k}: \ \ \mathbf{y}_k^i(l) = \mathbf{n}_k(l) \tag{5.1}$$

$$H_{1,k}: \ \ \mathbf{y}_k^i(l) = \mathbf{S}_k^i(l) + \mathbf{n}_k(l) \tag{5.2}$$

where $\mathbf{y}_k^i(l) \in \mathbb{C}^{L\times 1}$ denotes the received signal, $\mathbf{n}_k(l) \in \mathbb{C}^{L\times 1}$ denotes additive background noise on the kth channel, which is assumed to be independent and identically distributed additive complex Gaussian with zero mean and variance $(\sigma_{k,n}^i)^2$, i.e. $\mathcal{N}(0, (\sigma_{k,n}^i)^2\mathbf{I})$, $\mathbf{S}_k^i(l) = \mathbf{G}_k^i\mathbf{s}_k(l)$ stands for the PU transmit signal in channel k, where $\mathbf{s}_k(l) \in \mathbb{C}^{L\times 1} \sim \mathcal{N}(0, \gamma_k\mathbf{I})$ is a column vector of L information symbols, γ_k is the variance of symbol $\mathbf{s}_k$, and $\mathbf{G}_k^i \in \mathbb{C}^{L\times L}$ is the channel matrix on channel k from PU to CR-Rx i. Then, the covariance matrix of received signal $\mathbf{y}_k^i$ is:

$$\mathbf{Y}_k^i = \gamma_k\mathbf{G}_k^i(\mathbf{G}_k^i)^H + (\sigma_{k,n}^i)^2\mathbf{I} \tag{5.3}$$

Let $L_s = tf_s$ denote the number of samples, where t is the sensing time and f_s represents the sampling frequency. Under an energy detection scheme, for each frequency channel k, the statistics is defined as the sum of the received energy over an interval of L_s samples over each channel, and the decision is based on:

$$\sum_{l=1}^{L_s} \mathrm{Tr}(\mathbf{Y}_k^i) \gtrless_{H_{0,k}}^{H_{1,k}} \tau_k^i, \quad k = 1, 2, \ldots, N. \tag{5.4}$$

where τ_k^i denotes the decision threshold. According to the Central Limit Theorem, for large L_s, $\mathbf{Y}_k^i$ are approximately normally distributed: $\mathbf{Y}_k^i \sim \mathcal{N}(\mu_{k,0}^i, (\sigma_{k,0}^i)^2)$ for $H_{0,k}$, and $\mathbf{Y}_k^i \sim \mathcal{N}(\mu_{k,1}^i, (\sigma_{k,1}^i)^2)$ for $H_{1,k}$, where:

$$\mathcal{N}(\mu_{k,0}^i,(\sigma_{k,0}^i)^2) = \begin{cases} \mu_{k,0}^i = L_s L(\sigma_{k,n}^i)^2 \\ (\sigma_{k,0}^i)^2 = L_s L^2(\sigma_{k,n}^i)^4 \end{cases} \tag{5.5}$$

$$\mathcal{N}(\mu_{k,1}^i,(\sigma_{k,1}^i)^2) = \begin{cases} \mu_{k,1}^i = L_s(L(\sigma_{k,n}^i)^2 + \gamma_k \operatorname{Tr}(\mathbf{G}_k^i(\mathbf{G}_k^i)^H)) \\ (\sigma_{k,1}^i)^2 = L_s(L(\sigma_{k,n}^i)^2 + \gamma_k \operatorname{Tr}(\mathbf{G}_k^i(\mathbf{G}_k^i)^H))^2 \end{cases} \tag{5.6}$$

The probabilities of detection $\mathcal{P}_{k,d}^i$ and false alarm $\mathcal{P}_{k,fa}^i$ on the kth channel for multiple antenna CR-Rx i, $i = 1, 2, ..., M$, are expressed in closed forms, respectively, as:

$$\mathcal{P}_{k,fa}^i(\tau_k^i, t) = \mathcal{Q}\left(\frac{\tau_k^i - \mu_{k,0}^i}{\sigma_{k,0}^i}\right) \tag{5.7}$$

$$\mathcal{P}_{k,d}^i(\tau_k^i, t) = \mathcal{Q}\left(\frac{\tau_k^i - \mu_{k,1}^i}{\sigma_{k,1}^i}\right) \tag{5.8}$$

5.2 Problem Formulation

Formally, for channel k, let $\mathbf{q}_k^i \in \mathbb{C}^{L\times 1}$ be a column vector of L information symbols sent from CR-Tx i to its destination node CR-Rx i. Each element of $\mathbf{q}_k^i$ belongs to one data stream. In our scenario, several CR users can simultaneously occupy the same channel, thus MUI is taken in to consideration. Specifically, for channel k, the received signal $\mathbf{z}_k^i \in \mathbb{C}^{L\times 1}$ at the CR-Rx i, $i = 1, 2, ..., M$, is given by:

$$\mathbf{z}_k^i = \mathbf{H}_k^{ii}\mathbf{q}_k^i + \sum\nolimits_{j=1, j\neq i}^{M} \mathbf{H}_k^{ji}\mathbf{q}_k^j + \mathbf{n}_k \tag{5.9}$$

where $\mathbf{H}_k^{ii} \in \mathbb{C}^{L\times L}$ is the channel matrix on channel k from CR-Tx i to the intended CR-Rx i, and $\mathbf{H}_k^{ji} \in \mathbb{C}^{L\times L}$ is the cross-channel matrix corresponding to channel k from CR-Tx j to CR-Rx i. The elements in $\mathbf{H}_k^{ii}$ and $\mathbf{H}_k^{ji}$ are complex Gaussian variables with zero mean and unit variance. The first term on the right-hand side is the desired signal sent from CR-Tx i, the second term represents the MUI from other CR-Tx that share the channel k. For the sake of simplicity, we consider here only the case where the channel matrices $\mathbf{H}_k^{ii}$ and $\mathbf{H}_k^{ji}$ are square nonsingular.

5.2.1 Total achievable throughput of the CR users

The opportunistic achievable throughput of the CR i for a given set of user's covariance matrices $\mathbf{Q}_k^1, \mathbf{Q}_k^2, ..., \mathbf{Q}_k^M$, is denoted as $R^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i)$, and can be formulated as:

$$\left(1 - \frac{t^i}{T}\right) \sum_{k=1}^{N} (1 - \mathcal{P}_{k,fa}^i(\tau_k^i, t^i)) \log\det\left(\mathbf{I} + (\mathbf{C}_k^i)^{-1} \mathbf{H}_k^{ii} \mathbf{Q}_k^i (\mathbf{H}_k^{ii})^H\right) \tag{5.10}$$

where $\mathbf{Q}^i = (\mathbf{Q}_k^i)_{k=1}^N$, $\boldsymbol{\tau}^i = (\tau_k^i)_{k=1}^N$, and $\mathbf{Q}_k^i$ denotes the covariance matrix of the symbols transmitted by CR-Tx i on channel k. $\mathbf{C}_k^i$ is the noise-plus-interference covariance matrix at CR-Rx i over channel k, given by:

$$\mathbf{C}_k^i = \mathbf{I} + \sum_{j=1, j\neq i}^{M} \mathbf{H}_k^{ji} \mathbf{Q}_k^j (\mathbf{H}_k^{ji})^H \tag{5.11}$$

Observe that $\mathbf{C}_k^i$ depends on the strategies of all the other CR-Txs, except CR -Tx i.

5.2.2 Constraints for the CR users

For CR-Tx i, the total transmit power over all channels should not exceed its maximum allowed power $P_{\max}^i$. Consequently, the power budget constraint can be formulated as:

$$\sum\nolimits_{k=1}^{N} \mathrm{Tr}(\mathbf{Q}_k^i) \leq P_{\max}^i \tag{5.12}$$

Furthermore, there are two methods to effectively protect the PU from harmful performance degradation in a MIMO network: null constraints and soft-shaping constraints [34]. A null constraint is given as:

$$\mathbf{V}_i^H \mathbf{Q}_i = 0 \tag{5.13}$$

where $\mathbf{V}_i^H$ is a strict tall matrix*, whose columns represent the spatial directions along which CR i is not allowed to transmit. The structure of the null constraint expresses the strict limitation imposed on CR users

*$\mathbf{A}_{m\times n}$ is a strict tall matrix, if $m > n$.

to prevent them from transmitting over the channels occupied by the PU. The use of the spatial domain can greatly improve the capabilities of CR users, as it allows them to transmit over the same channel without interfering to each other. This is possible became the CR-Tx has an antenna array and uses a beamforming that sets nulls over the directions identified with the PU.
The soft-shaping constraint allows interference in specific channels, that is, generated from the transmissions of the CR, as long as it falls below a certain power mask for PUs. It can be considered as a relaxed form of (5.13), where the CR can transmit over some channels occupied by the PUs, provided that the interference to the PUs is bounded. There are two kinds of soft-shaping constraint: individual and global. The individual constraint requires that the transmit power of each CR on channel k is always less than a given power mask. Instead of specifying an individual constraint on the transmit powers of each CR, the global constraint adapts the transmit powers of each CR-Tx depending also on the activity of other CR users that share the channel, so that the accumulated interference from all CR users at the PU does not exceed a threshold. Though the global constraint may result in higher network throughput, it requires coordination among CR users with a large information exchange between them.
Considering that the sensing information is not always reliable and that the CR users are not willing to exchange information, in our scenario, the individual soft-shaping constraint is denoted as:

$$(1 - \mathcal{P}^i_{k,d}(\tau^i_k, t^i)) \, \mathrm{Tr}(\mathbf{G}^i_k \mathbf{Q}^i_k (\mathbf{G}^i_k)^H) \leq P_{mask,k} \tag{5.14}$$

where $\mathbf{G}^i_k \in \mathbb{C}^{L \times L}$ is the channel matrix for CR-Tx i on channel k, and $\mathbf{P}_{mask} = [P_{mask,k}]_{k=1}^N$ denote the power mask on all channels. In addition, based on the target sensing accuracy, we have the following linear constraint:

$$\tau^i_{k,min} \leq \tau^i_k \leq \tau^i_{k,\max} \tag{5.15}$$

where $\tau^i_{k,min} = \mu^i_{k,0}$, $\tau^i_{k,\max} = \mu^i_{k,1}$.

5.2.3 Optimization problem

Under the above discussion, the optimization problem of maximizing the total opportunistic throughput of CR i over all channels can be formulated as problem $P5.1$:

$$\begin{aligned}
\max_{\mathbf{Q}^i,\boldsymbol{\tau}^i,t^i} \quad & R^i(\mathbf{Q}^i,\boldsymbol{\tau}^i,t^i) \\
\text{s. t.} \quad & (a1)\ \ (1-\mathcal{P}^i_{k,d}(\tau^i_k,t^i))\,\mathrm{Tr}(\mathbf{G}^i_k\mathbf{Q}^i_k(\mathbf{G}^i_k)^H) \le P_{mask,k} \\
& (b1)\ \ \sum\nolimits_{k=1}^{N} \mathrm{Tr}(\mathbf{Q}^i_k) \le P^i_{\max} \\
& (b2)\ \ \tau^i_{k,min} \le \tau^i_k \le \tau^i_{k,\max} \\
& (b3)\ \ 0 \le t^i \le T, \quad k = 1,2,\ldots,N.
\end{aligned} \tag{5.16}$$

where $(a1)$ are the non-convex constraint sets, and $(b1),(b2),(b3)$ are the convex constraint sets. Each CR i aims at maximizing its own throughput under the power budget constraint (b1) and the interference constraint (a1). Both the power budget constraint and the interference constraint are individual constraints, meaning that the CR users are allowed to choose their power allocation individually and there is no information exchange between CR users.

5.3 QNE for Non-Convex Game in MIMO CRNs

Due to the non-convex nature of the problem, the traditional methods proposed in the literature can not address our problem. In addition, to the best of our knowledge, there is no literature considering the joint power allocation and detection optimization problem in multiuser MIMO CRNs. Theoretically speaking, the throughput region can still be found by an exhaustive search through all possible feasible covariance matrices. However, the computational complexity of this approach is prohibitively high, given the large number of variables and CR users involved in the optimization problem.

In Chapter 4, we introduced the new concept QNE, for the non-convex problem in SISO CRNs, and proved the existence of the QNE. In this section, we provide the sufficient conditions to ensure the existence and uniqueness of the QNE for the non-convex problem in MIMO CRNs by using the VI method. In addition, the PDIP method is proposed for

MIMO CRNs (PDIPM), which is shown in the simulations to converge to the QNE.

5.3.1 Equivalent reformulation of game theory

According to the inherently competitive nature of distributed multiuser MIMO CRNs, game theory is adopted to solve the non-convex non-cooperative problem for MIMO CRNs. Using the concept of QNE as the competitive optimality criterion, the resource allocation problem among CR users is then reformulated as a strategic non-cooperative game. In order to simplify the game, we start from the two multidimensional variables case, i.e., $\mathbf{Q}^i$ and $\boldsymbol{\tau}^i$, the sensing time t^i is not considered as a multidimensional variable, and finally we optimize t^i by exhaustive search. In general, the optimal sensing times of CR users are different. For the sake of simplicity, in problem $P5.2$ we ignore the degradation in the sensing process due to the interference generated by the transmitting CR-Tx.

Assume that there are M players, corresponding to the M CR-Txs, each one controlling the variables $\mathbf{x}^i = (\mathbf{Q}^i, \boldsymbol{\tau}^i)$, $i = 1, ..., M$. Let $\mathbf{x}^{-i} = (\mathbf{x}^1, ..., \mathbf{x}^{i-1}, \mathbf{x}^{i+1}, ..., \mathbf{x}^M)$ be the set of strategies from all the CR-Txs, except CR-Tx i. The utility function for each CR-Tx is the total opportunistic throughput, given as:

$$U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) = t_c^i \sum_{k=1}^{N} (1 - \mathcal{P}_{k,fa}^i(\tau_k^i)) \log\det\left(\mathbf{I} + (\mathbf{C}_k^i)^{-1} \mathbf{H}_k^{ii} \mathbf{Q}_k^i (\mathbf{H}_k^{ii})^H\right) \tag{5.17}$$

where $t_c^i = 1 - \frac{t^i}{T}$, is considered as a constant to be optimized at a later step. Each CR-Tx competes against the others by choosing the transmit covariance matrix $\mathbf{Q}^i$ and the associated threshold $\boldsymbol{\tau}^i$ (i.e., its strategies) that maximize its own total throughput, given the constraints imposed by the presence of the PU, besides the usual constraint on transmit power budget. Each individual non-cooperative game can be formulated as problem $P5.2$:

$$\begin{aligned} \max_{\mathbf{Q}^i, \boldsymbol{\tau}^i} \quad & U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) \\ \text{s. t.} \quad & (1 - \mathcal{P}_{k,d}^i(\tau_k^i)) \operatorname{Tr}(\mathbf{G}_k^i \mathbf{Q}_k^i (\mathbf{G}_k^i)^H) \leq P_{mask,k}, \end{aligned}$$

$$\sum_{k=1}^{N} \mathrm{Tr}(\mathbf{Q}_k^i) \leq P_{\max}^i,$$
$$\tau_{k,\min}^i \leq \tau_k^i \leq \tau_{k,\max}^i, \quad k = 1, 2, \ldots, N. \tag{5.18}$$

5.3.2 VI method and KKT conditions

In the case of a convex game in MIMO CRNs, where all the frame length is used for transmission, there is always a NE for any set of channel matrices and power constraints, when each CR-Tx, given the strategy profiles of the other CR-Txs, does not get any throughput increase by unilaterally changing its own strategy. The best response of each CR-Tx for the convex game can be efficiently computed via MIMO waterfilling like solutions. However, the absence of convexity leads to the non-existence of a NE, and no mathematical tool is currently available to show the existence of the NE. In the following, we use the concept of QNE, introduced in Chapter 4, and prove that the proposed non-convex game in MIMO CRNs always admit a unique QNE, which coincides with the NE.

From the original optimization problem $P5.1$, we denote the non-convex individual constraints $(a1)$ as $\mathbf{H}_{\mathbb{C}}(\mathbf{x}) = [h_{\mathbb{C}}^i(\mathbf{x}^i)]_{i=1}^M$. The convex individual constraints $(b1), (b2)$ are denoted as $\tilde{\mathbf{G}}_{\mathbb{C}}(\mathbf{x}) = [(\tilde{g}_k^i(\mathbf{x}^i))_{k=1}^N]_{i=1}^M$, and embeded in the definition set of $\mathbf{x}^i$, denoted as $\mathbf{X}_{\mathbb{C}}^i$. Thus, the non-convex game $\mathcal{G}_{\mathbb{C}}(\mathbf{H}_{\mathbb{C}}, \tilde{\mathbf{G}}_{\mathbb{C}})$ can be denoted as problem $P5.3$:

$$\begin{aligned} \max_{\mathbf{Q}^i, \boldsymbol{\tau}^i} \quad & U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) \\ \text{s. t.} \quad & h_{\mathbb{C}}^i(\mathbf{x}^i) < 0, \mathbf{x}^i \in \mathbf{X}_{\mathbb{C}}^i \end{aligned} \tag{5.19}$$

Let $\mathcal{Y}_{\mathbb{C}}^i$ denote the feasible strategy set of each CR i, which can be written as:

$$\mathcal{Y}_{\mathbb{C}}^i = \{\mathbf{x}^i \in \mathbf{X}_{\mathbb{C}}^i \mid h_{\mathbb{C}}^i(\mathbf{x}^i) \leq 0\}, \; 1 \leq k \leq N. \tag{5.20}$$

Instead of explicitly accounting all the multipliers as variables of the KKT conditions for each CR-Tx's optimization problem, we introduce multipliers only for the non-convex constraints $h_{\mathbb{C}}^i(\mathbf{x}^i) \leq 0$, and the convex constraints are embedded in the defining set $\mathbf{X}_{\mathbb{C}}^i$. Denoting by α_k^i the multipliers associated with the non-convex constraints $h_{\mathbb{C}}^i(\mathbf{x}^i) \leq 0$ of CR-Tx i, for CR i, the Lagrange function of the problem $P5.3$ can be written as:

$$L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i) = -U^i(\mathbf{x}^i) + \boldsymbol{\alpha}^i h^i_{\mathbb{C}}(\mathbf{x}^i) \tag{5.21}$$

Furthermore, the KKT conditions for CR i are given by:

$$\begin{aligned}
&-\nabla_{\mathbf{Q}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) + \alpha^i_k (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) \operatorname{Tr}(\mathbf{G}^i_k (\mathbf{G}^i_k)^H) = 0 \\
&-\nabla_{\boldsymbol{\tau}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) - \alpha^i_k \nabla_{\tau^i_k} \mathcal{P}^i_{k,d}(\tau^i_k) \operatorname{Tr}(\mathbf{G}^i_k \mathbf{Q}^i_k (\mathbf{G}^i_k)^H) = 0 \\
&\alpha^i_k \left[(1 - \mathcal{P}^i_{k,d}(\tau^i_k)) \operatorname{Tr}(\mathbf{G}^i_k \mathbf{Q}^i_k (\mathbf{G}^i_k)^H) - P_{mask,k} \right] = 0
\end{aligned} \tag{5.22}$$

where $\nabla_{\mathbf{Q}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i), \nabla_{\boldsymbol{\tau}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i)$ denote the complex matrix derivative of $U^i(\mathbf{x}^i)$ with respect to $\mathbf{Q}^i$ and $\boldsymbol{\tau}^i$, respectively, which are given by:

$$\begin{aligned}
&\nabla_{\mathbf{Q}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) \\
&= t^i_c \sum\nolimits_{k=1}^{N} (1 - \mathcal{P}^i_{k,fa}(\tau^i_k)) \mathbf{H}^{ii}_k \left(\mathbf{C}^i_k + \mathbf{H}^{ii}_k \mathbf{Q}^i_k (\mathbf{H}^{ii}_k)^H \right)^{-1} (\mathbf{H}^{ii}_k)^H
\end{aligned} \tag{5.23}$$

$$\begin{aligned}
&\nabla_{\boldsymbol{\tau}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) \\
&= t^i_c \sum\nolimits_{k=1}^{N} -\nabla_{\tau^i_k} \mathcal{P}^i_{k,fa}(\tau^i_k) \log\det\left(I + (\mathbf{C}^i_k)^{-1} \mathbf{H}^{ii}_k \mathbf{Q}^i_k (\mathbf{H}^{ii}_k)^H \right)
\end{aligned} \tag{5.24}$$

The transmission covariance matrix $\mathbf{Q}^i$, the detection threshold $\boldsymbol{\tau}^i$ and the constraint multipliers $\boldsymbol{\alpha}^i[\alpha^i_k]^N_{k=1}$ are accounted as variables of the problem. Note that, if some CQs are not satisfied, the KKT conditions may not be valid necessary conditions for the non-convex game. More specifically, if $\mathbf{x}^\star$ are the stationary solutions of game $\mathcal{G}_{\mathbb{C}}(\mathbf{H}_{\mathbb{C}}, \tilde{\mathbf{G}}_{\mathbb{C}})$, and some CQ holds at $\mathbf{x}^\star$, the KKT conditions (5.22) can be reformulated to the equivalent form:

$$\begin{pmatrix} \mathbf{Q} - \mathbf{Q}^\star \\ \boldsymbol{\tau} - \boldsymbol{\tau}^\star \\ \alpha_k - \alpha^\star_k \end{pmatrix}^T \underbrace{\begin{pmatrix} \nabla_{\mathbf{Q}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) - \alpha^i_k (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) \operatorname{Tr}(\mathbf{G}^i_k (\mathbf{G}^i_k)^H) \\ \nabla_{\boldsymbol{\tau}^i} U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i) + \alpha^i_k \nabla_{\tau^i_k} \mathcal{P}^i_{k,d}(\tau^i_k) \operatorname{Tr}(\mathbf{G}^i_k \mathbf{Q}^i_k (\mathbf{G}^i_k)^H) \\ -P_{mask,k} + (1 - \mathcal{P}^i_{k,d}(\tau^i_k)) \operatorname{Tr}(\mathbf{G}^i_k \mathbf{Q}^i_k (\mathbf{G}^i_k)^H) \end{pmatrix}^M_{i=1}}_{\boldsymbol{\Theta}_{\mathbb{C}}(\mathbf{x}^\star, \boldsymbol{\alpha}^\star)}$$

$$\leq 0, \quad (\mathbf{x}^i, \boldsymbol{\alpha}^i) \in \underbrace{\prod_{i=1}^{M} \mathbf{X}^i_{\mathbb{C}} \times \mathbb{R}^r_+}_{\mathbf{Y}_{\mathbb{C}}} \tag{5.25}$$

The above system of inequalities defines a VI problem with variables $(\mathbf{x}, \boldsymbol{\alpha})$, denoted as $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, where the vector function $\boldsymbol{\Theta}_{\mathbb{C}}$ and feasible set $\mathbf{Y}_{\mathbb{C}}$ are defined in (5.25). This $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ is an equivalent reformulation of the KKT conditions (5.22). The convex constraints $(b1), (b2)$ are embedded in the complex defining set $\mathbf{Y}_{\mathbb{C}}$, $\mathbf{Y}_{\mathbb{C}} = \prod_{i=1}^{M} \mathbf{X}_{\mathbb{C}}^{i} \times \mathbb{R}_{+}^{r}$, where $\mathbf{X}_{\mathbb{C}}^{i}$ stands for the complex convex constraints $(b1), (b2)$ defined in the problem $P5.1$, and r is the total number of multipliers $\boldsymbol{\alpha}$.
In order to show that the KKT conditions are valid necessary conditions for an optimal solution of problem $P5.2$, as shown in [98], we need to verify that the some CQs hold. Since the gradients of the constraints (5.12), (5.14), and (5.15) are linearly independent at every $\mathbf{x}^{i,\star}$, following a similar approach as given in Chapter 4, we can show that the LICQ holds at $\mathbf{x}^{i,\star}$ [20, 98], and we can conclude that the KKT conditions are necessary conditions for an optimal solution of problem $P5.3$.

Definition 4. *The game $P5.3$ is equivalent to the VI problem in the complex domain, denoted as $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, which consists in finding complex variables: $\mathbf{x}^{\star}$ and multipliers $\boldsymbol{\alpha}^{\star}$ of the non-convex constraints* (5.14) *such that $< \mathbf{z} - \mathbf{z}^{\star}, \boldsymbol{\Theta}_{\mathbb{C}}(\mathbf{z}^{\star}) > \geq 0$, $\mathbf{z}^{\star} = (\mathbf{x}^{\star}, \boldsymbol{\alpha}^{\star})$ for all $\mathbf{x} \in \mathbf{X}_{\mathbb{C}}$.*

where $< .,. >$: $\mathbb{C}^{L\times L} \times \mathbb{C}^{L\times L}$ is the inner product, defined as $< \mathbf{A}, \mathbf{B} > = \mathrm{Re}(\mathrm{Tr}(\mathbf{A}^{H}\mathbf{B}))$, and $\mathrm{Re}(\mathbf{A})$ denotes the real part of complex matrix $\mathbf{A}$. In addition, if $(\mathbf{x}^{\star}, \boldsymbol{\alpha}^{\star})$ is the solution of the $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, there exists $\boldsymbol{\beta}^{\star}$ such that $(\mathbf{x}^{\star}, \boldsymbol{\alpha}^{\star}, \boldsymbol{\beta}^{\star})$ is a solution of the game, $\boldsymbol{\beta}^{\star}$ are the multipliers associated with the CR-Txs' convex constraints $(b1), (b2)$ [32].

5.3.3 The existence and the uniqueness of QNE

In this section we focus on the existence and the uniqueness of the solution of problem $P5.3$. According to **Theorem 1** in Chapter 4 for the SISO CRNs, we could see that the existence of the QNE for problem $P5.3$ is associated with the properties of the Hessian matrix $\nabla^{2}_{\mathbf{x}^{i}} h^{i}_{\mathbb{C}}(\mathbf{x}^{i})$, as well as with the feasible set of the variables $\mathbf{x}^{i}$. Hence, we have the following theorem for MIMO CRNs.

Theorem 3. *The $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ has a solution, thus the game $\mathcal{G}_{\mathbb{C}}(\mathbf{H}_{\mathbb{C}}, \tilde{\mathbf{G}}_{\mathbb{C}})$ has a QNE, which is nontrivial.*

Proof. See Appendix D. □

The uniqueness of the QNE for the problem $P5.3$ needs an appropriate second-order sufficiency condition. We follow a similar approach as in [21, 33], and provide the following theorem.

Theorem 4. *If the Hessian matrix of (5.21), denoted as* $\nabla^2_{\mathbf{x}^i} L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ *is positive definite for all* $\mathbf{x}^i \in \mathbf{X}^i_{\mathbb{C}}$ *and* $\boldsymbol{\alpha}^i \in \mathbb{R}^r_+$*, then, the non-convex optimization problem* $P5.3$ *for each CR* i *has a unique optimal solution* $\mathbf{x}^{i,\star} \in \mathbf{X}^i_{\mathbb{C}}$*. In addition, the* $\nabla^2_{\mathbf{x}^i} L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ *is positive definite, if the following sufficient condition is satisfied:*

$$\frac{\max\limits_{k=1,\ldots,N} \frac{\boldsymbol{\alpha}^i}{\sqrt{2\pi}} \operatorname{Tr}(\mathbf{G}^i_k (\mathbf{G}^i_k)^H)}{\min\left(\operatorname{Re}\left(\lambda_{\min}(-\nabla^2_{\mathbf{Q}^i} U^i(\mathbf{x}^i)), \lambda_{\min}(-\nabla^2_{\boldsymbol{\tau}^i} U^i(\mathbf{x}^i))\right)\right)} < 1 \tag{5.26}$$

Proof. See Appendix E. □

$\lambda_{\min}(-\nabla^2_{\mathbf{Q}^i} U^i(\mathbf{x}^i))$ and $\lambda_{\min}(-\nabla^2_{\boldsymbol{\tau}^i} U^i(\mathbf{x}^i))$ denote the minimum eigenvalues of matrix $-\nabla^2_{\mathbf{Q}^i} U^i(\mathbf{x}^i)$ and $-\nabla^2_{\boldsymbol{\tau}^i} U^i(\mathbf{x}^i)$, respectively. This condition quantifies how much MUI can be tolerated by the systems to guarantee the existence and the uniqueness of the QNE, meaning that when the interference from the CR-Tx to the PU is sufficiently small (satisfying the condition (5.26)), the non-convex problem $P5.3$ has a unique solution. Hence, when condition (5.26) is satisfied, the $VI_{\mathbb{C}}(\mathbf{X}_{\mathbb{C}}, \mathbf{F}_{\mathbb{C}})$ admits a unique solution. From the definition of QNE in Chapter 4, we can state that the non-cooperative MIMO game always admits a unique QNE, which coincides with the NE.

5.3.4 Equi-sensing time for all the CR users

The decision model proposed so far is based on the assumption that only the PUs' signals are involved in the detection process performed by the CR-Rx, implying that the CR-Rx are somehow able to distinguish between PU and CR signaling. This can be naturally accomplished if there is a common sensing time during which all the CR users stay silent while sensing the spectrum. However, the joint optimization of the sensing and transmission parameters proposed in this chapter, in general, leads to different optimal sensing times of the CR users, implying that some CR users may start transmitting while some others are still in the sensing phase. Since the energy based detection is not able to discriminate between different received energy, this interference generated by the transmitting CR users in the same frequency channel would confuse the energy detector, and thus introduce a significant performance degradation in the sensing information.

To overcome this issue, two different directions can be explored. A first approach could be using more sophisticated signal processing techniques for the spectrum sensing that look into a PU signal (e.g., modulation type, data rate, pulse shaping, or other signal feature) to improve the detector robustness, at the cost of an increased complexity. This would allow the CR users to differentiate between PU signals, background noise, and multiuser interference in each channel. Depending on what a priori knowledge of the PU signal is known to the CR users, different feature detectors can be applied under different scenarios and complexity requirements [122]. For example, Advanced Television Systems Committee (ATSC) digital TV signal has narrow pilots for audio and video carriers; Code Division Multiple Access (CDMA) systems have dedicated spreading codes for pilot and synchronization channels; OFDM packets have preambles for packet acquisition. Under such a priori information, the optimal detection is given by the matched filter [40]. However, the better sensing performance of the matched filter is obtained at the cost of additional hardware complexity: the CR users would need a dedicated receiver for every PU class.

The second approach we propose is suitable for scenarios where feature detection is not implementable, and thus the energy detector is the only available option. In such a case, the only way for the CR users to distinguish the PU from the CR signals is to avoid overlapping CR transmissions during the sensing phase. This can be done by "forcing" the same sensing time for all the CR users, which still needs to be optimized. This equi-sensing model can be realized by two ways: centralized or distributed. In centralized optimization of sensing time, we could follow the exhausted search used in the previous section, and find the optimal sensing time when the total throughput of all the CR users is maximized. In our work, in order to keep the distributed nature of the optimization, based on the equi-sensing model in SISO CRNs [32], we modify the original game $P5.3$ as the following problem $P5.4$:

$$\begin{aligned} \max_{\mathbf{Q}^i,\boldsymbol{\tau}^i} \quad & U^i(\mathbf{Q}^i,\boldsymbol{\tau}^i) - c(t^i - \frac{1}{M}\sum_{j=1}^{M} t^j)^2 \\ \text{s. t.} \quad & h^i_{\mathbb{C}}(\mathbf{x}^i) < 0, \quad \mathbf{x}^i \in \mathbf{X}^i_{\mathbb{C}} \end{aligned} \tag{5.27}$$

The difference with respect to the previous formulation in problem $P5.3$ is that now, in the objective function of each CR-Tx, there is an additional term that works like a penalization in using different sensing times for the CR-Txs. Because of this penalization, we would expect that, for sufficiently large c, the equilibrium of the game tends to equal sensing times. This solution indeed is the one that minimizes the loss induced by the penalization in the payoff function of each CR-Tx.
Note that all the original sensing time t^i must be equal to the optimal sensing time $t^{i,\star}$, implying that there always exists a sufficiently large c such that each CR-Tx can reach a NE of game $P5.4$, when its sensing time is equal to other CR-Txs' sensing time and without the penalization. Moreover, such a common sensing time is the optimal time of maximizing the sum throughput of all the CR users in the original game $P5.4$.

5.4 Primal-Dual Interior Point Optimization in MIMO CRNs

In this section, we use the PDIPM, which follows a similar approach as for the SISO CRN. Firstly, the $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, given by (5.25) is reformulated to the equivalent constrained equations and solved by the PDIPM algorithm. The PDIPM algorithm combines a line search step that computes iterative steps by factoring the primal-dual equations, and a trust region step. Furthermore, the PDIPM algorithm requires no information exchange between CR users, thus, it can be regarded as a distributed solution.
We outline the PDIPM algorithm in **Algorithm 6**, where N_b is the maximum number of backtracking search steps. For our problem, we choose $\eta = 10^{-8}$, $\varepsilon = 10^{-6}$, and $N_b = 4$. The resulting algorithm is ensured to have global convergence, thus achieving a QNE of the game $P5.3$. For more details, we refer to Chapter 4, and [18, 99, 100].

5.4.1 Complexity analysis of the PDIPM algorithm

The complexity of the iterative PDIPM algorithm is dominated by the procedure of line search iteration steps and trust region iteration steps, as well as the size of the MIMO CRN. The number of antennas will also affect the time complexity of the algorithm. The total complexity of the PDIPM algorithm is given by $O_{PDIPM} = O\Bigg(\ln(\frac{1}{\varepsilon})M\sqrt{L(2N+M)}\Bigg) \sim O\Bigg(\ln(\frac{1}{\varepsilon})M((N_b+1)\sqrt{L(2N+M)} + L(2N+M))\Bigg)$.

Algorithm 6 Primal-Dual Interior Point Optimization in MIMO CRNs

Initialize $\mathbf{x}^i(0) = (\mathbf{Q}^i(0), \boldsymbol{\tau}^i(0))$, $\mathbf{z}^i(0) = (\mathbf{x}^i(0), \mathbf{s}^i(0))$. Compute initial values for the multipliers $\mathbf{u}^i(0) = (\boldsymbol{\alpha}^i(0), \boldsymbol{\beta}^i(0))$, the trust-region radius $\Upsilon^i(0) > 0$ and the barrier parameter $\mathbf{v}^i(0) > 0$.
repeat
  for i =1: M
    repeat
      repeat
      Compute the number N_e^i, set LS = 0
        if $N_e^i \leq 3N$
        Get the search direction $d(p) = (d_{\mathbf{z}^i}(p), d_{\mathbf{u}^i}(p))$
        Compute $\rho^i_{\mathbf{z}^i}$, $\rho^i_{\mathbf{u}^i}$
          if $\min\{\rho^i_{\mathbf{z}^i}, \rho^i_{\mathbf{u}^i}\} > \varepsilon$
          Set $j = 0, \rho^i_T = 1$
            repeat
              if $M_{c^i}(\mathbf{z}^i(p) + \rho^i_T \rho^i_{\mathbf{z}^i} d_{\mathbf{z}^i}(p))$
              $\leq M_{c^i}(\mathbf{z}^i(p)) + \eta \rho^i_T \rho^i_{\mathbf{z}^i} D_{M_{c^i}(\mathbf{z}^i(p))}$
              Update $\rho^i_{\mathbf{z}^i} = \rho^i_T \rho^i_{\mathbf{z}^i}$, $\rho^i_{\mathbf{u}^i} = \rho^i_T \rho^i_{\mathbf{u}^i}$
              Update $\mathbf{z}^i(p+1), \mathbf{u}^i(p+1)$
              Update $\Upsilon^i(p+1)$ and set LS = 1
              else
              Update $j = j + 1$, choose a smaller value of ρ^i_T
              endif
            until $j > N_b$ Or $\rho^i_T < \varepsilon$ Or LS == 1
          endif
        endif
        if LS == 0
        Compute $\mathbf{z}^i(p+1), \mathbf{u}^i(p+1)$ using the trust region method
        Compute $\Upsilon^i(p+1)$
        endif
        Set $\mathbf{v}^i(p+1) = \mathbf{v}^i(p)$
      until $\|\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \mathbf{u}^i)\|_\infty \leq \varepsilon$ and $\|\mathbf{S}^i e \Lambda^i - \mathbf{v}^i e\|_\infty \leq \varepsilon$
      Reset the barrier parameters, so that $\mathbf{v}^i(p+1) < \mathbf{v}^i(p)$
    until $\|\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \mathbf{u}^i)\|_\infty \leq \varepsilon$ and $\|\mathbf{S}^i \Lambda^i\|_\infty \leq \varepsilon$
    Update $\mathbf{x}^i(p_0) = \mathbf{x}^i(p)$
  endfor
until $\|\mathbf{x}^i(p_0) - \mathbf{x}^i(p_0 - 1)\|_2 \leq \varepsilon$

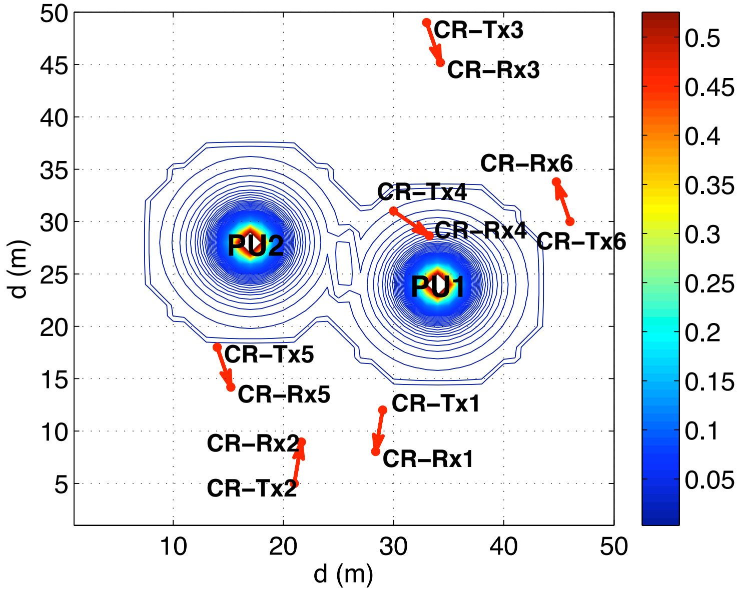

Figure 5.3: Radio environment map of two PUs and six CR Tx-Rx pairs

5.5 Simulation Results

5.5.1 Scenario description

Table 5.2: **Simulation parameters**

Symbol	Value
Frame length T	100ms
Antenna array size L	4
Sampling frequency, f_s	$2MHz$
Transmit power budget of CR i, $P^i_{\max}/(\sigma^i_{k,n})^2 d_{ii}$	$5dB$
Minimum SNR from PU at the CR-Rx γ^i_k	$-20dB$
Interference mask $\mathbf{P}_{mask}$	10^{-4}

We consider a CRN with $M = 6$ CR Tx-Rx pairs and $N = 2$ PU channels. The antenna array size is $L = 4$. All the PUs and CR users are randomly placed in a 50 meter $\times$ 50 meter square. The radio environment map is shown in Figure 5.3, where the color-bar shows the

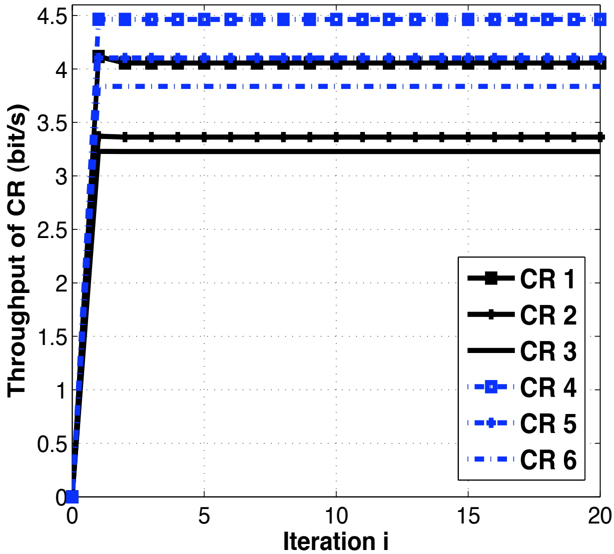

Figure 5.4: Throughput versus iteration number for different CR users

received power from PUs in Watt. We use the channel model from the 3GPP Indoor scenario for LTE [115]. The distance-dependent path loss is given by $PL_{dB} = 7 + 56\log_{10}(d)$, where $d = d_{ji}/d_{ii}$ (m) is the relative distance between CR-Tx j and CR-Rx i, where d_{ii} and d_{ji} are the distances between CR-Tx i and CR-Rx i, CR-Tx j and CR-Rx i, respectively. We consider the shadowing as a lognormal variable with variance $10dB$, $T = 100ms$, $f_s = 2MHz$, $(\sigma^i_{k,n})^2 = 1$, according to [123]. The minimum received SNR from PU in channel k at the CR-Rx i is equal to $\gamma^i_k = 10\log_{10}(P_{pu}/(\sigma^i_{k,n})^2 d_{\max}) = -20dB$, where P_{pu} is the transmission power of PU, and $d_{\max}$ is the longest relative distance between CR-Rx and PU. The maximum power at each CR-Tx is equal to $10\log_{10}(P^i_{\max}/(\sigma^i_{k,n})^2 d_{ii}) = 5dB$, and $\mathbf{P}_{mask} = 10^{-4}$ on all channels. As mentioned before, the noise from PU transmissions is treated as floor noise that together with the thermal noise are normalized to a unit variance. We assume that the sensing environment is stable during the time where the optimization process takes place. The main parameters used in the simulation are given in Table 5.2.

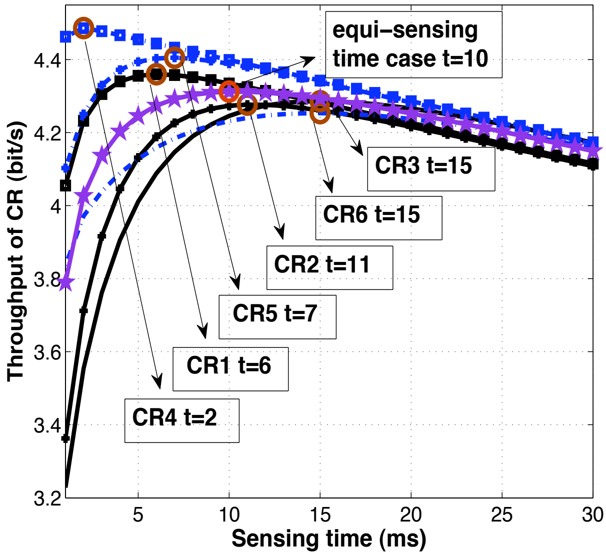

Figure 5.5: Throughput versus optimal sensing time for different CR users

5.5.2 Simulation results analysis

Figure 5.4 shows that all the CR users are able to achieve the QNE within a few iterations. Specifically, the nearby CR users, which are closer to the PUs, are able to achieve higher throughputs compared with the distant CR users, which are far from the PUs. Because all the CR users are bounded by the power budget constraint, the CR users can exhaust all the power budget without causing harmful interference to the PUs. Hence, the distant CR users have to increase the sensing time and decrease the detection threshold to satisfy the target $\mathcal{P}_{k,d}$ and $\mathcal{P}_{k,fa}$, yielding a decrease in the data transmission time and the throughput.
Figure 5.5 shows the (individual and equal) optimal sensing time versus the achievable throughput for each CR. The optimal t^i is highlighted in the figure. According to the result, there exists an optimal t^i for each CR at which its throughput is maximized. Moreover, as expected, the optimal t^i for the nearby CR users are smaller than the distant ones due to the target sensing accuracy, since higher sensing accuracy needs longer sensing time. Specifically, in the figure, we highlight the optimal equi-sensing time for all the CR users from the problem $P5.4$, when the

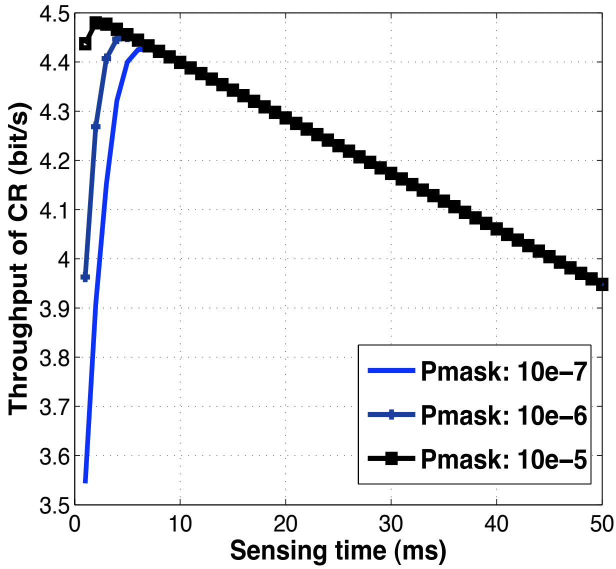

Figure 5.6: Throughput versus sensing time for different values of P_{mask}

average throughput of the CRNs, denoted as $(1/M)\sum_{i=1}^{M} U^i$, is maximized. As it is shown in the figure, in general, the optimal equi-sensing time in the problem $P5.4$ is not the same as the optimal sensing time for each CR. The problem $P5.4$ focus more on the sum-throughput of all the CR users in the original problem $P5.3$, while keeping all the constraints fixed.

In Figure 5.6, we plot the throughput achieved at QNE by one CR versus the optimal sensing time for different values of the interference constraint bound. It can be observed that the optimal sensing time increases as the interference constraints become more stringent. More specifically, more stringent interference constraints impose lower missed detection probabilities as well as false alarm rates, which require more accurate detection information by increasing the sensing time, leading to the degradation in the throughput.

Figure 5.7 compares the performance achieved by the proposed sensing based game with those achieved by the deterministic game in [34]. For the deterministic game, all the frame length is used for the transmission, the sensing information is not considered as a part of optimization, and deterministic interference constraints are applied to all the CR users.

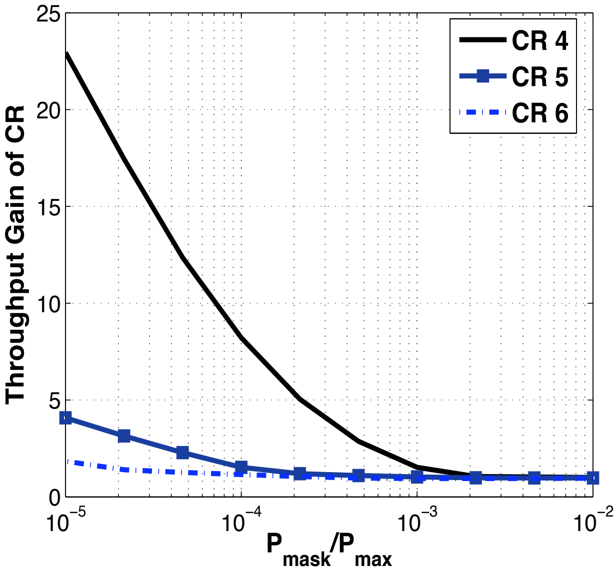

Figure 5.7: Throughput gain $\frac{U}{U_d}$ versus normalized interference $\frac{P^i_{mask}}{P^i_{\max}}$.

To quantify the throughput gain achievable at the QNE of the proposed game, we plot the ratio U/U_d versus the normalized interference constraint bound $P^i_{mask}/P^i_{\max}$, where U_d is the throughput achievable at the NE of the deterministic game. From the figure, it can be seen that the proposed joint optimization of the sensing information and transmission power yields a considerable performance improvement with respect to the disjoint case, especially when the normalized interference constraint bound is stringent. Moreover, the performance improvement becomes more significant with the nearby CR users. This is because a more stringent normalized interference constraint bound imposes lower transmission power for the nearby CR users in the deterministic game, while higher transmission power is allowed due to the accurate sensing information in the proposed game.

5.6 Conclusions

In this chapter, we extended the work from Chapter 4, and proposed a sensing-based non-cooperative game for a multiuser MIMO CRN. To deal with the non-convexity of the game, we used a new relaxed equilibrium concept, namely, QNE. In particular, we theoretically prove the sufficient conditions of the existence and the uniqueness of the QNE for the proposed game. Furthermore, the possible extension of this work considering an equal sensing time is also discussed in this chapter.

Chapter 6

Conclusion and Future Work

Throughout this thesis, the resource optimization problems in single user SISO CRNs, multiuser SISO CRNs, and multiuser MIMO CRNs have been studied. The contributions of this thesis include the scenario modeling problem, formulations, certifications and solutions. This work sheds a light on the future deployment of joint power optimization problems with sensing information in CR technology. In this chapter, we will draw the conclusions of this thesis, and present the future work.

6.1 Conclusions

In this thesis, we have explored an interweave communication in CRNs where the overall objective is to maximize the sum-rate of each CR user by optimizing jointly both the detection operation based on sensing and the power allocation across channels that can be used for transmitting, taking into account the influence of the sensing accuracy and the interference limitation to the PUs. The optimization problem is addressed in single and multiuser CRNs for both SISO and MIMO channels.

6.1.1 Joint optimization of detection and power allocation in single user SISO CRNs

The first scenario considered in this thesis is the resource allocation optimization problem for single user CRNs, where joint power allocation

and spectrum detection is one of the most important issues. In single user CRNs, one pair of CR Tx-Rx performs the spectrum sensing before accessing the channel. We consider the OSA model under the interweave system. In the OSA model, CR users are allowed to transmit over the channel of interest when all the PUs are not transmitting at this channel. One essential enabling technique for OSA-based CRNs is spectrum sensing, where the CR users individually detect active PU transmissions over the channel, and decide to transmit if the sensing results indicate that the PU is inactive at this channel.
In Chapter 3, we consider an OFDM based communication system and present the EPD and APD algorithms to maximize the sum-rate of the CR user by optimizing jointly both the detection operation and the power allocation. The problem can be formulated as a two-variable problem and solved by operating sequentially over power allocation and detection threshold. Both of these algorithms operate basically in two regimes depending on the constraints involved. A novel interference constraint, denoted as rate-loss gap constraint, is proposed to design the power allocation, ensuring that the performance degradation of the PU is bounded.

6.1.2 Joint optimization of detection and power allocation in multiuser SISO CRNs

Secondly, the resource allocation problem among multiple CR users for the SSS scheme is analyzed as a strategic non-cooperative game in Chapter 4, where each CR user is selfish and strives to use the available spectrum in order to maximize its own sum-rate by considering the effect of imperfect sensing information. The resulting game-theoretical formulations belong to the class of non-convex games, where the non-convexity occurs at both the objective functions and feasible sets of the CR users' optimization problems. Therefore, traditional mathematical tools are not applicable to show the existence of an equilibrium for this game.
In order to deal with the non-convexity of the game, we start with the ADOS algorithm, and prove that the local NE is achieved by the ADOS algorithm. In the second step, we use a new relaxed equilibrium concept, namely, QNE, instead of the traditional NE for the convex game. We show the sufficient conditions for the existence of a QNE for the proposed game, by making use of the VI method. Meanwhile, we show that, under the so-called LICQ, the achieved QNE coincides with the

NE. In addition, a distributed cooperative sensing scheme based on a consensus algorithm is considered in the proposed game for a SSS scenario.
Finally, an iterative PDIPS algorithm that converges to a QNE of the proposed game is provided here. The PDIPS algorithm can run at each node in parallel, since it requires only the local information of each CR user, and hence, it can be regarded as a distributed solution. Simulation results show that the PDIPS algorithm yields a considerable performance improvement, in terms of the sum-rate of each CR user, with respect to previous state-of-the-art methods, such as ADOS algorithm and the DG algorithm.

6.1.3 Joint optimization of detection and power allocation in multiuser MIMO CRNs

In Chapter 5, we move a step ahead from Chapter 4, and consider a OSA scenario in a MIMO CRN. In order to reduce the complexity of the optimization problem, we only consider the throughput of CR users under the correct sensing information, and exclude the throughput due to the erroneous decision of the CR user to transmit over occupied channels. The optimization problem is analyzed as a strategic NCG, where the transmit covariance matrix, sensing time, and detection threshold are considered as variables to be optimized. The resulting game is nonconvex, hence, we also use the new relaxed equilibria concept QNE, and prove that the proposed game can achieve the QNE under certain conditions, by making use of the VI method. In particular, we prove theoretically the sufficient condition of the existence and the uniqueness of the QNE for the proposed game.
Furthermore, a possible extension of this work considering equal sensing time is also discussed in this chapter. From the simulation results, the PDIPM algorithm is shown to be an efficient solution that converges to the QNE of the proposed game.

6.2 Future Work

There are several directions of research that we consider for future work:

- Use of more advanced detection methods, increasing the accuracy of the spectrum sensing.
  The reliability of the PU detection at the CR-Tx is limited by several factors, such as the attenuation due to path loss, as well as shadowing and fading. Therefore, decisions made by independent CR-Txs with local sensing capability either most probably will generate harmful interference to the primary system or will use very conservative allocation policies with unnecessary transmission back-off. Cooperative sensing among CR users is one efficient way to improve the sensitivity of the spectrum sensing, which is more robust against fading and hidden terminal problem. In the following work, we will work on more robust detection methods based on cooperative sensing among CR users to improve the efficiency of the spectrum sensing. Furthermore, multi-antenna detection, which shows better performance than single-antenna detection, with cooperative sensing among CR users also will be considered.

- Price policy in objective functions for multiuser CRNs.
  Since each CR-Tx in the game maximizes the function of utility in a distributed fashion and the CR-Txs act selfishly, a NE point is not necessarily the best operating point from a social point of view. Pricing policy appears to be a powerful tool for achieving a more socially desirable result, which drives the CR-Tx to have a "social welfare". An optimal pricing policy is one that yields the game to a NE that is identical to the globally optimal solution of the problem. In economics, the pricing function can take various forms to account for various marketing and pricing policies, e.g., volume discount, coupon discount, etc. In the context of network resource allocation, both linear (e.g., [124–126]) and nonlinear pricing functions have been proposed. Recently, a price factor associated with the global interference constraint to the PU is discussed in [32, 33].
  Deriving such a pricing function is often difficult for two reasons. First, it is hard to characterize the optimal, making it not possible to quantify the performance gap between these policies and the achieved NE. Second, an optimal pricing function that requires

global network information is impractical for a distributed network. To improve the efficiency of the NE, the pricing functions in the literature are usually based on heuristics. In order to simplify the problem, we did not consider the pricing mechanism in this thesis, but we leave this issue open as part of our future work.

- Resource allocation problem in the presence of spatial interference maps.
  Instead of using more advanced detection methods, spatial interference maps will be considered in our future work. These maps could manage the presence of the PU in a soft manner (level of presence), instead of deciding “Yes” or “No” as in conventional detection methods.

- Demonstration on Testbeds
  In this thesis, we have verified our algorithms and schemes by simulations. In the future, we plan to implement our algorithms on Testbeds.

Appendix A

Definition of the Tangent Cone

Definition 5. *Given a subset* $\mathbf{X}$ *of* $\mathbb{R}_n$ *and a vector* $\mathbf{x} \in \mathbf{X}$*, a vector* $\mathbf{y}$ *is said to be a tangent of* $\mathbf{X}$ *at* $\mathbf{x}$ *if either* $\mathbf{y} = 0$ *or there exists a sequence* $x_k \in \mathbf{X}$ *such that* $x_k \neq \mathbf{x}$ *for all* k *and* $x_k \to \mathbf{x}$ *[5],*

$$\frac{x_k - \mathbf{x}}{||x_k - \mathbf{x}||} \to \frac{\mathbf{y}}{||\mathbf{y}||} \tag{A.1}$$

The set of all tangents of $\mathbf{X}$ *at* $\mathbf{x}$ *is a cone called the tangent cone of* $\mathbf{X}$ *at* $\mathbf{x}$*, and is denoted by* $T(\mathbf{X};\mathbf{x})$*.*

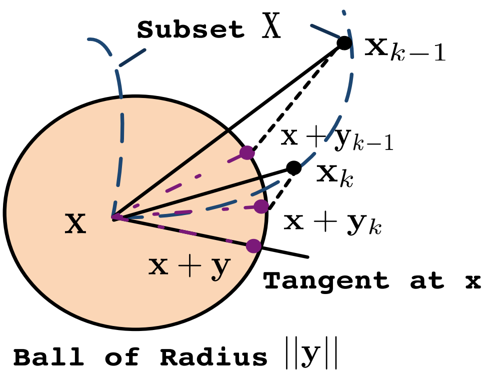

Figure A.1: Tangent cone at $\mathbf{x}$ [5]

Figure A.1 Illustrates a tangent $\mathbf{x}$ at a vector $\mathbf{x} \in \mathbf{X}$. If there is a sequence $x_k \in \mathbf{X}$ that converges to $\mathbf{x}$ and is such that the normalized direction sequence $\frac{x_k - \mathbf{x}}{||x_k - \mathbf{x}||}$ converges to $\frac{\mathbf{y}}{||\mathbf{y}||}$, the normalized direction of $\mathbf{y}$, or equivalently, the sequence:

$$\mathbf{y} = \frac{||\mathbf{y}||(x_k - \mathbf{x})}{||x_k - \mathbf{x}||} \tag{A.2}$$

converges to $\mathbf{y}$, as illustrated in Figure A.1.

Appendix B

Basic Concepts of Variational Inequality Theory

Definition 6. *Given a subset* $\mathbf{X}$ *of the Euclidean n-dimensional space* $\mathbb{R}_n$ *and a mapping* $\mathbf{F}$*, the VI problem, denoted* $VI(\mathbf{X}, \mathbf{F})$*, is to find a vector* $\mathbf{x}^\star \in \mathbf{X}$ *such that [83]*

$$(\mathbf{x} - \mathbf{x}^\star)^T \mathbf{F}(\mathbf{x}^\star) \geq 0, \quad \forall \mathbf{x} \in \mathbf{X} \tag{B.1}$$

The set of solutions to this problem is denoted $SOL(\mathbf{X}, \mathbf{F})$. Several standard problems in nonlinear programming, game theory and nonlinear analysis can be naturally formulated as a VI problem, for example:

- Solution of systems of equations. The simplest example of VI is the problem of solving a system of equations. In fact, if $\mathbf{X} = \mathbb{R}_n$ then $VI(\mathbb{R}_n, \mathbf{F})$ is equivalent to finding a $\mathbf{x}^\star \in \mathbb{R}_n$ such that $\mathbf{F}(\mathbf{x}^\star) = 0$. As special case, if the mapping $\mathbf{F}$ is affine, i.e., $\mathbf{F}(\mathbf{x}) = \mathbf{A}\mathbf{x} - b$ the previous problem is equivalent to the classical system of equation $\mathbf{A}\mathbf{x}^\star = b$.

- Fixed-point problems. Given a closed and convex set $\mathbf{X}$ and a mapping $\mathbf{T}$, the fixed-point problem is to find a vector such that $\mathbf{T}(\mathbf{x}^\star) = \mathbf{x}^\star$. This problem can be converted into a VI format, simply by defining $\mathbf{F}(\mathbf{x}) = \mathbf{x} - \mathbf{T}(\mathbf{x})$.

- Constrained and unconstrained optimization. If the problem is convex and the mapping $\mathbf{F}$ in $VI(\mathbf{X}, \mathbf{F})$ is the gradient of a real-valued function f, then $VI(\mathbf{X}, \mathbf{F})$ represents the necessary conditions of optimality for the following optimization problem: find a point $\mathbf{x}^\star \in \mathbf{X}$ such that $f(\mathbf{x}^\star) \leq f(\mathbf{x})$, for all $\mathbf{x} \in \mathbf{X}$. Also, if the function f is convex, a point $\mathbf{x} \in \mathbf{X}$ minimizes f over $\mathbf{X}$ if and only if it is a solution to $VI(\mathbf{X}, \nabla f)$, where ∇ denotes the gradient of function f (the VI coincides with the first-order necessary and sufficient optimality conditions of a convex differentiable function).

- Game theory problems. Consider a strategic noncooperative game, where player i's problem is to determine, for each fixed but arbitrary tuple $\mathbf{x}^{-i}$ of the other players' strategies, an optimal strategy profile $\mathbf{x}^\star$ that solves the following optimization:

$$\begin{aligned} \max_{\mathbf{x}^i} \quad & U^i(\mathbf{x}^i, \mathbf{x}^{-i}) \\ \text{s. t.} \quad & \mathbf{x}^i \in \mathbf{X}^i \end{aligned} \tag{B.2}$$

  Suppose that each $\mathbf{X}^i \in \mathbb{R}_n$ is convex and closed, and $U^i(\mathbf{x}^i, \mathbf{x}^{-i})$ is convex and continuously differentiable in $\mathbf{x}^i$. By convexity and the first-order optimality conditions, we infer that a strategy profile is NE if and only if $(\mathbf{x}^i - \mathbf{x}^{i,\star})\nabla_{\mathbf{x}^i} U^i(\mathbf{x}^{i,\star})$, for each user i, where $\nabla_{\mathbf{x}^i} U^i(\mathbf{x}^i)$ denotes the gradient of $U^i(\mathbf{x}^i)$ with respect to $\mathbf{x}^i$. Summing these conditions and taking into account the Cartesian product structure of the strategy set of the game, we can conclude that this set of inequalities is equivalent to the $VI(\mathbf{X}, \mathbf{F})$, with $\mathbf{X} = [\mathbf{X}^i]_{i=1}^M$ and $\mathbf{F} = [\nabla_{\mathbf{x}^i} U^i(\mathbf{x}^i)]_{i=1}^M$ [83].

- Complementarity problems. When the set $\mathbf{X}$ is a cone (i.e., $\mathbf{x} \in \mathbf{X}, a\mathbf{x} \in \mathbf{X}$, for all scalars $a \geq 0$), the VI admits an equivalent form known as a complementarity problem, denoted by $CP(\mathbf{X}, \mathbf{F})$, which is to find a vector such that [83]:

$$\mathbf{x} \perp \mathbf{F} \in \mathbf{X}^\star \tag{B.3}$$

where $\mathbf{X}^\star$ is the dual cone of $\mathbf{X}$, defined as $\mathbf{X}^\star = \{\mathbf{d} \in \mathbb{R}_n | \mathbf{vd} \geq 0\}$, when $\mathbf{X}$ is the nonnegative orthant of $\mathbb{R}_n$. In such a case the $CP(\mathbb{R}_n^+, \mathbf{F})$ is known as nonlinear complementarity problem (NCP) and denoted $NCP(\mathbf{F})$. Recognizing that the dual cone of the nonnegative orthant is the nonnegative orthant itself, the $NCP(\mathbf{F})$ is to find a vector such that [83]

$$0 \leq \mathbf{x} \perp \mathbf{F} \geq 0 \tag{B.4}$$

Equivalent reformulation of the KKT system and the VI problems

Let $\mathbf{X}$ be represented by finitely many differentiable inequalities and equations, i.e., $\mathbf{X} = \{\mathbf{x} \in \mathbb{R}_n | \mathbf{h}(\mathbf{x}), \mathbf{g}(\mathbf{x}) \geq 0\}$, with $\mathbf{h} : \mathbb{R}_n \rightarrow \mathbb{R}_l$ and $\mathbf{g} : \mathbb{R}_n \rightarrow \mathbb{R}_m$ being vector-valued continuously differentiable functions. The following two statements are valid [83]:

- Let $\mathbf{x} \in SOL(\mathbf{X}, \mathbf{F})$. Under mild conditions on the constraints, there exist multipliers $\boldsymbol{\alpha} = [\alpha_j]_{j=1}^l$ and $\boldsymbol{\beta} = [\beta_i]_{i=1}^m$ such that:

$$0 = \mathbf{F} + \sum_{j=1}^{l} \alpha_j \nabla h_j(\mathbf{x}) + \sum_{i=1}^{m} \beta_i \nabla g_i(\mathbf{x}) \tag{B.5}$$

$$0 = \mathbf{h}(\mathbf{x}) \tag{B.6}$$

$$0 \leq \boldsymbol{\beta} \perp \mathbf{g}(\mathbf{x}) \leq 0 \tag{B.7}$$

- Conversely, if each function $h_j(\mathbf{x})$ is affine and each function $g_i(\mathbf{x})$ is convex, and if $(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ satisfies (B.5), then $\mathbf{x} \in SOL(\mathbf{X}, \mathbf{F})$.

Existence and uniqueness of the solution to a VI problem

The theory and solution methods for various kinds of VI problems allow one to choose a suitable way to investigate each particular problem under consideration. Here, we mention some of the basic conditions for the

existence and uniqueness of the solution to a VI problem.
In terms of existence of solutions, the $VI(\mathbf{X}, \mathbf{F})$ is solvable if [83]:

- $\mathbf{X}$ is a nonempty, convex and compact subset of a finite-dimensional Euclidean space;
- $\mathbf{F}$ is a continuous mapping [83]. The solution to $VI(\mathbf{X}, \mathbf{F})$ is unique if $\mathbf{F}$ is continuous and strongly monotone on the convex and closed set $\mathbf{X}$ [83] (the strong monotonicity of $\mathbf{F}$ is sufficient also for the existence of a solution);
- Uniqueness conditions can be weakened if the set $\mathbf{X} \in \mathbb{R}_n$ has a Cartesian structure, i.e., $\mathbf{X} = [\mathbf{X}^i]_{i=1}^M$, $\mathbf{X}^i \in \mathbb{R}_n$, if each $\mathbf{X}^i$ is closed and convex and $\mathbf{F}$ is strongly monotone function, then $VI(\mathbf{X}, \mathbf{F})$ has a unique solution [83].

Several solution methods along with their convergence properties have been proposed for VI in the literature. A good reference on parallel and distributed algorithms and their convergence for optimization problems and VI can be found in [127]. A comprehensive and more advanced treatment can be found in the monograph [83].

Appendix C

Proof of Theorem 2

The Hessian matrix $\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)$ is given by:

$$\begin{pmatrix} 0 & \Gamma^i_{k,c}|h^i_{k,cp}|^4 & -\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 \\ \Gamma^i_{k,c}|h^i_{k,cp}|^4 & 0 & \mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 \\ -\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 & \mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 & \mathcal{P}^i_{k,d}(\tau^i_k)''(I^{i,p}_{k,1} - I^{i,p}_{k,0}) \end{pmatrix} \tag{C.1}$$

where $\Gamma^i_{k,c} = (1 - \Gamma_k)/(\sigma^i_{k,n})^2$. In order to check that conditions (C1), (C2) and (C3) are satisfied, we assume that $P^{i,ref}_{k,0} = 0$, $P^{i,ref}_{k,1} = 0$ and $\tau^{i,ref}_k = \tau^i_{k,min}$, where $\tau^i_k \in [\tau^i_{k,min}, \tau^i_{k,max}]$. It follows that $\mathbf{x}^{i,ref} = [P^{i,ref}_{k,0}, P^{i,ref}_{k,1}, \tau^{i,ref}_k]^N_{k=1}$, and we have:

$$\begin{aligned} &\begin{pmatrix} P^i_{k,0} - P^{i,ref}_{k,0} \\ P^i_{k,1} - P^{i,ref}_{k,1} \\ \tau^i_k - \tau^{i,ref}_k \end{pmatrix}^T \nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i) \begin{pmatrix} P^i_{k,0} - P^{i,ref}_{k,0} \\ P^i_{k,1} - P^{i,ref}_{k,1} \\ \tau^i_k - \tau^{i,ref}_k \end{pmatrix} \\ &= 2\Gamma^i_{k,c} P^i_{k,0} P^i_{k,1} |h^i_{k,cp}|^4 + 2(\tau^i_k - \tau^{i,ref}_k)\mathcal{P}^i_{k,d}(\tau^i_k)'(P^i_{k,1} - P^i_{k,0})|h^i_{k,cp}|^2 \\ &+ (\tau^i_k - \tau^{i,ref}_k)^2 \mathcal{P}^i_{k,d}(\tau^i_k)''(I^{i,p}_{k,1} - I^p_{k,0}) \geq 0 \end{aligned} \tag{C.2}$$

Notice that $P^i_{k,1} < P^i_{k,0}$, $I^{i,p}_{k,1} < I^p_{k,0}$, $\mathcal{P}^i_{k,d}(\tau^i_k)' < 0$ and $\mathcal{P}^i_{k,d}(\tau^i_k)'' < 0$. All the terms are positive, thus the Hessian matrix of $\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)$ is copositive. Similarly, we can show that the Hessian matrix of function $h^i(\mathbf{x}^i)$ is copositive. Thus, conditions (C1), (C2) and (C3) are satisfied.

For condition (C4), we need to show that the player's variables $\mathbf{x} = (\mathbf{P}_0, \mathbf{P}_1, \boldsymbol{\tau})$ are bounded. For every CR i, from power budget constraint (4.20) we can get:

$$0 \leq P_{k,0}^i \leq \frac{P_{\max}^i}{(1 - \mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i) - \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i))} \leq \frac{P_{\max}^i}{A_{k,0}^i} \tag{C.3}$$

$$0 \leq P_{k,1}^i \leq \frac{P_{\max}^i}{(\mathcal{P}(H_{0,k})\mathcal{P}_{k,fa}^i(\tau_k^i) + \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i(\tau_k^i))} \leq \frac{P_{\max}^i}{A_{k,1}^i} \tag{C.4}$$

where:

$$A_{k,0}^i = 1 - \frac{1}{2}\mathcal{P}(H_{0,k}) - \mathcal{P}(H_{1,k})\mathcal{Q}\left(\frac{\mu_{k,0}^i - \mu_{k,1}^i}{\sigma_{k,1}^i}\right) \tag{C.5}$$

$$A_{k,1}^i = \frac{1}{2}\mathcal{P}(H_{1,k}) + \mathcal{P}(H_{0,k})\mathcal{Q}\left(\frac{\mu_{k,1}^i - \mu_{k,0}^i}{\sigma_{k,0}^i}\right) \tag{C.6}$$

In addition, $\boldsymbol{\tau}$ is bounded by the constraint (4.21), and we can conclude that the condition (C4) is also satisfied. Therefore, the $VI(\mathbf{Q}, \boldsymbol{\Theta})$ has a solution, and the game $\mathcal{G}(\mathbf{H}, \mathbf{G})$ has a QNE. Moreover, every QNE is nontrivial, a trivial QNE can not satisfy (4.51).
Constraint (4.17) versus Constraint (4.19): For Constraint (4.17), denoted as $g_{k,c}^i(\mathbf{x}^i)$, we have:

$$\begin{aligned}
&(\mathbf{x}^i - \mathbf{x}^{i,ref})^T \nabla_{\mathbf{x}^i}^2 g_{k,c}^i(\mathbf{x}^i)(\mathbf{x}^i - \mathbf{x}^{i,ref}) \\
&= (P_{k,0}^i - P_{k,0}^{i,ref})^2(1 - \mathcal{P}_{k,d}^i(\tau_k^i))U_{k,0}^{i,p} + (P_{k,1}^i - P_{k,1}^{i,ref})^2\mathcal{P}_{k,d}^i(\tau_k^i)U_{k,1}^{i,p} \\
&+ 2(P_{k,0}^i - P_{k,0}^{i,ref})(\tau_k^i - \tau_k^{i,ref})\mathcal{P}_{k,d}^i(\tau_k^i)'|h_{k,cp}^i|^2((I_{k,0}^{i,p} + |S_k^i|^2)^{-1} - (I_{k,0}^{i,p})^{-1}) \\
&+ 2(P_{k,1}^i - P_{k,1}^{i,ref})(\tau_k^i - \tau_k^{i,ref})\mathcal{P}_{k,d}^i(\tau_k^i)'|h_{k,cp}^i|^2((I_{k,1}^{i,p})^{-1} - (I_{k,1}^{i,p} + |S_k^i|^2)^{-1}) \\
&+ (\tau_k^i - \tau_k^{i,ref})^2\mathcal{P}_{k,d}^i(\tau_k^i)'' \log_2((1 + |S_k^i|^2/I_{k,1}^{i,p})/(1 + |S_k^i|^2/I_{k,0}^{i,p}))
\end{aligned} \tag{C.7}$$

where:

$$U_{k,0}^{i,p} = |h_{k,cp}^i|^4((I_{k,0}^{i,p} + |S_k^i|^2)^{-2} - (I_{k,0}^{i,p})^{-2}) \tag{C.8}$$

$$U_{k,1}^{i,p} = |h_{k,cp}^i|^4((I_{k,1}^{i,p} + |S_k^i|^2)^{-2} - (I_{k,1}^{i,p})^{-2}) \tag{C.9}$$

The first and the second term on the right side are negative, the fifth term is positive, and the sum of the third and the forth term can be proved to be positive. Hence, assuming $U_{k,0}^{i,p} > U_{k,1}^{i,p}$, the $\nabla^2_{\mathbf{x}^i} g^i_{k,c}(\mathbf{x}^i)$ is copositive if the following inequality is satisfied:

$$
\begin{aligned}
&(\tau_k^i - \tau_k^{i,ref})^2 \mathcal{P}^i_{k,d}(\tau_k^i)'' \log_2(1 + |S_k^i|^2 / I_{k,1}^{i,p})/(1 + |S_k^i|^2 / I_{k,0}^{i,p})) \\
&> ((\mathcal{P}^i_{k,d}(\tau_k^i) - 1)(P_{k,0}^i)^2 - \mathcal{P}^i_{k,d}(\tau_k^i)(P_{k,1}^i)^2) U_{k,0}^{i,p}
\end{aligned} \tag{C.10}
$$

However, this condition depends on the values of the system parameters as well as the action of the CR i, which is uncertain. In order to simplify the analysis, we use constraint (4.19) instead of constraint (4.17), which is more suitable for a general network, and offers a better protection for PU, as shown in the simulation results of Chapter 4.

Appendix D

Proof of Theorem 3

The Hessian matrix $\nabla^2_{\mathbf{x}^i} h^i_{\mathbb{C}}(\mathbf{x}^i)$ is a block diagonal matrix with N diagonal blocks, the k-th of which is given by:

$$\begin{bmatrix} 0 & -\mathcal{P}^i_{k,d}(\tau^i_k)' \operatorname{Tr}(\mathbf{G}^i_k(\mathbf{G}^i_k)^H) \\ -\mathcal{P}^i_{k,d}(\tau^i_k)' \operatorname{Tr}(\mathbf{G}^i_k(\mathbf{G}^i_k)^H) & -\mathcal{P}^i_{k,d}(\tau^i_k)'' \operatorname{Tr}(\mathbf{G}^i_k\mathbf{Q}^i_k(\mathbf{G}^i_k)^H) \end{bmatrix} \tag{D.1}$$

In order to check that conditions (C1) and (C2) are satisfied, we suppose that all $h^i_k(0,\boldsymbol{\tau}) \leq 0$ for all $\boldsymbol{\tau} = [(\tau^i_k)^N_{k=1}]^M_{i=1}$, where $\tau^i_k \in [\tau^i_{k,min}, \tau^i_{k,max}]$. Let $\mathbf{Q}^{i,ref}_k = 0$ and $\tau^{i,ref}_k = \tau^i_{k,min}$, it follows that $\mathbf{x}^{i,ref} = [\mathbf{Q}^{i,ref}_k, \tau^{i,ref}_k]^N_{k=1}$, $\mathbf{x}^{i,ref} \in \mathbf{X}^i_{\mathbb{C}}$, and we have that:

$$\begin{aligned}
&\begin{bmatrix} \mathbf{Q}^i_k - \mathbf{Q}^{i,ref}_k \\ \tau^i_k - \tau^{i,ref}_k \end{bmatrix}^T \nabla^2_{\mathbf{x}^i} h^i_{\mathbb{C}}(\mathbf{x}^i) \begin{bmatrix} \mathbf{Q}^i_k - \mathbf{Q}^{i,ref}_k \\ \tau^i_k - \tau^{i,ref}_k \end{bmatrix} \\
&= -2(\tau^i_k - \tau^{i,ref}_k)\mathcal{P}^i_{k,d}(\tau^i_k)' \operatorname{Tr}(\mathbf{G}^i_k\mathbf{Q}^i_k(\mathbf{G}^i_k)^H) \\
&- (\tau^i_k - \tau^{i,ref}_k)^2\mathcal{P}^i_{k,d}(\tau^i_k)'' \operatorname{Tr}(\mathbf{G}^i_k\mathbf{Q}^i_k(\mathbf{G}^i_k)^H) \geq 0
\end{aligned} \tag{D.2}$$

Thus, conditions (C1) and (C2) hold for the function $h^i_{k,\mathbb{C}}(\mathbf{x}^i)$. For condition (C3), we need to find the necessary condition for the existence of a solution of the $VI_{\mathbb{C}}(\mathbf{X}_{\mathbb{C}}, \mathbf{F}_{\mathbb{C}})$. According to [83], it is sufficient to show that the player's variables $\mathbf{x} = (\mathbf{Q}, \boldsymbol{\tau})$ are bounded. Clearly, the variable $\mathbf{Q}$ is bounded by constraint (b1) in problem $P5.1$, whereas the variable $\boldsymbol{\tau}$ is bounded by constraint (b2) in problem $P5.1$: for every player i, $i = 1, ..., M$, we have that:

$$\tau^i_{k,min} \leq \tau^i_k \leq \tau^i_{k,max} \quad and \quad \sum\nolimits_{k=1}^{N} \mathrm{Tr}(\mathbf{Q}^i_k) \leq P_{max} \tag{D.3}$$

Hence, we can conclude that the condition (C3) is also satisfied. Therefore, the $VI_{\mathbb{C}}(\mathbf{Y}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ has a solution, and the game $\mathcal{G}_{\mathbb{C}}(\mathbf{H}_{\mathbb{C}}, \tilde{\mathbf{G}}_{\mathbb{C}})$ has a QNE. Moreover, every QNE is not trivial, a trivial QNE can not satisfy (5.25).

Appendix E

Proof of Theorem 4

From [32,33], we have that if the Hessian matrix $\nabla^2_{\mathbf{x}^i} L^i_k(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ is positive definite for all $(\mathbf{x}^i, \boldsymbol{\alpha}^i) \in \mathbf{Y}^i_{\mathbb{C}}$, then, the optimization problem for player i has a unique optimal solution, which is necessarily nontrivial. For optimization problem $P5.3$, the Hessian matrix $\nabla^2_{\mathbf{x}^i} L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ is given as:

$$\nabla^2_{\mathbf{x}^i} L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i) = -\nabla^2_{\mathbf{x}^i} U^i(\mathbf{x}^i) + \boldsymbol{\alpha}^i \nabla^2_{\mathbf{x}^i} h^i_{\mathbb{C}}(\mathbf{x}^i) \tag{E.1}$$

We start on each term in (E.1) separately. For the complex matrix $\nabla^2_{\mathbf{x}^i} U^i(\mathbf{x}^i)$:

$$\nabla^2_{\mathbf{x}^i} U^i(\mathbf{x}^i) = \begin{pmatrix} \nabla^2_{\mathbf{Q}^i} U^i(\mathbf{x}^i) & \nabla_{\mathbf{Q}^i \boldsymbol{\tau}^i} U^i(\mathbf{x}^i) \\ \nabla_{\mathbf{Q}^i \boldsymbol{\tau}^i} U^i(\mathbf{x}^i) & \nabla^2_{\boldsymbol{\tau}^i} U^i(\mathbf{x}^i) \end{pmatrix} \tag{E.2}$$

where we have:

$$\begin{aligned} \nabla^2_{\mathbf{Q}^i} U^i(\mathbf{x}^i) = -t^i_c \Big(&(1 - \mathcal{P}^i_{k,fa}(\tau^i_k)) \mathbf{H}^{ii}_k \mathbf{H}^{ii}_k \left(\mathbf{C}^i_k + \mathbf{H}^{ii}_k \mathbf{Q}^i_k (\mathbf{H}^{ii}_k)^H \right)^{-2} \\ &(\mathbf{H}^{ii}_k)^H (\mathbf{H}^{ii}_k)^H \Big)_{k=1}^{N} \end{aligned} \tag{E.3}$$

$$\nabla^2_{\boldsymbol{\tau}^i} U^i(\mathbf{x}^i) = -t^i_c \Big(\mathcal{P}^i_{k,fa}(\tau^i_k)'' \log\det \left(\mathbf{I} + (\mathbf{C}^i_k)^{-1} \mathbf{H}^{ii}_k \mathbf{Q}^i_k (\mathbf{H}^{ii}_k)^H \right) \Big)_{k=1}^{N} \tag{E.4}$$

$$\nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} U^i(\mathbf{x}^i) = -t_c^i \bigg(\mathcal{P}_{k,fa}^i(\tau_k^i)' \mathbf{H}_k^{ii} \left(\mathbf{C}_k^i + \mathbf{H}_k^{ii} \mathbf{Q}_k^i (\mathbf{H}_k^{ii})^H \right)^{-1} (\mathbf{H}_k^{ii})^H \bigg)_{k=1}^N \tag{E.5}$$

Notice that $\mathcal{P}_{k,fa}^i(\tau_k^i)'' > 0, \mathcal{P}_{k,fa}^i(\tau_k^i)' < 0$, thus $\mathrm{Diag}(-\nabla_{\mathbf{x}^i}^2 U^i(\mathbf{x}^i)) > 0$. The minimum eigenvalue of the diagonal elements are denoted as $\lambda_{\min}^i(-\nabla_{\mathbf{Q}^i}^2 U^i(\mathbf{x}^i))$, $\lambda_{\min}^i(-\nabla_{\boldsymbol{\tau}^i}^2 U^i(\mathbf{x}^i))$, respectively. For complex matrix $\nabla_{\mathbf{x}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i)$, we have:

$$\nabla_{\mathbf{x}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i) = \begin{pmatrix} \nabla_{\mathbf{Q}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i) & \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} h_{\mathbb{C}}^i(\mathbf{x}^i) \\ \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} h_{\mathbb{C}}^i(\mathbf{x}^i) & \nabla_{\boldsymbol{\tau}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i) \end{pmatrix} \tag{E.6}$$

where:

$$\nabla_{\mathbf{Q}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i) = 0 \tag{E.7}$$

$$\nabla_{\boldsymbol{\tau}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i) = -\mathcal{P}_{k,d}^i(\tau_k^i)'' \operatorname{Tr}(\mathbf{G}_k^i \mathbf{Q}_k^i (\mathbf{G}_k^i)^H) \tag{E.8}$$

$$\nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} h_{\mathbb{C}}^i(\mathbf{x}^i) = -\mathcal{P}_{k,d}^i(\tau_k^i)' \operatorname{Tr}(\mathbf{G}_k^i (\mathbf{G}_k^i)^H) \tag{E.9}$$

With (E.2) and (E.6), the matrix $\nabla_{\mathbf{x}^i}^2 L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ can be formulated as follows:

$$\nabla_{\mathbf{x}^i}^2 L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i) = Diag(-\nabla_{\mathbf{Q}^i}^2 U^i(\mathbf{x}^i), -\nabla_{\boldsymbol{\tau}^i}^2 U^i(\mathbf{x}^i) + \boldsymbol{\alpha}^i \nabla_{\boldsymbol{\tau}^i}^2 h_{\mathbb{C}}^i(\mathbf{x}^i)) + \nabla_{\mathbf{x}^i}^2 \mathbf{D}_{\mathbb{C}}^i \tag{E.10}$$

where the matrix $\nabla_{\mathbf{x}^i}^2 \mathbf{D}_{\mathbb{C}}^i$ is given by:

$$\begin{pmatrix} 0 & \boldsymbol{\alpha}^i \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} h_{\mathbb{C}}^i(\mathbf{x}^i) - \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} U^i(\mathbf{x}^i) \\ \boldsymbol{\alpha}^i \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} h_{\mathbb{C}}^i(\mathbf{x}^i) - \nabla_{\mathbf{Q}^i\boldsymbol{\tau}^i} U^i(\mathbf{x}^i) & 0 \end{pmatrix} \tag{E.11}$$

We introduce a symmetric matrix $\nabla_{\mathbf{x}^i}^2 \tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$, having the property that:

$$[\nabla_{\mathbf{x}^i}^2 \tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)]_{ij} = \begin{cases} \leq \nabla_{\mathbf{x}^i}^2 L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i) & if \quad i = j, \\ \leq -|\nabla_{\mathbf{x}^i}^2 L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)| & if \quad i \neq j. \end{cases} \tag{E.12}$$

which quarantees that if the matrix $\nabla_{\mathbf{x}^i}^2 \tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ is positive definite, so is $\nabla_{\mathbf{x}^i}^2 L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$. Note that for a complex matrix $\mathbf{A}$, if the real part of $\mathbf{A}$, denoted as $\mathrm{Re}(\mathbf{A})$, is positive definite, the complex matrix $\mathbf{A}$ is called positive definite. The following matrix $\nabla^2_{\mathbf{x}^i}\tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$, satisfying the condition (E.12), can be defined as:

$$\nabla^2_{\mathbf{x}^i}\tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i) = \mathrm{Re}\left(\mathrm{Diag}(\lambda_{\min}(-\nabla^2_{\mathbf{Q}^i}U^i(\mathbf{x}^i)), \lambda_{\min}(-\nabla^2_{\boldsymbol{\tau}^i}U^i(\mathbf{x}^i)))\right) - \max_{k=1,\dots,N}\frac{\boldsymbol{\alpha}^i}{\sqrt{2\pi}}\mathrm{Tr}(\mathbf{G}^i_k(\mathbf{G}^i_k)^H)\mathbf{I} \tag{E.13}$$

We first introduce the definition of diagonal dominant matrix. A matrix is said to be diagonally dominant if for every row of the matrix, the magnitude of the diagonal entry in a row is larger than or equal to the sum of the magnitudes of all the other (non-diagonal) entries in that row. More precisely, the matrix $\mathbf{A}$ is diagonally dominant if:

$$|a_{ii}| \leq \sum_{j\neq i}|a_{ij}| \tag{E.14}$$

where a_{ij} denotes the entry in the i-th row and j-th column. Furthermore, we have that diagonal dominant matrices with real-nonnegative diagonal entries are positive definite. Based on this definition, the matrix $\nabla^2_{\mathbf{x}^i}\tilde{L}^i(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ is positive definite if the following condition is satisfied:

$$\frac{\max_{k=1,\dots,N}\frac{\boldsymbol{\alpha}^i}{\sqrt{2\pi}}\mathrm{Tr}(\mathbf{G}^i_k(\mathbf{G}^i_k)^H)}{\min\left(\mathrm{Re}\left(\lambda_{\min}(-\nabla^2_{\mathbf{Q}^i}U^i(\mathbf{x}^i)), \lambda_{\min}(-\nabla^2_{\boldsymbol{\tau}^i}U^i(\mathbf{x}^i))\right)\right)} < 1 \tag{E.15}$$

Consequently, according to (E.12), the $\nabla^2_{\mathbf{x}^i}L^i_k(\mathbf{x}^i, \boldsymbol{\alpha}^i)$ is also positive definite under condition (E.15).

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