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THE ECONOMICS OF PLATFORM COMPETITION: COMPATIBILITY, STANDARDIZATION AND PIRACY

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Chapter 0. Introducción

Las externalidades de red pueden convertirse en un recurso muy importante para las empresas en la era tecnológica. El éxito o el fracaso de un producto pueden depender claramente de este factor sin obviar otros elementos importantes para las empresas y consumidores en este tipo de mercados, pero pueden ser determinantes y marcar la diferencia. Las externalidades directas de red aparecen asociadas a una base de usuarios de un producto o a las expectativas que los consumidores tengan sobre el éxito o fracaso de un producto determinado frente a su rival. De este modo, en este tipo de mercados, un producto con una calidad inferior y/o un precio superior podría imponerse si los consumidores lo creen e incluyen en sus expectativas que lo hará.

Podemos encontrar externalidades directas cuando la valoración de un producto para un comprador potencial aumenta a medida que es mayor su base de usuarios. Un ejemplo de estos efectos utilizado por muchos autores es el de Betamax frente a VHS en 1976, que acabó con Betamax fuera del mercado, imponiéndose VHS como el estándar a pesar de ser un producto tecnológicamente inferior. No hace falta ir muy lejos en el tiempo para encontrar ejemplos de externalidades directas de red. En nuestros días los mercados de redes sociales también están afectados por estos efectos: un usuario potencial estará más dispuesto a unirse en red social que cuente con un amplia base instalada.

Las externalidades indirectas, aparecen cuando la valoración de un producto para un comprador potencial aumenta a medida que lo hace la oferta de productos complementarios del producto principal. Este es el caso de los mercados de videojuegos, donde las externalidades indirectas juegan un papel crucial, ya que no es concebible una videoconsola sin juegos. Otro factor clave en los mercados tecnológicos es la piratería. La piratería se define como la reproducción y distribución de copias de obras protegidas por el derecho de autor, así como su transmisión al público o su puesta a disposición en redes de comunicación en línea, sin la autorización de los propietarios legítimos, cuando dicha autorización resulte necesaria legalmente. La piratería afecta a obras de distintos tipos, como la música, la literatura, el cine, los programas informáticos, los videojuegos, los programas y las señales audiovisuales.

Ante la importancia de este tipo de mercados y su rápido crecimiento en las última décadas, el objetivo de esta tesis es el estudio de la competencia entre plataformas. Así, se analizará teóricamente el comportamiento de las empresas ante la presencia tanto de externalidades directas de red, externalidades indirectas y piratería, para poder comprender mejor el comportamiento estratégico y ver cuáles pueden ser sus consecuencias tanto para los consumidores como para las empresas que desarrollan complementos.

Como acaba de señalarse, a lo largo de toda esta tesis nos centramos en un contexto de competencia entre dos empresas o dos plataformas, debido a que la mayoría de mercados tecnológicos son oligopolios (por ejemplo el de sistemas operativos para ordenadores, el de sistemas operativos para móviles, los mercados de videoconsolas. . .). Estos mercados suelen estar abastecidos por un pequeño número de empresas de alto contenido tecnológico. Un factor diferencial en este tipo de mercados respecto a mercados más tradicionales es la necesidad de tener una amplia gama de complementos. Es decir, un consumidor no estará dispuesto a pagar por una videoconsola si no dispone de juegos para utilizar en la plataforma, y esto ocurre con la mayoría de productos en este tipo de mercados: las plataformas necesitan a los desarrolladores de aplicaciones y los desarrolladores necesitan a las plataformas. Cuanto mas amplía sea la red de una plataforma más posibilidades tiene de apoderarse de todo el mercado.

Desde la aparición de los primeros servidores de correo electrónico o incluso desde la aparición de las primeras máquinas de fax las externalidades de red han jugado un papel determinante en estos mercados. Más recientemente con la aparición de los llamados smartphones, tanto las externalidades de red como la estandarización de productos y la piratería de contenidos tecnológicos han estado presentes más pronto o más tarde en este tipo de mercados.

La estructura de esta tesis doctoral nos permite comprender la evolución de los mercados tecnológicos siguiendo en cada capítulo una etapa diferente de esta evolución. En un primer capítulo se ofrecerá una revisión de la literatura relevante y a continuación, en tres capítulos, los distintos modelos que se proponen.

En el segundo capítulo analizamos como, en un principio, las plataformas eligen el grado de compatibilidad de su producto con el producto rival. Este podía ser el caso de los servidores de correo electrónico en sus inicios, donde era muy complicado enviar correos entre servidores. A tal efecto, las externalidades directas juegan un papel crucial si no existe compatibilidad. El objetivo es analizar los incentivos de las plataformas a elegir ciertos grados de compatibilidad en presencia de externalidades directas de red.

En el tercer capítulo aparecen los desarrolladores de complementos para cada una de las plataformas. De este modo se puede entender a las plataformas como los sistemas operativos para los ordenadores, o bien Windows o bien Mac OS X, y los desarrolladores de complementos como cada una de las aplicaciones de software que existen para cada sistema operativo. Estamos pues ante un escenario con dos mercados que interactúan en las plataformas, es decir, un escenario de "two-sided markets". Se estudiará los incentivos de las plataformas a establecer un estándar en el mercado.

El cuarto capítulo cuenta con los mismos actores que el anterior pero se introduce una problemática bastante actual, la piratería. Los consumidores pueden adquirir software ilegal para utilizar en una de las plataformas. Los consumidores de aplicaciones de teléfonos móviles pueden comprarlas en las tiendas on line o pueden obtener copias piratas gratuitas. Estas copias piratas funcionan bien en un teléfono móvil iPhone o bien en uno Android, los cuales harían las veces de plataformas en este ejemplo. Se quiere establecer quién gana con la piratería y si realmente las empresas que desarrollan las aplicaciones están dispuestas a impedirla.

Objetivos, estructura y contenido.

La tesis está estructurada en cuatro capítulos. Tras esta introducción general se presenta la literatura relacionada con la presente tesis. En los tres capítulos siguientes se desarrollan tres modelos teóricos con externalidades de red y competencia entre plataformas. A continuación resumimos los capítulos 2, 3 y 4, sus principales resultados y posibles líneas de investigación futuras.

En el segundo capítulo "*Compatibility, vertical differentiation and network effects*", se analiza cómo la presencia de externalidades directas de red afecta a decisiones estratégicas de las empresas referentes a la compatibilidad e intensidad de la competencia. Este es un marco interesante para observar la evolución de los mercados con externalidades de red en sus inicios, ya que, al principio no existía compatibilidad entre productos, dejando todo en manos de las expectativas de los consumidores; si los consumidores esperan que un producto monopolice el mercado, casi con toda seguridad acabará haciéndolo. Aunque las calidades son exógenas, la elección del grado de compatibilidad endogenizará en cierto modo el grado de diferenciación vertical ya que, como se verá, éste afecta a la disposición a pagar por el producto.

Se plantea un modelo de diferenciación vertical donde cada plataforma ofrece un producto de distinta calidad. El problema de decisión de las plataformas se modeliza como un juego en dos etapas. En la primera etapa las plataformas eligen simultánea y no cooperativamente el grado de compatibilidad de su producto con el producto de la empresa rival y en la segunda etapa del juego compiten en precios. La decisión de los consumidores se verá afectada por sus expectativas sobre el número de usuarios de cada producto (externalidad directa de red), además de la intensidad de este efecto. En un mercado tradicional la intensidad de la externalidad de red sería cero, reduciendo el modelo a un modelo estandar de diferenciación vertical.

La presentación distingue tres casos dependiendo de las expectativas de los consumidores. En primer lugar nos referimos al caso en que las expectativas de los consumidores son simétricas. La empresa con el bien de calidad alta elige un grado de compatibilidad y un precio mayores que los que elige la empresa con el bien de calidad baja. Un segundo análisis contempla que las expectativas de la empresa de alta calidad sean relativamente grandes: entonces si la diferencia entre calidades es mínima, la empresa de baja calidad escoge nula compatibilidad y, además, no producirá en el mercado. Por otra parte, si la diferencia entre calidades aumenta; elegirá cierto grado de compatibilidad, pudiendo llegar a ser superior al elegido por la empresa rival. En tercer lugar se considera el caso donde la expectativa del tamaño de red del bien de calidad alta es relativamente baja. Si la diferencia entre calidades es mínima; la empresa de baja calidad se apodera de todo el mercado estableciendo incluso un alto grado de compatibilidad. Esta situación podría llevarnos a posibles pérdidas de eficiencia del tipo de las asociadas a una mala elección del estándar, ya que se trata del producto de baja calidad. Si la diferencia entre calidades aumenta, la empresa de alta calidad elegirá cierto grado de compatibilidad, pudiendo llegar a ser superior al elegido por la empresa rival. Lo que puede concluirse es que el factor determinante son las externalidades directas de red junto con las expectativas de los consumidores. Ante productos relativamente homogéneos (mínima diferencia entre calidades) la empresa cuyas expectativas sean mayores se apoderará de todo el mercado; en cambio, ante productos heterogéneos (aumentos en la diferencia de calidades) ambas empresas participan en el mercado. Podemos concluir que las empresas con un elevado tamaño de red siempre prefieren incompatibilidad.

En el tercer capítulo, "*Standardization and Network Effects*", se extiende el modelo analizado en el capítulo anterior. Se analiza el efecto de las externalidades directas de red y se introducen las externalidades indirectas de red. Para ello se plantea un modelo donde interactúan desarrolladores de aplicaciones, consumidores y plataformas. Se analizan las decisiones estratégicas de las plataformas en cuanto a intensidad de la competencia, innovación y estandarización de producto. En este capítulo, el problema de decisión de las plataformas se modeliza como un juego seis etapas. En la primera etapa las plataformas eligen simultánea y no cooperativamente si adoptan un estándard comun en el mercado o no, en la segunda etapa del juego eligen el nivel de innovación tecnológico de sus productos y en la tercera etapa competirán en precios. En la cuarta etapa las empresas que desarrollan los complementos decidirán si participan en el mercado o no, en la quinta etapa eligen el precio de sus complementos y finalmente los consumidores deciden a que plataforma se suscriben y compran un complemento.

Así se propone un modelo donde las plataformas están diferenciadas horizontalmente a la Hotelling mientras que las empresas que desarrollan complementos están diferenciadas a la Salop; al final de cada extremo de la línea de Hotelling se encuentra el círculo donde están localizadas las empresas que desarrollan complementos. Los complementos están diseñados para ser utilizados en una única plataforma. Un consumidor que se encuentra situado a lo largo de la línea de Hotelling, primero decide qué plataforma adquirir; una vez ha decidido la plataforma se encontrará identificado en un círculo de Salop donde estarán las empresas que desarrollan los complementos y decidirá que complemento comprar. Por lo tanto, a la hora de decidir qué plataforma adquirir, el consumidor tendrá en cuenta las externalidades directas de red y, además, las externalidades indirectas de red, relacionadas con el número de empresas de complementos asociadas con cada plataforma. De este modo quedan introducidas las externalidades indirectas de red basándonos en que, cuanto mayor sea el número de desarrolladores de complementos de una plataforma menos costoso será para un consumidor adquirir dicho complemento (en términos de utilidad indirecta). Se tendrá en cuenta que los consumidores adquieren una plataforma y un complemento de una empresa que los desarrolla.

En este capítulo van a distinguirse dos casos atendiendo de las expectativas de los consumidores, bien sean éstas del tipo "fulfilled expectations", bien se considere que las expectativas están centradas en una sola plataforma ("non-fulfilled expectations").

Cuando se consideran "fulfilled expectations", la decisión de estandarizar el mercado depende del grado de diferenciación en el mercado de complementos. Si el mercado de complementos está muy diferenciado, las plataformas no adoptan el estándar. Por el contrario, si el mercado de complementos está poco diferenciado, las plataformas adoptan el estándar. También se obtiene que la propensión para estandarizar el mercado cuando el número de empresas de complementos de cada plataforma es distinto aumenta cuando i) la intensidad de la externalidad directa de red disminuye, ii) el número de empresas de complementos de la propia plataforma disminuye, iii) el número de empresas de complementos de la plataforma rival aumenta y, iv) el coste de transporte de las plataformas aumenta.

En la segunda parte del capítulo se considera que las expectativas se centran todas en una sola plataforma. En este caso las externalidades de red constituyen un factor clave en la decisión de los consumidores, pero no son el único factor que influye en su decisión; la plataforma sin expectativas participa en el mercado gracias a su bajo precio. En este caso se obtiene el mismo resultado que en el segundo capítulo: la plataforma con un elevado tamaño de red nunca querrá establecer un estándar, debido a que puede establecer un mayor precio y obtener un mayor beneficio si no lo hace.

Para finalizar el capitulo se analizan los efectos sobre el bienestar. Dependiendo del grado de diferenciación de los complementos el bienestar será mayor con o sin estándar; los que salen perjudicados con la adopción de un estándar son los consumidores. Para entender este resultado podemos aludir al efecto precio y al efecto variedad: al establecer un estándar se pierde la competencia en precios de las plataformas por imponer su producto o sistema, todos los consumidores se encuentran en la misma red y se pierde una variedad de producto al adoptar un estándar.

En el cuarto capítulo, "*Platforms, Software and Piracy in a two-sided market*" se estudia uno de los problemas más frecuentes que afecta a los mercados con alto contenido tecnológico y externalidades de red: la piratería. Al igual que en el capitulo anterior, plataformas, complementos y consumidores están presentes en el mercado. Las plataformas seguirán estando diferenciadas horizontalmente en la línea de Hotelling. Sin embargo, se supone que los complementos están uniformemente distribuidos en la línea unitaria con respecto a su preferencia por las plataformas, al igual que los consumidores. La piratería aparece en el mercado donde los consumidores adquieren los complementos. Por lo tanto, los consumidores deciden si compran el software original o si obtienen una copia gratuita, teniendo en cuenta que la copia puede ser de menor calidad y que las empresas de software tienen la posibilidad de invertir en mecanismos antipiratería. En la primera etapa las plataformas eligen simultánea y no cooperativamente el precio de sus productos y la licencia para los desarrolladores de complementos, en la segunda etapa del juego los consumidores decidirán a que plataforma se suscriben y finalmente si copian o compran el complemento original.

El objetivo es analizar como la piratería afecta a las decisiones de las plataformas referentes a la elección de los precios que se cargan a consumidores y a desarrolladores. Se pretende identificar, en este contexto bastante realista, quién gana con la piratería y si los desarrolladores de aplicaciones realmente están dispuestos a prevenirla.

En la primera parte de este capítulo se distinguen cuatro tipos diferentes de escenarios dependiendo de la afiliación de los consumidores y desarrolladores: i) consumidores multi-homing (afiliados a ambas plataformas) y desarrolladores single-homing, ii) ambos pueden afiliarse a una única plataforma (single-homing), iii) desarrolladores multi-homing y consumidores single-homing y iv) cuando ambos están afiliados a las dos plataformas (todo el mercado es multi-homing). Se analiza la decisión estratégica de las plataformas en cuanto a precios para los consumidores y licencias para los desarrolladores en cada uno de los escenarios.

En una segunda parte se supone que un regulador social interviene en el mercado para solucionar el problema de la piratería, y se analizan los mismos escenarios que en el caso anterior pero con regulación.

Exceptuando el caso donde los consumidores son multi-homing y los desarrolladores single-homing, los desarrolladores de complementos obtienen una mayor utilidad con piratería. Este resultado nos deja entrever que los desarrolladores de aplicaciones no necesariamente van a ser contrarios a la piratería. Por otro lado vemos que, si un regulador social corrige el problema, los desarrolladores obtienen un mayor nivel de utilidad. Si los desarrolladores de aplicaciones no pueden prevenir la piratería completamente deberían invertir en mejorar su producto e intentar explotar las posibles ventajas de la piratería en vez de invertir en mecanismos antipiratería.

Chapter 1. Related Literature

The literature that studies competition between systems that merge goods or complementary components is extensive and, among other topics, it has studied questions referring to decisions of technology adoption and consumer choice. Part of the literature, which is related to the present thesis, has been devoted to study decisions about compatibility and network effects.

The literature usually distinguishes between two types of network effects: direct network effects that arise in horizontal networks and indirect network effects that arise in vertical networks.

The literature dealing with direct network externalities is quite large and rather recent. Katz and Shapiro (1985) and Farrell and Saloner (1985) were the first to develop models of competition between systems, where each firm offers only one product or component that combined by consumers to form a system. There is a network effect since a consumer's utility increases with the number of other consumers who consume the system. These papers draw attention to the feature of "excess of inertia": the natural tendency of the market is towards the survival of a system or a technology. As for the decision about compatibility, Katz and Shapiro (1985) found that companies with a good reputation or with a high enough network size do not want compatibility. Note that when markets become standardized consumers experience a reduction in their choice set (there is less variety). Besides, there might be an additional inefficiency stemming from the possible imposition of an inferior technology or a product of a lower quality.

Other papers focus on situations in which each company produces all the components, such as Matutes and Regibeau (1988) and Economides (1989). Both articles characterize equilibrium prices and profits with and without compatibility. In this context it must be observed that consumers prefer greater variety because they can merge the components from different companies and form a higher number of systems (mix-and-match). In contrast to the aforementioned papers, this type of companies (that offer all the components) prefer compatibility because it generates higher prices and higher profits. This is true without network externalities and the private and social incentives would coincide if the variety effect dominates the price effect.

The introduction of product differentiation in models of technological innovation is more recent. Belleflamme (1998) develops a representative consumer model of product differentiation (non-address approach); the companies choose between two technologies with network externalities (marginal cost decreases with the number of companies that adopt the same technology) and are Cournot competitors. He concludes that it is more probable that the companies adopt the same technology the more differentiated the products are.

This result is in contrast with that obtained in settings in which product differentiation continues on a path approach (address approach). In this way, Baake and Boom (2001) demonstrate that, when there is high degree of differentiation, the companies will agree the provision of an adapter that links compatible technologies and stresses the externality. In their model, product quality is endogenous and, in equilibrium, it is always decided in favor of compatibility. Sarkar (2005) develops a model of vertical differentiation, where quality can be high or low, and standardization is the only form of reaching compatibility; this does not always happen in equilibrium. On the other hand, the article by Sääskillathti (2006) features horizontal differentiation and she studies R&D investment decisions with direct network externalities. Her focus is on the interaction between R&D externalities and compatibility, although the latter is not decided strategically. When R&D investments and the network effect are important, the price may be reduced, which increases competition intensity. Therefore, previous results in the literature are reverted, pointing out that compatibility is anti-competitive.

Indirect network effects can generate an element of vertical differentiation; there are papers based on the same idea, yet using different models. Thus, Church and Gandal (1992) and Katz and Shapiro (1994) examine the incentives of platforms at the time to standardize the market, there is competition in a hardware-software environment, and the companies of complements develop complements that work exclusively with one of the platforms. An indirect effect arises for products where hardware sales influence software sales and vice versa. In Economides (1994) network externalities are then internalized and take the form of economies of scope. In software production, economies of scale are often present, which can result in inefficient technology adoption if development costs for software are lower for one hardware technology, Church and Gandal (1993). Clements (2004) develops a model with horizontal differentiation with both direct and indirect externalities, but he does not analyze the two effects at the same time. This author first assumes that the indirect network externality does not exist, analyzing in this way the effects of direct externalities; he then assumes the opposite and he analyzes the propensity for the market to standardize. Church, Gandal and Krause (2008) examine the effects of indirect network externalities in a horizontally differentiated market, where we find hardware, software and companies that produce other goods. They study the choices of the complements companies. In the work of Casadesus and Ruiz (2009), a model of horizontal differentiation in two-sided markets is raised in the presence of direct network externalities. Compatibility is represented by the complement, that is to say, the platforms will be compatible if the complements can work for both platforms.

Other overviews and discussions of the network effects literature can be found in the survey paper by Economides (1996) and in the books by Shy (2001) and Shapiro and Varian (1999). Economides (1996) has a strong focus on the theoretical literature and, like the literature in general at that stage, focuses in particular on standardization, compatibility and industry structure aspects. The books tend to focus on the theoretical literature, Shy (2001) or on the managerial implications of these findings, Shapiro and Varian (1999).

The discussion on market intermediaries is the main distinction between the literature on two-sided markets and the literature on network effects, and on indirect network effects in particular. In the two-sided markets literature (Armstrong, 2006; Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006), the price structure depends on the indirect network externalities between user groups (two-sided network externalities). The classic example is provided by credit card services. In the last years, many research works have addressed diverse issues related to two-sided markets, and have considered variants of assumptions about timing, price instruments and externalities. In two-sided markets will find the possibility of consumers multi-homing and/or developers multi-homing, Choi (2010) analyzes the effects of tying on market competition in two-sided markets with multi-homing, he incorporates the possibility of multi-homing on both sides of the market. Belleflamme and Peitz (2010b) study the seller investment incentives in a two-sided framework with consumers multi-homing and with sellers multi-homing.

Software piracy influences the two-sided network externalities. In this thesis, when piracy is considered, a contribution to previous research is made since we analyze conditions under which piracy may not be beneficial to firms, which is contrast with the received literature that basically looks at when piracy is beneficial. In previous papers it is shown that, in the presence of network externalities, developers may benefit from software piracy.

In a monopoly structure, Conner and Rumelt (1991) found that increased protection raised price and profit and Takeyama (1994) that copying lead to greater firm profits and can produce a Pareto improvement in social welfare.

In oligopoly settings we found similar results as demonstrated in Shy and Thisse (1999), a coordinated software industry should choose not to protect the software when the network effects are strong, and Peitz (2004). Our fourth chapter differs from these contributions by taking a specific two-sided market view, we focus on platform behavior and we analyze four different types of scenarios[1]. Peitz and Wael-

[1] The first case is where consumers use a multi-homing strategy and developers single-home. Then, we consider that consumers and developers single-home. Then, the case of only developers multi-homing is analyzed and finally we look into the case of both agents multi-homing.

broeck (2006b) show us that consumers are willing to pay for the original although they could consume the download for free because they can make more informed with the illegal copy.

Finally, Rasch and Wenzel (2013) study the impact of software piracy in a single-homing two sided-market. Developers benefit from protection if the impact on the developers' immediate legal sales is small and the impact on consumer surplus from software consumption is large and platforms' profits decreases if the impact on legal sales is large, because they reduce the license fees.

Chapter 2. Compatibility, vertical differentiation and network effects

2.1. Introduction

In this chapter we study the role played by expectations, the strength of the network externality and product differentiation in the strategic decisions of the platforms regarding compatibility and price competition. Network externalities are represented by consumers expectations about the network size of each platform. Although their qualities are exogenous, the choice of the degree of compatibility endogenizes vertical differentiation since it affects the willingness to pay for the product. In industries with network externalities, the degree of compatibility can generate an element of vertical differentiation when consumers perceive incompatible products with different market shares that represent different levels of quality.

An example of the issues that we will be addressed in this chapter is the internet providers, specifically e-mail providers. In the beginning, America Online and Delphi Compuserver used their own e-mail systems. It was very difficult to send e-mails between systems of different companies. Consumers ended up using the systems with more consumers taking into account the quality of the service, consumers valued the quality / network externalities ratio.

Network externalities play a crucial role in consumers decision making, not to mention that quality also exerts an influence on this decision. If the mail systems were compatible, consumers would focus on their quality, because all consumers would indeed belong to the same network.

However, a high quality product might fail to prevail ending up with a relatively small network if consumers have low expectations, despite it being the high quality product. This happened to Digital with its microprocessor Alpha. The Alpha chip provided a higher quality than its rival Intel, they were faster. Even so Digital only sold 300000 chips while Intel sold 65 million. Digital applied compatibility with Intel and tried to benefit from the large network.

Another example is given by the word processors. Transfer files from WordPerfect to Word is a process where the errors are abundant. In these products there exists partial compatibility. The Word network size is greater than WordPerfect and its quality is superior. If Word made its product incompatible then WordPerfect would disappear.

A network is composed by a group of consumers of the same good or a compatible good. The network provides benefits or positive externalities if the utility enjoyed by each individual consumer is greater with more users. The modeling in a market with direct network effects requires the consideration that the willingness to pay for the product depends on the number of users. Demand changes each time that price changes and a different price implies a different number of users, and the willingness to pay is a function of the number of users. Therefore, the consumers expectation about the number of users (and other features like compatibility, quality, and so on) are one of the distinctive features in markets with network effects.

The coordination between platforms is another important element: the introduction of a new microprocessor will be useless unless software that works with it is provided. Coordination gives value to consumers if they have the expectation that this coordination will occur. Thus, the combination of hardware and software allows us to obtain a system. The compatibility decision - no, partial or complete - is related with the intensity of competition and affect the variety of products available for the consumer.

The decision problem is modeled as a two-stage game. In the first stage platforms simultaneously and non-cooperatively choose the degree of compatibility and in the second stage platforms compete in prices. We obtain the following results. When the expectations about network size are symmetrical, the high quality platform always chooses compatibility and fixes a higher price. The result changes when consumers have different expectations about networks size: if the difference between qualities is minimal, only the company whose expectations of network size are larger serves the market. If the products are minimally vertically differentiated, quality becomes irrelevant and externalities become decisive. When the difference between qualities increases, perhaps the degree of compatibility chosen by the high quality platform is not necessarily higher than the degree of compatibility chosen by the low quality platform and vice versa.

2.2. The Model

We consider a market formed by two vertically differentiated platforms, A and B. Each platform develops a good with quality s_A and s_B respectively; we assume that $s_A > s_B > 0$.

Each platform produces a good with specific characteristics that can make it compatible with the rival product or not. The platforms choose the degree of compatibility of their products. This degree is given by $\{t_A, t_B\}$, where $t_A, t_B \epsilon [0, 1]$.

Thus,

t_A measures the degree of compatibility of product A with B, whereas

t_B measures the degree of compatibility of product B with A.

There are two extreme cases;

$\{t_A = 0, t_B = 0\}$totally incompatible products and

$\{t_A = 1, t_B = 1\}$totally incompatible products.

If products are totally compatible $\{t_A = 1, t_B = 1\}$, them firms compete in the market. Consumers get the full benefits of the total network and platforms specific network sizes become irrelevant in consumers´ decision making.

If products are totally incompatible $\{t_A = 0, t_B = 0\}$, them firms compete for the market and each platform network size is important for consumers decisions.

This modelling allows for the possibility of partial compatibility in the case where one of the two firms chooses $t_i, i = A, B$, equal to zero.

There is a continuum of heterogeneous consumers in their willingness to pay for quality. It is assumed that consumers are uniformly distributed in $\theta \epsilon [0, 1]$ with density $f(\theta) = 1$; market size equals one. Each consumer consumes only one unit of one of the two products. All consumers have common expectations about the number of consumers of each product, these expectations are given by:

$$n_i \epsilon (0, 1) \text{ where } i \epsilon \{A, B\} \; i \neq j \text{ and } n_i + n_j = 1$$

The indirect utility of a consumer is given by

$$U_\theta = \begin{cases} V + n_A v + n_B t_A v + s_A \theta - p_A & \text{if buys from platform A} \\ V + n_B v + n_A t_B v + s_B \theta - p_B & \text{if buys from platform B} \end{cases} \tag{2.1}$$

We assume that V, the reservation utility level, is large enough to ensure that all consumers will participate in the market. The market is covered. The parameter n_A is the expected number of A consumers and the parameter n_B the expected number of B consumers, v corresponds with the strength of the direct network externality. It is assumed $v \epsilon [0, 1]$, finally p_A and p_B are the platforms product prices. For a consumer θ the willingness to pay increases if the quality is greater and if the expectations about the network size increase.

We can observe that this specification allows to distinguish the effect of the externalities from the same network (intranet externalities) from the externalities from the other network (internet externalities) with the variable t_i, that reflects the degree of compatibility chosen. It is logical to assume that with more consumers in the network the users obtain more utility than in the competing network. Summarizing, consumers are heterogeneous in their valuation of the quality and are identical in their expectations about the number of users in each network.

The analysis of competition between platforms is modeled as a three stage non-cooperative game, where platforms behave as profit maximizers. In the first stage, platforms simultaneously choose the levels of compatibility. In the second stage the firms simultaneously choose the prices for consumers and in the third stage, consumers decide which platform to join. The solution concept is Subgame Perfect Nash Equilibrium, we solve the model by backward induction.

Equalling the two branches of the indirect utility function there is one consumer located at θ who is just indifferent between buying from firm A or B. This consumer is given by

$$V + n_A v + n_B t_A v + s_A \theta - p_A = V + n_B v + n_A t_B v + s_B \theta - p_B \tag{2.2}$$

$$\overline{\theta} = \frac{p_A - p_B - n_A v + n_B v - n_B t_A v + n_A t_B v}{s_A - s_B} \tag{2.3}$$

The market demands are given by:

$$Q_A = 1 - \overline{\theta} = 1 - \frac{p_A - p_B - n_A v + n_B v - n_B t_A v + n_A t_B v}{s_A - s_B} \tag{2.4}$$

$$Q_B = \overline{\theta} = \frac{p_A - p_B - n_A v + n_B v - n_B t_A v + n_A t_B v}{s_A - s_B} \tag{2.5}$$

The choice of the degree of compatibility t_i endogenizes vertical differentiation between platforms: given a difference in qualities, if firm A increases t_A the indifferent consumer is moved to the left and Q_A will be greater.

The costs are quadratic with the compatibility level that they choose and linear with output:

$$C(t_i) = \gamma \frac{t_i^2}{2} \text{ where } t_i \epsilon \{A, B\} \text{ and it is assumed that } \gamma > \frac{4}{9}v \tag{2.6}$$

$$C(Q_i) = cQ_i, \text{ where } c \geq 0,\ Q_i \epsilon \{A, B\} \tag{2.7}$$

With the assumption $C(t_i) = \gamma \frac{t_i^2}{2}$ we reflect the existence of diminishing returns to compatibility investment and $\gamma > \frac{4}{9}v$ guarantees that the degrees of compatibility chosen are between 0 and 1.

Now, the platforms profit are given by:

$$\pi_A = (p_A - c)Q_A - \gamma \frac{t_A^2}{2} \tag{2.8}$$

$$\pi_B = (p_B - c)Q_B - \gamma \frac{t_B^2}{2} \tag{2.9}$$

In the next sections we solve the second and first stage of the game. The platforms decide the price charged to the consumers and the degrees of compatibility. The equilibrium is characterized by $(p_A^*, p_B^*, t_A^*, t_B^*)$.

2.3. Platforms Decision: The Price stage

In the second stage the firms simultaneously choose the profit maximizing price that is charged to consumers. The first order conditions are given by:

$$\frac{\partial \pi_A}{\partial p_A} = 1 - \frac{p_A - p_B - n_A v + n_B v - n_B t_A v + n_A t_B v}{s_A - s_B} - \frac{p_A - c}{s_A - s_B} = 0 \tag{2.10}$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{p_A - p_B - n_A v + n_B v - n_B t_A v + n_A t_B v}{s_A - s_B} - \frac{p_B - c}{s_A - s_B} = 0 \tag{2.11}$$

And the second order conditions are given by

$$\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{2}{s_A - s_B} < 0 \tag{2.12}$$

which guarantee a maximum since $s_A > s_B$.

From (2.10) and (2.11) the best response functions are given by:

$$p_A(p_B) = \frac{1}{2}(c + p_B + s_A - s_B + v(n_A(1 - t_B) - n_B(1 - t_A))) \tag{2.13}$$

$$p_B(p_A) = \frac{1}{2}(c + p_A + s_A - s_B + v(n_B(1 - t_A) - n_A(1 - t_B))) \tag{2.14}$$

Where it is easy to establish that prices are strategic complements. The Nash equilibrium pair of prices is the following:

$$p_A^* = \frac{1}{3}(3c + 2(s_A - s_B) + v(n_A(1 - t_B) - n_B(1 - t_A))) \tag{2.15}$$

$$p_B^* = \frac{1}{3}(3c + s_A - s_B + v(n_B(1 - t_A) - n_A(1 - t_B))) \tag{2.16}$$

We can write the difference between p_A^* and p_B^* and we obtain:

$$p_A^* - p_B^* = \frac{1}{3}(s_A - s_B + 2v(n_A(1 - t_B) - n_B(1 - t_A))) \tag{2.17}$$

If the difference in qualities is greater than certain threshold the price of the firm with the high quality product is greater than the price of the firm with the low quality product. This limit enhances, once again, the importance of the interaction between the expectations about network size and the degree of compatibility. Unlike a standard model of vertical differentiation, the price of the high quality product is not necessarily greater than the price of the low quality product.

From the equilibrium prices it is easy to obtain the following comparative static results;

LEMMA 1.

(1.1) *If the products are not fully compatible, an increase in the expectations of own network size will lead to an increase in own price.*

$$\frac{\partial p_A^*}{\partial n_A} = \frac{1}{3}v(1 - t_B) > 0 \tag{2.18}$$

$$\frac{\partial p_B^*}{\partial n_B} = \frac{1}{3}v(1 - t_A) > 0 \tag{2.19}$$

Note that the willingness to pay for a product increases as more users purchase that product. If there is an increase in the expectations about network size of one of the two products, then every potential consumer is willing to pay more for that product.

(1.2) *If the products are not fully compatible, increases in the expectation of the rival network causes a decrease in own price.*

$$\frac{\partial p_A^*}{\partial n_B} = \frac{1}{3}v(t_A - 1) < 0 \tag{2.20}$$

$$\frac{\partial p_B^*}{\partial n_A} = \frac{1}{3}v(t_B - 1) < 0 \tag{2.21}$$

The smaller the network that one firm has, the more it has to reduce prices to compensate the effect of the externality and attract consumers.

LEMMA 2. *Under full compatibility, expectations of network size of any firm do not affect the prices.*

$$\frac{\partial p_A^*}{\partial n_A} = \frac{\partial p_B^*}{\partial n_B} = \frac{\partial p_A^*}{\partial n_B} = \frac{\partial p_B^*}{\partial n_A} = 0 \tag{2.22}$$

Further, we always have that $p_A^* > p_B^*$.

This case implies that consumers perceive one network, the difference between platforms is determined by the quality levels. With complete compatibility in the products, the network size is the sum of $n_A + n_B = 1$, *network externalities are not important when users decide on which product to purchase. If the expectation of a particular network is practically zero, but the products are fully compatible, then network externalities would be irrelevant.*

The equilibrium prices change depending on the strength of the externality.

LEMMA 3. *The strength of the network externality does not affect prices if the expectations differ but the products are fully compatible and if the expectations are the same and the products are either fully compatible or totally incompatible.*

$$\frac{\partial p_A^*}{\partial v} = \frac{1}{3}(n_A(1-t_B) - n_B(1-t_A)) \tag{2.23}$$

$$\frac{\partial p_B^*}{\partial v} = \frac{1}{3}(n_B(1-t_A) - n_A(1-t_B)) \tag{2.24}$$

LEMMA 4. *Changes in own degree of compatibility cause an increase in own price and a decrease in that of the rival.*

$$\frac{\partial p_A^*}{\partial t_A} = \frac{n_B v}{3} > 0,\ \frac{\partial p_A^*}{\partial t_B} = -\frac{n_A v}{3} < 0 \tag{2.25}$$

$$\frac{\partial p_B^*}{\partial t_A} = -\frac{n_B v}{3} < 0,\ \frac{\partial p_B^*}{\partial t_B} = \frac{n_A v}{3} > 0 \tag{2.26}$$

If one firm increases the degree of compatibility of its product then it is indeed increasing the willingness to pay for its product. Ceteris paribus, the firm can set a higher price without losing customers. If instead the rival firm increases the degree of compatibility then the externality between networks is higher forcing a decrease in price for made the product more attractive. As companies decide strategically the degrees of compatibility, it is clear that the final effect on the prices depends on the expectations about the network size.

REMARK. *We wish to add the following comment. The prices in these markets, generally, will not be a determining factor, despite it a key one on users decisions. For new products not subject to externalities a skimming strategy is usually recommended, with high prices that are acceptable to consumers less price sensitive, and allow rapid recovery of investment. However, in the presence of the network externalities, this strategy can be counterproductive because it slows the development of a users base. Moreover, it is recommended a penetration pricing strategy (being able to even get to give away the product) that maximizes the creation of value linked with the installed base. In these markets, even a monopolist has strong incentives to launch a low-priced product, even below cost in order to attract a sufficiently large number of users.*

2.4. Platforms Decision: The Compatibility stage

In the first stage, platforms will choose the degrees of compatibility that maximize their profits. Fort he sake of exposition we will employ $\triangle$ to denote the quality difference $s_A - s_B$. Making use of (2.15)-(2.16) we can write the following expressions:

$$\pi_A = \frac{(2\triangle + v(n_A(1-t_B) - n_B(1-t_A)))^2}{9\triangle} - \gamma\frac{t_A^2}{2} \tag{2.27}$$

$$\pi_B = \frac{(\triangle + v(n_B(1-t_A) - n_A(1-t_B)))^2}{9\triangle} - \gamma\frac{t_B^2}{2} \tag{2.28}$$

The first order conditions are:

$$\frac{\partial \pi_A}{\partial t_A} = \frac{2n_B v(2\triangle + v(n_A(1-t_B) - n_B(1-t_A)))}{9\triangle} - \gamma t_A = 0 \tag{2.29}$$

$$\frac{\partial \pi_B}{\partial t_B} = \frac{2n_A v(\triangle + v(n_B(1-t_A) - n_A(1-t_B)))}{9\triangle} - \gamma t_B = 0 \tag{2.30}$$

and the second order conditions are the following:

$$\frac{\partial^2 \pi_A}{\partial t_A^2} = \frac{2n_B^2 v^2}{9\triangle} - \gamma < 0 \tag{2.31}$$

$$\frac{\partial^2 \pi_B}{\partial t_B^2} = \frac{2n_A^2 v^2}{9\triangle} - \gamma < 0 \tag{2.32}$$

which impose a restriction, restriction as follows

$$s_A > s_B + \frac{2v^2}{9\gamma} Max\{n_A^2, n_B^2\} \tag{2.33}$$

The platform A quality must be greater than platform B quality plus an amount, this amount correspond with $\frac{2v^2}{9\gamma} Max\{n_A^2, n_B^2\}$.

The best response functions are given by:

$$t_A(t_B) = \frac{2n_B v(2\triangle - (n_B - n_A(1-t_B))v)}{9\triangle\gamma - 2n_B^2 v^2} \tag{2.34}$$

$$t_B(t_A) = \frac{2n_A v(\triangle - (n_A - n_B(1-t_A))v)}{9\triangle\gamma - 2n_A^2 v^2} \tag{2.35}$$

The Nash equilibrium pair of degrees of compatibility is the following:

$$t_A^* = \frac{6n_B v\gamma(2\triangle + v(n_A - n_B)) - 4n_A^2 n_B v^3}{3\gamma(9\triangle\gamma - 2v^2(n_A^2 + n_B^2))} \epsilon[0,1] \tag{2.36}$$

$$t_B^* = \frac{6n_A v\gamma(\triangle + v(n_B - n_A)) - 4n_B^2 n_A v^3}{3\gamma(9\triangle\gamma - 2v^2(n_A^2 + n_B^2))} \epsilon[0,1] \tag{2.37}$$

We verify the nature of the strategic variables chosen by platforms because they are vital for the analysis. Reaction curves for strategic complements have a positive slope, while it is negative slope for strategic substitutes

$$\frac{\partial^2 \pi_A}{\partial t_A \partial t_B} = \frac{\partial^2 \pi_B}{\partial t_B \partial t_A} = -\frac{2n_A n_B v^2}{9\triangle} < 0 \tag{2.38}$$

It follows that the degrees of compatibility are strategic substitutes, the reaction functions are downward sloping.

Next, we define the following levels of expectations:

$$n_A = \frac{4v - 9\gamma + \sqrt{(2v - 9\gamma)(4v - 9\gamma)}}{2v} \text{ and } n_B = 1 - \frac{4v - 9\gamma + \sqrt{(2v - 9\gamma)(4v - 9\gamma)}}{2v} \tag{2.39}$$

These levels of expectations make the choice of t_A^* and t_B^* independent of any value of product qualities. This means: when the levels of expectations are given by (2.39), the difference between qualities, $\triangle$, disappears from the equations (2.36) and (2.37). The choice of the level of compatibility is not affected by the difference between qualities when $n_A = \frac{4v - 9\gamma + \sqrt{(2v - 9\gamma)(4v - 9\gamma)}}{2v}$.

We can see the values of the consumers expectations about n_A when strength of the network externality takes its extreme values

$$lim_{v \to 0} \frac{4v - 9\gamma + \sqrt{(2v - 9\gamma)(4v - 9\gamma)}}{2v} = \frac{1}{2} \text{ and}$$

$$lim_{v\to 1}\frac{4v-9\gamma+\sqrt{(2v-9\gamma)(4v-9\gamma)}}{2v}=\frac{1}{2}(4-9\gamma+\sqrt{81\gamma^2-54\gamma+8})$$

From now on we will use these levels of expectations to measure the size of the consumers expectations, when $n_A > \frac{4v-9\gamma+\sqrt{(2v-9\gamma)(4v-9\gamma)}}{2v}$ the expectations of the high quality platform are relatively large and when $n_A < \frac{4v-9\gamma+\sqrt{(2v-9\gamma)(4v-9\gamma)}}{2v}$ the expectations of the high quality platform are relatively small.

Now, remember that the degrees of compatibility must be bounded between $(0,1)$. The equilibrium pair t_A^* and t_B^* will belong to $(0,1)$ as long as the following conditions are met:

Condition 1. When the expectations about network size of the high quality product platform are relatively large: $s_A \geq s_B + \frac{1}{3\gamma}(2n_B^2v^2 + 3v\gamma(n_A - n_B))$.

The above condition is always satisfied for n_A is greater than n_B ; however, for high levels for the strength of the externality, v, we can find that $n_B > n_A$. Furthermore, this condition is applied in the particular case of $n_A = n_B = 1/2$.

Condition 2. When the expectations about network size of the high quality product platform are relatively small: $s_A \geq s_B + \frac{1}{6\gamma}(2n_A^2v^2 + 3v\gamma(n_A - n_B))$.

In this case n_B is always greater than n_A.

If these conditions are satisfied, then the second order condition and the stability condition will also be satisfied, the latter condition tells us that

$$\frac{2n_A^2n_B^2v^4}{(9\gamma(s_B-s_A)+2n_A^2v^2)(9\gamma(s_B-s_A)+2n_B^2v^2)} < 1 \tag{2.40}$$

2.5. Comparative statics

We will analyze how the choice of the degree of compatibility varies depending on whether consumer expectations about network size and the difference between qualities. Also we will study how equilibrium prices and demands are affected.

We distinguish two different cases, considering that consumers expect that network sizes are the same or different. Moreover, in the latter case, we have two separate subcases depending whether the expectation about the high quality platform is relatively large or small.

2.5.1. Case 1. $n_A = n_B = 1/2$.

If consumers have an expectation that network sizes are equal, then expressions (2.15), (2.16), (2.36) and (2.37), become

$$t_A^* = \frac{v(12\triangle\gamma - v^2)}{6\gamma(9\triangle\gamma - v^2)}$$

$$t_B^* = \frac{v(6\triangle\gamma - v^2)}{6\gamma(9\triangle\gamma - v^2)}$$

$$p_A^* = c + \frac{1}{6}(4\triangle + \frac{\triangle v^2}{9\triangle - v^2})$$

$$p_B^* = c + \frac{1}{6}(2\triangle - \frac{\triangle v^2}{9\triangle - v^2})$$

$$Q_A^* = \frac{1}{6}(4 + \frac{v^2}{9\triangle - v^2})$$

$$Q_B^* = \frac{1}{6}(2 - \frac{v^2}{9\triangle - v^2})$$

It is easy to see that the ordering of the degrees of compatibility, equilibrium prices and demands is determined by vertical differentiation $\triangle$, since

$$t_A^* - t_B^* = \frac{6v\triangle\gamma}{6\gamma(9\triangle\gamma - v^2)}$$

$$p_A^* - p_B^* = \frac{1}{3}\triangle$$

$$Q_A^* - Q_B^* = \frac{1}{3}(1 + \frac{v^2}{9\triangle\gamma - v^2})$$

Which are always positive because $\triangle > 0$. Thus, the platform with a high quality good chooses a high degree of compatibility and sets a higher price than the rival platform.

With symmetric expectations p_A^* is greater than p_B^*, this produces a shift of the indifferent consumer to the right, however the high quality platform sets a level of compatibility greater than the low quality platform and the indifferent consumer moves to the left. We conclude that the compatibility effect is stronger than the price effect.

LEMMA 5. *For the same level of expectations $n_A = n_B = 1/2$, the high quality platform fixes a higher price and a higher degree of compatibility and obtains a greater demand.*

If the difference between qualities is small is possible that the low quality product decides to make it incompatible. That is to say, the numerator in t_B^* vanishes if the strength of the network externality, v, is strong or if gamma, is relatively large. With low product differentiation competition between platforms will be more intense. This parameter combination leads to the low quality platform to set price equal to marginal cost. The choice of a certain degree of compatibility by the high quality platform promotes the externality within the network that even consumers with a low willingness to pay can afford to purchase the high quality platform. This is a particular case of one way compatibility, only one of the two platforms serves the market.

The above comparisons suggest the importance of the difference between qualities. We study how change the degrees of compatibility, prices and demands in terms of $\triangle$ if platforms have the same expectations about the network size.

$$\frac{\partial t_A^*}{\partial \triangle} = -\frac{v^3}{(2v^2 - 9\triangle\gamma)^2} < 0$$

$$\frac{\partial t_B^*}{\partial \triangle} = \frac{v^3}{(2v^2 - 9\triangle\gamma)^2} < 0$$

$$\frac{\partial p_A^*}{\partial \triangle} = \frac{2}{3} - (\frac{v^4}{6(v^2 - 9\triangle\gamma)^2}) > 0$$

$$\frac{\partial p_B^*}{\partial \triangle} = \frac{1}{3} + (\frac{v^4}{6(v^2 - 9\triangle\gamma)^2}) > 0$$

$$\frac{\partial Q_A^*}{\partial \triangle} = -\frac{3\gamma v^3}{(2v^2 - 9\triangle\gamma)^2} < 0$$

$$\frac{\partial Q_B^*}{\partial \triangle} = \frac{3\gamma v^3}{(2v^2 - 9\triangle\gamma)^2} > 0$$

The effects of changes in the difference between qualities are enumerated in the following lemma;

LEMMA 6. *For the same level of expectations $n_A = n_B = 1/2$: An increase in $\triangle = s_A – s_B$ produce: i) a decrease in the compatibility of the firm A, t_A^*, ii) an* *increase of the compatibility of the firm B, t_B^*, iii) a decrease in the quantity sold by firm A, iv) an increase in the quantity sold by firm B and v) increases in the prices of both firms.*

Recall that the degree of compatibility are strategic substitutes. Increases in the difference between qualities move the functions of reaction outward. We can deduce that the magnitude of the shift is more strong for the low quality platform. Only thus we can be understood that the effect on t_A^* is negative and the effect on t_B^* is positive. Platform prices will be higher by the direct effect from the difference between qualities[1]. Increases in the quality differential allows both platforms obtain a positive market shares.

2.5.2. Case 2. $n_A \neq n_B$.

The discussion on pages 34 and 35 allows us to distinguish two subcases depending on whether the expectations about network size of the high quality product are relatively large or not.

2.5.2.1. When n_A relatively large

The graph below illustrates a relatively large expectation compared with the strength of the network externality when $\gamma = 1/2$. We can observe that the values of n_A will almost always be higher than n_B, but for very high levels of it may not be so.

[1] In (2.15)-(2.16) we can seen that the difference between qualities enters directly and indirectly through t_A^* and t_B^*. In fact, the price p_A^* depends positively from $t_A^* - t_B^*$. As $t_A^* > t_B^*$, p_A^* price increases. However, the price p_B^* depends negatively from $t_A^* - t_B^*$. The direct effect dominates.

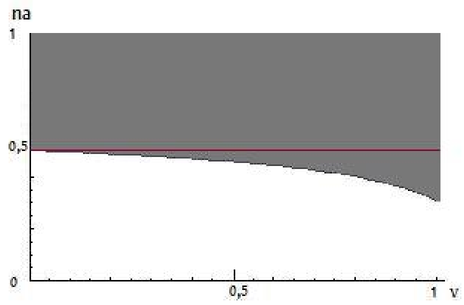

Figure 2.1: n_A relatively large.

Minimum difference between qualities: we know that the Condition 1 is the condition that must be fulfilled for the degrees of compatibility to belong to $(0,1)$. Let us define the following lower limit:

$$s^* = s_B + \frac{1}{3\gamma}(2n_B^2 v^2 + 3v\gamma(n_A - n_B)) \tag{2.41}$$

to study, through limits, how the equilibrium degrees of compatibility respond when the quality difference is minimal:

$$lim_{s_A \to s^*} t_A^* = \frac{2n_B v}{3\gamma}$$

$$lim_{s_A \to s^*} t_B^* = 0$$

The platform with a high quality product sets a degree of compatibility that depends on the expectations of rival network size, the strength of the externality and the compatibility cost. When the difference between qualities is minimal, the bigger the expectations about the rival´s network size the greater the degree of compatibility that the high quality platform will choose. This is its strategic response to promote an increase in the willingness to pay for its product by means of the effect of the externality between networks (internet).

As consumer expectations are favorable to the high quality platform and the difference between qualities is minimal, the low quality platform chooses incompatibility and it does not produce in the market; all consumers go to platform A.

Besides;

$$lim_{s_A \to s^*} p_A^* = c + n_A v + \frac{n_B v(2n_B v - 3\gamma)}{3\gamma}$$

$$lim_{s_A \to s^*} p_B^* = c$$

$$lim_{s_A \to s^*} Q_A^* = 1$$

$$lim_{s_A \to s^*} Q_B^* = 0$$

This limiting case leads to the same qualitative conclusions that are reached under symmetric expectations.

Increases in the difference between qualities: if the difference between qualities is important, the degree of compatibility chosen by the high quality platform is not necessarily greater than the degree chosen by the other platform. Specifically, we have $t_B^* > t_A^*$ when:

$$s_A > \frac{3\gamma(n_A s_B + n_A^2 v - n_B(2s_B + n_B v) - 2n_A n_B v^2(n_A - n_B)}{3\gamma(n_A - 2n_B)} > s^* \tag{2.42}$$

Furthermore, as shall be seen below, increases in the quality difference leads high quality platform to reduce its degree of compatibility while the low quality platform increases it. From the above discussion we can conclude:

LEMMA 7. *If the expectations about the network size of the high quality platform are sufficiently high, the platform that offers the low quality product chooses incompatibility when the difference between qualities is minimal. However the plat-*

form that offers the low quality product chooses some degree of compatibility when the difference between qualities increases, it may even be superior to the rival ´s.

2.5.2.2. When n_A relatively small

Here we analyze the case where the expected network size of the high quality product is relatively low. The graph below (when $\gamma = 1/2$) shows that n_A will take values below 50%. Specifically, the values of n_A will be lower tan n_B regardless of the strength of the network externality, v.

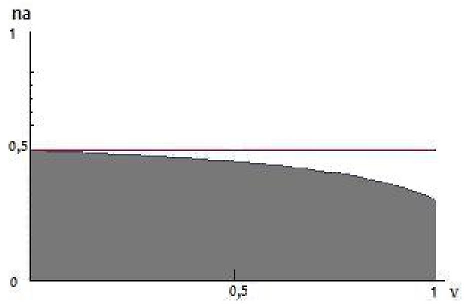

Figure 2.2: n_A relatively small.

Minimum difference between qualities: we know that the Condition 2 is the condition that must be fulfilled for the degrees of compatibility to belong to $(0, 1)$. Let us define the following lower limit:

$$s^{**} = s_B + \frac{1}{6\gamma}(2n_A^2 v^2 + 3v\gamma(n_A - n_B)) \tag{2.43}$$

to study, through limits, how the equilibrium degrees of compatibility respond when the quality difference is minimal:

$$lim_{s_A \to s^{**}} t_A^* = 0$$

$$lim_{s_A \to s^{**}} t_B^* = \frac{2n_A v}{3\gamma}$$

In this case, as in the previous one, the platform with a low quality product sets a degree of compatibility that depends on the expectations about the rival´s network size, the strength of the network externality and the compatibility cost. If the difference between qualities is minimal, the greater the expectations, the greater the degree of compatibility that platform B will choose.

As consumer expectations are favorable to the low quality platform and the difference between qualities is minimal, the high quality platform chooses incompatibility and it will not produces in the market. Consumers, with nearly equal qualities, opt for the platform whose expectations are larger, in this case the platform B.

Furthermore;

$$lim_{s_A \to s^{**}} p_A^* = c$$

$$lim_{s_A \to s^{**}} p_B^* = c + \frac{v(2n_A^2 + v - 3\gamma(n_A - n_B))}{6\gamma}$$

$$lim_{s_A \to s^{**}} Q_A^* = 0$$

$$lim_{s_A \to s^{**}} Q_B^* = 1$$

In this situation the low quality platforms serves all the market establishing a positive high degree of compatibility. This situation, if sustained over time, can lead to possible efficiency losses of the type associated with a poor choice of the standard, because the product in the market is the low quality product. Even though the quality difference is minimal. The expectations about network size of platform B are greater than the rivals; if the difference between qualities is small consumers, before two undifferentiated products, choose the platform with higher expectations. In fact, in markets subject to network externalities it is not surprising that a technology is overtaken by another technically inferior if the expectations about network size are favorable. The key to such the inefficient adoption is the dependence on the initial conditions, small differences in the initial market shares (by the expectations) can make a big difference in the evolution of the market. In fact, platform B ends up monopolize it.

Increases in the difference between qualities: Is worth noting the complementary case stage when n_A is relatively large. Now, the degree of compatibility chosen by the low quality platform is not necessarily greater than the chosen by the rival. Specifically, $t_A^* > t_B^*$ if:

$$s_A > \frac{3\gamma(n_A s_B + n_A^2 v - n_B(2s_B + n_B v) - 2n_A n_B v^2(n_A - n_B)}{3\gamma(n_A - 2n_B)} > s^{**} \qquad (2.44)$$

Besides, increases in the quality difference leads low quality platform to reduce its degree of compatibility while the high quality platform increases it.

LEMMA 8. *If the expectations about the network size of the high quality platform are sufficiently small, then it chooses incompatibility when the difference between qualities is minimal. However the platform that offers the high quality product chooses some degree of compatibility when the difference between qualities increases.*

However, when the difference between qualities increases it chooses some degree of compatibility and may be greater than the rival.

It is deduced from the Lemma 7 and Lemma 8 that: if the difference between qualities is minimal, only the platform with more expectations about network size sold in the market, independently of whether it is high or low quality platform.

This emphasizes the importance of the network externalities in consumer decisions in technological markets, if products are minimally vertically differentiated, quality becomes irrelevant, the important thing is network externalities.

With the minimum degree of vertical differentiation, the network externalities become the platform with more expectations in a monopoly. Increases in the levels of quality produce more competition.

It is not clear the final effect on welfare, because the market structure and the fact that consumers value other features of the platforms.

In contrast, if the difference between qualities increases, the company that has a higher expectations decreases their degree of compatibility, while the rival platform increases it and both platforms serve the market.

As we can see, even if there are network externalities, the value of the different types of technologies by consumers and vertical product differentiation can make that multiple networks coexist.

Consumers may prefer the intrinsic advantages of a product in spite of belonging to a smaller network. This markets usually end up with the adoption of standard, especially if the products are totally incompatible.

Network effects generate a process of positive feedback that usually end up with a monopoly: for example Economides (2000), in the presence of network externalities and with incompatibility products, the natural situation is the existence of different market shares. This is one of the reason whereby the Microsoft Windows operating system currently holds 95% market share.

When exist minimal vertical differentiation, the network externalities entails a number of disadvantages, there is a reduction in the variety of products, even if different consumers have different needs, the market are less likely to opt for only one of the two products, this happen when the quality differential increases.

If the low quality platform ends up monopolizing the market, we have a loss of efficiency by the choice of a poorer quality platform.

2.6. Conclusion

In this chapter, unlike the related literature on network externalities, it has been assumed that the consumer expectations about the network size is exogenous. These expectations are a key factor in consumers decision, in the network models the technology available is valued attention to other features besides price. Our results support those obtained by Katz and Shapiro (1985), where firms with a large network always prefer incompatibility and Sarkar (2005), who finds a mixed strategy equilibrium where firms are differentiated vertically and exist incompatibility. Specifically, in our model, with minimal differentiation, only the platform with high expectations serves the market establishing incompatibility and if the expectation is one and the difference between qualities is minimal we are in a case with incompatible platforms. By contrast, as found Baake and Boom (2001), firms may prefer compatibility but prefer vertically differentiation. In our model, without minimal difference between qualities and expectations are not focused in one of the two platforms, the platform with a highest expectations reduces its degree of compatibility, while rival platform increase it, but both platforms remain partial compatibility levels. Furthermore, when increase the difference between qualities the total welfare is higher despite the increases in prices. These price increases can be justified, for example, the regulation in the telecommunications markets and especially in the interconnection between the networks of operators, for example telephone and Internet operators. Therefore, the regulation on interconnection networks is a very important factor to encourage the growth of interconnected networks and prevent the development of standards and parallel networks that generate productive inefficiencies. Although the network externalities are hardly solvable through the competition, these issues deserve to be analyzed. A modeling competition in the hardware and software context and network effects would require a dynamic model. Consumers decision on what hardware purchase in initial periods makes them captives in subsequent periods because the key is the dependence on the initial conditions, small differences in initial market shares make a big difference in the subsequent market developments. This scenario allows us a better assessment of the strategic decisions of firms, because network externalities produce a positive feedback, which makes the rate of penetration and technology growth are different from other markets. The approach of a dynamic model would allow the study of the duration of each phase of the feedback, telecommunications markets are markets subject to these effects. These products often have a long phase of settlement, followed by a rapid growth phase where increasing the number of users, the greater the number of users who wish to adopt the technology, and eventually the product becomes the standard. Although this work has been approached from the perspective of static equilibrium, a dynamic environment would provide more information on the behavior of agents. Another possible extension would be to consider that consumers are also differentiated in their willingness to pay for the network externality. The study of innovation systems in markets is also a promising field of analysis. Another interesting line of analysis would consider technological substitution processes of products subject to network externalities, analyzing whether to offer support in one way or offer full compatibility of the products.

Chapter 3. Standardization and Network Effects

3.1. Introduction

This chapter contributes to the study of two-sided market modeling competition between platforms and developers (but our approach is different from the normal two sided market analysis). Strategic decisions are taken regarding standardization, technological innovation and market competition in the presence of direct and indirect network effects in consumption. The model considers that the products that platforms and developers offered are horizontally differentiated; the products offer by the platforms are of a different quality, i.e. are vertically differentiated.

Relative to the received literature the analysis relates the decisions of platforms to the developers market and analyzes simultaneously the two types of externalities in the standardization problem. Our main finding is that the market propensity to standardize depends on the developers' degree of differentiation costs. Specifically, whenever the developers are sufficiently differentiated, the platforms will not adopt the standard, whereas they will do so whenever differentiation is sufficiently low. In particular, platforms will not adopt the standard under the extreme case of consumers only having expectations on a single platform in the market.

In the welfare analysis we obtain that the effect of the standard is an increase in the price paid by consumers to subscribe to an existing platform and a decrease in the intensity of competition. As a result the price increase, consumers are worst, but platforms increase its profits and consumers losses, the final effect is an increase in total welfare.

These results suggest that in markets with network externalities, the adoption of a standard may result in a decrease of competition. Besides, consumer surplus is always higher without the standard. These occur due to the effects that follow the adoption of the standard: the variety effect and the price effect. The variety effect appears when the platforms adopt the same system because we loss one variety and the price effect appears because when the platforms adopt the standard they maintain a monopoly power.

On the consumers side, network effects (direct and indirect) play a crucial role when deciding what platform to subscribe to, a decision also shape by the level of technological innovation and prices. If consumers place low expectations about the network share of the product with better technology, then the high quality platform could end up with a relatively small network. Consequently, an improved technological innovation does not always guarantee a higher market share.

The market of personal computers offers good examples of several types of platforms and several types of complements. Operating systems like Mac OS X and Windows 7 compete to attract the highest number of consumers. In this competition, direct externalities are present (although with decreasing influence) and indirect (with a more relevant role). There is a large number of complements for each platform, but some of them are compatible for both platforms and some other ones only work with one of the two operating systems. The different types of software packages are complements for a best use of the platform. Some complements can be used on both platforms: OpenOffice.org and Microsoft Office, others only work with Windows (Microsoft Works or Easy Office) or with Mac (i Work or Apple Works).

A similar example is the Web browsers. Internet Explorer only works in Windows, but others Web browsers are totally compatible with all the platforms. For example: Mozilla Firefox, Google Chrome, Opera and Safari. These last four Web browsers would be an obvious example of a market formed by platforms that do not establish a standard and work with totally compatible complements.

We only consider incompatible complements to analyze the total effects of the indirect network externalities. These externalities provoke an increase in the willingness to pay for consumers when they purchase one of the two systems, that is, a consumer will be more disposed to purchase, for example, Windows 7, if he knows that Windows has a wide range of complements. The consumer will evaluate the number of complements for each platform and how much they cost. If a platform has a variety of complements but are very difficult to obtain (for the price or for the distance) the consumer could consider buying the rival platform. Therefore, the indirect externalities are a key factor in the consumers' decision making. The utility of the consumers using the same network increased when an additional user was linked to it. All consumers belong to the same network, independently of the platform, and then they only take notice of the price. It is clear, therefore, that when platforms adopt a standard, then direct externalities will become irrelevant. This happens at present in the markets of DVD players or blue ray as opposed to what happened with platforms Betamax and VHS. A frequent problem that is observed in these markets is the interoperability, this is, the capability of a product or system, whose interfaces are totally known, of running in another product or existing system or future system, without access restriction or implementation. This problem is diminishing with the appearance of free software.

Finally, the last example is the learning of a word processor. It is convenient to choose one with high market share or one that is expected to be adopted by the majority of consumers, because it will allow accomplishing file interchanges and it will be easier to work with another authors' documents. The utility directly increases with the number of other people also working with it. The adoption of a standard and the existence of incompatible complements is investigated in this work. With incompatible complements, each company of complements works with one platform, producing exclusive complements for this platform. For instance, consider video-games where each game only works on the platform for which it has been designed. Besides this, we analyze a rational expectations model under two types of expectations: when consumers hope that both platforms will serve the market and expectations on a single platform,when consumers hope that one platform will serve the market.

There exists a direct relationship between direct and indirect externalities. With a big network it is more probable to have a large number of complements and with more developers it is more probable to have a large number of consumers. The largest companies remain dominant and the smaller remain small. The modeling of a market with direct and indirect network effects requires the consideration that the willingness to pay for platforms depends on the users' number and on the number of developers for each platform. Therefore, the consumers' expectations with respect number of users (and other characteristics such as the availability, quality) constitute one of the distinctive aspects in the markets with network effects.

Finally, compatibility between components allows user to exploit complementarities and indeed benefit from a larger network. We find a large number of factors that fit in these markets that they affect the final consumer's decision making. There exist key factors; the control of a consumers installed base, intellectual property rights, capability to innovate, the advantage to be the first in the market, capability of manufacture, the fortress, the number of complements, the reputation and name of the brand. We are going to solve the game in three stages; at stage one, platforms decide whether to adopt the standard, at stage two they choose the size of the technological innovation and in the third stage they compete in prices. Finally, consumers subscribe to one of the platforms and purchase a complement.

Section 3.2 describes the assumptions of the model. In section 3.3 the model is analyzed and section 3.4 offers the welfare analysis. The last section concludes.

3.2. The Model

3.2.1. Players

We consider a model of competition between platforms with direct and indirect network externalities. The game has three different types of players: two platforms which offer horizontally differentiated products and an oligopolistic market of developers and a continuum of consumers.

- Platforms. There are two platforms that are differentiated in two dimensions, a horizontal and a vertical one. Regarding the former, platforms are horizontally differentiated on the Hotelling line. This dimension can be related with design. Furthermore, the platforms offer products with a different levels of technological innovation, they are captured by parameters θ_A and θ_B, we assume that $\theta_i \epsilon [0, \overline{\theta}], i \epsilon \{A, B\}\, i \neq j$. The two platforms are denoted A and B, they compete to attract consumers and developers. They set prices $\{p_A, p_B\}$ to consumers and license fees $\{l_A, l_B\}$ to developers; the latter can be interpreted as what developers have to pay to enter and compete in the market. We assume that license fees $\{l_A, l_B\}$ are positive. Platforms can adopt a standard in the market. They choose an element of $Z = \{0, 1\}$, where $Z = 0$ means not adopting the standard and $Z = 1$ means adopting it[2]. The adoption of a common standard requires mutual agreement by platforms. If the standard is adopted , $Z = 1$, consumers get the full benefits of the total network and platforms specific network sizes become irrelevant in consumers´ decision making. There is a homogeneous products industry with just competition within the market. In case the standard is not adopted, $Z = 0$, each platform network size is important for consumers decisions. There is a differentiated products industry where platforms compete for the market.

- Developers. They first decide whether to enter the market. If so, they choose which platform to work with and they set prices. We use n_i to denote the number of developers that trade with platform i. Developers are located around a unit circumference. Each developer supplies one complement that is suitable only for one platform; complements are incompatible with each other. Developers incur license fees $\{l_A, l_B\}$ and marginal cost c^c. The complements to platforms A and B have prices $\{p^s_A, p^s_B\}$ for consumers. Developers are Bertrand competitors and in equilibrium we have n_A platform A developers and n_B plat-

[2]Some authors use the term standardization when the market is monopolized. In this model we have the possibility of having a duopoly in a standardized market, with only one system. Other authors consider the market standardized when one of the platforms obtained 95% of the market.

form B developers. Developers competition is modeled a lá Salop (1979). The assumptions allow the characterization of the market for each platform separately.

- Consumers. Consumers are uniformly distributed on the unit line with respect to their preference for platforms and uniformly distributed around the unit circumference with respect to their preference for developers. The location of a consumer with respect to platforms is denoted by x,$x\epsilon\,[0,1]$. The total mass of consumers is 1, and their demand is inelastic. The respective unit transport cost associated with platforms and developers are denoted t^p and t^c, respectively. These parameters measure the degree of differentiation. For given n_i, let dn_i be the distance to the nearest type i developer. So consumers enjoy larger utility the higher the number of developers. Consumer choose one platform and then they choose one complement. Consumers have expectations about network size of each platform, which are given by $y_i\epsilon(0,1)$, where $i\epsilon\,\{A,B\}\;i\neq j$ and $y_i+y_j=1$.

Two types of expectations are considered: *i*) fulfilled expectations and *ii*) non-fulfilled expectations under the extreme case of expectations just on a single platform. Note that the model exhibits two dimensions of product differentiation. The platforms are vertically differentiated because of the potentially different technological innovation levels θ_A and θ_B that are determined at the second stage of the game. Horizontal product differentiation is captured by a transportation cost that is linear in the distance between each consumer's most preferred point x and the location of the platform he buys from. The degree of horizontal product differentiation in the developers market is reflected by the parameter $t^c>0$ and in the consumers market is reflected by $t^p>0$.

The final objective is to study platform competition when the standard can be adopted and face a market of incompatible complements. The market structure is displayed in figure 3.1 below.

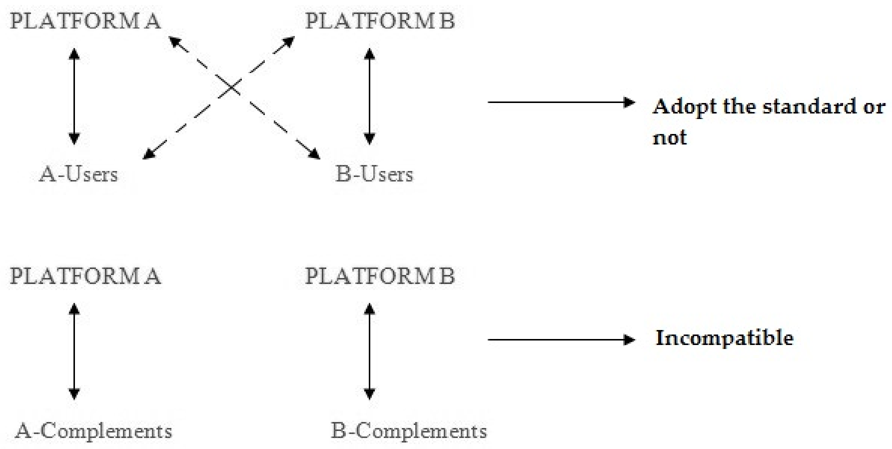

Figure 3.1: Market structure.

As we can observe in figure 3.1, when platforms do not adopt the standard the type A-users are only connected with other type A-users in platform A. Similarly for type B-users in platform B. This relationship is represented with the bold arrows. Alternatively, when the standard is adopted the type A-users are connected with the type B-users and vice versa since they now belong to the same network (the dotted arrows appear). Competing platforms are said to be standardized if the utility of a platform A user is enhanced by an increase in the number of platform B users, and vice versa.

In the developers market, given incompatibility, type A-complements are only connected with platform A and type B-complements are only connected with platform B. Regardless of platforms adoption of the standard, complements are assumed to be incompatible throughout.

The timing of moves is as follows:

In the platforms market:

1. At stage 1, platforms decide whether to adopt the standard $\{Z = 1\}$ or not $\{Z = 0\}$.

2. At stage 2, each platform i, $i\epsilon \{A, B\}$ can make an investment θ_i in technological innovation at cost $\frac{\gamma}{2}\theta_i^2$, where $\gamma > 0$ is a constant[3].

3. At stage 3, platforms set license fees l_i, $i\epsilon \{A, B\}$and choose prices p_i, $i\epsilon \{A, B\}$.

In the developers market:

1. At stage 4, developers decide whether to enter or not.

2. At stage 5, developers pay license fee li, $i\epsilon \{A, B\}$and choose prices p_i^s, $i\epsilon \{A, B\}$.

3. At stage 6, consumers subscribe to a platform and buy a complement.

The analysis of competition between platforms is modeled as a three stage non-cooperative game. As already noted the developers market is characterized by competition a lá Salop. The equilibrium is characterized by $(p_A^{s*}, p_B^{s*}, Z^*, \theta_A^*, \theta_B^*, l_A^*, l_B^* p_A^*, p_B^*)$.

The solution concept is Subgame Perfect Nash Equilibrium, we solve the model by backward induction.

3.2.2. Relationship Between Players

Recall that it is assumed that complements are incompatible, this means that the same complement does not serve for both platforms, it is impossible to use a complement in platform A and later in B. Each complement is specifically designed for use on a single platform.

We have developers of complements of type A,n_A, and developers of complements of type B, n_B. Developers are distributed around two different unit circles. Both circumferences are independent and are not correlated. Thus, the distance from one developer to another is $\frac{1}{n_i}$.

[3]This assumption is common in R&D literature to reflect the existence of diminishing returns to R&D investments (see, among many others, d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Suzumura, 1992).

The three different players are illustrated in figure 3.2:

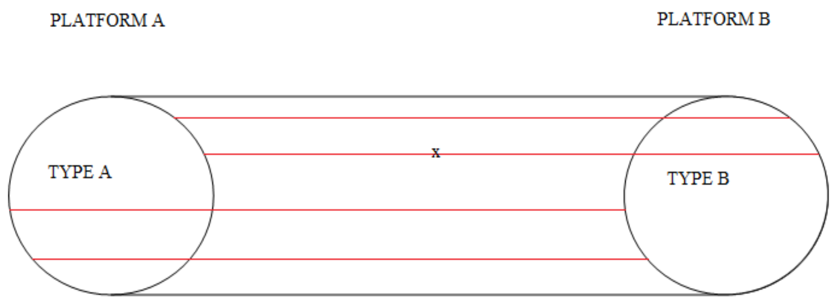

Figure 3.2: Consumers, developers and platforms.

The consumers are located on one of the horizontal lines. We assume that platform A is located on the left end, at zero, and platform B is located on the right end, at one. We assume that the length of the segments is 1. Depending on the location of the segment, consumers are located at a point in the developer circumference.

Once consumer i takes the decision on which platform to choose, he meets the circumference where the developers are present.

For a consumer who buys from platform A, when he reaches the developers market he is located at distance dn_A from the nearest type A developer and a consumer who buys from platform B when he reaches the developers market he is located at distance dn_B from the nearest type B developer. It happens that $dn_A \leq \frac{1}{2n_A}$ and $dn_B \leq \frac{1}{2n_B}$; the distance that a consumer has to travel to the nearest developer is less than or equal to the distance travelled by the indifferent consumer. It is assumed that $\frac{\partial dn_A}{\partial n_A} < 0$ and $\frac{\partial dn_B}{\partial n_B} < 0$; an increase in the number of developers produces a reduction in the distance travelled.

First of all we analyze the developers market; incompatible complements produce an effect on the purchasing decisions of consumers.

The profit of type A developers and type B developers are given by

$$\pi_A^c = \left(\frac{\hat{x}}{n_A}\right)(p_A^s - c^c) - l_A \tag{3.1}$$

$$\pi_B^c = \left(\frac{1-\hat{x}}{n_B}\right)(p_B^s - c^c) - l_B \tag{3.2}$$

The benefits of the developers are given by the number of consumers who purchased a platform A or platform B, divided by the number of type A or B developers, where $\widehat{x}$ is the indifferent consumer. Changes in variables of the developers market will affect the platform market.

We consider $\{l_A, l_B\}$ as the portion of profits that developers pay to platforms in the form of license fees to design complements. For example, in the video games market the companies that develop video games have to pay a license fees to make games for the platforms.

The developers market is a free entry market, the profits of both types of developers will be zero.

Developing the model, we obtain the prices that the developers charge consumers[4]

$$p_A^s = c^c + \frac{t^c}{n_A} \tag{3.3}$$

$$p_B^s = c^c + \frac{t^c}{n_B} \tag{3.4}$$

As more developers enter the market, consumers have to pay a lower price to purchase one complement[5].

[4]As in Salop´s (1979) circular city model who obtains the same price but each type of developers market behaves as an independent model, because we have two circumferences.

[5]We are assuming that, if a single software firm breaks off and forms a smaller network, it follows the same pricing pattern that multiple firms follow, we always have: $p_i^c = c^c + \frac{t^c}{n_i}$

By the zero profits condition we obtain the license fees. Setting (3.1) and (3.2) equal to zero and clearing for l_A and l_B yields

$$l_A = \frac{t^c \hat{x}}{n_A^2} \tag{3.5}$$

$$l_B = \frac{t^c (1-\hat{x})}{n_B^2} \tag{3.6}$$

The developers profits (which are zero by the free entry condition) and license fees depend on the number of consumers that buy from each of the platforms. It is assumed that the platform charging the lowest l_i, $i \epsilon \{A, B\}$ where $i \neq j$ attracts more developers, that is, developers prefer the platform whose license fees are lower.

We now move to analyze the strategic decisions of platforms. Rather naturally, these depend on whether consumers expect that the market for platforms be a monopoly or a duopoly.

Therefore, we analyze two types of equilibrium: the first one, when consumers expect that the market is a duopoly, and the second one, when consumers expect that the market is a monopoly:

Fulfilled Nash equilibrium: consumer expectations are distributed between both platforms: $y_A = \widehat{x}$, $y_B = (1 - \widehat{x})$.

Non-fulfilled Nash equilibrium: consumer expectations focused only on platform A: $y_A = 1$, $y_B = 0$ or focused only on platform B: $y_A = 0$, $y_B = 1$. In section 3.3.2 we consider that consumers expectations focus on platform A.

A consumer located at x who buys platform A and a type A complement obtains a utility of

$$R - p_A^s - t^c dn_A - t^p x - p_A + \theta_A + v y_A + v y_B Z \tag{3.7}$$

and a consumer located at x who buys platform B and a type B complement obtains a utility of

$$R - p_B^s - t^c dn_B - t^p (1-x) - p_B + \theta_B + v y_B + v y_{BA} Z \tag{3.8}$$

Substituting (3.3) and (3.4) in the utility functions

$$R - c^c - \frac{t^c}{n_A} - t^c dn_A - t^p x - p_A + \theta_A + v y_A + v y_B Z \tag{3.9}$$

$$R - c^c - \frac{t^c}{n_B} - t^c dn_B - t^p (1-x) - p_B + \theta_B + v y_B + v y_A Z \tag{3.10}$$

We consider that consumers obtain a reservation utility level, R, which is large enough to ensure that all consumers will participate in the market. The market is covered. The consumers obtain extra utility if the number of developers that work with the same platform that the consumers will buy is high, this effect is reflected by the parameters dn_i that measures the distance to the nearest type i developer, i, $i \epsilon \{A, B\}$ and by the variable p_i^s, the complement type i price, $i \epsilon \{A, B\}$.

When we consider fulfilled Nash equilibrium the indirect utility function is the same that we can observe above and when we consider non-fulfilled Nash equilibrium y_B disappears because consumers expectations are focused on platform A and platform B network size is zero. We introduce the direct network effects with the values of y_A, y_B and v, and we introduce the indirect network effects with the values of dn_A, dn_B and t^c. An increase in the number of developers produces a distance reduction and consequently an increment in consumer utility.

Many products have little or no value in isolation, but generate value when combined with its complement. In this utility function we observe that even if no interaction is possible (if direct or indirect network effects disappear), consumers can still derive value from quality or from product features[6].

3.3. Platforms Decision

3.3.1. Fulfilled Nash equilibrium

We know from the developers market that $p_A^s = c^c + \frac{t^c}{n_A}$ and $p_B^s = c^c + \frac{t^c}{n_B}$; the respective distance to the nearest type i developer i, $i\epsilon\{A, B\}$ is given by dn_A and dn_B and we are assuming fulfilled expectations, that is $y_A = \hat{x}$ and $y_B = (1 - \hat{x})$. Thus both platforms have a positive market share, there is one consumer located at $\widehat{x}$ who is just indifferent between buying from platform A or B. Equating (3.9) and (3.10) and clearing x the indifferent consumer is obtained:

$$\widehat{x} = \frac{t^c\left(n_A - n_B\right) - t^c n_A n_B\left(dn_A - dn_B\right) + n_A n_B\left(p_B - p_A + t^p + v(Z-1) + \theta_A - \theta_B\right)}{2n_A n_B\left(t^p + v\left(Z-1\right)\right)} \tag{3.11}$$

The location of the indifferent consumer is a function of the number of developers of each type, the distance to the nearest developer, the degree of differentiation in the developers market and the adoption of a common standard.

Next, we calculate the license fees. Substituting $\hat{x}$ in (3.5) and (3.6) we obtain:

[6] In other models, if consumers do not buy some complement his utility is zero. See for example Clements (2004)

$$l_A = \frac{t^c \hat{x}}{n_A^2} = \frac{t^{c2}\left(n_A - n_B\right) - t^{c2} n_A n_B \left(dn_A - dn_B\right) + t^c n_A n_B \left(p_B - p_A + t^p + v(Z-1) + \theta_A - \theta_B\right)}{2 n_A^3 n_B (t^p + v(Z-1))} \tag{3.12}$$

$$l_B = \frac{t^c (1-\hat{x})}{n_B^2} = \frac{t^{c2}\left(n_B - n_A\right) - t^{c2} n_A n_B \left(dn_B - dn_A\right) + t^c n_A n_B \left(p_A - p_B + t^p + v(Z-1) + \theta_B - \theta_A\right)}{2 n_A n_B^3 (t^p + v(Z-1))} \tag{3.13}$$

The relationship between the number of developers and the license fees is as follows. An increase in the number of developers produces a reduction in the license fees. And an increase in the license fees produces a reduction in the number of developers. Therefore, in practical terms, to consider an increase in the number of developers is equivalent to considering a reduction in the licenses fees, they are inversely related.

If the developers travel cost increases, the license fees will also decrease. If the developers travel cost were zero, there would not be indirect network effects. It is required that $t^c > 0$, in other words, we need differentiation in the developers market to analyze the effect of the indirect network externalities.

Differentiating $\{l_A, l_B\}$ with respect to $\{p_A, p_B\}$ we observe: an increase in the platform price with which developers work, produces a reduction in the licenses fees, $\{l_A, l_B\}$, and an increase in the rival platform price produces an increase in the licenses fees imposed by the platform that the developer is working with.

A platform is aware that an increase in its price reduces demand. Due to the presence of the strength of the network externality there is a decrease in its network size and, if platforms do not adopt the standard, the reduction in its demand is bigger. Considering the presence of the indirect network effects the platform can compensate this effect via reductions in the license fees. Thus if platforms reduce the license fees there will be an increase in their network of developers. More developers would add to its network and this effect would compensate for the decrease in the number of consumers due to the price increase; the final effect is a growth in the demand. The platform can obtain higher profit because revenues from consumers go up, in spite of the reduction in the revenues from developers.

To clarify this argument, suppose that platform A increases its price. Then the indifferent consumer moves to the left; demand for platform A is lower and the effect of direct externality moves the indifferent consumer even further to the left. Nevertheless, the reduction in the license fee produces an increase in the number of developers; this moves the indifferent consumer to the right. The latter effect is stronger; the final effect on the indifferent consumer is a displacement to the right.

For simplicity let us assume that the marginal cost of production is zero, and therefore, the profits of platforms A and B are given by:

$$\pi_A = p_A\left(\frac{t^c\left(n_A-n_B\right)-t^c n_A n_B\left(dn_A-dn_B\right)+n_A n_B\left(p_B-p_A+t^p+v(Z-1)+\theta_A-\theta_B\right)}{2n_A n_B\left(t^p+v\left(Z-1\right)\right)}\right)$$
$$+n_A\left(\frac{t^{c2}\left(n_A-n_B\right)-t^{c2}n_A n_B\left(dn_A-dn_B\right)+t^c n_A n_B\left(p_B-p_A+t^p+v(Z-1)+\theta_A-\theta_B\right)}{2n_A^3 n_B(t^p+v(Z-1))}\right)-\frac{\gamma}{2}\theta_A^2 \tag{3.14}$$

$$\pi_B = p_B\left(\frac{t^c\left(n_B-n_A\right)-t^c n_A n_B\left(dn_B-dn_A\right)+n_A n_B\left(p_A-p_B+t^p+v(Z-1)+\theta_B-\theta_A\right)}{2n_A n_B(t^p+v(Z-1))}\right)$$

$$+n_B\left(\frac{t^{c2}\left(n_B-n_A\right)-t^{c2}n_A n_B\left(dn_B-dn_A\right)+t^c n_A n_B\left(p_A-p_B+t^p+v(Z-1)+\theta_B-\theta_A\right)}{2n_A n_B^3(t^p+v(Z-1))}\right)-\frac{\gamma}{2}\theta_B^2 \tag{3.15}$$

The profits are divided into two parts. The revenue that the platforms obtain from developers and the revenue that they receive from consumers.

•The pricing decision of platforms

At stage 3, platforms simultaneously choose the profit maximizing price. The solution to $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$ yields:

$$p_A = \frac{1}{3}(3t^p - \frac{3t^c}{n_A} + 3v(Z-1) + \theta_A - \theta_B + t^c(dn_B - dn_A)) \tag{3.16}$$

$$p_B = \frac{1}{3}(3t^p - \frac{3t^c}{n_B} + 3v(Z-1) + \theta_B - \theta_A + t^c(dn_A - dn_B)) \tag{3.17}$$

Note that, equilibrium prices are a function of the strength of the network externality, the number of developers in each platform, the developer travel cost, the platform travel cost, the technological innovation and the difference between developers´ distances. When we introduce an incompatible market of developers, the prices are subject to the degree of differentiation in this market and the distance to the nearest developer. This effect is caused by the presence of the indirect network effects.

Let us now take a look at the comparative statics exercise, to study how prices change as a function of the direct and indirect network effects.

i) If the platforms do not adopt the standard, an increase in the strength of the network externality produces a price reduction in both platforms. In the contrary case this effect disappears.

ii) An increase in the number of platform i developers produces a price increase in platform i and an increase in the number of platform j developers produces a price reduction in platform i.

iii) If $n_i > n_j$ where $i, j = a, b$ $i \neq j$ an increase in the developer travel cost produces either an increase in platform i price if $n_i > \frac{3}{dn_j - dn_i}$ or a reduction if $n_i < \frac{3}{dn_j - dn_i}$.

If platforms do not adopt the standard, an increase in the strength of the network externality produces a price reduction in both platforms. A reduction in the price leads a platform to increase its market share and one increase in the market share produces one increment in its own network size; this last effect accompanied with the strength of the network externality produces one gain in consumer utility that buys this platform and produces a growth in the demand. Therefore, an increase in the strength of the network externality results in stronger price competition. The final objective of each platform is to reduce the price to attract more consumers and attempt to monopolize the market; in this case platforms compete for the market.[7]

The second part of *i)* is intuitive too. If both platforms adopt the standard, then all consumers are in the same network and platforms need not reduce their prices to impose its technology or system; in this case platforms compete within the market.

This arises because the demand curve under standardization it is more inelastic and without standardization is more elastic. Without a common standard positive network externalities have the same effect as a reduction in platform travel cost.

Also, an increase in the number of developers increases consumers´ willingness to pay. A platform can increase its price and capture demand at the same time. Moreover, if platforms do not adopt the standard and we add the effect of the strength of network externality, its demand increment by more. Its network will be of greater size and more consumers want to be in this network.

[7]This result is similar to that obtained Baake and Boom (2001) when firms compete without adapter and also to that in Navon, Shy and Thisse (1995).

•The technological innovation decision

At stage 2, each platform makes an investment in technological innovation, substituting equations (3.13) and (3.14) into (3.11) and (3.12) and maximizing with respect θ_A and θ_B respectively, the equilibrium level of technological innovation for both platforms is given by:

$$\theta_A = \frac{9\gamma(t^p + v(Z-1)) + 3t^c\gamma(dn_B - dn_A) - 2}{3\gamma(9\gamma(t^p + v(Z-1) - 2))} \tag{3.18}$$

$$\theta_B = \frac{9\gamma(t^p + v(Z-1)) + 3t^c\gamma(dn_A - dn_B) - 2}{3\gamma(9\gamma(t^p + v(Z-1) - 2))} \tag{3.19}$$

The levels of technological innovation are a function of platforms travel cost, the strength of the network externality, developer travel cost and the difference between developers´ distances. If platforms do not adopt the standard, to the choice of the level of technological innovation, direct and indirect network effects are affected. On the other hand, if platforms adopt the standard, only indirect network effects are affected. It is curious how the level of technological innovation does not depend only on market structure in platforms but also on developers market. Changes in the developers market affect the innovation decisions. If one platform has a higher network of developers, it invests more. The intuition tells us that one platform invest more to maintain or increase its network of developers, if one platform is a technological leader, more consumers and developers want to be in its network.

Substituting equations (3.16), (3.17), (3.18) and (3.19) into (3.14) and (3.15) we obtain the platforms profits functions as follows:

$$\pi_A = \frac{(9(t^p + v(Z-1))\gamma - 1)(9(t^p + v(Z-1))\gamma - 1 + 3t^c\gamma(dn_B - dn_A))^2}{18\gamma(2 - 9\gamma(t^p + v(Z-1)))^2} \quad (3.20)$$

$$\pi_B = \frac{(9(t^p + v(Z-1))\gamma - 1)(9(t^p + v(Z-1))\gamma - 1 + 3t^c\gamma(dn_A - dn_B))^2}{18\gamma(2 - 9\gamma(t^p + v(Z-1)))^2} \quad (3.21)$$

Note that, by the second-order condition, for a maximum, the following assumption must be satisfied:

Assumption 1. *For* $(\theta_A^*, \theta_B^*, p_A^*, p_B^*)$ *to be a maximum, it must be satisfied that:*
If platforms do not adopt the standard $t^p > v$ and $\gamma > \frac{2}{9(t^p - v)}$
If platforms adopt the standard $\gamma > \frac{2}{9t^p}$
See Appendix.

Further note that for positive technological innovation, θ_A, θ_B, the following assumption must be satisfied:

Assumption 2. *Both platforms invest in technological innovation:*

If platforms do not adopt the standard

And $n_i > n_j \longrightarrow t^c < \frac{9\gamma(t^p - v) - 2}{3\gamma(dn_j - dn_i)} = t_i^c(Z = 0)$, with $i, j = A, B$ where $i \neq j$

If platforms adopt the standard

And $n_i > n_j \longrightarrow t^c < \frac{9\gamma t^p - 2}{3\gamma(dn_j - dn_i)} = t_i^c(Z = 1)$, with $i, j = A, B$ where $i \neq j$

See Appendix.

3.3.1.1. The standardization stage

In the following two subsections we report the values of technological innovation, prices, license fees and platforms profit. We consider first the case where platforms do not adopt the standard and secondly the case when they adopt it.

• ***Without the standard***

$$\theta_i = \frac{9\gamma(t^p - v) + 3t^c\gamma(dn_j - dn_i) - 2}{3\gamma(9\gamma(t^p - v) - 2)} \; i, j = A, B \text{ where } i \neq j \tag{3.22}$$

$$p_i = \frac{(n_i(t^p - v) - t^c)(9\gamma(t^p - v) - 2) + 3n_i t^c \gamma(t^p - v)(dn_j - dn_i)}{n_i(9\gamma(t^p - v) - 2)} \; i, j = A, B \text{ where } i \neq j \tag{3.23}$$

$$l_i = \frac{t^c(9\gamma(t^p - v) + 3t^c\gamma(dn_j - dn_i) - 2)}{2n_i^2(9\gamma(t^p - v) - 2)} \; i, j = A, B \text{ where } i \neq j \tag{3.24}$$

$$\pi_i = \frac{(9\gamma(t^p - v) - 1)(9\gamma(t^p - v) + 3t^c\gamma(dn_j - dn_i) - 2)^2}{18\gamma(2 - 9\gamma(t^p - v))^2} \; i, j = A, B \text{ where } i \neq j \tag{3.25}$$

• ***With the standard***

$$\theta_i = \frac{9\gamma t^p + 3t^c\gamma(dn_j - dn_i) - 2}{\gamma(27\gamma t^p - 6)} \; i,j = A,B \text{ where } i \neq j \tag{3.26}$$

$$p_i = \frac{(n_i t^p - t^c)(9t^p\gamma - 2) + 3n_i t^c t^p \gamma(dn_j - dn_i)}{n_i(9t^p\gamma - 2)} \; i,j = A,B \text{ where } i \neq j \tag{3.27}$$

$$l_i = \frac{t^c(9t^p\gamma + 3t^c\gamma(dn_j - dn_i) - 2)}{2n_i^2(9t^p\gamma - 2)} \; i,j = A,B \text{ where } i \neq j \tag{3.28}$$

$$\pi_i = \frac{(9t^p\gamma - 1)(9t^p\gamma + 3t^c\gamma(dn_j - dn_i) - 2)^2}{18\gamma(2 - 9t^p\gamma)^2} \; i,j = A,B \text{ where } i \neq j \tag{3.29}$$

Before moving to the first stage, we can observe the difference in the equilibrium variables just presented. The main difference is the presence of the strength of the direct network externality; when platforms adopt the standard this parameter vanishes because consumers are in the same network, independently of the strength of the network externality. The result for the other variables of the model is going to be symmetrical.

It is straightforward to see the role played by an unequal number of developers. If one of the platforms has a higher number of developers, the consumers distance to the nearest developer is lower than in the rival platform. As we can observe above, if one of the platforms has a higher number of developers it sets a higher price, a higher level of technological innovation and obtains higher profit independently of the standard being adopted.

Intuitively, if a platform has a higher number of developers, this will increase consumers' willingness to pay to belong to that network. There is a kind of vertical differentiation element, where the most differentiated platform sets a higher price. This effect is reinforced by the presence of the strength of the network externality. Also, the levels of technological innovation are the same if both platforms have the same number of developers. The effect on the level of technological innovation is the same if indirect network effects are not present or both platforms have the same number of developers. The indirect network effect disappears if the developers market is not differentiated or both platforms have the same number of developers. The intuition of this result goes as follows: for a potential consumer it is equally valuable, for example, nine compatible complements or that each platform has nine incompatible complements. If this happens, consumers will value other characteristics of the developers, such as quality, technology and so on.

We now solve the first stage of the game. We analyze the platforms profit when they adopt the standard and when they do not adopt it.

First, we define[8]:

$$H = \frac{(9t^p\gamma - 2)(9\gamma(t^p - v) - 2)}{\gamma^2(dn_A - dn_B)(9t^p(t^p - v) - 2t^p + v)} \tag{3.30}$$

$$G = \sqrt{\frac{(2 - 9t^p\gamma)^2(9t^p\gamma - 1)(9\gamma(t^p - v) - 1)(2 - 9\gamma(t^p - v))^2}{\gamma^4(dn_A - dn_B)^2(9t^p\gamma(t^p - v) - 2t^p + v)^2}} \tag{3.31}$$

For platform A, its profit will be higher with the standard if:

$t^c < \frac{1}{27}\left(H + G\right) = \widetilde{t}^c_A$

For platform B, its profit will be higher with the standard if:

[8] The detailed calculation is available from the author

$t^c < \frac{1}{27}(G-H) = \widetilde{t}^c_B$

Where:

$\widetilde{t}^c_A > \widetilde{t}^c_B$ if $n_B > n_A$

$\widetilde{t}^c_A < \widetilde{t}^c_B$ if $n_B < n_A$

The areas of equilibrium with or without the standard[9]:

If $n_A = n_B$:

.Platforms always adopt the standard:

$\pi_A(Z=1) > \pi_A(Z=0)$ and $\pi_B(Z=1) > \pi_B(Z=0)$

If $n_A > n_B$:

.Platforms always adopt the standard if $t^c < \widetilde{t}^c_A < t^c_A(Z=1)$:

$\pi_A(Z=1) > \pi_A(Z=0)$ and $\pi_B(Z=1) > \pi_B(Z=0)$

.Platforms never adopt the standard if $\widetilde{t}^c_A < t^c < t^c_A(Z=0)$[10]:

$\pi_A(Z=0) > \pi_A(Z=1)$ and $\pi_B(Z=1) > \pi_B(Z=0)$

If $n_B > n_A$:

.Platforms always adopt the standard if $t^c < \widetilde{t}^c_B < t^c_B(Z=1)$:

$\pi_A(Z=1) > \pi_A(Z=0)$ and $\pi_B(Z=1) > \pi_B(Z=0)$

.Platforms never adopt the standard if $\widetilde{t}^c_B < t^c < t^c_B(Z=0)$[11]:

$\pi_A(Z=1) > \pi_A(Z=0)$ and $\pi_B(Z=0) > \pi_B(Z=1)$

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[9] The term Z=1 in parentheses means that platforms adopt the standard and the term Z=0 that they do not adopt it.

[10] This only occurs when $t^p > \frac{5}{3}v$ and $\frac{2}{9(t^p-v)} < \gamma < \frac{4t^p+v}{18t^p v} + \frac{1}{18}\sqrt{\frac{t^p(16t^p-24v)+v^2}{t^{p2}v^2}}$

[11] This only occurs when $t^p > \frac{5}{3}v$ and $\frac{2}{9(t^p-v)} < \gamma < \frac{4t^p+v}{18t^p v} + \frac{1}{18}\sqrt{\frac{t^p(16t^p-24v)+v^2}{t^{p2}v^2}}$

If one of the two platforms has a lower number of developers it is ready to establish the standard for a wider range of differentiation in the developers market. This arises from the interaction of direct and indirect network effects. Indirect network externalities through the travel cost in the developers market plays a crucial role in the platforms decisions. As can be observed in figures 3.3 and 3.4, the platform with a lower number of developers is ready to establish the standard when the developers travel cost is high enough, but the platform with a higher number of developers obtains a higher profit if it does not adopt the standard. When the developers travel cost is low, both platforms adopt the standard. In the beginning, if the developers travel cost is very high, more consumers will want to purchase the platform with more developers and this platform will not want to adopt the standard. The effect of the direct network effect increases with the decrease in the developer ´s travel cost and the effect of the indirect network effects also decreases with the decreasing in the developers travel cost.

The following lemma summarizes the earlier argument:

LEMMA 1. *If the degree of differentiation in the developers market is sufficiently low, then the platforms adopt the standard. Alternatively, if the degree of differentiation in the developers market is sufficiently high, then the platforms do not adopt the standard.*

.*If* $n_i > n_j$ *and* $t^c < \widetilde{t}_i^c < t_i^c(Z=1)$ *where* $i, j \epsilon \{A, B\}$ $i \neq j$ *or* $n_i = n_j$*, platforms always adopt the standard.*

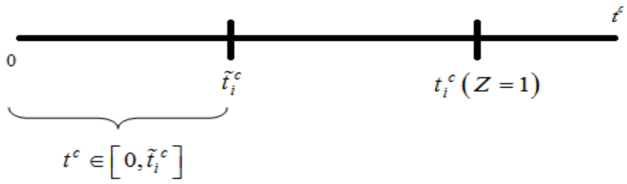

Figure 3.3: Equilibrium with standard.

•If $n_i > n_j$ and $\widetilde{t_i}^c < t^c < t_i^c(Z = 0)$ where $i, j \epsilon \{A, B\}$ $i \neq j$, platforms never adopt the standard.

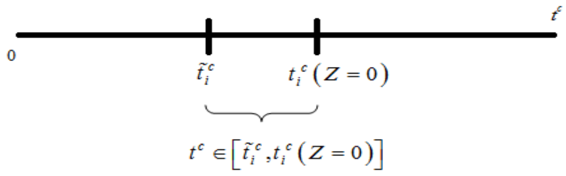

Figure 3.4: Equilibrium without standard.

If the degree of differentiation in the developers market is sufficiently low, then the platforms adopt the standard, in this case the platforms network shares are important to consumers. Both platforms can obtain higher profits because they can establish higher prices. It is the same that happens if indirect network effects are not present, consumers accept to pay a higher price to be in the same network. When the developer travel cost is low, the developers market is less important.

On the contrary case, if the degree of differentiation in the developers market is sufficiently high, then the platforms do not adopt the standard, in this case the developers market becomes important to consumers. The platform with a lower number of developers is at a disadvantage with respect to the other platform, more consumers will want to purchase the rival platform, and by the presence of consumers' expectations it will have a smaller network size. One enters in a spiral of higher number of developers, higher demand and higher network size that finishes with the platform with fewer developers out of the market. This platform can reduce this effect if the standard is adopted, but the rival platform never wants to adopt it.

The propensity for the market to standardize with fulfilled expectations is subject to three factors:

LEMMA 2. *In the presence of direct and indirect network effects, if $n_i > n_j$*

$i = A, B$ where $i \neq j$, *the propensity to standardize the market increases if: (i) the strength of the network effect decreases, (ii) the number of type i developers decreases, (iii) the number of type j developers increases, and (iii) the platform travel cost increases.*

If $n_i > n_j$ $i = A, B$ where $i \neq j$, the condition for higher profits in platform i with the standard is given by:

$$\widetilde{t_i^c} = \frac{1}{27}\left(\frac{(9t^p\gamma-2)(9\gamma(t^p-v)-2)}{\gamma^2(dn_i-dn_j)(9t^p(t^p-v)-2t^p+v)} + \sqrt{\frac{(2-9t^p\gamma)^2(9t^p\gamma-1)(9\gamma(t^p-v)-1)(2-9\gamma(t^p-v))^2}{\gamma^4(dn_i-dn_j)^2(9t^p\gamma(t^p-v)-2t^p+v)^2}}\right) \tag{3.32}$$

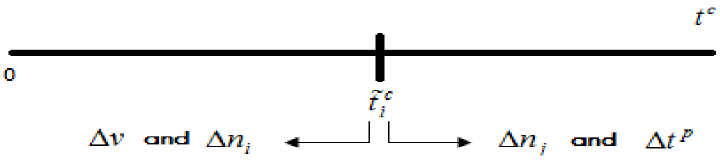

Figure 3.5: Propensity for the market to standardize.

On the left side of this figure both platforms adopt the standard and on the right side they do not adopt it. Also, (ii) and (iii) imply that standardization is less or more likely for a greater number of software firms. The same result is found by Church and Gandal (1992): standardization is more likely before an increase in type j developers. More recently, Clements (2004) obtains a similar result.[12].

The first factor is the strength of the network externality: an increase in the strength of the externality reduces the incentive to adopt a standard. If one platform becomes a monopolist the profit is enormous. The second factor is the platform travel cost: an increase in the platform travel cost produces an incentive to adopt a standard increases, the consumers will not accept paying a higher travel cost,

[12]The reason for the difference is that software prices in Clements (2004) model depend on the number of firms in the industry and software prices in Church and Gandal's model do not depend on the number of firms in the industry.

so platforms can reduce this effect with the standard. And the third factor is the difference between the number of developers:

If the difference between the number of developers increases, the propensity for the market to standardize decreases and if the difference between the number of developers decreases, the propensity for the market to standardize increases. The more equals the number of developers between both platforms, the more incentives they have to adopt the standard because the platforms can obtain higher profits.

Taking into account lemma 2; when the strength of network externality is increased or the number of type i developers is increased, the function $\widetilde{t}_i^c$ moves to the left and decreases the range where both platforms obtain higher profits with the standard. On the other hand, when platforms travel cost is increased or the number of type j developers is increased, the function $\widetilde{t}_i^c$ moves to the right and increases the range where both platforms obtain higher profits with standard.

Increases in v and n_i reduce the propensity for the market to standardize and increases in t^p and n_j increase the propensity for the market to standardize.

This result is the same as the one obtained by Bender and Schmidt (2007), these authors found that if the strength of the network externality is high, it is less likely to adopt the standard. The model structure of these authors is similar to our model, except for the presence of developers markets. They analyse three different types of equilibria, the first equilibrium they analyze is identical to ours, a symmetric equilibrium where both platforms are used in the market. Then they consider the case where only one of the two platforms involved in the market. And finally they consider a combination of the earlier two. Their results depend on platforms transportation cost, with a higher cost, it is less likely that the platforms adopt the standard. Therefore, for these authors, increases in strength of the network externality increases the travel cost required for the adoption of the standard and therefore reduces the possibility to adopt the standard. The difference is the specification in the indirect utility function, but in order to model development it can be seen that the results are identical.

In contrast, in the work of Clements (2004), we have the opposite result; increases in the strength of the network externality increases the possibility to adopt the standard. In this chapter the utility functions is different. The main change is that if consumers do not buy a complement, in the utility is zero. When indirect network externalities are not present, there are two possible equilibrium depending on the platforms travel cost, there exists a threshold in the platforms travel cost where they do not adopt the standard, and by increasing the strength of the network externality this threshold is higher.

3.3.2 Non-Fulfilled Nash equilibrium

In this section we consider that consumers expectations focus on platform A ($y_A = 1, y_B = 0$). The same results would be obtained if consumers expectations focus on platform B ($y_A = 0, y_B = 1$), but inversely.

Assuming that both platforms have a positive market share there is one consumer located at $\widehat{x}$ who is just indifferent between buying from A or B. This consumer is given by

$$\hat{x} = \frac{t^c (n_A - n_B) + t^c n_A n_B (dn_B - dn_A) + n_A n_B (p_B - p_A + t^p + v(1-Z) + \theta_A - \theta_B)}{2 n_A n_B t^p} \tag{3.33}$$

We can calculate the license fees, substituting $\hat{x}$ into (3.5) and (3.6) as follows:

$$l_A = \frac{t^c \widehat{x}}{n_A^2} = \frac{t^{c2} (n_A - n_B) + t^{c2} n_A n_B (dn_B - dn_A) + t^c n_A n_B (p_B - p_A + t^p + v(1-Z) + \theta_A - \theta_B)}{2 n_A^3 n_B t^p} \tag{3.34}$$

$$l_B = \frac{t^c(1-\widehat{x})}{n_B^2} = \frac{t^{c2}\left(n_B - n_A\right) + t^{c2} n_A n_B (dn_A - dn_B) + t^c n_A n_B (p_A - p_B + t^p + v(Z-1) + \theta_B - \theta_A)}{2 n_A n_B^3 t^p} \quad (3.35)$$

The presence of direct network externalities affects not only consumers and platforms, but also developers:

If platforms do not adopt the standard, an increase in the strength of the network externality, v, produces an increase in type A licenses fees and a reduction in type B licenses fees. In the contrary case this effect disappears.

•The pricing decision of platforms

At stage 3, platforms simultaneously choose the profit maximizing price. The solution to $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$ yields:

$$p_A = \frac{1}{3}\left(3t^p + v\left(1-Z\right) + \theta_A - \theta_B - \frac{3t^c}{n_A} + t^c\left(dn_B - dn_A\right)\right) \quad (3.36)$$

$$p_B = \frac{1}{3}\left(3t^p + v\left(Z-1\right) + \theta_B - \theta_A - \frac{3t^c}{n_B} + t^c\left(dn_A - dn_B\right)\right) \quad (3.37)$$

Note that the prices are a function of the strength of the network externality, the number of developers of each platform, the developers travel cost, the platform travel cost, technological innovation and the difference between developers ´ distances.

If platforms do not adopt the standard, an increase in the strength of the network externality produces one increase in platform A price and one reduction in platform B price. In the contrary case this effect disappears.

In case platforms adopt the standard we get the same result that in fulfilled Nash equilibrium because standardization eliminates the expected advantage of platform A[13]. In contrast to the type 1 Nash equilibrium, if platforms do not adopt the standard, an increase in the strength of the network externality produces an increase in platform A price, and a reduction in platform B price. If platform B has no expectations from consumers and there is an increase in the strength of the network externality, then its demand decreases. To compensate this effect platform B responds by lowering price to attract consumers; in spite of it, the decrease in price is not reflected in an increment in its network size because consumers' expectations are focused on platform A.

The only way that platform B can compete in the market is through price, in a standard common market thus would lead to a demand increase. However, direct network effects become a key factor, albeit not the only one that consumers to care when making decisions. Taking this into account, platform B can compete in the market with a rather small market share.

In the developers market we have similar findings as in fulfilled Nash equilibrium.

•The technological innovation decision

At stage 2, each platform make an investment in technological innovation, substituting equations (3.36) and (3.37) in the profits functions and maximizing with respect θ_A and θ_B, the equilibrium level of technological innovation for both platforms is given by:

$$\theta_A = \frac{2 - 9t^p\gamma + 3v(Z-1)\gamma - 3t^c\gamma(dn_B - dn_A)}{3\gamma(2 - 9t^p\gamma)} \tag{3.38}$$

[13]The results are given by equations (3.26), (3.27), (3.28) and (3.29).

$$\theta_B = \frac{2 - 9t^p\gamma + 3v(1-Z)\gamma - 3t^c\gamma(dn_A - dn_B)}{3\gamma(2 - 9t^p\gamma)} \tag{3.39}$$

In this case, the levels of technological innovation are a function of platforms travel cost, the strength of the network externality, developer travel cost and the difference between developers´ distances. If platforms do not adopt the standard, platform A will choose more level of technological innovation. On the other hand, if platforms adopt the standard occur the same that in fulfilled Nash equilibrium.

Regardless of standardization happening or not, and of fulfilled expectations or not the investment in technological innovation will always be the same. The aggregate level of technological innovation is constant throughout all the model.It is given by $\theta_A^* + \theta_B^* = \frac{2}{3\gamma}$. The aggregate level of innovation is independent of direct and indirect network effects.

Note that, by the second-order condition, to obtain a maximum, the next assumption is required.

Assumption 3. *For* $(\theta_A^*, \theta_{B,}^* p_A^*, p_B^*)$ *to be a maximum, it must be satisfied that:*

$\gamma > \frac{1}{9t^p}$

Further note that for positive technological innovation, θ_A, θ_B, the following assumption must be satisfied:

Assumption 4. *Both platforms invest in technological innovation if:*

$t^p > \frac{v}{3}$, $\gamma > \frac{2}{3(3t^p - v)}$ *and:*

$n_i > n_j \longrightarrow t^c < \frac{2-3\gamma(3t^p-v)}{3\gamma(dn_i-dn_j)}$, $i, j = A, B$ where $i \neq j$

$n_j > n_i \longrightarrow t^c < \frac{2-3\gamma(3t^p-v)}{3\gamma(dn_j-dn_i)}$, $i, j = A, B$ where $i \neq j$

3.3.2.1. The standardization stage

We will not discuss the case in which platforms adopt the standard ($Z = 1$), the results obtained are the same as for the fulfilled Nash equilibrium. If platforms adopt the standard, consumer´s expectations are irrelevant, every user is in the same network independently of which platform has more consumers.

Therefore, we only analyze the case when platforms do not adopt the standard ($Z = 0$). The equilibrium levels of technological innovation, prices and profits are given by:

$$\theta_A = \frac{2 - 9\gamma(t^p + \frac{v}{3}) - 3t^c\gamma(dn_B - dn_A)}{\gamma(6 - 27t^p\gamma)} \tag{3.40}$$

$$\theta_B = \frac{2 - 9\gamma(t^p - \frac{v}{3}) - 3t^c\gamma(dn_A - dn_B)}{\gamma(6 - 27t^p\gamma)} \tag{3.41}$$

$$p_A = \frac{t^c(2 - 9t^p\gamma) + n_At^p(3\gamma(3t^p + v) - 2) + 3n_At^ct^p\gamma(dn_B - dn_A)}{n_A(9t^p\gamma - 2)} \tag{3.42}$$

$$p_B = \frac{t^c(2 - 9t^p\gamma) + n_Bt^p(3\gamma(3t^p - v) - 2) + 3n_Bt^ct^p\gamma(dn_A - dn_B)}{n_B(9t^p\gamma - 2)} \tag{3.43}$$

$$\pi_A = \frac{(9t^p\gamma - 1)(3\gamma(3t^p + v + t^c(dn_B - dn_A)) - 2)^2}{18\gamma(2 - 9t^p\gamma)^2} \tag{3.44}$$

$$\pi_B = \frac{(9t^p\gamma - 1)(3\gamma(3t^p - v + t^c(dn_A - dn_B)) - 2)^2}{18\gamma(2 - 9t^p\gamma)^2} \tag{3.45}$$

The profits comparison with and without the standard, equation (3.29) with (3.44) and (3.45), yields:

$\pi_A\,(Z = 0) > \pi_A(Z = 1)$

$\pi_B\,(Z = 1) > \pi_B(Z = 0)$

Platform A always obtains a higher profit if it does not adopt the standard, as it can establish a higher price because consumer expectations are focused on platform A. On the other hand, platform B always prefers to adopt the standard, since standardization eliminates the expected advantage, it can fix a higher price. All consumers want to be in platform A, but if platforms adopt the standard then consumer who buy platform A or B is in the same network. In conclusion, if consumer expectations are focused in one of the platforms, the standard will never be adopted, because in this case, platform A never wants to adopt it.

LEMMA 3. *If consumers expect that only one of the platforms serves the market, the standard will never be adopted.*

The Subgame Perfect Nash Equilibrium when consumers´expectations focus on platform A are given by the values of $(p_A^*, p_B^*, \theta_A^*, \theta_B^*, Z^*)$, where:

$$Z^* = 0$$

$$\theta_A^* = \frac{2 - 9\gamma(t^p + \frac{v}{3}) - 3t^c\gamma(dn_B - dn_A)}{\gamma(6 - 27t^p\gamma)}$$

$$\theta_B^* = \frac{2 - 9\gamma(t^p - \frac{v}{3}) - 3t^c\gamma(dn_A - dn_B)}{\gamma(6 - 27t^p\gamma)}$$

$$p_A^* = \frac{t^c(2 - 9t^p\gamma) + n_A t^p(3\gamma(3t^p + v) - 2) + 3n_A t^c t^p\gamma(dn_B - dn_A)}{n_A(9t^p\gamma - 2)}$$

$$p_B^* = \frac{t^c(2 - 9t^p\gamma) + n_B t^p(3\gamma(3t^p - v) - 2) + 3n_B t^c t^p\gamma(dn_A - dn_B)}{n_B(9t^p\gamma - 2)}$$

Although platform B has no expectations from consumers, it can still have a positive market share due to its low price. In an extreme case it could monopolize the market.

In particular, this may happen if platform B establishes a price such that

$$p_B \leq \frac{t^c(2 - 9t^p\gamma) + 2n_B t^p(3\gamma(3t^p - v) - 2) + 6n_B t^c t^p\gamma(dn_A - dn_B)}{n_B(9t^p\gamma - 2)} \tag{3.46}$$

This price is so low, that allows platform B to be present in the market. Direct and indirect network effects are key factors in consumers' decisions, but such a low price compensates the zero expectations on platform B network size. Platform B takes a "judo economics" strategy; he takes advantage of its smaller size and price to take part in the market. Platform B is capable of convincing platform A, by its insignificant market share, that it does not suppose a threat. The presence of the indirect network effects can make this threat real, when $n_B > n_A$ and $\frac{v}{dn_A - dn_B} < t^c < \frac{3\gamma(3t^p+v)-2}{3\gamma(dn_A-dn_B)}$.

Specifically, when $n_B > n_A$ and $t^c = \frac{v}{dn_A - dn_B}$, platforms obtain the same profit, the same demand and choose the same level of technological innovation. The equilibrium values of $(\theta_A, \theta_B, p_A, p_B)$ and platforms profit are:

$p_A = t^p - \frac{v}{n_A(dn_A - dn_B)}$, $p_B = t^p + \frac{v}{n_B(dn_B - dn_A)}$, $\theta_A = \theta_B = \frac{1}{3\gamma}$, $D_A = D_B = \frac{1}{2}$,
$\pi_A = \pi_B = \frac{9t^p\gamma - 1}{18\gamma}$

where, $p_B > p_A$ and $l_A > l_B$. Platform B obtains a higher revenue from consumers, while platform A obtains a higher revenue from developers. Platform A has a smaller number of developers and it charges higher license fees. Consider the level of developers travel cost, $t^c = \frac{v}{dn_A - dn_B}$. Below this cost the developer market will have a greater effect on consumer decisions than direct effects.

This effect is due to a greater presence of type B developers. If platform B had a higher number of developers and indirect network effects were extremely strong, it could monopolize the market. Thus, if the developers market is sufficiently differentiated, the platform B sets a higher price, a higher level of technological innovation and it obtains a higher demand, despite the fact that consumers' expectations are focused on platform A.

Platforms do not adopt the standard because platform A is able to set a higher price without a standard, even in the case where platform B has a higher market share.

Corollary. *If consumers expect that both platforms will serve the market, then the decision to adopt a standard depends on the degree of differentiation in the developers market. Alternatively, if consumers expect that only one of the platform serves the market, platforms will not adopt the standard.*

Summary;

•*If consumers expect that both platforms serve the market (fulfilled expectations),* $y_A = \widehat{x}$, $y_B = (1 - \widehat{x})$, *the Subgame Perfect Nash Equilibrium is given by:*

. *When* $n_i > n_j$ *and* $t^c < \widetilde{t_i^c} < t_i^c(Z=1)$, $i,j = A,B$ where $i \neq j$:

$Z^* = 1$

$\theta_i^* = \frac{9\gamma t^p + 3t^c\gamma(dn_j - dn_i) - 2}{\gamma(27\gamma t^p - 6)}$; $i,j = A,B$ where $i \neq j$

$p_i^* = \frac{(n_i t^p - t^c)(9t^p\gamma - 2) + 3n_i t^c t^p \gamma(dn_j - dn_i)}{n_i(9t^p\gamma - 2)}$; $i,j = A,B$ where $i \neq j$

. *When* $n_i > n_j$ *and* $\widetilde{t_i}^c < t^c < t_i^c(Z=0)$, $i,j = A,B$ where $i \neq j$:

$Z^* = 0$

$\theta_i^* = \frac{9\gamma(t^p - v) + 3t^c\gamma(dn_j - dn_i) - 2}{3\gamma(9\gamma(t^p - v) - 2)}$; $i,j = A,B$ where $i \neq j$

$p_i^* = \frac{(n_i(t^p - v) - t^c)(9\gamma(t^p - v) - 2) + 3n_a t^c \gamma(t^p - v)(dn_j - dn_i)}{n_i(9\gamma(t^p - v) - 2)}$; $i,j = A,B$ where $i \neq j$

•*If consumers expect that only one of the platforms serves the market,* $y_A = 1$, $y_B = 0$ *, the Subgame Perfect Nash Equilibrium is given by:*

$Z^* = 0$

$\theta_A^* = \frac{2 - 9\gamma(t^p + \frac{v}{3}) - 3t^c\gamma(dn_B - dn_A)}{\gamma(6 - 27t^p\gamma)}$

$\theta_B^* = \frac{2 - 9\gamma(t^p - \frac{v}{3}) - 3t^c\gamma(dn_A - dn_B)}{\gamma(6 - 27t^p\gamma)}$

$p_A^* = \frac{t^c(2 - 9t^p\gamma) + n_A t^p(3\gamma(3t^p + v) - 2) + 3n_A t^c t^p \gamma(dn_B - dn_A)}{n_A(9t^p\gamma - 2)}$

$p_B^* = \frac{t^c(2 - 9t^p\gamma) + n_B t^p(3\gamma(3t^p - v) - 2) + 3n_B t^c t^p \gamma(dn_A - dn_B)}{n_B(9t^p\gamma - 2)}$

If consumers expect that both platforms will serve the market, depending on the degree of differentiation in the developers market, platforms adopt the standard or not. On the other hand, if consumers expect that only one platform will serve the market, platforms never adopt the standard. Katz and Shapiro (1985) obtain that the platform with a large network does not adopt the standard, although their model does not consider indirect network effects.

3.4. Welfare Analysis

In this section we study the welfare implications of fulfilled Nash equilibrium with fulfilled expectations. We compare the welfare when platforms do not adopt the standard and when they adopt it; to get a better grasp of the welfare changes we analyze separately consumer and producer surplus, which consist of platforms and developers profits.

3.4.1. Total Welfare

Proposition 3. *Total welfare is higher with the standard if*

$$t^c < \frac{1}{\sqrt{3}}\sqrt{\frac{(2-9t^p\gamma)^2(2-9\gamma(t^p-v))^2}{\gamma^2(9\gamma(4v+t^p(45\gamma(t^p-v)-8))-4)(dn_A-dn_B)^2}} = t^{cw} \tag{3.47}$$

3.4.2. Consumer surplus

Proposition 4. *Consumer surplus is always higher without the standard*

$EC(Z=0) > EC(Z=1)$

3.4.3. Producer surplus

Proposition 5. *Producer Surplus is higher with the standard if*

$$t^c < \frac{1}{9}\sqrt{\frac{(2-9t^p\gamma)^2(2-9\gamma(t^p-v))^2}{\gamma^3(v-2t^p+9t^p\gamma)(t^p-v))(dn_A-dn_B)^2}} = t^{cPS} \tag{3.48}$$

where $t^{cPS} > t^{cw}$.

Taking into account propositions 3 and 5 we distinguish three areas:

When $t^c < t^{cw} < t^{cPS}$, total welfare is higher with the standard because producers gain is greater than consumers losses . ***(Area 1)***

When $t^{cw} < t^c < t^{cPS}$, total welfare is higher without the standard because if platforms adopt the standard consumer loss is greater than the producers gain. ***(Area 2)***

When $t^{cw} < t^{cPS} < t^c$, total welfare is higher without the standard. Consumer and producer surplus is higher without the standard. ***(Area 3)***

C:/Users/Ignacio/Desktop/Latest/Capitulos/Chapter 3-Standardization and Network

Figure 3.6: Welfare.

As can be observed in figure 3.6, total welfare is explained by what happens to producer surplus since consumer surplus is always higher without the standard. This occurs because when platforms adopt the standard we observe two effects: the variety and the price effect.

The variety effect arises since one variety is lost when a standard is imposed. If platforms do not adopt the standard we will have two systems in the market. There is a trade-off between standardization and variety. The second one is the price effect, when platforms adopt the standard, they maintain a high monopoly power and charge high prices to obtain higher profits. In these markets consumers pay a higher price when they all belong to the same network. The adoption of a common standard produces limitations in variety and increases in prices. If platforms do not adopt the standard, price competition will be stronger because each platform will want to impose its system in the market. If platforms adopt the standard, price competition dampens; all consumers are in the same network and one system is imposed.

We can see that some standardization choices of platforms are inefficient. Taking into account propositions 4 and 5 and equation (3.32) there exist two inefficient areas[14]:

The first case, when $t^{cw} < t^c < \widetilde{t}_i^c$

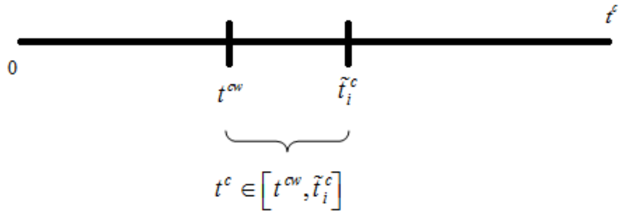

Figure 3.7: The welfare is higher without the standard.

In this case platforms adopt the standard but welfare is higher without it; $W(Z = 0) > W(Z^* = 1)$.

The second case, when $\widetilde{t}_i^c < t^c < t^{cw}$

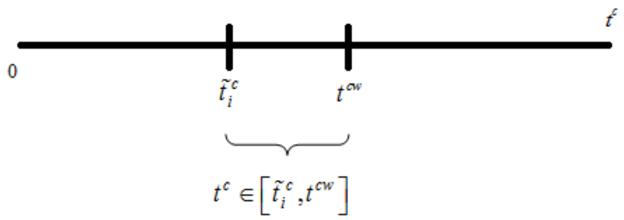

Figure 3.8: The welfare is higher with the standard.

In this case platforms do not adopt the standard but welfare is higher with it; $W(Z = 1) > W(Z^* = 0)$.

[14]This result is opposite to the welfare result in Clement (2004). The difference is due to the modeling of consumer preferences. He obtains that there is no equilibrium in which there is inefficient standardization with direct network externalities.

These inefficient standardization choices and the increases in prices when platforms adopt the standard and when they do not adopt it can justify the regulation in the telecommunications and technological markets, although the network effects are very hardly resoluble through the defense mechanisms of the competition.

3.6. Conclusion

In the first part of the chapter we have considered fulfilled expectations, in this setting, if the degree of differentiation in the developers market is sufficiently high; then the platforms will not adopt a standard. Alternatively, if the degree of differentiation in the developers market is sufficiently low, then the platforms will adopt a standard. With fulfilled expectations the propensity for the market to standardize is subject to the platforms travel cost, the strength of the network externality and the difference between the number of developers. In the second part, we analyze the extreme case of expectations of a single platform. In this case platforms will not adopt a standard. Finally, we analyze total welfare under the fulfilled expectations setting. We observe two effects, the variety and the price effect; as a consequence, the consumer surplus is always higher without the standard. In this chapter, we consider the presence of direct and indirect network externalities in the same model. The direct network externalities are present by the consumers' expectations on the platforms network sizes, the above mentioned expectations are a key factor in the consumer's decision making. In the network models the available technology is valued taking into account other features in addition to the price. The indirect network externalities appear by the market of developers, the presence of developers in a platform increase consumers' willingness to pay for this platform.

If the platforms do not adopt the standard, then the travel costs are important to the consumers, given that the consumers will rather prefer the largest network;

consumers will be ready to pay a higher travel cost to belong to the same network. The presence of the indirect network externalities, in certain conditions, can do that a platform that has no expectation from the consumers finishes with a high market share, it can become a monopoly. Unlike what happens in traditional markets, in these markets, one increases in the price does not reflected in reductions in the demand. Increases in the strength of the network externality or increases in the number of developers, lead to a platform to increase its price and produce a growth in its demand. Also, we observe that before the presence of indirect network effects, the platforms are capable of giving away for free (to fix negative price), this can be observed in the real life in the markets telecommunications, in operating systems markets, web browsers, mobile applications... Our results when the consumers expect that both platforms will serve the market support the obtained by Casadesus and Masanell (2009), when the consumers are heterogeneous, the platforms adopt the standard, establish higher prices and obtain a higher profits.

We analyze the propensity for the market to standardize with fulfilled expectations, and we observe that it is subject to three factors, i) if the difference between the number of developers increases, the propensity for the market to standardize decreases and ii) if the difference between the number of developers decreases, the propensity for the market to standardize increases. The more equals the number of developers between both platforms, the more incentives they have to adopt the standard because the platforms can obtain higher profits. And iii) if the platform travel cost increases, the propensity for the market to standardize increases.

If the platforms adopt the standard, they maintain high monopoly power and charge high prices and obtain higher profits. These increases in prices can justify the regulation in the telecommunication markets and especially in the interconnection between operators, telephone or Internet operators. When we have compared the total welfare, it is higher with the standard, producers gain is greater than the consumer's losses. The consumer surplus is always higher without the standard. These occur because when platforms adopt the standard we observe two effects:

Variety and Price effects. When the platforms adopt the standard one system is imposed, we loss one variety. If platform will not adopt the standard we will have two systems in the market. The consumers pay a higher price by all belonging to the same network. We can observe a trade off, Variety Vs Standardization. The adoption of a common standard produces limitations in variety and increases in prices.

Finally, we observe some inefficient standardization choices that can justify the regulation in the telecommunications and technological markets

3.7. Appendix

3.7.1. Proof of Assumption 1

By the second order condition from the technological innovation decision we obtain:

$$\frac{\partial^2 \pi_A}{\partial \theta_A^2} = \frac{1}{9(t^p + v(Z-1))} - \gamma$$

and we need that $\frac{\partial^2 \pi_A}{\partial \theta_A^2} < 0$, it will be less than zero when

$$\frac{1}{9(t^p + v(Z-1))} - \gamma < 0 \longrightarrow \frac{1}{9(t^p + v(Z-1))} < \gamma \longrightarrow \gamma > \frac{1}{9(t^p - v(1-Z))}$$

When platforms adopt the standard, $Z = 1$, $\gamma > \frac{1}{9t^p}$ and
when they do not adopt it, $Z = 0$, $\gamma > \frac{1}{9(t^p - v)}$ and $t^p > v$.

3.7.2. Proof of Assumption 2

From the technological innovation decision we have that $\theta_i = \frac{9\gamma(t^p+v(Z-1))+3t^c\gamma(dn_j-dn_i)-2}{3\gamma(9\gamma(t^p+v(Z-1)-2))}$ and we assume that both platform invest a positive amount:

$$\frac{9\gamma(t^p + v(Z-1)) + 3t^c\gamma(dn_j - dn_i) - 2}{3\gamma(9\gamma(t^p + v(Z-1) - 2))} > 0 \longrightarrow 9\gamma(t^p + v(Z-1)) + 3t^c\gamma(dn_j - dn_i) - 2 > 0 \longrightarrow$$

$$3t^c\gamma(dn_j - dn_i) > -9\gamma(t^p + v(Z-1)) + 2 \longrightarrow t^c > \frac{-9\gamma(t^p + v(Z-1)) + 2}{3\gamma(dn_j - dn_i)} \longrightarrow t^c < \frac{9\gamma(t^p + v(Z-1)) - 2}{3\gamma(dn_j - dn_i)}$$

When platforms adopt the standard, $Z = 1$, $t^c < \frac{9\gamma t^p - 2}{3\gamma(dn_i - dn_j)}$ and
when they do not adopt it, $Z = 0$, $t^c < \frac{9\gamma(t^p - v) - 2}{3\gamma(dn_i - dn_j)}$.

Chapter 4. Platforms, Software and Piracy in a two-sided market

4.1. Introduction

Many technological markets are formed mainly by platforms, software developers and consumers. Consumers usually need to buy software products to use the platforms, but in these types of markets the consumers can get illegal copies of software. Many of these platforms will not work without software applications; if consumers purchase a platform alone they will not obtain any utility, they need to combine it with applications. For example, in the console market the consumers need to buy the console and games.

These types of markets are two-sided, with the presence of network externalities in consumption, specifically by two-sided externalities. In the console market the platforms (Xbox 360 and PlayStation) compete to attract game developers and to attract consumers. If one platform has a large variety of games, more consumers are willing to buy it and if one platform has a large number of consumers, more game developers want to develop games for this platform. Now, the network externalities do not affect the same side of the market, on the consumer side they base their decision by taking into account the number of games and not according to the number of consumers who have the same console.

A further example is the market of mobile operating systems. Consumers decide whether to buy a mobile with Android or a mobile with iOS, from Google or from Apple and the software developers decide if they create applications for one of the two operating systems or for both. If a developer wants to work with Apple he will pay $99[15] per year and if he wants to work with Google they will pay $25[16] per year. Consumers can buy the software developers applications in the online stores, App Store or Google Play, but they can also obtain illegal copies. In these markets

[15] https://developer.apple.com/programs/which-program

[16] In this case Google also charged a royalty of 30% by application sold. http://www.android.com/uk/developer-distribution-agreement.html#showlanguages

we find applications that work exclusively with Apple or with Google but the most common is that the software developers in the mobile markets use a multi-homing strategy and produce software for both platforms.

In the personal computer market we find a similar case: both platforms, Mac and Windows, compete to attract consumers and software developers.

Finally, we may consider the market of e-books with, for example, Amazon or with Smashwords[17]. The platforms obtain income from selling books and from authors who want to publish their books. In this case the developers are the authors[18].

A common problem for software developers in the markets for digital goods is piracy. Piracy of digital content is considered a serious problem by content companies. In this type of markets, it is relatively easy obtain a copy. Then users only face the moral dilemma about violating the property rights.

The different software developers are members of different associations against piracy. For example, in the Business Software Alliance (BSA) are Adobe, Apple, Intel, Microsoft and McAfee, among others[19].

The different types of software associations declare their position against piracy but we can find other developers who confess that piracy may be good.

For example, the company Rovio Mobiles said in a conference that “piracy may not be a bad thing: it can get us more business”, they believe that it allows them to attract new users. The Nero Boss declares that “piracy has made us grow, so instead of investing time and effort in anti-piracy mechanisms, we prefer to improve product quality”. Unity Technologies has declared that: “Piracy can be a way to seed a market. Nobody will ever fully conquer the piracy problem, but we can certainly turn it to our advantage” and the Mojang A B boss said: “Desperate to try the blocky indie sensation Minecraft but can’t afford the $ 26.95 price tag? Just

[17]Smashwords is an e-book publishing and distribution platform for e-book authors, publishers, agents and readers. More information at http://www.smashwords.com

[18]In this case the platforms charge royalties to the publishers, while in our model we only consider license fees. See for price information https://kdp.amazon.com/self-publishing/help?topicId=A29FL26OKE7R7B and http://www.smashwords.com/about/how_to_publish_on_smashwords

[19]All members can be found at http://www.bsa.org/country/BSA%20and%20Members/Our%20Members.aspx

pirate it".

A 2009 survey conducted by TIGA[20] about the piracy demonstrates that developers view the actual threat of piracy to their business survival as low (60%) with only 20% ranking the threat as medium and only 10% considering the threat to be high (10% had no view). For these developers piracy is a problem, but not a threat to survival.

The problem of piracy was analyzed by the Business Software Alliance (BSA), [21]in the last report[22] they estimated the losses in 2010 due to piracy and estimated that the worldwide piracy rate is 42%. According to this association, about half of the world population pirate or copy software products. The problem of this information is that it cannot be real because piracy losses are calculated with the following methodology: Number of Unlicensed Units Installed multiplied by Average Software Unit Price equal to The Commercial Value of Pirated Software.

The problem of this approach arises because not everyone that copies software would buy it if he had to pay for the product. Therefore, to consider pirated software as a lost sale can be misleading.

In this chapter we analyze the piracy problem in a two-sided market. Four possible scenarios are distinguished depending on whether consumers and/or developers are following single-homing or multi-homing strategies. The first scenario is where only consumers use a multi-homing strategy whereas developers stick to a single platform; in the second, consumers and developers follow a single-homing strategy (and work with just one platform); the third scenario considers that only developers use a multi-homing strategy; and the fourth scenario has both agents multi-homing and use both platforms. We wish to characterize the equilibrium prices paid by

[20]TIGA is the non-profit trade association representing the UK's games industry. See http://www.tiga.org/

[21]Business Software Alliance (BSA) is a nonprofit trade association created to advance the goals of the software industry and its hardware partners. It is the foremost organization dedicated to promoting a safe and legal digital world. BSA acts to protect software providers' intellectual property rights, enforce software copyright legislation, and encourage compliance. It receives thousands of reports from end users, resellers, law enforcement, members, and affiliate associations detailing the alleged use of unlicensed and illegal software. You can find more information about BSA at "http://www.bsa.org"

[22]http://portal.bsa.org/globalpiracy2011/downloads/study_pdf/2011_BSA_Piracy_Study-Standard.pdf

consumers and the equilibrium license fees borne by developers in the presence of software piracy, and to study whether the software developers actually lose with piracy. We consider a two-sided market where platforms interact with software developers and consumers and another market where consumers decide whether to buy the original software from the developers or obtain an illegal copy. Finally, the model considers the possibility that developers invest in an anti-piracy mechanism to protect their software.

Our main results reveal that piracy may be a problem for the software developers depending on the type of scenario. An decrease in the platform license fee is realized in the first scenario where only consumers use a multi-homing strategy. There are developers who are against piracy and others not. This reflects why in the real world some developers might be in favour and others against piracy.

In a single-homing scenario more piracy will reduce the developers license fees, therefore the platforms obtain a lower profit because they have to reduce the license fees to attract more developers. Software developers are better off with more piracy, but platforms are against it.

In the most common scenario in real life - consumers only buy or join one platform and developers may be single-homing and multi-homing -, the variations in the price, license fee and in platforms profits depend on several factors.

In the latter case, with consumers and developers multi-homing, we found that platform profits always increase with more piracy.

Finally, in the last part of the chapter we provide a welfare analysis under all scenarios, it is shown that in three of the four scenarios the piracy is desirable, and calculate the social optimum when a social planner regulates the consumers and the developers market.

4.2. The Model

This section introduces a model of software piracy in a two-sided market.

4.2.1. Platforms

There are two horizontally differentiated platforms in the Hotelling line $[0, 1]$, A and B. We assume that platform A is located on the left end and platform B is located on the right end. For the sake of the exposition we assume that the marginal cost of production is zero. The platforms´ income is divided into two parts; the revenue that the platforms obtain from developers and the revenue that they obtain from consumers; platforms charge an access price to consumers, denoted by $\{p_A, p_B\}$ and a license fee to the software developers, denoted by $\{l_A, l_B\}$.

4.2.2. Software Developers and platforms

Developers are uniformly distributed on the unit line with respect to their preference for platforms. The total mass of developers in the market is one. The location of a developer with respect to platforms is denoted by z, $z\epsilon\,[0, 1]$.

The indirect utility of a developer z who joins either platform is given by

$$V(z) = \begin{cases} \beta - l_A - t^c z + u_A v, & \text{if joining A} \\ \beta - l_B - t^c(1 - z) + u_B v, & \text{if joining B} \end{cases} \tag{4.1}$$

Software developers obtain a reservation utility level, β. It is assumed that it is large enough to ensure that all developers will work with one of the platforms. Thus, the software developers market is covered.

Developers pay a license fee to work with one of the two platforms. For example, in the video games market, the platforms charge a per unit license fee on the game that the software developers produce for one of the two platforms; in this model such per unit license fee is represented by l_i, $i \epsilon \{A, B\}$.

The software developers decision is affected by the share of consumers subscribed to one of the two platforms. Thus, the effect of the network externality in the developers market is accounted for, because developers want to work with the platform with the highest number of users. It is clear that if the number of platform A users is higher, more developers will wish to work with platform A and vice versa for platform B. This is represented by the number of users of each platform, u_i, $i \epsilon \{A, B\}$ and v, where $v > 0$, corresponds with the strength of the network externality and measures the importance of the number of users for the software developers. Finally, t^c measures the degree of differentiation between platforms from the point of view of the developers.

4.2.3. Consumers and platforms

Consumers are uniformly distributed on the unit line with respect to their preference for platforms. The location of a consumer with respect to platforms is denoted by x,$x \epsilon [0, 1]$. The total mass of consumers is one. The indirect utility of a consumer x who joins either platform is given by

$$U(x) = \begin{cases} V - t^p x - p_A + n_A \lambda, & \text{if joining A} \\ V - t^p(1-x) - p_B + n_B \lambda, & \text{if joining B} \end{cases} \tag{4.2}$$

Consumers obtain a reservation utility level, V. It is assumed to be large enough to ensure that all consumers will buy from one of the platforms. Thus, the consumers market is covered. The consumer obtains extra utility if the share of software developers that work with the same platform is high. This effect is reflected by the parameter n_i, $i \epsilon \{A, B\}$ and by the strength of the network externality, λ, where

$\lambda > 0$. Finally, consumers will pay a price p_i, $i\epsilon\{A, B\}$ and the parameter t^p measures the degree of differentiation of the platforms from the consumers point of view.

The market is seen from different perspectives by consumers and developers and we consider that the degrees of differentiation t^c and t^p are different.

4.2.4. Consumers and software developers

We assume that consumers can copy a developer´s application already in the market[23]. Therefore they decide whether to buy the legal software or to obtain a copy for free yet it may be of lesser quality[24].

We use a framework where legal and illegal software are vertically differentiated and where the software developers have the possibility to invest in an anti-piracy mechanism. This mechanism can stop the creation of a copy. Software developers can invest a quantity k, where $k\epsilon(0, 1)$, in an anti-piracy mechanism at a cost $c(k)$, where $\frac{\partial c(k)}{\partial k} = \overline{c} > 0$.

We assume that each software developer behaves like a monopoly and that users may buy one unit of software from each software developer working with the same platform to which the consumer is subscribed; alternatively, they may get an illegal copy for free.

Consumers differ in their valuation θ for software. It is assumed that θ is uniformly distributed on $[0, 1]$. The indirect utility of user θ for each software unit is given by

$$U(\theta) = \begin{cases} \theta - p^o \text{ if buys the original product} \\ \theta\delta(1-k) \text{ if gets a copy} \end{cases} \tag{4.3}$$

[23] A survey on piracy of digital products is available by Peitz and Waelbroeck (2006a) and Belleflamme and Peitz (2010a).

[24] We use a similar approach comparable to Yoon (2002), Belleflamme (2003) and Bae and Choi (2006)

If the consumer buys the original software, he obtains a utility $\theta - p^o$ where p^o is the original software price charged by the software developers. If the consumer gets a copy, he obtains a utility of $\theta\delta(1-k)$, where $\delta \epsilon (0,1)$ measures the quality degradation of an illegal copy. With this, if $\delta = 0$ or $k = 1$, that is if the quality of the copy is terrible or the software developer makes the product impossible to copy, the utility that the consumer obtains from the copy is zero. It is further assumed that the cost of investment is [25] $c(k) < \frac{1+\delta(k-1)}{4}$, $c(0) = 0$ and when $\delta = 0$ the cost is $c(k) = 0$ because $k = 0$.

4.2.5. Timing

The timing of the game is as follows: in the first stage, platforms simultaneously choose the price charged to consumers $\{p_A, p_B\}$ and the license fees for developers $\{l_A, l_B\}$. In the second stage, consumers and software developers decide which platform to join and in the third stage consumers decide whether to buy or get a copy. We solve the model by backward induction and for four types of scenarios. We shall begin with the case where consumers use a multi-homing strategy and developers single-home, that is, consumers may buy from both platforms whereas each developer only buys or works with one of the two platforms. Then, we consider that consumers and developers single-home. Then, the case of only developers multi-homing is analyzed and finally we look into the case of both agents multi-homing.

Next, we solve the third stage of the game.

The indifferent user between buying and copying is given by $\widehat{\theta} = \frac{p^o}{1-\delta(1-k)}$ and the demand for the original product is $D^o = \frac{1-p^o-\delta(1-k)}{1-\delta(1-k)}$. The users from zero to $\widehat{\theta}$ get a copy and the users from $\widehat{\theta}$ to one buy the original product.

Developer $i\acute{}s$ profit is given by

$\pi_i^d = D^o p_i^o - c(k)$

[25]This condition guarantees that the developers obtain positive profits in this market.

The developers choose the profit-maximizing price, which is equal to:

$p^{o*} = \frac{1}{2}(1 - \delta(1-k))$

The developers compensate a better quality copy with a lower price. Therefore a reduction in the investment in an anti-piracy mechanism has a similar effect to an increase in the quality of the copy.

Each software developer earns profits of $\pi^d = \frac{1}{4}(1 - \delta(1-k) - 4c(k))$ per buyer; these profits correspond with the parameter v in (4.1).

User surplus for each software product is $\frac{1}{8}(1 + 3\delta(1-k))$, which corresponds to the network parameter governing the benefit users get from each additional software developer. User surplus is equalized to λ and developer surplus to v, which are the variables that have an influence in the platforms market[26]

$$\lambda = \frac{1}{8}(1 + 3\delta(1-k)) \tag{4.4}$$

$$v = \frac{1}{4}(1 - \delta(1-k) - 4c(k)) \tag{4.5}$$

To measure the level of piracy we make use of the quality of the copy and the investment in the anti-piracy mechanism, δ and k. A high level of piracy will involve an increase in δ and a decrease in k. A low level of piracy will be related with a decrease in δ and an increase in k.

In this setting, the consumers additional valuation of each developer (λ) always increases with more piracy and the developers additional valuation of each consumer (v) increases when $\bar{c} > \frac{1}{4}(1 + \delta - k)$ and decreases when $\bar{c} < \frac{1}{4}(1 + \delta - k)$.

The following assumption is required to guarantee an interior solution.

A.0. $t^c > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$

Assumption **A.0.** tells us that the network effects must not be too large compared

[26]The details of the calculation is relegated to the appendix.

with the horizontal differentiation parameter. It is assumed that **A.0.** holds in the remainder of the chapter.

See Appendix.

4.3. Platforms decision

In this section we solve the second and first stages of the game: the platforms, the consumers and the developers choice. Platforms decide simultaneously the price charged to consumers and the license fee charged to software developers for each type of market. For the sake of the presentation, the main text presents the above referred to as first scenario[27]. Finally, we examine the utility that developers obtain in each scenario.

4.3.1. Scenario 1: Consumers Multi-homing and Developers Single-homing

In this subsection we assume that the consumers side of the market is multi-homing while the developers side is single-homing. Consumers can buy from both platforms at the same time but developers only work with one of the platforms. For example, some consumers may buy a mobile with Android and a mobile with iOS, others users only buy a mobile with Android or with iOS. In this case, the application developers only make them for one mobile operating system.

When we consider that the consumers can follow a multi-homing strategy we find three types of consumers. From zero to x_A the consumers only buy from platform A, from x_A to x_B consumers buy from both platforms and from x_B to one

[27]The results before the piracy stage are similar to Belleflame and Peitz (2010b), except when we consider whole market multi-homing.

consumers only buy from platform B. We assume that $0 < x_A < x_B < 1$and report the necessary condition for this ordering below. In this case x_A corresponds with the share of users that only buy from platform A and $1 - x_B$ corresponds with the share of users that only buy from platform B. Therefore, $x_B - x_A$ corresponds with the share of users that buy from both platforms. Note that with consumers multi-homing the total user demand for platforms is greater than 1.

The indirect utility of a consumer x subscribing to a platform is given by

$$U(x) = \begin{cases} V - t^p x_A - p_A + n_A\lambda & \text{if buy platform A} \\ V - t^p(1 - x_B) - p_B + n_B\lambda & \text{if buy platform B} \\ 2V + \lambda - p_A - p_B - t^p & \text{if buy both platforms} \end{cases} \tag{4.6}$$

Fulfilled expectations for both sides of the market implies that $u_A = x_B$, $u_B = (1 - x_A)$ and $n_A = \widehat{z}$, $n_B = (1 - \widehat{z})$.

Taking into account these relationships, the location of the indifferent developer between joining platform A or platform B is given by

$$\widehat{z} = \frac{l_B - l_A + t^c + v(x_A + x_B - 1)}{2t^c} \tag{4.7}$$

A group of developers goes to platform A and the other group goes to platform B, developers are single-home.

To obtain the indifferent consumer, we first need to know the location of the indifferent consumer between joining platform A or both platforms and secondly the location of the indifferent consumer between joining platform B or both platforms.

The indifferent consumers (x_A and x_B) and the multi-homing consumers ($x_B - x_A$) are given by

$$x_A = \frac{\lambda(\widehat{z}-1) + p_B + t^p - V}{t^p} \tag{4.8}$$

$$x_B = \frac{V + \lambda\widehat{z} - p_A}{t^p} \tag{4.9}$$

$$(x_B - x_A) = \frac{\lambda + 2V - p_A - p_B - t^p}{t^p} \tag{4.10}$$

The number of developers does not affect the multi-homing consumers because they can purchase and use the two types of software. The effect of the externality disappears for these users.

The multi-homing users base their decision taking into account the price of both platforms. An increase in platform A (B) price produces a reduction of $\frac{1}{t^p}$ in the number of multi-homing consumers and an increase in platform B (A) user by the same amount.

Taking these equations into account, we solve for x_A, x_B and $\widehat{z}$ simultaneously. We solve the system of three equations to express the indifferent developer and the indifferent consumers in terms of prices and license fees[28]. The indifferent consumers are given by

$$x_A = \frac{\lambda^2 v + 2t^c t^p(p_B + t^p - V) - \lambda(v(p_A + p_B) + t^p(l_A - l_B + t^c + 2v) - 2vV)}{2(t^c t^p - \lambda v)} \tag{4.11}$$

$$x_B = \frac{\lambda^2 v + \lambda((l_B - l_A + t^c)t^p + v(p_A + p_B - 2V)) + 2t^c t^p(V - p_A)}{2(t^c t^p - \lambda v)} \tag{4.12}$$

And the indifferent developer is given by

[28] The detailed calculations are given in the appendix.

$$\widehat{z} = \frac{t^p(t^c + l_B - l_A) + v(p_B - p_A - \lambda)}{2(t^c t^p - \lambda v)} \tag{4.13}$$

Therefore, we derive total demand for either side of the market. The consumers demand, denoted by $D^c_{CMA} = x_B$, $D^c_{CMB} = (1 - x_A)$ and the developers demand, denoted by $D^d_{CMA} = \widehat{z}$, $D^d_{CMB} = (1 - \widehat{z})$ are given by

$$D^c_{CMi}(p_i, p_j, l_i, l_j) = \frac{\lambda(t^p(l_j - l_i + t^c) + v(p_j + p_i - 2V)) + 2t^c t^p(V - p_i) - \lambda^2 v}{2(t^c t^p - \lambda v)}, i, j = A, B, i \neq j \tag{4.14}$$

$$D^d_{SHi}(p_i, p_j, l_i, l_j) = \frac{t^p(t^c + l_j - l_i) - v(p_i - p_j + \lambda)}{2(t^c t^p - \lambda v)},\ i, j = A, B,\ i \neq j \tag{4.15}$$

Platforms choose prices and license fees so as to maximize their total revenues

$\pi_{CMA} = D^c_{CMA}(p_A, p_B, l_A, l_B)p_A + D^d_{CMA}(p_A, p_B, l_A, l_B)l_A$

$\pi_{CMB} = D^c_{CMB}(p_A, p_B, l_A, l_B)p_B + D^d_{CMB}(p_A, p_B, l_A, l_B)l_B$

The symmetric Nash equilibrium price for users and the symmetric Nash equilibrium license fee for developers are

$$p^*_{CM} = \frac{1}{2}V + \frac{1}{4}(\lambda - v) \tag{4.16}$$

$$l^*_{CM} = t^c - \frac{\lambda(\lambda + 3v + 2V)}{4t^p} \tag{4.17}$$

On the developers side, the platforms will charge the standard Hotelling[29] license fee less a term that depends on the strength of the network externality, if $\lambda = 0$ the license fee will be $l^*_{CM} = t^c$.

[29] The standard Hotelling license fee is the same that the price when we are in the standard hotelling model with zero cost: $l^* = t^c$

The equilibrium platforms profits are

$$\pi^*_{CM} = \frac{1}{2}t^c + \frac{V^2}{4t^p} - \frac{1}{16t^p}(\lambda^2 + v^2 + 6\lambda v) \tag{4.18}$$

The platform profit is negatively influenced by the two type of externalities.

4.3.1.1. Piracy

Next, we analyze the effect of piracy on the equilibrium variables. We know from the analysis of the third stage that [30] $\lambda = \frac{1}{8}(1+3\delta(1-k))$, $v = \frac{1}{4}(1-\delta(1-k)-4c(k))$ and the final values of $(p^*_{CM}, l^*_{CM}, \pi^*_{CM})$ with the possibility of piracy are as follows

$$p^*_{CM} = \frac{1}{2}V + \frac{1}{4}c(k) + \frac{1}{32}(5\delta(1-k) - 1) \tag{4.19}$$

$$l^*_{CM} = t^c - \frac{(1+3\delta(1-k))(7-24c(k)+16V-3\delta(1-k))}{256t^p} \tag{4.20}$$

$$\pi^*_{CM} = \frac{1}{2}t^c + \frac{1}{4t^p}V^2 - \frac{1}{16t^p}c(k)^2 + \frac{1}{64t^p}c(k)(5+7\delta(1-k)) - \frac{1}{1024t^p}(\delta(1-k)(22-23\delta(1-k))+17) \tag{4.21}$$

On the developers side, the platforms charge the Hotelling standard license fee less a term that is always positive. With more piracy, the latter term increases and produces a reduction in the license fee. The platforms charge a lower license fee to attract the largest number of software developers, also the consumer valuation of additional software increases with more piracy.

On the consumer side, when $\overline{c} < \frac{5}{8}(1-k+\delta)$ the platform increases the price and when $\overline{c} > \frac{5}{8}(1-k+\delta)$ it decreases the price.

[30] From section 4.2.5.

Now, the platform profits can either increase or decrease with more piracy. We observe separately the revenue that the platforms obtain from developers (π^{pd}) and the revenue from consumers (π^{pc}). If $\overline{c} < \frac{1}{8}(1-k+\delta)$ the platform profit can increase or decrease with more piracy; under this condition π^{pc} always increases with more piracy, but π^{pd} always decreases with more piracy. There are two possible cases; if $\triangle\pi^{pc} > \nabla\pi^{pd}$, the platform profit increases with more piracy and if $\triangle\pi^{pc} < \nabla\pi^{pd}$, the profit decreases with more piracy.

When $\frac{1}{8}(1-k+\delta) < \overline{c} < \frac{1}{4}(1-k+\delta)$, we always have that $\triangle\pi^{pc} < \nabla\pi^{pd}$, the reduction in revenue from developers is greater than the increase in the revenue from consumers. The platform profit decreases with more piracy.

Finally, if $\overline{c} > \frac{1}{4}(1-k+\delta)$, the platform profit always decreases with more piracy because, either we have that $\triangle\pi^{pc} < \nabla\pi^{pd}$ or that the two revenues decrease with more piracy.

In the following figure we have all the effects

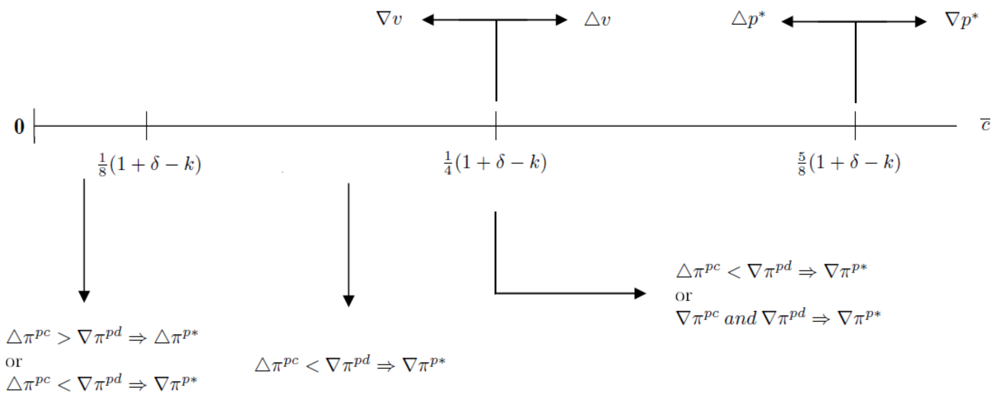

Figure 4.1: The effect of an increase in piracy when consumers use a multi-homing strategy.

Proposition 1. *If $\overline{c} < \frac{5}{8}(1-k+\delta)$, the price increases with more piracy and if $\overline{c} > \frac{5}{8}(1-k+\delta)$, the price decreases with more piracy. If $\overline{c} > \frac{1}{8}(1-k+\delta)$, the platform profit decreases with more piracy and if $\overline{c} < \frac{1}{8}(1-k+\delta)$, the platform profit can either increase or decrease with more piracy. The license fee always decreases* *with more piracy.*

Now, we analyze the effect of piracy on the share of consumers

At equilibrium $n_A^* = n_B^* = 1/2$ and u_A and u_B are given by

$$u_A^* = u_B^* = \frac{3 - 8c(k) + 16V + \delta(1-k)}{32t^p} \tag{4.22}$$

And the number of multi-homing consumers

$$(x_B - x_A) = \frac{3 - 8c(k) - 16t^p + 16V + \delta(1-k)}{16t^p} \tag{4.23}$$

The number of consumers multi-homing is higher with more piracy despite the fact that it can produce an increase in platform price.

The number of single-homing users is given by

$$x_A = 1 - x_B = \frac{8c(k) - 3 + 32t^p - 16V - \delta(1-k)}{32t^p} \tag{4.24}$$

The final demand of each platform when consumers can be multi-homing is the same. The number of single-homing consumers decreases with more piracy.

On the one hand, an increase in the quality of the copy produces an increment in the number of multi-homing consumers in $\frac{1-k}{16t^p}$. On the other hand, a decrease in the investment produces an increment in the number of multi-homing consumers in $\frac{\delta}{16t^p}$. Logically, the effect on the number of single-homing consumers is the opposite, before an increase in the quality of the copy the number of single-homing consumers decreases in $\frac{1-k}{16t^p}$ and were there an increase in the investment in an anti-piracy mechanism the number of single-homing consumers would increase in $\frac{\delta}{16t^p}$ in each platform.

The following assumption is required for the consistency of the above argument with $0 < x_A < x_B < 1$.

A.1. $\frac{1}{32}(3 - 8c(k) + 16V + \delta(1-k)) < t^p < \frac{1}{16}(3 - 8c(k) + 16V + \delta(1-k))$

See Appendix.

4.3.2. Scenario 2: Platforms Decisions with Single-homing

In this subsection we assume a market where users only buy from one of the platforms and the developers only work with one of the platforms. To illustrate, users would decide whether to buy a Mac or a PC and the developers only develop software for one of the two systems. Consumers also decide if they want to purchase the original software for a particular platform or prefer, if possible, to get an illegal copy.

Both sides of the market are single-homing. In this setting, fulfilled expectations on the consumers side imply that $n_A = \widehat{z}$ and $n_B = (1 - \widehat{z})$, and on the developers side $u_A = \widehat{x}$ and $u_B = (1 - \widehat{x})$. In equilibrium the consumers and the software developers expectations about the number of users and the number of developers in each platform are equal to the platforms market share on both sides.

Now, we analyze the effects of piracy in a single-homing market. To do it, we study what happens when the quality of the copy increases and when the investment in an anti-piracy mechanism decrease.

We know from the analysis of the third stage that $\lambda = \frac{1}{8}(1 + 3\delta(1-k))$ and $v = \frac{1}{4}(1 - \delta(1-k) - 4c(k))$, therefore the final values of $(p^*_{SH}, l^*_{SH}, \pi^*_{SH})$ are as follows

$$p^*_{SH} = t^p + c(k) + \frac{1}{4}(\delta(1-k) - 1) \tag{4.25}$$

$$l^*_{SH} = t^c - \frac{3}{8}\delta(1-k) - \frac{1}{8} \tag{4.26}$$

With more piracy the platforms reduce the license fee. An increase in piracy produces an increment $\frac{3}{8}(1 + \delta - k)$ in the consumer valuation of additional software. If the platforms reduce the license fee they will attract more developers and consequently more consumers. The developers have lost sales from the market where they sell the software, but that fall is compensated by a reduction in the license fee.

If $\overline{c} > \frac{1}{4}(1 - k + \delta)$ more piracy leads platforms to reduce the price because this level of marginal cost produces an increment in the developers additional valuation of each consumer (v) $\frac{1}{4}(k - 1 - \delta) + \overline{c} > 0$. Platforms can attract more developers if, first, they attract more consumers.

In contrast, if $\overline{c} < \frac{1}{4}(1 - k + \delta)$, such additional valuation decreases with more piracy , with more consumers the platforms will attract less developers, and they compensate this effect with higher prices.

At equilibrium $u^*_A = u^*_B = 1/2$ and $n^*_A = n^*_B = 1/2$, the two platforms share the market equally and make the following profit

$$\pi^*_{SH} = \frac{1}{2}(t^c + t^p) - \frac{1}{2}c(k) - \frac{1}{16}(16\delta(1-k) + 3) \tag{4.27}$$

Is easy to observe from the profit function that more piracy produces a reduction in platform profits. Although we consider the case where $\overline{c} > \frac{1}{4}(1 - k + \delta)$, the reduction in the license fee is so strong that in spite of the increase in price the final effect is a profit reduction. The decrease in the license fee produces a loss of $\frac{3}{16}(1+\delta-k)$ in the platform profit and the increase in the price a gain in $\frac{1}{8}(1+\delta-k)-\overline{c}$, the other term always exceeds the latter.

Proposition 2. *If $\bar{c} < \frac{1}{4}(1-k+\delta)$, the price increases with more piracy and if $\bar{c} > \frac{1}{4}(1-k+\delta)$, the price decreases with more piracy. The license fee and the platform profits always decrease with more piracy.*

4.3.3. Scenario 3: Platforms Decisions with Developers Multi-homing and consumers Single-homing

In this subsection we assume that the consumers side of the market is single-homing and the developers side is multi-homing. Developers can work with both platforms at the same time but consumers only buy from one of the two platforms. This is a rather common case as shown in the following example. Consider the console market with PlayStation and Xbox, the most common habit is that each user buys one of the two consoles. On the developers side we find a large variety of game developers that develop games for both consoles, for example Electronic Arts, Ubisoft Entertainment, Konami Corporation or Square Enix, to mention a few. Besides we find game developers that only work with one of the video consoles, for example Novarama, Bungie Studios, Game Freak, Insomniac Games...

When we consider that the developers employ a multi-homing strategy we find three types of developers. From zero to z_A the developers only work with platform A, from z_A to z_B developers work with both platforms and from z_B to one developers only work with platform B. We assume that $0 < z_A < z_B < 1$, and report the necessary condition below.

We know from the analysis of the third stage that $\lambda = \frac{1}{8}(1+3\delta(1-k))$ and $v = \frac{1}{4}(1-\delta(1-k)-4c(k))$, the final values of $(p^*_{DM}, l^*_{DM}, \pi^*_{DM})$:

$$p^*_{DM} = t^p - \frac{(4c(k) + \delta(1-k) - 1)(8c(k) - 5 - 16\beta - 7\delta(1-k))}{128t^c} \tag{4.28}$$

$$l^*_{DM} = \frac{1}{2}\beta - \frac{1}{4}c(k) - \frac{1}{32}(5\delta(1-k) - 1) \tag{4.29}$$

$$\pi^*_{DM} = \frac{1}{2}t^p + \frac{1}{4t^c}\beta^2 - \frac{1}{16t^c}c(k)^2 + \frac{1}{64t^c}c(k)(5+7\delta(1-k)) - \frac{1}{1024t^c}(\delta(1-k)(22-23\delta(1-k))+17) \tag{4.30}$$

Now, platform profits can either increase or decrease with more piracy. We observe separately the revenue that the platforms obtain from developers (π^{pd}) and the revenue from consumers (π^{pc}). In this case the difference with the previous section is that the license fee can increase or decrease with more piracy.

When $\bar{c} < \frac{1}{8}(1-k+\delta)$ the platform profit increases or decreases with more piracy, there are three possible cases; first, if $\triangle\pi^{pc} > \nabla\pi^{pd}$ the platform profit increases, second, if $\triangle\pi^{pc} < \nabla\pi^{pd}$ the platform profit decreases and the last one, when both revenues decrease and logically platform profit decreases.

If $\frac{1}{8}(1-k+\delta) < \bar{c} < \frac{1}{4}(1-k+\delta)$, the platform profit always decreases with more piracy because the two possibilities that we have are $\triangle\pi^{pc} < \nabla\pi^{pd}$ or that both revenues decrease.

Finally, if we consider that $\bar{c} > \frac{1}{4}(1-k+\delta)$, we have that $\triangle\pi^{pc} < \nabla\pi^{pd}$ or that both revenues decrease.

In the following figure we have all the effects

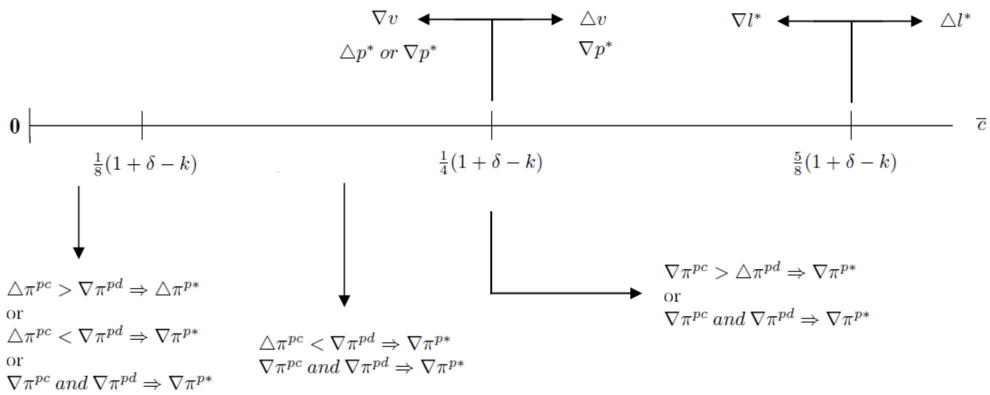

Figure 4.2: The effect of an increase in piracy when developers use a multi-homing strategy.

Proposition 3. *If $\bar{c} > \frac{5}{8}(1-k+\delta)$, the license fee increases with more piracy and if $\bar{c} < \frac{5}{8}(1-k+\delta)$, the license fee decreases with more piracy. If $\bar{c} > \frac{1}{4}(1-k+\delta)$, the price decreases with more piracy and if $\bar{c} < \frac{1}{4}(1-k+\delta)$, the price can increase or decrease with more piracy. If $\bar{c} > \frac{1}{8}(1-k+\delta)$, the platform profit decreases with more piracy and if $\bar{c} < \frac{1}{8}(1-k+\delta)$, the platform profit can increase or decrease with more piracy.*

We know that at equilibrium $u_A^* = u_B^* = 1/2$ and n_A^* and n_B^* at equilibrium are:

$$n_A^* = n_B^* = \frac{3 - 8c(k) + 16\beta + \delta(1-k)}{32t^c} \tag{4.31}$$

The number of single-homing developers is given by:

$$z_A = (1 - z_B) = \frac{8c(k) - 3 + 32t^c - 16\beta - \delta(1-k)}{32t^c} \tag{4.32}$$

And the number of multi-homing developers:

$$(z_B - z_A) = \frac{3 - 8c(k) - 16t^c + 16\beta + \delta(1-k)}{16t^c} \tag{4.33}$$

The following assumption is required for the consistency of the above argument with $0 < z_A < z_B < 1$.

A.3. $\frac{1}{32}(3 - 8c(k) + 16\beta + \delta(1-k)) < t^c < \frac{1}{16}(3 - 8c(k) + 16\beta + \delta(1-k))$

See Appendix.

4.3.4. Scenario 4: Platforms Decision with Multi-homing

In this subsection we consider that consumers and developers are multi-homing. We find three types of consumers , from zero to x_A the set of consumers that only buy from platform A, from x_A to x_B those that buy from both platforms and from x_B to one the set of consumers who only buy from platform B. And we assume that all developers are multi-homing. In this setting, we must have that $0 < x_A < x_B < 1$ and $z_A = 0$ and $z_B = 1$; we express the specific condition for this below.

We know from the analysis of the third stage that $\lambda = \frac{1}{8}(1 + 3\delta(1-k))$ and $v = \frac{1}{4}(1 - \delta(1-k) - 4c(k))$, the final values of (p_M^*, l_M^*, π_M^*) :

$$p_M^* = \frac{1}{2}V + \frac{1}{2}c(k) + \frac{1}{16}(5\delta(1-k) - 1) \tag{4.34}$$

$$l_M^* = \frac{1}{4t^p}c(k)^2 + \frac{1}{2}\beta + \frac{V}{32t^p} - \frac{1}{8t^p}c(k)(1+2V-\delta(1-k)) - \frac{1}{128t^p}\delta(1-k)(7+20V) - \frac{1}{256t^p}((5\delta^2(1-k)^2)-3) \tag{4.35}$$

$$\pi_M^* = \frac{1}{2}\beta + \frac{1}{4t^p}V(V - c(k)) + \frac{V}{32t^p}(\delta(1-k)+3) \tag{4.36}$$

In the following figure we have all the effects

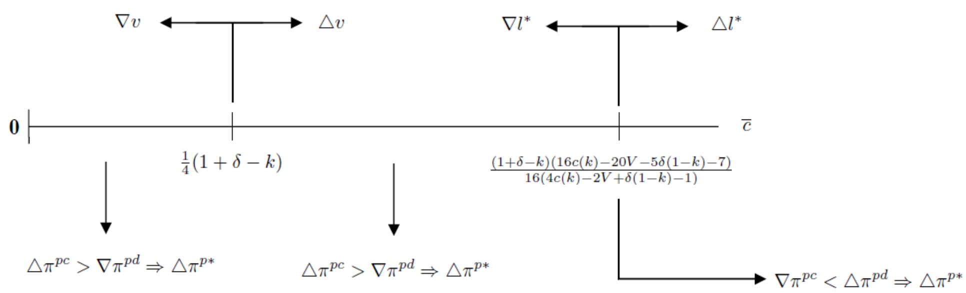

Figure 4.3: The effect of an increase in piracy when the whole the market is multi-homing.

Proposition 4. *If* $\overline{c} > \frac{1}{8}(5-5k+5\delta)$*, the price increases with more piracy and if* $\overline{c} < \frac{1}{8}(5-5k+5\delta)$*, the price decreases with more piracy. If* $\overline{c} > \frac{(1+\delta-k)(16c(k)-20V-5\delta(1-k)-7)}{16(4c(k)-2V+\delta(1-k)-1)}$*, the license fee increases with more piracy and if* $\overline{c} < \frac{(1+\delta-k)(16c(k)-20V-5\delta(1-k)-7)}{16(4c(k)-2V+\delta(1-k)-1)}$ *the license fee decreases with more piracy. Platform profits increase with more piracy.*

We assume that $n_A^* = n_B^* = 1$ and u_A^* and u_B^* at equilibrium are:

$$u_A^* = u_B^* = \frac{3-8c(k)+8V+\delta(1-k)}{16t^p} \tag{4.37}$$

The number of single-homing users is given by

$$x_A = (1-x_B) = \frac{8c(k)-3+16t^p-8V-\delta(1-k)}{16t^p} \tag{4.38}$$

and the multi-homing one is

$$(x_B - x_A) = \frac{3-8ck-8t^p+8V+\delta(1-k)}{8t^p} \tag{4.39}$$

The following assumption[31] is required for the consistency of the above argument

[31] The detailed calculation is available from the author

with $0 < x_A < x_B < 1$ and $z_A = 0$ and $z_B = 1$.

A.4. $\frac{1}{32}(3 - 8c(k) + 8V + \delta(1-k)) < t^p < \frac{1}{16}(3 - 8c(k) + 8V + \delta(1-k))$ and $t^c = \frac{64c(k)^2 + 128t^p\beta + (3+(1-k)\delta)^2 + 8V(3+\delta(1-k)) - 16c(k)(3+4V+\delta(1-k))}{256t^p}$

4.3.5. Comparing the developer utility

Now, we calculate the developers utility in the four types of scenarios to establish when developers do not want piracy and when they are not against.

The total utility[32] of a developer in the four scenarios is divided into two parts, the utility coming from consumers and the utility coming from platforms.

In scenario 1 the utility that the developers obtain from their sales to consumers is given by $\frac{1}{128t^p}(8c(k) - 16V - \delta(1-k) - 3)(4c(k) + \delta(1-k) - 1)$ and the utility from working with a platform is given by $\frac{1}{2}(2\beta - 3t^c) + \frac{3}{32t^p}c(k)(1 + 3\delta(1-k)) + \frac{1}{256t^p}(1 + 3\delta(1-k))(7 + 16V - 3\delta(1-k))$. Total developer utility is given by

$\pi^d_{CM} = \frac{1}{256t^p}(1 + 9\delta^2(k-1)^2 + 6\delta(1-k)(1 + 16v + 8V)) + \frac{1}{16t^p}(V + 2v(1 + 2v + 4V)) - \frac{1}{4}(4c(k) + 6t^c + \delta(1-k) - 1)$

From the consumers, the utility can increase or decrease with more piracy due to the existence of consumer's multi-homing, in the single-homing case this utility always decreases. From the platforms, the utility increases because the platforms establish a lower license fee independently of the presence of consumer's multi-homing.

When the market is single-homing, scenario 2, the utility that the developers obtain is decomposed into $\frac{1}{8}(1 - 4c(k) - \delta(1-k))$ from sales to consumers and $\frac{1}{8}(1 - 12t^c + 8\beta + 3\delta(1-k))$ from working with a platform. Total developer utility is given by

[32] From now on we will use the notation π^d

$\pi^d_{SH} = \frac{1}{8}(3 - 8c(k) - 12t^c + 4v + \delta(1-k))$

From the consumers, utility goes down with more piracy because developers establish a lower price to compensate for the effect of piracy. From the platforms, utility increases because platforms establish a lower license fee.

In scenario 3, the utility derived from the consumers, if they are single-homing is $\frac{1}{8}(1 - 4c(k) - \delta(1-k))$ and if they are multi-homing is $\frac{1}{4}(1 - 4c(k) - \delta(1-k))$. And the utility derived from the platforms, if they are single-homing $\frac{1}{16}(1 - 16t^c + 16\beta + 3\delta(1-k))$ and if they are multi-homing $\frac{1}{16}(8c(k) - 1 - 16t^c + 16\beta + 5\delta(1-k))$. Total developer utility is given by

$\pi^d_{DM} = \frac{1}{16}(3 - 8c(k) - 16t^c + 16\beta + \delta(1-k))$.

Now, since developers may be multi-homing we have two different utilities, it depends on whether the developers are single-homing or multi-homing. As we can observe the multi-homing developers obtain the double of utility than the single-homing from the direct sales to the consumers. But this effect is compensated in the utility that they obtain from the platforms and all the developers obtain the same final utility.

When all the market is multi-homing the utility that the developers obtain from their direct sales to the consumers is given by $\frac{1}{32t^p}(8c(k) - 8V - \delta(1-k) - 3)(4c(k) + \delta(1-k) - 1)$ and the usefulness of working with a platform is given by $\frac{1}{2}\beta + \frac{1}{16t^p}c(k)(7 + 12V - 3\delta(1-k)) + \frac{1}{256t^p}(3 + \delta(1-k))(9\delta(1-k) - 5) - \frac{1}{32t^p}V(5 - 9\delta(1-k)) - \frac{3}{4t^p}c(k)^2$. The total developer utility is given by

$\pi^d_{M} = \frac{1}{4t^p}c(k)^2 + \frac{1}{2}\beta - \frac{1}{16t^p}c(k)(3 + 4V + \delta(1-k)) + \frac{1}{32t^p}V(3 + \delta(1-k)) + \frac{1}{256t^p}((1-k)\delta + 3)^2$

When whole the market is multi-homing, we have the same results in developers utility that in the first and the second scenario.

Except when consumers can be multi-homing (in which case developer utility can be greater or lesser). The developers always obtain a higher utility with more piracy. If we are in a market with piracy, the developers obtain a higher utility with more piracy.

4.3.6. Summary

We next summarize the effect of piracy in the four scenarios earlier considered. Let us first look at the effect of an increase in the quality of the copies, δ, and secondly at the effect of an increase in the anti-piracy mechanism, k.

Increases in δ

	p^*	l^*	π^*	π^{d*}
Cosumers Multi − homing	(+)	(−)	(+/−)	(+/−)
Single − homing	(+)	(−)	(−)	(+)
Developers Multi − homing	(+/−)	(−)	(+/−)	(+)
Multi − homing	(+)	(−)	(+)	(+)

Increases in k

	p^*	l^*	π^*	π^{d*}
Cosumers Multi − homing	(+/−)	(+)	(+/−)	(+/−)
Single − homing	(+/−)	(+)	(+)	(−)
Developers Multi − homing	(+/−)	(+/−)	(+/−)	(−)
Multi − homing	(+/−)	(+/−)	(−)	(−)

We observe that, when there is an increase the quality of the copy, the license fee decreases in all cases. Developers obtain lower revenue from the sales to consumers and platforms compensate this effect with reductions in the license fee to attract the largest possible variety of developers. In contrast, the price almost always increases because platforms compensate the reductions in the license fee with higher prices. When the whole the market follows a multi-homing strategy platforms always obtain a higher profit, the effect of the price increases is greater than the reduction in the license fee.

4.4. Welfare Analysis

Now, we are interested in assessing the welfare effects on piracy under the various scenarios

4.4.1. Scenario 1: Consumers Multi-homing and Developers Single-homing

The expressions for consumer surplus, CS, and developer surplus, DS, in equilibrium are:

$$CS^*_{CM} = \frac{1}{1024t^p}(3 - 8c(k) + 16V + \delta(1-k))^2 \tag{4.40}$$

$$DS^*_{CM} = \frac{1}{4t^p}c(k)^2 + \frac{3}{16t^p}V + \frac{1}{4}(4\beta - 5tc) + \frac{1}{128t^p}\delta(1-k)(7+8V) - \frac{1}{4t^p}c(k)(1+2V+\delta(1-k)) + \frac{1}{256tp}(13 - 11\delta^2(1-k)^2) \tag{4.41}$$

Adding up platform profits from the previous section, total welfare can be expressed as follows

$$W^*_{CM} = \frac{3}{16t^p}c(k)^2 - \frac{1}{4}(t^c - 4\beta) - \frac{3}{64t^p}c(k)(3 + 16V + \delta(1-k) + \frac{1}{1024t^p}3(3 + 16V + \delta(1-k))^2 \tag{4.42}$$

The next proposition provides some comparative statics results

Proposition 5. *Consumer surplus and total welfare increase with more piracy. If $\overline{c} > \frac{1}{16}(1-k+\delta)$, developer surplus increases with more piracy and if $\overline{c} < \frac{1}{16}(1-k+\delta)$, developer surplus can increase or decrease with more piracy.*

4.4.2. Scenario 2: Platforms Decision with Single-homing

The expressions for CS and DS in equilibrium are:

$$CS^*_{SH} = \frac{1}{16}(5 - 16c(k) - 20t^p + 16V - \delta(1-k)) \tag{4.43}$$

$$DS^*_{SH} = \frac{1}{4}(1 - 5t^c + 4\beta + \delta(1-k)) \tag{4.44}$$

Adding up platform profits, total welfare can be expressed as

$$W^*_{SH} = \frac{1}{16}(3 - 4t^c - 4t^p + 16V + 16\beta + \delta(1-k)) \tag{4.45}$$

The next proposition provides some comparative statics results

Proposition 6. *If $\overline{c} > \frac{1}{16}(1-k+\delta)$ consumer surplus increases with more piracy and if $\overline{c} < \frac{1}{16}(1-k+\delta)$ consumer surplus decreases with more piracy. Developer surplus and total welfare increase with more piracy.*

4.4.3. Scenario 3: Platforms Decision with Developers Multi-homing and consumers Single-homing

The expressions for CS and DS in equilibrium are:

$$CS^*_{DM} = \frac{1}{4t^c}c(k)^2+\frac{3}{16t^c}\beta+\frac{1}{4}(4V-5tc)+\frac{1}{128t^c}\delta(1-k)(7+8\beta)-\frac{1}{4t^c}c(k)(1+2\beta+\delta(1-k))+\frac{1}{256t^c}(13-11\delta^2(1-k)^2) \tag{4.46}$$

$$DS^*_{DM} = \frac{1}{1024t^c}(3-8c(k)+16\beta+\delta(1-k))^2 \tag{4.47}$$

Adding up platform profits from the previous section, total welfare can be expressed as follows

$$W^*_{DM} = 2t^p-\frac{1}{64t^c}(12c(k)^2+c(k)(16\beta-17)-2\beta(3+40\beta))+\frac{1}{512t^c}\delta(1-k)(216c(k)+16\beta-41)+\frac{1}{1024t^c}(93\delta^2(1-k)^2-59) \tag{4.48}$$

The next proposition provides some comparative statics results

Proposition 7. *If $\bar{c} > \frac{1}{16}(1-k+\delta)$, consumer surplus increases with more piracy and if $\bar{c} < \frac{1}{16}(1-k+\delta)$, consumer surplus can increase or decrease with more piracy. Developer surplus increases with more piracy. Total welfare can increase or decrease with more piracy.*

4.4.4. Scenario 4: Platforms Decision with Multi-homing

The expressions for CS and DS in equilibrium are:

$$CS^*_{M} = \frac{1}{256t^p}(3-8c(k)+8V+\delta(1-k)^2 \tag{4.49}$$

$$DS^*_M = \frac{1}{2}\beta + \frac{1}{4t^p}c(k)^2 + \frac{1}{256t^p}(3+\delta(1-k))^2 + \frac{1}{32t^p}V(3+\delta(1-k)) - \frac{1}{16t^p}c(k)(3+4V+\delta(1-k)) \quad (4.50)$$

Adding up platform profits from the previous section, total welfare can be expressed as follows

$$W^*_M = \frac{1}{4t^p}c(k)^2 + \frac{V^2}{tp} + \frac{5}{2}\beta + \frac{1}{256t^p}(3+\delta(1-k))^2 + \frac{5}{32t^p}V(3+\delta(1-k)) - \frac{1}{16t^p}c(k)(3+20V+\delta(1-k)) \quad (4.51)$$

The next proposition provides some comparative statics results

Proposition 8. *Consumer surplus, developer surplus and total welfare increase with more piracy.*

The main result that we observe is that total welfare increases with more piracy, except when developers are multi-homing; in this case the effect is ambiguous. The increase in developer and consumer surplus compensate the possible effect of the reduction in platform profits.

We summarize the main effects in the following table when the quality of the copy increases and when the investment in an anti-piracy mechanism increases:

	CS^*		DS^*		π^*		W^*
$Consumers\, Multi-homing$	$(+)$	$+$	$(+/-)$	$+$	$(+/-)$	$\Rightarrow$	$(+)$
$Single-homing$	$(+/-)$	$+$	$(+)$	$+$	$(-)$	$\Rightarrow$	$(+)$
$Developers\, Multi-homing$	$(+/-)$	$+$	$(+)$	$+$	$(+/-)$	$\Rightarrow$	$(+/-)$
$Multi-homing$	$(+)$	$+$	$(+)$	$+$	$(+)$	$\Rightarrow$	$(+)$

4.5. What can a social planner do to improve welfare?

Now consider the case where a social planner can mediate with the piracy problem. To do it, the authority will regulate the developers market. We focus our attention on the market where software developers and consumers interact.

The social planner will choose the software prices that maximize total welfare defined by the sum of consumer and developer surplus.

Thus, consumer surplus is given by $CS = \int_0^{\theta} \theta\delta(1-k)d\theta + \int_{\theta}^{1} \theta - p^o d\theta$ and developer surplus by $DS = (1-\widehat{\theta})p^o - c(k)$. We may then write total welfare in this market as $W = \frac{1}{2} - \frac{p^{o2}}{2-2\delta(1-k)} - c(k)$. The social planner chooses the final price that maximizes W with piracy. We obtain that $p^{*ow} = 0$, that is, the social planner establishes a price that blocks piracy because the cost of the original software is the same as that of the illegal version. All consumers buy the software and the demand for the original software is $D^o = 1$.

Now, with this regulation consumer surplus for each software product is $\frac{1}{2}$ and developer surplus is $-c(k)$ but developers need not invest in any anti-piracy mechanism, therefore developer surplus is zero because $c(0) = 0$. In conclusion, we have: $\lambda = \frac{1}{2}$ and $v = 0$.

Next, we proceed to examine all scenarios when the social planner regulates the software market for $\lambda = \frac{1}{2}$ and $v = 0$. The final values of $(\widetilde{p}^*, \widetilde{l}^*, \widetilde{\pi}^*)$[33] in the four types of scenarios.

4.5.1. Scenario 1: Consumers Multi-homing and Developers Single-homing

Platforms establish the profit maximizing price and license fee as follows

$$\widetilde{p}^*_{CM} = \frac{1}{8}(1 + 4V - 2v) \tag{4.52}$$

[33] We use an upper bar to identify a variable in the social optimum setting.

$$\widetilde{l}^*_{CM} = \frac{16t^c t^p - 6v - 4V - 1}{16t^p} \tag{4.53}$$

Both platforms make the following profit

$$\widetilde{\pi}^*_{CM} = \frac{32t^c t^p - 4v(3+v) + 16V^2 - 1}{64t^p} \tag{4.54}$$

The number of single-homing consumers and the multi-homing consumers is given by

$$\widetilde{x}_a = (1 - \widetilde{x}_b) = \frac{8t^p - 2v - 4V - 1}{8t^p} \tag{4.55}$$

$$(\widetilde{x}_b - \widetilde{x}_a) = \frac{1 - 4t^p + 2v + 4V}{4t^p} \tag{4.56}$$

And total welfare is equal to

$$\widetilde{W}^*_{CM} = \frac{3(1 + 2v + 4V)^2 - 16t^c t^p}{64t^p} \tag{4.57}$$

We can now compare the results with regulation and without regulation

The price $p^*_{CM} - \widetilde{p}^*_{CM} < 0$

The license fee $l^*_{CM} - \widetilde{l}^*_{CM} \lesseqgtr 0$

The platforms profit $\pi^*_{CM} - \widetilde{\pi}^*_{CM} \lesseqgtr 0$

The welfare $W^*_{CM} - \widetilde{W}^*_{CM} < 0$

The number of multi-homing users is higher with the social planner regulation.

Regulation results in a welfare improvement. If a social planner regulates the consumers and developers market then the price paid by consumers and platforms profit are higher, whereas the license fee paid by developers is lower.

4.5.2. Scenario 2: Consumers and Developers Single-homing

Platforms establish the profit maximizing price and license fee as follows

$$\widetilde{p}^{*}_{SH} = t^{p} - v \tag{4.58}$$

$$\widetilde{l}^{*}_{SH} = t^{c} - \frac{1}{2} \tag{4.59}$$

Both platforms make the following profit

$$\widetilde{\pi}^{*}_{SH} = \frac{1}{4}(2(t^{c} + t^{p} - v) - 1) \tag{4.60}$$

Total welfare with single-homing and regulation is given by

$$\widetilde{W}^{*}_{SH} = \frac{1}{4}(1 - t^{c} - t^{p} + 2v) + V \tag{4.61}$$

We can now compare the results with regulation and without regulation

The price $p^{*}_{SH} - \widetilde{p}^{*}_{SH} < 0$

The license fee $l^{*}_{SH} - \widetilde{l}^{*}_{SH} > 0$

The platforms profit $\pi^{*}_{SH} - \widetilde{\pi}^{*}_{SH} > 0$

The welfare $W^{*}_{SH} - \widetilde{W}^{*}_{SH} < 0$

4.5.3. Scenario 3: Developers Multi-homing and Consumers Single-homing

Platforms establish the profit maximizing price and license fee as follows

$$\widetilde{p}^{*}_{DM} = \frac{8t^{c}t^{p} - v(3 + 2v)}{8t^{c}} \tag{4.62}$$

$$\widetilde{l}^*_{DM} = \frac{1}{8}(2v - 1) \tag{4.63}$$

Both platforms make the following profit

$$\widetilde{\pi}^*_{DM} = \frac{32t^c t^p - 4v(3 + v) + 16V^2 - 1}{64t^c} \tag{4.64}$$

The number of single-homing and multi-homing developers is given by

$$\widetilde{z}_a = (1 - \widetilde{z}_b) = \frac{8tc - 2v - 1}{8tc} \tag{4.65}$$

$$(\widetilde{z}_b - \widetilde{z}_a) = \frac{1 - 4t^c + 2v}{4t^c} \tag{4.66}$$

Total welfare is equal to

$$\widetilde{W}^*_{CM} = \frac{128t^c t^p - 4v(11 + 3v) - 3}{64tc} \tag{4.67}$$

We can now compare the results with regulation and without regulation

The price $p^*_{DM} - \widetilde{p}^*_{DM} \lesseqgtr 0$

The license fee $l^*_{DM} - \widetilde{l}^*_{DM} > 0$

The platforms profit $\pi^*_{DM} - \widetilde{\pi}^*_{DM} \lesseqgtr 0$

The welfare $W^*_{DM} - \widetilde{W}^*_{DM} \lesseqgtr 0$

The number of multi-homing developers is higher without the social planner regulation.

We now find that regulation results in a welfare decrease. A social planner that regulates the consumers and developers market produces an increase in the license fee paid by developers; the effect on prices and platform profits is ambiguous.

4.5.4. Scenario 4: Platforms Decision with Multi-homing

Platforms establish the profit maximizing price and license fee as follows

$$\widetilde{p}_M^* = \frac{1}{4}(1 + 2V) \tag{4.68}$$

$$\widetilde{l}_M^* = \frac{8t^p\beta - 2V - 1}{16t^p} \tag{4.69}$$

Both platforms make the following profit

$$\widetilde{\pi}_M^* = \frac{V + 2V^2 + 4t^p\beta}{8t^p} \tag{4.70}$$

The number of multi-homing consumers is given by[34]

$$\widetilde{x}_a = (1 - \widetilde{x}_b) = \frac{4t^p + 2V - 1}{4t^p} \tag{4.71}$$

$$(\widetilde{x}_b - \widetilde{x}_a) = \frac{1 - 2t^p + 2V}{2t^p} \tag{4.72}$$

Total welfare is equal to

$$\widetilde{W}_M^* = \frac{1 - 8t^c t^p + 6V + 8V^2 + 24t^p\beta}{8t^p} \tag{4.73}$$

We can now compare the results with regulation and without regulation

The price $p_M^* - \widetilde{p}_M^* < 0$

The license fee $l_M^* - \widetilde{l}_M^* \gtreqless 0$

The platforms profit $\pi_M^* - \widetilde{\pi}_M^* < 0$

The total welfare $W_M^* - \widetilde{W}_M^* \gtreqless 0$

[34] Recall that all the developers are multi-homing $\widetilde{z}_A = 0$ and $\widetilde{z}_B = 1$.

4.5.5. Comparing developer profits with and without regulation

A social planner that regulates the market makes piracy disappear. In this subsection we analyze developer profits with and without piracy to conclude that the developers are usually better off without piracy but, when it exists, their profits are higher with more piracy.

When we consider a market where consumers can be multi-homing the effect on developer profits is ambiguous. With regulation profits are $\widetilde{\pi}^d_{CM} = \beta - \frac{3t^c}{2} + \frac{1+4V}{16t^p}$. This profit level can be higher or lower than without regulation depending on certain conditions.

With a single-homing market without regulation developer profits can be seen in section 4.3.5. When a social planner regulates the market these profits are $\widetilde{\pi}^d_{SH} = \beta + \frac{1}{2} - \frac{3t^c}{2}$. With a simple subtraction we obtain $\pi^d_{SH} - \widetilde{\pi}^d_{SH} = \frac{1}{4}(\delta(1-k) - 1 - 2c(k))$ that is always negative; this means that software developers obtain a higher profit without piracy in the market.

Another case is when developers can be multi-homing. In this setting developer profits with regulation are $\widetilde{\pi}^d_{DM} = \beta + \frac{1}{4} - t^c$ and without regulation are given in section 4.3.5.. The difference between profits $\pi^d_{DM} - \widetilde{\pi}^d_{DM} = \frac{1}{16}(\delta(1-k) - 1 - 8c(k))$, this difference is always negative. As in the single-homing case, the software developers obtain a higher profit without piracy.

The last case is when all the market is multi-homing. Developer profits when a social planner regulates is given by $\widetilde{\pi}^d_{M} = \frac{1+2V+8t^p\beta}{16tp}$. It can be checked that developer profits is always higher with regulation.

The developers always obtain a higher utility with more piracy, but if a social planner regulates the market (making piracy disappear), the developers always obtain a higher utility without piracy[35]. If software developers cannot prevent piracy completely, they should invest in improving their products and attempt to exploit the potential advantages of piracy rather than investing in anti-piracy mechanisms (if you cannot beat them, join them). In this regard, we can find developers who confess that piracy may be good ("except when consumers can be multi-homing. The developers always obtain a higher utility with more piracy") and other developers who want to prevent it completely ("if piracy disappears, developers always obtain a higher utility"). Software developers obtain higher utilities without piracy, but like or not, we live in a world with piracy and, with this, they obtain a higher utility with more piracy.

4.6. Conclusion

This chapter has studied the impact of software piracy in a two-sided market, in particular, its effect on the platforms and profit behaviour since platforms compete to attract consumers and software developers.

In our approach piracy appears in the market where consumers and developers interact; consumers can buy the original software or can get an illegal copy. Furthermore, the software developers have the possibility to invest in an anti-piracy mechanism. We wish to establish whether developers win or lose with piracy.

To this end we have developed a model with platforms, software developers and consumers. On the one hand, consumers and developers decide which platform to join in four different scenarios. On the other hand, consumers decide whether to buy the original software or obtain an illegal version, taking into account the quality of the copy and if the developers invest to avoid piracy.

There appear externalities that transmit across markets. This means that whatever may happen on one side of the market will eventually affect the other side.

[35]Except when consumers can be multi-homing because profit can be higher or lower than without regulation depending on certain conditions

Consumers want to purchase from the platform with a large variety of software developers and software developers want to develop software for the platform with a larger user base.

Our results support what we actually observe in these markets. Many developers are positioned against piracy, but there is a range of developers that are not against. We also identify a conflict between platforms and software developers in the anti-piracy strategies because we have found cases where developers gain with more piracy, but platforms lose profits.

When consumers use a multi-homing strategy and software developers a single-homing strategy, developers may gain or lose with more piracy.

In the second scenario, software developers obtain a higher utility with more piracy despite it affecting negatively the sales from original software; such a negative effect is compensated by a lower license fee. Platforms obtain lower profit with more piracy because they reduce the license fee to try and attract the largest number of developers.

In third scenario, the results are more ambiguous. When the developers side can be multi-homing we find conditions under which platforms might not be against piracy. Software developers gain with more piracy.

In the last scenario is the only one scenario where software developers and platforms gain with more piracy.

Finally, we have analyzed total welfare and we have calculated the social optimum to explore the role of a social planner with the piracy problem.

Total welfare increases with more piracy, except when developers are multi-homing; in this case the effect is ambiguous. The increase in developer and consumer surplus compensate the possible effect of the reduction in platform profits. When developers follow a multi-homing strategy the effect of piracy in total welfare depends on the marginal cost of the investment in anti-piracy, total welfare can increase or decrease with more piracy. A social planner should intervene when consumers use a multi-homing strategy and developers a single-homing strategy, and when the market is single-homing, in the last two scenarios depend on several conditions.

Finally, the developers always obtain a higher utility with more piracy, but if a social planner regulates the market (making piracy disappear), the developers always obtain a higher utility without piracy.

4.7. Appendix

4.7.1 Proof of user surplus and the developer surplus

Consumer surplus in the software market is given by

$$CS = \int_0^{\widehat{\theta}} \theta\delta(1-k)d\theta + \int_{\widehat{\theta}}^1 \theta - p^o d\theta = \frac{1}{2}(\widehat{\theta}(1-\widehat{\theta}) + \delta(1-k)(1-\widehat{\theta}(1-\widehat{\theta}))$$

We know that in equilibrium $\widehat{\theta} = \frac{1}{2}$ and then consumer surplus becomes

$$CS = \frac{1}{8}(1+3\delta(1-k)) = \lambda$$

Such consumer surplus level corresponds with the parameter λ, the network parameter governing the benefit users get from each additional software developer.

Developer surplus is given by the developers profits in this market

$$\pi^d = \frac{1}{4}(1-4c(k)-\delta(1-k)) \geqslant 0$$

where the parameter $c(k) < \frac{1+\delta(k-1)}{4}$ guarantees that the developers obtain a positive profit in the market; if we assumed that $c(k) > \frac{1+\delta(k-1)}{4}$ no developer would invest in the anti-piracy mechanism because $\pi^d = \frac{1}{4}(1-4c(k)-\delta(1-k)) < 0$ and the investment would be zero.

4.7.2 Proof of Assumption 0

Assumption A.0 follows from the second order condition for profit maximization. We calculate the second order condition in the four types of scenarios. We obtain more than one condition but the stronger condition is given by $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$.

4.7.2.1. When consumers side can be multi-homing

The second order condition for the consumer side is given by

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{8tc}{v(1+3\delta(1-k))8 - t^c t^p} - \frac{1}{t^p}$$

We must have that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, this condition is only satisfied if $\frac{1}{t^p} > \frac{8tc}{v(1+3\delta(1-k))8 - t^c t^p}$ and this happen when $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$ and when $0 < t^p < \frac{v(1+3\delta(1-k))}{16t^c}$.

The second order condition for the developer side is given by

$$\frac{\partial^2 \pi_i}{\partial l_i^2} = \frac{8tc}{v(1+3\delta(1-k))8 - t^c t^p}$$

We must have that $\frac{\partial^2 \pi_i}{\partial l_i^2} < 0$, this condition is met if $t^c t^p > v(1+3\delta(1-k))8$ and this happen when $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$.

4.7.2.2. When both sides of the market are single-homing

The second order condition for the consumer side is given by

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{2t^p}{\frac{1}{4}v(1+3\delta(1-k)) - 2t^c t^p}$$

We must have that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, this condition is only satisfied if $2t^c t^p > \frac{1}{4}v(1 + 3\delta(1-k))$ and this happen when $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$.

The second order condition for the developer side is the same, consequently the condition is the same.

4.7.2.3. When developers side can be multi-homing

The second order condition for the consumers side is given by

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{8tc}{v(1+3\delta(1-k))8 - t^c t^p}$$

We must have that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, this condition is only met if $t^c t^p > v(1+3\delta(1-k))8$ and this happen when $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$.

The second order condition for the developer side is given by

$$\frac{\partial^2 \pi_i}{\partial l_i^2} = \frac{8tc}{v(1+3\delta(1-k))8 - t^c t^p} - \frac{1}{t^p}$$

We must have that $\frac{\partial^2 \pi_i}{\partial l_i^2} < 0$, this condition is only met if $\frac{1}{t^p} > \frac{8tc}{v(1+3\delta(1-k))8-t^c t^p}$ and this happen when $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$ and when $0 < t^p < \frac{v(1+3\delta(1-k))}{16t^c}$.

The only condition that is satisfied in the four cases and in both sides of the market is $t^p > \frac{(1+3\delta(1-k))(1-4c(k)-\delta(1-k))}{32t^p}$, therefore we impose this condition which guarantees that the second order conditions will be fulfilled.

4.7.3. Proof of consumers multi-homing

We consider the possibility of consumers multi-homing, we calculate the indirect utility of a consumer $\widehat{x}$ who buys platform A or platform B or both platforms.

A consumer who is indifferent between buy platform A or buy both platforms is given by $U(x)_A = U(x)_A + U(x)_B$, $V - t^p x_A - p_A + \widehat{z}\lambda = 2V + \lambda - p_A - p_B - t^p$. We solve x_A, and we obtain $x_A = \frac{\lambda(\widehat{z}-1)+p_b+t^p-V}{t^p}$.

A consumer who is indifferent between buy platform B or buy both platforms is given by $U_B = U_A + U_B$, $V - t^p(1-x_B) - p_B + (1-\widehat{z})\lambda = 2V + \lambda - p_A - p_B - t^p$. We solve x_B, and we obtain $x_B = \frac{\lambda\widehat{z}+V-p_A}{t^p}$.

The indifferent developer is $\beta - l_A - t^c z + x_B v = \beta - l_B - t^c(1-z) + (1-x_A)v$, we solve for $\widehat{z}$ and obtain the indifferent developer $\widehat{z} = \frac{l_B - l_A + t^c + v(x_A + x_B - 1)}{2t^c}$. As we know the values of x_A and x_B, substituting these values into the equation we have

$$\widehat{z} = \frac{l_B - l_A + t^c + v((\frac{\lambda(\widehat{z}-1)+p_B+t^p-V}{t^p}) + (\frac{\lambda\widehat{z}+V-p_A}{t^p}) - 1)}{2t^c}$$

We solve for $\widehat{z}$ and we have the indifferent developer

$$\widehat{z} = \frac{t^p(t^c + l_B - l_A) - v(\lambda + p_A - p_B)}{2(t^c t^p - \lambda v)}$$

We can solve the indifferent consumers by substituting the value of $\widehat{z}$ and we obtain

$$x_A = \frac{\lambda(\widehat{z} - 1) + p_B + t^p - V}{t^p} = \frac{\lambda(\frac{t^p(t^c + l_B - l_A) - v(\lambda + p_A - p_B)}{2(t^c t^p - \lambda v)} - 1) + p_B + t^p - V}{t^p} =$$

$$= \frac{\lambda^2 v + 2t^c t^p(p_B + t^p - V) - \lambda(v(p_A + p_B) + t^p(l_A - l_B + t^c + 2v) - 2vV)}{2(t^c t^p - \lambda v)}$$

$$x_B = \frac{\lambda\widehat{z} + V - p_a}{t^p} = \frac{\lambda\frac{t^p(t^c + l_B - l_A) - v(\lambda + p_A - p_B)}{2(t^c t^p - \lambda v)} + V - p_A}{t^p} =$$

$$= \frac{\lambda^2 v + \lambda((l_B - l_A + t^c)t^p + v(p_A + p_B - 2V)) + 2t^c t^p(V - p_A)}{2(t^c t^p - \lambda v)}$$

4.7.4. Proof of Assumption 1

To be consistent with consumer side multi-homing we need $0 < x_A < x_B < 1$, with the final values in equilibrium we can rewrite this condition as

$$0 < \frac{8c(k) - 3 + 32t^p - 16V - \delta(1-k)}{32t^p} < \frac{3 - 8c(k) + 16V + \delta(1-k)}{32t^p} < 1$$

We focus first on the left hand of the inequalities. Rearranging terms we obtain $0 < (x_B - x_A)$:

$$0 < \frac{3 - 8c(k) - 16t^p + 16V + \delta(1-k)}{16t^p}$$

$$0 < 3 - 8c(k) - 16t^p + 16V + \delta(1-k)$$

$$16t^p < 3 - 8c(k) + 16V + \delta(1-k)$$

$$t^p < \frac{1}{16}(3 - 8c(k) + 16V + \delta(1-k))$$

Now, we focus on the right hand side to obtain $(x_B - x_A) < 1$

$$\frac{3 - 8c(k) - 16t^p + 16V + \delta(1-k)}{16t^p} < 1$$

$$3 - 8c(k) - 16t^p + 16V + \delta(1-k) < 16t^p$$

$$3 - 8c(k) + 16V + \delta(1-k) < 32t^p$$

$$t^p > \frac{1}{32}(3 - 8c(k) + 16V + \delta(1-k))$$

The condition $0 < x_A < x_B < 1$ is satisfied when $\frac{1}{32}(3-8c(k)+16V+\delta(1-k)) < t^p < \frac{1}{16}(3 - 8c(k) + 16V + \delta(1-k))$

4.7.5. Proof of developers multi-homing

When considering the possibility of developers multi-homing, the indirect utility of a developer z who works with platform A or platform B or both platforms is given by

$$U(z) = \begin{cases} \beta - t^c z_A - l_A + u_A v & \text{if joining A} \\ \beta - t^c(1 - z_B) - l_B + u_B v & \text{if joining B} \\ 2\beta + v - p_A - p_B - t^c & \text{if joining both platforms} \end{cases}$$

A developer who is indifferent between working with platform A or working with both platforms is given by$U(z)_A = U(z)_A + U(z)_B$, specifically we have that $\beta - t^c z_A - l_A + \widehat{x}v = 2\beta + v - l_A - l_B - t^c$. We solve for z_A, and we obtain $z_A = \frac{l_B + t^c + v(\widehat{x}-1) - \beta}{t^c}$.

A developer who is indifferent between working with platform B or working with both platforms is given by $U(z)_B = U(z)_A + U(z)_B$, specifically we have that $\beta - t^c(1 - z_B) - l_B + (1 - \widehat{x})v = 2\beta + v - l_A - l_B - t^c$. We solve for z_B, and we obtain $z_B = \frac{v\widehat{x}+\beta-l_A}{t^c}$.

The indifferent consumer between A and B is given by $U_A = U_B$,
$V - t^p x - p_A + z_B\lambda = V - t^p(1 - x) - p_B + (1 - z_A)\lambda$. We solve for $\widehat{x}$ and obtain the indifferent consumer $\widehat{x} = \frac{p_B - p_A + t^p + \lambda(z_A + z_B - 1)}{2t^p}$. As we know the values of z_A and z_B, substituting these values into the equation we have

$$\widehat{x} = \frac{p_B - p_A + t^p + \lambda((\frac{l_B + t^c + v(\widehat{x}-1)-\beta}{t^c}) + (\frac{v\widehat{x}+\beta-l_A}{t^c}) - 1)}{2t^p}$$

We solve for $\widehat{x}$ and we have the indifferent consumer as a function of prices and license fees $\widehat{x} = \frac{t^c(t^p+p_B-p_A)-\lambda(l_A-l_B+v)}{2(t^c t^p-\lambda v)}$.

We can solve the indifferent type of developers by substituting the value of $\widehat{x}$ to obtain

$$z_A = \frac{l_B + t^c + v(\widehat{x} - 1) - \beta}{t^c} = \frac{l_B + t^c + v(\frac{t^c(t^p+p_B-p_A)-\lambda(l_A-l_B+v)}{2(t^c t^p-\lambda v)} - 1) - \beta}{t^c} =$$

$$= \frac{2t^{c2}t^p - t^c v(2\lambda + t^p + p_A - p_B) + l_B(2t^c t^p - \lambda v) - 2t^c t^p \beta + \lambda v(v + 2\beta - l_A)}{2t^c(t^c t^p - \lambda v)}$$

$$z_B = \frac{v\widehat{x} + \beta - l_A}{t^c} = \frac{v(\frac{t^c(t^p+p_B-p_A)-\lambda(l_A-l_B+v)}{2(t^c t^p-\lambda v)}) + \beta - l_A}{t^c} =$$

$$= \frac{l_A(\lambda v - 2t^c t^p) + \lambda v(l_B - v - 2\beta) + tcv(p_B - p_A + t^p) + 2t^p t^c \beta}{2t^c(t^c t^p - \lambda v)}$$

4.7.6. Proof of Assumption 3

To be consistent with consumers multi-homing we need $0 < z_A < z_b < 1$. With the final values in equilibrium, the ordering becomes

$$0 < \frac{8c(k) - 3 + 32t^c - 16\beta - \delta(1-k)}{32t^c} < \frac{3 - 8c(k) + 16\beta + \delta(1-k)}{32t^c} < 1$$

We focus first on the left hand of the inequalities. Rearranging terms we obtain $0 < (z_B - z_A)$:

$$0 < \frac{3 - 8c(k) - 16t^c + 16\beta + \delta(1-k)}{16t^c}$$

$$0 < 3 - 8c(k) - 16t^c + 16\beta + \delta(1-k)$$

$$16t^c < 3 - 8c(k) + 16\beta + \delta(1-k)$$

$$t^c < \frac{1}{16}(3 - 8c(k) + 16\beta + \delta(1-k))$$

Now, we focus on the right hand side to obtain $(z_B - z_A) < 1$

$$\frac{3 - 8c(k) - 16t^c + 16\beta + \delta(1-k)}{16t^c} < 1$$

$$3 - 8c(k) - 16t^p + 16\beta + \delta(1-k) < 16t^c$$

$$3 - 8c(k) + 16\beta + \delta(1-k) < 32t^c$$

$$t^c > \frac{1}{32}(3 - 8c(k) + 16\beta + \delta(1-k))$$

The condition $0 < z_A < z_B < 1$ is satisfy when $\frac{1}{32}(3 - 8c(k) + 16\beta + \delta(1-k)) < t^c < \frac{1}{16}(3 - 8c(k) + 16\beta + \delta(1-k))$

4.7.7. Proof of Welfare

4.7.7.1. Consumers can be multi-homing

When consumers are multi-homing we have three segments of consumers and three segments of surplus, the surplus from platform A users, the surplus from platform B users, and the surplus from users that buy both platform, $CS = \int_0^{x_A} V - t^p x_A - p_A + n_A \lambda dx_A + \int_{x_A}^{x_B} 2V + \lambda - p_A - p_B - t^p dx \int_{x_B}^{1} V - t^p(1 - x_B) - p_B + n_B \lambda dx_B$.
Developers surplus is
$DS = \int_0^{\widehat{z}} \beta - t^c \widehat{z} - l_A + u_A v dz + \int_{\widehat{z}}^{1} \beta - t^c(1 - \widehat{z}) - l_B + u_B v dz$.

4.7.7.2. Single-homing

First of all we compute the consumer surplus
$CS = \int_0^{\widehat{x}}(V - t^p \widehat{x} - p_A + n_A \lambda)dx + \int_{\widehat{x}}^{1}(V - t^p(1 - \widehat{x}) - p_B + n_B \lambda)dx =$
$V - p_A \widehat{x} + (\widehat{x} - 1)(p_B - t^p \widehat{x}) + \lambda(1 - \widehat{z} + \widehat{x}(2\widehat{z} - 1)) - \frac{t^p}{2}$. Developers surplus is
$DS = \int_0^{\widehat{z}}(\beta - t^c \widehat{z} - l_A + u_A v)dz + \int_{\widehat{z}}^{1}(\beta - t^c(1 - \widehat{z}) - l_B + u_B v)dz =$
$\beta + v(1 - x) + l_B(\widehat{z} - 1) - l_A \widehat{z} + v\widehat{z}(2x - 1) + t^c(\widehat{z} - \widehat{z}^2 - \frac{1}{2})$.

4.7.7.3. Developers can be multi-homing

Consumer surplus $CS = \int_0^{\widehat{x}} V - t^p \widehat{x} - p_A + n_A \lambda dx + \int_{\widehat{x}}^{1} V - t^p(1 - \widehat{x}) - p_B + n_B \lambda dx$ and developer surplus $DS =$
$\int_0^{z_A} \beta - t^c z_A - l_A + u_A v dz_A + \int_{z_A}^{z_B} 2\beta + v - l_A - l_b - t^c dz + \int_{z_B}^{1} \beta - t^c(1 - z_b) - l_b + u_B v dz_B$.

4.7.8. Proof of Comparisons social optimum

4.7.8.1. Comparisons consumers multi-homing case

Now, we compare the results with consumers multi-homing.

The equilibrium price is $p^*_{CM} = \frac{1}{32}(1 + 16V - 8v + 3\delta(1 - k))$, the socially optimal price is $\widetilde{p}^*_{CM} = \frac{1}{8}(1 - 2v + 4V)$, the difference is given by $p^*_{CM} - \widetilde{p}^*_{CM} = -\frac{3}{32}(1 + \delta(k - 1))$.

The license fees are different, in the market equilibrium we have $l^*_{CM} = t^c - \frac{(3\delta(1-k)+1)(1+24v+16V+3\delta(1-k))}{256t^p}$ and in the social optimum $\widetilde{l}^*_{CM} = \frac{1}{8}(2v - 1)$. It turns out that $l^* - \widetilde{l}^* = \frac{3(1-\delta(1-k))(5+24v+16V+3\delta(1-k))}{256tp}$.

Platforms profit in the market equilibrium is

$$\pi^*_{CM} = \frac{1}{2}t^c - \frac{48v(3\delta(1-k)+1)+(1+3\delta(1-k))^2+64v^2-256V^2}{1024t^p}$$

and in the social optimum $\widetilde{\pi}^*_{CM} = \frac{32t^ct^p-4v(3+v)+16V^2-1}{64t^c}$, the difference between them is

$$\pi^*_{CM} - \widetilde{\pi}^*_{CM} = \frac{3(5+48v+3\delta(1-k))(1-\delta(1-k))}{1024t^p}$$

Total welfare in the market equilibrium is

$$W^*_{CM} = \frac{3(1+8v+16V+3\delta(1-k))^2 - 256t^p(4c(k)+t^c+\delta(1-k)-1)}{1024t^p}$$

and in the social optimum is $\widetilde{W}^*_{CM} = \frac{3(1+2v+4V)^2-16t^ct^p}{64t^p}$, the difference

$$W^*_{CM} - \widetilde{W}^*_{CM} = -\frac{((9(1-\delta(1-k)\delta)(5+16v+32V+3\delta(1-k))+256tp(4c(k)+\delta(1-k)-1)}{1024t^p}$$

4.7.8.2. Comparisons single-homing case

Now, we compare the results with single-homing.

The equilibrium price and the socially optimal price are the same.

The license fees are different, in the former case we have $l^*_{SH} = t^c - \frac{3}{8}\delta(1-k) - \frac{1}{8}$ and in the social optimum $\widetilde{l}^*_{SH} = t^c - \frac{1}{2}$. We can easily subtraction one from another to obtain $l^*_{SH} - \widetilde{l}^*_{SH} = \frac{3}{8}(1+\delta(k-1))$.

Platforms profit is $\pi^* = \frac{1}{2}(t^c + t^p) - \frac{1}{2}\lambda - \frac{1}{2}v$ and in the social optimum is $\widetilde{\pi}^* = \frac{1}{4}(2(t^c+t^p-v)-1)$, the difference between them is $\pi^*_{SH} - \widetilde{\pi}^*_{SH} = \frac{3}{16}(1+\delta(k-1))$.

Total welfare is $W^*_{SH} = \frac{1}{16}(5-4(t^c+t^p)+12v-\delta(1-k))$ and in the social optimum $\widetilde{W}^*_{SH} = \frac{1}{4}(1-t^c-t^p+2v)+V$, the difference is$W^*_{SH} - \widetilde{W}^*_{SH} = \frac{1}{16}(1+\delta(k-1))$. As we can observe the license fee, platform profit and total welfare with single-homing is always higher in the market equilibrium.

4.7.8.3. Comparisons developers multi-homing case

Now, we compare the results with developers multi-homing.

The equilibrium price is $p^*_{DM} = t^p - \frac{v(7+8v+5\delta(1-k)-16c(k))}{32tc}$, the socially optimal price is $\widetilde{p}^*_{DM} = \frac{8t^c t^p - v(3+2v)}{8t^c}$, the difference is $p^*_{DM} - \widetilde{p}^*_{DM} = -\frac{3}{32}(1+\delta(k-1))$.

The license fees are different, in the market equilibrium we have $l^*_{DM} = \frac{1}{32}(3+8v-7\delta(1-k)-16c(k))$ and in the social optimum $\widetilde{l}^*_{DM} = \frac{1}{8}(2v-1)$ we can subtraction one from another to obtain $l^*_{DM} - \widetilde{l}^*_{DM} = \frac{3(1-\delta(1-k))(5+24v+16V+3\delta(1-k))}{256t^p}$.

Platform profit in the market equilibrium is

$$\pi^*_{DM} = \frac{512t^c t^p + 15 + 128c(k)(2c(k)-1) - 16v(3+4v) + \delta(k-1)(38 - 128c(k) + 144v + 7\delta(k-1))}{1024t^c}$$

and in the social optimum $\widetilde{\pi}^*_{DM} = \frac{32t^c t^p - 4v(3+v) + 16V^2 - 1}{64t^c}$, the difference between them is

$$\pi^*_{DM} - \widetilde{\pi}^*_{DM} = \frac{(256c(k)^2 - 128c(k)(1+\delta(-1+k)) + (1+\delta(-1+k))(31+144v-7\delta(1-k))}{1024t^c}$$

Total welfare in the market equilibrium is

$$W^*_{DM} = \frac{85 + 1280c(k)^2 + 2048t^c t^p - 16v(7+12v) - 2\delta(1-k)(81+296v) + 29\delta^2(1-k)^2 - 32c(k)(21+8v-17\delta(1-k))}{1024t^c}$$

and in the social optimum $\widetilde{W}^*_{DM} = \frac{128t^c t^p - 4v(11+3v) - 3}{64tc}$, the difference is given by

$$W^*_{DM} - \widetilde{W}^*_{DM} = \frac{1280c(k)^2 - 32c(k)(21+8v-17\delta(1-k)) + (1-\delta(1-k))(133+592v-29\delta(1-k))}{1024t^c}$$

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