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# **Economic Analysis of the International Cooperation to Face Global Environmental Problems**

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## CHAPTER 1

### INTRODUCTION: SPANISH AND ENGLISH

#### 1.1 Motivación y objetivos de la tesis

Debido a las serias dudas, hoy ya confirmadas, sobre la eficacia del Protocolo de Kyoto, distintos académicos especialistas en economía ambiental se han planteado en los últimos años si se podrían diseñar otros tipos de acuerdos internacionales que fuesen más eficaces y permitiesen una reducción de las emisiones de gases de efecto invernadero (GEI) lo suficientemente importante como para controlar y/o revertir el cambio climático. En esta línea, la principal motivación de esta tesis es el estudio de la rentabilidad, estabilidad y eficiencia de acuerdos ambientales internacionales que permitan hacer frente al problema global del cambio climático.

Una idea sería concentrarse en mejoras tecnológicas que permitan reducir los costes de mitigación, es decir, los costes económicos de reducir las emisiones de GEI, ya que esto puede aumentar la disponibilidad de un país para realizar reducciones significativas de emisiones. Por ejemplo, podría ser beneficioso completar un acuerdo del tipo Kyoto, es decir un acuerdo exclusivamente de emisiones, con acuerdos sobre el desarrollo de tecnologías más limpias sobre todo si el desarrollo tecnológico no depende sólo de la propia inversión en inversión y desarrollo (I + D) de un país, sino también de I + D de otros países a través de la difusión tecnológica entre países, como por ejemplo ya han planteado Carraro y Siniscalco (1997). Incluso, sin un acuerdo explícito sobre las emisiones, un acuerdo tecnológico que condujese al desarrollo y adopción de tecnologías menos contaminantes podría conducir a una reducción de las emisiones. Este es el argumento detrás de distintas propuestas, como la de Barrett

(2006), de un acuerdo climático basado en el desarrollo tecnológico, como por ejemplo en Barrett (2006). Existen ya distintas propuestas internacionales para promover la I+D en tecnologías menos contaminantes como el *Carbon Sequestration Leadership Forum* (con 21 países miembros más la Comisión Europea), la *International Partnership for the Hydrogen Economy* (17 países más la Comisión Europea) y el proyecto ITER (*International Thermonuclear Experimental Reactor*), aunque este proyecto no se puede ver exclusivamente como un proyecto tecnológico para combatir el cambio climático. Una revisión de los acuerdos tecnológicos que se centran en el estudio de las posibilidades de estos acuerdos para eliminar los incentivos de los países a actuar como polizones, es decir, para quedarse al margen en las negociaciones sobre el clima se puede encontrar en el muy interesante artículo de Coninck et al. (2008).

En esta línea de investigación, la tesis se plantea el análisis de diferentes tipos de acuerdos internacionales para combatir el cambio climático con el objetivo de poder determinar cuál de ellos podría ser más eficaz en la lucha contra el calentamiento global. El análisis pasa por determinar en primer lugar el nivel de participación que los distintos acuerdos podrían promover y en segundo lugar su eficiencia, medida en términos de las reducciones de emisiones y de costes que cada acuerdo permitiría implementar.

El examen de la cooperación internacional en el desarrollo tecnológico como un complemento a la cooperación internacional sobre la reducción de emisiones de GEI, es el objetivo principal del segundo capítulo donde se analizan cuatro tipos diferentes de acuerdos de reducción de emisiones. Los cuatro acuerdos tienen en común que los países firmantes actúan cooperativamente en la fijación de los niveles de emisiones. Sin embargo, se consideran distintas hipótesis en los que se refiere a la inversión en I+D y a su difusión entre distintos países. En todos los acuerdos con un componente tecnológico, se supone que la inversión efectiva en un país depende de la cantidad invertida en I + D en ese país, así como de la inversión en I + D en el resto de países

a través de un factor de difusión (*spillovers*). El factor de difusión es endógeno de manera que los países firmantes pueden, si así lo acuerdan, internalizar completamente los efectos externos de la inversión I+D intercambiando información sobre las actividades de I+D. En este caso el factor de difusión para los firmantes es máximo y por lo tanto igual a la unidad. En cambio, para los países no firmantes del acuerdo tecnológico, el factor de difusión es exógeno e inferior a la unidad.

El examen de la cooperación internacional en el desarrollo tecnológico como una alternativa a la cooperación internacional para la reducción de emisiones de GEI, es el principal objetivo del tercer capítulo, donde se analizan tres tipos de acuerdos tecnológicos. Los tres tipos de acuerdos: Acuerdo de I + D sin intercambio de información; Acuerdo de I + D sólo de intercambio de información (*Research joint venture*); Acuerdo de I + D con intercambio de información, comparten el principal aspecto de que los países signatarios actúan de forma unilateral en la determinación del nivel de emisiones, es decir, existe cooperación a nivel tecnológico pero cada país es libre de fijar su nivel de emisiones. Una comparación entre todos los tipos de acuerdos (analizado en el segundo y el tercero capítulos), se introduce también en este capítulo.

El cuarto capítulo de la tesis tiene como objeto evaluar la robustez de los resultados obtenidos en el tercer capítulo considerando distintas hipótesis sobre las estructura de costes. En concreto se evalúa la robustez de los resultados obtenidos para costes de inversión y daños medioambientales cuadráticos cuando. En el modelo utilizado en los capítulos dos y tres se suponen que los costes son lineales.

## **1.2 Metodología**

Como el objetivo de la tesis es el estudio de la formación de coaliciones con externalidades, la metodología utilizada ha sido la teoría de juegos. En concreto, la estabilidad (participación) del acuerdo se ha analizado resolviendo un juego no coop-

erativo en tres etapas. En la primera etapa, los países deciden de forma no cooperativa si firman o no un acuerdo de cooperación. En la segunda etapa, deciden sobre los niveles de inversión en función de la decisión tomada en la primera etapa. Los países firmantes actúan de forma cooperativa en esta segunda etapa de manera que eligen los niveles de inversión que minimizan los costes de reducir las emisiones del acuerdo. Sin embargo, los países no firmantes actúan de forma unilateral y fijan sus niveles de inversión teniendo en cuenta solamente los costes nacionales. En esta segunda etapa, los países firmantes pueden compartir o no la información asociada a las actividades de I + D dependiendo de lo que se haya establecido en la etapa anterior. En cambio los países no firmantes no comparten ninguna información y sólo se pueden beneficiar de los efectos externos de las actividades de I + D recogidos en el factor de difusión. En la tercera etapa se fijan los niveles de emisiones. En esta etapa puede haber cooperación o no dependiendo del tipo de acuerdo que se haya firmado en la primera etapa. Si se firma un acuerdo exclusivamente de reducción de emisiones. Sólomente habrá cooperación en esta etapa. Si se firma un acuerdo de reducción de emisiones y cooperación tecnológica, los países firmantes actuarán colectivamente tanto en la segunda como en la tercera etapa. Finalmente, si sólo se firma un acuerdo tecnológico, los países firmantes cooperarán sólo en la segunda etapa. Como es habitual en estos juegos por etapas, la solución se obtiene por inducción hacia atrás, resolviendo primero la última etapa, para resolver después la segunda y finalmente determinar la participación. En la primera etapa el nivel de participación lo fija el equilibrio de Nash donde las estrategias son firmar o no firmar. El acuerdo está formado por todos los países que deciden firmar. En la segunda y tercera etapa, los niveles de inversión y de emisiones quedan determinados por un equilibrio de Nash de acuerdo parcial en el que los países firmantes actúan de forma no cooperativa con respecto a los países no firmantes pero de forma cooperativa con respecto al resto de firmantes.

### 1.3 Principales conclusiones

De acuerdo con el análisis presentado en esta tesis, las principales conclusiones que se obtienen son:

i) Cooperar en las emisiones, incluso si va acompañado con la cooperación en inversión, sin intercambio de información, no es suficiente para eliminar los incentivos de los países para actuar como free-rider.

ii) Compartir las inversiones en  $I + D$  y evitar la duplicación de actividades de  $I + D$  es suficiente para estabilizar la gran coalición en los altos niveles de daños marginales. Este resultado se explica por la asimetría en el factor de difusión entre los signatarios y no signatarios. Como el acuerdo, que incluye el intercambio de información, implica que los signatarios internalizar completamente los efectos de spillovers de sus inversiones en  $I + D$ , los signatarios pueden eliminar las emisiones utilizando menos recursos que los países no signatarios. El resultado es que para los signatarios los costes de inversión son más bajos que para los no signatarios y por lo tanto los costes totales son también menores. Por otra parte, hay externalidades negativas de la cooperación para los no signatarios, es decir, la cooperación aumenta los costes totales de los no signatarios. Por eso, si un país abandona la gran coalición, sus costes totales aumentan debido al incremento en los costes de inversión, que hace que la gran coalición sea estable.

iii) Si los países signatarios invierten al nivel máximo de inversión en  $I + D$  para eliminar completamente las emisiones de gases de efecto invernadero, la cooperación en la tercera etapa del juego (las emisiones) no afecta ni el nivel de cooperación, ni los costes totales.

iv) Para niveles altos de daños marginales, se ha encontrado que los acuerdos que incluyen tanto la cooperación en inversión en  $I + D$  como intercambio de información son los acuerdos dominantes. Sin embargo, para valores bajos de daños marginales,

los acuerdos dominantes cambian según los diferentes intervalos de daños marginales como se explica en detalle en el tercer capítulo.

v) Al examinar la robustez de los diferentes supuestos del modelo, se encontró que los dos supuestos de rendimientos constantes a escala de la inversión en I + D (costes de inversión lineales) y el supuesto de de los daños ambientales lineales no son críticos para obtener el resultado de que l gran coalición es estable y beneficiosa para altos niveles de daños marginales.



## 1.4 Introduction

Climate change is becoming an important issue in human lives. Many changes have been observed in global climate over the past century, including the increment in global average temperature and sea level, a warmer and more acidic ocean, atmospheric moisture and the human activities have led to large increases in the greenhouse gas emissions (GHG). Greenhouse gases include water vapor, carbon dioxide, methane, nitrous oxide, ozone and a variety of manufactured chemicals that help capture the heat by absorbing infrared radiation, which in turn causes an elevation of the average surface temperature above what it would be in the absence of these gases.

Due to the absence of a supra-national authority that can enforce environmental policies to control greenhouse gas (GHG) emissions on a global scale, countries have had to negotiate an international environmental agreement (IEA), the Kyoto Protocol, to address this problem. The aim of the Kyoto Protocol was to achieve a reduction in GHG emissions of 5% taking as reference the level of 1990 for countries of Annex B in the commitment period 2008-2012. However, this target of abating GHG emissions was not achieved for that period. Limited coverage and moderate emission reductions requirements are two limitations that reduced the effectiveness of the agreement.

## 1.5 Work Motivation

Because of the doubts about the effectiveness of the Kyoto Protocol, several scholars have asked whether other types of agreements can be designed to achieve large reductions of GHG emissions. Designing a profitable and stable international environmental agreement (IEA) that deals with the shortcomings of Kyoto-type agreement is the main motivation of this work.

One idea would be to focus on technology improvements in order to reduce abatement costs, as this might increase a country's willingness to undertake significant emission reductions. For example, it could be beneficial to supplement a Kyoto-type agreement with technology elements if technological development depends not only on a country's own R&D investment but also on R&D by other countries through cross-country technology spillovers, see for instance, Carraro and Siniscalco (1997). Even with no explicit agreement on emissions, a technology agreement leading to increased R&D in clean technologies, and thus to lower abatement costs, might yield a reduction in emissions. This is the argument behind the proposals of a climate agreement on technology development, see for instance, Barrett (2006).

## 1.6 Literature Survey

The analysis of profitability and stability of IEAs has been studied since the seminal paper by Carraro and Siniscalco (1993). They show that an IEA is formed when conditions of profitability and stability are satisfied. They consider a model where signatories of IEA may choose means of self-enforced transfers to induce non-signatories to join the IEA. However, expanding coalitions requires some form of commitment, but such commitment is inconsistent with the notion of self-enforcement as claimed by Barrett (1994).

Then, the importance of studying self-enforcing IEA has been highlighted since the seminal paper by Barrett (1994), which employed some concepts from game theory to explore the properties of self-enforcing IEAs. Using two different modelling approaches, Barrett has concluded that self-enforcing IEAs, which establish rules for managing shared environmental resources, may not be able to improve substantially upon the non-cooperative outcome. The first model of self-enforcing IEA analyzed by Barrett solves jointly for the number of signatories, the terms of IEA and the actions of non-signatories. In this model, it has been concluded that the self-enforcing IEA

can sustain a large number of signatories only when the difference in the net benefits between the non-cooperative and full cooperative outcome is very small. Otherwise, a self-enforcing IEA may not exist or it may not be able to sustain more than two or three countries. The second model analyzed by Barrett, which takes the IEA to be an equilibrium to an infinity repeated game, shows that full cooperative outcome can be sustained by large number of countries only when the difference in global net benefits between the non-cooperative and full cooperative outcome is small. Otherwise, the full cooperative outcome can be sustained by only a few countries or by non at all.

Since then, different strands of the literature about self-enforcing IEAs and how to overcome the free-riding incentives to stabilize an IEA with high level of cooperation have emerged.

These strands of the literature are including, but not limited to, the following points:

### **1.6.1 Transfers**

One strand of the literature is concentrated on the possibility of using self-financed transfers to compensate the countries that might lose by joining the environmental agreements (see for instance, Carraro and Siniscalco (1993), Hoel (1994), Carraro et al. (2006) and Fuentes-Albero and Rubio (2010)).

As mentioned before, Carraro and Siniscalco (1993) show that using means of self-transfers induces non-signatories to join the IEA which is close to one of the main conclusions by Hoel (1994), that cooperating countries should try to induce the non-cooperating countries through appropriate transfers in order to tax the consumption and production of carbon at the same rate as in the cooperating countries.

Carraro et al. (2006) develop a general framework which allows the study of transfers role in IEAs by proposing transfers using both internal and external financial resources, to achieve both self-enforcing and welfare optimal agreements.

Fuentes-Albero and Rubio (2010) show that when there is differences in the environmental damages among countries, the level of cooperation, that can be bought through a self-financed transfers scheme, increases with the degree of asymmetry.

### 1.6.2 Policy Tools

Another strand of the literature is concentrated on the choice of the policy tools to form an IEA (see for instance, Hoel (1993), Moher (1995), Finus and Rundshagen (1998), Carraro and siniscalco (1998), Barrett and Stavins (2003) and Turunen-Red and Woodland (2004)).

Hoel (1993) uses some kind of carbon tax to examine the need for harmonization of carbon taxes across countries inside the IEA. Hoel also discusses a similar arrangement through a system of emission permits which are internationally tradable between agreements, while Moher (1995) study the linking of international environmental problems to the international permit trade. Moher analyzes the impact of trade opportunities on countries incentives to continue environmental cooperation.

Finus and Rundshagen (1998) analyze the coalition formation process with asymmetric countries. They show that the total welfare is higher by comparison to the Nash equilibrium under the policies of uniform emission reduction quota and an effluent tax. Carraro and Siniscalco (1998) discuss mechanisms and strategies to offset the free-riding incentives and increasing welfare. They conclude that partial coalitions and multiple agreements tend to prevail among subsets of players, and the agreements such as the grand coalition are most unlikely to exist.

Barrett and Stavins (2003) study the role of protocols and policy architectures to induce participation and compliance in IEA, while Turunen-Red and Woodland (2004) show that a policy of emission tax reforms accompanied with reforms in tariffs can be used as alternative to the emission tax reforms accompanied by international income transfers which in turn is going to achieve the desirable welfare outcome.

### 1.6.3 Linkage

The strand of the literature which is related to our work can be defined under the concept of (linkage). In particular, to link the unstable emission IEA with an agreement on technological cooperation (the issue of the second chapter in this thesis) which is shown to be profitable and stable. This has been analyzed in the seminal paper by Carraro and Siniscalco (1997), by employing a numerical example to show that overcoming the free-ride incentive can be obtained through this kind of linkage. The model used in this paper considers the interactions between the government and domestic firms in one country and the governments in different countries.

The authors conclude that linking the investment in R&D with the environmental negotiation increases the stability of the environmental cooperation, number of signatories and the total welfare of cooperating countries.

Even with no explicit agreement on emissions, a technology agreement leading to increased R&D in clean technologies, and thus to lower abatement costs might yield a reduction in emissions (the issue of the third chapter in this thesis). This idea has been highlighted recently by Barrett (2006). In this paper, an alternative treaty system to Kyoto has been proposed. This system is a system of two treaties, one promoting cooperative R&D and the other encouraging the adoption of breakthrough technology emerging from this R&D.

Barrett concluded that breakthrough technologies can not improve the performance with the exception of breakthrough technologies that exhibit increasing returns to scale. Ruis and Zeeuw (2010) give support to this idea in a framework of a model with quadratic investment costs and without spillovers effects. The main reason that explains the difference of the results obtained in our thesis with those obtained by Barrett (2006) is that while Barrett assumes global investment in R&D to be a *perfect public good*, we assume that some imperfections exist and introduce an asymmetry between signatories and non-signatories as regards the degree of spillovers. This change

is enough to reverse the nature of the game when emissions are completely eliminated and hence the fact that different results are obtained. The idea that the degree of spillovers is different among countries which cooperate than among countries which do not cooperate can be also found in Xepapadeas (1995) and Carraro and Siniscalco (1997). In Xepapadeas (1995), it is assumed that when all countries enter into an international agreement, the level of technology is common to all countries (it is a perfect public good). Carraro and Siniscalco (1997) normalize to zero the spillover effects for non-signatories. The model of the present thesis assumes that spillover effects are positive for non-signatories and are fully internalized for signatories under the types of agreements which include R&D information exchange.

Hoel and de Zeeuw (2010) show that a focus on the R&D phase in the development of breakthrough technologies can change the result obtained by Barrett (2006). Assuming that the cost of adoption decreases with respect to the level of R&D, they find that even without increasing returns to scale, a technology agreement can yield better results than those obtained by focusing on abatement targets, although the first best cannot be achieved. This result is obtained when the non-cooperative equilibrium with full adoption exists and for a different timing of the game. Hoel and de Zeeuw (2010) assume that the agreement chooses R&D expenditures after the participation stage. Hong and Karp (2012) explore a similar idea but in the framework of the standard model of an IEA formation with linear payoff. The authors assume that the cost of abatement decreases with respect to the level of R&D. Moreover, they assume, as in Barrett (2006), that countries individually decide whether to invest in a public good that reduces abatement costs before the participation stage. Their findings show that using mixed strategies at the participation stage the standard result mentioned above reverses: membership can be large but only when the treaty does make all countries substantially better off. Mixed strategies create endogenous risk so that risk aversion increases the equilibrium probability of participation. In the fourth

chapter, we extend this research to the case of quadratic abatement and investment costs but assuming the timing proposed by Hoel and de Zeeuw (2010) and focusing on pure strategies at the participation stage.

Barrett (2009) introduces a survey of the possibilities of developing breakthrough technologies that are needed to reduce emissions dramatically. The breakthrough technologies analyzed in this paper include wind energy, solar energy and nuclear energy. Hoffert et al. (2002) introduce technical discussion of some of the breakthrough technologies. The authors survey possible future sources, evaluated for their capability to supply massive amounts of carbon emission-free energy and for their potential for large-scale commercialization. They concluded that a broad range of intensive R&D is needed to produce technological options that can allow both stabilization and economic development.

The issue of linkage has been highlighted by many scholars in other different ways. de Coninck et al. (2008) provide an overview of technology-oriented agreements stressing their potential role in addressing the free-riding incentives in climate negotiations. They find that the technology-oriented agreements which aim to knowledge sharing and coordination and R&D could increase the overall efficiency and effectiveness of international climate cooperation. However, those types of agreements have limited environmental effectiveness on their own as concluded by the authors. Nevertheless, the results of this paper indicate that technology-oriented agreements could potentially provide a valuable contribution to the global response to climate change depending on their design, implementation and their expected role relatively to other components of policy portfolio.

The effect of technology spillovers on the stability of international climate coalition has been studied in Nagashima and Dellink (2008). They address the effects of asymmetric spillovers, that affect the marginal abatement cost curve, on the participation in an emission agreement. Their results show that spillovers do not substantially in-

crease the success of IEAs. However, in their model the size of the spillovers cannot be controlled by the signatories, as their state of technology is exogenous. More recently, Nagashima et al. (2011) have extended this analysis by relaxing the assumption of exogenous technological, but do not consider knowledge spillovers. Unlike the results obtained in the thesis, their results continue being pessimistic, stable coalitions are smaller when the gains from cooperation are large.

Investing in R&D to overcome the free-riding incentives in IEA taking into account the spillovers effects has been also analyzed in different ways in Heal and Tarui (2010), Benchekroun et al (2011) and Harstad (2012).

Heal and Tarui (2010) study the incentives to develop advanced pollution abatement technology when technology may spillover across agents and pollution abatement is a public good. In a framework of a general model, the authors examine how the effect of technological innovation on the cost structure of emission abatement influences the agents, incentives to reduce emissions and to invest in R&D in new technologies. Benshekron et al (2011) investigate how the success of the attempt to mitigate emissions of GHG emissions through international negotiations, depends on the adaptive measures taken by different countries to reduce the negative effects of climate change. They show that the increase in effectiveness of adaptation diminishes the individual countries incentives to free-ride on global agreement over emissions. A dynamic game has been presented by Harstad (2012) where players can contribute to a public bad, invest in technologies and write incomplete contracts.

Next, it is worth adding that different empirical papers give support to the idea that supplementing an emission agreement with technology elements or replacing an emission agreement with a technology agreement can have positive effects on the participation into the agreement. See for instance the papers written by Buchner et al. (2005), Kemfert (2004), Buchner and Carraro (2005) and Lessmann and Edenhofer (2011).



Buchner et al. (2005) study whether linking the cooperation on climate change with cooperation on technological innovation and diffusion can motivate the US to sign the Kyoto or not. The paper analyzes mainly two points. First, the incentives for Europe, Japan and Russia to adopt the linkage strategy. Second, the incentives for US to join a coalition which cooperates on GHG emission control and on R&D investment and technological diffusion.

Following the analysis of the same issue, Kemfert (2004) studies whether incentives exist for non-cooperation nations like USA to join a coalition based upon linkage, taking into account the spillover effects. The model used in this paper concluded the existence of incentives for climate control coalitions coupled with issue linkage of technological innovations. Thus, there is an incentive for US to join a full coalition or small coalition on climate control and technological improvements with Europe, Japan and Russia.

Buchner and Carraro (2005) study the idea of replacing international cooperation on GHG emission control with international cooperation on climate-related technological innovation and diffusion. It has been concluded in this paper that technological cooperation, without any commitment to emission control, may not lead to a sufficient abatement of GHG concentrations.

Lessman and Edenhofer (2011), by using a numerical model, analyze the differences of several technology agreements and how they interact. They conclude that participation in and environmental effectiveness of the IEA are raised less effectively when the technology oriented agreement focuses on research cooperation in mitigation technology rather than cooperation on augmenting productivity in the private good sector.

Finally, it is important to highlight the literature survey about the investment in R&D with spillovers and the research joint ventures. The seminal paper, concerning R&D spillover, by D'Aspremont and Jacquemin (1988) analyzed a symmetric duopoly

model of R&D and spillovers in order to compare three different games, depending on whether firms cooperate or compete in the different stages of the game played. In the first game, it has been assumed that firms act non-cooperatively in both output and R&D. In the second game, it is assumed that firms act cooperatively in the R&D stage while compete in the output stage. In the third game, monopoly game has been analyzed assuming that firms act cooperatively in both stages. The main conclusion of this paper is that cooperation in R&D increases both expenditures in R&D and quantities of production, with respect to the non-cooperative solution whenever the spillover is large enough and vice versa. D' Aspremont and Jacquemin (1988) paper has been commented by Henriques (1990) by showing that comparing the pure cooperative and the pure non-cooperative solutions as defined by d' Aspremont and Jacquemin is only meaningful when the non-cooperative solution is stable, which means that spillovers are not too small.

The seminal paper by Kamien et al. (1992), concerning the research joint ventures, which our specification to the countries effective investment is built on, has analyzed the effects of R&D cartelization and research joint ventures on oligopolistic competition by using four different scenarios. The different scenarios are depending on whether countries are coordinating their R&D investments and sharing R&D efforts or not. It is shown that creating a competitive research joint venture reduces the equilibrium level of technological improvements and increases equilibrium prices compared to when firms conduct R&D independently.

The cooperation on technological development by assuming the endogeneity of the R&D spillovers has been analyzed by Katsoulacos and Ulph (1998), while Yi and Shin (2000) examine the endogenous formation of research coalitions with high spillovers among symmetric firms. Katsoulacos and Ulph (1998) examine the effects of research joint ventures on all aspects of innovative performance in the case where R&D spillovers are endogenously chosen both in the absence of a research joint venture

and once the research joint venture has formed. The number of research coalitions and their sizes are determined endogenously in Yi and Shin (2000). The stability of the grand research coalition and effects on stable research structure of an increase in research, have been also analyzed in this paper.

Greenlee (2005) examines the effects of research sharing on welfare and on firm incentives to form joint ventures. In contrast to Kamien et al. (1992), Greenlee allows for imperfect research sharing among partners and assumes that firms optimally adjust individual research intensities in response to changes in the membership of any joint venture.

Kamien and Zang (2000) propose a representation of a firm's effective R&D effort level that incorporates absorptive capacity as a strategic variable into the existing research joint venture models. The analysis in this paper reveals that if the firms can cooperate in setting their R&D budgets, then they choose identical R&D approaches and dissimilar approaches if they cannot.

Amir et al. (2008) conclude that a given R&D investment should always produce more cost reduction if devoted to one lab rather than two independent labs operated under natural spillovers. The conclusion of this paper coincides with one of the important results of this thesis, that an agreement which doesn't allow for sharing R&D efforts can't enhance the non-cooperative outcome.

## **1.7 Scope and Work Organization**

The aim of the present work is to examine different types of international environmental agreements in order to determine what would be the dominant agreement (at the different levels of marginal damages) with respect to both the total costs of signatories countries and the level of cooperation. The model used in the analysis is a three-stage static model (explained in chapter 2), where the membership game is

played in the first stage, the investment game is played in the second stage and finally the emission game is played in the third stage.

Examining the international cooperation on technological development as a supplement to international cooperation on GHG emission reductions, is the main objective of the second chapter where four different types of emission agreements are analyzed. The four agreements share the main aspect that signatories countries act cooperatively in the third stage of the game (emission game). However, the second stage of the game (investment game) differs from one type to another depending on whether signatories are sharing R&D efforts and coordinating their R&D activities or not. In all agreements, it is assumed that effective investment in one country depends on the amount invested in R&D in that country as well as on the investment in R&D undertaken in all countries through technological spillovers.

In the types of agreements that include information exchange (emission agreement with information exchange and emission and R&D agreement with information exchange), the technological spillovers is perfect among signatories, which means that signatories countries avoid the duplication of R&D efforts. However, the technological spillovers is not perfect among signatories in the other two types (emission agreement and emission and R&D agreement without information exchange).

Examining the international cooperation on technological development as an alternative to international cooperation on GHG emission reductions, is the main objective of the third chapter where three different types of technological agreements are analyzed. The three different types of agreements (R&D agreement without information exchange, research joint venture agreement and R&D agreement with information exchange), share the main aspect that signatories countries act non-cooperatively in the third stage of the game (emission game). A comparison between all types of agreements (analyzed in second and third chapters), is also introduced in this chapter.

An extension of the analysis which examines the robustness of the model for the dominant agreements at the high levels of marginal damages, is the main objective of the fourth chapter. The quadratic investment costs and the quadratic environmental damages are the two different assumptions that have been analyzed in this chapter.

## 1.8 Main Conclusions

According to the analysis introduced in this thesis, the main conclusions can be summarized as follows

- Cooperating on emissions even if it is accompanied with cooperating on investment without information exchange, is not enough to eliminate countries incentives to act as free-rider.
- Sharing R&D investments and avoid duplication of R&D activities is enough to stabilize the grand coalition at the high levels of marginal damages. This result is explained by the asymmetry in the spillovers parameter between signatories and non-signatories. As the agreement, which includes information exchange, implies that signatories fully internalize the spillover effects of their investments in R&D, signatories can eliminate emissions using less resources than non-signatories. The result is that the signatories' investment costs are lower than the non-signatories' investment costs and hence the total costs are also lower. Moreover, there are negative externalities for non-signatories stemming from cooperation, i.e. cooperation increases the total costs of non-signatories. In this framework, if one country abandons the grand coalition, its total costs increase because of the increase in investment costs, which makes the grand coalition stable.
- As far as signatories countries invest at the maximum level of R&D investment to eliminate completely the GHG emissions, cooperation in the third stage of

the game (emissions) does not affect neither the level of cooperation nor the total costs.

- At high levels of marginal damages, it is found that the agreements which include both cooperation on R&D investment and information exchange are the dominant agreements. However, for low values of marginal damages, the dominant agreements are changing according to the different intervals of marginal damages as explained in details in the third chapter.
- By examining the robustness of the different assumptions of the model, it is found that both of the assumptions of constant returns to scale of the R&D investment (linear investment costs) and the assumption of linear environmental damages are not critical for achieving the result that grand coalition is stable and profitable, at the high levels of marginal damages.

## CHAPTER 2

### EMISSION AGREEMENTS WITH DIFFERENT TYPES OF TECHNOLOGICAL COOPERATION

#### 2.1 The Model

We develop a static model with  $N$  countries that pollute the atmosphere and negotiate the control of greenhouse gases (GHG) emissions, taking into account the effects of spillovers in R&D from one country to another. The model is based on Golombek and Hoel (2005). We assume that the effective investment in a country  $i$ ,  $y_i$ ,  $i = 1, \dots, N$ , depends on the amount invested in R&D in that country,  $x_i$ , and also the investments in R&D undertaken in all other countries. However, technological spillovers is not perfect, only a part of the R&D investments undertaken in other countries is beneficial for country  $i$ . Hence, the effective investment of country  $i$  is given by

$$y_i = x_i + \gamma X_{-i}, \quad \gamma \in [0, 1], \quad (2.1)$$

where  $X_{-i} = \sum_{l \neq i} x_l$ . This specification for effective R&D investment was introduced by Spence (1984) and it has been recently used by Golombek and Hoel (2005, 2008, 2011) in the analysis of climate policy under technology spillovers.<sup>1</sup> However, following the approach adopted by Kamien et al.(1992) in their analysis of the effects of R&D cartelization and research joint ventures on oligopolistic competition,

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<sup>1</sup>Golombeck and Hoel (2005, 2008, 2011) analyze the effects on R&D investments, emissions and welfare of different types of agreements implemented by different types of instruments, including a technology agreement implemented by a subsidy but they do not study the stability of the agreements.

we assume that countries can coordinate by pooling their R&D efforts so as to fully internalize the spillovers effects which implies that in this case  $\gamma = 1$  for signatories.

In the absence of any explicit abatement activities, emissions in each country depend only on the technology level of the country. So, the business as usual emissions (BAU) for a level of effective investment equal to  $y_i$  is defined as  $\bar{E}(y_i) = \delta - \alpha y_i$ , with  $\delta, \alpha > 0$ , and  $\alpha$  is representing the emissions abatement per each unit invested in clean technologies. According to that, we can define the abatement of country  $i$  as  $A_i = \bar{E}(y_i) - E_i = \delta - \alpha y_i - E_i$  where  $E_i$  stands for the current emissions generated by country  $i$ . Thus, abatement costs depend on both the level of abatement and the level of effective investment. The effective R&D investment reduces the abatement costs because it reduces the intensity of emissions in the production of goods and services for a country. The greater the effective R&D investment, the lower the ratio of GHG emissions over the GDP of the country and, consequently, the lower the abatement costs. For this specification, there exists a critical value for  $y_i$  given by

$$y_i = \frac{\delta}{\alpha}, \quad (2.2)$$

for which GHG emissions are completely eliminated, in other words, fossil fuels could be completely substituted by other non-polluting energies. We assume that abatement costs are quadratic

$$C(A_i) = \frac{c}{2} A_i^2 = \frac{c}{2} (\delta - \alpha y_i - E_i)^2, \quad c > 0. \quad (2.3)$$

Following Golombek and Hoel (2005), the price of R&D investments is normalized to one and investment is irreversible. So, the cost of investing in R&D is  $R(x_i) = x_i$ .

Finally, the environmental damages in each country depend on the sum of total emissions,  $E = \sum_{i=1}^N E_i$ . We assume that the environmental damages are linear:



$D(E) = dE$ ,  $d > 0$ . Thus, we can write the total costs of controlling GHG emissions for the representative country as follows

$$TC_i = \frac{c}{2}(\delta - \alpha y_i - E_i)^2 + dE + x_i, \quad (2.4)$$

where  $y_i = x_i + \gamma X_{-i}$ , with  $\gamma \in [0, 1]$  and  $E = \sum_{i=1}^N E_i$ .

The fully non-cooperative equilibrium will be analyzed in the next section, in order to examine whether the different types of agreements, that will be studied later, are profitable to signatories in comparison with the non-cooperative equilibrium or not.

## 2.2 Fully Non-Cooperative Equilibrium

In the fully non-cooperative equilibrium, players make decisions independently. Thus, under the assumption of our model, countries don't cooperate neither in the emission nor in the investment game. The fully non-cooperative equilibrium can be calculated as the equilibrium of a two-stage game. In the first stage, countries decide the level of investment in R&D. In the second stage they decide about emissions. In both stages, the Nash equilibrium is calculated. Solving by backward induction, we begin analyzing the equilibrium of the second stage.

For a given technology, the optimal emissions can be calculated by minimizing the total cost function given by (2.4) which yields for the representative country<sup>2</sup>

$$E_i^{nc} = \bar{\delta} - \alpha y_i. \quad (2.5)$$

So that, the effective investment which yields zero emissions for each country is given by

$$y_i^{nc} = \frac{\bar{\delta}}{\alpha}, \quad (2.6)$$

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<sup>2</sup>In order to simplify the notation,  $\bar{\delta}$  will be used to represent the difference  $\delta - (d/c)$ . Thus, for  $d \geq \delta c$  the model has a trivial solution with  $y_i^{nc} = 0$ ,  $E_i^{nc} = 0$ .

which is lower than the critical value for the effective investment given by (2.2) that eliminates completely the GHG emissions. Therefore, the level of emissions could be positive or zero depending on the level of effective investment in the following way

$$E_i^{nc} = \left\{ \begin{array}{ll} \bar{\delta} - \alpha y_i & \text{if } y_i^{nc} \in (0, \bar{\delta}/\alpha] \\ 0 & \text{if } y_i^{nc} \in (\bar{\delta}/\alpha, \delta/\alpha] \end{array} \right\}.$$

Using (2.5), the global emissions can be calculated as follows

$$E = \sum_{i=1}^N E_i = N\bar{\delta} - \alpha Y, \quad (2.7)$$

where  $Y$  is the global effective investment in R&D given by

$$Y = \sum_{i=1}^N y_i = \sum_{i=1}^N (x_i + \gamma X_{-i}). \quad (2.8)$$

By substituting both (2.5) and (2.7) in (2.4), the total costs function can be written as

$$TC_i = \frac{d^2}{2c} + d(N\bar{\delta} - \alpha Y) + x_i. \quad (2.9)$$

Observe that the global effective investment in R&D becomes a *public good*. Any investment made by a country reduces the total costs of all countries.

Now, we calculate the equilibrium for the first stage. As the cost of investing in R&D is linear, there is a *linear programming problem* defined for the representative country as follows

$$\min_{\{x_i\}} TC_i = \frac{d^2}{2c} + d(N\bar{\delta} - \alpha Y) + x_i, \quad (2.10)$$

$$s.t. \quad x_i \geq 0, \quad (2.11)$$

where  $Y$  is given by (2.8).

By taking the first derivative of total costs function given by (2.10) with respect to investment in R&D, taking into account that  $\partial Y/\partial x_i = 1 + \gamma(N - 1)$ , the following critical value of marginal damages defined by condition  $\partial TC_i/\partial x_i = 0$  is obtained

$$\hat{d}^{nc} = \frac{1}{\alpha(1 + \gamma(N - 1))}, \quad (2.12)$$

such that when  $d \geq \hat{d}^{nc}$ , total costs are *decreasing* with respect to  $x_i$ , and the countries invest in R&D until emissions are completely eliminated, while for  $d < \hat{d}^{nc}$ , total costs are *increasing* with respect to  $x_i$ , and countries decide not to invest in R&D.

First, assuming that  $d \geq \hat{d}^{nc}$ , as countries invest to eliminate emissions, both  $E$  and  $E_i$  are substituted by zero in the total costs functions given by (2.4). Thus, the optimization problem for each country can be represented as follows

$$\min_{\{x_i^{nc}\}} TC_i^{nc} = \frac{c}{2} (\delta - \alpha y_i)^2 + x_i, \quad (2.13)$$

$$s.t. \ y_i = x_i + \gamma X_{-i} \geq \frac{\bar{\delta}}{\alpha}, \quad (2.14)$$

$$y_i = x_i + \gamma X_{-i} \leq \frac{\delta}{\alpha}. \quad (2.15)$$

The solution of this optimization problem yields the following level of investment for each country

$$x_i^{nc} = \frac{c\alpha\delta - 1}{c\alpha^2(1 + \gamma(N - 1))}, \quad (2.16)$$

while the effective investment is given by

$$y_i^{nc} = \frac{c\alpha\delta - 1}{c\alpha^2}. \quad (2.17)$$

which satisfies the constraint on effective investment given by (2.15). However, in order to investigate whether the constraint on effective investment given by (2.14) is satisfied or not, the difference between the levels of effective investment given by

(2.17) and (2.6) is taken and it is found that the constraint is satisfied at any level of marginal damages higher than

$$\tilde{d}^{nc} = \frac{1}{\alpha}, \quad (2.18)$$

which is higher than  $\hat{d}^{nc}$ .

Therefore, it is concluded that the decisions about the level of investment for each country and the corresponding total costs functions depend on the level of marginal damages. Thus, for  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , as the constraint given by (2.14) is not satisfied, countries invest at the level of effective investment given by (2.6) and the level of investment for each country in this case is given by

$$x_i^{nc} = \frac{\bar{\delta}}{\alpha(1 + \gamma(N - 1))}. \quad (2.19)$$

Finally, doing the substitution of both investment and effective investment in the total costs function given by (2.13), the following expression is obtained

$$TC_i^{nc} = \frac{d^2}{2c} + \frac{\bar{\delta}}{\alpha(1 + \gamma(N - 1))}. \quad (2.20)$$

However, if  $d \geq \tilde{d}^{nc}$ , countries increase their investment to the maximum level that eliminates the GHG emissions and the level of effective investment is now given by (2.17). Doing the substitution in the total costs function given by (2.13), the following expression is obtained

$$TC_i^{nc} = \frac{1}{2c\alpha^2} + \frac{c\alpha\delta - 1}{c\alpha^2(1 + \gamma(N - 1))}. \quad (2.21)$$

Observe that although the effective investment is independent of the technology diffusion parameter, both the investment in R&D and the total costs decrease as the technology spillovers increase.

Finally, if  $d < \hat{d}^{nc}$ , as the optimal policy for the countries is not to invest, the emissions increase to the BAU emissions level given by  $E_i^{nc} = \bar{\delta}$  and the total costs are given by

$$TC_i^{nc} = \frac{d^2}{2c} + dN\bar{\delta}, \quad (2.22)$$

which corresponds to the outcome of the standard model of emissions abatement for a given technology.

### 2.3 Emission Agreement

The formation of emission agreement is modeled as a three-stage game. Each game will be described briefly in a reverse order as the subgame-perfect equilibrium of this three stage game is computed by backward induction.

Given the level of participation in the agreement and the investment in R&D of all countries, at the third stage, the emission game, non-signatory countries choose their emissions acting non-cooperatively and taking the emissions of all other countries as given in order to minimize their own costs of controlling pollution. On the other hand, signatories countries choose the emissions acting cooperatively in order to minimize the aggregate costs of the agreement. At the second stage, the R&D investment game, each country acting non-cooperatively decides its own R&D level given the R&D investments of other countries. As there is no information exchange, the marginal abatement costs of all countries (signatories and non-signatories) are decreased by the country's R&D effect in addition to some spillover from other countries' R&D. In other words, the spillover is not increased because of the agreement. Thus, the effective investment is given by (2.1) but this expression can be written as follows

$$y_j^s = x_j^s + \gamma \left( \sum_{k=1}^{n-1} x_k^s + \sum_{l=1}^{N-n} x_l^f \right), \quad j = 1, \dots, n,$$

for signatories, and

$$y_i^f = x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n-1} x_l^f \right), \quad i = 1, \dots, N - n,$$

for non-signatories. Where  $n$  stands for the number of signatories,  $s$  for a signatory country and  $f$  for a non-signatory country.

Signatories and non-signatories choose their R&D investment simultaneously. Thus, R&D investments are provided by the *partial agreement Nash equilibrium* with respect to a coalition defined by Chander and Tulkens (1995). Finally, we assume that at the first stage, countries play a *simultaneous open membership game with a single binding agreement*. In a single agreement formation game, the strategies for each country are to sign or not to sign and the agreement is formed by all players who have chosen to sign. Under open membership, any country is free to join the agreement if interested. Finally, we assume that the signing of the agreement is binding on signatories. They therefore acquire a commitment to stay and implement the agreement during the second stage of the game so that full compliance is achieved. The game finishes when the emission sub-game is over.

### 2.3.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved by backward induction assuming that in the first stage  $n$  countries with  $n \geq 2$ , have signed the agreement.

As we have supposed that there is no cooperation between non-signatories countries in the emission game, the solution of this stage for non-signatories is given exactly as in the fully non-cooperative equilibrium.

For signatories countries, as they select emissions minimizing the joint costs of all signatories and taking into account the environmental damages of all countries in the agreement, then emissions for signatories should be calculated by minimizing the following total costs function

$$ATC = \sum_{j=1}^n TC_j^s = \sum_{j=1}^n \left( \frac{c}{2} (\delta - \alpha y_j^s - E_j^s)^2 + dE \right), \quad j = 1, \dots, n,$$

where  $ATC$  stands for the total costs of the agreement, which yields<sup>3</sup>

$$E_j^s = \delta - \frac{nd}{c} - \alpha y_j^s. \quad (2.23)$$

In this case, the effective investment which yields zero emissions for each signatory country is given by

$$y_j^s = \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right). \quad (2.24)$$

Thus, the level of emissions for signatories countries could be positive or zero depending on their level of effective investment in the following way

$$E_j^s = \left\{ \begin{array}{ll} \delta - (nd/c) - \alpha y_j^s & \text{if } y_j^s \in (0, \frac{1}{\alpha} (\delta - \frac{nd}{c})] \\ 0 & \text{if } y_j^s \in (\frac{1}{\alpha} (\delta - \frac{nd}{c}), \delta/\alpha] \end{array} \right\}.$$

Using (2.5) and (2.23), the global emissions can be written as follows

$$E = N\delta - \frac{d}{c} (N + n^2 - n) - \alpha Y, \quad (2.25)$$

where  $Y$  is given by the sum of effective investment for both non-signatories and signatories as follows

$$Y = \sum_{i=1}^{N-n} \left( x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n-1} x_l^f \right) \right) + \sum_{j=1}^n \left( x_j^s + \gamma \left( \sum_{k=1}^{n-1} x_k^s + \sum_{l=1}^{N-n} x_l^f \right) \right). \quad (2.26)$$

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<sup>3</sup>For  $d \geq \delta c/n$ , the model has a trivial solution with  $y_j^s = 0$ ,  $E_j^s = 0$ . As the level of  $d$  that yields the trivial solution for signatories countries is lower than that yields the trivial solution for non-signatories countries, the focus will be on the values of  $d$  below  $\delta c/n$ , and on the values of  $d$  below  $\delta c/N$  if full cooperation is achieved.

By substituting the emissions in the total costs for signatories and non-signatories countries, we obtain

$$TC_i^f(n) = \frac{d^2}{2c} + d \left( N\delta - \frac{d}{c}(N + n^2 - n) - \alpha Y \right), \quad i = 1, \dots, N - n, \quad (2.27)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + d \left( N\delta - \frac{d}{c}(N + n^2 - n) - \alpha Y \right), \quad j = 1, \dots, n, \quad (2.28)$$

where the first term of total costs represents the abatement costs and the second term represents the environmental damages. Observe that the global effective investment in R&D becomes a *public good*. Any investment made by a country reduces the total costs of all countries.

Next, the partial agreement Nash equilibrium of the investment game is calculated. As all countries, signatories and non-signatories do not cooperate at this stage, each non-signatory country selects investment to minimize the following expression of the total costs

$$\min_{\{x_i^f\}} TC_i^f(n) = \frac{d^2}{2c} + d \left( N\delta - \frac{d}{c}(N + n^2 - n) - \alpha Y \right) + x_i^f, \quad (2.29)$$

$$s.t. \quad x_i^f \geq 0, \quad i = 1, \dots, N - n. \quad (2.30)$$

For signatories countries, the optimization problem that yields the optimal investment is

$$\min_{\{x_j^s\}} TC_j^s(n) = \frac{n^2 d^2}{2c} + d \left( N\delta - \frac{d}{c}(N + n^2 - n) - \alpha Y \right) + x_j^s, \quad (2.31)$$

$$s.t. \quad x_j^s \geq 0, \quad j = 1, \dots, n. \quad (2.32)$$

Although these two linear programming problems are not identical to that of the fully non-cooperative equilibrium, the effect of investment on total costs is the same as



that obtained in the fully non-cooperative equilibrium which yields the same critical value of marginal damages given by (2.12), such that

$$\text{if } d \begin{cases} > \\ = \\ < \end{cases} \hat{d}^{nc}, \text{ then } \frac{\partial TC_i^f}{\partial x_i^f}, \frac{\partial TC_j^s}{\partial x_j^s} \begin{cases} < \\ = \\ > \end{cases} 0.$$

Therefore, when  $d$  is greater than  $\hat{d}^{nc}$ , total costs are *decreasing* with respect to the investment in R&D for both signatories and non-signatories independently of the level of participation, and both signatories and non-signatories invest in R&D until emissions are completely eliminated.<sup>4</sup> However, when  $d$  is lower than  $\hat{d}^{nc}$ , total costs are *increasing* with respect to the investment in R&D for both signatories and non-signatories countries and the optimal policy is not to invest.

Thus, when  $d \geq \hat{d}^{nc}$ , as both types of countries invest to eliminate completely the emissions, the optimization problem for non-signatories is the same presented by (2.13)-(2.15). However, for signatories countries, the optimization problem is represented by

$$\min_{\{x_j^s\}} TC_j^s = \frac{c}{2} (\delta - \alpha y_j^s)^2 + x_j^s, \quad (2.33)$$

$$\text{s.t. } y_j^s = x_j^s + \gamma X_{-j} \geq \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right), \quad (2.34)$$

$$y_j^s = x_j^s + \gamma X_{-j} \leq \frac{\delta}{\alpha}. \quad (2.35)$$

Assuming symmetry,  $X_{-i}$ ,  $X_{-j}$  can be written as

$$X_{-i} = (N - n - 1)x_i^f + nx_j^s, \quad X_{-j} = (N - n)x_i^f + (n - 1)x_j^s,$$

---

<sup>4</sup>We assume that  $\delta$  is high enough, in particular,  $\delta$  is higher than  $N/(\alpha c(1 + \gamma(N - 1)))$  in order to allow for the analysis at some levels of marginal damages higher than  $\hat{d}^{nc}$  and lower than the value of  $d = \delta c/N$  which yields the trivial solution.

By solving the optimization problems for both non-signatories and signatories, the following pair of reaction functions of investment in R&D are obtained

$$x_i^f = \frac{c\alpha (\delta - \alpha\gamma n x_j^s) - 1}{c\alpha^2 (1 + \gamma(N - n - 1))}, \quad (2.36)$$

$$x_j^s = \frac{c\alpha (\delta - \alpha\gamma (N - n) x_i^f) - 1}{c\alpha^2 (1 + \gamma(n - 1))}, \quad (2.37)$$

which establishes that the increase in investment of one type of countries reduces the investment of the other type, in other words, the investments in R&D are *strategic substitutes*.

The solution to the previous system yields the same optimal level of investment in R&D and effective investment, for both non-signatories and signatories, as those given by (2.16) and (2.17). It is known already that the constraint on effective investment for non-signatories is satisfied for any level of marginal damages higher than or equal to  $\tilde{d}^{nc}$  given by (2.18). However, as the constraint on effective investment for signatories countries given by (2.34) is different from the constraint on effective investment for non-signatories given by (2.14), it is important now to examine whether the level of effective investment given by (2.17) satisfies the constraint on effective investment for signatories countries given by (2.34) or not. It is found that the constraint is only satisfied at any level of marginal damages higher than or equal to

$$\tilde{d}^s(n) = \frac{1}{\alpha n}, \quad (2.38)$$

which is always lower than  $\tilde{d}^{nc}$  and higher than or equal to  $\hat{d}^{nc}$  provided that  $\gamma \geq \check{\gamma}$ , where

$$\check{\gamma} = \frac{n - 1}{N - 1}, \quad (2.39)$$

and vice versa.

As the order of the critical values of marginal damages will be changed depending on the value of the diffusion parameter  $\gamma$ , two possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}^s(n) > \hat{d}^{nc}.$$

Second, if  $\gamma < \check{\gamma}$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}^s(n).$$

In both cases, if  $d \geq \tilde{d}^{nc}$ , as both types of countries invest at the maximum to eliminate the GHG emissions, total costs are given by the same expression of the fully non-cooperative equilibrium given by (2.21) and the following proposition is concluded

**Proposition 1** *At the high levels of marginal damages, in particular if  $d \geq \tilde{d}^{nc}$ , total costs of both signatories and non-signatories countries of the emission agreement are the same as total costs of the fully non-cooperative equilibrium.*

Next, if  $\gamma \geq \check{\gamma}$ , and  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , as any level of marginal damages in this interval is lower than the levels given by (2.18) and (2.38), thus the level of effective investment given by (2.17) doesn't satisfy the constraints on effective investment for both types of countries in this interval. According to that, both non-signatories and signatories reduce their effective investment to the levels given by (2.6) and (2.24) respectively. In this case, the levels of investment for both types are obtained as follows

$$x_i^f = \frac{\bar{\delta}}{\alpha} - \gamma X_{-i}, \quad x_j^s = \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right) - \gamma X_{-j},$$

and the following pair of reaction functions are obtained

$$x_i^f = \frac{\frac{\bar{\delta}}{\alpha} - n\gamma x_j^s}{1 + \gamma(N - n - 1)}, \quad (2.40)$$

$$x_j^s = \frac{\frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right) - (N - n)\gamma x_i^f}{1 + \gamma(n - 1)}. \quad (2.41)$$

The solution to the previous system yields the following optimal level of investments

$$x_j^s = \frac{\delta(1 - \gamma) + \frac{d}{c}(\gamma n^2 - (1 + \gamma N)n + N\gamma)}{\alpha(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.42)$$

$$x_i^f = \frac{\delta(1 - \gamma) + \frac{d}{c}(\gamma n^2 - \gamma n - (1 - \gamma))}{\alpha(1 - \gamma)(1 + \gamma(N - 1))}. \quad (2.43)$$

In order to investigate the effect of cooperation on the level of investment for both signatories and non-signatories, the first derivatives of the levels of investment given by (2.42) and (2.43) are taken as follows

$$\frac{\partial x_j^s}{\partial n} = -\frac{d(1 + \gamma(N - 2n))}{c\alpha(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.44)$$

$$\frac{\partial x_i^f}{\partial n} = \frac{d\gamma(2n - 1)}{c\alpha(1 - \gamma)(1 + \gamma(N - 1))} > 0. \quad (2.45)$$

By analyzing (2.44), it is concluded that the effect of cooperation on the level of investment in R&D of a signatory country changes depending on the level of spillovers. Thus, it is sufficient that for any  $\gamma \leq 1/N$ , there is a negative relationship between the level of cooperation and the investment in R&D by a signatory country. For a non-signatory country, analyzing (2.45), it is obvious that there is always a positive relationship between the level of cooperation and the level of investment in R&D.

In order to guarantee positive levels of investment for both signatories and non-signatories, the levels of investments given by (2.42) and (2.43) are analyzed and the next proposition is concluded

**Proposition 2** *Non-signatories' investment is positive for all  $\gamma$  provided that  $d > \hat{d}^{nc}$  but this is not the case for signatories if  $\gamma > 1/N$ .*

Proof: Appendix 1.

This means that in the emission agreement, at the range of marginal damages given by  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , non-signatories will invest in R&D regardless the level of the spillovers. Nevertheless, the non-negative constraint applies for signatories countries (i.e. signatories invest at zero level) at the high levels of spillovers as the investment done by non-signatories will be enough to eliminate the emissions.

By taking the difference between the levels of investments given by (2.42) and (2.43)

$$x_j^s - x_i^f = - \left( \frac{d}{c\alpha(1-\gamma)} \right) (n-1) < 0,$$

it is obvious that the level of investment by a non-signatory country is higher than the level of investment by a signatory country. The explanation of this result returns to the fact that signatories reduce their investment than non-signatories because they have to do more efforts in reducing emissions as they cooperate in the third stage of the game. Using (2.26), the global effective investment can be written as follows

$$Y = \frac{N\bar{\delta}}{\alpha} - \frac{d}{\alpha c}(n^2 - n), \quad (2.46)$$

where the level of cooperation has a negative effect on the global effective investment. The logic behind this is that while the level of cooperation has no effect on the effective investment of non-signatories, it affects negatively the effective investment of signatories, thus, the net effect is the reduction of the global effective investment as the cooperation increases.

Doing the substitutions for effective investment, the total costs for non-signatories and signatories countries become

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{\delta(1-\gamma) + \frac{d}{c}(\gamma n^2 - \gamma n - (1-\gamma))}{\alpha(1-\gamma)(1+\gamma(N-1))}, \quad (2.47)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + \frac{\delta(1-\gamma) + \frac{d}{c}(\gamma n^2 - (1+\gamma N)n + N\gamma)}{\alpha(1-\gamma)(1+\gamma(N-1))}, \quad (2.48)$$

where the first term represents the abatement costs and the second term represents the investment costs. It can be directly concluded that cooperation has a positive effect on the total costs of non-signatories as cooperation in the third stage is not affecting the abatement cost of non-signatories, therefore, the effect of cooperation on the total costs of their total costs is given by the same expressions as (2.45).

In order to investigate the profitability of joining the emission agreement in the range of marginal damages  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , the total costs function of a signatory country given by (2.48) should be compared by the total costs function of playing fully non-cooperatively given by (2.20).

Now, the difference between (2.48) and (2.20) is taken as follows

$$TC_j^s(n) - TC_i^{nc} = \frac{d}{c} \left( (n^2 - 1) \frac{d}{2} + \frac{1}{\alpha(1+\gamma(N-1))} \left( \frac{\gamma n^2 - (1+\gamma N)n + \gamma N + 1 - \gamma}{1-\gamma} \right) \right), \quad (2.49)$$

which is increasing in  $d$ . Thus, substituting for  $d = \hat{d}^{nc}$  in (2.49), the difference in total costs becomes

$$TC_j^s(n) - TC_i^{nc} = \frac{n^2(1+\gamma) - 2(1+\gamma N)n + 2N\gamma + (1-\gamma)}{2c\alpha^2(1-\gamma)(1+\gamma(N-1))^2}. \quad (2.50)$$

This difference in the total costs is analyzed and the following proposition is concluded

**Proposition 3** *The emission agreement is not profitable in the range of marginal damages  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ .*

Proof: Appendix 2.

Thus, the emission agreement is dominated by the fully non-cooperative equilibrium in the range of marginal damages  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ .

Next, if  $\gamma \geq \check{\gamma}$  and  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , as any level of marginal damages in this interval is higher than the level of marginal damages given by (2.38), thus the level of effective investment given by (2.17) satisfies the constraint on effective investment for signatories countries only, while it doesn't satisfy the constraint on effective investment for non-signatories. According to that, the reaction functions of investment for non-signatories and signatories in this case are given by (2.40) and (2.37) respectively. The solution of these reaction functions yields the following levels of investment

$$x_i^f = \frac{(\delta c\alpha - \alpha d)(1 - \gamma) + \gamma n(1 - \alpha d)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.51)$$

$$x_j^s = \frac{(\delta c\alpha - 1)(1 - \gamma) - \gamma(N - n)(1 - \alpha d)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.52)$$

such that the level of effective investment for non-signatories is still given by (2.6), while the level of effective investment for signatories is given by (2.17). In order to investigate the effect of cooperation on the level of investment for both non-signatories and signatories, the first derivatives of the levels of investment given by (2.51) and (2.52) are taken and it is found that cooperation has a positive effect on the investment for both non-signatories and signatories provided that  $d < \tilde{d}^{nc} = 1/\alpha$  which is satisfied in this interval of marginal damages.

Doing the substitutions for effective investment in the total costs functions of non-signatories and signatories, the following expressions are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{(\delta c\alpha - \alpha d)(1 - \gamma) + \gamma n(1 - \alpha d)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.53)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2} + \frac{(\delta c\alpha - 1)(1 - \gamma) - \gamma(N - n)(1 - \alpha d)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.54)$$

where the first term represents the abatement costs and the second term represents the investment costs.

In order to investigate the profitability of joining the emission agreement at  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , the total costs function of a signatory country given by (2.54) should be compared by the total costs function of playing fully non-cooperatively. By substituting  $n = 1$  in (2.53), the total costs function of the fully non-cooperative equilibrium in this case are obtained as follows

$$TC_i^{mc} = \frac{d^2}{2c} + \frac{(\delta c\alpha - \alpha d)(1 - \gamma) + \gamma(1 - \alpha d)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}. \quad (2.55)$$

Now the difference between (2.55) and (2.54) is taken as follows

$$TC_i^{mc} - TC_j^s(n) = \frac{(1 - \alpha d)}{2c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))} (\gamma^2 A + \gamma B + 1 - \alpha d), \quad (2.56)$$

where  $A = (1 + \alpha d)(N - 1)$ , and  $B = N(1 - \alpha d) - 2(n - 1 - \alpha d)$ .

By analyzing the difference in total costs given by (2.56), it is found that total costs of signatory country are higher than total costs of the non-cooperative equilibrium for any level of cooperation higher than  $n^*$ , where  $n^*$  is given by

$$n^* = \frac{1}{2\gamma} ((N - 1)(1 + \alpha d)\gamma^2 + (N(1 - \alpha d) + 2(1 + \alpha d))\gamma + 1 - \alpha d). \quad (2.57)$$

Thus, for any level of cooperation higher than  $n^*$ , the emission agreement is not profitable in this range of marginal damages. Next, it is important to examine the profitability of the grand coalition by substituting for  $n = N$  in total costs function given by (2.54) which yields

$$TC_j^s(N) = \frac{1}{2c\alpha^2} + \frac{(\delta c\alpha - 1)(1 - \gamma)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.58)$$



and then taking the difference between (2.55) and (2.58) as follows

$$TC_i^{mc} - TC_j^s(N) = \frac{(1 - \alpha d)(\gamma^2(N - 1)(1 + \alpha d) - \gamma(N - 2)(1 + \alpha d) + 1 - \alpha d)}{2c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.59)$$

which is negative for any  $N > N^*$ , where  $N^*$  is given by

$$N^* = \frac{\gamma(1 + \alpha d)(2 - \gamma) + (1 - \alpha d)}{\gamma(1 - \gamma)(1 + \alpha d)}. \quad (2.60)$$

By taking the difference between  $N$  and  $N^*$  as follows

$$N - N^* = \frac{(1 + \alpha d)(N(1 - \gamma) - \gamma(2 - \gamma)) - (1 - \alpha d)}{\gamma(1 - \gamma)(1 + \alpha d)},$$

it is found that the difference is positive for any  $d$  higher than  $d^*$ , where  $d^*$  is given by

$$d^* = \frac{\gamma^2 N(\gamma - 1) + \gamma(2 - \gamma) + 1}{\alpha\gamma(N - 2) - \gamma^2\alpha(N - 1) + \alpha}. \quad (2.61)$$

Next, we compare  $d^*$  with  $\hat{d}^{nc}$  as follows

$$\hat{d}^{nc} - d^* = \frac{\gamma(N(1 - \gamma) + \gamma - 3)}{\alpha(1 - \gamma)(1 + \gamma(N - 1))},$$

which is positive for any  $\gamma > N - 3/N - 1$ , that converts to 1 at the high values of  $N$ . According to that, the differences in the total costs given by (2.59) is negative for any level of marginal damages higher than  $\hat{d}^{nc}$  and the following proposition is concluded

**Proposition 4** *The grand coalition of the emission agreement is not profitable in the range of marginal damages  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , while it could be profitable at some level of cooperation lower than the grand coalition.*

The following numerical example proves this result. In this numerical example, the total costs of both non-signatories and signatories given by (2.53) and (2.54) are calculated in the interval of marginal damages  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , under the following assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.5, \gamma = 0.75$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	2.9012	2.998	3.0948	3.1915	3.2883	3.385	3.4812	3.579
$TC_j^s$	2.0887	2.1855	2.2823	2.379	2.4758	2.573	2.669	2.766
<i>prof.</i>	0.619	0.5222	0.4254	0.3287	0.2319	0.135	0.0383	-0.058

Table 2.1: Profitability of emission agreement at  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ .

Note: The profitability (*prof.*) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium given by  $TC_i^{nc} = 2.7077$  and the total costs of signatories countries.

Notice that if  $\gamma < \check{\gamma}$  and  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the analysis is identical to that presented above and the previous proposition is applied here. The following numerical example proves this result using the same total costs functions as the previous numerical example, but now assuming that  $\gamma < \check{\gamma}$ .

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.5, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	6.2163	6.2420	6.2676	6.2933	6.3189	6.3446	6.3702	6.396
$TC_j^s$	6.0705	6.0962	6.1218	6.1474	6.1731	6.1987	6.2244	6.25
<i>prof.</i>	0.0946	0.0689	0.0433	0.0177	-0.008	-0.034	-0.059	-0.085

Table 2.2: Profitability of emission agreement at  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ .

Note: The profitability (*prof.*) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium given by  $TC_i^{nc} = 6.1651$  and the total costs of signatories countries.

Finally, when  $d$  is lower than  $\hat{d}^{nc}$ , total costs are increasing with respect to the investment in R&D for both non-signatories and signatories and the optimal policy is not to invest. In this case, the total costs are given as follows

$$TC_i^f(n) = \frac{d^2}{2c} + dN\delta - \frac{d^2}{c} (N + n^2 - n), \quad (2.62)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + dN\delta - \frac{d^2}{c} (N + n^2 - n). \quad (2.63)$$

It is immediate that total costs for signatories are always higher than total costs for non-signatories at any level of cooperation and we are in the standard model of emissions abatement with linear damages.

### 2.3.2 The Nash Equilibrium of the Membership Game

We use stability conditions to investigate which is the level of participation the emission agreement can achieve. First, we present the definition of coalitional stability from d'Aspremont et al. (1983), which has been extensively used in the literature on international environmental agreements.<sup>5</sup>

**Definition 5** *An agreement consisting of  $n$  signatories is stable if  $TC_j^s(n) \leq TC_i^f(n-1)$  for  $j = 1, \dots, n$  and  $TC_i^f(n) \leq TC_j^s(n+1)$  for  $i = 1, \dots, N - n$ .*

The first inequality, which is also known as the *internal stability condition*, simply means that any signatory country is at least as well-off staying in the agreement as

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<sup>5</sup>We avoid to use the term *self-enforcing* in the definition because as has been pointed out by McEvoy and Stranlund (2009) is a bit misleading. The concept refers to the stability of cooperative agreements, not to enforcing compliance with these agreements once they are signed.

withdrawing from it, assuming that all other countries do not change their membership decisions. The second inequality, which is also known as the *external stability condition*, similarly requires any non-signatory to be at least as well-off remaining a non-signatory as joining the agreement, assuming once again, that all other countries do not change their membership decisions. To check the stability conditions the auxiliary function  $\Omega(n) = TC_j^s(n) - TC_i^f(n-1)$  is used. If  $\Omega(n) = 0$  has a unique positive solution and  $\Omega(n)$  is increasing around this positive solution, then there is a self-enforcing agreement given by the greatest natural number on the left of the positive solution to equation  $\Omega(n) = 0$  provided that this number is equal to or lower than  $N$ . If we represent this number by  $\tilde{n}$ , we have that  $\Omega(\tilde{n})$  is negative and the internal stability condition is satisfied. Moreover, as  $\Omega(n)$  is an increasing function,  $\Omega(\tilde{n} + 1)$ , where  $\tilde{n} + 1$  is the lowest natural number on the right of the positive solution to equation  $\Omega(n) = 0$ , must be positive which means that  $TC_j^s(\tilde{n} + 1)$  is greater than  $TC_i^f(\tilde{n})$  which according to Definition 1 means that an agreement consisting of  $\tilde{n}$  countries is also externally stable.<sup>6</sup> If  $N$  is lower than  $\tilde{n}$ , the grand coalition could be stable provided that  $\Omega(N)$  is negative. If  $\Omega(n) = 0$  has more than one positive solutions, we could have more than one self-enforcing agreement.

Next, the stability analysis is performed to investigate whether there exists a stable emission agreement. As the emission agreement is not profitable except for some level of participation when the level of marginal damages is in the range of  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , we study next the stability of the agreement in this range only, as no country will have the incentive to participate in the agreement in the other ranges of marginal damages. Thus, the auxiliary function  $\Omega(n)$  is built using the total costs functions given by (2.53) and (2.54).

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<sup>6</sup>If the positive solution to  $\Omega(n) = 0$  is a natural number. The self-enforcing agreement consists of a number of signatories equal to the solution to the equation and the internal stability condition is satisfied as an equality.

$$\Omega(n) = TC_j^s(n) - TC_i^f(n-1),$$

$$\Omega(n) = \frac{-(1-\alpha d)(\gamma^2(N-1)(1+\alpha d) + \gamma(N-2)(1-\alpha d) + (1-\alpha d))}{2c\alpha^2(1-\gamma)(1+\gamma(N-1))} < 0.$$

This means that the internal stability condition is satisfied for any level of cooperation regardless the level of  $\gamma$ . By investigating the stability of the grand coalition, we find that  $\Omega(N) = \Omega(n)$ , which means that the internal stability condition for the grand coalition is satisfied and the following proposition is concluded<sup>7</sup>

**Proposition 6** *The grand coalition of the emission agreement is stable for any level of marginal damages  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ .*

This proposition is already proofed in table 2.1, as it is clear that  $TC_j^s(n) < TC_i^{nc}(n-1)$  for any level of cooperation. However, the grand coalition of the emission agreement will not be signed because it is not profitable for signatories.

According to the previous analysis, it is concluded that allowing for some technological spillovers (although it is not perfect) in the emission agreement reduces the incentives of countries to deviate and act as free-rider, which in turn increases the participation for some levels higher than three countries (as in the standard model) but lower than the grand coalition which is not profitable in this case.

In the following sections, we analyze other types of agreements that include technological cooperation in different ways in order to investigate if this technological cooperation has an effect on the profitability of signatories and their decisions on the participation or not.

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<sup>7</sup>Notice that for the grand coalition, there is no need to investigate the external stability condition.

## 2.4 Emission and R&D Agreement without Information Exchange

An emission and R&D agreement without information exchange is modeled as a three-stage game as the emission agreement. Given the level of participation and the investment in R&D of all countries, at the third stage, the emission game, non-signatory countries choose their emissions acting non-cooperatively and taking the emissions of all other countries as given in order to minimize their own costs of controlling pollution. On the other hand, signatories countries choose the emissions acting cooperatively in order to minimize the aggregate costs of the agreement. At the second stage, the R&D investment game, unlike the emission agreement, signatories countries act cooperatively so as to minimize agreement total costs, while each non-signatory country acts non-cooperatively to decide its own R&D level given the R&D investments of other countries. The marginal costs of abatement, as in the emission agreement, are decreased by the country's R&D efforts in addition to some spillover from other countries' R&D. Finally, the membership game is played in the first stage of the game.

### 2.4.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved, as done in the emission agreement, by backward induction assuming that in first stage  $n$  countries with  $n \geq 2$  have signed the agreement. As signatories cooperate while non-signatories countries act non-cooperatively at the third stage of the game, the emissions, the solution of this stage is identical to the solution that has been developed in the emission agreement.

Next, the partial agreement Nash equilibrium of the investment game is calculated. As the non-signatories countries do not cooperate at this stage, the solution is identical to the solution that has been developed in the emission agreement for

non-signatories. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$  where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, as they are minimizing the agreement total costs given by

$$ATC = \sum_{j=1}^n TC_j^s = \frac{n^3 d^2}{2c} + d \left( nN\delta - \frac{dn}{c}(N + n^2 - n) - \alpha nY \right) + \sum_{j=1}^n x_j^s,$$

their optimization problem which yields the optimal investment is given as follows

$$\min_{\{x_1^s, \dots, x_n^s\}} ATC = \frac{n^3 d^2}{2c} + d \left( nN\delta - \frac{dn}{c}(N + n^2 - n) - \alpha nY \right) + \sum_{j=1}^n x_j^s, \quad (2.64)$$

$$s.t. \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right) - \gamma X_{-j} \geq x_j^s, \quad (2.65)$$

$$x_j^s \geq 0, \quad (2.66)$$

where  $Y$  is given by (2.26).

By taking the first derivative of the agreement total costs function given by (2.64) with respect to investment in R&D, taking into account that  $\partial Y / \partial x_j^s = 1 + \gamma(N - 1)$ , the following critical value of marginal damages defined by condition  $\partial ATC / \partial x_j^s = 0$  is obtained

$$\hat{d}_1^s(n) = \frac{1}{\alpha n(1 + \gamma(N - 1))}. \quad (2.67)$$

Now, we have that the critical value of  $d$  which triggers investment in R&D depends on the number of signatories. It is easy to see that  $\hat{d}_1^s(n)$  decreases with respect to the level of cooperation and takes values between

$$\hat{d}_1^s(N) = \frac{1}{\alpha N(1 + \gamma(N - 1))} \leq \hat{d}_1^s(n) \leq \hat{d}_1^s(2) = \frac{1}{2\alpha(1 + \gamma(N - 1))}.$$

Therefore, when  $d$  is greater than  $\hat{d}_1^s(2)$ , the total costs of the agreement are *decreasing* with respect to investment independently of the level of participation, and

signatories invest in R&D until emissions are completely eliminated. However, when  $d$  is in the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the total costs of the agreement are decreasing depending on the number of signatories. In this case, for a given value of  $d$ , it is necessary a minimum of cooperation, given by  $d = \hat{d}_1^s(n)$ , to make the investment in R&D profitable. If this is not the case, signatories do not invest in clean technologies and cooperation is not enough to promote the replacement of fossil fuels. The condition  $d = \hat{d}_1^s(n)$  yields the following solution for  $n$

$$\hat{n}_1 = \frac{1}{\alpha d(1 + \gamma(N - 1))}. \quad (2.68)$$

Thus, given  $d$  in the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , participation in the agreement must be at least equal to the lowest natural number on the right of  $\hat{n}_1$  to make it profitable for signatories to invest in R&D. Moreover, as  $\hat{d}_1^s(n)$  is a decreasing function with respect to  $n$ , the minimum level of participation decreases as the marginal damages increase.

As we have clarified, the decision on investing in R&D for both signatories and non-signatories depends critically on the value of marginal damages. Taking into account this result, the optimal decision of the countries can be characterized as follows: If the damages are great enough, in particular when  $d > \hat{d}^{nc}$ , both signatories and non-signatories invest in R&D. If  $d$  belongs to the interval  $(\hat{d}_1^s(2), \hat{d}^{nc}]$ , only signatories invest in R&D independently of the number of signatories. However, if  $d$  belongs to the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , signatories will invest in R&D provided that the participation is greater than the critical value given by (2.68). Finally, if  $d$  is equal or lower than  $\hat{d}_1^s(N)$ , both signatories and non-signatories countries are not going to invest. Figure 2.1 shows which is the pattern of investment for signatories and non-signatories depending on the marginal damages and the number of signatories.



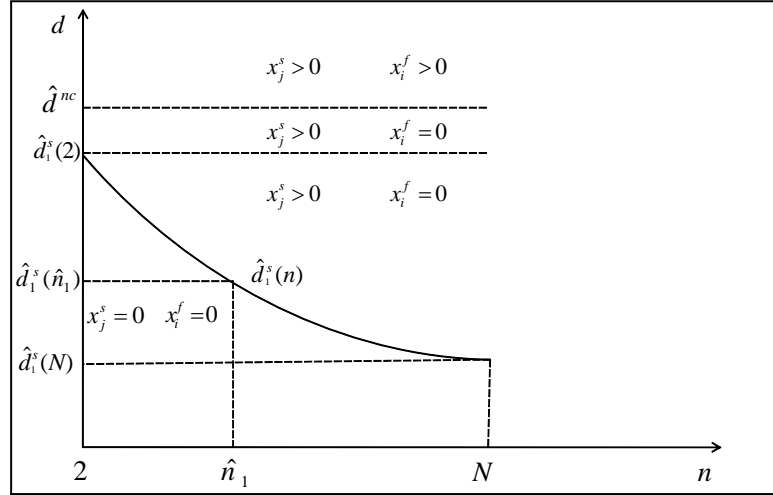


Figure 2.1: Decisions on investment in R&D for signatories and non-signatories of the emission and R&D agreement without information exchange.

**Conjecture 7**  $\hat{d}_1^s(n)$  defines the stable participation.

Next, the level of investment at the different levels of marginal damages is calculated. Three possibilities must be considered. First, both signatories and non-signatories invest in R&D. Second, only signatories invest. Third, both signatories and non-signatories do not find profitable to invest in R&D.

When  $d > \hat{d}^{nc}$ , both types of countries invest in R&D to eliminate completely the emissions. In this case, the optimization problem of non-signatories' countries is the same presented by (2.13)-(2.15). Nevertheless, for signatories countries, as they minimize the agreement total costs, the optimization problem is given as follows

$$\min_{\{x_1^s, \dots, x_j^s\}} ATC = \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + \sum_{j=1}^n x_j^s, \quad (2.69)$$

where the conditions on effective investment are the same given by (2.34)-(2.35). The solution of the optimization problem for signatories yields the following reaction function of signatories' investment

$$x_j^s = \frac{c\alpha n (\delta - \alpha\gamma (N - n) x_i^f) - 1}{c\alpha^2 n (1 + \gamma (n - 1))}, \quad (2.70)$$

while the reaction function of non-signatories' investment is still given by (2.36) which confirms again that the investments in R&D are *strategic substitutes*.

The solution of both (2.36) and (2.70) yields the optimal level of investments for both non-signatories and signatories as follows

$$x_i^f = \frac{(c\alpha\delta - 1)(1 - \gamma) - \gamma(n - 1)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.71)$$

$$x_j^s = \frac{(c\alpha\delta n - 1)(1 - \gamma) + \gamma(N - n)(n - 1)}{c\alpha^2 n(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.72)$$

such that the effective investment for signatories is now given by

$$y_j^s = \frac{c\alpha\delta n - 1}{c\alpha^2 n}, \quad (2.73)$$

while the effective investment for non-signatories is still given by (2.17).

We already know from the solution of fully non-cooperative equilibrium that non-signatories' effective investment given by (2.17) satisfies the constraint on effective investment given by (2.14) for any level of marginal damages higher than  $\tilde{d}^{nc}$  which is given by (2.18). However, for signatories countries, it is found that the level of effective investment given by (2.73) satisfies the constraint given by (2.34) at any level of marginal damages higher than or equal to

$$\tilde{d}_1^s(n) = \frac{1}{\alpha n^2}, \quad (2.74)$$

which is lower than  $\tilde{d}^{nc}$  and higher than  $\hat{d}^{nc}$  provided that  $\gamma \geq \check{\gamma}_1$  where

$$\check{\gamma}_1 = \frac{n^2 - 1}{N - 1}, \quad (2.75)$$

and vice versa. By comparing  $\tilde{d}_1^s(n)$  with  $\hat{d}_1^s(2)$ , it is found that  $\tilde{d}_1^s(n)$  is higher than  $\hat{d}_1^s(2)$  for any  $\gamma \geq \gamma_1^*$  where

$$\gamma_1^* = \frac{n^2 - 2}{2(N - 1)}, \quad (2.76)$$

and vice versa.<sup>8</sup>

As the order of the critical values of marginal damages will be changed depending on the value of the diffusion parameter, three possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}_1$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}_1^s(n) > \hat{d}^{nc} > \hat{d}_1^s(2) > \hat{d}_1^s(N).$$

Second, if  $\gamma \in (\gamma_1^*, \check{\gamma}_1]$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}_1^s(n) > \hat{d}_1^s(2) > \hat{d}_1^s(N),$$

and finally, if  $\gamma < \gamma_1^*$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_1^s(2) > \tilde{d}_1^s(n) > \hat{d}_1^s(N).$$

Notice that for high levels of cooperation, the value of  $\check{\gamma}_1$  is going to be very high, even higher than 1 for some levels of cooperation. As it is more suitable to assume that  $\gamma$  is not very high, our analysis for profitability of joining the agreement will be focused on the second and third cases when  $\gamma \in (\gamma_1^*, \check{\gamma}_1]$  and  $\gamma < \gamma_1^*$ . However, the levels of investments and the corresponding total costs functions of non-signatories and signatories countries will be analyzed for the different three cases.

In all cases, if  $d \geq \tilde{d}^{nc}$ , as the levels of effective investment for both non-signatories and signatories given by (2.17) and (2.73) satisfy, in this interval of marginal damages,

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<sup>8</sup>Notice that  $\check{\gamma}_1 > \gamma_1^*$ , then any  $\gamma > \check{\gamma}_1$  is also higher than  $\gamma_1^*$ .

the constraints on effective investment for both types given by (2.14) and (2.34) respectively, the following expressions of total costs are obtained <sup>9</sup>

$$TC_i^f(n) = \frac{1}{2c\alpha^2} + \frac{(c\alpha\delta - 1)(1 - \gamma) - \gamma(n - 1)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.77)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2 n^2} + \frac{(c\alpha\delta n - 1)(1 - \gamma) + \gamma(N - n)(n - 1)}{c\alpha^2 n(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.78)$$

where the first term represents the abatement costs and the second term represents the investment cost.

Next, we analyze the case when  $\gamma \geq \check{\gamma}_1$ . If  $d \in (\tilde{d}_1^s(n), \tilde{d}^{nc}]$ , as any level of marginal damages in this interval is higher than the level of marginal damages given by (2.74), thus the level of effective investment given by (2.73) satisfies the constraint on effective investment for signatories countries given by (2.34). However, the level of effective investment for non-signatories given by (2.17) doesn't satisfy, in this interval of marginal damages, the constraint on their effective investment given by (2.14), and therefore non-signatories countries reduce their effective investment to the level given by (2.6). According to that, the reaction function of signatories' investment is the same given by (2.70), while the reaction function for non-signatories is given as follows

$$x_i^f = \frac{\delta c - d - \alpha c \gamma n x_j^s}{\alpha c (1 + \gamma(N - n - 1))}. \quad (2.79)$$

The solution of both reaction functions yields the following levels of investment for non-signatories and signatories

$$x_i^f = \frac{(c\alpha\delta - \alpha d)(1 - \gamma) + \gamma(1 - \alpha dn)}{c\alpha^2(1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.80)$$

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<sup>9</sup>We avoid analyzing the profitability in this case, as it will be concluded in the analysis of the Nash equilibrium of the membership agreement that the agreement is not stable in this interval of marginal damages.

$$x_j^s = \frac{c\alpha\delta n(1-\gamma) + \gamma\alpha dn(N-n) - (1+\gamma(N-n-1))}{c\alpha^2 n(1-\gamma)(1+\gamma(N-1))}. \quad (2.81)$$

Doing the substitutions for effective investment in the total costs functions of non-signatories and signatories, the following expressions are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{(c\alpha\delta - \alpha d)(1-\gamma) + \gamma(1-\alpha dn)}{c\alpha^2(1-\gamma)(1+\gamma(N-1))}, \quad (2.82)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2 n^2} + \frac{c\alpha\delta n(1-\gamma) + \gamma\alpha dn(N-n) - (1+\gamma(N-n-1))}{c\alpha^2 n(1-\gamma)(1+\gamma(N-1))}, \quad (2.83)$$

where the first term represents the abatement costs and the second term represents the investment costs.

Next, if  $d \in (\hat{d}^{nc}, \tilde{d}_1^s(n)]$ , the levels of effective investment for non-signatories and signatories are given by (2.6) and (2.24) respectively. In this case the solution of the second stage is the same as that developed in the emission agreement where the total costs of non-signatories and signatories are given by (2.47) and (2.48) respectively and the same proposition of the emission agreement is directly concluded here

**Proposition 8** *Emission and R&D agreement without information exchange is not profitable in the range of marginal damages  $d \in (\hat{d}^{nc}, \tilde{d}_1^s(n)]$ .*

Now, if  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$ , as marginal damages are low enough to make it unprofitable for non-signatories to invest in R&D, then the level of investment for signatories countries can be obtained as follows

$$x_j^s = \frac{(\delta - \frac{nd}{c})}{\alpha(1+\gamma(n-1))}. \quad (2.84)$$

Notice that the level of investment is decreasing with the level of cooperation.

The effective level of investment for signatories is still the same given by (2.24), while for the non-signatories countries it becomes

$$y_i^f = \gamma n x_j^s = \frac{\gamma n (\delta - \frac{nd}{c})}{\alpha(1 + \gamma(n - 1))}. \quad (2.85)$$

while the global effective investment is given by

$$Y = \frac{n (\delta - \frac{nd}{c}) (1 + \gamma(N - 1))}{\alpha(1 + \gamma(n - 1))}. \quad (2.86)$$

By substituting (2.86) in (2.25), the global emissions, which is the sum of non-signatories emissions, is given as follows

$$E = \frac{(N - n)(\bar{\delta}(1 - \gamma) + \frac{d}{c}(n^2 - n)\gamma)}{(1 + \gamma(n - 1))}. \quad (2.87)$$

Doing the substitution of the effective investments in the total costs functions, the following expressions for the total costs of non-signatories and signatories are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{d(N - n)(\bar{\delta}(1 - \gamma) + \frac{d}{c}(n^2 - n)\gamma)}{(1 + \gamma(n - 1))}, \quad (2.88)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + \frac{d(N - n)(\bar{\delta}(1 - \gamma) + \frac{d}{c}(n^2 - n)\gamma)}{(1 + \gamma(n - 1))} + \frac{(\delta - \frac{nd}{c})}{\alpha(1 + \gamma(n - 1))}, \quad (2.89)$$

where the first term represents the abatement costs and the second term represents the environmental damages, while the third term in the total costs of signatories countries represents the investment costs.

However, if  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.88) and (2.89) until we reach to the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$  in figure 2.1, at this point the total costs function of signatories countries is still the same given by (2.89), while non-signatories' total costs function should be changed taking into account that by moving from the

area above the curve  $\hat{d}_1^s(n)$  in figure 2.1 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by

$$TC_i^f(n) = \frac{d^2}{2c} + d \left( N\delta - \frac{d}{c} (N + n^2 - n) \right). \quad (2.90)$$

Second, if  $\gamma \in (\gamma_1^*, \check{\gamma}_1]$  and  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs for non-signatories are given by (2.82), while given by (2.83) for signatories countries. However, if  $d \in (\tilde{d}_1^s(n), \hat{d}^{nc}]$ , signatories countries invest to eliminate the emissions while non-signatories countries don't invest at all. In this case, the global level of emissions is given by the sum of non-signatories emissions as follows

$$E = \sum_{i=1}^{N-n} E_i^f = \sum_{i=1}^{N-n} \left( \delta - \frac{d}{c} - \alpha y_i^f \right) = (N-n) \left( \delta - \frac{d}{c} \right) - \alpha \sum_{i=1}^{N-n} y_i^f, \quad (2.91)$$

where

$$\sum_{i=1}^{N-n} y_i^f = Y^f = \sum_{i=1}^{N-n} \left( \gamma \sum_{j=1}^n x_j^s \right).$$

Now, the optimization problem of the second stage for signatories countries should be represented as follows

$$\begin{aligned} \min_{\{x_1^s, \dots, x_n^s\}} ATC &= \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + dn \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + \sum_{j=1}^n x_j^s, \\ s.t. \ y_j^s &= (1 + \gamma(n-1)) x_j^s \geq \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right), \end{aligned} \quad (2.92)$$

$$y_j^s = (1 + \gamma(n-1)) x_j^s \leq \frac{\delta}{\alpha}. \quad (2.93)$$

The solution of the previous optimization problem yields the following level of investment and effective investment for signatories countries

$$x_j^s = \frac{\alpha n (d\gamma(N-n) + c\delta) - 1}{\alpha^2 cn (1 + \gamma(n-1))}, \quad (2.94)$$

$$y_j^s = \frac{\alpha n (d\gamma(N-n) + c\delta) - 1}{\alpha^2 cn}, \quad (2.95)$$

such that the condition on effective investment given by (2.92) is satisfied at any level of marginal damages higher than

$$\check{d}_1^s(n) = \frac{1}{\alpha n (n + \gamma (N - n))}, \quad (2.96)$$

which is lower than  $\hat{d}_1^s(n)$ . The critical value of marginal damages  $\check{d}_1^s(n)$  differs from  $\tilde{d}_1^s(n)$  given by (2.74), because  $\check{d}_1^s(n)$  is calculated assuming that investment of non-signatories equal to zero, while  $\tilde{d}_1^s(n)$  was calculated assuming that non-signatories investment is positive.

As  $\check{d}_1^s(n) < \hat{d}_1^s(n) < \hat{d}_1^s(2)$ , it can be concluded that the solution in the range of marginal damages given by  $d \in (\hat{d}_1^s(2), \hat{d}_1^{nc}]$  is the same as presented above, such that the total costs functions are given by

$$TC_i^f(n) = \frac{d^2}{2c} + d \left( (N - n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right), \quad (2.97)$$

$$TC_j^s(n) = \frac{(1 - \alpha n d \gamma (N - n))^2}{2\alpha^2 c n^2} + d \left( (N - n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + x_j^s, \quad (2.98)$$

where the first term represents the abatement costs and the second term represents the environmental damages, while the third term in the total costs of signatories countries represents the investment costs. Notice that  $x_j^s$  is given by (2.94), while  $Y^f$  is given as follows

$$Y^f = \gamma (N - n) \left( \frac{\alpha n (d \gamma (N - n) + c \delta) - 1}{\alpha^2 c (1 + \gamma (n - 1))} \right). \quad (2.99)$$

However, if  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.97) and (2.98) until we reach to the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$  in figure 2.1, at this point the total costs function of signatories countries is still the same given by (2.98), while non-signatories'



total costs function is given by (2.90) assuming that signatories investment equal to zero.

Finally, if  $\gamma < \gamma_1^*$ , it is clear that for any level of marginal damages lower than  $\hat{d}^{nc}$ , the critical value of marginal damages given by (2.74) doesn't play role in satisfying the constraint over signatories' effective investment given by (2.92) and the new critical value of marginal damages (2.96) is the one that plays this role. Thus, it can be concluded that the analysis, at the different intervals of marginal damages, under the assumption  $\gamma < \gamma_1^*$  is exactly the same as the analysis developed under the assumption  $\gamma \in (\gamma_1^*, \check{\gamma}_1]$ .

Next, the profitability of joining the emission and R&D agreement without information exchange is analyzed, numerically for any  $\gamma < \check{\gamma}_1$ , at the different levels of marginal damages. First, In order to investigate the profitability of joining the emission and R&D agreement without information exchange at  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs function of a signatory country given by (2.83) should be compared by the total costs function of playing fully non-cooperatively. By substituting for  $n = 1$  in (2.82), the total costs function of the fully non-cooperative equilibrium in this case is the same given by (2.55). By taking the difference between (2.55) and (2.83), the following proposition is concluded

**Proposition 9** *Emission and R&D agreement without information exchange is profitable at  $d \in (\tilde{d}_1^s(n), \tilde{d}^{nc}]$  only for the high levels of cooperation.*

The following numerical example proofs this result where

$$\alpha = 1, \quad c = 2, \quad \delta = 20, \quad N = 10, \quad d = 0.5, \quad \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	6.1138	6.088	6.062	6.037	6.011	5.986	5.9599	5.934
$TC_j^s$	6.1902	6.2079	6.21	6.204	6.192	6.177	6.1598	6.141
$prof.$	-0.025	-0.043	-0.045	-0.038	-0.027	-0.012	0.0053	0.024

Table 2.3: Profitability of emission and R&D agreement without information exchange at  $d \in (\hat{d}^{nc}, \hat{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 6.1651$ ) and the total costs of signatories countries.

Second, the profitability is examined at both intervals of marginal damages  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$  and  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$  and the following proposition is concluded

**Proposition 10** *Emission and R&D agreement without information exchange is profitable at  $d \in (\hat{d}_1^s(N), \hat{d}^{nc}]$ .*

The following numerical example proofs this proposition for  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$  by using the total costs of both non-signatories and signatories given by (2.97) and (2.98), under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.27, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	18.6	13.61	9.903	7.035	5.942	2.889	1.336	0.0182
$TC_j^s$	31.96	25.07	19.92	15.93	12.75	10.15	7.977	6.139
$prof.$	4.3	11.19	16.34	20.33	23.51	26.11	28.28	30.121

Table 2.4, Profitability of emission and R&D agreement without information exchange at  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 36.259$ ) and the total costs of signatories countries.

Finally, the previous proposition is proofed for  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$  using the total costs of both non-signatories and signatories given by (2.97) and (2.98), until we reach

to the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$ , the total costs function of non-signatories is replaced by (2.90). Under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, \gamma = 0.25,$$

if it is assumed that  $d = 0.11$ , the minimum level of cooperation needed to make the investment in R&D profitable is given by

$$\hat{n}_1 = 2.7972.$$

According to that, the profitability of  $n = 3$ , is examined using total costs functions given by (2.90) and (2.98) as follows

$$TC_i^{nc} = 21.943, TC_i^f(2) = 21.93, TC_j^s(3) = 20.971, prof. = 0.972.$$

However, the profitability of any level of cooperation higher than  $n = 3$  is examined using total costs functions given by (2.97) and (2.98) and the results are shown in next table

$n$	4	5	6	7	8	9	10
$TC_i^f$	5.6399	4.1085	2.9205	1.9718	1.1965	0.55042	0.00303
$TC_j^s$	17.041	14.090	11.794	9.9568	8.4535	7.2001	6.1385
$prof.$	-2.149	0.802	3.098	4.9352	6.4385	7.6919	8.7535

Table 2.5, Profitability of emission and R&D agreement without information exchange at  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 14.982$ ) and the total costs of signatories countries.

### 2.4.2 The Nash Equilibrium of the Membership Game

In this section, the stability analysis is studied to investigate whether there exist a stable emission and R&D agreement without information exchange or not. For  $d \geq \tilde{d}^{nc}$ , the auxiliary function  $\Omega(n)$  is built using the total costs functions given by (2.77) and (2.78)

$$\Omega(n) = \frac{n^2 (1 + \gamma(N - 2) + \gamma^2(N - 1)) - 2n(1 + \gamma(N - 1)) + 1 + \gamma(N - 2) - \gamma^2(N - 1)}{2n^2 c \alpha^2 (1 - \gamma)(1 + \gamma(N - 1))}, \quad (2.100)$$

which is a convex function that has a minimum at

$$n = \frac{(1 + \gamma(N - 1))}{(1 + \gamma(N - 2) + \gamma^2(N - 1))}. \quad (2.101)$$

At the minimum given by (2.101), the auxiliary function  $\Omega(n)$  given by (2.100) is positive, which means that the agreement is not stable for any level of cooperation. Thus, for the grand coalition, the analysis of the auxiliary function  $\Omega(N)$  concludes the same result as follows

$$\Omega(N) = \frac{\gamma^2(N^2 - 1)(N - 1) + \gamma(N^2(N - 4) + 3N - 2) + N^2 - 2N + 1}{2N^2 c \alpha^2 (1 - \gamma)(1 + \gamma(N - 1))} > 0, \quad (2.102)$$

and the following proposition is concluded

**Proposition 11** *At the high values of marginal damages, in particular if  $d \geq \tilde{d}^{nc}$ , the emission and R&D agreement without information exchange is not stable for any level of cooperation.*

By analyzing the previous numerical example, looking at the total costs of non-signatories and signatories at the different levels of marginal damages, the following proposition is concluded

**Proposition 12** *Emission and R&D agreement without information exchange is only stable at the level of cooperation given by the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$  in the interval  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ .*

## 2.5 Emission Agreement with Information Exchange on R&D Investment

The emission agreement with information exchange on R&D investment is modeled as a three stage game as the emission agreement. The main difference between the two agreements is that the signatories countries in the emission agreement with information exchange share their R&D efforts and avoid the duplication of R&D activities. As a result of sharing R&D efforts, the marginal costs of abatement of signatories countries are decreased by the sum of all R&D efforts in the agreement, in addition to some spillover from non-signatories countries' R&D investments, i.e. spillover is increased for signatories because of the agreement. Then if all countries are in the agreement, the effective investment in R&D is given by

$$y_i = X = \sum_{j=1}^N x_j, \quad i = 1, \dots, N.$$

### 2.5.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved, as done in the emission agreement, by backward induction assuming that in first stage  $n$  countries with  $n \geq 2$  have signed the agreement. As signatories cooperate while non-signatories countries act non-cooperatively at the third stage of the game, the emissions, the solution of this stage is identical to the solution that has been developed in the emission agreement. Nevertheless, the global level of investment is now given by

$$Y = \sum_{i=1}^{N-n} \left( x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n-1} x_l^f \right) \right) + \sum_{j=1}^n \left( \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \right), \quad (2.103)$$

Next, the partial agreement Nash equilibrium of the investment game is calculated. As the non-signatories countries do not cooperate at this stage, the solution is identical to the solution that has been developed in the emission agreement for non-signatories. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$  where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, acting non-cooperatively at this stage, the optimization problem that yields the optimal investment is given as follows

$$\min_{x_j^s} TC_j^s(n) = \frac{n^2 d^2}{2c} + d \left( N\delta - \frac{d}{c}(N + n^2 - n) - \alpha Y \right) + x_j^s, \quad (2.104)$$

$$s.t. \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right) - X_{-j}^s - \gamma X^f \geq x_j^s, \quad (2.105)$$

$$x_j^s \geq 0, \quad j = 1, \dots, n, \quad (2.106)$$

where  $Y$  is given by (2.103).

By taking the first derivative of the total costs function given by (2.104) with respect to investment in R&D, taking into account that  $\partial Y / \partial x_j^s = n + \gamma(N - n)$ , the following critical value of marginal damages defined by condition  $\partial TC_j^s / \partial x_j^s = 0$  is obtained

$$\hat{d}_2^s(n) = \frac{1}{\alpha(n + \gamma(N - n))}. \quad (2.107)$$

As in the emission and R&D agreement without information exchange, the critical value of  $d$  which triggers investment in R&D depends in the number of signatories. It is easy to see that  $\hat{d}_2^s(n)$  decreases with respect to the level of cooperation and takes values between

$$\hat{d}_2^s(N) = \frac{1}{\alpha N} \leq \hat{d}_2^s(n) \leq \hat{d}_2^s(2) = \frac{1}{\alpha(2 + \gamma(N - 2))}.$$

When  $d$  is greater than  $\hat{d}_2^s(2)$ , the total costs of the agreement are *decreasing* with respect to investment independently of the level of participation, and signatories invest in R&D until emissions are completely eliminated. However, when  $d$  is in the interval  $(\hat{d}_2^s(N), \hat{d}_2^s(2)]$ , the total costs of the agreement are decreasing depending on the number of signatories. In this case, it is necessary a minimum of cooperation, given by  $d = \hat{d}_2^s(\hat{n}_2)$ , to make the investment in R&D profitable. If this is not the case, signatories do not invest in clean technologies and cooperation is not enough to promote the replacement of fossil fuels. The condition  $d = \hat{d}_2^s(\hat{n}_2)$  yields the following solution for  $n$

$$\hat{n}_2 = \frac{1}{1-\gamma} \left( \frac{1}{\alpha d} - N\gamma \right). \quad (2.108)$$

The decision on investing in R&D for both signatories and non-signatories depends critically on the marginal damages in the same way as declared in the emission and R&D agreement without information exchange. Figure 2.2 shows which is the pattern of investment for signatories and non-signatories depending on the marginal damages and the number of signatories.

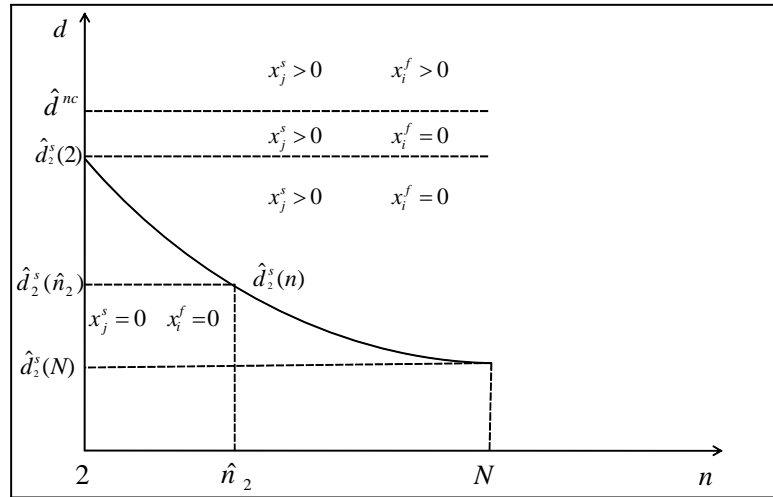


Figure 2.2: Decisions on investment in R&D for signatories and non-signatories of the emission agreement with information exchange on R&D investment.

**Conjecture 13**  $\hat{d}_2^s(n)$  defines the stable participation.

Next, the level of investment at the different levels of marginal damages is calculated. The same three possibilities that have been considered in the emission and R&D agreement without information exchange are considered here. First, both signatories and non-signatories invest in R&D. Second, only signatories invest. Third, both signatories and non-signatories do not find profitable to invest in R&D.

When  $d > \hat{d}^{nc}$ , both types of countries invest in R&D to eliminate completely the emissions. In this case, the optimization problem of non-signatories' countries is the same presented by (2.13)-(2.15). Nevertheless, for signatories countries, the optimization problem is given as follows

$$\min_{x_j^s} TC_j^s = \frac{c}{2} (\delta - \alpha y_j^s)^2 + x_j^s, \quad (2.109)$$

$$s.t. \ y_j^s = nx_j^s + \gamma(N-n)x_i^f \geq \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right), \quad (2.110)$$

$$y_j^s = nx_j^s + \gamma(N-n)x_i^f \leq \frac{\delta}{\alpha}. \quad (2.111)$$

The solution of the optimization problem for signatories yields the following reaction function of signatories' investment

$$x_j^s = \frac{c\alpha \left( \delta - \alpha\gamma(N-n)x_i^f \right) - 1}{c\alpha^2 n}, \quad (2.112)$$

while the reaction function of non-signatories' investment is still given by (2.36) such that the investments in R&D are *strategic substitutes*.

The solution of both (2.36) and (2.112) yields the optimal levels of investments for both non-signatories and signatories as follows

$$x_i^f = \frac{c\alpha\delta - 1}{c\alpha^2(1 + \gamma(N-n))}, \quad (2.113)$$

$$x_j^s = \frac{c\alpha\delta - 1}{c\alpha^2 n(1 + \gamma(N-n))}. \quad (2.114)$$



Although signatories' investment is less than non-signatories' investment, the effective investment for both types of countries is the same, as in the fully non-cooperative equilibrium, given by (2.17). We already know from the solution of fully non-cooperative equilibrium that non-signatories' effective investment given by (2.17) satisfies the constraint on effective investment given by (2.14) for any level of marginal damages higher than  $\tilde{d}^{nc}$  which is given by (2.18). However, for signatories countries, it is found that the level of effective investment given by (2.17) satisfies the constraint given by (2.110) at any level of marginal damages higher than or equal to  $\tilde{d}^s(n)$  given by (2.38). It is known from the analysis of emission agreement that  $\tilde{d}^s(n)$  is lower than  $\tilde{d}^{nc}$  and higher than or equal to  $\hat{d}^{nc}$  provided that  $\gamma \geq \check{\gamma}$  and by comparison, it is found that  $\tilde{d}^s(n) \geq \hat{d}_2^s(n)$  for any  $\gamma \geq \gamma_2^*$  where

$$\gamma_2^* = \frac{n-2}{N-2}, \quad (2.115)$$

and vice versa.

As the order of the critical values of marginal damages will be changed depending on the value of the diffusion parameter, three possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}^s(n) > \hat{d}^{nc} > \hat{d}_2^s(2) > \hat{d}_2^s(N).$$

Second, if  $\gamma \in (\gamma_2^*, \check{\gamma}]$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}^s(n) > \hat{d}_2^s(2) > \hat{d}_2^s(N),$$

and finally, if  $\gamma < \gamma_2^*$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_2^s(2) > \tilde{d}^s(n) > \hat{d}_2^s(N).$$

For high levels of cooperation, the value of  $\check{\gamma}$  is going to be very high, reaches to 1 at the level of full cooperation. As it is more suitable to assume that  $\gamma$  is not very high, our analysis for profitability of joining the agreement will be focused on the second and third cases when  $\gamma \in (\gamma_2^*, \check{\gamma}]$  and  $\gamma < \gamma_2^*$ . However, the levels of investments and the corresponding total costs functions of non-signatories and signatories countries will be analyzed for the different three cases.

In all cases, if  $d \geq \tilde{d}^{nc}$ , as the level of effective investment for both non-signatories and signatories given by (2.17) satisfies, in this interval of marginal damages, the constraints on effective investment for both types given by (2.14) and (2.110) respectively, the following expressions of total costs are obtained

$$TC_i^f(n) = \frac{1}{2c\alpha^2} + \frac{c\alpha\delta - 1}{c\alpha^2(1 + \gamma(N - n))}, \quad (2.116)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2} + \frac{c\alpha\delta - 1}{c\alpha^2 n(1 + \gamma(N - n))}. \quad (2.117)$$

In order to investigate the profitability of joining the agreement at this interval of marginal damages, the total costs function of a signatory country given by (2.117) is compared by the total costs function of playing fully non-cooperatively given by substituting for  $n = 1$  in (2.116) which is the same given by (2.21). The comparison yields the following expression

$$TC_j^s(n) - TC_i^{mc} = -\frac{(c\alpha\delta - 1)(n - 1)(1 + \gamma(N - n - 1))}{c\alpha^2 n(1 + \gamma(N - n))(1 + \gamma(N - 1))} < 0,$$

which means that total costs of signatories are lower than total costs of playing fully non-cooperatively and the following proposition is concluded

**Proposition 14** *The emission agreement with information exchange on R&D investment is profitable at  $d \geq \tilde{d}^{nc}$ .*

First, we analyze the case when  $\gamma \geq \check{\gamma}$ . If  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , as any level of marginal damages in this interval is higher than the level of marginal damages given by (2.38), thus the level of effective investment given by (2.17) satisfies the constraint on effective investment for signatories countries given by (2.110). However, this level of effective investment doesn't satisfy, in this interval of marginal damages, the constraint on non-signatories' effective investment given by (2.14), and therefore non-signatories countries reduce their effective investment to the level given by (2.6). According to that, the reaction function of non-signatories' investment is the same given by (2.79), while for signatories it is given as follows

$$x_j^s = \frac{c\alpha \left( \delta - \alpha\gamma(N-n)x_i^f \right) - 1}{c\alpha^2 n}. \quad (2.118)$$

The solution of the reaction functions yields the following levels of investment for non-signatories and signatories

$$x_i^f = \frac{c\alpha\delta(1-\gamma) + \gamma - \alpha d}{c\alpha^2(1-\gamma)(1+\gamma(N-n))}, \quad (2.119)$$

$$x_j^s = \frac{c\alpha\delta(1-\gamma) + \gamma\alpha d(N-n) - (1+\gamma(N-n-1))}{c\alpha^2 n(1-\gamma)(1+\gamma(N-n))}. \quad (2.120)$$

Doing the substitutions for effective investment in the total costs functions of non-signatories and signatories, the following expressions are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{c\alpha\delta(1-\gamma) + \gamma - \alpha d}{c\alpha^2(1-\gamma)(1+\gamma(N-n))}, \quad (2.121)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2} + \frac{c\alpha\delta(1-\gamma) + \gamma\alpha d(N-n) - (1+\gamma(N-n-1))}{c\alpha^2 n(1-\gamma)(1+\gamma(N-n))}, \quad (2.122)$$

where the first term represents the abatement costs and the second term represents the investment costs.

Next, if  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , the levels of effective investment for non-signatories and signatories are given by (2.6) and (2.24) respectively. In this case, the reaction functions for non-signatories and signatories are given by

$$x_i^f = \frac{\delta c - d - \alpha c \gamma n x_j^s}{\alpha c (1 + \gamma (N - n - 1))}, \quad (2.123)$$

$$x_j^s = \frac{\delta c - nd - \alpha c \gamma (N - n) x_i^f}{\alpha c n}. \quad (2.124)$$

The solution to these reaction functions yields the following levels of investment

$$x_i^f = \frac{\delta c (1 - \gamma) + d (\gamma n - 1)}{\alpha c (1 - \gamma) (1 + \gamma (N - n))}, \quad (2.125)$$

$$x_j^s = \frac{\delta c (1 - \gamma) + dn (\gamma n - 1) - dN\gamma (n - 1)}{\alpha c n (1 - \gamma) (1 + \gamma (N - n))}. \quad (2.126)$$

According to that, total costs are given by

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{\delta c (1 - \gamma) + d (\gamma n - 1)}{\alpha c (1 - \gamma) (1 + \gamma (N - n))}, \quad (2.127)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + \frac{\delta c (1 - \gamma) + dn (\gamma n - 1) - dN\gamma (n - 1)}{\alpha c n (1 - \gamma) (1 + \gamma (N - n))}. \quad (2.128)$$

Now, if  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$ , as marginal damages are low enough to make it unprofitable for non-signatories to invest in R&D, then the level of investment for signatories countries can be obtained as follows

$$x_j^s = \frac{1}{\alpha n} \left( \delta - \frac{nd}{c} \right), \quad (2.129)$$

where the level of investment is decreasing with the level of cooperation.

The effective level of investment for signatories is still the same given by (2.24), while for the non-signatories countries it becomes

$$y_i^f = \frac{\gamma}{\alpha} \left( \delta - \frac{nd}{c} \right), \quad (2.130)$$

which is increasing with the spillover, while the global effective investment is given by

$$Y = \frac{1}{\alpha} \left( \left( \delta - \frac{nd}{c} \right) (n(1 - \gamma) + \gamma N) \right). \quad (2.131)$$

Finally, adding the emissions for non-signatories, we obtain the global emissions as follows

$$E = (N - n) \left( \bar{\delta} - \gamma \left( \delta - \frac{nd}{c} \right) \right). \quad (2.132)$$

Doing the substitution of the effective investments in the total costs functions, the following expressions for the total costs of non-signatories and signatories are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + d(N - n) \left( \bar{\delta} - \gamma \left( \delta - \frac{nd}{c} \right) \right), \quad i = 1, \dots, N - n, \quad (2.133)$$

$$TC_j^s(n) = \frac{n^2 d^2}{2c} + d(N - n) \left( \bar{\delta} - \gamma \left( \delta - \frac{nd}{c} \right) \right) + \frac{1}{\alpha n} \left( \delta - \frac{nd}{c} \right), \quad j = 1, \dots, n, \quad (2.134)$$

where the first term represents the abatement costs and the second term represents the environmental damages, while the third term in the total costs of signatories countries represents the investment costs.

However, if  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.133) and (2.134) until we reach to the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$  in figure 2.2, at this point the total costs function of signatories countries is still the same given by (2.134), while non-signatories' total costs function should be changed taking into account that by

moving from the area above the curve  $\hat{d}_2^s(n)$  in figure 2.2 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by (2.90).

Second, if  $\gamma \in (\gamma_2^*, \check{\gamma}]$  and  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs for non-signatories are given by (2.121), while given by (2.122) for signatories countries. However, if  $d \in (\check{d}^s(n), \hat{d}^{nc}]$ , signatories countries invest to eliminate the emissions while non-signatories countries don't invest. In this case, the global level of emissions is given by (2.91). Now, the optimization problem of the second stage for signatories countries should be represented as follows

$$\begin{aligned} \min_{\{x_j^s\}} TC_j^s(n) &= \frac{c}{2} (\delta - \alpha y_j^s)^2 + d \left( (N - n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + x_j^s, \\ \text{s.t. } y_j^s &= nx_j^s \geq \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right), \end{aligned} \quad (2.135)$$

$$y_j^s = nx_j^s \leq \frac{\delta}{\alpha}. \quad (2.136)$$

The solution of the previous problem yields the following level of investment and effective investment for signatories countries

$$x_j^s = \frac{\alpha (d\gamma (N - n) + c\delta) - 1}{\alpha^2 cn}, \quad (2.137)$$

$$y_j^s = \frac{\alpha (d\gamma (N - n) + c\delta) - 1}{\alpha^2 c}, \quad (2.138)$$

such that the condition on effective investment given by (2.135) is satisfied at any level of marginal damages higher than

$$\check{d}_2^s(n) = \frac{1}{\alpha (n + \gamma (N - n))}, \quad (2.139)$$

which is equal to  $\hat{d}_2^s(n)$ . The critical value of marginal damages  $\check{d}_2^s(n)$  differs from  $\tilde{d}_2^s(n)$  given by (2.38), because  $\check{d}_2^s(n)$  is calculated assuming that investment of non-signatories equal to zero, while  $\tilde{d}_2^s(n)$  was calculated assuming that non-signatories

investment is positive. As  $\check{d}_2^s(n) = \hat{d}_2^s(n) < \hat{d}_2^s(2)$ , it can be concluded that the solution in the range of marginal damages given by  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$  is the same as presented above, and the total costs functions are given by

$$TC_i^f(n) = \frac{d^2}{2c} + d \left( (N - n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right), \quad (2.140)$$

$$TC_j^s(n) = \frac{(1 - \alpha d \gamma (N - n))^2}{2\alpha^2 c n^2} + d \left( (N - n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + x_j^s, \quad (2.141)$$

where the first term represents the abatement costs and the second term represents the environmental damages, while the third term in the total costs of signatories countries represents the investment costs. Notice that  $x_j^s$  is given by (2.137), while  $Y^f$  is given as follows

$$Y^f = \gamma (N - n) \left( \frac{\alpha (d \gamma (N - n) + c \delta) - 1}{\alpha^2 c} \right). \quad (2.142)$$

However, if  $d \in (\hat{d}_2^s(N), \hat{d}_2^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.140) and (2.141) until we reach to the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$  in figure 2.2, at this point the total costs function of signatories countries is still the same given by (2.141), while non-signatories' total costs function is given by (2.90) assuming that signatories investment equal to zero.

Finally, if  $\gamma < \gamma_2^*$ , it is clear that for any level of marginal damages lower than  $\hat{d}^{nc}$ , the critical value of marginal damages given by (2.38) doesn't play role in satisfying the constraint over signatories' effective investment given by (2.135) and the new critical value of marginal damages (2.139) is the one that plays this role. Thus, it can be concluded that the analysis, at the different intervals of marginal damages, under the assumption  $\gamma < \gamma_2^*$  is exactly the same as the analysis developed under the assumption  $\gamma \in (\gamma_2^*, \check{\gamma}]$ .

Next, the profitability of joining the emission agreement with information exchange is analyzed, numerically for any  $\gamma < \check{\gamma}$ , at the different levels of marginal damages and the following proposition is concluded

**Proposition 15** *Emission agreement with information exchange on R&D investment is profitable at any level of marginal damages.*

The following numerical examples proof this result.

First, in order to investigate the profitability of joining emission agreement with information exchange at  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs function of a signatory country given by (2.122) should be compared by the total costs function of playing fully non-cooperatively. By substituting for  $n = 1$  in (2.121), the total costs function of the fully non-cooperative equilibrium in this case are the same given by (2.55). Under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.5, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	7.275	7.996	8.877	9.979	11.396	13.29	15.93	19.896
$TC_j^s$	2.543	2.15	1.946	1.847	1.8214	1.861	1.976	2.2
$prof.$	3.622	4.015	4.219	4.318	4.3437	4.304	4.189	3.9651

Table 2.6: Profitability of emission agreement with information exchange at  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 6.1651$ ) and the total costs of signatories countries.

Second, the profitability is examined at the interval of marginal damages  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$  using the total costs of both non-signatories and signatories given by (2.140) and (2.141), under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.27, \gamma = 0.25$$



$n$	3	4	5	6	7	8	9	10
$TC_i^f$	28.24	24.22	20.198	16.17	12.14	8.104	4.063	0.018
$TC_j^s$	34.81	29.13	24.118	19.43	14.925	10.53	6.218	1.953
$prof.$	1.453	7.126	12.141	16.83	21.334	25.73	30.04	34.31

Table 2.7: Profitability of emission agreement with information exchange at  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 36.259$ ) and the total costs of signatories countries.

Finally, the profitability is examined at the interval of marginal damages  $d \in (\hat{d}_2^s(N), \hat{d}_2^s(2)]$  using the total costs of both non-signatories and signatories given by (2.140) and (2.141), until we reach to the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$ , the total costs function of non-signatories is replaced by (2.90). Under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, \gamma = 0.25,$$

if it is assumed that  $d = 0.11$ , the minimum level of cooperation needed to make the investment in R&D profitable is given by

$$\hat{n}_2 = 8.7879.$$

According to that, the profitability of  $n = 9$ , is examined using total costs functions given by (2.90) and (2.141) as follows

$$TC_i^{nc} = 21.943, TC_i^f(8) = 21.604, TC_j^s(9) = 3.8255, prof. = 18.118.$$

However, the profitability of any level of cooperation higher than  $n = 9$  is examined using total costs functions given by (2.140) and (2.141) as follows

$$TC_i^{nc} = 14.892, TC_i^f(9) = 1.6603, TC_j^s(10) = 1.9525, prof. = 12.940.$$

### 2.5.2 The Nash Equilibrium of the Membership Game

In this section, the stability analysis is studied to investigate whether there exist a stable emission agreement with information exchange on R&D investment or not. For  $d \geq \tilde{d}^{nc}$ , by examining directly the stability of the grand coalition, the auxiliary function  $\Omega(N)$  is built using the total costs functions given by (2.116) and (2.117)

$$\Omega(N) = -\frac{(c\alpha\delta - 1)(N - 1 - \gamma)}{c\alpha^2 N(1 + \gamma)} < 0.$$

Finally, by analyzing the previous numerical example, looking at the total costs of non-signatories and signatories at the different levels of marginal damages lower than  $\tilde{d}^{nc}$ , the following proposition is concluded

**Proposition 16** *The grand coalition of emission agreement with information exchange is stable at for any  $d \geq \hat{d}_2^s(2)$ . However, in the interval  $d \in (\hat{d}_2^s(N), \hat{d}_2^s(2)]$ , the unique stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$ .*

## 2.6 Emission and R&D Agreement with Information Exchange

An emission and R&D agreement with information exchange is modeled as a three-stage game as the emission and R&D agreement without information exchange. The main difference between the two agreements is that the signatories countries in the emission and R&D agreement with information exchange share their R&D efforts and avoid the duplication of the R&D activities. As a result of sharing the R&D efforts, the marginal costs of abatement of signatories countries are decreased by the sum of all R&D efforts in the agreement, in addition to some spillover from non-signatories countries' R&D investments, i.e. the spillover is increased for the signatories because of the agreement.

### 2.6.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved, as done in the emission agreement by backward induction assuming that in first stage  $n$  countries with  $n \geq 2$  have signed the agreement. As signatories cooperate while non-signatories countries act non-cooperatively at the third stage of the game, the emissions, the solution of this stage is identical to the solution that has been developed in the emission agreement. Nevertheless, the global level of investment is now given by (2.103).

Next, the partial agreement Nash equilibrium of the investment game is calculated. As non-signatories countries do not cooperate at this stage, the solution is identical to the solution that has been developed in the emission agreement for non-signatories. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$  where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, as they are minimizing the agreement total costs, the optimization problem that yields the optimal investment is the same given by (2.64), while the constraints on investments are given by (2.105)-(2.106).

By taking the first derivative of the agreement total costs function given by (2.64) with respect to investment in R&D, taking into account that  $\partial Y/\partial x_j^s = n + \gamma(N - n)$ , the following critical value of marginal damages defined by condition  $\partial ATC/\partial x_j^s = 0$  is obtained

$$\hat{d}_3^s(n) = \frac{1}{\alpha n (n + \gamma(N - n))}. \quad (2.143)$$

As in the emission and R&D agreement without information exchange, the critical value of  $d$  which triggers investment in R&D depends in the number of signatories. It is easy to check that  $\hat{d}_3^s(n)$  decreases with respect to the level of cooperation and takes values between

$$\hat{d}_3^s(N) = \frac{1}{\alpha N^2} \leq \hat{d}_3^s(n) \leq \hat{d}_3^s(2) = \frac{1}{2\alpha(2 + \gamma(N - 2))}.$$

When  $d$  is greater than  $\hat{d}_3^s(2)$ , the total costs of the agreement are *decreasing* with respect to investment independently of the level of participation, and signatories invest in R&D until emissions are completely eliminated. However, when  $d$  is in the interval  $(\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , the total costs of the agreement are decreasing depending on the number of signatories. If this is not the case, signatories do not invest in clean technologies and cooperation is not enough to promote the replacement of fossil fuels. This minimum level of cooperation, solved by the condition  $d = \hat{d}_3^s(n)$ , is given by the positive root of the following expression

$$d\alpha(1-\gamma)\hat{n}_3^2 + d\alpha\gamma N\hat{n}_3 - 1 = 0. \quad (2.144)$$

The decision on investing in R&D for both signatories and non-signatories depends critically on the marginal damages in the same way as declared in the emission and R&D agreement without information exchange. Figure 2.3 shows which is the pattern of investment for signatories and non-signatories depending on the marginal damages and the number of signatories.

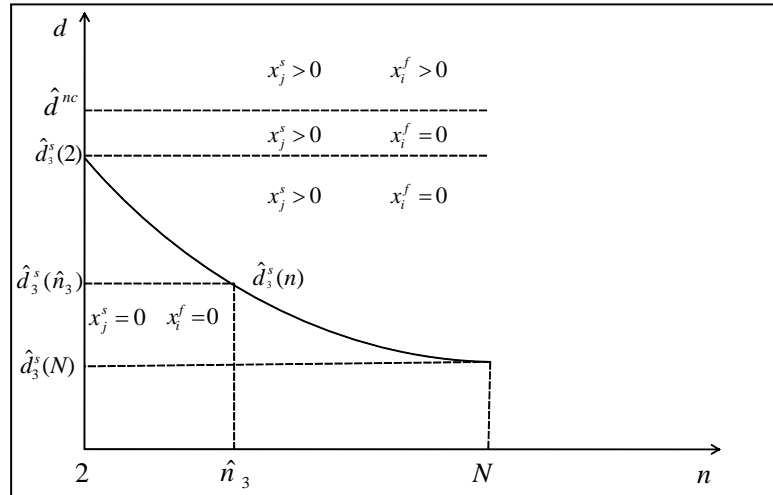


Figure 2.3: Decisions on investment in R&D for signatories and non-signatories of emission and R&D agreement with information exchange.

**Conjecture 17**  $\hat{d}_3^s(n)$  defines the stable participation.

Next, the level of investment for different levels of marginal damages is calculated. The same three possibilities that have been considered in the emission and R&D agreement without information exchange are considered here. First, both signatories and non-signatories invest in R&D. Second, only signatories invest. Third, both signatories and non-signatories do not find profitable to invest in R&D. When  $d > \hat{d}^{nc}$ , both types of countries invest in R&D to eliminate completely the emissions. In this case, the optimization problem of non-signatories' countries is the same presented by (2.13)-(2.15). Nevertheless, for signatories countries, the optimization problem is given by (2.69), while the constraints on their effective investment are given by (2.110)-(2.111).

The solution to the optimization problem for signatories yields the following reaction function of signatories' investment

$$x_j^s = \frac{c\alpha n \left( \delta - \alpha\gamma (N - n) x_i^f \right) - 1}{c\alpha^2 n^2}, \quad (2.145)$$

while the reaction function of non-signatories' investment is given by (2.36).

The solution of both (2.36) and (2.145) yields the optimal levels of investments for both non-signatories and signatories as follows

$$x_i^f = \frac{c\alpha\delta n (1 - \gamma) - n + \gamma}{c\alpha^2 n (1 - \gamma) (1 + \gamma (N - n))}, \quad (2.146)$$

$$x_j^s = \frac{c\alpha\delta n (1 - \gamma) + n\gamma (N - n) - (1 + \gamma (N - n - 1))}{c\alpha^2 n^2 (1 - \gamma) (1 + \gamma (N - n))}, \quad (2.147)$$

such that the level of effective investment for non-signatories is given by (2.17), while it is given by (2.73) for signatories countries.

We already know from the solution of fully non-cooperative equilibrium that non-signatories' effective investment given by (2.17) satisfies the constraint on effective investment given by (2.14) for any level of marginal damages higher than  $\tilde{d}^{nc}$  which is

given by (2.18), while for signatories countries, the constraint on effective investment given by (2.110) is satisfied for any level of marginal damages higher than  $\tilde{d}_1^s(n)$  which is given by (2.74). It is known from the analysis of emission and R&D agreement without information exchange that  $\tilde{d}_1^s(n)$  is lower than  $\tilde{d}^{nc}$  and higher than or equal to  $\hat{d}^{nc}$  provided that  $\gamma \geq \check{\gamma}_1$  and by comparison, it is found that  $\tilde{d}_1^s(n) \geq \hat{d}_3^s(n)$  for any  $\gamma \geq \gamma_3^*$  where

$$\gamma_3^* = \frac{n^2 - 4}{2(N - 2)}, \quad (2.148)$$

and vice versa.

As the order of the critical values of marginal damages will be changed depending on the value of the diffusion parameter, three possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}_1$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}_1^s(n) > \hat{d}^{nc} > \hat{d}_3^s(2) > \hat{d}_3^s(N).$$

Second, if  $\gamma \in (\gamma_3^*, \check{\gamma}_1]$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}_1^s(n) > \hat{d}_3^s(2) > \hat{d}_3^s(N),$$

and finally, if  $\gamma < \gamma_3^*$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_3^s(2) > \tilde{d}_1^s(n) > \hat{d}_3^s(N).$$

As concluded in the emission and R&D agreement without information exchange, for high levels of cooperation, the value of  $\check{\gamma}_1$  is going to be very high, even higher than 1 for some levels of cooperation. Thus, our analysis for profitability of joining the agreement will be focused on the second and third cases when  $\gamma \in (\gamma_3^*, \check{\gamma}_1]$  and  $\gamma < \gamma_3^*$ . However, the levels of investments and the corresponding total costs functions

of non-signatories and signatories countries will be analyzed for the different three cases.

In all cases, if  $d \geq \tilde{d}^{nc}$ , as the levels of effective investment for both non-signatories and signatories given by (2.17) and (2.73) satisfy, in this interval of marginal damages, the constraints on effective investment for both types given by (2.14) and (2.110) respectively, the following expressions of total costs are obtained

$$TC_i^f(n) = \frac{1}{2c\alpha^2} + \frac{c\alpha\delta n(1-\gamma) - n + \gamma}{c\alpha^2 n(1-\gamma)(1+\gamma(N-n))}, \quad (2.149)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2 n^2} + \frac{c\alpha\delta n(1-\gamma) + \gamma n(N-n) - (1+\gamma(N-n-1))}{c\alpha^2 n^2(1-\gamma)(1+\gamma(N-n))}, \quad (2.150)$$

where the first term represents the abatement costs and the second term represents the investment cost.

Next, we analyze the case when  $\gamma \geq \check{\gamma}_1$ . If  $d \in (\tilde{d}_1^s(n), \tilde{d}^{nc}]$ , as any level of marginal damages in this interval is higher than the level of marginal damages given by (2.74), thus the level of effective investment given by (2.73) satisfies the constraint on effective investment for signatories countries given by (2.110). However, the level of effective investment for non-signatories given by (2.17) doesn't satisfy, in this interval of marginal damages, the constraint on their effective investment given by (2.14), and therefore non-signatories countries reduce their effective investment to the level given by (2.6). According to that, the reaction function of non-signatories' investment is the same given by (2.79), while for signatories countries it is given as follows

$$x_j^s = \frac{c\alpha n(\delta - \alpha\gamma(N-n)x_i^f) - 1}{c\alpha^2 n^2}. \quad (2.151)$$

The solution of the reaction functions yields the following levels of investment for non-signatories and signatories

$$x_i^f = \frac{c\alpha\delta n(1-\gamma) - \alpha dn + \gamma}{c\alpha^2 n(1-\gamma)(1+\gamma(N-n))}, \quad (2.152)$$

$$x_j^s = \frac{c\alpha\delta n(1-\gamma) + \alpha d\gamma n(N-n) - (1+\gamma)(N-n-1)}{c\alpha^2 n^2(1-\gamma)(1+\gamma(N-n))}. \quad (2.153)$$

Doing the substitutions for effective investment in the total costs functions of non-signatories and signatories, the following expressions are obtained

$$TC_i^f(n) = \frac{d^2}{2c} + \frac{c\alpha\delta n(1-\gamma) - \alpha dn + \gamma}{c\alpha^2 n(1-\gamma)(1+\gamma(N-n))}, \quad (2.154)$$

$$TC_j^s(n) = \frac{1}{2c\alpha^2 n^2} + \frac{c\alpha\delta n(1-\gamma) + \alpha d\gamma n(N-n) - (1+\gamma)(N-n-1)}{c\alpha^2 n^2(1-\gamma)(1+\gamma(N-n))}, \quad (2.155)$$

where the first term represents the abatement costs and the second term represents the investment costs.

Next, if  $d \in (\hat{d}^{nc}, \tilde{d}_1^s(n)]$ , the levels of effective investment for non-signatories and signatories are given by (2.6) and (2.24) respectively. In this case the solution of the second stage is the same as that developed in the emission agreement with information exchange where the total costs of non-signatories and signatories are given by (2.127) and (2.128) respectively.

Now, if  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$ , as marginal damages are low enough to make it unprofitable for non-signatories to invest in R&D, then the level of investment for signatories countries is the same given by (2.129), and the total costs for non-signatories and signatories are given by (2.133) and (2.134) respectively.

However, if  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.133) and (2.134) until we reach to the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$  in figure 2.3, at this point the total costs function of signatories countries is still the same given by (2.134), while non-signatories' total costs function should be changed taking into account that by moving from the area above the curve  $\hat{d}_2^s(n)$  in figure 2.3 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by (2.90).



Second, if  $\gamma \in (\gamma_3^*, \check{\gamma}_1]$  and  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs for non-signatories are given by (2.154), while given by (2.155) for signatories countries. However, if  $d \in (\tilde{d}_1^s(n), \hat{d}^{nc}]$ , signatories countries invest to eliminate the emissions while non-signatories don't invest. In this case, the global level of emissions is given by (2.91). Now, the optimization problem of the second stage for signatories countries should be represented as follows

$$\begin{aligned} \min_{\{x_1^s, \dots, x_n^s\}} ATC &= \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + dn \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + \sum_{j=1}^n x_j^s, \\ \text{s.t. } y_j^s &= nx_j^s \geq \frac{1}{\alpha} \left( \delta - \frac{nd}{c} \right), \end{aligned} \quad (2.156)$$

$$y_j^s = nx_j^s \leq \frac{\delta}{\alpha}. \quad (2.157)$$

The solution of the previous problem yields the following level of investment for signatories countries

$$x_j^s = \frac{\alpha n (d\gamma (N-n) + c\delta) - 1}{\alpha^2 cn^2}. \quad (2.158)$$

The level of signatories' effective investment is the same given by (2.95), such that the condition on effective investment given by (2.156) is satisfied for any level of marginal damages higher than  $\check{d}_1^s(n)$  given by (2.96) which is equal to  $\hat{d}_3^s(n)$ . As  $\check{d}_1^s(n) = \hat{d}_3^s(n) < \hat{d}_3^s(2)$ , it can be concluded that the solution in the range of marginal damages given by  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$  is the same as presented above, and the total costs functions are given by

$$TC_i^f(n) = \frac{d^2}{2c} + d \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right), \quad (2.159)$$

$$TC_j^s(n) = \frac{(1 - \alpha nd\gamma (N-n))^2}{2\alpha^2 cn^2} + d \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + x_j^s, \quad (2.160)$$

where the first term represents the abatement costs and the second term represents the environmental damages, while the third term in the total costs of signatories

countries represents the investment costs. Notice that  $x_j^s$  is given by (2.158), while  $Y^f$  is given as follows

$$Y^f = \gamma(N - n) \left( \frac{\alpha n (d\gamma(N - n) + c\delta) - 1}{\alpha^2 cn} \right). \quad (2.161)$$

However, if  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , the total costs functions for non-signatories and signatories are still the same given by (2.159) and (2.160) until we reach to the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$  in figure 2.3, at this point the total costs function of signatories countries is still the same given by (2.160), while non-signatories' total costs function is given by (2.90) assuming that signatories investment equal to zero.

Finally, if  $\gamma < \gamma_3^*$ , it is clear that for any level of marginal damages lower than  $\hat{d}_3^{nc}$ , the critical value of marginal damages given by (2.74) doesn't play role in satisfying the constraint over signatories' effective investment given by (2.156) and the new critical value of marginal damages (2.96) is the one that plays this role. Thus, it can be concluded that the analysis, at the different intervals of marginal damages, under the assumption  $\gamma < \gamma_3^*$  is exactly the same as the analysis developed under the assumption  $\gamma \in (\gamma_3^*, \check{\gamma}_1]$ .

Next, the profitability of joining the emission and R&D agreement with information exchange is analyzed, numerically for any  $\gamma < \check{\gamma}_1$ , at the different levels of marginal damages and the following proposition is concluded

**Proposition 18** *Emission and R&D agreement with information exchange is profitable for any level of marginal damages.*

The following numerical examples proof this result.

First, in order to investigate the profitability of joining emission and R&D agreement with information exchange at  $d \geq \tilde{d}^{nc}$ , the total costs function of a signatory country given by (2.150) should be compared by the total costs function of playing

fully non-cooperatively which is obtained by substituting for  $n = 1$  in (2.149). Under the following assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 2, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	7.301	8	8.857	9.93	11.31	13.15	15.73	19.6
$TC_j^s$	2.526	2.08	1.838	1.71	1.667	1.6897	1.789	1.998
$prof.$	3.724	4.17	4.412	4.54	4.583	4.5603	4.461	4.253

Table 2.8: Profitability of emission and R&D agreement with information exchange at  $d \geq \hat{d}^{nc}$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 6.25$ ) and the total costs of signatories countries.

Second, the profitability is examined at the interval of marginal damages  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , using the total costs of both non-signatories and signatories given by (2.154) and (2.155), under the following assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.5, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	7.234	7.946	8.818	9.9097	11.31	13.19	15.81	19.75
$TC_j^s$	2.455	2.028	1.801	1.6852	1.647	1.676	1.782	1.998
$prof.$	3.7097	4.137	4.364	4.4799	4.519	4.489	4.383	4.168

Table 2.9: Profitability of emission and R&D agreement with information exchange at  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 6.1651$ ) and the total costs of signatories countries.

Third, the profitability is examined at the interval of marginal damages  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$  using the total costs of both non-signatories and signatories given by (2.159) and (2.160), assuming that

$$\alpha = 1, c = 2, \delta = 20, N = 10, d = 0.27, \gamma = 0.25$$

$n$	3	4	5	6	7	8	9	10
$TC_i^f$	28.08	24.068	20.063	16.058	12.053	8.045	4.033	0.018
$TC_j^s$	34.76	29.075	24.063	19.386	14.899	10.53	6.238	1.998
$prof.$	1.499	7.184	12.196	16.873	21.36	25.73	30.022	34.26

Table 2.10: Profitability of emission and R&D agreement with information exchange at  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 36.259$ ) and the total costs of signatories countries.

Finally, the profitability is examined at the interval of marginal damages  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$  using the total costs of both non-signatories and signatories given by (2.159) and (2.160), until we reach to the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$ , the total costs function of non-signatories is replaced by (2.90). Under the assumptions

$$\alpha = 1, c = 2, \delta = 20, N = 10, \gamma = 0.25,$$

if it is assumed that  $d = 0.11$ , the minimum level of cooperation needed to make the investment in R&D profitable is given by

$$\hat{n}_3 = 2.1933,$$

According to that, the profitability of  $n = 3$ , is examined using total costs functions given by (2.90) and (2.160) as follows

$$TC_i^{nc} = 21.943, TC_i^f(2) = 21.93, TC_j^s(3) = 18.165, prof. = 3.778.$$

However, the profitability of any level of cooperation higher than  $n = 3$  is examined using total costs functions given by (2.159) and (2.160) and the results are shown in the next table

$n$	5	6	7	8	9	10
$TC_i^f$	8.2271	6.5819	4.9374	3.2929	1.6481	0.0303
$TC_j^s$	12.218	9.9075	7.7872	5.7855	3.8627	1.995
$prof.$	2.674	4.9845	7.1048	9.1065	11.029	12.897

Table 2.11: Profitability of emission and R&D agreement without information exchange at  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$

Note: The profitability ( $prof.$ ) is calculated by taking the difference between the total costs of fully non-cooperative equilibrium (in this case given by  $TC_i^{nc} = 14.982$ ) and the total costs of signatories countries.

### 2.6.2 The Nash Equilibrium of the Membership Game

In this section, the stability analysis is studied to investigate whether there exist a stable emission and R&D agreement with information exchange or not. By analyzing the previous numerical example, looking at the total costs of non-signatories and signatories at the different levels of marginal damages, the following proposition is concluded

**Proposition 19** *The grand coalition of emission and R&D agreement with information exchange is stable at any level of marginal damages higher than or equal to  $\hat{d}_3^s(2)$ . However, in the interval  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , the unique stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$ .*

## 2.7 Conclusion

In this chapter, a model of three-stage game has been used to analyze four different types of emission agreements. The different agreements share the main aspect that signatories countries act cooperatively in the third stage of the game (emission game). However, the second stage of the game (investment game) differs from one type to another depending on whether signatories are sharing R&D efforts and coordinating their R&D activities or not. In all agreements, it is assumed that effective investment

in one country depends on the amount invested in R&D in that country as well as on the investment in R&D undertaken in all countries through technological spillovers.

In the types of agreements that include information exchange (emission agreement with information exchange and emission and R&D agreement with information exchange), the technological spillovers is perfect among signatories, which means that signatories countries avoid the duplication of R&D efforts.

According to the analysis introduced in this chapter for the different agreements, it is found that cooperating on emissions and investment without information exchange is not enough to eliminate countries incentives to act as free-rider. It is found also that emission agreement is not profitable at the high levels of cooperation. However, cooperating in the second stage of the game (investment game), as in the emission and R&D agreement without information exchange, increases the profitability for the high levels of cooperation and increases at the same time countries incentives to deviate. Therefore, emission and R&D agreement without information exchange is not stable except for the level of participation given by the lowest natural number on the right of  $\hat{d}_1^s(n)$  in the interval of marginal damages given by  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ .

In both emission agreement with information exchange and emission and R&D agreement with information exchange, it is concluded that sharing R&D efforts is enough to stabilize high levels of cooperation, even the grand coalition, at the high levels of marginal damages where countries incentives to free-ride over the investment of other countries is eliminated as a result of sharing information.

## 2.8 Appendices

### 2.8.1 Appendix 1: Proof of proposition 2

By analyzing the level of investment for signatories given by (2.42), there is a critical value for the level of the marginal damages for the signatories that can be obtained as follows

$$d = -\frac{c\delta(1-\gamma)}{\gamma n^2 - (1+\gamma N)n + \gamma N}, \quad (2.162)$$

such that if the marginal damages are higher than this critical value, the non-negative constraint applies. Substituting for  $n = N$  in (2.162), it is obtained that

$$d(N) = \frac{c\delta}{N}, \quad (2.163)$$

which is the critical value that separates the trivial solution from the relevant one.

While if  $n$  is substituted by 2, it is obtained that

$$d(2) = \frac{c\delta(1-\gamma)}{(N-4)\gamma + 2}. \quad (2.164)$$

Notice that the function which represents the critical value of  $d$  given by (2.162) is a convex function in the interval  $[2, N]$  with a minimum given by

$$n^* = \frac{1}{2\gamma} + \frac{N}{2}.$$

It is obvious that  $n^* > 2$ . Now, we must investigate the relationship between  $n^*$  and  $N$

$$N - n^* = \frac{\gamma N - 1}{2\gamma}, \quad (2.165)$$

so that, two cases should be analyzed (when  $\gamma > 1/N$  and when  $\gamma < 1/N$ ).

First, when  $\gamma > 1/N$ , then  $n^* < N$ . Provided that  $n^* < N$ , the level of marginal damages given by (2.162) must be increasing on the left of  $N$  which implies that the minimum is lower than  $c\delta/N$ . In order to conclude the characteristics of (2.162), the difference between  $d(2)$  and  $d(N)$  is taken

$$d(2) - d(N) = c\delta \left( \frac{N - 2 - \gamma(2N - 4)}{(N - 4)\gamma + 2} \right),$$

which is equal to zero at  $\gamma = 1/2$ . Thus,  $d(2)$  will be higher than  $d(N)$  for any  $\gamma < 1/2$  (Figure 2.4 illustrates this result) and vice versa.

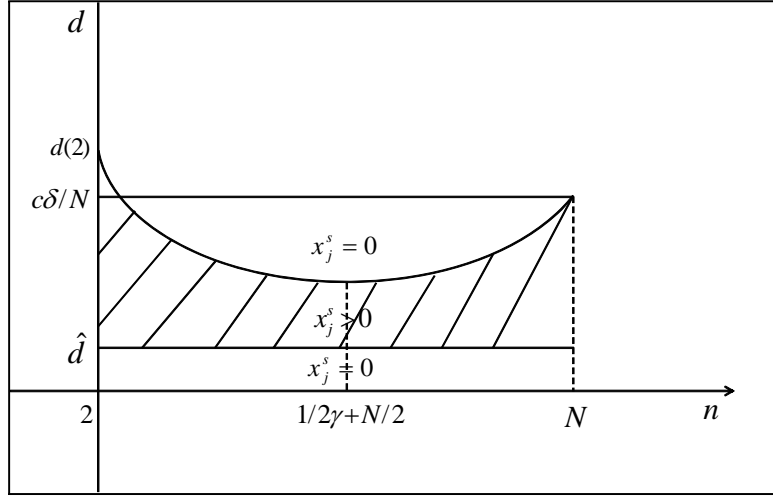


FIGURE 2.4

Second, when  $\gamma \leq 1/N$ , then  $n^* \geq N$ . It is immediate that  $d(n) > d(N)$  for all  $n \in (2, N - 1)$  (Figure 2.5 illustrates this result). In this case the investment of signatories is positive for  $n \in (2, N)$  and  $d \in (0, c\delta/N)$ .

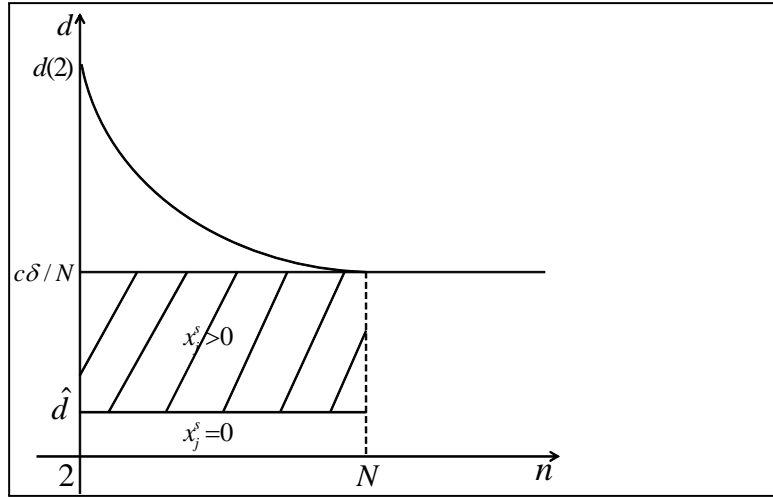


FIGURE 2.5

Following the same analysis for the non-signatories countries, there is also a critical value for the level of the marginal damages for non-signatories that can be obtained from (2.43) as follows

$$d = -\frac{c\delta(1-\gamma)}{\gamma n^2 - \gamma n - (1-\gamma)}. \quad (2.166)$$

Substituting for  $n = N$  in (2.166), it is obtained

$$d(N) = -\frac{c\delta(1-\gamma)}{\gamma(N^2 - N + 1) - 1}, \quad (2.167)$$



that is going to be negative if

$$\gamma > \frac{1}{N^2 - N + 1}, \quad (2.168)$$

and positive if the contrary applies.

On the other hand, substituting for  $n = 2$ , it is obtained

$$d(2) = \frac{c\delta(1-\gamma)}{1-3\gamma}, \quad (2.169)$$

that is going to be negative if  $\gamma > 1/3$  and positive if the contrary applies. In order to conclude the characteristics of (2.166), the difference between  $d(2)$  and  $c\delta/N$  is taken

$$d(2) - \frac{c\delta}{N} = c\delta \left( \frac{(1-\gamma)N - 1 + 3\gamma}{(1-3\gamma)N} \right),$$

that is going to be negative for any  $\gamma > 1/3$  and vice versa.

As we have two critical values of the spillovers parameter, we have to distinguish three different cases, when  $\gamma \in (0, \frac{1}{N^2-N+1})$ , when  $\gamma \in (\frac{1}{N^2-N+1}, \frac{1}{3})$  and when  $\gamma \in (\frac{1}{3}, 1)$ .

First, when  $\gamma \in (0, \frac{1}{N^2-N+1})$ , then both  $d(2)$ ,  $d(N) > 0$  and  $d(2) > c\delta/N$ . Thus,  $d(n)$  is positive in the interval  $[2, N]$ . Assuming the denominator of (2.166)

$$f(n) = \gamma n^2 - \gamma n - (1 - \gamma),$$

then, the first derivative of  $d$  becomes

$$d' = \frac{c\delta(1-\gamma)f'(n)}{f(n)^2} > 0,$$

and the second derivative

$$d'' = c\delta(1-\gamma) \cdot \frac{f(n)}{f(n)^2} \left( f''(n)f(n) - 2f'(n)^2 \right) > 0.$$

Then  $d(n)$  is a convex function and the non-signatories invest positively in R&D (Figure 2.6 illustrates this result).

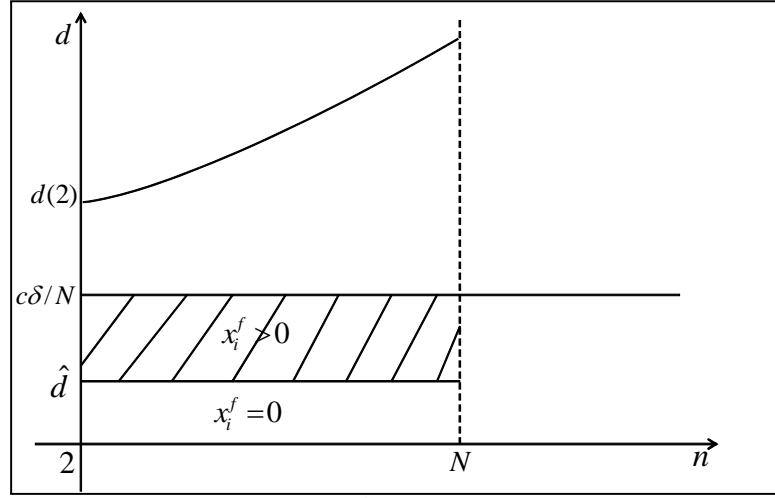


FIGURE 2.6

Second, when  $\gamma \in (\frac{1}{N^2-N+1}, \frac{1}{3})$ , then  $f(n') = 0$  and in this case  $d(n) > 0$  for  $2 \leq n < n'$ , while  $d(n) < 0$  for  $N \geq n > n'$ . The analysis when  $d(n) > 0$  is the same like the previous case. Now, the analysis when  $d(n) < 0$  must be investigated.

$$d' = c\delta(1-\gamma) \frac{\gamma(2n-1)}{(\gamma n^2 - \gamma n - (1-\gamma))^2},$$

$$d'' = c\delta(1-\gamma) \frac{2\gamma(\gamma n^2 - \gamma n - (1-\gamma))^2 - 2\gamma^2(2n-1)(\gamma n^2 - \gamma n - (1-\gamma))(2n-1)}{(\gamma n^2 - \gamma n - (1-\gamma))^4}.$$

Analyzing the numerator

$$\begin{aligned} & (\gamma n^2 - \gamma n - (1-\gamma)) (2\gamma(\gamma n^2 - \gamma n - (1-\gamma)) - 2\gamma^2(2n-1)^2) \\ &= -2\gamma(\gamma n^2 - \gamma n - (1-\gamma)) (3\gamma n(n-1) + 1) < 0, \end{aligned}$$

thus  $d'' < 0$  and the function  $d(n)$  in the interval  $[n', N]$  is concave. Consequently, the non-signatories invest positively in R&D (Figure 2.7 illustrates this result).

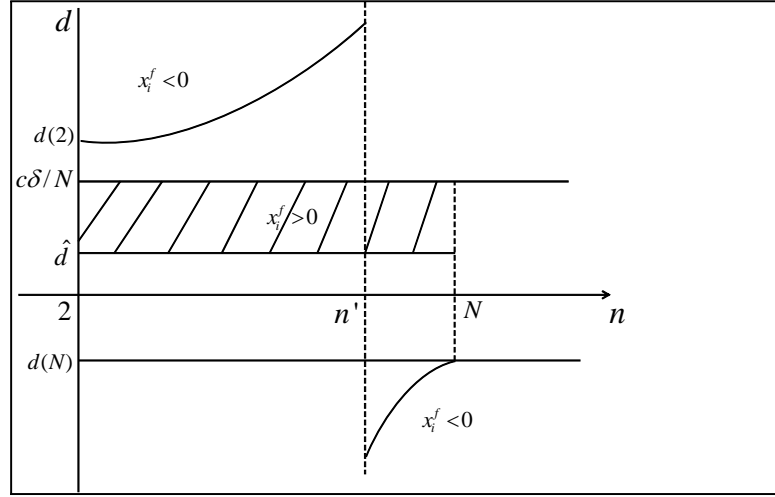


FIGURE 2.7

Finally, when  $\gamma \in (\frac{1}{3}, 1)$ , then  $d(n)$  for all  $n \in [2, N]$  and the non-signatories invest positively in R&D.

### 2.8.2 Appendix 2: Proof of proposition 3

By taking the first derivative of (2.50) with respect to the level of cooperation, it is found that this difference is convex function with a minimum given by

$$n^* = \frac{1 + \gamma N}{1 + \gamma}. \quad (2.170)$$

By substituting the minimum given by (2.170) in the numerator of (2.50) as follows

$$\left(\frac{1 + \gamma N}{1 + \gamma}\right)^2 (1 + \gamma) - 2(1 + \gamma N) \left(\frac{1 + \gamma N}{1 + \gamma}\right) + 2N\gamma + (1 - \gamma) = -\frac{\gamma^2}{1 + \gamma} (N - 1)^2 < 0.$$

Thus, the total costs of the fully non-cooperative equilibrium is higher than the total costs of a signatory country at the minimum. Here, it is important to investigate the relation between the minimum given by (2.170) and both of  $(N, 2)$  in order to have a complete view about the relation between the total costs given by (2.50) at any level of cooperation. First,  $n^*$  is compared with  $N$  as follows  $N - n^* = \frac{N-1}{1+\gamma} > 0$ .

When  $n$  is substituted by  $N$  in the numerator of (2.50) as follows

$$(N)^2(1 + \gamma) - 2(1 + \gamma N)(N) + 2N\gamma + (1 - \gamma) = (1 - \gamma)(N - 1)^2 > 0,$$

it is concluded that the total costs of the fully non-cooperative equilibrium are lower than the total costs of signatories at the level of full cooperation. In other words, the full cooperation is not profitable for the emission agreement. Next,  $n^*$  is compared with 2 as follows

$$n^* - 2 = \frac{-1 + \gamma(N - 1)}{1 + \gamma} < 0 \quad \forall \gamma' \leq \frac{1}{N - 1}.$$

According to proposition 2, we assume that  $\gamma < 1/N$  in order to guarantee a positive investment in R&D for signatories countries. Thus, it is obvious that any level of  $\gamma < 1/N$  is lower than  $\gamma'$  and consequently  $n^* < 2$ . When  $n$  is substituted by 2 in the numerator of (2.50) as follows

$$(2)^2(1 + \gamma) - 2(1 + \gamma N)(2) + 2N\gamma + (1 - \gamma) = 3\gamma - 2N\gamma + 1 > 0 \quad \forall \gamma'' \leq \frac{1}{2N - 3}.$$

Again, as  $\gamma = 1/N$  is lower than  $\gamma''$  for any  $N \geq 3$ , then the difference in the total costs given by (2.50) is positive at  $n = 2$ , which means that the total costs of a signatory country are higher than the total costs of a fully non-cooperative equilibrium at any level of cooperation.

## CHAPTER 3

# EVALUATION OF DIFFERENT TYPES OF TECHNOLOGICAL AGREEMENTS AS AN ALTERNATIVE TO EMISSION AGREEMENTS

### 3.1 R&D Agreement Without Information Exchange

R&D agreement without information exchange is modeled as a three stage game as the emission and R&D agreement without information exchange that has been analyzed in the second chapter. The main difference between the two agreements is that all countries, in the R&D agreement without information exchange, are acting non-cooperatively at the third stage of the game (emission game).

#### 3.1.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved by backward induction assuming that in the first stage  $n$  countries, with  $n \geq 2$ , have signed the agreement. As we have supposed that there is no cooperation in the third stage of the game (emissions), total emissions and total costs supported by all countries are given, as in the fully non-cooperative equilibrium, by (2.7) and (2.9) while the global effective investment in R&D is given by (2.26).

Next, the partial agreement Nash equilibrium of the investment game is calculated. As the non-signatories countries do not cooperate at this stage, the solution for non-signatories is identical to the solution that has been developed in the fully non-cooperative equilibrium. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$ , where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, as they are minimizing the agreement total costs given by

$$ATC = \sum_{j=1}^n TC_j^s = \sum_{j=1}^n \left( \frac{d^2}{2c} + d(N\bar{\delta} - \alpha Y) + x_j^s \right),$$

the optimization problem that yields the optimal investment is given as follows

$$\min_{\{x_1^s, \dots, x_n^s\}} ATC = \frac{nd^2}{2c} + dnN\bar{\delta} - \alpha dnY + \sum_{j=1}^n x_j^s, \quad (3.1)$$

$$s.t. \quad \frac{\bar{\delta}}{\alpha} - \gamma X_{-j} \geq x_j^s, \quad (3.2)$$

$$x_j^s \geq 0, \quad (3.3)$$

where  $Y$  is given by (2.26).

Although the optimization problem is different from the optimization problem of signatories of emission and R&D agreement without information exchange given by (2.64)-(2.66), the effect of investment on total costs function is the same. Thus, the critical value of marginal damages which defines the stable R&D agreement without information exchange is the same  $\hat{d}_1^s(n)$  given by (2.67). Therefore, the decision on investing in R&D for both signatories and non-signatories are defined as in Figure 2.1.

When  $d \geq \hat{d}^{nc}$ , as signatories invest in R&D to eliminate completely the GHG emissions, their optimization problem is given by (2.69), while the constraints on effective investment are given by (2.14)-(2.15).

As the optimization problem is the same as that presented in the emission and R&D agreement without information exchange, the solution of this stage yields the same levels of investments for non-signatories and signatories given by (2.71) and (2.72) respectively, and the effective investment for signatories is now given by (2.73).

However, as the constraint on signatories' effective investment is different, the critical value of marginal damages for which the constraint on effective investment given by (2.14) is satisfied, is now  $\tilde{d}^s(n)$  given by (2.38) instead of (2.74). It is

already known that  $\tilde{d}^s(n) \geq \hat{d}^{nc}$  provided that  $\gamma \geq \check{\gamma}$  where  $\check{\gamma}$  is given by (2.39). By comparison, it is also found that  $\tilde{d}^s(n) \geq \hat{d}_1^s(2)$  for any  $\gamma \geq \hat{\gamma}_1$  where

$$\hat{\gamma}_1 = \frac{n-2}{2(N-1)}, \quad (3.4)$$

and vice versa.

According to that, three possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}^s(n) > \hat{d}^{nc} > \hat{d}_1^s(2) > \hat{d}_1^s(N).$$

Second, if  $\gamma \in (\hat{\gamma}_1, \check{\gamma}]$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}^s(n) > \hat{d}_1^s(2) > \hat{d}_1^s(N),$$

and finally, if  $\gamma < \hat{\gamma}_1$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_1^s(2) > \tilde{d}^s(n) > \hat{d}_1^s(N).$$

Notice that for high levels of cooperation, the value of  $\check{\gamma}$  is going to be very high, reaches to 1 at the level of full cooperation. Thus, our analysis for profitability of joining the agreement will be focused the two cases when  $\gamma \in (\hat{\gamma}_1, \check{\gamma}]$  and when  $\gamma < \hat{\gamma}_1$ . However, the levels of investments and the corresponding total costs functions of non-signatories and signatories countries will be analyzed for the different three cases.

In all cases, when  $d \geq \tilde{d}^{nc}$ , total costs for non-signatories and signatories are given by (2.77) and (2.78) respectively. Next, we analyze the case when  $\gamma \geq \check{\gamma}$ . If  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , total costs for non-signatories and signatories are given by (2.82) and (2.83) respectively. However, if  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , as the effective investment given by

(2.73) doesn't satisfy, for any  $d \leq \tilde{d}^s(n)$ , the constraint on effective investment given by (2.66), signatories countries reduce their effective investment to the same level as non-signatories' effective investment given by (2.6). According to that, total costs of both non-signatories and signatories are the same given by (2.20).

Next, if  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$ , as marginal damages are low enough to make it unprofitable for non-signatories to invest in R&D, the optimal investment level for signatories is given by

$$x_j^s = \frac{\bar{\delta}}{\alpha(1 + \gamma(n - 1))}, \quad (3.5)$$

where the level of investment in R&D is decreasing with the number of signatories countries. The effective investment for signatories countries is still the same given by (2.6) as their emissions are zero, but the effective level of investment for non-signatories countries changes to

$$y_i^f = \frac{n\gamma\bar{\delta}}{\alpha(1 + \gamma(n - 1))}, \quad (3.6)$$

which is lower than the effective investment of the signatories and, consequently, lower than the effective investment of non-signatories at the higher levels of the marginal damages. Using (2.26), global effective investment becomes

$$Y = (N - n)y_i^f + ny_j^s = \frac{\bar{\delta}n(1 + \gamma(N - 1))}{\alpha(1 + \gamma(n - 1))}. \quad (3.7)$$

Finally, Using (2.23), global emissions can be written as

$$E = \frac{\bar{\delta}(N - n)(1 - \gamma)}{1 + \gamma(n - 1)}, \quad (3.8)$$

which decrease with cooperation. Doing the substitutions for the effective investments in the total cost functions, the total costs for non-signatories and signatories are



obtained as follows

$$TC_i^f = \frac{d^2}{2c} + \frac{\bar{\delta}d(N-n)(1-\gamma)}{1+\gamma(n-1)}, \quad i = 1, \dots, N-n, \quad (3.9)$$

$$TC_j^s = \frac{d^2}{2c} + \frac{\bar{\delta}d(N-n)(1-\gamma)}{1+\gamma(n-1)} + \frac{\bar{\delta}}{\alpha(1+\gamma(n-1))}, \quad j = 1, \dots, n, \quad (3.10)$$

where the first term represents the abatement costs, the second term represents the environmental damages and the third term, in the total costs of signatories countries, represents the investment costs. It is easy to notice that cooperation has a negative effect on total costs for both non-signatories and signatories.

However, if  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , total costs functions are still the same given by (3.9) and (3.10) until we reach to the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$  in Figure 2.1, at this point the total costs function of signatories countries is still the same given by (3.10), while non-signatories' total costs function should be changed taking into account that by moving from the area above the curve  $\hat{d}_1^s(n)$  in Figure 2.1 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by (2.22).

Second, if  $\gamma \in (\hat{\gamma}_1, \check{\gamma}]$  and  $d \in (\hat{d}^{nc}, \check{d}^{nc}]$ , the total costs for non-signatories are given by (2.82), while given by (2.83) for signatories countries. However, if  $d \in (\check{d}^s(n), \hat{d}^{nc}]$ , as signatories countries invest to eliminate the emissions, total costs function are given by (2.97) and (2.98) for non-signatories and signatories respectively. Nevertheless, as the constraint on effective investment is now given by

$$y_j^s = (1 + \gamma(n-1))x_j^s \geq \frac{\bar{\delta}}{\alpha}, \quad (3.11)$$

instead of (2.92), the critical value of marginal damages which satisfies this constraint is now given by

$$\hat{d}_1^s(n) = \frac{1}{\alpha n(1 + \gamma(N-n))}. \quad (3.12)$$

Notice that  $\hat{d}_1^s(n)$  is higher than  $\hat{d}_1^s(n)$  for all  $n \geq 2$ , while it is lower than  $\hat{d}_1^s(2)$  for high levels of cooperation. If  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$ , the solution is the same as presented above provided that  $d \geq \hat{d}_1^s(n)$ . Otherwise, total costs will be given by (3.9) and (3.10). For  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , as  $\hat{d}_1^s(n) > \hat{d}_1^s(n)$ , the solution is the same as that presented for  $\gamma \geq \check{\gamma}$  and  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ .

Finally, if  $\gamma < \hat{\gamma}_1$ , it is clear that for any level of marginal damages lower than  $\hat{d}^{nc}$ , the critical value of marginal damages given by (2.38) doesn't play role in satisfying the constraint over signatories' effective investment given by (2.92) and the new critical value of marginal damages (3.12) is the one that plays this role. Thus, it can be concluded that the analysis, at the different intervals of marginal damages, under the assumption  $\gamma < \hat{\gamma}_1$  is exactly the same as the analysis developed under the assumption  $\gamma \in (\hat{\gamma}_1, \check{\gamma}]$ .

Next, the profitability of joining the R&D agreement without information exchange is analyzed for any level of  $\gamma < \check{\gamma}$ . Notice that as the total costs functions at the high levels of marginal damages are the same as those obtained in the analysis of the emission and R&D agreement without information exchange, the numerical example given by Table 2-3 is applied here and the following proposition is concluded

**Proposition 20** *R&D agreement without information exchange is profitable for  $d \geq \hat{d}^{nc}$ .*

However, if  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$  and  $d < \hat{d}_1^s(n)$  and for  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$  at any level of cooperation higher than the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$ , where total costs functions are given by (3.9) and (3.10), the profitability is not analyzed as it will be shown, analytically, in the analysis of the Nash equilibrium of the membership game that the agreement is not stable in this interval of marginal damages.

### 3.1.2 The Nash Equilibrium of the Membership Game

In this section, the stability analysis is studied to investigate whether there exist a stable R&D agreement without information exchange or not. As the stability at high values of marginal damages can be concluded directly from Table 2-3 and Table 2-4, we analyze here the cases when  $d \in (\hat{d}_1^s(2), \hat{d}^{nc}]$  and  $d < \hat{d}_1^s(n)$  and when  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$  where total costs functions are given by (3.9) and (3.10).

Now, the auxiliary function  $\Omega(n)$  is built using total costs for non-signatories given by (3.9) and total costs for signatories given by (3.10)

$$\check{\Omega}(n) = \frac{\bar{\delta}}{1 + \gamma(n-1)} \left( \frac{1}{\alpha} - \frac{d(1-\gamma)(1+\gamma(N-1))}{1 + \gamma(n-2)} \right), \quad (3.13)$$

where  $\check{\Omega}(n)$  is an increasing concave function and the solution to  $\check{\Omega}(n) = 0$  yields

$$\check{n} = \frac{1}{\gamma} (\alpha d (1-\gamma)(1+\gamma(N-1)) - 1) + 2. \quad (3.14)$$

In the light of our assumption that  $\hat{d}_1^s(2) < d < \hat{d}^{nc}$ , if  $d$  is substituted by  $\hat{d}^{nc}$  in (3.14), then  $\check{n} = 1$ . Thus, for any  $d < \hat{d}^{nc}$ ,  $\check{n}$  will be lower than 1 and the following proposition is concluded

**Proposition 21** *If marginal damages are not great enough, in particular if  $d$  belongs to the interval  $(\hat{d}_1^s(2), \hat{d}^{nc}]$ , there will not be any stable R&D agreement without information exchange.*

When  $d$  is in the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the previous proposition also applies until we reach to the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$  in Figure 2.1 where total costs are given by (2.22) and (3.10).

Now, the auxiliary function  $\Omega(n)$  is given by

$$\hat{\Omega}(n) = \frac{\bar{\delta}}{1 + \gamma(n-1)} \left( \frac{1}{\alpha} - dn(1 + \gamma(N-1)) \right), \quad (3.15)$$

where  $\hat{\Omega}(n)$  is a decreasing linear function. Doing  $\hat{\Omega}(n) = 0$ , the solution for  $\hat{n}$  becomes the same given by (2.68). Thus, the curve  $\hat{d}^s(n)$  in Figure 2.1 also represents all the values of  $n$  for different values of  $d$  for which (3.15) is zero. Thus, to check the internal stability condition for the lowest natural number on the right of curve  $\hat{d}_1^s(n)$ , function (3.15) must be used. So, we need to know which is the relative position of functions (3.15) and (3.13) to advance in the stability analysis. The difference between  $\hat{\Omega}(n)$  and  $\check{\Omega}(n)$  is given by the following expression

$$\hat{\Omega}(n) - \check{\Omega}(n) = \frac{-\bar{\delta}d(1 + \gamma(N-1))}{1 + \gamma(n-1)} \left( \frac{n-1 + \gamma(1 + n(n-2))}{1 + \gamma(n-2)} \right) < 0,$$

which is negative for all  $n \geq 2$ . So that, it can be concluded that  $\hat{\Omega}(n) < \check{\Omega}(n)$  and consequently  $\hat{n}_1$  is lower than  $\check{n}$ . Let us now call  $\tilde{n}$  to the lowest natural number on the right to the of curve  $\hat{d}^s(n)$ . Then, the following relationship is obtained:  $\hat{n}_1 < \tilde{n} < N < \check{n}$ . According to the function  $\check{\Omega}(n)$ , none of the values from  $\tilde{n} + 1$  to  $N$  satisfy the internal stability condition, but the internal stability condition for  $\tilde{n}$  must be checked using the function  $\check{\Omega}(n)$ , and as  $\hat{n}_1$  is lower than  $\tilde{n}$  we find that  $\check{\Omega}(\tilde{n})$  is negative and  $\tilde{n}$  satisfies the internal stability condition. Moreover, as  $\check{\Omega}(\tilde{n} + 1)$  is positive, the external stability condition is also satisfied and then an agreement consisting of a number of signatories equal to  $\tilde{n}$  is the only stable R&D agreement without information exchange. Figure 3.1 illustrates this argument.

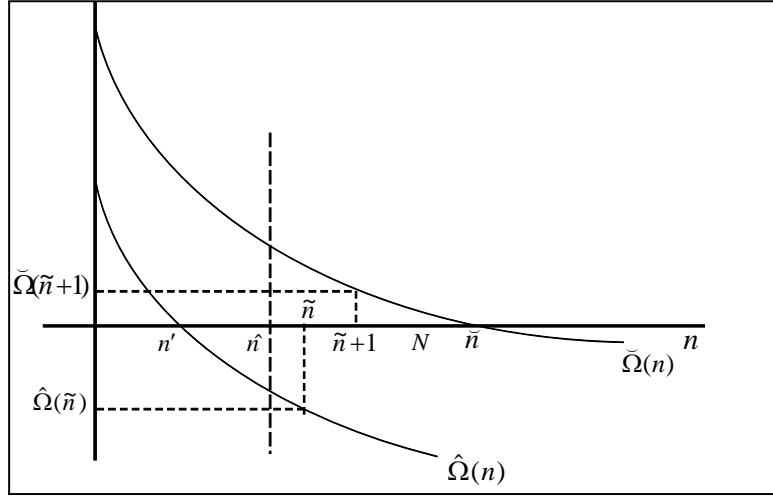


Figure 3.1 Stability conditions

The following proposition summarizes this result.

**Proposition 22** *If marginal damages are not great enough, in particular if  $d$  belongs to the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the lowest natural number greater than  $\hat{n}_1$  is the unique stable R&D agreement without information exchange.*

Moreover, as  $\hat{n}_1$  decreases when the marginal damages increase according to function  $\hat{d}_1^s(n)$  we obtain that

**Corollary 23** *If marginal damages are not great enough, in particular if  $d$  belongs to the interval  $(\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , the greater are the marginal damages, the lower is the level of participation in R&D agreement without information exchange.*

A standard result in the literature of international environmental agreements.

### 3.2 Research Joint Venture Agreement (RJV)

The research joint venture agreement is modeled as a three stage game as the emission agreement with information exchange that has been analyzed in the second chapter. The main difference between the two agreements is that all countries, non-signatories and signatories of the research joint venture agreement, are acting non-cooperatively at the third stage of the game (emission game).

### 3.2.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved by backward induction assuming that in the first stage  $n$  countries, with  $n \geq 2$ , have signed the agreement. As we have supposed that there is no cooperation in the third stage of the game (emissions), total emissions and total costs supported by all countries are given, as in the fully non-cooperative equilibrium, by (2.7) and (2.9) while the global effective investment in R&D is given by (2.103).

Next, the partial agreement Nash equilibrium of the investment game is calculated. As the non-signatories countries do not cooperate at this stage, the solution for non-signatories is identical to the solution that has been developed in the fully non-cooperative equilibrium. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$  where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, acting non-cooperatively at this stage, the optimization problem that yields the optimal investment is given as follows

$$\min_{x_j^s} TC_j^s = \frac{d^2}{2c} + dN\bar{\delta} - \alpha dY + x_j^s, \quad (3.16)$$

$$s.t. \quad \frac{\bar{\delta}}{\alpha} - X_{-j}^s - \gamma X^f \geq x_j^s, \quad (3.17)$$

$$x_j^s \geq 0, \quad (3.18)$$

where  $Y$  is given by (2.103).

Although the optimization problem is different from the optimization problem of signatories of emission agreement with information exchange given by (2.104)-(2.106), the effect of investment on total costs function is the same. Thus, the critical value of marginal damages which defines the stable research joint venture agreement is the same  $\hat{d}_2^s(n)$  given by (2.107). Therefore, the decision on investing in R&D for both signatories and non-signatories are defined as in figure 2.2.

When  $d \geq \hat{d}^{nc}$ , as signatories invest in R&D to eliminate completely the GHG emissions, their optimization problem is given by (2.109), while the constraints on effective investment are given by (2.14)-(2.15).

As the optimization problem is the same as that presented in the emission agreement with information exchange, the solution of this stage yields the same levels of investments for non-signatories and signatories given by (2.113) and (2.114) respectively, and the effective investment for signatories is the same for non-signatories given by (2.17).

However, the critical value of marginal damages for which the constraint on effective investment given by (2.14) is satisfied, is the same  $\tilde{d}^{nc}$  given by (2.18) instead of (2.38). Thus, the range of marginal damages can be defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_2^s(2) > \hat{d}_2^s(N).$$

First, when  $d \geq \tilde{d}^{nc}$ , total costs for non-signatories and signatories are given by (2.116) and (2.117). Next, if  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , both signatories and non-signatories find it profitable to invest at the level of effective investment given by (2.6) which yields the following levels of investment for non-signatories and signatories

$$x_i^f = \frac{\bar{\delta}}{\alpha(1 + \gamma(N - n))}, \quad (3.19)$$

$$x_j^s = \frac{\bar{\delta}}{\alpha n(1 + \gamma(N - n))}. \quad (3.20)$$

Notice that the signatories' investment is always lower than the non-signatories' investment. Moreover, for non-signatories, investment increases as participation increases. However, for signatories, it depends on the number of signatories and the scope of the spillovers.

Now, by substituting the effective investments in the total cost functions, the following expressions for the total costs are obtained

$$TC_i^f = \frac{d^2}{2c} + \frac{\bar{\delta}}{\alpha(1 + \gamma(N - n))}, \quad i = 1, \dots, N - n, \quad (3.21)$$

$$TC_j^s = \frac{d^2}{2c} + \frac{\bar{\delta}}{\alpha n(1 + \gamma(N - n))}, \quad j = 1, \dots, n. \quad (3.22)$$

Where the first term represents the abatement costs and the second term represents the investment costs. Observe that the signatories' total costs are always lower than the non-signatories' total costs and that there are *negative* spillovers for non-signatories stemming from cooperation, i.e. cooperation increases the cost of non-signatories. Nevertheless, global total costs decrease as cooperation increases.

When  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$ , as the marginal damages are low enough to make unprofitable the investment in R&D for non-signatories countries, the optimal investment level for signatories countries is given by

$$x_j^s = \frac{\bar{\delta}}{\alpha n}, \quad (3.23)$$

which is decreasing with the level of cooperation.

As signatories eliminate emissions completely, while non-signatories do not, the effective investment for signatories is given by

$$y_i^f = \frac{\gamma \bar{\delta}}{\alpha}, \quad (3.24)$$

while global emissions are

$$E = \bar{\delta}(1 - \gamma).$$

Notice that the greater the spillovers, the lower the non-signatories' emissions.



Adding the effective investment for signatories and non-signatories, the global effective investment is obtained as follows

$$Y = \frac{\bar{\delta}}{\alpha}(n + \gamma(N - n)). \quad (3.25)$$

Finally, adding the emissions for non-signatories, we obtain that global emissions decrease with cooperation as follows

$$E = \bar{\delta}(N - n)(1 - \gamma). \quad (3.26)$$

Now, by substituting effective investment and emissions in the total cost functions, the following expressions are obtained

$$TC_i^f = \frac{d^2}{2c} + d\bar{\delta}(N - n)(1 - \gamma), \quad i = 1, \dots, N - n, \quad (3.27)$$

$$TC_j^s = \frac{d^2}{2c} + d\bar{\delta}(N - n)(1 - \gamma) + \frac{\bar{\delta}}{\alpha n}, \quad j = 1, \dots, n. \quad (3.28)$$

Where the first term represents the abatement costs, the second term represents the environmental damages and the third term, in the total costs of signatories, represents the investment costs. Contrary to the previous case, the total costs of signatories are greater than the total costs of non-signatories regardless of the level of cooperation. Moreover, *positive* spillovers now stem from cooperation. This difference in the sign of spillovers from cooperation explains, as we will see in the next section, the different results as regards participation in a stable agreement depending on the level of marginal damages.

However, if  $d \in (\hat{d}_1^s(N), \hat{d}_1^s(2)]$ , total costs functions are still the same given by (3.27) and (3.28) until we reach to the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$  in Figure 2.2, at this point the total costs function of signatories countries is

still the same given by (3.28), while non-signatories' total costs function should be changed taking into account that by moving from the area above the curve  $\hat{d}_2^s(n)$  in Figure 2.2 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by (2.22).

### 3.2.2 The Nash Equilibrium of the Membership Game

Next, the stability analysis is developed to investigate whether there exists a stable research joint venture agreement. It is already known from the analysis of the emission agreement with information exchange that the grand coalition is stable for any level of marginal damages higher than or equal to  $\tilde{d}^{nc}$ . However, if  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , we calculate  $\Omega(n)$  using the total costs functions given by (3.21) and (3.22) as following

$$\Omega(n) = \frac{\bar{\delta}}{\alpha} \left( \frac{1 + \gamma(N - n + 1) - n(1 + \gamma(N - n))}{n(1 + \gamma(N - n))(1 + \gamma(N - n + 1))} \right). \quad (3.29)$$

As the denominator is positive for any level of cooperation, the solution to equation  $\Omega(n)$  is given by the number that does the numerator equal to zero. Developing the numerator, the following function of  $n$  is obtained

$$f(n) = \gamma n^2 - (1 + \gamma(N + 1))n + 1 + \gamma(N + 1). \quad (3.30)$$

It is easy to show that  $f(n) = 0$  has two real positive roots provided that  $N$  is equal to or greater than three, and that the function is decreasing around the lowest root and increasing around the greatest root. Then, an agreement consisting of a number of signatories equal to the greatest natural number on the left of the highest root is self-enforcing provided that this number is lower than  $N$ . We call to this number  $\tilde{n}$ . In order to ascertain whether this is the case, we only need to substitute  $N$  in (3.30). The result is that  $f(N)$  is negative for  $N \geq 2$  which means that  $N \leq \tilde{n}$ , and implies that the grand coalition is the unique stable agreement. Remember that

for the grand coalition, it is only necessary to check the internal stability condition to ascertain whether it is stable or not. Therefore, the following proposition is concluded

**Proposition 24** *The grand coalition is the unique stable research joint venture agreement for any level of marginal damages higher than or equal to  $\hat{d}^{nc}$ , independently of the degree of spillover effects.*

Next, stability conditions are analyzed when it is not optimal for non-signatories to invest, i.e. when marginal damages are equal to or lower than  $\hat{d}^{nc}$ . Now, the auxiliary function  $\check{\Omega}(n)$  is built using the total costs for non-signatories given by (3.27) and the total costs for signatories given by (3.28) as follows

$$\check{\Omega}(n) = \bar{\delta} \left( \frac{1}{\alpha n} - d(1 - \gamma) \right). \quad (3.31)$$

The solution to the equation  $\check{\Omega}(n) = 0$  is

$$\check{n} = \frac{1}{\alpha d(1 - \gamma)}. \quad (3.32)$$

As the slope of  $\check{\Omega}(n)$  is negative, when  $\check{\Omega}(n) = 0$ , the only stable agreement is the grand coalition provided that  $N$  is greater than  $\check{n}$ . Thus, the difference

$$N - \check{n} = N - \frac{1}{\alpha d(1 - \gamma)},$$

should be positive or zero for the grand coalition to be stable. The difference is positive when

$$d \geq \check{d} = \frac{1}{\alpha(1 - \gamma)N}. \quad (3.33)$$

In order to advance in the analysis of the stability conditions, the properties of the function  $\check{d}(\gamma)$  defined by the r.h.s of (3.33) must be studied. It is easy to show that

$\check{d}(\gamma)$  is an increasing convex function that take the value  $1/\alpha N$  for  $\gamma = 0$  and tends to infinite when  $\gamma$  tends to one. Moreover,  $\check{d}(\gamma)$  is equal to  $\hat{d}^{nc}$  when  $\check{\gamma} = (N-1)/(2N-1)$  so that for  $\gamma$  in the interval  $[0, (N-1)/(2N-1))$ ,  $\check{d}(\gamma)$  is lower than  $\hat{d}^{nc}$ . Then we can conclude that when  $\gamma$  is lower than or equal to  $(N-1)/(2N-1)$ , if the marginal damages are larger or equal to  $\check{d}(\gamma)$ , the grand coalition is the only stable research joint venture agreement as the internal stability condition will be satisfied for  $N$ . When this is not the case and the marginal damages are lower than  $\check{d}(\gamma)$  for all values of  $\gamma$ , two cases can be distinguished. First, when marginal damages are lower than  $\check{d}(\gamma)$  and they belong to the interval  $(\hat{d}_2^s(2), \hat{d}^{nc}]$  which requires that  $\gamma$  is greater than  $\bar{\gamma}_1 = (N-2)/2(N-1)$ .<sup>1</sup> In this case, signatories invest regardless of the number of countries that belong to the agreement and only the grand coalition can be stable, but as condition (3.33) is not satisfied because the marginal damages are lower than  $\check{d}(\gamma)$ , it must be concluded that there does not exist any stable agreement for these values of marginal damages. The second case to analyze is when marginal damages are lower than  $\check{d}(\gamma)$  and they belong to the interval  $(\hat{d}_2^s(N), \hat{d}_2^s(2)]$ .<sup>2</sup> As in the previous case, the grand coalition is not stable. But for these values of the marginal damages, signatories invest in R&D provided that a minimum of participation given by (2.108) is reached. Otherwise, signatories' investments are zero. Given this difference with the previous case, it should be investigated whether an agreement consisting of a number of signatories lower than  $N$  may be stable. In particular, whether an agreement that satisfies the minimum of participation defined by the curve  $\hat{d}_2^s(n)$  in Figure 2.2 may be stable. Notice that if membership moves from the area above the curve  $\hat{d}_2^s(n)$  to

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<sup>1</sup>This critical value  $\bar{\gamma}_2$  is obtained doing  $\check{d}(\gamma)$  equal to  $\hat{d}_2^s(2)$ . Remember that  $\check{d}(\gamma)$  is an increasing convex function so that for  $\gamma > (N-2)/2(N-1)$ ,  $\check{d}(\gamma)$  is going to be greater than  $\hat{d}_2^s(2)$ .

<sup>2</sup>Notice that if  $\gamma$  belongs to the interval  $[0, (N-2)/2(N-1)]$ , then when the marginal damages are lower than  $\check{d}(\gamma)$  they are also lower than  $\hat{d}_2^s(2)$ . However, this is not true when  $\gamma$  is greater than  $(N-2)/2(N-1)$ . For this reason we have to impose this second condition.

the area below, the signatories will react to the exit by reducing investment to zero and this may be enough to deter the exit and stabilize the agreement.

Thus, now the difference in costs that must be used to check the internal stability condition is given by the difference of expression (3.28) for signatories and expression (2.22) for non-signatories corresponding to the fully non-cooperative equilibrium as follows

$$\hat{\Omega}(n) = \bar{\delta} \left( d((N - n)(1 - \gamma) - N) + \frac{1}{\alpha n} \right). \quad (3.34)$$

Using  $\hat{\Omega}(n) = 0$ , the following second degree equation is obtained

$$\alpha d(1 - \gamma)n^2 + \alpha \gamma d N n - 1 = 0, \quad (3.35)$$

which has two real roots, one is negative and the other is positive. Moreover, (3.34) is a decreasing convex function for  $n > 0$ . Remember that only at  $n > \hat{n}_2$ , where  $\hat{n}_2$  is given by (2.108), signatories countries will find that it is profitable to invest in R&D. It is easy to check that the positive root of (3.35) is lower than  $\hat{n}_2$ . Therefore, it can be concluded that only at the lowest natural number higher than  $\hat{n}_2$ , the stability condition given by (3.34) can be applied and the agreement will be stable only if  $N > \hat{n}_2$ . So, the difference between  $N$  and  $\hat{n}_2$  is taken as follows

$$N - \hat{n} = N - \frac{1}{1 - \gamma} \left( \frac{1}{\alpha d} - N\gamma \right) = N\alpha d - 1$$

which is higher than zero for all  $d > \hat{d}_2^s(N)$ .

Finally, to confirm whether the difference in costs (3.34) can be used to check the internal stability condition, we must ascertain the relative position of functions (3.31) and (3.34). The difference between  $\hat{\Omega}(n)$  and  $\check{\Omega}(n)$  is given by the following expression

$$\hat{\Omega}(n) - \check{\Omega}(n) = -d\bar{\delta} ((n - 1)(1 - \gamma) + \gamma N) < 0,$$

which is negative for all  $n \geq 2$ . So that, the same argument which illustrated by Figure 3.1 in the analysis of the R&D agreement without information exchange is also applied. Thus, it can be concluded that if the marginal damages are lower than  $\hat{d}(\gamma)$  for all  $\gamma$  and they belong to the interval  $(\hat{d}_2^s(N), \hat{d}_2^s(2)]$ , the lowest natural number greater than  $\hat{n}_2$  is the unique stable research joint venture agreement.

The following proposition summarizes these results that are represented in Figure 3.2.

**Proposition 25** *If marginal damages are not sufficiently large, in particular if  $d \in (\hat{d}_2^s(N), \hat{d}^{nc}]$ , the membership of stable agreement depends on the level of marginal damages and the scope of spillover effects. Three cases can be distinguished: i) if  $d \geq \check{d}(\gamma)$ , then the grand coalition is the unique stable research joint venture agreement. This condition can be satisfied only for  $\gamma \in [0, (N-1)/(2N-1)]$ ; ii) if  $d \leq \check{d}(\gamma)$  and  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$ , then there does not exist any stable research joint venture agreement. These two conditions can be satisfied only for  $\gamma > (N-2)/2(N-1)$ ; iii) if  $d \leq \check{d}(\gamma)$  and  $d \in (\hat{d}_2^s(N), \hat{d}_2^s(2)]$ , then the lowest natural number greater than  $\hat{n}_2$  is the unique stable research joint venture agreement. These two conditions can be satisfied for all  $\gamma$ .*

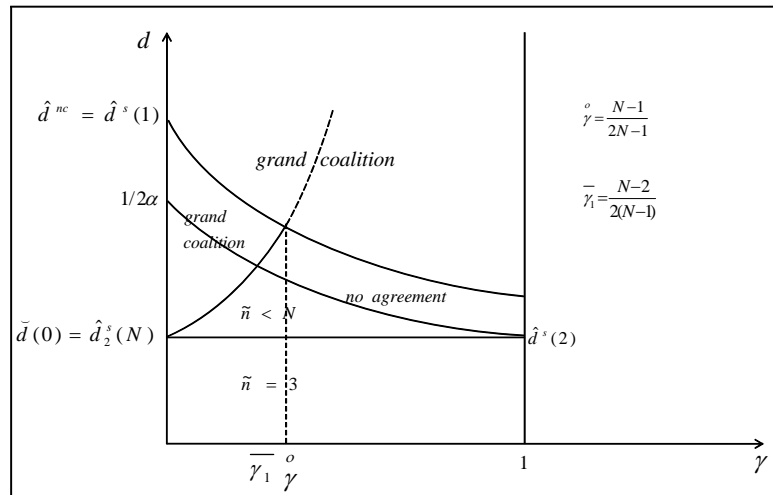


Figure 3.2 Membership of stable agreement

The previous analysis clearly establishes that it is impossible to stabilize the grand coalition if the spillover effects are greater than  $1/2$  or if the marginal damages are lower than  $1/\alpha N$  when non-signatories do not invest in R&D.

Moreover, as  $\hat{n}_2$  decreases when the marginal damages increase according to function  $\hat{d}^s(n)$  it is obtained that

**Corollary 26** *If  $d$  belongs to the interval  $(\hat{d}_2^s(N), \hat{d}_2^s(2)]$  and is lower than  $\check{d}(\gamma)$ , the greater the marginal damages, the lower the level of participation in the research joint venture agreement.*

### 3.3 R&D Agreement with Information Exchange

The R&D agreement with information exchange is modeled as a three stage game as the emission and R&D agreement with information exchange that has been analyzed in the second chapter. The main difference between the two agreements is that all countries, signatories and non-signatories of the R&D agreement with information exchange, are acting non-cooperatively at the third stage of the game (emission game).

#### 3.3.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three are solved by backward induction assuming that in the first stage  $n$  countries, with  $n \geq 2$ , have signed the agreement. As we have supposed that there is no cooperation in the third stage of the game (emissions), total emissions and total costs supported by all countries are given, as in the fully non-cooperative equilibrium, by (2.7) and (2.9) while the global effective investment in R&D is given by (2.103).

Next, the partial agreement Nash equilibrium of the investment game is calculated. As the non-signatories countries do not cooperate at this stage, the solution for non-signatories is identical to the solution that has been developed in the fully non-

cooperative equilibrium. Thus, non-signatories countries will invest in R&D provided that the marginal damages are higher than  $\hat{d}^{nc}$  where  $\hat{d}^{nc}$  is given by (2.12).

For signatories countries, acting cooperatively at this stage, the optimization problem that yields the optimal investment is given by (3.1), while the constraints on investments are given by (3.17)-(3.18). Thus, the critical value of marginal damages which defines the stable R&D agreement with information exchange is the same  $\hat{d}_3^s(n)$  given by (2.143). Therefore, the decision on investing in R&D for both signatories and non-signatories are defined as in Figure 2.3.

However, the critical value of marginal damages for which the constraint on signatories' effective investment given by (3.17) is satisfied, is the same  $\tilde{d}^s(n)$  given by (2.38). By comparing  $\tilde{d}^s(n)$  with  $\hat{d}_3^s(2)$ , it is found that  $\tilde{d}^s(n) \geq \hat{d}_1^s(2)$  for any  $\gamma \geq \hat{\gamma}_3$  where

$$\hat{\gamma}_3 = \frac{n-4}{2(N-2)}, \quad (3.36)$$

and vice versa

According to that, three possibilities should be analyzed. First, if  $\gamma \geq \check{\gamma}$  where  $\check{\gamma}$  is given by (2.39), the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \tilde{d}^s(n) > \hat{d}^{nc} > \hat{d}_3^s(2) > \hat{d}_3^s(N).$$

Second, if  $\gamma \in (\hat{\gamma}_3, \check{\gamma}]$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \tilde{d}^s(n) > \hat{d}_3^s(2) > \hat{d}_3^s(N),$$

and finally, if  $\gamma < \hat{\gamma}_3$ , the range of marginal damages is defined as follows

$$\tilde{d}^{nc} > \hat{d}^{nc} > \hat{d}_3^s(2) > \tilde{d}^s(n) > \hat{d}_3^s(N).$$

For the same reason concluded in the solution of R&D agreement without information exchange, that the value of  $\check{\gamma}$  is going to be very high, reaches to 1 at the



level of full cooperation, our analysis for profitability of joining the agreement will be focused on the two cases when  $\gamma \in (\hat{\gamma}_3, \check{\gamma}]$  and when  $\gamma < \hat{\gamma}_3$ . However, the levels of investments and the corresponding total costs functions of non-signatories and signatories countries will be analyzed for the different three cases.

In all cases, when  $d \geq \tilde{d}^{nc}$ , total costs for non-signatories and signatories are given as in the emission and R&D agreement with information exchange by (2.149) and (2.150) respectively. Next, we analyze the case when  $\gamma \geq \check{\gamma}$ . If  $d \in (\tilde{d}^s(n), \tilde{d}^{nc}]$ , total costs for non-signatories and signatories are given by (2.154) and (2.155). However, if  $d \in (\hat{d}^{nc}, \tilde{d}^s(n)]$ , total costs for non-signatories and signatories are given as in the research joint venture agreement by (3.21) and (3.22) respectively. If  $d \in (\hat{d}_3^s(2), \tilde{d}^s(n)]$ , total costs for non-signatories and signatories are given by (3.28) and (3.27). However, if  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , total costs functions are still the same given by (3.27) and (3.28) until we reach to the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$  in Figure 2.3, at this point the total costs function of signatories countries is still the same given by (3.28), while non-signatories' total costs function should be changed taking into account that by moving from the area above the curve  $\hat{d}_3^s(n)$  in Figure 2.3 to the area below, signatories countries will react to the exit reducing investment to zero. For zero investment, the total costs of non-signatories is given by (2.22).

Second, if  $\gamma \in (\hat{\gamma}_3, \check{\gamma}]$  and  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , the total costs for non-signatories are given by (2.154), while given by (2.155) for signatories countries. However, if  $d \in (\tilde{d}^s(n), \hat{d}^{nc}]$ , total costs are given by (2.159) and (2.160). As the new constraint on effective investment given by

$$y_j^s = nx_j^s \geq \frac{\bar{\delta}}{\alpha}, \quad (3.37)$$

is satisfied for any level of marginal damages higher than  $\hat{d}_1^s(n)$  given by (3.12). Notice that  $\hat{d}_1^s(n)$  is higher than  $\hat{d}_3^s(n)$  for all  $n \geq 2$ , while it is lower than  $\hat{d}_3^s(2)$  for high levels of cooperation. If  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$ , the solution is the same as presented above provided that  $d \geq \hat{d}_1^s(n)$ . Otherwise, total costs will be given by (3.27) and (3.28).

For  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , as  $\hat{d}_1^s(n) > \hat{d}_3^s(n)$ , the solution is the same as that presented for  $\gamma \geq \check{\gamma}$  and  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ .

Finally, if  $\gamma < \hat{\gamma}_3$ , it is clear that for any level of marginal damages lower than  $\hat{d}^{nc}$ , the critical value of marginal damages given by (2.38) doesn't play role in satisfying the constraint over signatories' effective investment given by (3.37) and the new critical value of marginal damages (3.12) is the one that plays this role. Thus, it can be concluded that the analysis, at the different intervals of marginal damages, under the assumption  $\gamma < \hat{\gamma}_3$  is exactly the same as the analysis developed under the assumption  $\gamma \in (\hat{\gamma}_3, \check{\gamma}]$ .

Next, the profitability of joining the R&D agreement with information exchange is analyzed for any level of  $\gamma < \check{\gamma}$ . Notice that as the total costs functions at the high levels of marginal damages are the same as those obtained in the analysis of the emission and R&D agreement with information exchange, the numerical examples given by Table 2-8 and Table 2-9 are applied here at the same levels of marginal damages and the following proposition is concluded

**Proposition 27** *R&D agreement with information exchange is profitable.*

### 3.3.2 The Nash Equilibrium of the Membership Game

Next, the stability analysis is developed to investigate whether there exist a stable agreement. At  $d \geq \hat{d}_3^s(2)$ , the same proposition of the emission and R&D agreement with information exchange is concluded

**Proposition 28** *The grand coalition of emission and R&D agreement with information exchange is stable at any level of marginal damages higher than or equal to  $\hat{d}_3^s(2)$ .*

However, for  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , as total costs are given by (3.27) and (3.28) as in the research joint venture agreement, the same auxiliary function  $\hat{\Omega}(n)$  given by

(3.34) is applied here. Observe that the positive solution to equation (3.35) is the same like the positive solution to equation (2.144), i.e. it coincides with  $\hat{n}_3$ , the value for which the total costs of the agreement are independent of the investment. Thus, the same proposition of the research joint venture agreement can be concluded here<sup>3</sup>

**Proposition 29** *If marginal damages are not sufficiently large, in particular if  $d \in (\hat{d}_3^s(N), \hat{d}^{nc}]$ , the membership of stable agreement depends on the level of marginal damages and the scope of spillover effects. Three cases can be distinguished: i) if  $d \geq \check{d}(\gamma)$ , then the grand coalition is the unique stable research joint venture agreement. This condition can be satisfied only for  $\gamma \in [0, (N-4)/(3N-4)]$ ; ii) if  $d \leq \check{d}(\gamma)$  and  $d \in (\hat{d}_3^s(2), \hat{d}^{nc}]$ , then there does not exist any stable research joint venture agreement. These two conditions can be satisfied only for  $\gamma > (N-4)/(3N-4)$ ; iii) if  $d \leq \check{d}(\gamma)$  and  $d \in (\hat{d}_3^s(N), \hat{d}_3^s(2)]$ , then the lowest natural number greater than  $\hat{n}_3$  is the unique stable research joint venture agreement. These two conditions can be satisfied for all  $\gamma$ .*

### 3.4 Comparison Between the Different Types of Agreements

In this section, a comparison between the different types of agreements, analyzed in this chapter and in the previous one, is introduced. It is clear from the analysis of all agreements, that under different levels of marginal damages, each agreement provides different level of cooperation and different total costs. According to that, we have to compare first all the critical values of marginal damages in order to determine, for a given level of marginal damages, which agreement is dominating the others with respect to the level of cooperation and total costs.

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<sup>3</sup>Now  $\check{d}(\gamma)$  is equal to  $\hat{d}_3^s(2)$  at  $\bar{\gamma}_2 = (N-4)/(3N-4)$ .

### 3.4.1 Comparison Between the Critical Values of Marginal Damages

By taking the differences between all the critical values of the marginal damages for the different agreements, it is found that the critical values of the marginal damages of both emission agreement with information exchange given by (2.107) are higher than the critical values of the marginal damages of the emission and R&D agreement without information exchange given by (2.67), which in turn are higher than those of the emission and R&D agreement with information exchange given by (2.143). Also, it is found that for any  $\gamma > \tilde{\gamma}$  (where  $\tilde{\gamma} = \frac{N-2}{2(N-1)}$ ),  $\hat{d}_2^s(N)$  is going to be higher than  $\hat{d}_1^s(2)$  and the relation between the critical values of the marginal damages is given as in Figure 3.3.

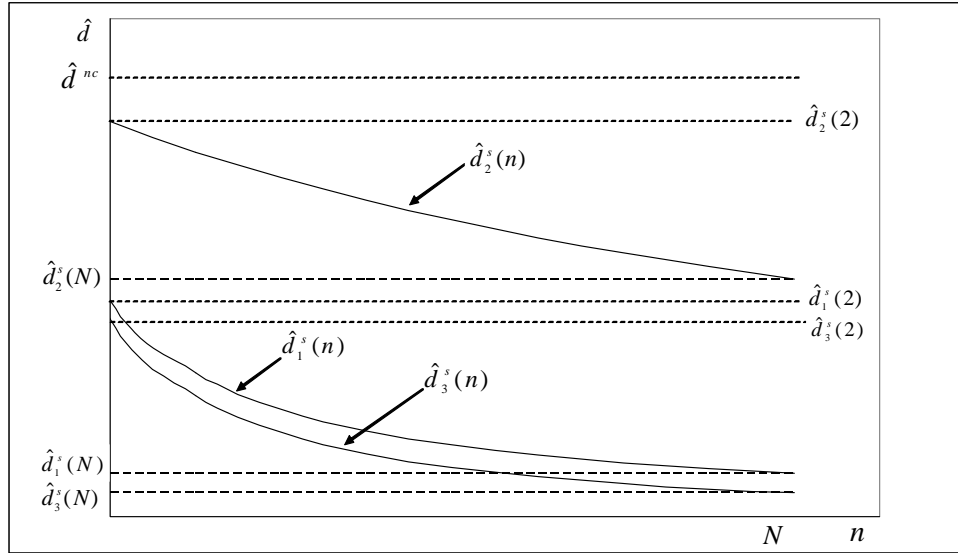


Figure 3.3: The relation between the critical values of marginal damages at  $\gamma > \tilde{\gamma}$

However for any  $\gamma < \tilde{\gamma}$  and at high values of  $N$ ,  $\hat{d}_1^s(2)$  is higher than  $\hat{d}_3^s(2)$  which is higher than  $\hat{d}_2^s(N)$  and the relation between the critical values of the marginal damages is given as in Figure 3.4. (See appendix 1).

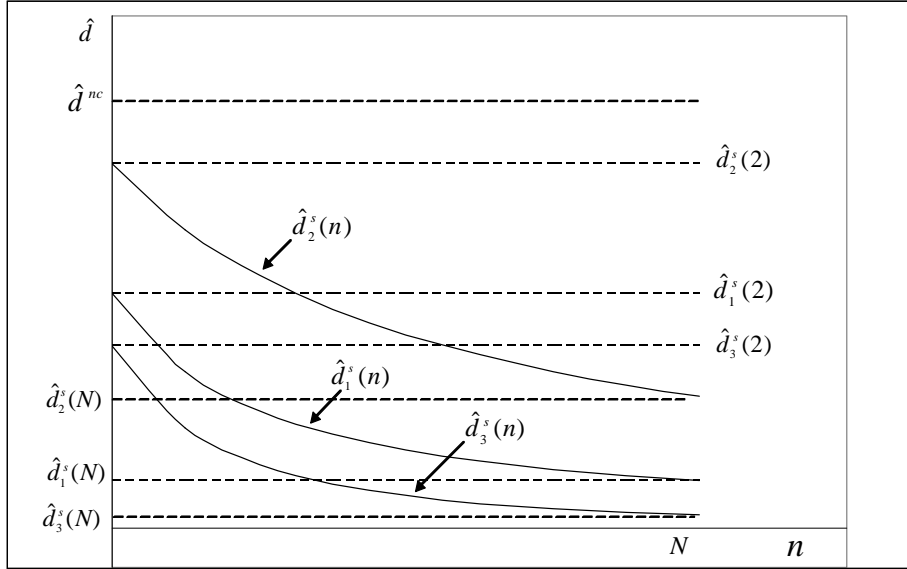


Figure 3.4: The relation between the critical values of marginal damages at  $\gamma < \check{\gamma}$

Notice that  $\check{\gamma} = \bar{\gamma}_1$  where  $\bar{\gamma}_1$  is the same value for which  $\check{d}(\gamma)$  equal to  $\hat{d}_2^s(2)$ . Thus, the relation between the different critical values of marginal damages given by Figure 3.3 and Figure 3.4 can be combined in Figure 3.5 which draws the relation between critical values of marginal damages with respect to the diffusion parameter.

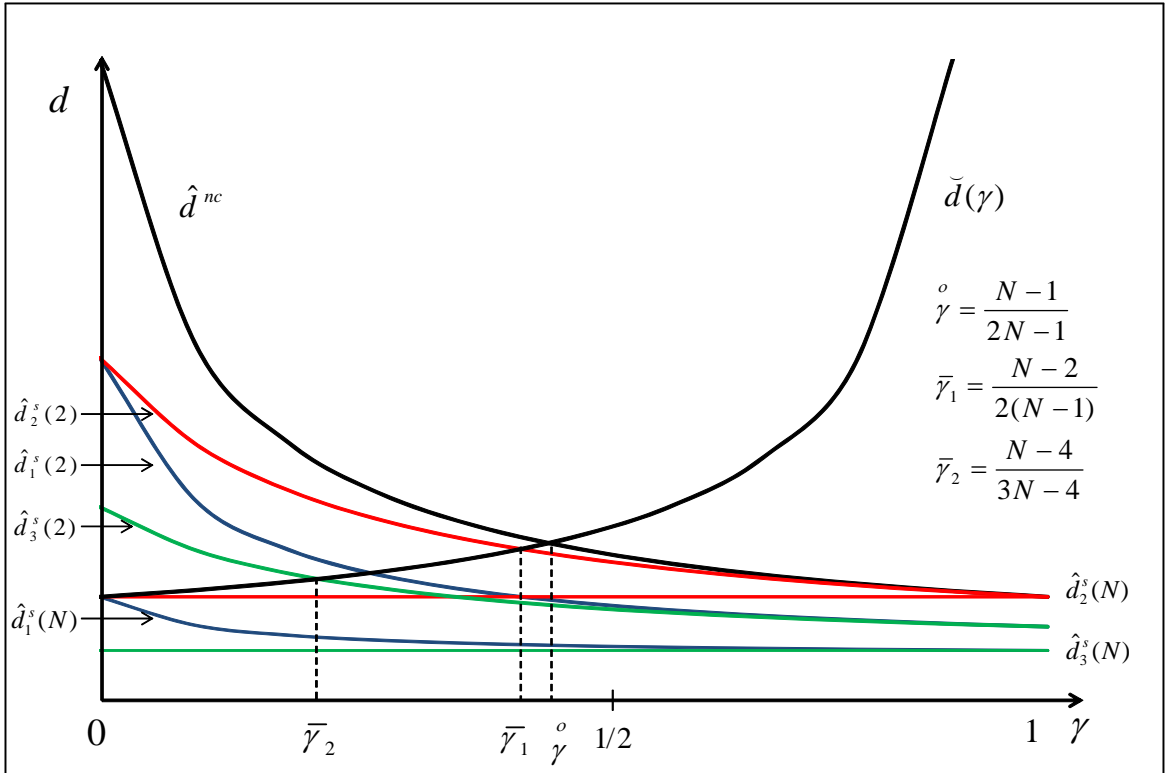


Figure 3.5: The relation between the critical values of marginal damages and  $\gamma$

Next, we compare the different types of agreements under the assumption that  $\gamma < \ddot{\gamma} = \bar{\gamma}_1$  where the critical values of marginal damages intersect as in Figure 3.4 and as in the left side of Figure 3.5. The comparison will be divided into two parts: First, the corner solution, i.e. at the high values of marginal damages,  $d \geq \hat{d}^{nc}$ , where all countries invest at the maximum level of investment to eliminate completely the GHG emissions. Second, the comparison at low values of marginal damages, i.e.  $d < \hat{d}^{nc}$ , where only signatories countries invest to eliminate completely the GHG emissions while non-signatories countries don't invest.

### 3.4.2 Comparison Between the Different Types of Agreements at the Corner Solution

We know from our analysis to the different types of agreements that at the high values of marginal damages, the grand coalition is stable only for the agreements that include information exchange, i.e. emission agreement with information exchange and emission and R&D agreement with information exchange in the second chapter, and both research joint venture agreement and R&D agreement with information exchange in the third chapter. Also, it is shown that cooperation in the third stage of the game, emission game, doesn't play role at the high values of marginal damages ( $d \geq \tilde{d}^{nc}$ ) on the level of cooperation and total costs of non-signatories and signatories, as all countries invest in R&D at the maximum level to eliminate completely the GHG emissions. Thus, at  $d \geq \tilde{d}^{nc}$ , both research joint venture agreement (as the emission agreement with information exchange) and R&D agreement with information exchange (as the emission and R&D agreement with information exchange) yield the grand coalition and dominate the other agreements with respect to the level of cooperation. However, the total costs of the grand coalition of the research joint venture agreement is given by  $TC_j^s(10) = 1.998$  as shown by the numerical example of Table 2.8 and the profitability is given by  $prof. = 4.253$ , while for the R&D

agreement with information exchange, by substituting in the total costs functions given by (2.116) and (2.117) with the same values for different parameter used in Table 2-8, it is found that  $TC_j^s(10) = 2.2$  and the profitability is given by  $prof. = 4.05$ . Thus, R&D agreement with information exchange dominate the other types of agreements with respect to total costs.

Next for  $d \in (\hat{d}^{nc}, \tilde{d}^{nc}]$ , emission agreement with information exchange and research joint venture agreement and R&D agreement with information exchange (as emission and R&D agreement with information exchange) yield the grand coalition and dominate the other agreements with respect to the level of cooperation. However, the total costs of the grand coalition of the emission agreement with information exchange is given by  $TC_j^s(10) = 2.2$  as shown by the numerical example of Table 2.6 and the profitability is given by  $prof. = 3.9651$ , while for the R&D agreement with information exchange, the total costs of the grand coalition is given by  $TC_j^s(10) = 1.998$  as shown by the numerical example of Table 2.9 and the profitability is given by  $prof. = 4.168$ . Finally, for the research joint venture agreement, by substituting in the total costs functions given by (3.21) and (3.22) with the same values for different parameter used in Table 2-9, it is found that  $TC_j^s(10) = 2.038$  and the profitability is given by  $prof. = 4.102$ . Thus, R&D agreement with information exchange dominate the other types of agreements with respect to total costs. According to that, the following proposition can be concluded

**Proposition 30** *Emission and R&D agreement with information exchange and R&D agreement with information exchange are the dominant agreements for any level of marginal damages higher than  $\hat{d}^{nc}$ .*

### 3.4.3 Comparison Between the Different Types of Agreements at Low Values of Marginal Damages

In this section, the comparison between the different types of agreements is analyzed at the low values of marginal damages, where  $d < \hat{d}^{nc}$ . It is known that for  $d < \hat{d}^{nc}$ , the decision of non-signatories, under any type of agreements, is not to invest in R&D.

#### 3.4.3.1 For $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$

In this interval of marginal damages, the grand coalition is stable for the emission agreement with information exchange, emission and R&D agreement with information exchange, R&D agreement with information exchange and the research joint venture agreement. By substituting for  $n = N$  in (2.160), total costs function of both emission and R&D agreement with information exchange and R&D agreement with information exchange is given as follows

$$TC_j^s(N) = \frac{1}{2\alpha^2 c N^2} + \frac{\alpha c \delta N - 1}{\alpha^2 c N^2}. \quad (3.38)$$

Nevertheless, by substituting for  $n = N$  in (3.28), it is found that total costs of signatories of the research joint venture agreement is the same given by (3.38) at  $d = \hat{d}_2^s(N) = 1/\alpha N$ . As total costs function given by (3.28) is increasing in the level of marginal damages, it is concluded that for  $d = \hat{d}_2^s(2) > \hat{d}_2^s(N)$ , the total costs function of the research joint venture agreement is higher than total costs function given by (3.38). Finally, by substituting for  $n = N$  in (2.141), the total costs function of the emission agreement with information exchange is given as follows

$$TC_j^s(N) = \frac{1}{2\alpha^2 c N^2} + \frac{\alpha c \delta - 1}{\alpha^2 c N}. \quad (3.39)$$

It is easy to check that total costs function given by (3.38) is higher than the total costs function given by (3.39) and the following proposition is concluded



**Proposition 31** *Emission agreement with information exchanges is the dominant agreement in the interval of marginal damages given by  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$ .*

**3.4.3.2 For  $d \in (\hat{d}_3^s(2), \hat{d}_2^s(2)]$**

In this interval of marginal damages, the grand coalition is stable for both emission and R&D agreement with information exchange and R&D agreement with information exchange, where the total costs function for both agreements is given by (3.38) and the following proposition is concluded

**Proposition 32** *Emission and R&D agreement with information exchange and R&D agreement with information exchange are the dominant agreements in the interval of marginal damages given by  $d \in (\hat{d}_3^s(2), \hat{d}_2^s(2)]$ .*

**3.4.3.3 For  $d \in (\hat{d}_2^s(N), \hat{d}_3^s(2)]$**

For any given  $d$  in this interval of marginal damages, it is known that for both emission and R&D agreement without information exchange and R&D agreement without information exchange, the only astable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$ . However, for both emission agreement with information exchange and research joint venture agreement, the stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_2^s(n)$ . Finally, for both emission and R&D agreement with information exchange and R&D agreement with information exchange, the stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$  in Figure 3.4. It is known from the numerical example solved in the emission agreement with information exchange, assuming that  $d = 0.11$ , that the level of cooperation is given by  $n = 9$  while  $TC_j^s(9) = 3.8255$ . Also, from the solution of numerical example solved for the emission and R&D agreement with information exchange, for  $d = 0.11$ , the level of cooperation is given by  $n = 3$  while  $TC_j^s(4) = 18.165$ . However, for the research joint venture agreement, by substituting with the same values of different parameter solved in the numerical examples,

assuming  $d = 0.11$ , in the total costs function given by (3.28) where  $n = 9$ , it is found that  $TC_j^s(9) = 3.8846$ . Doing the same for R&D agreement with information exchange, where total costs are the same given by (3.28) and  $n = 3$ , it is found that  $TC_j^s(3) = 18.194$ . Finally, for both emission and R&D agreement without information exchange and R&D agreement without information exchange, where total costs functions are given by (2.89) and (3.10) respectively, where  $n = 3$  and  $d = 0.11$ , it is found that total costs of the emission and R&D agreement without information exchange are given by  $TC_j^s(3) = 20.971$ , while total costs of the R&D agreement without information exchange are given by  $TC_j^s(3) = 20.979$ . By analyzing the different levels of cooperation and the corresponding total costs for each agreement, the following proposition is concluded

**Proposition 33** *Emission agreement with information exchange is the dominant agreement in the interval of marginal damages given by  $d \in (\hat{d}_2^s(N), \hat{d}_3^s(2)]$ .*

#### 3.4.3.4 For $d \in (\hat{d}_1^s(N), \hat{d}_2^s(N)]$

For any given  $d$  in this interval of marginal damages, it is known that for both emission and R&D agreement without information exchange and R&D agreement without information exchange, the only stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_1^s(n)$ . Finally, for both emission and R&D agreement with information exchange and R&D agreement with information exchange, the stable agreement is given by the lowest natural number on the right of the curve  $\hat{d}_3^s(n)$  in Figure 3.4. It is obvious from the previous analysis that the agreements that include cooperation in the third stage yield lower costs than the agreements that don't include cooperation. Thus, both emission and R&D agreement without information exchange and emission and R&D agreement with information exchange are dominating all the other types of agreements. However, by assuming that ( $d = 0.11$ ,  $\gamma = 0.25$ ), total costs of the emission and R&D agreement without information exchange are given by

$TC_j^s(7) = 8.8807$  for the level of cooperation given by  $n = 7$ , while the total costs function of the emission and R&D agreement with information exchange are given by  $TC_j^s(4) = 9.4773$  for the level of cooperation given by  $n = 4$ . According to that, the following proposition is concluded

**Proposition 34** *Emission and R&D agreement without information exchange is the dominant agreement in the interval of marginal damages given by  $d \in (\hat{d}_1^s(N), \hat{d}_2^s(N)]$ .*

### 3.4.3.5 For $d < \hat{d}_1^s(N)$

The unique stable agreement is given by the emission and R&D agreement with information exchange and R&D agreement with information exchange and the following proposition can be concluded directly

**Proposition 35** *Emission and R&D agreement with information exchange is the dominant agreement for any level of marginal damages lower than  $\hat{d}_1^s(N)$ .*

## 3.5 Conclusion

In this chapter, the three different types of agreements studied in the second chapter (emission and R&D agreement without information exchange, emission agreement with information exchange and emission and R&D agreement with information exchange) are analyzed again, but assuming that signatories countries act non-cooperatively in the third stage of the game (emission game).

Although the optimization problems of signatories of the different types of agreements are different from the optimization problems of signatories of the corresponding types of agreements that have been solved in the second chapter, the effect of investment on the total costs function is the same. Thus, the critical values of marginal damages which define the stable agreements are the same as those obtained in the second chapter.

In the analysis of this chapter, it is found that as far as signatories invest at the maximum level of R&D investment to eliminate completely the GHG emissions, the solution of each agreement at the high levels of marginal damages, yields the same level of cooperation and total costs functions as those obtained in the corresponding type of agreement solved in the second chapter where signatories act cooperatively in the third stage of the game (emission game). However, acting non-cooperatively in the third stage of the game has an effect on the critical values of marginal damages which satisfy the constraints on signatories' effective investment, and this explains the changes that occur between the different types of agreements from those solved in the second chapter at the low values of marginal damages, i.e. when signatories are not investing at the maximum level of investment to eliminate completely the GHG emissions.

The main result of the second chapter, that exchanging R&D information is enough to eliminate countries incentives to act as free-rider, is also concluded in this chapter. Thus, the grand coalition is found to be stable and profitable only for the types of agreements that allow information exchange (research joint venture agreement and R&D agreement with information exchange).

By comparing the different types of agreements that have been analyzed in this chapter and in the second chapter, at the low values of technological spillovers, it is found that at the corner solution, when all countries invest in R&D, emission and R&D agreement with information exchange and R&D agreement with information exchange are the dominant agreements. However, for low values of marginal damages, when it is not profitable for non-signatories to invest in R&D, emission agreement with information exchange becomes the dominant agreement in the interval of marginal damages given by  $d \in (\hat{d}_2^s(2), \hat{d}^{nc}]$ . Then, emission and R&D agreement with information exchange and R&D agreement with information exchange return to be the dominant agreements in the interval of marginal damages given by  $d \in (\hat{d}_3^s(2), \hat{d}_2^s(2)]$ .

For any  $d < \hat{d}_3^s(2)$ , the grand coalition is not stable anymore under any type of agreements. Thus, for  $d \in (\hat{d}_2^s(N), \hat{d}_3^s(2)]$ , emission agreement with information exchange return to be the dominant agreement as it stabilizes higher level of cooperation and lower total costs than the other types.

Emission and R&D agreement without information exchange appears ad a dominant agreement only in the interval of marginal damages given by  $d \in (\hat{d}_1^s(N), \hat{d}_2^s(N)]$ . Finally, for any level of  $d < \hat{d}_1^s(N)$ , emission and R&D agreement with information exchange becomes the dominant agreement.

## 3.6 Appendices

### 3.6.1 Appendix 1: The comparison between all the critical values of the marginal damages

In order to investigate the relation between all the critical values, we take the differences between those values as following;

First, we will rewrite all the critical values for the different agreements to make the comparison easier.

The critical values of the R&D agreement without information exchange

$$\hat{d}_1^s(2) = \frac{1}{2\alpha(1 + \gamma(N - 1))}, \quad \hat{d}_1^s(N) = \frac{1}{\alpha N(1 + \gamma(N - 1))}.$$

The critical values of the RJV agreement

$$\hat{d}_2^s(2) = \frac{1}{\alpha(2 + \gamma(N - 2))}, \quad \hat{d}_2^s(N) = \frac{1}{\alpha N}.$$

The critical values of the R&D agreement with information exchange

$$\hat{d}_3^s(2) = \frac{1}{2\alpha(2 + \gamma(N - 2))}, \quad \hat{d}_3^s(N) = \frac{1}{\alpha N^2}.$$

It is pretty obvious that  $\hat{d}_3^s(2) < \hat{d}_2^s(2)$  and  $\hat{d}_3^s(N) < \hat{d}_2^s(N)$ . Now, we take the differences between  $\hat{d}_1^s(2)$  and  $\hat{d}_2^s(2)$

$$\hat{d}_1^s(2) - \hat{d}_2^s(2) = \frac{1}{\alpha} \left( \frac{-\gamma N}{2(1 + \gamma(N - 1))(2 + \gamma(N - 2))} \right) < 0,$$

which means that  $\hat{d}_1^s(2) < \hat{d}_2^s(2)$ . Next, we take the differences between  $\hat{d}_1^s(N)$  and  $\hat{d}_2^s(N)$

$$\hat{d}_1^s(N) - \hat{d}_2^s(N) = \frac{1}{\alpha N} \left( \frac{-\gamma(N - 1)}{1 + \gamma(N - 1)} \right) < 0,$$

which means that  $\hat{d}_1^s(N) < \hat{d}_2^s(N)$ . Now, we have to take the differences between  $\hat{d}_1^s(2)$  and  $\hat{d}_3^s(2)$

$$\hat{d}_1^s(2) - \hat{d}_3^s(2) = \frac{1}{2\alpha} \left( \frac{1 - \gamma}{(1 + \gamma(N - 1))(2 + \gamma(N - 2))} \right) > 0,$$

which means that  $\hat{d}_3^s(2) < \hat{d}_1^s(2)$ . Then, the differences between  $\hat{d}_1^s(N)$  and  $\hat{d}_3^s(N)$

$$\hat{d}_1^s(N) - \hat{d}_3^s(N) = \frac{1}{\alpha N} \left( \frac{(N - 1)(1 - \gamma)}{(1 + \gamma(N - 1))} \right) > 0,$$

which means that  $\hat{d}_3^s(N) < \hat{d}_1^s(N)$ .

As  $\hat{d}_2^s(2) > \hat{d}_2^s(N)$ ,  $\hat{d}_2^s(N) > \hat{d}_1^s(N)$  and  $\hat{d}_1^s(N) > \hat{d}_3^s(N)$ , then  $\hat{d}_2^s(2) > \hat{d}_1^s(N) > \hat{d}_3^s(N)$ .

Also, as  $\hat{d}_1^s(2) > \hat{d}_1^s(N)$  and  $\hat{d}_1^s(N) > \hat{d}_3^s(N)$ , then  $\hat{d}_1^s(2) > \hat{d}_3^s(N)$ . Now, the relations that are left to be checked, in order to have a complete comparison between all the critical values, are the relations between  $[\hat{d}_2^s(N), \hat{d}_1^s(2)]$ ,  $[\hat{d}_3^s(2), \hat{d}_1^s(N)]$  and  $[\hat{d}_2^s(N), \hat{d}_3^s(2)]$ .

First,

$$\hat{d}_2^s(N) - \hat{d}_1^s(2) = \frac{1}{\alpha} \left( \frac{2(1 + \gamma(N - 1)) - N}{2N(1 + \gamma(N - 1))} \right).$$

As the denominator is always positive, we check numerator

$$2 + 2\gamma N - 2\gamma - N,$$

is going to be positive for any

$$\gamma > \bar{\gamma}_1 = \frac{N - 2}{2(N - 1)}.$$

Second,

$$\hat{d}_3^s(2) - \hat{d}_1^s(N) = \frac{1}{\alpha} \left( \frac{N(1 + \gamma(N - 1)) - 2(2 + \gamma(N - 2))}{2N(2 + \gamma(N - 2))(1 + \gamma(N - 1))} \right).$$

As the denominator is always positive, we check numerator

$$\gamma N^2 + N - 3\gamma N + 4\gamma - 4,$$

observer that the derivative of this numerator with respect to  $\gamma$  is

$$N^2 - 3N + 4 > 0 \quad \forall N \geq 1.$$

This means that  $(\hat{d}_3^s(2) - \hat{d}_1^s(N))$  is increasing with respect to  $\gamma$ . Now, assuming that  $\gamma = 0$ , then this numerator becomes

$$N - 4 > 0 \quad \forall N > 4,$$

and for  $\gamma = 1$ , it becomes

$$N^2 - 2N > 0 \quad \forall N > 2,$$

then we can conclude that ant any  $N > 4$ ,  $(\hat{d}_3^s(2) - \hat{d}_1^s(N))$  is positive regardless the value of  $\gamma$ . In other words, we can say that it is sufficient condition that  $N > 4$  to have  $\hat{d}_3^s(2) > \hat{d}_1^s(N)$ .

Finally, we compare  $[\hat{d}_2^s(N), \hat{d}_3^s(2)]$ . Notice that if the condition  $\gamma > \bar{\gamma}_1$  is satisfied, which means that  $\hat{d}_2^s(2) > \hat{d}_1^s(2)$  and as  $\hat{d}_1^s(2) > \hat{d}_3^s(2)$ , then  $\hat{d}_2^s(N) > \hat{d}_3^s(2)$ . But if this is not the case and the condition  $\gamma < \bar{\gamma}_1$  is not satisfied, then we have to take the difference between  $[\hat{d}_2^s(N), \hat{d}_3^s(2)]$  as follows

$$\hat{d}_2^s(N) - \hat{d}_3^s(2) = \frac{1}{\alpha} \left( \frac{2(2 + \gamma(N - 2)) - N}{2N(2 + \gamma(N - 2))} \right).$$

As the denominator is always positive, we check numerator

$$4 + 2\gamma N - 4\gamma - N,$$

is going to be positive for any

$$\gamma > \tilde{\gamma} = \frac{N - 4}{2(N - 2)}.$$

So for any

$$\bar{\gamma}_1 > \gamma > \tilde{\gamma} \Rightarrow \hat{d}_2^s(N) > \hat{d}_3^s(2),$$

and if

$$\gamma < \tilde{\gamma} \Rightarrow \hat{d}_3^s(2) > \hat{d}_2^s(N).$$

If we compare  $\bar{\gamma}_1$  and  $\tilde{\gamma}$ ,

$$\bar{\gamma}_1 - \tilde{\gamma} = \frac{N}{2(N^2 - 3N + 3)},$$

taking the derivative of this difference with respect to  $N$

$$= \frac{-2N^2 + 6}{(2(N^2 - 3N + 3))^2} < 0 \quad \forall N > 1.$$



So the higher is  $N$  the lower is the difference between  $\bar{\gamma}_1$  and  $\tilde{\gamma}$ . Thus, at a higher value of  $N$ , we will find that  $\hat{d}_3^s(2) > \hat{d}_2^s(N)$ .

So, if  $\gamma > \bar{\gamma}_1$ , then Figure 3.3 concludes the comparison between the critical values of the marginal damages. However if this condition is not satisfied and  $N$  is high enough to decrease enough the difference between  $\bar{\gamma}_1$  and  $\tilde{\gamma}$ , then Figure 3.4 concludes the comparison between the critical values of the marginal damages.

## CHAPTER 4

### EXTENSION OF THE ANALYSIS (EXAMINATION OF ROBUSTNESS)

In this chapter, the robustness of our model assumptions is examined, mainly, the assumption of the linear investment costs and the assumption of linearity of environmental damages. In particular, we examine whether these assumptions are critical for achieving the result that grand coalition is stable at high levels of marginal damages or not.

It is concluded from the analysis of the previous two chapters that the grand coalition is stable, at the high levels of marginal damages, and yields the lowest total costs for the agreements that include information exchange, i.e. both emission agreement with information exchange and emissions and R&D agreement with information exchange analyzed in the second chapter, and both research joint venture agreement and R&D agreement with information exchange analyzed in the third chapter. Nevertheless, we concluded that at the high levels of marginal damages, cooperating in the third stage of the game, doesn't play any role in reducing the total costs of signatories as far as they invest at the maximum level of investment to completely eliminate the GHG emission. Thus, at the high levels of marginal damages, both emission agreement with information exchange yield the same solution, while both emission and R&D agreement with information exchange and R&D agreement with information exchange yield the same solution. According to that, our analysis in this chapter will be focused on both research joint venture agreement and R&D agreement with information exchange.

## 4.1 Quadratic Investment Costs

In order to check whether the assumption of the linear investment cost is critical for achieving the result that the grand coalition is stable at the high levels of marginal damages, in this chapter, we introduce the same model presented in the second chapter, but now assuming that the investment costs are quadratic.

According to that, the total costs function of controlling GHG emissions for the representative country, instead of (2.4), is now given by

$$TC_i = \frac{c}{2}(\delta - \alpha y_i - E_i)^2 + dE + \frac{r}{2}x_i^2, \quad (4.1)$$

where  $y_i = x_i + \gamma X_{-i}$ , with  $\gamma \in [0, 1]$ .

Before analyzing the research joint venture agreement and R&D agreement with information exchange, we introduce both the fully non-cooperative equilibrium and the efficient solution.

### 4.1.1 Fully Non-Cooperative Equilibrium

The fully non-cooperative equilibrium can be calculated as the equilibrium of a two-stage game. In the first stage, countries decide the level of investment in R&D. In the second stage they decide about emissions. In both stages, the Nash equilibrium is calculated. Solving by backward induction, we begin analyzing the equilibrium of the second stage.

For a given technology, the optimal emissions can be calculated by minimizing the following total cost function

$$TC_i = \frac{c}{2}(\delta - \alpha y_i - E_i)^2 + dE, \quad i = 1, \dots, N.$$

The solution of the third stage yields the same solution of the fully non-cooperative equilibrium analyzed in the second chapter, According to that, total costs can be

written as

$$TC_i = \frac{d^2}{2c} + d(N\bar{\delta} - \alpha Y) + \frac{r}{2}x_i^2, \quad (4.2)$$

where the first term represents abatement costs, the second term stands for environmental damages and the third term for investment costs.

Now we calculate the equilibrium for the first stage, when  $y_i \in (0, \bar{\delta}/\alpha]$ , as follows

$$\min_{\{x_i\}} TC_i = \frac{d^2}{2c} + d(N\bar{\delta} - \alpha Y) + \frac{r}{2}x_i^2, \quad (4.3)$$

$$s.t. Y = \sum_{i=1}^N (x_i + \gamma X_{-i}) . \quad (4.4)$$

Observe that global effective investment in R&D becomes a public good. Any investment made by a country reduces the total costs of all countries because of the reduction in global emissions. Thus, in the second stage of the game, countries have to decide which is the provision of a *public bad* whereas in the first stage they have to decide about the provision of a *public good*.

The first-order condition for an *interior solution* is

$$\frac{\partial TC_i}{\partial x_i} = -\alpha d \frac{\partial Y}{\partial x_i} + r x_i = 0,$$

where  $\partial Y / \partial x_i = 1 + \gamma(N - 1)$ , so that

$$\alpha d (1 + \gamma(N - 1)) = r x_i,$$

where the left-hand side represents marginal revenue of investment while the right hand side represents marginal cost. Thus, the level of investment of the fully non-cooperative equilibrium is given by

$$x_i^{nc} = \frac{\alpha d}{r} (1 + \gamma(N - 1)). \quad (4.5)$$

If we focus on the symmetric solution, the effective investment is

$$y_i^{nc} = x_i^{nc} + \gamma X_{-i}^{nc} = x_i^{nc} + \gamma(N-1)x_i^{nc} = x_i^{nc}(1 + \gamma(N-1)),$$

$$y_i^{nc} = \frac{\alpha d}{r} (1 + \gamma(N-1))^2, \quad (4.6)$$

while global effective investment is given by

$$Y^{nc} = N y_i^{nc} = \frac{\alpha d N}{r} (1 + \gamma(N-1))^2. \quad (4.7)$$

Notice that effective investment increases with marginal damages and spillover effects. Finally, the level of global emissions is now given by

$$E_i^{nc} = N\bar{\delta} - \alpha Y^{nc} = N\bar{\delta} - \frac{\alpha^2 d N}{r} (1 + \gamma(N-1))^2. \quad (4.8)$$

The total costs in this case are given by

$$TC_i^{nc} = dN\delta - \frac{d^2(2N-1)}{2cr} (r + \alpha^2 c(1 + \gamma(N-1))^2), \quad (4.9)$$

where the first term represents abatement costs, the second term the environmental damages and the last term the investment costs.

As the effective investment given by (4.6) increases with respect to marginal damages whereas the effective investment which yields zero emissions given by (2.6) decreases, there will be a threshold value for marginal damages for which the level given by (2.6) becomes operative. This threshold value for marginal damages is given by<sup>1</sup>

$$\hat{d}^{nc} = \frac{\delta cr}{r + c\alpha^2(1 + \gamma(N-1))^2}. \quad (4.10)$$

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<sup>1</sup>Notice that by comparing  $\hat{d}^{nc}$  with the level of  $d = \delta c$  which yield the trivial solution, it is easy to find that  $\hat{d}^{nc} < \delta c$ .

Thus, for  $d \geq \hat{d}^{nc}$ , the level of emissions and global emissions are equal to zero, and countries invest in the interval  $y_i \in (\bar{\delta}/\alpha, \delta/\alpha]$  in order to eliminate completely the GHG emissions. Now, the equilibrium for the first stage is given as follows

$$\min_{\{x_i\}} TC_i = \frac{c}{2} (\delta - \alpha y_i)^2 + \frac{r}{2} x_i^2, \quad (4.11)$$

$$s.t. \ y_i = x_i + \gamma X_{-i} \geq \frac{\bar{\delta}}{\alpha}. \quad (4.12)$$

The first-order condition is

$$\frac{\partial TC_i}{\partial x_i} = -c\alpha (\delta - \alpha x_i - \alpha\gamma X_{-i}) + r x_i = 0,$$

where  $X_{-i} = (N - 1)x_i$ , so that

$$\alpha c (\delta - \alpha x_i - \alpha\gamma X_{-i}) = r x_i,$$

where the left-hand side represents the marginal revenue of investment while the right hand side represents the marginal cost. Thus, the level of investment of the fully non-cooperative equilibrium is given by

$$x_i^{nc} = \frac{\alpha c \delta}{r + \alpha^2 c (1 + \gamma (N - 1))} > 0. \quad (4.13)$$

If we focus on the symmetric solution, the effective investment is

$$y_i^{nc} = x_i^{nc} (1 + \gamma (N - 1)) = \frac{\alpha c \delta (1 + \gamma (N - 1))}{r + \alpha^2 c (1 + \gamma (N - 1))}, \quad (4.14)$$

while global effective investment is given by

$$Y^{nc} = N y_i^{nc} = \frac{\alpha c \delta N (1 + \gamma (N - 1))}{r + \alpha^2 c (1 + \gamma (N - 1))}. \quad (4.15)$$

In order to investigate whether the constraint on the effective investment given by (4.12) is satisfied or not, the difference between the levels of effective investments given by (4.14) and (2.6) is taken as follows

$$y_i^{nc} - \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) = -\frac{\delta r}{r + \alpha^2 c (1 + \gamma (N - 1))} + \frac{d}{\alpha c}.$$

So, it is clear that the condition on the effective investment is only satisfied at the level of marginal damages higher than

$$\tilde{d}^{nc} = \frac{\delta cr}{r + \alpha^2 c (1 + \gamma (N - 1))}, \quad (4.16)$$

which is higher than the level of marginal damages  $\hat{d}^{nc}$ .<sup>2</sup>

According to that, it is concluded that in the range of the marginal damages ( $\hat{d}^{nc}$ ,  $\tilde{d}^{nc}$ ], the level of effective investment is given by (2.6). By substituting this level of effective investment in total costs function given by (4.11), the total costs can be written as follows

$$TC_i^{nc} = \frac{(r + \alpha^2 c (1 + \gamma (N - 1)))^2 d^2 - 2\delta crd + rc^2\delta^2}{2\alpha^2 c^2 (1 + \gamma (N - 1))^2}, \quad (4.17)$$

while in the range of marginal damages ( $\tilde{d}^{nc}$ ,  $\delta c$ ], the level of effective investment is given by (4.14), and the total costs function in this case can be written as follows

$$TC_i^{nc} = \frac{\delta^2 cr (r + \alpha^2 c)}{2 (r + \alpha^2 c (1 + \gamma (N - 1)))^2}, \quad (4.18)$$

where the GHG emissions are completely eliminated.

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<sup>2</sup>Notice that the level of marginal damage given by  $\tilde{d}^{nc}$  is lower than the level of marginal damages  $d = \delta c$  which yields the trivial solution.

### 4.1.2 The Efficient Solution

In order to characterize the efficient solution, the game is solved again in two stages, but on this occasion assuming that countries minimize global total costs in both stages. We begin analyzing the solution of the second stage. Given the technology, countries select emissions to minimize the global total costs

$$GTC = \sum_{i=1}^N TC_i = \sum_{i=1}^N \left( \frac{c}{2} (\delta - \alpha y_i - E_i)^2 + dE \right), \quad i = 1, \dots, N.$$

The solution to the optimization problem is<sup>3</sup>

$$E_i = \delta - \alpha y_i - \frac{Nd}{c}. \quad (4.19)$$

So that, the effective investment which yields zero emissions for each country is given by

$$\hat{y}_i^e = \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right), \quad (4.20)$$

while the critical value of effective investment which eliminates completely the business as usual emissions (BAU) is still given by (2.2). Thus, the range of effective investment and the corresponding levels of emissions can be determined as follows

$$E_i = \left\{ \begin{array}{ll} \left( \delta - \frac{Nd}{c} \right) - \alpha y_i & \text{if } y_i \in \left( 0, \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right) \right] \\ 0 & \text{if } y_i \in \left( \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right), \frac{\delta}{\alpha} \right] \end{array} \right\}.$$

Using (4.19), global emissions can be calculated

$$E = \sum_{i=1}^N E_i = N \left( \delta - \frac{Nd}{c} \right) - \alpha Y, \quad (4.21)$$

---

<sup>3</sup>Again, in this case for  $d \geq \delta c/N$ , the model has a trivial solution. For this reason, we will limit the analysis in this paper to the interval of values for  $d$  between zero and  $\delta c/N$ . Notice that if  $d \geq \delta c/N$ , the damages are so large that it is not necessary to invest in cleaner technologies to eliminate completely the emissions.



where  $Y$  is global effective investment in R&D which is given by<sup>4</sup>

$$Y = \sum_{i=1}^N y_i = \sum_{i=1}^N (x_i + X_{-i}). \quad (4.22)$$

Using (4.19) and (4.21), total costs for the representative country can be written as

$$TC_i = \frac{d^2 N^2}{2c} + d \left( N \left( \delta - \frac{Nd}{c} \right) - \alpha Y \right) + \frac{r}{2} x_i^2, \quad (4.23)$$

where the first term represents abatement costs, the second term stands for environmental damages and the third term for investment costs.

Next, in the first stage, countries select the level of investment to minimize the global total costs of controlling emissions that are given by the following expression

$$\begin{aligned} GTC &= \sum_{i=1}^N TC_i = \sum_{i=1}^N \left( \frac{d^2 N^2}{2c} + d \left( N \left( \delta - \frac{Nd}{c} \right) - \alpha Y \right) + \frac{r}{2} x_i^2 \right) \\ &= \frac{d^2 N^3}{2c} + dN \left( N \left( \delta - \frac{Nd}{c} \right) - \alpha Y \right) + \frac{r}{2} \sum_{i=1}^N x_i^2. \end{aligned} \quad (4.24)$$

Now we calculate the equilibrium for the first stage, when  $y_i \in (0, \frac{1}{\alpha} (\delta - \frac{Nd}{c})]$ , as follows

$$\min_{\{x_1, \dots, x_N\}} GTC = \frac{d^2 N^3}{2c} + dN \left( N \left( \delta - \frac{Nd}{c} \right) - \alpha Y \right) + \frac{r}{2} \sum_{i=1}^N x_i^2, \quad (4.25)$$

$$s.t. Y = \sum_{i=1}^N (x_i + X_{-i}). \quad (4.26)$$

The first-order condition for an *interior solution* is

$$\frac{\partial GTC}{\partial x_i} = -\alpha dN \frac{\partial Y}{\partial x_i} + r x_i = 0,$$

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<sup>4</sup>We assume that when countries cooperate they pool their R&D investment so as to fully internalize the spillover effects, i.e.  $\gamma = 1$  for the efficient solution.

where  $\partial Y/\partial x_i = N$ , so that, since  $\gamma = 1$

$$\alpha dN^2 = rx_i,$$

where the left hand side represents the marginal revenue of investment while the right hand side represents the marginal cost. Thus, the level of investment of the efficient solution is given by

$$x_i^e = \frac{\alpha d}{r} N^2, \quad (4.27)$$

and the level of effective investment, focusing on the symmetric solution, is

$$y_i^e = x_i^e + X_{-i}^e = Nx_i^e = \frac{\alpha d}{r} N^3, \quad (4.28)$$

while global effective investment is given by

$$Y^e = Ny_i^e = \frac{\alpha d}{r} N^4. \quad (4.29)$$

Observe that investment increases with marginal damages.

Finally, the level of global emissions is now given by

$$E^e = N \left( \delta - \frac{Nd}{c} \right) - \frac{\alpha^2 d}{r} N^4. \quad (4.30)$$

The total costs in this case are given by

$$TC_i^e = dN\delta - \frac{d^2 N^2}{2cr} (r + c\alpha^2 N^2). \quad (4.31)$$

As the effective investment given by (4.28) increases with respect to marginal damages whereas the effective investment which yields zero emissions given by (4.20) decreases, there will be a threshold value for marginal damages for which the level

given by (4.20) becomes operative. This threshold value for marginal damages is given by condition  $\hat{y}_i^e = y_i^e$  and is equal to<sup>5</sup>

$$\hat{d}^e = \frac{cr\delta}{N(c\alpha^2N^2 + r)}. \quad (4.32)$$

Thus, for  $d \geq \hat{d}^e$ , the level of emissions and global emissions are equal to zero, and countries invest in the interval  $y_i \in (\frac{1}{\alpha}(\delta - Nd/c), \delta/\alpha]$  in order to eliminate completely the business as usual emissions (BAU). Now, the global total costs of controlling emissions are given by the following expression

$$GTC = \sum_{i=1}^N TC_i = \sum_{i=1}^N \frac{c}{2} (\delta - \alpha y_i)^2 + \frac{r}{2} \sum_{i=1}^N x_i^2. \quad (4.33)$$

Next, the equilibrium for the first stage is calculated as follows

$$\min_{\{x_1, \dots, x_N\}} GTC = \frac{c}{2} \sum_{i=1}^N (\delta - \alpha y_i)^2 + \frac{r}{2} \sum_{i=1}^N x_i^2, \quad (4.34)$$

$$s.t. \ y_i = X = \sum_{i=1}^N x_i \geq \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right). \quad (4.35)$$

As the first-order condition is

$$\frac{\partial GTC}{\partial x_i} = -c\alpha N (\delta - \alpha y_i) + r x_i = 0,$$

so that,

$$c\alpha N (\delta - \alpha y_i) = r x_i,$$

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<sup>5</sup>By comparing  $\hat{d}^e$  with the level of  $d = \delta c/N$  which yield the trivial solution, it is easy to find that  $\hat{d}^e < \delta c/N$ .

where the left hand side represents marginal revenue of investment while the right hand side represents marginal cost. Thus, the level of investment of the efficient solution is given by

$$x_i^e = \frac{\alpha c \delta N}{r + \alpha^2 c N^2} > 0. \quad (4.36)$$

If we focus on the symmetric solution, the effective investment is

$$y_i^e = N x_i^e = \frac{\alpha c \delta N^2}{r + \alpha^2 c N^2}, \quad (4.37)$$

while the global effective investment is given by

$$Y^e = N y_i^e = \frac{\alpha c \delta N^3}{r + \alpha^2 c N^2}. \quad (4.38)$$

In order to investigate whether the condition given by (4.35) is satisfied or not, the difference between the levels of effective investment given by (4.37) and (4.20) is taken as follows

$$\begin{aligned} y_i^e - \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right) &= \frac{\alpha c \delta N^2}{r + \alpha^2 c N^2} - \frac{1}{\alpha} \left( \delta - \frac{Nd}{c} \right) \\ &= -\frac{\delta r}{\alpha (r + \alpha^2 c N^2)} + \frac{Nd}{\alpha c}. \end{aligned}$$

So, it is clear that the condition on the effective investment is only satisfied at the level of marginal damages higher than

$$\tilde{d}^e = \hat{d}^e = \frac{cr\delta}{N(c\alpha^2 N^2 + r)}. \quad (4.39)$$

Thus, it is concluded that for any level of marginal damages higher than  $\hat{d}^e$ , all countries will invest at the level of investment given by (4.36), and the total costs function in this case can be written as follows

$$TC_i^e = \frac{cr\delta^2}{2(\alpha^2 c N^2 + r)}, \quad (4.40)$$

where both the level of emissions and the level of business as usual emissions are completely eliminated.

By comparing the efficient outcome with the fully non-cooperative equilibrium, the following proposition is concluded

**Proposition 36** *The level of effective investment of the efficient solution is higher than the level of effective investment of the fully non-cooperative equilibrium, while the total costs of the fully non-cooperative equilibrium are higher than the total costs of the efficient solution for all levels of marginal damages.*

Proof: Appendix 1.

#### **4.1.3 Research Joint Venture Agreement (RJV)**

In order to check whether the assumption of the linear investment cost is critical for achieving the result that the grand coalition is stable at the high levels of marginal damages, the research joint venture agreement (RJV) is solved again in this section, assuming the linearity of the environmental damages while considering a quadratic investment costs (decreasing returns to scale of the R&D efforts).

In this case, the total cost function is given by (4.1), where the three stages of the game and the levels of effective investment for signatories and non-signatories are defined in the same way as in the second chapter with linear environmental damages and linear investment costs.

##### **4.1.3.1 The Partial Agreement Nash Equilibrium of the Investment Game**

In this section, stages two and three of the research joint venture agreement (RJV) are solved backward induction assuming that in the first stage  $n$  countries with  $n \geq 2$  have signed the agreement. As the only change in the analysis of the RJV agreement in this case from the RJV agreement that has been analyzed in the previous chapter occurs in the second stage of the game (the investment game), the levels of emissions

for both non-signatories and signatories are the same like those obtained in the previous chapter. However, as we have supposed that there is no cooperation in the third stage of the game (emissions), the level of investment for non-signatories countries is the same as in the fully non-cooperative equilibrium given by (4.5). For signatories countries, as  $\partial Y/\partial x_j^s = n + \gamma(N - n)$ , the level of investment for a signatory country is given by

$$x_j^s = \frac{\alpha d}{r} (n + \gamma(N - n)). \quad (4.41)$$

If we focus on the symmetric solution for each type of country, the effective investment of non-signatories is

$$\begin{aligned} y_i^f &= x_i^f + \gamma(X_{-i}^f + X^s) = (1 + \gamma(N - n - 1))x_i^f + \gamma n x_j^s \\ &= \frac{\alpha d}{r} ((1 + \gamma(N - 1))(1 + \gamma(N - n - 1)) + \gamma n (n + \gamma(N - n))), \end{aligned} \quad (4.42)$$

However, the effective investment for the signatories is

$$\begin{aligned} y_j^s &= X^s + \gamma X^f = n x_j^s + \gamma(N - n)x_i^f \\ &= \frac{\alpha d}{r} (n(n + \gamma(N - n)) + \gamma(N - n)(1 + \gamma(N - 1))). \end{aligned} \quad (4.43)$$

It is easy to show that the effective investment of both signatories and non-signatories increases with the number of signatories. Moreover, if we compare the investment done by each type of country using (4.5) and (4.41), the following expression is obtained

$$x_j^s - x_i^f = \frac{\alpha d}{r} (1 - \gamma)(n - 1) > 0.$$

Thus, signatories devote more resources for R&D than non-signatories for any level of participation. The same occurs for the effective investment.

Finally, in order to calculate the total costs, we aggregate the effective investment of the different countries to obtain the global effective investment in R&D:

$$Y = (N - n) y_i^f + n y_j^s,$$

which yields

$$Y = \frac{\alpha d}{r} \left( (N - n) (1 + \gamma (N - 1))^2 + n (n + \gamma (N - n))^2 \right),$$

so that global emissions are given by

$$E = N\bar{\delta} - \alpha Y = N\bar{\delta} - \frac{\alpha^2 d}{r} \left( (N - n) (1 + \gamma (N - 1))^2 + n (n + \gamma (N - n))^2 \right). \quad (4.44)$$

The first derivative of the global emissions with respect to  $n$  is negative for  $n \geq 2$ . Global emissions decrease as the international cooperation increases.

Thus, total costs of non-signatories are given by

$$TC_i^f = \frac{d^2}{2c} + dE + \frac{r}{2} (x_i^f)^2, \quad (4.45)$$

where  $E$  is given by (4.44) and investment by (4.5). The total costs of signatories are given by the same kind of expression, but with investment defined by (4.41)

$$TC_j^s = \frac{d^2}{2c} + dE + \frac{r}{2} (x_j^s)^2. \quad (4.46)$$

The comparison of the total costs is immediate because we have established above that signatories invest more resources in R&D. Thus, as the abatement costs and environmental damages are the same, it is the difference in investment that explains the difference in the total costs. The signatories invest more and support a larger

cost for controlling pollution. Moreover, there are *positive* externalities for non-signatories stemming from cooperation, i.e. cooperation decreases the total costs of non-signatories. The incorporation of one country to the agreement reduces global emissions and has no effect on the non-signatories' investment.

Now by substituting the levels of investment in the total costs functions, the following expressions for total costs are obtained

$$TC_i^f = \frac{d^2}{2c} + dN\bar{\delta} - \frac{\alpha^2 d^2}{2r} (2n(n + \gamma(N - n))^2 + (1 + \gamma(N - 1))^2(2(N - n) - 1)), \quad (4.47)$$

for non-signatories, and

$$TC_j^s = \frac{d^2}{2c} + dN\bar{\delta} - \frac{\alpha^2 d^2}{2r} (2(N - n)(1 + \gamma(N - 1))^2 + (n + \gamma(N - n))^2(2n - 1)), \quad (4.48)$$

for signatories.

In order to examine the profitability of joining the RJV agreement, the total costs function of a signatory country given by (4.48) should be compared by the total costs function of playing non-cooperatively which is obtained by substituting for  $n = 1$  in (4.47). The total costs function of playing non-cooperatively is given by

$$TC_i^{nc} = \frac{d^2}{2c} + dN\bar{\delta} - \frac{\alpha^2 d^2}{2r} (2(1 + \gamma(N - 1))^2 + (1 + \gamma(N - 1))^2(2(N - 1) - 1)). \quad (4.49)$$

Now, the difference between (4.48) and (4.49) is taken as follows

$$TC_j^s - TC_i^{nc} = -\frac{\alpha^2 d^2}{2r} (1 - \gamma)(2n^2 - 3n + 1)(2N\gamma + (n + 1)(1 - \gamma)) < 0,$$

and the following proposition is concluded

**Proposition 37** *The research joint ventures agreement is profitable for signatories.*



#### 4.1.3.2 The Nash Equilibrium of the Membership Game

As was declared from the previous proposition that the investment in R&D by signatories countries of the RJV agreement is profitable, it is important now to examine the stability of the agreement under the assumption of decreasing returns to scale of the R&D efforts.

Using the total costs functions given by (4.47) and (4.48), the stability condition can be calculated as follows

$$\Omega(n) = TC_j^s(n) - TC_i^f(n-1),$$

$$\Omega(n) = -\frac{\alpha^2 d^2}{2r} (1-\gamma)(n-1) ((1-\gamma)(5n-1) + 6N\gamma) < 0 \forall \gamma < 1,$$

and for the grand coalition

$$\Omega(N) = -\frac{\alpha^2 d^2}{2r} (1-\gamma)(N-1)(5N-1 + \gamma(1+N)) < 0 \forall \gamma < 1,$$

and the following proposition can be concluded

**Proposition 38** *The grand coalition is the unique stable research joint venture agreement, under the assumption of quadratic investment cost, regardless the level of marginal damages and the degree of spillover effects.*

#### 4.1.4 R&D Agreement with Information Exchange

In order to check whether the assumption of the linear investment cost is critical for achieving the result that the grand coalition is stable at the high levels of marginal damages, the R&D agreement with information exchange is solved again in this section, assuming the linearity of the environmental damages while considering a quadratic investment costs (decreasing returns to scale of the R&D efforts).

The total cost is the same given by (4.1), where the three stages of the game and the levels of effective investment for signatories and non-signatories are defined in

the same way as in the second chapter with linear environmental damages and linear investment costs.

#### 4.1.4.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three of the R&D agreement with information exchange are solved backward induction assuming that in the first stage  $n$  countries with  $n \geq 2$  have signed the agreement. As the only change in the analysis of the R&D agreement with information exchange in this case from the R&D agreement with information exchange that has been analyzed in the previous chapter occurs in the second stage of the game (the investment game), the levels of emissions for both non-signatories and signatories are the same like those obtained in the previous chapter. However, as we have supposed that there is no cooperation in the third stage of the game (emissions), the level of investment for non-signatories countries is the same as in the fully non-cooperative equilibrium given by (4.5).

For signatories, as they are minimizing the agreement total costs function given by

$$\min_{\{x_1, \dots, x_n\}} ATC = \frac{nd^2}{2c} + dn(N\bar{\delta} - \alpha Y) + \frac{r}{2} \sum_{i=1}^n x_j^2. \quad (4.50)$$

The first-order condition for an *interior solution* is

$$\frac{\partial ATC}{\partial x_j^s} = -\alpha dn \frac{\partial Y}{\partial x_j^s} + rx_j^s = 0,$$

where  $\partial Y / \partial x_j^s = n + \gamma(N - n)$ , so that the interior solution is given by the following expression<sup>6</sup>

$$x_j^s = \frac{\alpha dn}{r} (n + \gamma(N - n)), \quad (4.51)$$

which is higher than the level of investment by non-signatories.

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<sup>6</sup>For  $n = N$ , this expression gives the level of investment corresponding to the efficient solution.

If we focus on the symmetric solution for each type of country, the effective investment of non-signatories is

$$\begin{aligned} y_i^f &= x_i^f + \gamma(X_{-i}^f + X^s) = (1 + \gamma(N - n - 1))x_i^f + \gamma nx_j^s \\ &= \frac{\alpha d}{r} \left( (1 + \gamma(N - 1))(1 + \gamma(N - n - 1)) + \gamma n^2(N - n) \right), \end{aligned} \quad (4.52)$$

However, the effective investment for the signatories is

$$\begin{aligned} y_j^s &= X^s + \gamma X^f = nx_j^s + \gamma(N - n)x_i^f \\ &= \frac{\alpha d}{r} \left( n^2(n + \gamma(N - n)) + \gamma(N - n)(1 + \gamma(N - 1)) \right). \end{aligned} \quad (4.53)$$

It is easy to show that the effective investment of both signatories and non-signatories for the interior solution increases with the number of signatories. Moreover, if we compare the investment done by each type of country using (4.5) and (4.51), the following expression is obtained

$$x_i^f - x_j^s = -\frac{\alpha d}{r} \left( (1 - \gamma)n^2 + \gamma Nn - (1 + \gamma(N - 1)) \right),$$

that is negative for  $n \geq 2$  and  $\gamma \in (0, 1)$ . Thus, signatories devote more resources for R&D than non-signatories for any level of participation. The same occurs for the effective investment.

Finally, in order to calculate the total costs, we aggregate the effective investment of the different countries to obtain the global effective investment in R&D:

$$Y = (N - n)y_i^f + ny_j^s,$$

which yields

$$Y = \frac{\alpha d}{r} \left( (N - n) (1 + \gamma (N - 1))^2 + n^2 (n + \gamma (N - n))^2 \right),$$

so that global emissions are given by

$$E = N\bar{\delta} - \alpha Y = N\bar{\delta} - \frac{\alpha^2 d}{r} \left( (N - n) (1 + \gamma (N - 1))^2 + n^2 (n + \gamma (N - n))^2 \right). \quad (4.54)$$

The first derivative of the global emissions with respect to  $n$  is negative for  $n \geq 2$ . Global emissions decrease as the international cooperation increases.

Thus, total costs of non-signatories are given by

$$TC_i^f = \frac{d^2}{2c} + dE + \frac{r}{2} (x_i^f)^2, \quad (4.55)$$

where  $E$  is given by (4.54) and investment by (4.5). The total costs of signatories are given by the same kind of expression, but with investment defined by (4.51)

$$TC_j^s = \frac{d^2}{2c} + dE + \frac{r}{2} (x_j^s)^2. \quad (4.56)$$

The comparison of the total costs is immediate because we have established above that signatories invest more resources in R&D. Thus, as the abatement costs and environmental damages are the same, it is the difference in investment that explains the difference in the total costs. The signatories invest more and support a larger cost for controlling pollution. Moreover, there are *positive* externalities for non-signatories stemming from cooperation, i.e. cooperation decreases the total costs of non-signatories. The incorporation of one country to the agreement reduces global emissions and has no effect on the non-signatories' investment. The result is a reduction in the cost of the countries that stay outside the agreement.

**4.1.4.1.1 Only the Signatories Eliminate Emissions** As it occurs for the fully non-cooperative equilibrium and efficient solution, there is a threshold value of marginal damages for both types of countries for which the level of effective investment given by (2.6), that yields zero emissions level, becomes operative. In order to calculate the threshold values, we write  $y_i = y_i^f$  where  $y_i^f$  is given by (4.52) and  $y_i = y_j^s$  where  $y_j^s$  is given by (4.53). The results are the following values for marginal damages

$$\hat{d}^f(n) = \frac{\delta cr}{\alpha^2 c(\gamma n^2(n + \gamma(N - n)) + (1 + \gamma(N - 1))(1 + \gamma(N - n - 1))) + r}, \quad (4.57)$$

$$\hat{d}^s(n) = \frac{\delta cr}{\alpha^2 c(\gamma(N - n)(1 + \gamma(N - 1)) + n^2(n + \gamma(N - n))) + r}. \quad (4.58)$$

The comparison of these threshold values is not so complicated because the numerators are the same. The comparison yields that  $\hat{d}^s(n)$  is lower than  $\hat{d}^f(n)$ . A result that is consistent with those obtained in the previous sections. Now, the difference is that the threshold values depend on the number of signatories.

Thus, we can conclude that in the interval of marginal damages  $d \in (0, \hat{d}^s(n)]$ , the interior solution presented above applies. For continuity, if marginal damages are greater than  $\hat{d}^s(n)$  but close to this value, the solution combines an interior solution for non-signatories and a corner solution, with zero emissions, for signatories. Thus, the global level of emissions is given now by the total level of emissions for non-signatories as follows

$$E = \sum_{i=1}^{N-n} E_i^f = \sum_{i=1}^{N-n} \left( \delta - \frac{d}{c} - \alpha y_i^f \right) = (N - n) \left( \delta - \frac{d}{c} \right) - \alpha \sum_{i=1}^{N-n} y_i^f, \quad (4.59)$$

where

$$\sum_{i=1}^{N-n} y_i^f = Y^f = \sum_{i=1}^{N-n} \left( x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n-1} x_l^f \right) \right). \quad (4.60)$$

Next, the equilibrium of the first stage for non-signatories is given by

$$\begin{aligned} \min_{x_i^f} TC_i^f &= \frac{d^2}{2c} + d \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + \frac{r}{2} (x_i^f)^2 \\ \text{s.t. } y_i^f &= x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n-1} x_l^f \right) \leq \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right), \\ x_i^f &\geq 0. \end{aligned}$$

The first order condition for an *interior* solution is

$$\frac{\partial TC_i^f}{\partial x_i^f} = -\alpha d \frac{\partial Y^f}{\partial x_i^f} + r x_i^f = 0,$$

where  $\partial Y^f / \partial x_i^f = 1 + \gamma(N - n - 1)$ , so that

$$\alpha d (1 + \gamma(N - n - 1)) = r x_i^f,$$

where the left hand side represents marginal revenue of investment while the right hand side represents marginal cost. Thus, the level of investment for a non-signatory country is given by

$$x_i^f = \frac{\alpha d}{r} (1 + \gamma(N - n - 1)). \quad (4.61)$$

For signatories countries, as they invest at the level of effective investment in the interval  $(\bar{\delta}/\alpha, \delta/\alpha]$  to eliminate completely the business as usual emissions, they select the level of investment to minimize the global total costs of controlling emissions that are given by the following expression

$$ATC = \sum_{j=1}^n TC_j^s = \sum_{j=1}^n \left( \frac{c}{2} (\delta - \alpha y_j^s)^2 + d \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) \right) + \frac{r}{2} \sum_{j=1}^n (x_j^s)^2, \quad (4.62)$$

where  $ATC$  stands for the total costs of the agreement.

Next, we calculate the equilibrium for the first stage as follows

$$\min_{\{x_1, \dots, x_n\}} ATC = \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + dn \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + \frac{r}{2} \sum_{j=1}^n (x_j^s)^2 \quad (4.63)$$

$$s.t. \ y_j^s = \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \geq \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right), \quad (4.64)$$

$$y_j^s = \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \leq \frac{\delta}{\alpha}, \quad (4.65)$$

$$x_j^s \geq 0, \quad j = 1, \dots, n.$$

By forming the Lagrangian function of the previous minimization problem as follows

$$\begin{aligned} L = & \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + dn \left( (N-n) \left( \delta - \frac{d}{c} \right) - \alpha Y^f \right) + \frac{r}{2} \sum_{j=1}^n (x_j^s)^2 \\ & + \lambda_1 \left( \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) - \sum_{k=1}^n x_k^s - \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \right) \\ & + \lambda_2 \left( \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) - \frac{\delta}{\alpha} \right), \end{aligned}$$

we obtain the following Kuhn-Tucker conditions

$$\begin{aligned} \frac{\partial L}{\partial x_j^s} &= -\alpha cn (\delta - \alpha y_j^s) - \alpha dn \frac{\partial Y^f}{\partial x_j^s} + r x_j^s - \lambda_1 - \lambda_2 \geq 0, \\ x_j^s &\geq 0, \quad x_j^s \frac{\partial L}{\partial x_j^s} = 0, \end{aligned}$$

where the conditions on the multipliers are given by

$$\begin{aligned} \frac{\partial L}{\partial \lambda_1} &= \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) - \sum_{k=1}^n x_k^s - \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \leq 0, \\ \lambda_1 &\geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda_2} &= \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) - \frac{\delta}{\alpha} \leq 0, \\ \lambda_2 &\geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.\end{aligned}$$

As

$$\frac{\partial L}{\partial \lambda_i} < 0, \text{ then } \lambda_i = 0, \quad i = 1, 2,$$

thus, we obtain

$$\alpha cn (\delta - \alpha y_j^s) + \alpha dn \frac{\partial Y^f}{\partial x_j^s} = r x_j^s,$$

where  $\partial Y^f / \partial x_j^s = \gamma (N - n)$ , so that

$$\alpha cn (\delta - \alpha y_j^s) + \alpha dn \gamma (N - n) = r x_j^s, \quad (4.66)$$

where the left hand side represents marginal revenue of investment while the right hand side represents marginal cost. By assuming the symmetry, and by substituting for

$$y_j^s = n x_j^s + \gamma (N - n) \frac{\alpha d}{r} (1 + \gamma (N - n - 1)), \quad (4.67)$$

in (4.66), it is obtained the following level of investment for signatories countries

$$x_j^s = \frac{\alpha n (\delta cr + d \gamma (N - n) (r - \alpha^2 c (1 + \gamma (N - n - 1))))}{r (r + \alpha^2 cn^2)}. \quad (4.68)$$

By analyzing the level of investment given by (4.68), it is found that this level of investment is positive for any value of marginal damages provided that

$$r \geq \alpha^2 c (1 + \gamma (N - n - 1)), \quad (4.69)$$

while if this condition is not satisfied, then the level of investment given by (4.68) is going to be positive only in the interval of marginal damages given by

$$d \in [\hat{d}^s(n), \check{d}(n)],$$



where  $\hat{d}^s(n)$  is given by (4.58) and<sup>7</sup>

$$\check{d}(n) = \frac{\delta cr}{\gamma(N-n)(\alpha^2 c(1+\gamma(N-n-1)) - r)}. \quad (4.70)$$

Next, by substituting (4.68) in (4.67), the effective investment for signatories is given by

$$y_j^s = \frac{\alpha(\delta cn^2 + \gamma(N-n)d(n^2 + 1 + \gamma(N-n-1)))}{r + c\alpha^2 n^2}. \quad (4.71)$$

In order to investigate whether the constraint on effective investment given by (4.64) is satisfied or not, the difference between the levels of effective investments given by (4.71) and (2.6) is taken and it is found that this constraint on investment is only satisfied at the values of marginal damages higher than  $\tilde{d}^s(n)$  which is given by

$$\tilde{d}^s(n) = \frac{\delta cr}{c\alpha^2((1+\gamma(N-n))(n^2 + \gamma(N-n)) - \gamma^2(N-n)) + r}, \quad (4.72)$$

which, by comparison, is higher than the level of marginal damages given by (4.58).

Therefore, it is concluded that in the interval  $d \in (\hat{d}^s(n), \tilde{d}^s(n)]$ , the level of effective investment is given by (2.6). In this range of marginal damages, the level of investment for signatories can be obtained by substituting the level of investment of non-signatories given by (4.61) in the following expression

$$\hat{y}_j^s = \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) = nx_j^s + \gamma(N-n)x_i^f,$$

which yields<sup>8</sup>

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<sup>7</sup>By comparison, it is easy to check that  $\tilde{d}^s(n) > \hat{d}^s(n)$ .

<sup>8</sup>This level of investment is positive at any level of  $d^* < \delta cr / (c\alpha^2\gamma(N-n)(1+\gamma(N-n-1)) + r)$ , which is higher than  $\tilde{d}^s(n)$  given by (4.72). Thus, it is concluded that the level of investment for signatories is positive in the interval  $d \in (\hat{d}^s(n), \tilde{d}^s(n)]$ .

$$x_j^s = \frac{\delta cr - d(r + \alpha^2 c \gamma (N - n) (1 + \gamma (N - n - 1)))}{\alpha c n r}. \quad (4.73)$$

Next, by substituting the level of investment of signatories in the following expression

$$y_i^f = \frac{\alpha d}{r} (1 + \gamma (N - n - 1))^2 + \gamma n x_j^s, \quad (4.74)$$

the effective investment of non-signatories can be calculated in both case, i.e. when  $d \in (\hat{d}^s(n), \tilde{d}^s(n)]$  and when  $d \geq \tilde{d}^s(n)$ . The calculations yield

$$y_i^f = \frac{\gamma \delta}{\alpha} + \frac{d}{r \alpha c} (\alpha^2 c (1 + \gamma (N - n - 1)) (1 + \gamma (N - n)) (1 - \gamma) - \gamma r), \quad (4.75)$$

when  $d \in (\hat{d}^s(n), \tilde{d}^s(n)]$ , and

$$y_i^f = \frac{\alpha}{r (r + c \alpha^2 n^2)} (\gamma n^2 \delta c r + d r ((1 + \gamma (N - n - 1))^2 + \gamma^2 n^2 (N - n)) + c \alpha^2 n^2 d (1 - \gamma) (1 + \gamma (N - n - 1)) (1 + \gamma (N - n))), \quad (4.76)$$

when  $d > \tilde{d}^s(n)$ . By taking the difference between (4.75) and (4.76), we find that they intersect at the level of marginal damages given by (4.72), such that  $y_i^f$  given by (4.76) is higher than  $y_i^f$  given by (4.75) for marginal damages higher than this level and vice versa.

**4.1.4.1.2 The Corner Solution** By comparison, we find that the effective investment of signatories given by (4.71) is higher than the effective investment of non-signatories given by (4.76) for any level of marginal damages lower than

$$\hat{d}(n) = \frac{\delta c r n^2}{(1 + \gamma (N - n - 1)) (r + c \alpha^2 n^2 (1 + \gamma (N - n))) - \gamma r n^2 (N - n)}, \quad (4.77)$$

which is higher than the level of marginal damages given by (4.72). Taking into account that both levels of non-signatories' effective investment given by (4.75) and

(4.76) intersect at the level of marginal damages given by (4.72), it is concluded that the critical value of marginal damages for non-signatories, which separate the corner solution of non-signatories from the interior one, will be on the right of  $\tilde{d}^s(n)$ . Therefore, this critical value of marginal damages for non-signatories is given by the difference between the levels of effective investment given by (4.76) and (2.6), which yields

$$\tilde{d}^f(n) = \frac{cr\delta(r + (1 - \gamma)c\alpha^2n^2)}{r^2 + \alpha^2crA + c^2\alpha^4n^2B}, \quad (4.78)$$

where  $A = ((1 + \gamma(N - n - 1))^2 + n^2(1 + \gamma^2(N - n)))$ ,

$$B = (1 - \gamma)(1 + \gamma(N - n))(1 + \gamma(N - n - 1)).$$

Then, for marginal damages larger than  $\tilde{d}^f(n)$ , the corner solution applies for both types of countries and the business as usual emissions are completely eliminated. Now, the equilibrium of the first stage for non-signatories is given by

$$\begin{aligned} \min_{x_i^f} TC_i^f &= \frac{c}{2} (\delta - \alpha y_i^f)^2 + \frac{r}{2} (x_i^f)^2, \\ s.t. \ y_i^f &= x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n} x_l^f \right) \geq \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right), \end{aligned} \quad (4.79)$$

$$y_i^f = x_i^f + \gamma \left( \sum_{k=1}^n x_k^s + \sum_{l=1}^{N-n} x_l^f \right) \leq \frac{\delta}{\alpha}. \quad (4.80)$$

The first order condition yields the following level of investment for non-signatories

$$x_i^f = \frac{\alpha c}{r} (\delta - \alpha y_i^f). \quad (4.81)$$

By substituting the level of investment given by (4.81) in the total costs function of non-signatories, their total costs can be written as

$$TC_i^f = \frac{c(r + \alpha^2c)}{2r} (\delta - \alpha y_i^f)^2. \quad (4.82)$$

For signatories countries, as they select the level of investment to minimize the agreement total costs of controlling emissions, the equilibrium of the first stage for them is given by

$$\begin{aligned} \min_{\{x_1, \dots, x_n\}} ATC &= \frac{c}{2} \sum_{j=1}^n (\delta - \alpha y_j^s)^2 + \frac{r}{2} \sum_{j=1}^n (x_j^s)^2, \\ \text{s.t. } y_j^s &= \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \geq \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right), \end{aligned} \quad (4.83)$$

$$y_j^s = \sum_{k=1}^n x_k^s + \gamma \left( \sum_{l=1}^{N-n} x_l^f \right) \leq \frac{\delta}{\alpha}. \quad (4.84)$$

The first order condition yields the following level of investment for signatories

$$x_j^s = \frac{\alpha c n}{r} (\delta - \alpha y_j^s). \quad (4.85)$$

By substituting the level of investment given by (4.85) in the total costs function of signatories, their total costs can be written as

$$TC_j^s = \frac{c(r + \alpha^2 c n^2)}{2r} (\delta - \alpha y_j^s)^2. \quad (4.86)$$

By substituting the levels of investment for non-signatories and signatories given by (4.81) and (4.85) in the levels of effective investment for both non-signatories and signatories, the following expressions for the effective investment are obtained

$$y_i^f = \frac{\alpha c \delta (r(1 + \gamma(N - n - 1) + \gamma n^2) + \alpha^2 c n^2 (1 - \gamma)(1 + \gamma(N - n)))}{r(r + \alpha^2 c(1 + \gamma(N - n - 1) + n^2)) + \alpha^4 c^2 n^2 (1 - \gamma)(1 + \gamma(N - n))}, \quad (4.87)$$

for non-signatories, and

$$y_j^s = \frac{\alpha c \delta (r(1 + \gamma(N - n) + n^2) + \alpha^2 c n^2 (1 - \gamma)(1 + \gamma(N - n)))}{r(r + \alpha^2 c(1 + \gamma(N - n - 1) + n^2)) + \alpha^4 c^2 n^2 (1 - \gamma)(1 + \gamma(N - n))}, \quad (4.88)$$

for signatories, such that the effective investment of signatories is higher than the effective investment of non-signatories. These levels of effective investment are satisfying the conditions given by (4.80) and (4.84).

Substituting the levels of effective investments given by (4.87) and (4.88) in the total costs functions given by (4.82) and (4.86), respectively, the following total costs functions are obtained

$$TC_i^f(n) = \frac{cr\delta^2 (r + \alpha^2 c) (r + \alpha^2 c (1 - \gamma) n^2)^2}{2 (r (r + \alpha^2 c (1 + \gamma (N - n - 1) + n^2)) + \alpha^4 c^2 n^2 (1 - \gamma) (1 + \gamma (N - n)))^2}, \quad (4.89)$$

for non-signatories, and

$$TC_j^s(n) = \frac{cr\delta^2 (r + \alpha^2 cn^2) (r + \alpha^2 c (1 - \gamma))^2}{2 (r (r + \alpha^2 c (1 + \gamma (N - n - 1) + n^2)) + \alpha^4 c^2 n^2 (1 - \gamma) (1 + \gamma (N - n)))^2}, \quad (4.90)$$

for signatories countries.

In order to investigate whether the constraint on effective investment given by (4.79) is satisfied or not, the difference between the levels of effective investment given by (4.87) and (2.6) is taken and it is found that this constraint on effective investment is only satisfied at the level of marginal damages higher than  $d''(n)$  which is given by

$$d''(n) = \frac{\delta cr (r + (1 - \gamma) \alpha^2 cn^2)}{r (r + \alpha^2 c (1 + \gamma (N - n - 1) + n^2)) + \alpha^4 c^2 n^2 (1 - \gamma) (1 + \gamma (N - n))}, \quad (4.91)$$

which, by comparison, is higher than the level of marginal damages given by (4.78).

Therefore, it is concluded that in the interval of marginal damages  $d \in (d^f(n), d''(n)]$ , the level of effective investment for non-signatories is given by (2.6). In this range of marginal damages, the level of investment for non-signatories can be obtained by substituting the level of investment of signatories given by (4.85) in the following expression

$$\hat{y}_i^f = \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) = (1 + \gamma(N - n - 1)) x_i^f + \gamma n x_j^s,$$

which yields

$$x_i^f = \frac{\delta cr - rd - \gamma \alpha^2 c^2 n^2 \delta + \gamma \alpha^3 c^2 n^2 y_j^s}{\alpha cr (1 + \gamma(N - n - 1))}. \quad (4.92)$$

Next, by substituting the level of investment of non-signatories in the following expression

$$y_j^s = \gamma(N - n) x_i^f + \frac{\alpha c n^2}{r} (\delta - \alpha y_j^s),$$

the effective investment of signatories, in the interval of marginal damages  $d \in (\tilde{d}^f(n), d''(n)]$ , is given by

$$y_j^s = \frac{r\gamma(N - n)(c\delta - d) + \alpha^2 c^2 n^2 \delta (1 - \gamma)(1 + \gamma(N - n))}{\alpha c (r(1 + \gamma(N - n - 1)) + \alpha^2 c n^2 (1 - \gamma)(1 + \gamma(N - n)))}, \quad (4.93)$$

which is decreasing with the level of marginal damages, and for  $d \geq d''(n)$  the level of effective investment is given by (4.88).

#### 4.1.4.2 The Nash Equilibrium of the Membership Game

In this section, we use stability conditions to investigate which is the level of participation the R&D agreement with information exchange can achieve under the assumption of the quadratic investment costs.

We begin with the corner solution, when the marginal damages are sufficiently large. We examine directly the stability of the grand coalition. For this case, the auxiliary function  $\Omega(N)$  given by

$$\Omega(N) = TC_j^s(N) - TC_i^f(N - 1),$$

can be obtained by substituting for  $n = N - 1$  in (4.89) which yields

$$TC_i^f(N - 1) = \frac{cr\delta^2 (r + \alpha^2 c) (r + \alpha^2 c (1 - \gamma) (N - 1)^2)^2}{2 (r^2 + r\alpha^2 c (1 + (N - 1)^2) + \alpha^4 c^2 (N - 1)^2 (1 - \gamma^2))^2},$$

and for  $n = N$  in (4.90) which yields

$$TC_j^s(N) = \frac{cr\delta^2 (r + \alpha^2 c N^2) (r + \alpha^2 c (1 - \gamma))^2}{2(r^2 + r\alpha^2 c (1 - \gamma + N^2) + \alpha^4 c^2 N^2 (1 - \gamma))^2}.$$

Thus, when the emissions are completely eliminated, The result is that having  $\gamma < 1/2$  is sufficient condition to have  $\Omega(N)$  negative which means that the grand coalition is stable agreement at the low values of the spillovers parameter. Remember that for the grand coalition is only necessary to check the internal stability condition to ascertain whether it is stable. Thus, the following proposition is concluded

**Proposition 39** *If the marginal damages are sufficiently large, in particular if  $d$  is bigger than  $d''(n)$ , the grand coalition is stable for low values of spillover effects.*

For values of marginal damages lower than the critical value  $d''(n)$ , the signatories' investment for  $n$  is larger than the non-signatories' investment for  $n - 1$  and the stability of the grand coalition is not guaranteed. The analysis of the stability becomes more complicated. Nevertheless, we do not expect a high degree of participation for low values of marginal damages because as we have commented in the previous section the properties of the solution change depending whether the solution is interior or a corner solution. For the corner solution, there are negative externalities for non-signatories coming from cooperation that play for cooperation. However, for the interior solution the sign changes and the positive externalities for non-signatories play against cooperation. In fact, the analysis of the stability for the interior solution yields the following result

**Proposition 40** *If marginal damages are sufficiently small, in particular if  $d$  is lower than  $\hat{d}^s(n)$ , the participation in an IEA increases as the spillover effects decrease although the membership upper bound is of six countries.*

**Proof.** In order to prove this result, we write the auxiliary function  $\Omega(n)$  for the interior solution

$$\Omega(n) = d(E(n) - E(n-1)) + \frac{r}{2} \left( x_j^s(n)^2 - x_i^f(n-1)^2 \right),$$

where global emissions are given by (4.54) and investment in R&D by (4.5) for non-signatories and by (4.51) for signatories. The difference in global emissions is

$$\begin{aligned} E(n) - E(n-1) &= \frac{\alpha^2 d}{r} \left( (1 + \gamma(N-1))^2 + (n-1)^2(1-\gamma)^2 \right. \\ &\quad \left. - 2(n-1)^2(1-\gamma)(n + \gamma(N-n)) - (2n-1)(n + \gamma(N-n))^2 \right), \end{aligned}$$

and the difference in investments is given by

$$x_j^s(n)^2 - x_i^f(n-1)^2 = \frac{\alpha^2 d^2}{r^2} \left( n^2(n + \gamma(N-n))^2 - (1 + \gamma(N-1))^2 \right).$$

Doing the substitution in  $\Omega(n)$ , the difference in total costs is

$$\begin{aligned} \Omega(n) &= \frac{\alpha^2 d^2}{2r} \left( (1-\gamma)(n^3 - 8n^2 + 10n - 4)(n + \gamma(N-n)) \right. \\ &\quad \left. + \gamma N(n^2 - 4n + 2)(n + \gamma(N-n)) + 2(n-1)^2(1-\gamma)^2 + (1 + \gamma(N-1))^2 \right). \end{aligned}$$

It is immediate that  $\Omega(n)$  is positive for  $n \geq 7$  since  $n^3 - 8n^2 + 10n - 4$  is positive for  $n \geq 7$  and  $n^2 - 4n + 2$  is positive for  $n \geq 4$  and the other terms are positive for all  $n$ . Thus, no agreement consisting of seven or more signatories is going to satisfy the internal stability condition. Next, we study the stability of a bilateral agreement.

For  $n = 2$ , the difference in costs is

$$\Omega(2) = -\frac{\alpha^2 d^2}{2r} \left( (N^2 - 10N + 13) \gamma^2 + (10N - 26)\gamma + 13 \right),$$



which is negative for  $N \geq 9$ . Thus, the internal stability condition is satisfied for any value of  $\gamma \in (0, 1)$ . In order to evaluate the external stability condition, we need to look at the sign of  $\Omega(3)$  :

$$\Omega(3) = \frac{2\alpha^2 d^2}{r} ((5N - 12)\gamma^2 - (5N - 24)\gamma - 12).$$

This expression is negative for  $\gamma$  in the interval  $(0, 1)$  and  $N \geq 9$ .<sup>9</sup> Thus, as the external stability condition requires that  $\Omega(3)$  be positive, a bilateral agreement cannot be stable. For an agreement with three countries, the internal stability condition is fulfilled for all  $\gamma$  because  $\Omega(3)$  is negative as we have just seen. On the other hand, the external stability condition requires that

$$\Omega(4) = \frac{3\alpha^2 d^2}{2r} ((N^2 + 6N - 31)\gamma^2 - (6N - 62)\gamma - 31)$$

be positive. Doing  $\Omega(4) = 0$ , we obtain a critical value for  $\gamma$  in the interval  $(0, 1)$  defined by the positive root of this equation

$$\bar{\gamma}(N; 4) = \frac{9.325N - 31}{N^2 + 6N - 31}$$

such that if  $\gamma$  is larger than or equal to  $\bar{\gamma}(N; 4)$ , the external stability condition is satisfied. Then, an agreement consisting of three countries is stable provided that  $\gamma$  is larger than or equal to  $\bar{\gamma}(N; 4)$ . For an agreement with four countries, the internal stability condition is fulfilled if  $\gamma$  is lower than  $\bar{\gamma}(N; 4)$  because then  $\Omega(4)$  is negative. Moreover, the external stability condition requires that

$$\Omega(5) = \frac{4\alpha^2 d^2}{r} ((N^2 - N - 14)\gamma^2 + (N + 28)\gamma - 14)$$

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<sup>9</sup>We do not investigate the stability of an IEA for  $N \leq 8$  because the focus of the paper is on global environmental problems that involve a great number of countries.

be positive. Doing now  $\Omega(5) = 0$ , we obtain a critical value for  $\gamma$  in the interval  $(0, 1)$  defined by the positive root of this equation

$$\bar{\gamma}(N; 5) = \frac{3.275N - 14}{N^2 - N - 14},$$

such that if  $\gamma$  is larger than or equal to  $\bar{\gamma}(N; 5)$ , the external stability condition for an agreement consisting of four countries is satisfied. Then, the agreement is stable provided that  $\bar{\gamma}(N; 5)$  is lower than  $\bar{\gamma}(N; 4)$ . It is not complicated to show that this is the case and therefore we can conclude that an agreement consisting of four countries is stable in the interval  $[\bar{\gamma}(N; 5), \bar{\gamma}(N; 4)]$ . For an agreement consisting of five countries the internal stability condition is satisfied for all  $\gamma$  lower than or equal to  $\bar{\gamma}(N; 5)$  because then  $\Omega(5)$  is negative. However, the external stability condition requires that

$$\Omega(6) = \frac{\alpha^2 d^2}{2r} ((15N^2 - 70N - 45)\gamma^2 + (70N + 90)\gamma - 45)$$

be positive. Doing  $\Omega(6) = 0$ , we obtain a critical value for  $\gamma$  in the interval  $(0, 1)$  defined by the positive root of this equation

$$\bar{\gamma}(N; 6) = \frac{8.59N - 45}{15N^2 - 70N - 45},$$

such that if  $\gamma$  is larger than or equal to  $\bar{\gamma}(N; 6)$ , the external stability condition for an agreement consisting of five countries is fulfilled. Then, as  $\bar{\gamma}(N; 6)$  is lower than  $\bar{\gamma}(N; 5)$  we can conclude that an agreement consisting of five countries is stable in the interval  $[\bar{\gamma}(N; 5), \bar{\gamma}(N; 6)]$ . Finally, an agreement consisting of six countries can be stable if  $\gamma$  is lower than or equal to  $\bar{\gamma}(N; 6)$  because the external stability condition is satisfied for all  $\gamma$ . Remember that  $\Omega(n)$  is positive for all  $n \geq 7$  regardless of the value of  $\gamma$ . ■

In order to illustrate this result, we have calculated the critical values for  $\gamma$  when  $N = 10$ . When there are only ten countries involved in the externality the critical values for  $\gamma$  are:  $\gamma(N = 10, n = 4) = 0.48$ ,  $\gamma(N = 10, n = 5) = 0.24$ ,  $\gamma(N = 10, n = 6) = 0.05$ . Then if  $\gamma \in (0, 0.05]$  and agreement consisting of six countries is stable. However, if  $\gamma \in (0.05, 0.24]$  the stable agreement is formed by five countries. For values of  $\gamma$  in the interval  $(0.24, 0.48]$ , the stable agreement consists of four countries. Finally, if  $\gamma > 0.48$ , only three countries can form a stable agreement. Table 4.1 shows the solution of the investment game for different values of participation. The selected set of values for parameters yields an interior solution for emissions for both types of countries. It can be seen that for all  $n$  between 1 (the fully non-cooperative equilibrium) and 10 (the grand coalition), the signatories' investment is larger than the non-signatories's investment and that this difference is increasing with membership. The same occurs with total costs. Moreover, at the aggregate level, total costs and global emissions decrease as the participation in the agreement increases.

$n$	$E$	$x_i^f$	$x_j^s$	$TC_i^f$	$TC_j^s$	$TC$
1	997.75	0.065		9.979		99.790
2	996.90	0.065	0.160	9.970	9.975	99.712
3	994.33	0.065	0.285	9.944	9.964	99.502
►4	988.92	0.065	0.440	9.890	9.938	99.093
5	979.28	0.065	0.625	9.794	9.891	98.422
6	963.74	0.065	0.840	9.638	9.814	97.437
7	940.37	0.065	1.085	9.405	9.698	96.101
8	906.96	0.065	1.360	9.071	9.532	94.398
9	861.04	0.065	1.665	8.611	9.303	92.343
10	798.67		2.000		8.987	89.870

$\alpha=1, \gamma=0.25, \delta=100, c=0.75, d=0.01, r=0.5, N=10$

Table 4.1: Numerical example with  $\gamma = 0.25$

In Table 4.2 we have recalculated the example for  $\gamma = 0.025$ . According to our results, the participation increases. In this example from four countries to six. Basically, what explains the increment in participation is that the reduction in the spillover effects soften the variations in investments caused by the exit of one country from the agreement. Except for  $n = \{9, 10\}$ , when one country leaves the agreement the reduction in investment that it achieves when  $\gamma = 0.025$  is lower than when  $\gamma = 0.25$ . Thus, when spillover effects are lower the incentive to act as a non-signatory is reduced because the saving in investment costs is then smaller. On the other hand, we find that the reduction in spillover effects has the same effects on global emissions. Except for  $n = \{8, 9, 10\}$ , when one country leaves the agreement the increase in global emissions that the exist causes when  $\gamma = 0.025$  is lower than when  $\gamma = 0.25$ . Thus, when spillover effects are lower the incentive to act as a non-signatory is augmented because the increment in environmental damages is in this case smaller. But for an interior solution, marginal damages are low and the first incentive dominates the second yielding a larger level of participation. Thus, although the increase in environmental damages is lower when spillover effects are lower, the decrease in investment costs is also lower and the net effect, because of the low marginal damages, is that the exit from the agreement becomes unprofitable for a larger level of signatories.

$n$	$E$	$x_i^f$	$x_j^s$	$TC_i^f$	$TC_j^s$	$TC$
1	999.57	0.024		9.996		99.960
2	999.24	0.024	0.088	9.993	9.994	99.930
3	997.84	0.024	0.190	9.979	9.987	99.813
4	994.18	0.024	0.332	9.942	9.969	99.530
5	986.58	0.024	0.512	9.866	9.931	98.988
▶6	972.96	0.024	0.732	9.730	9.864	98.101
7	950.72	0.024	0.990	9.507	9.752	96.790
8	916.86	0.024	1.288	9.169	9.583	95.005
9	867.89	0.024	1.624	8.680	9.339	92.734
10	798.67		2.000		8.987	89.870

$\alpha=1, \gamma=0.025, \delta=100, c=0.75, d=0.01, r=0.5, N=10$

Table 4.2: Numerical example with  $\gamma = 0.025$

According to the previous analysis of both research joint venture agreement and R&D agreement with information exchange, the following proposition is concluded

**Proposition 41** *The assumption of constant returns to scale of the R&D investment (linear investment costs) is not critical for achieving the result that grand coalition of both the research joint venture agreement and R&D agreement with information exchange is stable, at the high levels of marginal damages.*

## 4.2 Quadratic Environmental Damages (Increasing Marginal Damages)

It is well known that with linear environmental damages, the emissions game has an equilibrium in dominant strategies, in other words, the reaction functions of the countries are orthogonal. In order to check whether this assumption is critical for achieving the result that the grand coalition is stable at the high levels of marginal

damages for the research joint venture agreement and R&D agreement with information exchange, the model is solved again for quadratic environmental damages. In this case, the total costs function is given by<sup>10</sup>

$$TC_i = \frac{c}{2}(\delta - \alpha y_i - E_i)^2 + d_0 E + \frac{d_1}{2} E^2 + x_i, \quad (4.94)$$

where the three stages of the game and the levels of effective investment for signatories and non-signatories are defined in the same way as in the previous chapter with linear environmental damages.

#### 4.2.1 Research Joint Venture Agreement (RJV)

In order to check whether the assumption of the linear environmental damages is critical for achieving the result that the grand coalition is stable at the high levels of marginal damages, the research joint venture agreement (RJV) is solved again in this section, assuming quadratic environmental damages while considering linear investment costs.

In this case, the total cost function is given by (4.94), where the three stages of the game and the levels of effective investment for signatories and non-signatories are defined in the same way as in the second chapter with linear environmental damages and linear investment costs.

##### 4.2.1.1 The Partial Agreement Nash Equilibrium of the Investment Game

In this section, stages two and three of the research joint ventures agreement (RJV) are solved by backward induction assuming that in the first stage  $n$  countries with  $n \geq 2$  have signed the agreement. The emission for non-signatories and signatories

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<sup>10</sup>Notice that now the marginal environmental damages are increasing and positive for zero emissions as it occurs for the linear case.

are given by the first order conditions of the total costs function given by (4.94) as follows

$$E_i^f = \bar{\delta} - \alpha y_i^f - \frac{d_1}{c} E, \quad i = 1, \dots, N - n, \quad (4.95)$$

$$E_j^s = \bar{\delta} - \alpha y_j^s - \frac{d_1}{c} E, \quad j = 1, \dots, n, \quad (4.96)$$

where  $\bar{\delta} = \delta - (d_0/c)$ . Thus, the reaction functions for signatories and non-signatories are

$$E_i^f = \frac{c(\bar{\delta} - \alpha y_i^f)}{c + d_1} - \frac{d_1}{c + d_1} E_{-i}, \quad i = 1, \dots, N - n, \quad (4.97)$$

$$E_j^s = \frac{c(\bar{\delta} - \alpha y_j^s)}{c + d_1} - \frac{d_1}{c + d_1} E_{-j}, \quad j = 1, \dots, n, \quad (4.98)$$

where  $E_{-i}$  and  $E_{-j}$  stand for global emissions minus non-signatory  $i$ 's emissions and minus signatory  $j$ 's emissions respectively. Thus, as the reaction functions have a negative slope, emissions are *strategic substitutes*.

By solving both (4.97) and (4.98), the following pair of reaction functions of emissions are obtained

$$E_i^f = \frac{c(\bar{\delta} - \alpha y_i^f) - d_1 n E_j^s}{c + d_1 (N - n)}, \quad (4.99)$$

$$E_j^s = \frac{c(\bar{\delta} - \alpha y_j^s) - d_1 (N - n) E_i^f}{c + d_1 n}, \quad (4.100)$$

such that the increase in emissions of one type of countries reduces the emissions of the other type.

The solution of both (4.99) and (4.100) yields the following levels of emissions for each type

$$E_i^f = \frac{c\bar{\delta} + d_1 n \alpha y_j^s - \alpha y_i^f (c + d_1 n)}{c + d_1 N}, \quad (4.101)$$

$$E_j^s = \frac{c\bar{\delta} + d_1 (N - n) \alpha y_i^f - \alpha y_j^s (c + d_1 (N - n))}{c + d_1 N}, \quad (4.102)$$

such that the increase in the effective investment of one type of countries increases the emissions of the other type, while the effective investment of each country reduces has a negative effect on it.

According to that, the following pair of reaction functions, representing the required levels of effective investment that lead to zero emissions for each type, are obtained

$$y_i^f = \frac{c\bar{\delta} + d_1 n \alpha y_j^s}{\alpha(c + d_1 n)}, \quad (4.103)$$

$$y_j^s = \frac{c\bar{\delta} + d_1(N - n)\alpha y_i^f}{\alpha(c + d_1 n)}. \quad (4.104)$$

The solution of both (4.103) and (4.104) yields the same level of effective investment for both non-signatories and signatories given by (2.6).

Aggregating for (4.95) and (4.96), the total emissions are obtained as follows

$$E = \sum_{i=1}^{N-n} E_i^f + \sum_{j=1}^n E_j^s = \sum_{i=1}^{N-n} \left( \bar{\delta} - \alpha y_i^f - \frac{d_1}{c} E \right) + \sum_{j=1}^n \left( \bar{\delta} - \alpha y_j^s - \frac{d_1}{c} E \right),$$

$$E = \frac{c(N\bar{\delta} - \alpha Y)}{c + Nd_1}, \quad (4.105)$$

where  $Y$  is the global effective investment in R&D that is given by (2.103).

Then, by substitution in (4.95) and (4.96), the emissions for non-signatories and signatories become

$$E_i^f = \bar{\delta} - \alpha y_i^f - \frac{d_1(N\bar{\delta} - \alpha Y)}{c + Nd_1}, \quad i = 1, \dots, N - n, \quad (4.106)$$

$$E_j^s = \bar{\delta} - \alpha y_j^s - \frac{d_1(N\bar{\delta} - \alpha Y)}{c + Nd_1}, \quad j = 1, \dots, n. \quad (4.107)$$

in order to ascertain the effect of investment on emissions, the derivative of both (4.106) and (4.107) with respect to investment is taken as follows



$$\begin{aligned}
\frac{\partial E_i^f}{\partial x_i^f} &= -\alpha \frac{\partial y_i^f}{\partial x_i^f} + \frac{d_1 \alpha}{c + Nd_1} \frac{\partial Y}{\partial x_i^f} \\
&= -\alpha + \frac{d_1 \alpha}{c + Nd_1} (1 + \gamma(N - 1)) \\
&= -\alpha \frac{d_1(N - 1)(1 - \gamma) + c}{c + Nd_1} < 0.
\end{aligned}$$

Thus, it is concluded that investment in R&D by a non-signatory country reduces its emissions. The same result is obtained for signatories

$$\begin{aligned}
\frac{\partial E_j^s}{\partial x_j^s} &= -\alpha \frac{\partial y_j^s}{\partial x_j^s} + \frac{d_1 \alpha}{c + Nd_1} \frac{\partial Y}{\partial x_j^s} \\
&= -\alpha + \frac{d_1 \alpha}{c + Nd_1} (n + \gamma(N - n)) \\
&= -\alpha \frac{d_1(N - n)(1 - \gamma) + c}{c + Nd_1} < 0.
\end{aligned}$$

Doing the substitution of (4.105), (4.106) and (4.107) in the total cost function and in the light of our assumption that there is no cooperation in the emission game, the total costs faced by all countries are given by

$$TC_l = \frac{d_0^2}{2c} + d_0 \left( \frac{c + d_1}{c + Nd_1} \right) (N\bar{\delta} - \alpha Y) + \frac{d_1}{2} \left( \frac{c(c + d_1)}{(c + Nd_1)^2} \right) (N\bar{\delta} - \alpha Y)^2 + x_l. \quad (4.108)$$

Next, the Nash equilibrium of the investment game is calculated using (4.108). By taking the derivative of the total cost function with respect to the investment in R&D of non-signatories countries, it is obtained that

$$\frac{\partial TC_i^f}{\partial x_i^f} = -\alpha d_0 \left( \frac{c + d_1}{c + Nd_1} \right) \frac{\partial Y}{\partial x_i^f} - \alpha d_1 \left( \frac{c(c + d_1)}{(c + Nd_1)^2} \right) (N\bar{\delta} - \alpha Y) \frac{\partial Y}{\partial x_i^f} + 1 = 0,$$

for  $i = 1, \dots, N - n$ . As  $\partial Y / \partial x_i^f = 1 + \gamma(N - 1)$ , the previous condition can be written as

$$\frac{\alpha(1 + (N - 1)\gamma)(c + d_1)}{c + Nd_1} \left( d_0 + d_1 \frac{c(N\bar{\delta} - \alpha Y)}{c + Nd_1} \right) = 1, \quad (4.109)$$

where the left-hand side represents the marginal benefits of investment for the non-signatories.<sup>11</sup>

Taking the derivative of the total cost function with respect to the investment in R&D of signatories countries, it is obtained that

$$\frac{\partial TC_j^s}{\partial x_j^s} = -\alpha d_0 \left( \frac{c + d_1}{c + Nd_1} \right) \frac{\partial Y}{\partial x_j^s} - \alpha d_1 \left( \frac{c(c + d_1)}{(c + Nd_1)^2} \right) (N\bar{\delta} - \alpha Y) \frac{\partial Y}{\partial x_j^s} + 1 = 0,$$

for  $j = 1, \dots, n$ . As  $\partial Y / \partial x_j^s = n + \gamma(N - n)$ , the previous condition yields

$$\frac{\alpha(n + (N - n)\gamma)(c + d_1)}{c + Nd_1} \left( d_0 + d_1 \frac{c(N\bar{\delta} - \alpha Y)}{c + Nd_1} \right) = 1, \quad (4.110)$$

where the left-hand side represents the marginal benefits of investment for signatories.

By comparing (4.109) and (4.110), it is pretty obvious that the marginal benefits of signatories are higher than the marginal benefits of the non-signatories countries for all  $n > 1$  for the same level of global effective investment.

Next, the conditions to get a corner solution, i.e. to eliminate completely the emissions, are studied. This occurs when the marginal benefits of investment are greater or equal to the marginal costs for zero global emissions. Then, using (4.109) and (4.110), a critical value for  $d_0$  can be obtained for both non-signatories and signatories, which makes the derivative of total costs equal to zero for zero global emissions as follows

$$\hat{d}_0^f = \frac{c + Nd_1}{\alpha(c + d_1)(1 + \gamma(N - 1))}, \quad (4.111)$$

$$\hat{d}_0^s(n) = \frac{c + Nd_1}{\alpha(c + d_1)(n + \gamma(N - n))}. \quad (4.112)$$

Observe that in this case, the critical values of  $d_0$  for both non-signatories and signatories are depending on  $c$  and  $d_1$ . Also, as in the linear model, the critical value

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<sup>11</sup>Notice that  $(d_0 + d_1 \frac{c(N\bar{\delta} - \alpha Y)}{c + Nd_1}) = d_0 + d_1 E$  stands for the marginal environmental damages.

of  $d_0$  for the signatories countries is decreasing with respect to the level of cooperation and takes values between

$$\hat{d}_0^s(N) = \frac{c + Nd_1}{\alpha N(c + d_1)} \leq \hat{d}_0^s(n) \leq \hat{d}_0^s(2) = \frac{c + Nd_1}{\alpha(c + d_1)(2 + \gamma(N - 2))}.$$

Moreover, it is obvious that  $\hat{d}_0^f = \hat{d}_0^s(1)$ , so that  $\hat{d}_0^s < \hat{d}_0^f$  for all  $n > 1$ . Therefore, for any  $d_0 > \hat{d}_0^f$ , both signatories and non-signatories will invest at the maximum level that leads to the elimination of the emissions. In this case, the optimization problems of non-signatories and signatories countries are the same as in the previous chapter given as follows

$$\min_{x_i^f} TC_i^f = \frac{c}{2} (\delta - \alpha y_i^f)^2 + x_i^f, \quad (4.113)$$

$$s.t. y_i^f = x_i^f + \gamma X_{-i} \geq \frac{\bar{\delta}}{\alpha}, \quad (4.114)$$

$$y_i^f = x_i^f + \gamma X_{-i} \leq \frac{\delta}{\alpha}, \quad (4.115)$$

for non-signatories, and

$$\min_{x_j^s} TC_j^s = \frac{c}{2} (\delta - \alpha y_j^s)^2 + x_j^s, \quad (4.116)$$

$$s.t. y_j^s = nx_j^s + \gamma(N - n)x_i^f \geq \frac{\bar{\delta}}{\alpha}, \quad (4.117)$$

$$y_j^s = nx_j^s + \gamma(N - n)x_i^f \leq \frac{\delta}{\alpha}, \quad (4.118)$$

for signatories. Thus, the previous optimization problems yield the same solutions obtained in the previous chapters with the levels of investments given by (2.113) for non-signatories and (2.114) for signatories, where the effective investment for both types of countries is given by (2.17).

We know from the solution of the previous chapter that the constraints on effective investment given by (4.114) and (4.117) are satisfied for any level of marginal damages higher than  $\tilde{d}^{nc}$  given by (2.18).

By comparing  $\tilde{d}^{nc}$  with  $\hat{d}_0^f$ , it is found that  $\tilde{d}^{nc} > \hat{d}_0^f$  for any  $\gamma > \tilde{\gamma}$  given by

$$\tilde{\gamma} = \frac{d_1}{c + d_1}, \quad (4.119)$$

and vice versa. According to that if  $\gamma > \tilde{\gamma}$ , then the total costs for any  $d \geq \tilde{d}^{nc}$  are given as in the previous chapter by (2.116) and (2.117) for non-signatories and signatories countries respectively. However, if  $d \in (\hat{d}_0^f, \tilde{d}^{nc}]$ , the levels of effective investment for both non-signatories and signatories will be given, as in the linear model, by

$$y_i^f = y_j^s = \frac{\bar{\delta}}{\alpha}, \quad Y = \frac{N\bar{\delta}}{\alpha},$$

except that now  $\bar{\delta} = \delta - (d_0/c)$ , and the same occurs for the investment in R&D:

$$x_i^f = \frac{\bar{\delta}}{\alpha(1 + \gamma(N - n))}, \quad x_j^s = \frac{\bar{\delta}}{\alpha n(1 + \gamma(N - n))}.$$

Finally, by substituting the levels of investment in the total costs functions, the following total costs functions are obtained

$$TC_i^f = \frac{d_0^2}{2c} + \frac{\bar{\delta}}{\alpha(1 + \gamma(N - n))}, \quad i = 1, \dots, N - n, \quad (4.120)$$

$$TC_j^s = \frac{d_0^2}{2c} + \frac{\bar{\delta}}{\alpha n(1 + \gamma(N - n))}, \quad j = 1, \dots, n, \quad (4.121)$$

where the first term represents the abatement cost and the second term represents the investment cost.

#### 4.2.1.2 The Nash Equilibrium of the Membership Game

As the total costs obtained here are the same like those obtained in the previous chapter, it is clear that the stability analysis when the level of the marginal damage is high enough will yield the same result as those obtained with the assumption of linear environmental damages. Thus, the following proposition is concluded

**Proposition 42** *If marginal damages are sufficiently large, in particular if  $d$  is bigger than  $\hat{d}_0^f$ , the grand coalition is the unique stable research joint ventures agreement even when the marginal damages are quadratic, regardless the degree of spillover effects.*

#### 4.2.2 R&D Agreement with Information Exchange

In order to check whether the assumption of the linear environmental damages is critical for achieving the result that the grand coalition is stable at the high levels of marginal damages, the R&D agreement with information exchange is solved again in this section, assuming quadratic environmental damages while considering linear investment costs. As obtained in the solution of the research joint ventures agreement, it is found that R&D agreement with information exchange, at the high levels of marginal damages yields the same levels of total costs for both non-signatories and signatories as those obtained for the R&D agreement with information exchange solved in the previous chapters. Thus, total costs for both non-signatories and signatories, for any  $d \geq \tilde{d}^{nc}$  are given by (2.149) and (2.150) respectively. However, taking into account that signatories countries cooperate at the second stage of the game, the critical value of marginal damages for signatories is now given by

$$\hat{d}_0^s(n) = \frac{c + Nd_1}{\alpha(c + d_1)(n + \gamma(N - n))}, \quad (4.122)$$

while for non-signatories it is still given by (4.111).

Also, as in the linear model, the critical value of  $d_0$  for the signatories countries is decreasing with respect to the level of cooperation and takes values between

$$\hat{d}_0^s(N) = \frac{c + Nd_1}{\alpha N^2(c + d_1)} \leq \hat{d}_0^s(n) \leq \hat{d}_0^s(2) = \frac{c + Nd_1}{2\alpha(c + d_1)(2 + \gamma(N - 2))}.$$

According to that, the following proposition is concluded

**Proposition 43** *If marginal damages are sufficiently large, in particular if  $d$  is bigger than  $\tilde{d}^{nc}$ , the grand coalition is the unique stable R&D agreement with information exchange even when the marginal damages are quadratic, regardless the degree of spillover effects.*

According to the previous analysis of both research joint venture agreement and R&D agreement with information exchange, the following proposition is concluded

**Proposition 44** *The assumption of linear environmental damages is not critical for achieving the result that grand coalition of both the research joint venture agreement and R&D agreement with information exchange is stable, at the high levels of marginal damages.*

### 4.3 Conclusion

In this chapter, the robustness of our model assumptions is examined, mainly, the assumption of the linear investment costs and the assumption of linearity of environmental damages. In particular, we examined whether these assumptions are critical for achieving the result that grand coalition is stable at high levels of marginal damages or not for the agreements that allow R&D information exchange.

Our analysis in this chapter is focused on both research joint venture agreement and R&D agreement with information exchange, as we concluded in the previous chapter that at the high levels of marginal damages, cooperating in the third stage of the game, doesn't play any role in reducing the total costs of signatories as far as they invest at the maximum level of investment to completely eliminate the GHG emission.

According to the analysis introduced in this chapter, it is found that both of the assumptions of constant returns to scale of the R&D investment (linear investment costs) and the assumption of linear environmental damages are not critical for achieving the result that grand coalition of both the research joint venture agreement and

R&D agreement with information exchange is stable, at the high levels of marginal damages.

It is concluded that under the assumption of decreasing returns to scale of the R&d efforts, exchanging R&D information of the break-through technologies is enough eliminate countries incentives to act as free-rider at any level of marginal damages. However, coordinating the R&D investment reduces the level of cooperation at the low values of marginal damages.

## 4.4 Appendices

### 4.4.1 Appendix1: The Proof of Proposition 36

In order to have a complete view about the comparison between the efficient outcome and the fully non-cooperative equilibrium, the range of the level of marginal damages should be divided into four parts as follows

$$d \in (0, \hat{d}^e), d \in [\hat{d}^e, \hat{d}^{nc}), d \in [\hat{d}^{nc}, \tilde{d}^{nc}) \text{ and } d \geq \tilde{d}^{nc},$$

as the levels of effective investment and the total costs change along those parts.

First: when  $d \in (0, \hat{d}^e)$ .

By comparing the effective investment of the fully non-cooperative equilibrium given by (4.6) with the effective investment of the efficient solution given by (4.28) as follows

$$\begin{aligned} y_i^e - y_i^{nc} &= \frac{\alpha d}{r} N^3 - \frac{\alpha d}{r} (1 + \gamma (N - 1))^2 \\ &= \frac{\alpha d}{r} (N^3 - (1 + \gamma (N - 1))^2) > 0 \text{ for } \gamma(0, 1), \end{aligned}$$

it is clear that

$$y_i^e > y_i^{nc}.$$

Next, the level of emissions of the fully non-cooperative equilibrium is compared by the level of emissions of the efficient solution as follows

$$\begin{aligned} E_i^{nc} - E_i^e &= \delta - \frac{d}{c} - \alpha y_i^{nc} - \delta + \frac{Nd}{c} + \alpha y_i^e \\ &= \frac{d}{c} (N - 1) + \alpha (y_i^e - y_i^{nc}) > 0, \end{aligned}$$

so, it is concluded that

$$E_i^{nc} > E_i^e.$$

Finally, the total costs of the fully non-cooperative equilibrium given by (4.9) are compared with the total costs of the efficient solution given by (4.31) as follows

$$\begin{aligned} TC_i^{nc} - TC_i^e &= dN\delta - \frac{d^2(2N-1)}{2cr} (r + \alpha^2c(1 + \gamma(N-1))^2) \\ &\quad - dN\delta + \frac{d^2N^2}{2cr} (r + c\alpha^2N^2) \\ &= \frac{d^2}{2cr} (r(N^2 - 2N + 1) + c\alpha^2(N^4 - (2N-1)(1 + \gamma(N-1))^2)) > 0, \end{aligned}$$

for any  $\gamma(0, 1)$ . So, it is concluded that

$$TC_i^{nc} > TC_i^e.$$

Second: when  $d \in [\hat{d}^e, \hat{d}^{nc})$ .

By comparing the effective investment of the fully non-cooperative equilibrium given by (4.6) with the effective investment of the efficient solution given by (4.37) as follows

$$y_i^e - y_i^{nc} = \frac{\alpha c \delta N^2}{r + \alpha^2 c N^2} - \frac{\alpha d}{r} (1 + \gamma(N-1))^2,$$

we find that  $y_i^e = y_i^{nc}$  at

$$d^* = \frac{\delta c r N^2}{(r + c \alpha^2 N^2) (1 + \gamma(N-1))^2}.$$



Thus, at any  $d$  lower than  $d^*$ , the level of effective investment of the efficient solution is higher than the level of effective investment of the fully non-cooperative. By comparing the level of marginal damages given by (4.10) with  $d^*$  as follows

$$\begin{aligned} d^* - \hat{d}^{nc} &= \frac{\delta cr N^2}{(r + c\alpha^2 N^2)(1 + \gamma(N-1))^2} - \frac{\delta cr}{r + c\alpha^2(1 + \gamma(N-1))^2} \\ &= \delta cr^2 \left( \frac{N^2 - (1 + \gamma(N-1))^2}{(r + c\alpha^2 N^2)(1 + \gamma(N-1))^2 (r + c\alpha^2(1 + \gamma(N-1))^2)} \right) > 0, \end{aligned}$$

it is clear that  $d^* > \hat{d}^{nc}$  which means that in the range of  $d \in [\hat{d}^e, \hat{d}^{nc})$

$$y_i^e > y_i^{nc}.$$

Finally, the total costs of the fully non-cooperative equilibrium given by (4.9) are compared with the total costs of the efficient solution given by (4.40) as follows

$$TC_i^{mc} - TC_i^e = dN\delta - \frac{d^2(2N-1)}{2cr} (r + \alpha^2 c(1 + \gamma(N-1))^2) - \frac{c\delta^2 r}{2(r + c\alpha^2 N^2)},$$

where the first derivative of the difference is given by

$$-\frac{d(2N-1)}{cr} (r + \alpha^2 c(1 + \gamma(N-1))^2) + N\delta = 0,$$

and the second derivative is given by

$$-\frac{(2N-1)}{2cr} (r + \alpha^2 c(1 + \gamma(N-1))^2) < 0.$$

Thus, it is clear that the function of the difference in the total costs has a maximum at

$$\dot{d} = \frac{N\delta cr}{(2N-1)(r + \alpha^2 c(1 + \gamma(N-1))^2)} > 0.$$

By assuming that  $d = \hat{d}^e$  and substituting in the differences of the total costs as follows

$$\Delta TC_i(\hat{d}^e) = \frac{\delta^2 cr}{2(c\alpha^2 N^2 + r)} \left( \frac{r(N-1)^2 + c\alpha^2(N^4 - (2N-1)(1 + \gamma(N-1))^2)}{N^2(c\alpha^2 N^2 + r)} \right) > 0,$$

and by comparison, it is easy to find that  $\hat{d} > \hat{d}^e$ .

Now, assume that  $d = \hat{d}^{nc}$  and substituting in the difference of the total costs

$$\Delta TC_i(\hat{d}^{nc}) = \frac{r\delta^2 c^2 \alpha^2 (N^2 - (1 + \gamma(N-1))^2)}{2(r + c\alpha^2(1 + \gamma(N-1))^2)} > 0,$$

and by comparison, it is easy to find that  $\hat{d}^{nc} > \hat{d}$ . So, it is concluded that for any  $d \in [\hat{d}^e, \hat{d}^{nc})$

$$TC_i^{nc} > TC_i^e.$$

Third: when  $d \in [\hat{d}^{nc}, \tilde{d}^{nc})$ .

By comparing the effective investment of the fully non-cooperative equilibrium given by (4.47) with the effective investment of the efficient solution given by (4.37) as follows

$$y_i^{nc} - y_i^e = \frac{1}{\alpha} \left( \delta - \frac{d}{c} \right) - \frac{\alpha c \delta N^2}{r + \alpha^2 c N^2},$$

we find that  $y_i^e = y_i^{nc}$  at

$$\tilde{d} = \frac{\delta cr}{(r + c\alpha^2 N^2)}.$$

Thus, at any  $d$  higher than  $\tilde{d}$ , the level of effective investment of the efficient solution is higher than the level of effective investment of the fully non-cooperative. By comparing the level of marginal damages given by (4.10) with  $\tilde{d}$  as follows

$$\begin{aligned} \tilde{d} - \hat{d}^{nc} &= \frac{\delta cr}{(r + c\alpha^2 N^2)} - \frac{\delta cr}{r + c\alpha^2(1 + \gamma(N-1))^2} \\ &= \delta c^2 r \alpha^2 \left( \frac{2(1 + \gamma(N-1))^2 - N^2}{(r + c\alpha^2 N^2)(r + c\alpha^2(1 + \gamma(N-1))^2)} \right) < 0, \end{aligned}$$

it is clear that  $\hat{d}^{nc} > \tilde{d}$  which means that in the range of  $d \in [\hat{d}^{nc}, \tilde{d}^{nc})$

$$y_i^e > y_i^{nc}.$$

Finally, concerning the comparison of the total costs, it is clear that at the level of marginal damages  $d = \hat{d}^{nc}$ , the total costs of the fully non-cooperative equilibrium are higher than the total costs of the efficient solution, as calculated previously.

Next, the total costs of the fully non-cooperative equilibrium in the range of  $\tilde{d}^{nc} > d > \hat{d}^{nc}$  are given by (4.17)

$$TC_i^{nc} = \frac{(r + \alpha^2 c (1 + \gamma (N - 1))^2) d^2 - 2\delta c r d + r c^2 \delta^2}{2\alpha^2 c^2 (1 + \gamma (N - 1))^2},$$

which has a minimum at

$$\check{d} = \frac{\delta c r}{r + \alpha^2 c (1 + \gamma (N - 1))^2} = \hat{d}^{nc},$$

so, the total costs of the fully non-cooperative equilibrium at this minimum is given by

$$TC_i^{nc} = \delta^2 c^2 r \left( \frac{\alpha^2 c (1 + \gamma (N - 1))^2}{r + \alpha^2 c (1 + \gamma (N - 1))^2} \right) > 0.$$

Thus, as the costs of the efficient solution are constant and it is already known from the comparison of costs at  $d = \hat{d}^{nc}$ , previously, that  $TC_i^{nc} > TC_i^e$ , it is concluded that  $TC_i^{nc} > TC_i^e$  in this range of marginal damages.

Fourth: when  $d \geq \tilde{d}^{nc}$ .

By comparing the effective investment of the fully non-cooperative equilibrium given by (4.14) with the effective investment of the efficient solution given by (4.37) as follows

$$\begin{aligned}
y_i^e - y_i^{nc} &= \frac{\alpha c \delta N^2}{r + \alpha^2 c N^2} - \frac{\alpha c \delta (1 + \gamma (N - 1))}{r + \alpha^2 c (1 + \gamma (N - 1))} \\
&= \alpha c \delta \left( \frac{r (N^2 - (1 + \gamma (N - 1)))}{(r + \alpha^2 c N^2) (r + \alpha^2 c (1 + \gamma (N - 1)))} \right) > 0,
\end{aligned}$$

which means that

$$y_i^e > y_i^{nc}.$$

Next, the total costs of the fully non-cooperative equilibrium given by (4.18) are compared with the total costs of the efficient solution given by (4.37) as follows

$$\begin{aligned}
TC_i^{nc} - TC_i^e &= \frac{\delta^2 c r (r + c \alpha^2)}{2 (r + \alpha^2 c (1 + \gamma (N - 1)))^2} - \frac{c \delta^2 r}{2 (r + c \alpha^2 N^2)} \\
&= \frac{\delta^2 c r}{2} \left( \frac{r c \alpha^2 (N^2 + 1 - 2 (1 + \gamma (N - 1))) + c^2 \alpha^4 (N^2 - (1 + \gamma (N - 1))^2)}{(r + c \alpha^2 N^2) (r + \alpha^2 c (1 + \gamma (N - 1)))^2} \right) >
\end{aligned}$$

which means that

$$TC_i^{nc} > TC_i^e.$$

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