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Quark-Lepton Mass Relation and CKM mixing in an A_4 Extension of the Minimal Supersymmetric Standard Model

S. Morisi,[1,*] M. Nebot,[2,†] Ketan M. Patel,[3,‡] E. Peinado,[4,§] and J. W. F. Valle[2,¶]

[1] *Institut für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany*
[2] *Departament de Física Teòrica & IFIC, Universitat de València/C.S.I.C., Edificio Institutos de Paterna, Apt 22085, E–46071 Valencia, Spain*
[3] *Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400 005, India*
[4] *INFN, Laboratori Nazionali di Frascati, Via Enrico Fermi 40, I-00044 Frascati, Italy*

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An interesting mass relation between down type quarks and charged leptons has been recently predicted within a supersymmetric $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ model based on the A_4 flavor symmetry. Here we propose a simple extension which provides an adequate full description of the quark sector. By adding a pair of vector-like up-quarks we show how the CKM entries V_{ub}, V_{cb}, V_{td} and V_{ts} arise from deviations of the unitarity. We perform an analysis including the most relevant observables in the quark sector, such as oscillations and rare decays of kaons, B_d and B_s mesons. In the lepton sector, model predicts an inverted hierarchy for the neutrino masses leading to a potentially observable rate of neutrinoless double beta decay.

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I. INTRODUCTION

Understanding the observed pattern of quark and lepton masses and mixing from first principles constitutes one of the deepest challenges in particle physics [1]. The recent robust experimental discovery of a nonzero value for the reactor mixing angle θ_{13} in the neutrino sector [2] may unveil surprises in the underlying theoretical structure of the flavour sector [3], opening also the door towards a new generation of experiments searching for CP violation in the leptons [4, 5]. An ever growing body of experimental evidence makes flavour in the quark sector a challenging playground for any extension of the Standard Model (SM).

Flavor symmetries provide a very useful approach towards reducing the number of free parameters describing the structure of the fermion sector. Non-Abelian discrete groups have played an important role in connection with the flavour problem. Their mathematics is somewhat less familiar to particle physicists as continuous non-Abelian symmetries, and have been extensively discussed in the recent book, Ref. [6]. Groups like A_4 [1] are especially useful in that they contain triplet irreducible representations, exactly the number of $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ generations.

In Ref. [9] a supersymmetric extension of the standard $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ has been proposed based on the A_4 group, where all the matter fields as well as the Higgs doublets were assigned to the same A_4 representation, namely, the triplet. This leads to an important theoretical prediction, namely a mass relation

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}} \;, \tag{1}$$

involving down-type quarks and charged lepton mass ratios. Such relation provides a generalization of the three Georgi-Jarlskog (GJ) mass relations [10],

$$m_b = m_\tau,\;\; m_s = m_\mu/3,\;\; m_d = 3m_e, \tag{2}$$

*Electronic address: stefano.morisi@gmail.com
†Electronic address: nebot@ific.uv.es
‡Electronic address: ketan@theory.tifr.res.in
§Electronic address: epeinado@ific.uv.es
¶URL: http://astroparticles.ific.uv.es/; Electronic address: valle@ific.uv.es

[1] A_4 is the group of even permutation of four objects, for pioneering work see [7, 8].

which arise within a particular ansatz for the SU(5) model and hold at the unification scale. In contrast to Eq. (2), our relation requires no unification group and holds at the electroweak scale. It would, in any case, be rather robust against renormalization effects as it involves only mass ratios [2].

A second prediction obtained in Ref. [9] involves the Cabibbo angle for the quarks which arises mainly from the down-type quark sector [11] with a correction coming from the up isospin diagonalization matrix. While this provides a successful prediction for the Cabibbo angle the corresponding predictions for V_{ub}, V_{cb}, V_{td} and V_{ts} were unacceptably small and require an extension.

Following the suggestion in the original paper [9] here we address such a possibility. Recently some of us considered the same problem in [12] where the CKM matrix is obtained by assuming a different A_4 quark assignment. In this paper we consider a variant of the first scheme in which extra fermions are added, namely a pair of up-type quarks, in which case the full CKM matrix will be a 4×3 matrix. The small V_{ij} mixings can be generated by means of violations of 3×3 unitarity. In the next section we introduce our model, giving a brief description of quark mixing as well as neutrino masses and mixing, in section III we address in detail the phenomenology of the quark sector of the model, and in section IV we give our summary and conclusions.

II. THE MODEL

The basic content in terms of MSSM matter and Higgs superfields is the same as that in [9]. It is assumed that all such fields transform as A_4 triplets. In order to generate correct quark mixing angles without changing the mass relations between down quarks and charged leptons, we introduce a pair of vector-like up-type quark multiplets T and T^c transforming as $(3, 1, 2/3)$ and $(\bar{3}, 1, -2/3)$ under the SM gauge group $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$. In addition, we introduce two A_4 triplet flavon fields σ and σ' imposing a Z_4 symmetry (with $\omega^4 = 1$) in order to distinguish between them. The various charge assignments are given in Table I where

Fields	L	E^c	Q	U^c	D^c	H^u	H^d	T	T^c	σ	σ'
$SU(2)_L$	2	1	2	1	1	2	2	1	1	1	1
A_4	3	3	3	3	3	3	3	1	1	3	3
Z_4	1	1	1	1	1	1	1	ω	ω^3	ω^3	ω

TABLE I: Field content and quantum numbers of the model.

The most general Yukawa superpotential allowed by the above symmetry is

$$w = w_Y + w_T, \tag{3}$$
$$w_Y = y^u_{ijk} Q_i H^u_j U^c_k + y^d_{ijk} Q_i H^d_j D^c_k + y^l_{ijk} L_i H^d_j E^c_k, \tag{4}$$
$$w_T = M T T^c + X T U^c_i \sigma_i + \frac{Y}{\Lambda} Q_i (H^u \cdot \sigma')_i T^c, \tag{5}$$

where X, Y and $y^{u,d,l}_{ijk}$ are the Yukawa couplings and we assume all $y^{u,d,l}_{ijk}$ to be real for simplicity. The product of two A_4 triplets denoted as $(\phi \cdot \chi)$ also transforms as a triplet. In the above Yukawa superpotential, we consider a dimension-5 non-renormalizable operator allowed by the symmetry of model and required for realistic quark masses and mixings as discussed later in this section. Such operator can arise from a renormalizable superpotential by introducing extra messenger fields such as, for instance, h^u and h^d with the same quantum numbers of $H^{u,d}$ under $SU(2)_L \times U(1)$ and transforming respectively as ω and ω^3 under Z_4. Then the extra terms $Q h^u T^c$, $H^u h^d \sigma'$ are allowed and, of course, the mass term $h^u h^d$. The dimension-5 operator in w_T arises after integrating out the heavy messenger fields h^u and h^d from the spectrum. The Z_4 symmetry forbids all the other dimension-5 operators in the above superpotential. In addition, we neglect corrections coming from operators of dimension-6 or greater. An alternative could be to replace $(H^u \cdot \sigma')$ with $H^{u'}$ transforming as σ' under Z_4 but as a Higgs doublet under SU(2). For anomaly cancellation we

[2] The mass relation in Eq. (1) can get modified due to finite supersymmetric threshold corrections to the fermion masses. However such corrections crucially depend on several details of the soft supersymmetry breaking parameters which are not constrained by the model presented here. In order to keep our discussion as model-independent as possible we defer the discussion of the details of soft supersymmetry breaking threshold effects to another publication.

must also add an $H^{d'}$ with opposite charges with respect to $H^{u'}$. One may forbid the higher order operators by replacing Z_4 with Z_N symmetry with sufficiently large N. We first review the main phenomenological consequences of the w_Y part of the superpotential w and in the next section we consider the w_T piece, which is the new part of the present work. By using A_4 product rules in the superpotential w_Y it is straightforward to show that the charged fermion mass matrices take the following universal structure [13]

$$M_f = \begin{pmatrix} 0 & y_1^f \left\langle H_3^f \right\rangle & y_2^f \left\langle H_2^f \right\rangle \\ y_2^f \left\langle H_3^f \right\rangle & 0 & y_1^f \left\langle H_1^f \right\rangle \\ y_1^f \left\langle H_2^f \right\rangle & y_2^f \left\langle H_1^f \right\rangle & 0 \end{pmatrix}, \tag{6}$$

where $f = l, u, d$ denotes charged leptons, up-type or down-type quarks respectively and $y_{1,2}$ are the only A_4 invariant contractions of the y^f_{ijk}. Following [9] we assume the general alignment form

$$\langle H^u \rangle = (v^u, \varepsilon_1^u, \varepsilon_2^u)^T \quad \text{and} \quad \langle H^d \rangle = (v^d, \varepsilon_1^d, \varepsilon_2^d)^T \ , \tag{7}$$

where $\varepsilon^u_{1,2} \ll v^u$ and $\varepsilon^d_{1,2} \ll v^d$, namely A_4 is completely broken as a result of explicit A_4 soft–breaking terms in the scalar potential (see for example, the discussion below Eqs. (5) in [9]). Note that, in addition to the "texture" zeros in the diagonal, one has additional relations among the off-diagonal elements in M_f. This may be seen explicitly by rewriting Eq. (6) as

$$M_f = \begin{pmatrix} 0 & a^f \alpha^f & b^f \\ b^f \alpha^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix}, \tag{8}$$

where $a^f = y_1^f \varepsilon_1^f$, $b^f = y_2^f \varepsilon_1^f$, with $y^f_{1,2}$ denoting the only two couplings arising from the A_4-tensor in Eq. (4), $r^f = v^f/\varepsilon_1^f$ and $\alpha^f = \varepsilon_2^f/\varepsilon_1^f$. Thanks to the fact that the same MSSM Higgs doublet H^d couples to the lepton and to the down-type quarks one has, in addition, the following relations

$$r^l = r^d, \qquad \alpha^l = \alpha^d, \tag{9}$$

involving down-type quarks and charged leptons.

Each of the mass matrices in Eq. (8) depends on just four parameters. We can express three of the parameters like for instance r^f, a^f, b^f in terms of the corresponding fermion masses and α^f. The results are

$$r^f \approx \frac{m_3^f}{\sqrt{m_1^f m_2^f}} \sqrt{\alpha^f}\ , \tag{10}$$

$$a^f \approx \frac{m_2^f}{m_3^f} \sqrt{\frac{m_1^f m_2^f}{\alpha^f}}, \tag{11}$$

$$b^f \approx \sqrt{\frac{m_1^f m_2^f}{\alpha^f}}\ . \tag{12}$$

where we have used the approximation $r^f \gg 1$ and $r^f \gg b^f/a^f$. From Eqs. (9) and (10) we obtain the mass relation given in Eq. (1). The diagonalization of $M_{d,l}$ and its phenomenological implications have already been discussed in Ref. [9]. For example, one finds

$$V_{12}^f \approx \sqrt{\frac{m_1}{m_2}} \frac{1}{\sqrt{\alpha^f}}, \tag{13}$$

$$V_{13}^f \approx \frac{m_2}{m_3^2} \sqrt{m_1 m_2} \frac{1}{\sqrt{\alpha^f}}, \tag{14}$$

$$V_{23}^f \approx \frac{m_1 m_2}{m_3^2} \frac{1}{\alpha^f}. \tag{15}$$

Using the above relations and the experimental values of the fermion masses we have $V_{12}^f \sim \mathcal{O}(\lambda_C)$, while V_{13}^f and V_{23}^f are too small. In order to generate adequate predictions for all entries of the quark mixing matrix we must modify the above scheme, as discussed in the next section. Note that we modify only the up-quark sector in order to maintain our mass relations in Eq. (1).

A. Quark mixing

In the previous section we have only considered the w_Y superpotential. We now take into account also the w_T superpotential terms. Assuming that the flavon fields σ and σ' take complex vacuum expectation values in completely random directions, we have

$$\langle\sigma\rangle = (\langle\sigma_1\rangle, \langle\sigma_2\rangle, \langle\sigma_3\rangle)^T \quad \text{and} \quad \langle\sigma'\rangle = (\langle\sigma'_1\rangle, \langle\sigma'_2\rangle, \langle\sigma'_3\rangle)^T. \tag{16}$$

The resulting 4×4 up-type quark mass matrix is

$$M_u = \begin{pmatrix} 0 & a^u\alpha^u & b^u & Y_1 \\ b^u\alpha^u & 0 & a^u r^u & Y_2 \\ a^u & b^u r^u & 0 & Y_3 \\ X_1 & X_2 & X_3 & M \end{pmatrix} \tag{17}$$

where $X_i = X\langle\sigma_i\rangle$ and $Y_i = Y\langle(H^u\cdot\sigma')_i\rangle/\Lambda$ are complex parameters. The down quark mass matrix is unchanged and is given by Eq. (8). Here we consider in detail the role of the (T, T^c) coupling in the up quark sector and how it makes it possible to account for the full structure of the CKM mixing matrix. This possibility has already been studied in the literature in the general case of flavour-blind models [14, 15]. Here we explore its role in the context of our flavor–symmetric model. In Sec. III, we give an example set of the above parameters which produces a viable quark mixing pattern without spoiling the charged leptons and down-type quarks mass relations. The idea is that the 3×3 sub-matrix of the 4×4 matrix which diagonalizes on the left the up–quark mass matrix, is not unitary. Unitarity deviations modify the relations (13), (14) and (15) allowing for an acceptable fit of the CKM matrix. In the next section we discuss this in details.

B. Neutrino masses and mixing

We now turn to neutrino mass generation, describing it effectively *a la Weinberg*. In this model neutrino masses are generated by the following dimension-5 operator

$$w_\nu = \frac{1}{\Lambda} L L H_u H_u. \tag{18}$$

This operator is given by the product of four A_4-triplets and contains many possible A_4 contractions. The general structure has already been studied in Ref. [13] and is given by

$$M_\nu \approx \begin{pmatrix} x & \kappa & \kappa\alpha^u \\ \kappa & y & 0 \\ \kappa\alpha^u & 0 & z \end{pmatrix}, \tag{19}$$

where x, y, z and κ are proportional to couplings that arise from each A_4 contraction. Note that M_ν is invariant under the μ-τ interchange symmetry if $y = z$ and $\alpha^u = 1$. Following [16], one can quantify the μ-τ breaking as

$$\epsilon_1 = \left|\frac{1-\alpha^u}{1+\alpha^u}\right| \quad \text{and} \quad \epsilon_2 = \left|\frac{y-z}{y+z}\right|. \tag{20}$$

The large but non-maximal atmospheric mixing angle as evidenced from the recent global fits to oscillation data [2] requires $y \approx z$. Interestingly, the parameter α^u enters also in the up quark mass matrix. We find from the quark sector (see section III for the details) that α^u lies in the range $[0.7,\ 1.8]$ at 3σ. This implies ϵ_1 in the range $[0,\ 0.3]$ which corresponds to small μ-τ breaking. As it is shown in [16], given the large value of θ_{13} such a small breaking of μ-τ symmetry is only consistent with either inverted or quasi-degenerate mass spectrum of neutrinos. Moreover, the neutrino mass matrix has one zero which forbids quasi-degenerate neutrinos and leads to a correlation between the effective mass parameter characterizing the $0\nu\beta\beta$ decay amplitude $m_{\beta\beta}$ and the solar mixing angle as displayed in Figure 1. For convenience, we also plot $m_{\beta\beta}$ as a function of the lightest neutrino mass eigenvalue. Clearly, the model predicts $m_{\beta\beta} \in [0.01,\ 0.03]$ eV which could be accessible in the next generation experiments. Note that we have performed a numerical exploration of the parameter space, in which we have also included the correction associated to the diagonalization of the charged lepton sector and given from Eqs. (13), (14), (15).

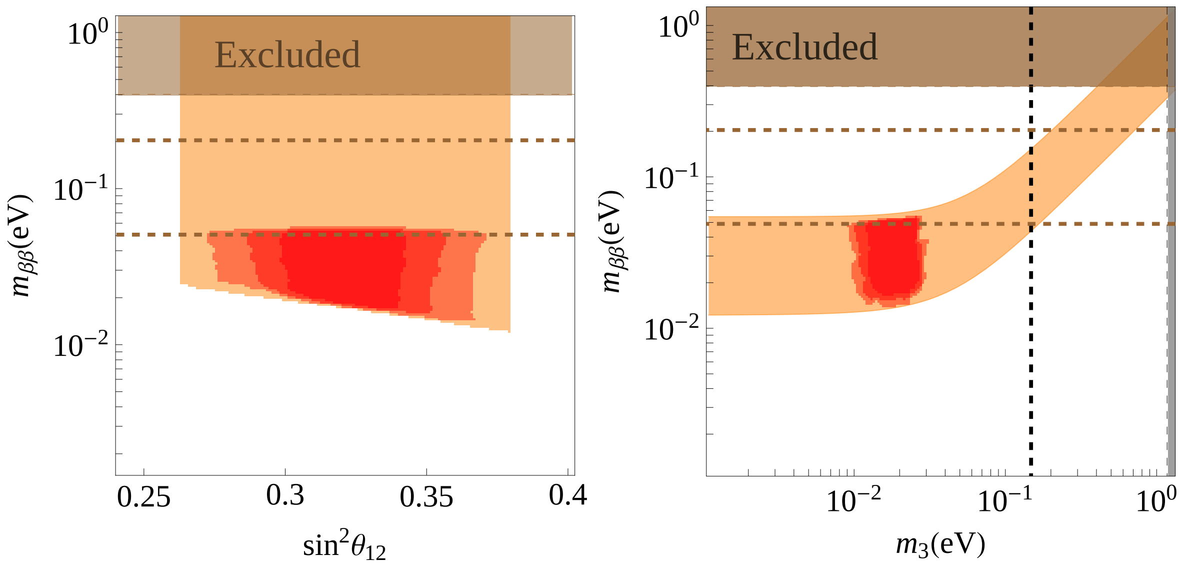

FIG. 1: Correlations between the neutrinoless double beta decay effective mass parameter $m_{\beta\beta}$ and the solar mixing angle (left panel) and the lightest neutrino mass (right panel). The red regions correspond to the model prediction while the orange ones correspond to the generically allowed regions for an inverted mass hierarchy. Darker to lighter tones correspond to 68%, 95% and 99% CL regions allowed by the model parameters. The dark brown region corresponds to the present experimental bound on $m_{\beta\beta}$. The upper and lower horizontal dashed lines correspond to the sensitivities of GERDA-I and CUORE experiments while the vertical dashed line in right panel shows the future sensitivity of KATRIN experiment.

III. PHENOMENOLOGY OF THE QUARK SECTOR

We now analyze the implications of the structure of the mass matrices for the quark sector. To do so we will first remind the modifications that arise in the couplings to W and Z bosons. Having set the stage, we will then give an overview of both the most relevant phenomenological consequences that should be explored and the experimental constraints to be considered. We will then show that the model can comply with present constraints. Since in some interesting observables this is no obstacle to accommodate predictions different from SM expectations, we will also pay some attention to such a possibility. As usual, rotating quark fields to the mass eigenstate basis yields a clash among left-handed up and down rotation matrices, which gives rise to the appearance of a mixing matrix V in the charged-current couplings,

$$\mathscr{L}_W = -\frac{g}{\sqrt{2}} \bar{\mathbf{u}}_L \, \gamma^\mu \, V \, \mathbf{d}_L \, W_\mu \, + \text{h.c.} \; . \tag{21}$$

However, since now $\mathbf{d} = (d, s, b)$ and $\mathbf{u} = (u, c, t, T)$, *i.e.* there is an additional up-type state, the mixing matrix is not anymore a 3×3 unitary matrix, it is a 4×3 matrix. This enlarged mixing matrix V can be embedded in a 4×4 unitary matrix U [3]. Non unitarity of V manifests in the couplings to the Z,

$$\mathscr{L}_Z = -\frac{g}{2\cos\theta_W} \left[\bar{\mathbf{u}}_L \, \gamma^\mu \, (VV^\dagger) \, \mathbf{u}_L - \bar{\mathbf{d}}_L \, \gamma^\mu \, \mathbf{d}_L - 2\sin^2\theta_W \, J^\mu_{em} \right] Z_\mu \; , \tag{22}$$

The model naturally includes flavour changing neutral couplings controlled by the deviations from 3×3 unitarity of the mixing matrix:

$$U = \left(\begin{array}{ccc|c} V_{ud} & V_{us} & V_{ub} & U_{u4} \\ V_{cd} & V_{cs} & V_{cb} & U_{c4} \\ V_{td} & V_{ts} & V_{tb} & U_{t4} \\ V_{Td} & V_{Ts} & V_{Tb} & U_{T4} \end{array}\right) , \quad U\,U^\dagger = U^\dagger\,U = \mathbf{1} \, , \qquad V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{pmatrix} , \quad (VV^\dagger)_{ij} = \delta_{ij} - U_{i4} U^*_{j4} \, . \tag{23}$$

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[3] This is exactly analogous to the rectangular structure of neutrino mixing within seesaw schemes [17].

Equations (21) and (22) are the cornerstones to study the phenomenological consequences of the model. According to them, from a departure from the standard 3×3 unitary mixing, we could expect

- modified effective vertices that involve virtual up-type quarks (in our scheme the T quark also runs in the loops),
- tree-level flavour-changing $Z\bar{u}_i u_j$ vertex (analogous to the non-diagonal $Z\nu\nu$ vertex in seesaw models [17]).

These would mainly affect

- oscillations in neutral meson systems such as kaons, B_d, B_s or D^0 mesons, yielding potential modifications of mass and width differences, and CP-violating asymmetries.
- Decays, in particular *rare* decays which are typically loop induced in the SM (through penguins or boxes, like the oscillations in the previous item). To mention a few, $B \to X_s\gamma$ (new contributions with virtual T quarks in the loop), $B_s \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$ (new contributions with virtual T quarks and Z flavour changing tree level couplings) or $t \to cZ$ (Z flavour changing tree level couplings).

Observables that are, essentially, tree-level induced, could be modified in this framework, since "enlarging" the mixing matrix necessarily implies modifications of its entries; the modifications are, nevertheless, typically small.

In addition to the observables associated to the phenomenology of mesons, electroweak precision data is also sensitive to the modifications that the additional up vector-like quark produce. We will consider the oblique parameters S and T (since the U parameter is typically of little importance).

For a more exhaustive description of the observables we refer the reader to [18, 19] and references therein for details and further technicalities associated to the numerical aspects of the exploration of the physics reach of the model [4]. In the following we will simply collect the most relevant observables . Tables II, III and IV display together the ranges (1) allowed within the model, (2) allowed within the SM and (3) experimentally determined (where available or appropriate), for a selected set of observables.

A. CKM from deviation of unitarity

Table II illustrates two important aspects [5]: (1) the mixing element 13, $|V_{ub}|$, is in agreement with the experimental constraint, (2) the mixing matrix departs from the 3×3 unitary case. It is important to underline that the model can produce an adequate value for $|V_{ub}|$: the difficulty raised in [9], where no T quark was included, is therefore cured. In addition, it might be larger than in the SM, and this is central if one is interested in providing some relief to the "tensions" that, over the last few years, have arised among the measurements of $|V_{ub}|$, the branching ratio $\mathrm{Br}(B^+ \to \tau^+\nu)$ (which is also displayed to further illustrate the issue) and the time-dependent CP asymmetry in B_d^0–$\bar{B}_d^0$ decays to $J/\Psi\, K_S$, $A_{J/\Psi K_S}$. Concerning the deviations from 3×3 unitarity that the model can accommodate, the physical (rephasing invariant) phases β and β_s [20], together with the mixing element $|V_{tb}|$, are displayed to illustrate the possibility of having significant departures from their SM expectations. This is particularly evident for β_s, which is important to describe the mixing in the $B_s^0 - \bar{B}_s^0$ system.

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[4] A systematic study of the parameter space of the model is conducted using Markov Chain MonteCarlo techniques. Notice in addition that one important difference which deserves attention: the analyses in [18, 19] address models with up vector like quarks generically, directly in terms of the parameters that describe the mixing matrix, without an underlying flavour symmetry as we have here.

[5] In Tables II to IV, an asterisk $*$ instead of the corresponding experimental measurements denotes that the quantity is either not directly measurable, or no relevant measurement exists yet. Physical bounds like $|V_{tb}| \leq 1$ (by definition) are displayed for reference with a thin vertical line.

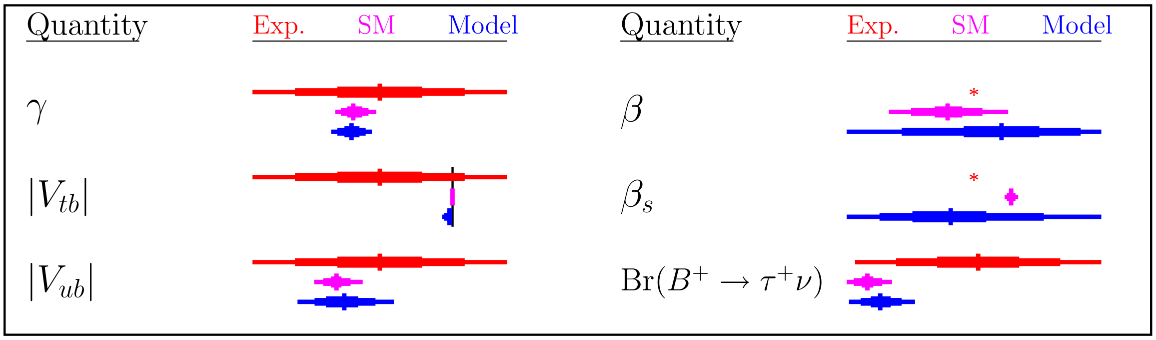

TABLE II: Mixing elements and phases.

B. Neutral meson observables

Table III displays several observables associated to neutral meson systems. The model agrees with the experimental constraints. A close look to the time-dependent CP violating asymmetry in $B_s^0 \to J/\Psi\Phi$, $A_{J/\Psi\Phi}$, shows that the allowed range in the model is much larger than the SM one: for this asymmetry the model could accommodate values incompatible with the SM; best of all, LHCb, which currently dominates the determination of this asymmetry, may attain precisions sufficient to distinguish such an hypothetical case.

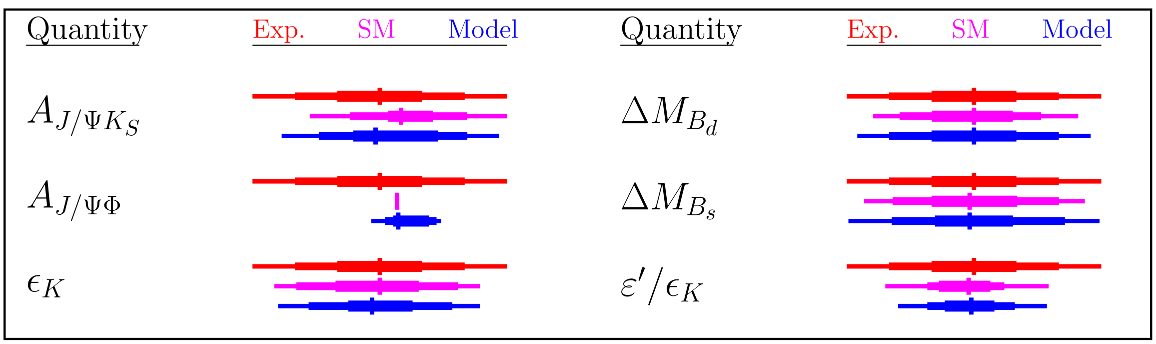

TABLE III: Observables related to the K^0–$\bar{K}^0$, B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$ systems.

C. Rare Decays

While Table III deals with oscillation observables, *i.e.* $\Delta F = 2$ processes, Table IV presents results for some $\Delta F = 1$ decays that are absent at tree level in the SM. Accordance with the experimental constraints is again complete, while the possibility to accommodate deviations from the SM expectations is clearly open in several decays where experimental progress is starting to shed light on SM territory (e.g. $K^+ \to \pi^+\nu\bar{\nu}$, $B_d \to \mu^+\mu^-$).

D. Additional information

The set of observables considered above does not exhaust nor the constraints to be imposed, neither the potentially interesting channels. Electroweak precision data has to been taken into account: to do so, agreement with the oblique parameters ΔT and ΔS is also incorporated. Concerning beyond SM signals, rare decays such as $t \to uZ$ and $t \to cZ$, which have highly suppressed branching ratios in the SM, may be raised to the $\mathcal{O}(10^{-5})$ level through the Z tree level flavour changing couplings, and such rates may be within reach of LHC experiments. Similar comments apply to the mixing of neutral D mesons. Nevertheless, since in that system long distance hadronic interactions are relevant, we do

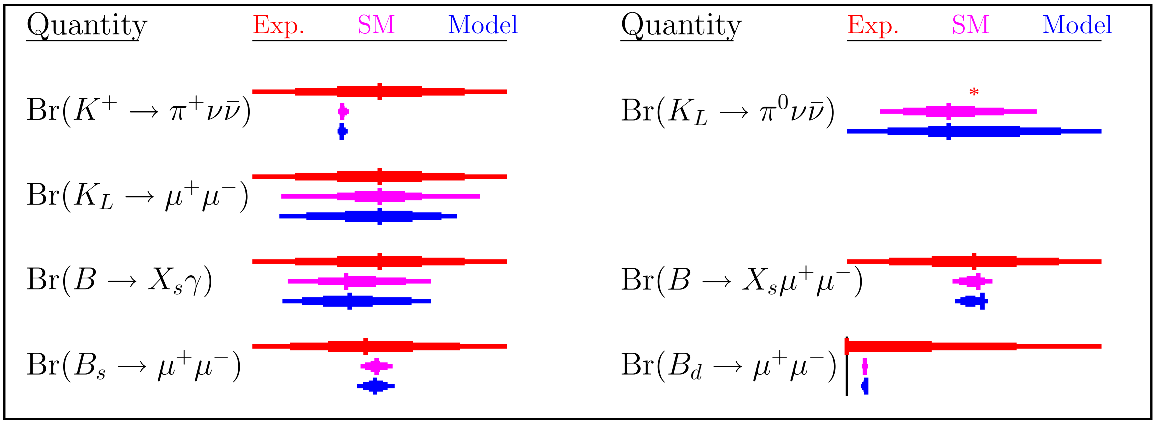

TABLE IV: Radiative decays of K, B_d and B_s mesons.

not elaborate and refer instead to [18, 19]. Although further information on the model expectations can be obtained from additional aspects like, for example, the study of correlations among different observables or the allowed values of the mass of the new quark T, Tables II to IV illustrate sufficiently that this model does indeed agree with the numerous constraints imposed by the phenomenology of the quark sector (and may even accommodate non standard predictions of observables to be explored in detail in the near future, as for example the time dependent CP asymmetry in $B_s^0 \to J/\Psi\Phi$). To further confirm this good agreement, both in the quark and the lepton sector, let us show a specific example.

E. Example

The parameters of our example point are the following[6]:

$$\begin{aligned}
&r^l = r^d = 243.7417\,, && \alpha^l = \alpha^d = 1.009666\,, && r^u = 17226.27\,, && \alpha^u = 1.307231\,,\\
&|a^l| = 4.286928 \cdot 10^{-4}\,, && \arg(a^l) = 0.480176\,, && |b^l| = 0.0072897\,, && \arg(b^l) = 0.018797\,,\\
&|a^u| = 5.626974 \cdot 10^{-5}\,, && \arg(a^u) = -1.486845\,, && |b^u| = 0.009015\,, && \arg(b^u) = 0.003155\,,\\
&|a^d| = 2.229758 \cdot 10^{-4}\,, && \arg(a^d) = -1.248927\,, && |b^d| = 0.012320\,, && \arg(b^d) = 0.035811\,,
\end{aligned} \tag{24}$$

$$\begin{aligned}
&|x| = 4.792066 \cdot 10^{-20}\,, && \arg(x) = -1.697607\,, && |y| = 6.327750 \cdot 10^{-20}\,, && \arg(y) = 1.864509\,,\\
&|z| = 4.708915 \cdot 10^{-20}\,, && \arg(z) = 1.173758\,, && |k| = 1.771198 \cdot 10^{-15}\,, && \arg(k) = 1.455252\,,
\end{aligned} \tag{25}$$

$$\begin{aligned}
&|X_1| = 1.030217\,, && \arg(X_1) = -0.215681\,, && |Y_1| = 0.549803\,, && \arg(Y_1) = 2.173908\,,\\
&|X_2| = 572.111824\,, && \arg(X_2) = -0.756832\,, && |Y_2| = 9.278986\,, && \arg(Y_2) = 2.252815\,,\\
&|X_3| = 35.42186\,, && \arg(X_3) = 2.799661\,, && |Y_3| = 105.8904\,, && \arg(Y_3) = -2.147215\,,\\
&|M| = 498.0447\,, && \arg(M) = -0.368877\,.
\end{aligned} \tag{26}$$

The corresponding masses and mixings are:

- In the lepton sector:

$$\begin{aligned}
&m_e = 0.5110 \text{ MeV}\,, && m_\mu = 0.1050 \text{ GeV}\,, && m_\tau = 1.7768 \text{ GeV}\,,\\
&m_{\nu_1} = 5.22468 \cdot 10^{-2} \text{ eV}\,, && m_{\nu_2} = 5.29683 \cdot 10^{-2} \text{ eV}\,, && m_{\nu_3} = 1.60681 \cdot 10^{-2} \text{ eV}\,,
\end{aligned} \tag{27}$$

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[6] Dimensionful parameters, that is: $|a^f|$, $|b^f|$ with $f = l, u, d$; $|X_i|$ and $|Y_i|$ with $i = 1, 2, 3$; $|x|$, $|y|$, $|z|$, $|k|$ and M, are given in GeVs.

$$|V_{\rm lep}| = \begin{pmatrix} 0.80835 & 0.56694 & 0.15857 \\ 0.28458 & 0.58005 & 0.7632 \\ 0.51535 & 0.58490 & 0.62634 \end{pmatrix} . \tag{28}$$

One can readily check that the mass differences are

$$\Delta m^2_{21} = 7.59156 \cdot 10^{-5}\ \text{eV}^2 , \qquad \Delta m^2_{13} = 2.47155 \cdot 10^{-3}\ \text{eV}^2 . \tag{29}$$

and the mixing angles

$$\begin{aligned} &\theta_{12} = 0.611636 , && \theta_{23} = 0.883605 , && \theta_{13} = 0.159244 . \\ &\sin^2\theta_{12} = 0.32971 , && \sin^2\theta_{23} = 0.59757 , && \sin^2\theta_{13} = 0.02514 . \end{aligned} \tag{30}$$

- In the quark sector, with m_T the mass of the new up eigenstate,

$$\begin{aligned} &m_d = 0.002686\ \text{GeV} , && m_s = 0.057036\ \text{GeV} , && m_b = 3.00288\ \text{GeV} , \\ &m_u = 0.001282\ \text{GeV} , && m_c = 0.621607\ \text{GeV} , && m_t = 173.1228\ \text{GeV} , \\ &m_T = 762.93\ \text{GeV} . \end{aligned} \tag{31}$$

and

$$|V_{CKM}| = \begin{pmatrix} 0.974171 & 0.225779 & 0.003685 \\ 0.225645 & 0.973318 & 0.040211 \\ 0.008512 & 0.040202 & 0.994269 \\ 0.001634 & 0.007730 & 0.098988 \end{pmatrix} . \tag{32}$$

IV. CONCLUSION

We have considered a supersymmetric model based on A_4 flavor symmetry where leptons as well as quarks belong to triplets representation of A_4. This kind of models predict a very interesting mass relation between charged leptons and down quarks at the level of the standard $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ gauge group, without unification. In this scenario we have studied the possibility of fitting the full structure of quark mixing. We have introduced a pair of vector-like up quarks. Then the up quark mass matrix is a 4×4 matrix instead of 3×3. Then the sub-block of the mixing matrix that diagonalizes the up-quark mass violates unitary. We use such a deviation in order to fit the V_{ub} and V_{cb} entries of the CKM matrix. A complete numerical analysis is performed to establish the validity of the model when relevant experimental constraints, including meson oscillations and rare decays in kaons, B_d and B_s mesons, are considered. In addition, potential deviations from SM expectations are briefly addressed. We have also analyzed the lepton sector and found that model predicts inverted neutrino mass spectrum leading to a potentially observable rate of neutrinoless double beta decay. The degree of predictivity of the model within this sector could still be enhanced within a full-fledged seesaw-type formulation of the model, to be taken up elsewhere.

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Appendix A: Experimental input

We collect the experimental information used in the analysis in Table V.

[1] Particle Data Group, J. Beringer *et al.*, Phys.Rev. **D86**, 010001 (2012).

$\sin^2\theta_{12}$	0.320 ± 0.017	$\sin^2\theta_{13}$	0.0248 ± 0.0029	$\sin^2\theta_{23}$	$(*)$
Δm^2_{21} (eV^2)	$(7.62 \pm 0.19) \cdot 10^{-5}$	$\lvert\Delta m^2_{13}\rvert$ (eV^2)	$(2.50 \pm 0.08) \cdot 10^{-3}$	$m_{0\nu\beta\beta}$ (eV)	< 0.6 eV (90% CL)
$\lvert V_{ud}\rvert$	0.97425 ± 0.00022	$\lvert V_{us}\rvert$	0.2252 ± 0.0009	$\lvert V_{ub}\rvert$	$(4.15 \pm 0.49) \cdot 10^{-3}$
$\lvert V_{cd}\rvert$	0.230 ± 0.011	$\lvert V_{cs}\rvert$	1.023 ± 0.036	$\lvert V_{cb}\rvert$	0.0406 ± 0.0013
γ	$(77 \pm 14)^\circ$	$\sin 2\bar{\alpha}$	0.00 ± 0.15	$\mathrm{Br}(B \to \tau\nu)$	$(11.3 \pm 2.3) \cdot 10^{-5}$
ΔM_{B_d} (ps^{-1})	(0.508 ± 0.004)	$A_{J/\Psi K_S}$	0.68 ± 0.02	$\Delta\Gamma_d/\Gamma_d$	-0.017 ± 0.021
ΔM_{B_s} (ps^{-1})	(17.725 ± 0.049)	$A_{J/\Psi\Phi}$	0.002 ± 0.087	$\Delta\Gamma_s$ (ps^{-1})	(0.116 ± 0.019)
A^d_{SL}	-0.0030 ± 0.0078	A^s_{SL}	-0.0024 ± 0.0063	A^b_{SL}	-0.00787 ± 0.00196
ϵ_K	$(2.228 \pm 0.011) \cdot 10^{-3}$	ϵ'/ϵ_K	$(1.67 \pm 0.16) 10^{-3}$		
$\mathrm{Br}(B \to X_s\gamma)$	$(3.56 \pm 0.25) \cdot 10^{-4}$	$\mathrm{Br}(B \to X_s\mu^+\mu^-)$	$(1.60 \pm 0.51) \cdot 10^{-6}$	$\mathrm{Br}(B_s \to \mu^+\mu^-)$	$(3.2\,^{+1.5}_{-1.2}) \cdot 10^{-9}$
$\mathrm{Br}(K_L \to \mu^+\mu^-)$	$(6.84 \pm 0.11) \cdot 10^{-9}$	$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$	$(1.73\,^{+1.15}_{-1.05}) \cdot 10^{-10}$	$\mathrm{Br}(B_d \to \mu^+\mu^-)$	$< 9.4 \cdot 10^{-10}$ (95% CL)
ΔT	0.05 ± 0.12	ΔS	0.02 ± 0.11	x_D	< 0.012 (95% CL)

TABLE V: Summary of experimental input [1, 21]. Gaussian profiles are typically used to model the uncertainties. $(*)$ For $\sin^2\theta_{23}$ we use a "double well" $\Delta\chi^2$ profile, around 0.515, to model the constraint taken from [2].

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