

# On the definition and observability of the neutrino charge radius.

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We present a brief summary of recent results concerning the unambiguous definition and experimental extraction of the gauge-invariant and process-independent neutrino charge radius.

## 1. Introduction

The neutrino electromagnetic form-factor and the neutrino charge radius (NCR) have constituted an important theoretical puzzle for over than three decades. Since the dawn of the Standard Model (SM) it had been pointed out that one-loop radiative corrections to the scattering amplitude of a charged particle (such as the electron) with a neutrino would generate an effective photon-neutrino vertex  $\Gamma_{A\nu\bar{\nu}}^\mu$ , which, in turn, would give rise to a non-zero NCR [1]. Traditionally (and, of course, rather heuristically) the NCR has been interpreted as a measure of the “size” of the neutrino  $\nu_i$  when probed electromagnetically, owing to its classical definition (in the static limit) as the second moment of the spatial neutrino charge density  $\rho_\nu(\mathbf{r})$ , i.e.  $\langle r_\nu^2 \rangle = e^{-1} \int d\mathbf{r} r^2 \rho_\nu(\mathbf{r})$ . Even though in the SM the one-loop computation of the *entire*  $S$ -matrix element describing the electron-neutrino scattering is conceptually straightforward, the identification of a *sub-amplitude*, which would serve as the effective  $\Gamma_{A\nu\bar{\nu}}^\mu$  has been faced with serious complications, associated with the simultaneous reconciliation of crucial requirements such as gauge-invariance, finiteness, and target-independence (for various references see [2]). The crux of the problem is that since in non-Abelian gauge theories individual off-shell Green’s functions are in general unphysical, the definition of quantities familiar from scalar theories or QED, such as effective charges and form-factors, is in general problematic. In the case of the NCR, various attempts to define its SM value through the one-

loop  $\gamma\nu\nu$  vertex calculated in the renormalizable ( $R_\xi$ ) gauges reveal that the corresponding electromagnetic form-factor depends explicitly on the gauge-fixing parameter  $\xi$  in a prohibiting way. In particular, even though in the static limit of zero momentum transfer,  $q^2 \rightarrow 0$ , the form-factor becomes independent of  $\xi$ , its first derivative with respect to  $q^2$ , which corresponds to the definition of the NCR, continues to depend on it. Similar (and some times worse) problems occur in the context of other gauges (e.g. unitary gauge). These complications have obscured the entire concept of an NCR, and have casted serious doubts on whether it can be regarded as a genuine physical observable.

In the last two years significant progress has been accomplished in our understanding of the NCR. To begin with, the field-theoretical difficulties mentioned above have been conclusively settled in [2], by resorting to the well-defined electroweak gauge-invariant separation of physical amplitudes into effective self-energy, vertex and box sub-amplitudes, implemented by the pinch technique formalism [3]. In addition, in a very recent work [5], the observable nature of the NCR was established. In particular it was shown that the probe-independent charge radius of the neutrino may be extracted from experiment, at least in principle. This was accomplished by expressing a set of experimental  $\nu_\mu - e$  cross-sections in terms of the NCR and two additional gauge- and renormalization-group-invariant quantities, corresponding to the electroweak effective charge and mixing angle. In what follows we will present a brief summary of these new developments.

## 2. On the PT definition of the NCR

The PT is a diagrammatic method which exploits the underlying symmetries encoded in a *physical* amplitude such as an  $S$ -matrix element, in order to construct effective Green's functions with special properties. The aforementioned symmetries, even though they are always present, they are usually concealed by the gauge-fixing procedure. The PT makes them manifest by means of a fixed algorithm, which does *not* depend on the gauge-fixing scheme one uses in order to quantize the theory, *i.e.* regardless of the set of Feynman rules used when writing down the  $S$ -matrix element. In particular, the PT uses the elementary Ward identities triggered by the longitudinal momenta appearing inside Feynman diagrams in order to enforce massive cancellations. The realization of these cancellations mixes non-trivially contributions stemming from diagrams of different kinematic nature (propagators, vertices, boxes). Thus, a given physical amplitude is reorganized into sub-amplitudes, which have the same kinematic properties as conventional  $n$ -point functions and, in addition, are endowed with desirable physical properties. Most importantly, they are independent of the gauge-fixing parameter, and satisfy naive (ghost-free, QED-like) Ward identities instead of the usual Slavnov-Taylor identities.

For the specific problem of the NCR, by resorting to the PT one can construct a genuine, target-independent form-factor endowed with the crucial properties of gauge-independence, gauge-invariance and finiteness, for arbitrary values of the momentum transfer. The essential ingredient for accomplishing this was the realization that, when the target fermions involved are considered to be massless, all gauge-dependent contributions stemming from box and vertex-like Feynman diagrams are effectively propagator-like, *i.e.* have no dependence on the kinematic properties or quantum numbers of the initial and final states, as boxes and vertices normally do. Thus, all gauge dependence stemming from vertices and boxes cancels precisely against the gauge-dependence stemming from the conventional one-loop self-energies. As a result, the remaining gauge-

independent structures retain their initial kinematic identity; in particular one can speak in terms of gauge-independent effective boxes, vertices, and self-energies. Thus, once the gauge-dependent pieces have been extracted from the box, the remaining gauge-independent “pure” box should not be considered as a part of the resulting form-factor, which should be entirely determined from the “pure” gauge-independent set of vertex graphs.

The aforementioned propagator-like pieces are extracted by tracking down the action of the longitudinal momenta appearing in the  $S$ -matrix element of  $F\bar{F} \rightarrow \nu\bar{\nu}$ . Longitudinal momenta originate from the tree-level gauge-boson propagators and tri-linear gauge-boson vertices appearing inside loops. In particular, in the  $R_\xi$ -scheme the gauge-boson propagators have the general form

$$\Delta_V^{\mu\nu}(k) = \left[ g^{\mu\nu} - \frac{(1 - \xi_V)k^\mu k^\nu}{k^2 - \xi_V M_V^2} \right] (k^2 - M_V^2)^{-1} \quad (1)$$

where  $V = W, N$  with  $N = Z, \gamma$  and  $M_\gamma^2 = 0$ ;  $k$  denotes the virtual four-momentum circulating in the loop. Clearly, in the case of  $\Delta_V^{\mu\nu}(k)$  the longitudinal momenta are those proportional to  $(1 - \xi_V)$ . The longitudinal terms arising from the tri-linear vertex may be identified by splitting  $\Gamma_{\alpha\mu\nu}(q, k, -k - q)$  appearing inside the one-loop diagrams into two parts ( $q$  denotes the physical four-momentum entering into the vertex):

$$\Gamma_{\alpha\mu\nu} = \Gamma_{\alpha\mu\nu}^F + \Gamma_{\alpha\mu\nu}^P, \quad (2)$$

with

$$\begin{aligned} \Gamma_{\alpha\mu\nu}^F &= (2k + q)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu}, \\ \Gamma_{\alpha\mu\nu}^P &= -(k + q)_\nu g_{\alpha\mu} - k_\mu g_{\alpha\nu}. \end{aligned} \quad (3)$$

The above decomposition assigns a special role to the  $q$ -leg, and allows  $\Gamma_{\alpha\mu\nu}^F$  to satisfy the Ward identity

$$q^\alpha \Gamma_{\alpha\mu\nu}^F = (k + q)^2 g_{\mu\nu} - k^2 g_{\mu\nu}, \quad (4)$$

All aforementioned longitudinal momenta originating from  $\Delta_V^{\mu\nu}(k)$  and  $\Gamma_{\alpha\mu\nu}^P$  trigger the following WI when contracted with the appropriate  $\gamma$  matrix appearing in the various elementary vertices. In the absence of fermion masses

$$\begin{aligned} \not{k} P_L &= (\not{k} + \not{p}) P_L - P_R \not{p}, \\ &= S_{F'}^{-1}(\not{k} + \not{p}) P_L - P_R S_F^{-1}(\not{p}) \end{aligned} \quad (5)$$

where  $P_{R(L)} = [1 + (-)\gamma_5]/2$  is the chirality projection operator and  $S_F$  is the tree-level propagator of the fermion  $F$ ;  $F'$  is the isodoublet-partner of the external fermion  $F$ . The result of this contraction is that the term in Eq.(5) proportional to  $S_{F'}^{-1}$ , i.e the inverse of the internal fermion propagator gives rise to a self-energy-like term, whose coupling to the external fermions is proportional to the vertex

$$\Gamma_{WF\bar{F}}^\mu = -i \left( \frac{g_w}{2} \right) \gamma_\mu P_L. \quad (6)$$

The effective vertex  $\Gamma_{WF\bar{F}}^\mu$  can be written as a linear combination of the two tree-level vertices  $\Gamma_{\gamma F\bar{F}}^\mu$  and  $\Gamma_{ZF\bar{F}}^\mu$  as follows:

$$\Gamma_{WF\bar{F}}^\mu = \left( \frac{s_w}{2T_z^F} \right) \Gamma_{\gamma F\bar{F}}^\mu - \left( \frac{c_w}{2T_z^F} \right) \Gamma_{ZF\bar{F}}^\mu \quad (7)$$

In the above formulas  $Q_F$  is the electric charge of the fermion  $F$ , and  $T_z^F$  its  $z$ -component of the weak isospin, with  $c_w = \sqrt{1 - s_w^2} = M_W/M_Z$ ,  $e = g_w s_w$ . The identity of Eq.(7) allows to combine the propagator-like parts emerging from box-diagrams and vertex-diagrams after the application of the WI in Eq.(5) with the conventional self-energy graphs  $\Pi_{\mu\nu}^{\gamma Z}$  and  $\Pi_{\mu\nu}^{ZZ}$ , by judiciously multiplying the former by  $D_N(q)D_N^{-1}(q)$ .

Finally, from the gauge-independent one-loop *proper* vertex  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$  constructed using this method one extracts the dimension-full electromagnetic form-factor  $\widehat{F}_{\nu_i}(q^2)$  as  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu = ieq^2 \widehat{F}_{\nu_i}(q^2) \gamma_\mu (1 - \gamma_5)$ . The NCR, to be denoted by  $\langle r_{\nu_i}^2 \rangle$ , is then defined as  $\langle r_{\nu_i}^2 \rangle = 6\widehat{F}_{\nu_i}(0)$ , and thus one obtains

$$\langle r_{\nu_i}^2 \rangle = \frac{G_F}{4\sqrt{2}\pi^2} \left[ 3 - 2 \log \left( \frac{m_i^2}{M_W^2} \right) \right], \quad (8)$$

where  $i = e, \mu, \tau$ ,  $m_i$  denotes the mass of the charged iso-doublet partner of the neutrino under consideration, and  $G_F$  is the Fermi constant.

### 3. Measuring the probe-independent NCR

After arriving at a physically meaningful definition for the NCR, the next crucial question is whether the NCR so defined constitutes a genuine physical observable. In the rest of this section we

will briefly discuss the method proposed in [5] for the extraction of the NCR from experiment.

It is important to emphasize that measuring the entire process  $f^\pm \nu \rightarrow f^\pm \nu$  does *not* constitute a measurement of the NCR, because by changing the target fermions  $f^\pm$  one will generally change the answer, thus introducing a target-dependence into a quantity which (supposedly) constitutes an intrinsic property of the neutrino. Instead, what we want to measure is the target-independent Standard Model NCR only, stripped of any target dependent contributions. Specifically, as mentioned above, the PT rearrangement of the  $S$ -matrix makes possible the definition of distinct, physically meaningful sub-amplitudes, one of which,  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$ , is finite and directly related to the NCR. However, the remaining sub-amplitudes, such as self-energy, vertex- and box-corrections, even though they do not enter into the definition of the NCR, still contribute numerically to the entire  $S$ -matrix; in fact, some of them combine to form additional physical observables of interest, most notably the effective (running) electroweak charge and mixing angle. Thus, in order to isolate the NCR, one must conceive of a combination of experiments and kinematical conditions, such that all contributions not related to the NCR will be eliminated.

Consider the elastic processes  $f(k_1)\nu(p_1) \rightarrow f(k_2)\nu(p_2)$  and  $f(k_1)\bar{\nu}(p_1) \rightarrow f(k_2)\bar{\nu}(p_2)$ , where  $f$  denotes an electrically charged fermion belonging to a different iso-doublet than the neutrino  $\nu$ , in order to eliminate the diagrams mediated by a charged  $W$ -boson. The Mandelstam variables are defined as  $s = (k_1 + p_1)^2 = (k_2 + p_2)^2$ ,  $t = q^2 = (p_1 - p_2)^2 = (k_1 - k_2)^2$ ,  $u = (k_1 - p_2)^2 = (k_2 - p_1)^2$ , and  $s + t + u = 0$ . In what follows we will restrict ourselves to the limit  $t = q^2 \rightarrow 0$  of the above amplitudes, assuming that all external (on-shell) fermions are massless. As a result of this special kinematic situation we have the following relations:  $p_1^2 = p_2^2 = k_1^2 = k_2^2 = p_1 \cdot p_2 = k_1 \cdot k_2 = 0$  and  $p_1 \cdot k_1 = p_1 \cdot k_2 = p_2 \cdot k_1 = p_2 \cdot k_2 = s/2$ . In the center-of-mass system we have that  $t = -2E_\nu E'_\nu (1 - x) \leq 0$ , where  $E_\nu$  and  $E'_\nu$  are the energies of the neutrino before and after the scattering, respectively, and  $x \equiv \cos \theta_{cm}$ , where  $\theta_{cm}$  is the scattering angle. Clearly, the condition

$t = 0$  corresponds to the exactly forward amplitude, with  $\theta_{cm} = 0$ ,  $x = 1$ .

At tree-level the amplitude  $f\nu \rightarrow f\nu$  is mediated by an off-shell  $Z$ -boson, coupled to the fermions by means of the bare vertex  $\Gamma_{Zf\bar{f}}^\mu = -i(g_w/c_w)\gamma^\mu [v_f + a_f\gamma_5]$  with  $v_f = s_w^2 Q_f - \frac{1}{2}T_z^f$  and  $a_f = \frac{1}{2}T_z^f$ .

At one-loop, the relevant contributions are determined through the PT rearrangement of the amplitude, giving rise to gauge-independent sub-amplitudes. In particular, the one-loop  $AZ$  self-energy  $\widehat{\Sigma}_{AZ}^{\mu\nu}(q^2)$  obtained is transverse, for both the fermionic and the bosonic contributions, i.e.  $\widehat{\Sigma}_{AZ}^{\mu\nu}(q^2) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\widehat{\Pi}_{AZ}(q^2)$ . Since the external currents are conserved, from the  $ZZ$  self-energy  $\widehat{\Sigma}_{ZZ}^{\mu\nu}(q^2)$  we keep only the part proportional to  $g^{\mu\nu}$ , whose dimension-full cofactor will be denoted by  $\widehat{\Sigma}_{ZZ}(q^2)$ . Furthermore, as is well-known, the one-loop vertex  $\widehat{\Gamma}_{ZF\bar{F}}^\mu(q, p_1, p_2)$ , with  $F = f$  or  $F = \nu$ , satisfies a QED-like Ward identity, relating it to the one-loop inverse fermion propagators  $\widehat{\Sigma}_F$ , i.e.  $q_\mu \widehat{\Gamma}_{ZF\bar{F}}^\mu(q, p_1, p_2) = \widehat{\Sigma}_F(p_1) - \widehat{\Sigma}_F(p_2)$ . It is then easy to show that, in the limit of  $q^2 \rightarrow 0$ ,  $\widehat{\Gamma}_{ZF\bar{F}}^\mu \sim q^2 \gamma^\mu (c_1 + c_2 \gamma_5)$ ; since it is multiplied by a massive  $Z$  boson propagator  $(q^2 - M_Z)^{-1}$ , its contribution to the amplitude vanishes when  $q^2 \rightarrow 0$ . This is to be contrasted with the  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$ , which is accompanied by a  $(1/q^2)$  photon-propagator, thus giving rise to a contact interaction between the target-fermion and the neutrino, described by the NCR.

We next proceed to eliminate the target-dependent box-contributions; to accomplish this we resort to the ‘‘neutrino–anti-neutrino’’ method. The basic observation is that the tree-level amplitudes  $\mathcal{M}_{\nu f}^{(0)}$  as well as the part of the one-loop amplitude  $\mathcal{M}_{\nu f}^{(B)}$  consisting of the propagator and vertex corrections (the ‘‘Born-improved’’ amplitude), are proportional to  $[\bar{u}_f(k_2)\gamma_\mu(v_f + a_f\gamma_5)u_f(k_1)][\bar{v}(p_1)\gamma_\mu P_L v(p_2)]$ , and therefore transform differently than the boxes under the replacement  $\nu \rightarrow \bar{\nu}$ . In particular, the coupling of the  $Z$  boson to a pair of on-shell anti-neutrinos may be written in terms of on-shell neutrinos provided that one changes the chirality projector from  $P_L$  to  $P_R$ , and supplies a relative

minus sign [4], i.e.

$$\bar{v}(p_1)\gamma_\mu P_L v(p_2) = -\bar{u}(p_2)\gamma_\mu P_R u(p_1) \quad (9)$$

Thus, under the above transformation,  $\mathcal{M}_{\nu f}^{(0)}$  +  $\mathcal{M}_{\nu f}^{(B)}$  reverse sign once, whereas the box contributions reverse sign twice. These distinct transformation properties allow for the isolation of the box contributions when judicious combinations of the forward differential cross-sections  $(d\sigma_{\nu f}/dx)_{x=1}$  and  $(d\sigma_{\bar{\nu} f}/dx)_{x=1}$  are formed. In particular,  $\sigma_{\nu f}^{(+)} \equiv (d\sigma_{\nu f}/dx)_{x=1} + (d\sigma_{\bar{\nu} f}/dx)_{x=1}$  does not contain boxes, whereas the conjugate combination  $\sigma_{\nu f}^{(-)} \equiv (d\sigma_{\nu f}/dx)_{x=1} - (d\sigma_{\bar{\nu} f}/dx)_{x=1}$  isolates the contribution of the boxes.

Finally, a detailed analysis shows that in the kinematic limit we consider, the Bremsstrahlung contribution vanishes, due to a completely destructive interference between the two relevant diagrams corresponding to the processes  $fA\nu(\bar{\nu}) \rightarrow f\nu(\bar{\nu})$  and  $f\nu(\bar{\nu}) \rightarrow fA\nu(\bar{\nu})$ . The absence of such corrections is consistent with the fact that there are no infrared divergent contributions from the (vanishing) vertex  $\widehat{\Gamma}_{ZF\bar{F}}^\mu$ , to be cancelled against.

$\sigma_{\nu f}^{(+)}$  receives contributions from the tree-level exchange of a  $Z$ -boson, the one-loop contributions from the ultraviolet divergent quantities  $\widehat{\Sigma}_{ZZ}(0)$  and  $\widehat{\Pi}^{AZ}(0)$ , and the (finite) NCR, coming from the proper vertex  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$ . The first three contributions are universal, i.e. common to all neutrino species, whereas that of the proper vertex  $\widehat{\Gamma}_{A\nu_i\bar{\nu}_i}^\mu$  is flavor-dependent.

To proceed, the renormalization of  $\widehat{\Sigma}_{ZZ}(0)$  and  $\widehat{\Pi}^{AZ}(0)$  must be carried out. It turns out that, by virtue of the Abelian-like Ward-identities enforced after the pinch technique rearrangement [3], the resulting expressions combine in such a way as to form manifestly renormalization-group invariant combinations [6]. In particular, after carrying out the standard re-diagonalization, two such quantities may be constructed:

$$\begin{aligned} \bar{R}_Z(q^2) &= \frac{\alpha_w}{c_w^2} \left[ q^2 - M_Z^2 + \Re e \{ \widehat{\Sigma}_{ZZ}(q^2) \} \right]^{-1} \\ \bar{s}_w^2(q^2) &= s_w^2 \left( 1 - \frac{c_w}{s_w} \Re e \{ \widehat{\Pi}^{AZ}(q^2) \} \right). \quad (10) \end{aligned}$$

where  $\alpha_w = g_w^2/4\pi$ , and  $\Re\{\dots\}$  denotes the real part. In addition to being renormalization-group invariant, both quantities defined in Eq.(10) are universal (process-independent);  $\bar{R}_Z(q^2)$  corresponds to the  $Z$ -boson effective charge, while  $\bar{s}_w^2(q^2)$  corresponds to an effective mixing angle. We emphasize that the renormalized  $\hat{\Pi}_{AZ}(0)$  *cannot* form part of the NCR, because it fails to form a renormalization-group invariant quantity on its own. Instead,  $\hat{\Pi}_{AZ}(0)$  must be combined with the appropriate tree-level contribution (which evidently does not enter into the definition of the NCR, since it is  $Z$ -mediated) in order to form the effective  $\bar{s}_w^2(q^2)$  acting on the electron vertex, whereas the finite NCR will be determined from the proper neutrino vertex only.

After recasting  $\sigma_{\nu f}^{(+)}$  in terms of manifestly renormalization-group invariant building blocks, one may fix  $\nu = \nu_\mu$ , and then consider three different choices for  $f$ : (i) right-handed electrons,  $e_R$ ; (ii) left-handed electrons,  $e_L$ , and (iii) neutrinos,  $\nu_i$  other than  $\nu_\mu$ , i.e.  $i = e, \tau$ . Thus, we arrive at the system

$$\begin{aligned}\sigma_{\nu_\mu \nu_i}^{(+)} &= s\pi\bar{R}^2(0) \\ \sigma_{\nu_\mu e_R}^{(+)} &= s\pi\bar{R}^2(0)\bar{s}_w^4(0) - 2\lambda s_w^2 \langle r_{\nu_\mu}^2 \rangle \\ \sigma_{\nu_\mu e_L}^{(+)} &= s\pi\bar{R}^2(0) \left( \frac{1}{2} - \bar{s}_w^2(0) \right)^2 \\ &\quad + \lambda(1 - 2s_w^2) \langle r_{\nu_\mu}^2 \rangle\end{aligned}\quad (11)$$

where  $\lambda \equiv (2\sqrt{2}/3)s\alpha G_F$ ,  $\alpha = e^2/4\pi$ .  $\bar{R}^2(0)$ ,  $\bar{s}_w^2(0)$ , and  $\langle r_{\nu_\mu}^2 \rangle$  are treated as three unknown quantities, to be determined from the above equations. The corresponding solutions are given by  $\bar{s}_w^2(0) = s_w^2 \pm \Omega^{1/2}$  and

$$\begin{aligned}\langle r_{\nu_\mu}^2 \rangle &= \lambda^{-1} \left[ \left( s_w^2 - \frac{1}{4} \pm \Omega^{1/2} \right) \sigma_{\nu_\mu \nu_i}^{(+)} \right. \\ &\quad \left. + \sigma_{\nu_\mu e_L}^{(+)} - \sigma_{\nu_\mu e_R}^{(+)} \right]\end{aligned}\quad (12)$$

where the discriminant  $\Omega$  is given by

$$\Omega = (1 - 2s_w^2) \left( \frac{\sigma_{\nu_\mu e_R}^{(+)}}{\sigma_{\nu_\mu \nu_i}^{(+)}} - \frac{1}{2}s_w^2 \right) + 2s_w^2 \frac{\sigma_{\nu_\mu e_L}^{(+)}}{\sigma_{\nu_\mu \nu_i}^{(+)}} \quad (13)$$

and must satisfy  $\Omega > 0$ . The actual sign in front of  $\Omega$  may be chosen by requiring that it correctly

accounts for the sign of the shift of  $\bar{s}_w^2(0)$  with respect to  $s_w^2$  predicted by the theory [6]. To extract the experimental values of the quantities  $\bar{R}^2(0)$ ,  $\bar{s}_w^2(0)$ , and  $\langle r_{\nu_\mu}^2 \rangle$ , one must substitute in Eq.(12) and Eq.(13) the experimentally measured values for the differential cross-sections  $\sigma_{\nu_\mu e_R}^{(+)}$ ,  $\sigma_{\nu_\mu e_L}^{(+)}$ , and  $\sigma_{\nu_\mu \nu_i}^{(+)}$ . This means that to solve the system one would have to carry out three different pairs of experiments.

In summary, we have seen that the neutrino charge radius can be defined unambiguously by means of the pinch technique, and that its observability has been established.

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