Model independent bounds on the tau lepton electromagnetic and weak magnetic moments

Gabriel A. González-Sprinberg [a], Arcadi Santamaria [b] and Jordi Vidal [b]

[a]*Instituto de Física, Facultad de Ciencias, Universidad de la República, Iguá 4225, 11400 Montevideo, Uruguay*

[b]*Departament de Física Teòrica, IFIC, CSIC-Universitat de València, E-46100 Burjassot, València, Spain*

Abstract

Using LEP1, SLD and LEP2 data, for tau lepton production, and data from $p\bar{p}$ colliders and LEP2, for W decays into tau leptons, we set model independent limits on non-standard electromagnetic and weak magnetic moments of the tau lepton. The most general effective Lagrangian giving rise to tau moments is used without further assumptions. Precise bounds (2σ) on the non-standard model contributions to tau electromagnetic $(-0.007 < a_\gamma < 0.005)$, tau Z-magnetic $(-0.0024 < a_Z < 0.0025)$ and tau W-magnetic $(-0.003 < \kappa^W < 0.004)$ dipole moments are set from the analysis.

1 Introduction

The present best bound on the tau lepton magnetic moment (a_γ) is indirect [1,2]. It comes from the observation that in general extensions of the standard model (SM) it is very difficult to generate a magnetic moment for a lepton without originating a coupling of the Z boson to the lepton of the same order of magnitude. This anomalous Z coupling (a_Z) is strongly bounded by LEP1, therefore, by assuming that the magnetic moment of the lepton (a_γ) has the same size, one obtains a rather strong bound on it.

While this argument is plausible, the complete amount of data coming from tau-lepton production at LEP1, SLD and LEP2, and data on W decays into tau leptons from LEP2 and $p\bar{p}$ colliders, allows for a more complete analysis of the magnetic moment couplings of the tau to the photon, the Z and the W bosons.

Following Ref. [1,3], in order to analyze tau magnetic moments, we will use an effective Lagrangian description[4]. Thus, in section 2 we describe the effective Lagrangian we use and fix the notation. LEP1 and SLD are sensitive only to the Z-magnetic moments, however LEP2 is sensitive to both Z- and photon- magnetic moments. Far from the resonance statistics decreases dramatically and the precision is not as good as the precision obtained at LEP1. However, since magnetic moment couplings are non-renormalizable couplings, their effects grow with energy and, therefore, LEP2 limits can still be relevant. This is especially true for electromagnetic couplings for which LEP1 does not give much information. That is also the reason why experiments performed at lower energies do not provide, in general, stringent bounds (see for instance the bounds obtained from tau decays [5,6]). To obtain relevant bounds at lower energies the suppression factor, $(E_{\text{low}}/m_Z)^2$, has to be compensated by higher precision in the experiment.

Tau magnetic moment contributions to tau production in LEP1-SLD and LEP2 are studied in section 3. In this respect we can classify the observables in three classes: i) universality test at LEP1, which are studied in 3.2, ii) total production rates at LEP2, considered in 3.3, and iii) transverse polarization asymmetries, studied in 3.4.

Tau magnetic moments flip chirality and in the standard model the only source of chirality flips are fermion masses. This means that any contribution of magnetic moments (weak or electromagnetic) to total rates are either suppressed by the fermion mass (relative to the electroweak scale) or need two operator insertions and then, they come as the square of the magnetic moments. In addition any new physics, not only that related with magnetic moments, will appear in total rates. Hence, in order to study magnetic moments it is interesting to look for observables that are exactly zero when chirality is conserved. Those observables will only be sensitive to fermion masses and to magnetic moments. In addition they will only depend linearly on magnetic moments. Some transverse tau polarization asymmetries [7] are observables of this type and they have been already measured at LEP1 [8] and SLD [9]. We will show in subsection 3.4 that they already give now the best and cleanest bounds on tau magnetic moments. It would be interesting to study these observables also at LEP2 to disentangle weak an electromagnetic magnetic moments.

Gauge invariance, after the spontaneous symmetry breaking, relates the magnetic moments of the Z, the photon, and the W gauge bosons. Therefore one can also gain some insight on tau magnetic moments by studying W decays into tau leptons. There are already rather good bounds on the universality of the leptonic decays of the W coming from LEP2, UA1, UA2, CDF and D0, those are studied in section 4. In section 5 we combine all bounds and obtain the best limits on the different magnetic moment couplings. Finally in section 6 we discuss the obtained results and compare with other bounds found in the

literature.

2 The effective Lagrangian for tau magnetic moments

The standard model gives a very good description of all physics at energies available at present accelerators. Therefore, one expects that any deviation from the standard model, at present energies, can be parametrized by an effective Lagrangian built with the standard model particle spectrum, having as zero order term just the standard model Lagrangian, and containing higher dimension gauge invariant operators suppressed by the scale of new physics Λ. The leading non-standard effects will come from the operators with the lowest dimension. Those are dimension six operators. There are only two operators of this type contributing to the tau magnetic moments:

$$\mathcal{O}_B = \frac{g'}{2\Lambda^2}\overline{L_L}\varphi\sigma_{\mu\nu}\tau_R B^{\mu\nu} \ , \tag{2.1}$$

and

$$\mathcal{O}_W = \frac{g}{2\Lambda^2}\overline{L_L}\vec{\tau}\varphi\sigma_{\mu\nu}\tau_R\vec{W}^{\mu\nu} \ . \tag{2.2}$$

Here $L_L = (\nu_L, \tau_L)$ is the tau leptonic doublet, φ is the Higgs doublet, $B^{\nu\nu}$ and $\vec{W}^{\mu\nu}$ are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ field strength tensors, and g' and g are the gauge couplings.

In principle, one could also write dimension six operators like

$$\frac{g'}{\Lambda^2}\overline{L_L}\sigma_{\mu\nu}\,\slashed{D}L_L B^{\mu\nu} \ , \tag{2.3}$$

with $\slashed{D} = \gamma_\mu D^\mu$. However, these operators reduce to the operator eq. (2.1) after using the standard model equations of motion. In doing so, the couplings will be proportional to the tau-lepton Yukawa couplings. Note that operators in eq. (2.1) and eq. (2.2) break chirality while the operator eq. (2.3) does not break it. Therefore, by using this last form we would implicitly assume that the only source of chirality breaking are Yukawa couplings and that any chirality breaking, including magnetic moments, should be proportional to fermion masses. In order to be more general we will assume the forms in eq. (2.1) and eq. (2.2), having in mind that we are introducing in the standard model an additional source of chirality breaking in addition to fermion masses. As we will see in the next sections this will become very important in looking for the right observables sensitive to magnetic moments.

Thus, we write our effective Lagrangian as,

$$\mathcal{L}_{eff} = \alpha_B \mathcal{O}_B + \alpha_W \mathcal{O}_W + \text{h.c.} \ , \tag{2.4}$$

where, for simplicity, we will take the couplings α_B and α_W real. Note that complex couplings will break CP conservation and lead to electric dipole moments.

If these operators come from a renormalizable theory, in the perturbative regime one expects, in general, that they arise only at one loop and therefore their contribution will be further suppressed. However, this does not need to be the case, therefore we leave the couplings α_B and α_W as free parameters without any further assumption.

After spontaneous symmetry breaking, the Higgs gets a vacuum expectation value $< \varphi^0 >= u/\sqrt{2}$ with $u = 1/\sqrt{\sqrt{2}G_F} = 246$ GeV, and the interactions (2.4) can be written in terms of the gauge boson mass eigenstates, A^μ and Z^μ, using that

$$
\begin{aligned}
B^\mu &= -s_W Z^\mu + c_W A^\mu \ , \\
W_3^\mu &= c_W Z^\mu + s_W A^\mu \ , \\
W^{+\mu} &= \frac{1}{\sqrt{2}} \left(W_1^\mu - i W_2^\mu \right) \ ,
\end{aligned} \tag{2.5}
$$

where, as usual, we define $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $\tan\theta_W = g'/g$ and $e = g s_W$. Thus, our Lagrangian, written in terms of the mass eigenstates, is [1]

$$
\begin{aligned}
\mathcal{L}_{eff} = {} & \epsilon_\gamma \frac{e}{2m_Z} \overline{\tau} \sigma_{\mu\nu} \tau F^{\mu\nu} + \epsilon_Z \frac{e}{2m_Z s_W c_W} \overline{\tau} \sigma_{\mu\nu} \tau Z^{\mu\nu} \\
& + \left(\epsilon_W \frac{e}{2m_Z s_W} \overline{\nu_{\tau L}} \sigma_{\mu\nu} \tau_R W_+^{\mu\nu} + \text{h.c.} \right) \ ,
\end{aligned} \tag{2.6}
$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ and $W_+^{\mu\nu} = \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+$ are the corresponding strength tensors for the Z and W gauge bosons. We have not written the non-abelian couplings involving more than one gauge boson because they are not relevant to our purposes. We have normalized all couplings to m_Z, the natural mass-scale at the energies we will consider, and we have defined the following dimensionless couplings

$$\epsilon_\gamma = (\alpha_B - \alpha_W) \frac{u m_Z}{\sqrt{2}\Lambda^2} \ , \tag{2.7}$$

---

[1] Similar results are found when using a non-linear realization of the gauge symmetry, as can be seen in Ref.[3]

$$\epsilon_Z = -(\alpha_W c_W^2 + \alpha_B s_W^2)\frac{u m_Z}{\sqrt{2}\Lambda^2} \, , \tag{2.8}$$

$$\epsilon_W = \alpha_W \frac{u m_Z}{\Lambda^2} = -\sqrt{2}\left(\epsilon_Z + s_W^2 \epsilon_\gamma\right) \, . \tag{2.9}$$

The Lagrangian (2.6) gives additional contributions to the electromagnetic moment of the tau, which usually is expressed in terms of the parameter a_γ. Similar parameters have been introduced in the literature for the corresponding weak magnetic moments for the Z-boson, a_Z [7] and the W-boson, κ^W [5]. They can be expressed in terms of our ϵ's as follows,

$$a_\gamma = \frac{2\,m_\tau}{m_Z}\,\epsilon_\gamma \, , \tag{2.10}$$

$$a_Z = \frac{2\,m_\tau}{m_Z}\frac{1}{s_W\,c_W}\,\epsilon_Z \, , \tag{2.11}$$

$$\kappa^W = \sqrt{2}\frac{2\,m_\tau}{m_Z}\,\epsilon_W \, . \tag{2.12}$$

Notice that, in the effective Lagrangian approach, exactly the same couplings that contribute to processes at high energies also contribute to the magnetic moment form factors, $F^{\text{new}}(q^2)$, at $q^2 = 0$. The difference $F^{\text{new}}(q^2) - F^{\text{new}}(0)$ only comes from higher dimension operators whose effect is suppressed by powers of q^2/Λ^2, as long as $q^2 \ll \Lambda$ as needed for the consistency of the effective Lagrangian approach.

In (2.6) we have all type of magnetic moment couplings except neutrino-neutrino couplings. This is due to the fact that we have not included right-handed neutrinos, ν_R, in the particle spectrum. If we include them we can add two additional operators that will give magnetic moment couplings of the neutrinos to the Z, and additional contributions to the W^+ magnetic moment couplings. However they will also give rise to an electromagnetic moment for the neutrinos which is extremely well bounded from a variety of sources, energy loss in red giants, supernova cooling, etc., so we are not going to consider them in our calculation.

Since the effect of the operators in (2.6) is suppressed at low energies the most interesting bounds will come from the highest precision experiments at the highest available energies. Presently this means LEP1 and SLD (Z decay rates and polarization asymmetries), LEP2 (cross sections and W decays rates), CDF and D0 (W decay rates). Consequently in the following we will study all those observables.

3 $e^+e^- \to \tau^+\tau^-$ in presence of electromagnetic and weak magnetic moments

In this section we will consider $e^+e^- \to \tau^+\tau^-$ collision in a range of energies from threshold to LEP2, therefore we will include both photon and Z-exchange with standard model vector and axial couplings to fermions, plus additional magnetic moment couplings given by eq. (2.6). The amplitude for the process can be written as:

$$
\mathcal{M} = i\,e^2 \sum_{k=\gamma,Z,\dots} \bar{v}(p_2,s_2)\,\gamma_\mu(v_e^k - a_e^k\gamma_5)\,u(p_1,s_1)\,\mathcal{P}_k
$$

$$
\bar{u}(p_3,s_3)\,[\gamma_\mu(v_\tau^k - a_\tau^k\gamma_5) - g^k\,\frac{i}{2m_\tau}\,\sigma^{\mu\nu}\,q_\nu]\,v(p_4,s_4)\,, \tag{3.1}
$$

where the momenta p_1, p_2, p_3, p_4, correspond to e^-, e^+, τ^-, τ^+ respectively, $q = p_1 + p_2 = p_3 + p_4$, and $\mathcal{P}_k$ are the propagators of the different gauge bosons that contribute to the process:

$$
\mathcal{P}_\gamma = \frac{1}{q^2} \qquad ; \qquad \mathcal{P}_Z = \frac{1}{(2s_W c_W)^2}\,\frac{1}{q^2 - m_Z^2 + i\,m_Z\Gamma_Z}\,. \tag{3.2}
$$

After squaring, summing and averaging over initial polarizations of electrons we obtain

$$
\overline{\sum}|\mathcal{M}|^2 = -\frac{1}{2}[\mathrm{Re}\,\{\mathcal{V}^\mu\mathcal{V}^{*\nu} + \mathcal{A}^\mu\mathcal{A}^{*\nu}\}\,(q^2 g_{\mu\nu} - q_\mu q_\nu + p_{i\mu}p_{i\nu}) +
$$

$$
\mathrm{Im}\,\{\mathcal{V}^\mu\mathcal{A}^{*\nu} + \mathcal{A}^\mu\mathcal{V}^{*\nu}\}\,i\,\epsilon_{\rho\sigma\mu\nu}\,q^\sigma\,p_i^\rho)]\,, \tag{3.3}
$$

where $p_i = p_2 - p_1$, $p_f = p_4 - p_3$, and $\mathcal{V}$, $\mathcal{A}$ carry all the coupling constants, final spin and momentum dependences. After some algebra one finds:

$$
\mathcal{V}^\mu = e\,\bar{u}(p_3,s_3)\sum_{k=\gamma,Z} v_e^i\left\{\gamma^\mu\left[(v_\tau^k - g^k) - a_\tau^k\gamma_5\right] - \frac{1}{2m_\tau}g^k\,p_f^\mu\right\}\mathcal{P}_k\,v(p_4,s_4)\,,
$$

$$
\mathcal{A}^\mu = e\,\bar{u}(p_3,s_3)\sum_{k=\gamma,Z} a_e^i\left\{\gamma^\mu\left[(a_\tau^k - g^k) - v_\tau^k\gamma_5\right] - \frac{1}{2m_\tau}g^k\,p_f^\mu\right\}\mathcal{P}_k\,v(p_4,s_4). \tag{3.4}
$$

Here the couplings are:

$$
v_e^\gamma = v_\tau^\gamma = -1 \qquad ; \qquad a_e^\gamma = a_\tau^\gamma = 0\,, \tag{3.5}
$$

$$
v_e^Z \equiv v_e = v_\tau^Z \equiv v_\tau = -\frac{1}{2} + 2s_W^2 \quad ; \qquad a_e^Z \equiv a_e = a_\tau^Z \equiv a_\tau = -\frac{1}{2}\,, \tag{3.6}
$$

$$g^\gamma = 2\frac{m_\tau}{m_Z}\epsilon_\gamma \qquad\qquad ; \qquad g^Z = \frac{4m_\tau}{m_Z}\epsilon_Z \ . \qquad\qquad (3.7)$$

3.1   Total rates and cross sections

If we are not interested in final polarization we can sum over the polarizations of the tau-leptons and obtain the angular distribution. This is usually written in the following notation:

$$\frac{d\sigma^0}{d\cos\theta} = \sigma^0(s)\frac{1}{2}(1+\cos^2\theta) + \sigma^m(s)\frac{1}{2}\sin^2\theta + \sigma^{\mathrm{FB}}(s)\cos\theta \ , \qquad (3.8)$$

where the coefficients σ^i are given by:

$$
\begin{aligned}
\sigma^0(s) = \frac{\pi\alpha^2}{s}\beta\Big\{ & 1 + 2\,v_\tau\,v_e\,\mathrm{Re}\,\{\chi\} + (a_e^2 + v_e^2)(v_\tau^2 + a_\tau^2\beta^2)\,|\chi|^2 \\
& +4\,r_Z\,[1 + v_\tau\,v_e\mathrm{Re}\,\{\chi\}]\,\epsilon_\gamma + 4\,r_Z^2\,\epsilon_\gamma^2 \\
& -8\,r_Z\,\Big[v_e\,\mathrm{Re}\,\{\chi\} + v_\tau\,(v_e^2 + a_e^2)\,|\chi|^2\Big]\,\epsilon_Z \qquad (3.9) \\
& + (4\,r_Z)^2\,(v_e^2 + a_e^2)\,|\chi|^2\,\epsilon_Z^2 - (4\,r_Z)^2\,v_e\,\mathrm{Re}\,\{\chi\}\,\epsilon_Z\,\epsilon_\gamma\,\Big\} \ ,
\end{aligned}
$$

$$
\begin{aligned}
\sigma^m(s) = \frac{\pi\alpha^2}{s}\beta\Big\{ & \frac{4\,m_\tau^2}{s}\Big[1 + 2v_\tau\,v_e\mathrm{Re}\,\{\chi\} + v_\tau^2(v_e^2 + a_e^2)|\chi|^2\Big] \\
& +4\,r_Z\,[1 + v_\tau\,v_e\mathrm{Re}\,\{\chi\}]\,\epsilon_\gamma + \frac{s}{m_Z^2}\epsilon_\gamma^2 \\
& -8\,r_Z\,\Big[v_\tau\,(v_e^2 + a_e^2)\,|\chi|^2 + v_e\,\mathrm{Re}\,\{\chi\}\Big]\,\epsilon_Z \qquad (3.10) \\
& + \frac{4\,s}{m_Z^2}(v_e^2 + a_e^2)|\chi|^2\epsilon_Z^2 - \frac{4\,s}{m_Z^2}\,v_\tau\,\mathrm{Re}\,\{\chi\}\,\epsilon_Z\,\epsilon_\gamma\Big\} \ ,
\end{aligned}
$$

$$
\begin{aligned}
\sigma^{\mathrm{FB}}(s) = \frac{\pi\alpha^2}{s}\beta^2\Big\{ & 2\,a_\tau\,a_e\,\Big[\mathrm{Re}\,\{\chi\} + 2\,v_\tau\,v_e\,|\chi|^2\Big] \\
& +4\,r_Z\,a_\tau\,a_e\,\mathrm{Re}\,\{\chi\}\,\epsilon_\gamma - 16\,r_Z\,a_\tau\,a_e\,v_e\,|\chi|^2\,\epsilon_Z\Big\} \ . \qquad (3.11)
\end{aligned}
$$

with

$$r_Z = \frac{m_\tau}{m_Z}, \qquad \beta = \beta(s) = \sqrt{1 - \frac{4m_\tau^2}{s^2}},$$

$$\chi = \chi(s) = \frac{1}{(2s_W c_W)^2}\frac{s}{(s - m_Z^2 + i\,\Gamma_Z m_Z)},$$

$s = q^2$, and θ being the angle between the linear momentum of the e^- and the τ^-.

In the limit of massless fermions ($\beta \to 1$) all linear terms in the dipole moments disappear and the coefficients σ^i simplify to:

$$\sigma^0(s)|_{m_\tau \to 0} = \frac{\pi \alpha^2}{s} \left\{ 1 + 2\, v_\tau\, v_e\, \mathrm{Re}\,\{\chi\} + (a_e^2 + v_e^2)(v_\tau^2 + a_\tau^2)\, |\chi|^2 \right\} , \qquad (3.12)$$

$$\sigma^m(s)|_{m_\tau \to 0} = \frac{\pi \alpha^2}{s} \left\{ \frac{s}{m_Z^2} \left[\epsilon_\gamma^2 + \frac{4\,s}{m_Z^2}\,(v_e^2 + a_e^2)\, |\chi|^2\, \epsilon_Z^2 \right. \right.$$

$$\left. \left. - 4\, v_\tau\, \mathrm{Re}\,\{\chi\}\, \epsilon_Z\, \epsilon_\gamma \right] \right\} , \quad (3.13)$$

$$\sigma^{\mathrm{FB}}(s)|_{m_\tau \to 0} = \frac{\pi \alpha^2}{s} \left\{ 2\, a_\tau\, a_e \left[\mathrm{Re}\,\{\chi\} + 2\, v_\tau\, v_e\, |\chi|^2 \right] \right\} . \qquad (3.14)$$

It is worth noticing that $\sigma^m(s)$, the term in $\sin^2 \theta$, is proportional to the tau mass in the standard model and, therefore, it vanishes for massless taus. However, in the presence of magnetic moment couplings this term remains, even in the massless limit, and in fact it carries all the information about the magnetic moments ϵ_Z, ϵ_γ. This contribution peaks in the angular region where the standard model contribution ($\frac{1}{2}(1 + \cos^2 \theta)\sigma^0|_{m_\tau \to 0}$) reaches its minimum.

By integrating out the angle in eq. (3.8) we obtain the total cross section:

$$\sigma(e^+ e^- \to \tau^+ \tau^-) = \frac{4}{3}\sigma^0(s) + \frac{2}{3}\sigma^m(s) . \qquad (3.15)$$

From eqs. (3.9–3.14) we find that there are basically three types of contributions: i) the standard model contribution $\sigma^0|_{m_\tau \to 0}$, which is the dominant one, ii) a contribution which is proportional to the tau mass. This comes together with an insertion of the magnetic moment operators (non-standard contribution to σ^m and σ^0) or with an insertion of another fermion mass (standard model contributions to σ^m) or both, two insertions of magnetic moment operators and two mass insertions (non-standard contributions to σ^0), and iii) a contribution free of masses but with two insertions of the magnetic moment operators (in σ^m).

This can be easily understood, since standard model couplings of gauge bosons to fermions conserve chirality, while mass terms and magnetic moment couplings change it, therefore interference of magnetic moment contributions with standard ones should be proportional to the fermion masses and only the square of magnetic moments can be independent of fermion masses. In the limit of zero tau mass only the contribution iii) is relevant, however there could be some range of the parameters in which contribution ii) is higher than iii). In fact for any finite value of the tau lepton mass it is obvious that for large enough Λ ii) will always dominate over iii). In order to be as general as

possible we will include all three contributions.

3.2 Bounds from $e^+e^- \to \tau^+\tau^-$ at LEP1 and SLD

Bounds on the new couplings ϵ_γ and ϵ_Z can be obtained from LEP1-SLD universality tests by assuming that only the tau lepton has anomalous magnetic moments (muon and electron electromagnetic moments have been measured quite precisely [2]). In order to compare with experimental data it is convenient to define the universality ratio:

$$R_{\tau\mu} = \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \mu^+\mu^-)}\bigg|_{s=m_Z^2} = \frac{\Gamma_{\tau\bar{\tau}}}{\Gamma_{\mu\bar{\mu}}} = \frac{R_\mu}{R_\tau} \equiv R_{SM} + R_1 + R_2 . \quad (3.16)$$

Here $R_\mu \equiv \Gamma_{\text{had}}/\Gamma_{\mu\bar{\mu}}$ and $R_\tau = \Gamma_{\text{had}}/\Gamma_{\tau\bar{\tau}}$ are the quantities measured directly [10], on the other hand, R_{SM} is the standard model contribution (including lepton-mass corrections), and R_1 and R_2 are the linear and quadratic terms, respectively, in the tensor couplings. In order to get bounds on the anomalous couplings, the theoretical expression for $R_{\tau\mu}$ can be easily computed from eqs. (3.8–3.14),

$$
\begin{aligned}
R_{SM} &= \sqrt{1 - 4\,r_Z^2}\,\left[1 + 2\,r_Z^2\,\frac{v^2 - 2a^2}{v^2 + a^2}\right] , \\
R_1 &= -12\,\sqrt{1 - 4\,r_Z^2}\,r_Z\,\frac{v}{v^2 + a^2}\,\epsilon_Z , \\
R_2 &= 2\,\sqrt{1 - 4\,r_Z^2}\,\left[1 + 8\,r_Z^2\right]\,\frac{1}{v^2 + a^2}\,\epsilon_Z^2 .
\end{aligned}
\quad (3.17)
$$

with $a \equiv a_e = a_\tau = a_\mu$, and $v \equiv v_e = v_\tau = v_\mu$. Notice that in the ratio eq. (3.16) electroweak radiative corrections cancel to large extend and, therefore, we can use tree-level formulae. However, if needed, the expressions in eq. (3.17) can be improved by using effective couplings [11]. Combining the very precise experimental LEP1 and SLD measurements [10],

$$R_\mu = 20.786 \pm 0.033, \quad R_\tau = 20.764 \pm 0.045,$$

we obtain

$$R_{\tau\mu} = 1.0011 \pm 0.0027 . \quad (3.18)$$

Comparing (3.18) with (3.16) and (3.17) one gets the condition for the ϵ_Z coupling to be

$$0.0007 \leq 7.967\,\epsilon_Z^2 + 0.037\,\epsilon_Z \leq 0.0061 \ . \tag{3.19}$$

This equation leads to the following bounds on ϵ_Z:

$$-0.030 \leq \epsilon_Z \leq -0.012 \quad \text{or} \quad 0.007 \leq \epsilon_Z \leq 0.025 \ , \tag{3.20}$$

so that, at 1σ, the zero value for ϵ_Z is excluded. This is so because the measured value of $R_{\tau\mu} = 1.0011$, given in eq. (3.18), excludes the SM (at 1σ) due to the fact that the SM mass correction (r_Z) to $R_{\tau\mu} = 1$ is negative while the measured value is larger than one. At 2σ the effect disappears.

It must be noticed that, in eq. (3.19), the linear term in ϵ_Z is strongly suppressed by the mass insertion r_Z –necessary to get a chirality even contribution to the observable– and also by the vector coupling v ($\frac{1}{4}$ effect), so that these bounds on ϵ_Z come almost entirely from the quadratic term in the coupling. Note also that, as expected from eq. (3.17), no bound is found on the γ-coupling ϵ_γ, due to the Z dominance at the Z-peak.

3.3 Bounds from $e^+e^- \to \tau^+\tau^-$ at LEP2

The situation is quite different at LEP2, where contributions coming from the photon-exchange are dominant over those coming from the Z-exchange. This is easily seen from the expression of the $e^+e^- \longrightarrow \tau^+\tau^-$ cross section given in eq. (3.9) to eq. (3.11).

Present limits from $e^+e^- \to \tau^+\tau^-$ cross section are much milder at LEP2 [12]. A combination of the LEP2 data on this cross section, for a value of s' (the invariant mass of the pair of tau leptons) so that $\sqrt{s'/s} > 0.85$, is listed in table 1. This combination of data has been only made for the 183 GeV and 189 GeV data-sets as they have the highest luminosities and center-of-mass energies. For comparison we also present the standard model prediction for the cross section. In both, experimental results and standard model predictions, initial-final state radiation photon interference is subtracted.

For LEP2 let us define the ratio $R_{\tau\bar{\tau}}$ as:

$$R_{\tau\bar{\tau}} \equiv \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \tau^+\tau^-)_{SM}} = 1 + F_1^\gamma(s)\,\epsilon_\gamma + F_2^\gamma(s)\,\epsilon_\gamma^2 + F_1^Z(s)\,\epsilon_Z$$

$$+ F_2^Z(s)\,\epsilon_Z^2 + F^{\gamma Z}(s)\,\epsilon_Z\epsilon_\gamma \ . \tag{3.21}$$

Table 1
Combined experimental data for the $\tau^+\tau^-$ cross section from ALEPH, DELPHI, L3 and OPAL at LEP2 energies. s'/s is cut in the invariant mass of the tau pair

$\sqrt{s}$ (GeV)	$\sigma_{\tau\bar\tau}^{\mathrm{SM}}$(pb)	$\sigma_{\tau\bar\tau}$(pb)	$\sqrt{s'/s} >$	Collaboration
182.7	3.45	3.43 ± 0.18	0.85	LEP Electroweak
188.6	3.21	3.135 ± 0.102	0.85	Working Group[10]

Table 2
Coefficients of the anomalous contributions to $R_{\tau\bar\tau}$, for the different center of mass measured energies at LEP2.

$\sqrt{s}$(GeV)	F_1^γ	F_2^γ	F_1^Z	F_2^Z	$F^{\gamma Z}$
130	0.079	0.682	0.028	5.258	0.286
136	0.083	0.784	0.026	5.152	0.304
161	0.092	1.221	0.022	5.272	0.384
172	0.094	1.427	0.021	5.497	0.424
183	0.096	1.642	0.020	5.789	0.467
189	0.096	1.765	0.019	5.971	0.491

For the range of energies used by LEP2 experiments, the coefficients $F_i(s)$, obtained from eqs. (3.8–3.11), are given in table 2. Direct comparison of eq. (3.21) with experimental data will provide bounds on anomalous couplings. We have checked that, even though coefficients in table 2 are obtained with no initial state radiation, its inclusion only changes the coefficients by about a 10% and this does not affect significantly the obtained bounds.

In order to see how well the new couplings can be bound from LEP2 let us find the limits on ϵ_γ obtained by using only the data at 189 GeV (All data are used independently in the global fit discussed in section 5). The experimental value for the ratio $R_{\tau\bar\tau}$ is:

$$R_{\tau\bar\tau}|_{\mathrm{exp}} = 0.978 \pm 0.032 \ , \tag{3.22}$$

which must be compared with our theoretical prediction (assuming that ϵ_Z is well bounded from LEP1-SLD, as it is)

$$R_{\tau\bar\tau}|_{\mathrm{th}} = 1.00 + 1.765\epsilon_\gamma^2 + 0.096\epsilon_\gamma \ . \tag{3.23}$$

From the two previous equations it is easy to find the following 1σ bound:

$$-0.10 < \epsilon_\gamma < 0.05 \ , \tag{3.24}$$

which is comparable to the one obtained in the final global fit given in section 5, where all available data have been included.

3.4 Tau lepton transverse polarization asymmetry

Although magnetic moments change chirality, total cross sections are chirality even observables. Thus, in the limit of massless taus, magnetic moment contributions to cross sections come always squared. In addition all kind of new chirality even physics will also contribute to total cross sections. Therefore, observables that vanish for massless taus are superior because they depend linearly on magnetic moments and therefore they are more sensitive to them. On the other hand they will not get contributions from physics conserving chirality. In that sense they are truly magnetic moment observables.

At LEP1[8], with the τ direction fully reconstructed in the semi-leptonic decays $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h_1^+ X \ h_2^- \nu_\tau$, $h_1^+ \bar{\nu}_\tau \ h_2^- X$ $(h_1, h_2 = \pi, \rho)$, it has been shown[7] that one can get relevant information about the anomalous weak magnetic moment just by measuring the following azimuthal asymmetry of the τ-decay products:

$$A_{cc}^{\mp} = \frac{\sigma_{cc}^{\mp}(+) - \sigma_{cc}^{\mp}(-)}{\sigma_{cc}^{\mp}(+) + \sigma_{cc}^{\mp}(-)} \ , \tag{3.25}$$

where $\sigma_{cc}^{\mp}$ is defined in the following angular regions

$$\sigma_{cc}^{\mp}(+) = \sigma\left(\cos\theta_{\tau^-} > 0, \cos\phi_{h^\mp} > 0\right) + \sigma\left(\cos\theta_{\tau^-} < 0, \cos\phi_{h^\mp} < 0\right) \ , \tag{3.26}$$
$$\sigma_{cc}^{\mp}(-) = \sigma\left(\cos\theta_{\tau^-} > 0, \cos\phi_{h^\mp} < 0\right) + \sigma\left(\cos\theta_{\tau^-} < 0, \cos\phi_{h^\mp} > 0\right) \ . \tag{3.27}$$

In the $\beta \rightarrow 1$ limit, the expression that one finds for the proposed asymmetry is:

$$A_{cc}^{\mp} = \mp \alpha_h \frac{1}{2} \frac{a}{v^2 + a^2} \left[-v\, r_Z + \epsilon_Z\right] \ , \tag{3.28}$$

where $\alpha_h = (m_\tau^2 - 2m_h^2)/((m_\tau^2 + 2m_h^2))$, is the polarization analyzer for each hadron channel $(h = \pi, \rho)$, $a \equiv a_e = a_\tau$, and $v \equiv v_e = v_\tau$. The formula shows that $A_{cc}^{\mp}$, being sensitive to the transverse polarization of the τ lepton, selects the leading contribution in the anomalous weak magnetic coupling ϵ_Z of the tau. In addition, the SM contribution to the observable is doubly suppressed with respect to the non-standard one: by the fermion vector coupling v ($\equiv -\frac{1}{2} + 2s_W^2$) and by the r_Z ($\equiv \dfrac{m_\tau}{m_Z}$) factor.

Within 1σ, the LEP1 [8] measurement of this asymmetry and the SLD[9] determination of the transverse tau polarization, translate in the following values for the ϵ_Z coupling

$$\epsilon_Z = \begin{cases} (0.0 \pm 1.7 \pm 2.4) \times 10^{-2} \text{ (LEP1)} , \\ (0.28 \pm 1.07 \pm 0.81) \times 10^{-2} \text{ (SLD)} \end{cases} \tag{3.29}$$

Combining these results one gets the bound:

$$\epsilon_Z = 0.002 \pm 0.012 . \tag{3.30}$$

Note that even though the transverse tau polarization has been measured at LEP1-SLD with a precision one order of magnitude worse than the universality test $R_{\tau\mu}$ (2-4% typically for the asymmetry, and 0.5% for the tau-muon cross section ratio), the obtained bound eq. (3.30) is as good as the one coming from universality eq. (3.20). This is so because the asymmetry depends linearly on the couplings. All the other observables depend mainly quadratically on the ϵ's, therefore, if the new-physics contributions to magnetic moments are ever found to be different from zero, the asymmetry will be the only observable able to disentangle the sign of the couplings. In addition, as commented before, the asymmetry $A_{cc}^{\mp}$ is also qualitatively a better observable since it is independent on any physics that does not break chirality.

At present we do not know any similar measurement at LEP2. However we think that this measurement will be very interesting since it will allow us to disentangle cleanly the γ components from the Z components of the magnetic moments.

4 Lepton universality in $W \to \tau\nu_\tau$

From eq. (2.6) we see that the same couplings that give rise to electromagnetic and Z-boson magnetic moments, also contribute to the couplings of the W gauge bosons to tau leptons. The couplings appear in a different combination than that in the photon or Z couplings, so their study gives us an additional independent information on magnetic moment couplings. As was noticed in Ref. [5] the best place to look for effects of the ϵ_W coupling is in the W decay widths.

Using our effective Lagrangian we can easily compute the ratio of the decay width of the W-gauge boson in tau-leptons (with magnetic moments) to the decay width of the W to electrons (without magnetic moments).

$$R_{\tau e}^W \equiv \frac{\Gamma(W \to \tau\nu)}{\Gamma(W \to e\nu)}$$

$$= (1 - r_W^2)^3 \left[1 + \frac{r_W^2}{2} + 3\sqrt{2}\, r_W\, c_W\, \epsilon_W + (1 + 2\, r_W^2)\epsilon_W^2 \right] , \qquad (4.1)$$

where $r_W = m_\tau / m_W$, and ϵ_W can be rewritten in terms of ϵ_γ and ϵ_Z as in eq. (2.9). Similar expression to eq. (4.1) was given in Ref. [5] but we found and extra global $(1 - r_W^2)$ missing, and a factor 3 instead of a $\left(-\frac{3}{2}\right)$ factor (in our notation) in the term linear in the coupling. Note that $R_{\tau e}^W$, like the cross sections studied in section 3.1, is a chirality even observable. Therefore, in the limit of massless taus, the only contribution from magnetic moments comes squared.

The decay of the W into leptons has been measured to a rather good precision at LEP2, UA1, UA2, CDF and D0. There, results are presented in the form of universality tests on the couplings (for a review on tau lepton universality tests see Ref. [13])

$$\frac{g_\tau}{g_e} = 0.987 \pm 0.025 \text{ (Colliders) } [14] , \qquad (4.2)$$

$$\frac{g_\tau}{g_e} = 1.010 \pm 0.019 \text{ (LEP2) } [10] . \qquad (4.3)$$

We combine these results and rewrite them as a measurement on the ratio $R_{\tau e}^W$ defined above

$$R_{\tau e}^W = 1.002 \pm 0.030 , \qquad (4.4)$$

where we have assumed that the small effect (0.12%) of the tau lepton mass has been subtracted to obtain the results shown in eq. (4.2) to eq. (4.3).

Then, from eq. (4.4) and eq. (4.1), we obtain the following limit on the W-boson magnetic moments.

$$-0.23 \leq \epsilon_W \leq 0.15 . \qquad (4.5)$$

5 Combined limits on electromagnetic and weak magnetic moments of the tau lepton

We have performed a global fit, as a function of the two independent couplings ϵ_γ and ϵ_Z, to the following studied observables :

- ratio of cross sections $R_{\tau\mu} = \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \mu^+\mu^-)}$ (eq. (3.16), Lepton Universality),

at LEP1 and SLD (eq. (3.18));

- the ratio of cross sections $R_{\tau\bar{\tau}} \equiv \frac{\sigma(e^+e^-\to\tau^+\tau^-)}{\sigma(e^+e^-\to\tau^+\tau^-)_{SM}}$ (eq. (3.21)), for the two highest energies measured at LEP2 (table 1);

- the transverse tau polarization and the tau polarization asymmetry $A_{cc}^{\mp} = \frac{\sigma_{cc}^{\mp}(+)-\sigma_{cc}^{\mp}(-)}{\sigma_{cc}^{\mp}(+)+\sigma_{cc}^{\mp}(-)}$ (eq. (3.25)) measured at SLD and LEP1 (eq. (3.30));

- and the ratios of decay widths of W-gauge bosons $R_{\tau e}^{W} \equiv \frac{\Gamma(W\to\tau\nu)}{\Gamma(W\to e\nu)}$ (eq. (4.1)) measured at LEP2 and $p\bar{p}$ colliders (eq. (4.4)).
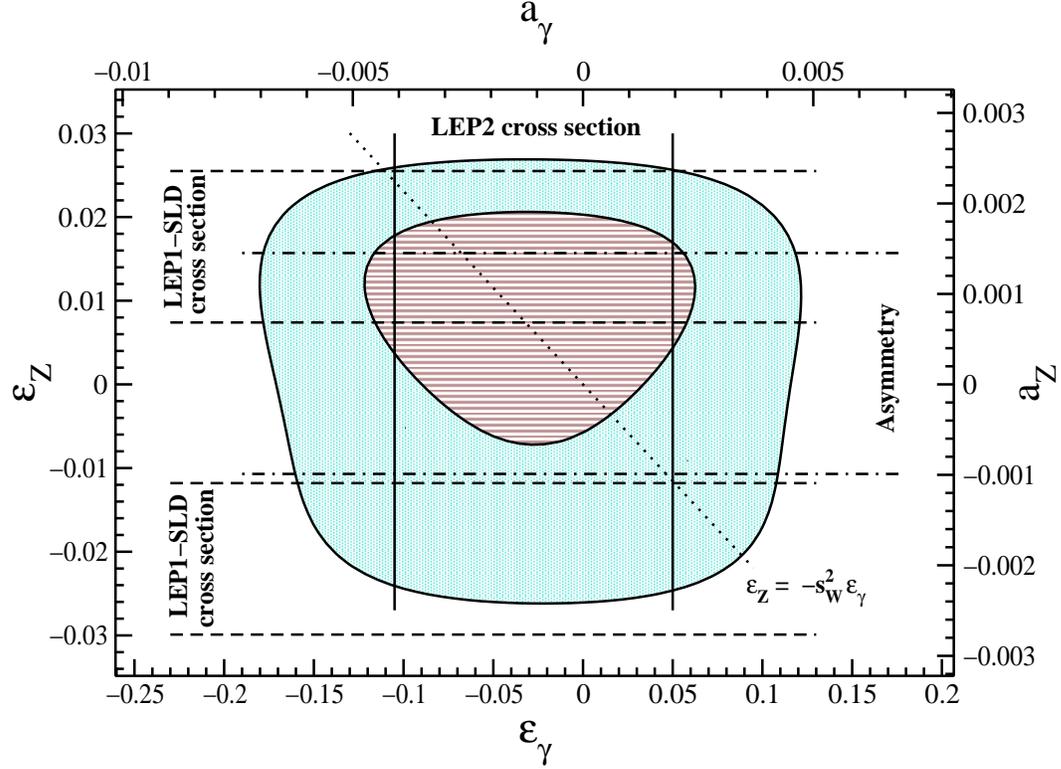

Fig. 1. Global fit including all constraints discussed in the paper. 95% CL and 68% CL contours are shown. The bands between straight lines show the allowed regions (1σ) coming from the different experiments: solid (LEP2-189 GeV), dashed (LEP1-SLD cross section), dot-dashed (asymmetry). We also have plotted the line $\epsilon_Z = -s_W^2\epsilon_\gamma$ (dotted line). This relationship appears when only the operator $\mathcal{O}_B$ contributes.

In fig. 1 we present, in the plane ϵ_γ–ϵ_Z (or a_γ–a_Z) the allowed region of parameters at 1σ and 2σ. For comparison we also present (at 1σ) the relevant limits set independently by the different observables, as discussed in the text. By projecting onto the axes one can read off the 1σ and 2σ limits on the different couplings

$$(1\sigma) \rightarrow \begin{cases} -0.12 < \epsilon_\gamma < 0.06 \ , \\ -0.0072 < \epsilon_Z < 0.021 \ , \end{cases} \tag{5.1}$$

$$(2\sigma) \rightarrow \begin{cases} -0.18 < \epsilon_\gamma < 0.12 \ , \\ -0.026 < \epsilon_Z < 0.027 \ . \end{cases} \qquad (5.2)$$

At 1σ the allowed region is far from elliptic. This is because the dominant quadratic dependence of the observables on the parameters ϵ_γ and ϵ_Z gives a tendency to provide two symmetric zones around two different minima. This is especially true for the bounds coming from universality tests at the Z-peak. In this situation the interpretation of contours as 68% CL contours is not clear. Probably one can combine the two minima and obtain more stringent bounds. However in order to be conservative we just quote the maximum allowed region. In any case, 2σ contours do not show this effect and we think they can be used reliably to obtain 95% CL limits.

Using the relationship among ϵ_γ, ϵ_Z, α_B and α_W at a given value of the scale of new physics, one can easily obtain bounds on α_B and α_W. Alternatively, by assuming that $\alpha_B/4\pi$ or $\alpha_W/4\pi$ are order unity one can obtain bounds on the scale of new physics Λ ($\Lambda > 9$ TeV).

Finally the limits on ϵ_γ and ϵ_Z can be immediately translated into limits on the non-standard contributions to the anomalous electromagnetic and weak magnetic moments a_γ, a_Z just by using eq. (2.12). Thus we have:

$$(1\sigma) \rightarrow \begin{cases} -0.005 < a_\gamma < 0.002 \ , \\ -0.0007 < a_Z < 0.0019 \ , \end{cases} \qquad (5.3)$$

$$(2\sigma) \rightarrow \begin{cases} -0.007 < a_\gamma < 0.005 \ , \\ -0.0024 < a_Z < 0.0025 \ . \end{cases} \qquad (5.4)$$

For a_γ these limits are only about one order of magnitude larger than the standard model contribution, $a_\gamma^{\mathrm{SM}} \sim 0.00117$.

6   Discussion and conclusions

The above bounds are completely model independent and no assumption has been made on the relative size of couplings α_B and α_W in the effective Lagrangian (2.4). For the sake of comparison with published data [1] we present now the limits that can be found by considering separately only operator $\mathcal{O}_B$ or only operator $\mathcal{O}_W$ in the Lagrangian (2.4). Consider that only $\mathcal{O}_B$ is present, as in Ref. [1], is equivalent to impose the relation $\epsilon_Z = -s_W^2 \epsilon_\gamma$. Thus, from fig. 1, it is straightforward to obtain that the bounds on the anomalous mag-

netic moment (at 2σ) are reduced to $-0.004 < a_\gamma < 0.003$, while little change is found on the weak-magnetic moment $-0.0019 < a_Z < 0.0024$.
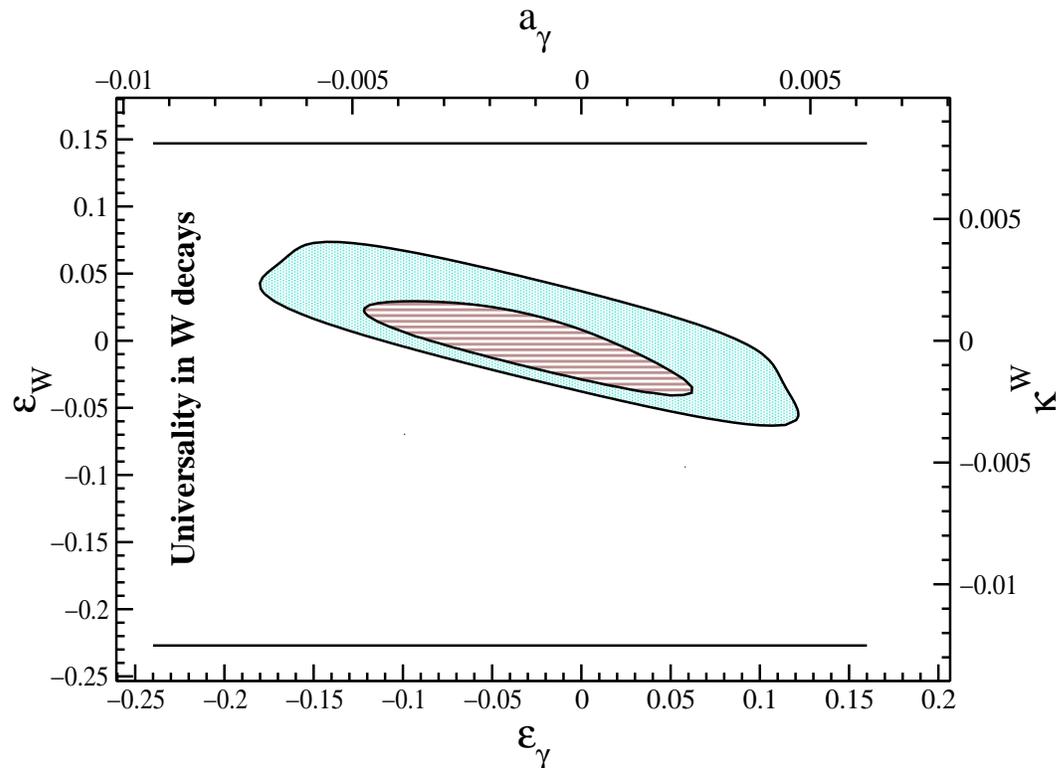

Fig. 2. The global fit of fig. 1 is now plotted in the plane ϵ_γ and ϵ_W to show the combined bounds on ϵ_W. As in fig. 1 95% CL and 68% CL contours are shown. For comparison we also draw as straight lines the direct 1σ bounds obtained from universality tests in W decays.

Universality tests in W decays do not provide any interesting constraint on the couplings ϵ_Z and ϵ_γ. However, because of the relationship (2.9), the LEP1-SLD and LEP2 constraints on ϵ_Z and ϵ_γ can be translated into constraints on ϵ_W (or κ^W defined in eq. (2.12)), the weak magnetic moment couplings of the W-gauge-boson to taus and neutrinos. In fig. 2 we present, in the plane ϵ_γ–ϵ_W (or a_γ–κ^W), the 1σ and 2σ regions obtained from our global fit to all data. Clearly LEP1-SLD limits on ϵ_Z coming from Z-decays and the asymmetry and LEP2 limits on ϵ_γ constrain ϵ_W very strongly. For comparison we also plot, as straight lines, the 1σ limits we have obtained from the universality tests in W decays.

From the figure 2, one immediately obtains the 95% CL limits

$$-0.06 < \epsilon_W < 0.07, \quad \text{or equivalently} \quad -0.003 < \kappa^W < 0.004 \ . \quad (6.1)$$

Bounds on the anomalous electromagnetic moment of the τ can also be obtained from the radiative decay $Z \to \tau^+ \tau^- \gamma$ at LEP1 [15,16]. There, only the

anomalous coupling a_γ is taken into account, while the contributions coming from the tau Z-magnetic coupling a_Z are neglected. Using this approach, with the inclusion of linear terms in a_γ in the cross section [16], and taking into account that the τ which emits the photon is off-shell, the analysis of the L3 [17] and OPAL [18] collaborations lead to the limits (at 95% CL):

$$-0.056 < a_\gamma < 0.044 \quad [\text{L3}] , \tag{6.2}$$
$$-0.068 < a_\gamma < 0.065 \quad [\text{OPAL}] . \tag{6.3}$$

As can be seen from eq. (5.4) our result, coming mainly from LEP2, is about one order of magnitude better than the ones obtained from the radiative Z-decay. In both cases some of the particles in the vertex are off-shell. The interpretation of off-shell form factors is problematic since they can hardly be isolated from other contributions and gauge invariance can be a problem. In the effective Lagrangian approach all those problems are solved because form factors are directly related to couplings in the effective Lagrangian, which is gauge invariant, and as discussed in section 2, the difference $F^{\text{new}}(q^2) - F^{\text{new}}(0)$ only comes from higher dimension operators whose effect is suppressed by q^2/Λ^2.

Concluding, we have shown that the use of all available data at the highest available energies (LEP1, SLD, LEP2, D0, CDF) allowed us to strongly constrain all the magnetic moments (weak and electromagnetic) of the tau lepton without making any assumption about naturalness or fine tuning. The obtained bounds (eq. (5.4) and eq. (6.1)), to our knowledge, are the best bounds that one can find in published data.

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