Bounds on the Tau Magnetic Moments: Standard Model and Beyond

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We obtain new bounds for the magnetic dipole moments of the tau lepton. These limits on the magnetic couplings of the tau to the electroweak gauge bosons (γ, W, Z) are set in a model independent way using the most general effective Lagrangian with the $SU(2)_L \otimes U(1)_Y$ symmetry. Comparison with data from the most precise experiments at high energies shows that the present limits are more stringent than the previous published ones. For the anomalous magnetic moment the bounds are, for the first time, within one order of magnitude of the standard model prediction.

1. INTRODUCTION

The fermion dipole moments are some of the best measured quantities in physics. While the limits for the CP-violating ones, the electric dipole moments, are impressive[1] ($10^{-26} - 10^{-25}$ e-cm for the electron and the neutron), the agreement between the measurement of the anomalous magnetic dipole moments of the electron and of the muon and its theoretical prediction is such that provides the best determination of the fine structure constant. Since the first prediction for the electron anomalous magnetic moment computed by Schwinger[2], the magnetic properties of the leptons played a central role in testing symmetries and the quantum prediction of the theory. This could be done thanks to the relativity long lifetime of the muon and the stability of the electron in ordinary atoms. For the heavy fermions the situation is very different. As short lived-particles, there is no possibility of such high precision experiments. Nevertheless, the magnetic dipole moments are, for on-shell particles, physical magnitudes that not only contain important information about the interactions, but

may also give some insights on the mechanism of mass generation. Due to the high mass of the tau lepton (and other heavy fermions) the measurement of the dipole moments is an interesting test for the standard model and possible new physics that may show up there. As expressed by M.Perl[3] in the section *Dreams and odd ideas in tau research*: "... It would be very nice to measure μ_τ with enough precision to check this [the Schwinger term $\alpha/2\pi$], as it was checked for the e and the μ years ago.*At present such precision is a dream.*" Previous bounds on the tau magnetic moment a_γ are indirect [4], or include a partial analysis of the amplitudes in the considered processes[5,6]. In the first case they come from the observation that in general extensions of the standard model (SM) it is very difficult to generate a weak magnetic moment, a_Z, for a lepton without originating a coupling of the Z boson to the lepton of the same order of magnitude. This weak magnetic coupling is strongly bounded by LEP1, therefore, by assuming that the magnetic moment of the lepton (a_γ) has the same size, one obtains a rather strong bound on it. On the other hand, far from the resonance the a_γ magnetic moment couplings of the tau is more important and should be taken into account in the theoretical and experimental analysis. Finally, the tau-W magnetic

coupling was first considered in the W decays into tau plus leptons[7] and also at low energies in the tau decays into leptons and pions[8].

While these arguments are plausible, a complete analysis of the data coming from tau-lepton production at LEP1, SLD and LEP2, and data on W decays into tau leptons from LEP2 and $p\bar{p}$ colliders, allows for a better understanding of the magnetic moment couplings of the tau to the photon, the Z and the W bosons. More technical details of what follows can be found in [9].

1.1. Dipole moments in the standard model and beyond

The anomalous magnetic moments of the tau lepton, as the rest of the leptons, receive the standard contribution from the Schwinger term. However, in the case of the tau lepton, larger contributions from new physics are still allowed. At the electroweak scale those can be studied within the effective Lagrangian approach. Magnetic dipole moments are usually defined with all particles in the magnetic moment vertex on-shell because *these are the gauge invariant quantities of the theory, as shown in the classical paper* [10].

For the a_γ tau magnetic dipole moment there is no available experiment satisfying the above requirement and to determine it one has to resort to general and consistent (gauge invariant) parameterizations of all physics in which the magnetic moment could play a significant role. The most general of those parameterizations is based on the effective field theory language which will allow us to relate off-shell matrix elements with the physical on-shell anomalous magnetic moment a_γ. Instead, the a_Z can be determined on-shell: we have tau pair production at the Z-peak at various experiments and specific observables proportional to the weak magnetic moment have been calculated[11] and measured[12].

The effective Lagrangian approach [13] we use is described in section 2 where we also fix the notation. Magnetic moment are generated by non-renormalizable couplings, their effects grow with energy and, therefore, high energy experiments, though not as precise as experiments at lower energies, can still be relevant. This is the case for LEP1 and SLD measurements as compared to LEP2 ones. Besides, while the former are only sensitive to the Z-magnetic couplings, LEP2 is sensitive to both Z and photon magnetic moments (a_Z and a_γ). Far from the resonance the statistics rapidly decreases and the precision is not as good. In general, lower energy experiments do not provide stringent bounds: the suppression factor $(E_{low}/m_Z)^2$ has to be compensated by higher precision in the experiment (see for instance the bounds obtained from tau decays [8]).

Observables are studied in section 3 where tau production is considered in universality test and transverse polarization at LEP1-SLD, total production rates at LEP2, and lepton universality in W decays in LEP2 and hadron colliders.

What are the observables related to the magnetic moments? There is no symmetry guide as, for example, in CP-violation by electric dipole moments, where specific CP-odd observables can be constructed. However, magnetic couplings flip chirality, so in order to select the physical effects of the magnetic moments one should take these properties into account. In the standard model the only source of chirality flips are fermion masses, much lower than the electroweak scale. This means that any contribution to inclusive observables are either suppressed by the fermion masses or need two operator insertions and then, they come as the electroweak magnetic moments squared. Furthermore, any new physics terms, not only that related to the magnetic moments will appear in these observables. It is therefore convenient to also include observables that are exactly zero when chirality is conserved: these will be only sensitive to fermion masses and/or magnetic moments and will depend linearly on the couplings. Up to now, observables of this kind have been studied at LEP1 but not at LEP2.

There are only two gauge invariant dimension six operators contributing to magnetic moments in the effective Lagrangian. Therefore gauge invariance imposes a relationship among the three weak and electromagnetic magnetic moments.Therefore one can achieve some insight on tau magnetic moments by studying W decays into tau leptons. There exist good bounds on universality of W decays into leptons coming from

LEP2, UA1, UA2, CDF and D0 that can be translated into limits on the magnetic moments.

In section 4 we present the combined analysis of the observables, compare them to other bounds found in the literature and discuss the results.

2. NEW PHYSICS AND THE EFFECTIVE LAGRANGIAN

The new physics phenomenology at low energy, *i.e.* at energies much lower than the scale of this new physics, can be parametrized by an effective Lagrangian built with the standard model particle spectrum and gauge symmetry, having as zero order term just the standard model Lagrangian, plus higher dimension gauge invariant operators suppressed by the scale of new physics Λ. The leading non-standard effects will come from the operators with the lowest dimension. Those are dimension six operators and among them there are only two operators that contribute to the tau magnetic moments:

$$\mathcal{O}_B = \frac{1}{\Lambda^2}\overline{L_L}\varphi\sigma_{\mu\nu}\tau_R B^{\mu\nu} \ , \tag{1}$$

$$\mathcal{O}_W = \frac{1}{\Lambda^2}\overline{L_L}\vec{\tau}\varphi\sigma_{\mu\nu}\tau_R\vec{W}^{\mu\nu} \ . \tag{2}$$

Here $L_L = (\nu_L, \tau_L)$ is the tau leptonic doublet, φ is the Higgs doublet, $B^{\nu\nu}$ and $\vec{W}^{\mu\nu}$ are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ field strength tensors. Other dimension six operators like

$$\frac{1}{\Lambda^2}\overline{L_L}\sigma_{\mu\nu}\not{D}L_L B^{\mu\nu} \ , \tag{3}$$

reduce to the operator of eq. (1) after the use of the standard model equations of motion. The effective Lagrangian is

$$\mathcal{L}_{eff} = \alpha_B\mathcal{O}_B + \alpha_W\mathcal{O}_W + \mathrm{h.c.} \ , \tag{4}$$

where we will take the couplings α_B and α_W real.

After spontaneous symmetry breaking, the Higgs gets a vacuum expectation value and, as usual, the interactions in (4) can be written in terms of the gauge boson mass eigenstates A^μ, Z^μ and W^μ. Our Lagrangian, written in terms of the mass eigenstates, is

$$\mathcal{L}_{eff} = \frac{a_\gamma e}{4m_\tau}\overline{\tau}\sigma_{\mu\nu}\tau F^{\mu\nu} + \frac{a_Z e}{4m_\tau}\overline{\tau}\sigma_{\mu\nu}\tau Z^{\mu\nu}$$

$$+ \left(\frac{\kappa^W e}{4\sqrt{2}m_\tau}\overline{\nu_{\tau L}}\sigma_{\mu\nu}\tau_R W_+^{\mu\nu} + \mathrm{h.c.}\right) \tag{5}$$

where one can recognize the usual couplings related to the magnetic moments. $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ (for $X = F, Z, W_+$) is the Abelian field strength tensor for the gauge bosons. We have not written the non-Abelian couplings involving more than one gauge boson because they do not contribute to magnetic moments at leading order. The above new physics magnetic moments can be written as

$$a_\gamma = (\alpha_B - \alpha_W)\frac{u\sqrt{2}m_\tau}{\Lambda^2} \ , \tag{6}$$

$$a_Z = -(\alpha_W c_W^2 + \alpha_B s_W^2)\frac{u\sqrt{2}m_\tau}{s_W c_W \Lambda^2} \ , \tag{7}$$

$$\kappa^W = \alpha_W\frac{u2\sqrt{2}m_\tau}{\Lambda^2} = -2s_W\left(c_W a_Z + s_W a_\gamma\right) \tag{8}$$

where $u/\sqrt{2}$ is the Higgs vacuum expectation value and $s_W = sin\theta_W$, $c_W = cos\theta_W$. The Lagrangian (5) gives additional contributions to the anomalous standard model electromagnetic moment of the tau $a_\gamma{}^{SM}$. The same is true for the other magnetic moments that have been introduced for the weak magnetic moments: a_Z for the Z-boson[11], and κ^W for the W-boson[7]. Sometimes it will be convenient to consider the following dimensionless couplings, proportional to the magnetic moments

$$a_\gamma = 2\, r_Z\, \epsilon_\gamma \ , \tag{9}$$

$$a_Z = 2\, \frac{1}{s_W\, c_W} r_Z\, \epsilon_Z \ , \tag{10}$$

$$\kappa^W = 2\sqrt{2}\, r_Z\, \epsilon_W \ . \tag{11}$$

where $r_Z = m_\tau/m_Z$. We would like to stress the following point. In the effective Lagrangian approach, *exactly* the same couplings that contribute to processes at high energies also contribute to the (magnetic moment) form factors, $F^{\mathrm{new}}(q^2)$, at $q^2 = 0$. The difference $F^{\mathrm{new}}(q^2) - F^{\mathrm{new}}(0)$ only comes from higher dimension operators whose effect is suppressed by powers of q^2/Λ^2, as long as $q^2 \ll \Lambda$ as needed for the consistence of the effective Lagrangian approach. This means that any measurement (bound) for the new physics contributions to magnetic moments, at an

energy scale where some of the gauge bosons or other particles are off-shell, can be directly added to the on-shell standard model prediction in order to get a value (limit) on the magnetic dipole moments.

The effects of the operators in (5) are suppressed at low energies. This means that the most interesting bounds will come from the highest precision experiments at the highest available energies. At present they are LEP1 and SLD (Z decay rates and polarization asymmetries), LEP2 (cross sections and W decays rates), CDF and D0 (W decay rates). Consequently in the following section we will study all those observables.

3. OBSERVABLES

In this section we will present the observables best suited to set bounds on the magnetic moments.

3.1. Universality in tau pair production at LEP1 and SLD

We begin with tau pair production in $e^+e^- \to \tau^+\tau^-$ from threshold to LEP2 energies. Therefore we will include both photon and Z-exchange with standard model vector and axial couplings to fermions, plus additional magnetic moment couplings given by eq. (5). The expressions for the cross section can be found in [9,11]. It can be separated into three types of contributions: i) the standard model tree level contribution , which is the dominant one, ii) a contribution which is proportional to the tau mass. This comes together with an insertion of the magnetic moment operators (non-standard contribution) or with an insertion of another fermion mass (standard model contributions) or both, two insertions of magnetic moment operators and two mass insertions (non-standard contributions), and iii) a contribution free of masses but with two insertions of the magnetic moment operators.

This can be easily understood, since standard model couplings of gauge bosons to fermions conserve chirality, while mass terms and magnetic moment couplings change it, therefore interference of magnetic moment contributions with standard ones should be proportional to the

fermion masses and only the square of magnetic moments can be independent of fermion masses. In the limit of zero tau mass only the contribution iii) is relevant, however there could be some range of the parameters in which contribution ii) is higher than iii). In fact for any finite value of the tau mass it is obvious that for large enough Λ ii) will always dominate over iii). In order to be as general as possible we will include all three contributions.

Bounds on the couplings a_γ and a_Z can be obtained from LEP1-SLD universality tests by assuming that only the tau lepton has anomalous magnetic moments (muon and electron electromagnetic moments have been measured quite precisely [1]). In order to compare with experimental data it is convenient to define the universality ratio:

$$
\begin{aligned}
R_{\tau\mu} &= \left. \frac{\sigma(e^+e^- \to \tau^+\bar{\tau})}{\sigma(e^+e^- \to \mu^+\mu^-)} \right|_{s=m_Z^2} = \frac{\Gamma_{\tau\bar{\tau}}}{\Gamma_{\mu\bar{\mu}}} \\
&= \frac{R_\mu}{R_\tau} \equiv R_{SM} + R_1 + R_2 . \qquad (12)
\end{aligned}
$$

Here $R_\mu \equiv \Gamma_{had}/\Gamma_{\mu\bar{\mu}}$ and $R_\tau = \Gamma_{had}/\Gamma_{\tau\bar{\tau}}$ are the quantities directly measured [14], R_{SM} is the standard model contribution (including lepton-mass corrections), and R_1 and R_2 are the linear and quadratic terms, respectively, in the tensor couplings. Notice that this quotient eliminates the hadronic part that, although very well determined in experiments, has much higher theoretical uncertainties than $R_{\tau\mu}$. The theoretical expression for $R_{\tau\mu}$ can be computed from the cross sections, and it is

$$
R_{SM} = \sqrt{1 - 4 r_Z^2} \left[1 + 2 r_Z^2 \frac{v^2 - 2a^2}{v^2 + a^2} \right] , \qquad (13)
$$

$$
R_1 = -12 \sqrt{1 - 4 r_Z^2} \, r_Z \frac{v}{v^2 + a^2} \epsilon_Z , \qquad (14)
$$

$$
R_2 = 2 \sqrt{1 - 4 r_Z^2} \left[1 + 8 r_Z^2 \right] \frac{1}{v^2 + a^2} \epsilon_Z^2 \qquad (15)
$$

with $a \equiv a_e = a_\tau = a_\mu$, $v \equiv v_e = v_\tau = v_\mu$. Notice that in the ratio eq. (12) the electroweak radiative corrections cancel to a large extent and, therefore, we can use tree-level formulae. However, if needed, the expressions in eqs. (13–15) can be improved by using effective couplings [15]. From the

Table 1
Combined experimental data for the $\tau^+\tau^-$ cross section from ALEPH, DELPHI, L3, OPAL at LEP2 energies (in GeV). $\sqrt{s'/s}$ is the cut in the invariant mass of the tau pair.

$\sqrt{s}$	$\sigma_{\tau\bar{\tau}}^{\mathrm{SM}}$(pb)	$\sigma_{\tau\bar{\tau}}$(pb)	$\sqrt{s'/s}$
182.7	3.45	3.43 ± 0.18	0.85
188.6	3.21	3.135 ± 0.102	0.85

very precise experimental LEP1 and SLD measurements [14] we obtain

$$R_{\tau\mu} = 1.0011 \pm 0.0027 . \qquad (16)$$

Comparing the equation (16) with (12) one gets

$$0.0007 \leq 7.967\,\epsilon_Z^2 + 0.037\,\epsilon_Z \leq 0.0061 \qquad (17)$$

that leads to the following two bands for ϵ_Z:

$$-0.030 \leq \epsilon_Z \leq -0.012 \qquad (18)$$

or

$$0.007 \leq \epsilon_Z \leq 0.025 , \qquad (19)$$

3.2. Tau pair production at LEP2

At LEP2, the contributions coming from the photon-exchange are dominant over those coming from the Z-exchange, and as a result both magnetic moments, a_γ and a_Z, will enter into the constraints.

Present experimental errors from $e^+e^- \to \tau^+\tau^-$ cross section are much milder at LEP2 than at LEP1. A combination of the LEP2 data [16] on this cross section, for s' (the invariant mass of the pair of tau leptons) so that $\sqrt{s'/s} > 0.85$, is listed in table 1. This combination of data has been only made for the 183 GeV and 189 GeV data-sets as they have the highest luminosity and center-of-mass energies. For comparison we also present the standard model prediction for the cross section. In both, experimental results and standard model predictions, initial-final state radiation photon interference is subtracted. For LEP2 let us define the ratio $R_{\tau\bar{\tau}}$ as:

$$
\begin{aligned}
R_{\tau\bar{\tau}} \equiv\; & \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \tau^+\tau^-)_{SM}} = 1 \\
& + F_1^\gamma(s)\,\epsilon_\gamma + F_2^\gamma(s)\,\epsilon_\gamma^2 + F_1^Z(s)\,\epsilon_Z \\
& + F_2^Z(s)\,\epsilon_Z^2 + F^{\gamma Z}(s)\,\epsilon_Z\epsilon_\gamma
\end{aligned} \qquad (20)
$$

Table 2
Coefficients of the anomalous contributions to $R_{\tau\bar{\tau}}$, for the different center of mass measured energies (in GeV) at LEP2.

$\sqrt{s}$	F_1^γ	F_2^γ	F_1^Z	F_2^Z	$F^{\gamma Z}$
130	0.079	0.682	0.028	5.258	0.286
136	0.083	0.784	0.026	5.152	0.304
161	0.092	1.221	0.022	5.272	0.384
172	0.094	1.427	0.021	5.497	0.424
183	0.096	1.642	0.020	5.789	0.467
189	0.096	1.765	0.019	5.971	0.491

For the range of energies used by LEP2 experiments, the coefficients $F_i(s)$, obtained from the cross sections, are given in table 2. Direct comparison of eq. (20) to experimental data will provide bounds on the anomalous couplings. We have checked that, even though coefficients in table 2 are obtained with no initial state radiation, its inclusion only changes the coefficients by about a 10% and this does not affect significantly the obtained bounds.

All data are used independently in the global fit discussed in the final section. Just as an example of how well the new couplings can be bound from LEP2 let us find the limits on ϵ_γ obtained by using only the data at 189 GeV. The experimental value for the ratio $R_{\tau\bar{\tau}}$ is in this case:

$$R_{\tau\bar{\tau}}|_{\exp} = 0.978 \pm 0.032 , \qquad (21)$$

to be compared with the theoretical prediction (assuming that ϵ_Z is well bounded from LEP1-SLD, as it is)

$$R_{\tau\bar{\tau}}|_{\mathrm{th}} = 1.00 + 1.765\epsilon_\gamma^2 + 0.096\epsilon_\gamma . \qquad (22)$$

From the two previous equations we find:

$$-0.10 < \epsilon_\gamma < 0.05 , \qquad (23)$$

This 1σ bound is comparable to the one obtained in the global fit given in the final section where all available data have been included.

3.3. Tau lepton transverse polarization

Chirality flipping observables vanish for massless taus and depend linearly on magnetic moments. On the other hand they will not get contributions from physics conserving chirality. In

that sense they are truly magnetic moment observables. At LEP1, with the τ direction fully reconstructed in the semi-leptonic decays it has been shown[11] that one can measure the anomalous weak magnetic moment a_Z by measuring an azimuthal asymmetry on the tau and tau decay products angles. This asymmetry selects the leading term in the weak magnetic moment in the tau pair production cross section, that appears in the P-odd, chirality flipping transverse tau polarization. The expression one finds for the proposed asymmetry is:

$$A_{cc}^{\mp} = \mp \alpha_h \frac{1}{2} \frac{a}{v^2 + a^2} \left[-v \, r_Z + \epsilon_Z \right] \ , \qquad (24)$$

where $\alpha_h = (m_\tau^2 - 2m_h^2)/((m_\tau^2 + 2m_h^2))$, is the polarization analyzer for each hadron channel ($h = \pi, \rho$), $a \equiv a_e = a_\tau$, and $v \equiv v_e = v_\tau$. In addition, the SM contribution to this observable is doubly suppressed with respect to the non-standard one: by the fermion-boson vector coupling v and by the r_Z factor.

Within 1σ, the LEP1 measurement of this asymmetry and the SLD determination of the transverse tau polarization[12], translate into the following values for the ϵ_Z coupling

$$\epsilon_Z = \left\{ \begin{array}{l} (0.0 \pm 1.7 \pm 2.4) \times 10^{-2} \ (\text{LEP1}) \ , \\ (0.28 \pm 1.07 \pm 0.81) \times 10^{-2} \ (\text{SLD}) \end{array} \right. \quad (25)$$

Combining these results one gets the bound:

$$\epsilon_Z = 0.002 \pm 0.012 \ . \qquad (26)$$

Note that even though the transverse tau polarization has been measured at LEP1-SLD with a precision one order of magnitude worse than the universality test $R_{\tau\mu}$ (2-4% typically for the asymmetry, and 0.5% for the tau-muon cross section ratio), the obtained bound eq. (26) is as good as the one coming from universality eq. (19). This is so because the asymmetry depends linearly on the couplings.

At present there does not exists a similar measurement at LEP2. This would allow to disentangle the γ components from the Z components of the magnetic moments.

3.4. Lepton universality in W decays.

The Lagrangian eq. (5) shows that the same couplings that give rise to electromagnetic and Z-boson magnetic moments, also contribute to the couplings of the W gauge bosons to tau leptons. The couplings appear in a different combination than that in the photon or Z couplings, so their study gives additional independent information on magnetic moment couplings. As was noticed in Ref. [7] the best place to look for effects of the ϵ_W coupling is in the W decay widths.

Using our effective Lagrangian we can easily compute the ratio of the decay width of the W-gauge boson in tau-leptons (with magnetic moments) to the decay width of the W to electrons (without magnetic moments).

$$R_{\tau e}^W \equiv \frac{\Gamma(W \to \tau\nu)}{\Gamma(W \to e\nu)} = (1 - r_W^2)^3 \times$$
$$\left[1 + \frac{r_W^2}{2} + 3\sqrt{2} \, r_W \, c_W \, \epsilon_W + (1 + 2 \, r_W^2)\epsilon_W^2 \right] \quad (27)$$

where $r_W = m_\tau/m_W$, and ϵ_W can be rewritten in terms of ϵ_γ and ϵ_Z as in eq. (8). Note that $R_{\tau e}^W$, like the cross sections studied in subsection (1), is a chirality even observable. The decay of the W into leptons has been measured to a rather good precision at LEP2, UA1, UA2, CDF and D0. There, results are presented in the form of universality tests on the couplings [14,17] that we rewrite as a measurement on the ratio $R_{\tau e}^W$ defined above

$$R_{\tau e}^W = 1.002 \pm 0.030 \ , \qquad (28)$$

Then, from eqs. (28–27), we obtain the following limit on the W-boson magnetic moments.

$$-0.23 \le \epsilon_W \le 0.15 \ . \qquad (29)$$

4. COMBINED ANALYSIS AND RESULTS

We have performed a global fit, as a function of the two independent couplings ϵ_γ and ϵ_Z, to the studied observables :

- Lepton universality $R_{\tau\mu} = \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \mu^+\mu^-)}$ at LEP1 and SLD;

- the ratio of cross sections $R_{\tau\bar{\tau}} \equiv \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \tau^+\tau^-)_{SM}}$, for the two highest energies measured at LEP2;

- the transverse tau polarization and polarization asymmetry $A_{cc}^{\mp}$ measured at SLD and LEP1 ;

- lepton universality of W decays $R_{\tau e}^{W} \equiv \frac{\Gamma(W \to \tau\nu)}{\Gamma(W \to e\nu)}$ measured at LEP2 and $p\bar{p}$ colliders.

In fig. 1 we present, in the plane a_γ–a_Z (or ϵ_γ–ϵ_Z) the allowed region of parameters at 1σ and 2σ. For comparison we also present (at 1σ) the
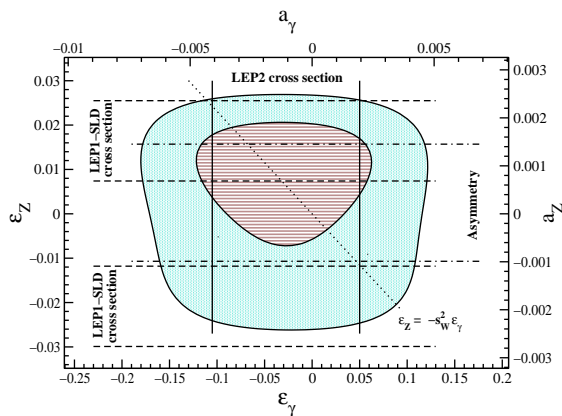

Figure 1. Global fit including all constraints; 95% CL and 68% CL contours are shown. The bands between straight lines show the allowed 1σ regions coming from the different experiments: solid (LEP2-189 GeV), dashed (LEP1-SLD cross section), dot-dashed (asymmetry). We also have plotted the line $\epsilon_Z = -s_W^2 \epsilon_\gamma$ (dotted line). This relationship appears when only the operator $\mathcal{O}_B$ contributes.

relevant limits set independently by the different observables, as discussed in the text. By projecting onto the axes one can read off the 1σ and 2σ limits on the different non-standard contributions to the anomalous electromagnetic and weak

magnetic moments a_γ, a_Z

$$(1\sigma) \to \begin{cases} -0.005 < a_\gamma < 0.002 \ , \\ -0.0007 < a_Z < 0.0019 \ , \end{cases} \quad (30)$$

$$(2\sigma) \to \begin{cases} -0.007 < a_\gamma < 0.005 \ , \\ -0.0024 < a_Z < 0.0025 \ . \end{cases} \quad (31)$$

Using the relationship among ϵ_γ, ϵ_Z, α_B and α_W at a given value of the scale of new physics, one can easily obtain bounds on α_B and α_W. Alternatively, by assuming that $\alpha_B/4\pi$ or $\alpha_W/4\pi$ are order unity one finds a bound on the scale of new physics: $\Lambda > 9$ TeV.

We note that, for the first time, the bound for a_γ is of the order of magnitude of the prediction computed long ago by Schwinger $a_\gamma^{QED} \sim 0.0012$.

Bounds on the anomalous electromagnetic moment for the τ have been considered and measured from the radiative decay $Z \to \tau^+ \tau^- \gamma$ at LEP1 [5,6]. There, only the anomalous coupling a_γ is taken into account, while the contributions coming from the tau Z-magnetic coupling a_Z or the effective 4-particle vertex are neglected. The interpretation of off-shell form factors is problematic since they can hardly be isolated from the other contributions and gauge invariance can be a problem. In the effective Lagrangian approach all those problems are solved because form factors are directly related to couplings in the effective Lagrangian, which is gauge invariant, and as discussed above, the difference $F^{new}(q^2) - F^{new}(0)$ only comes from higher dimension operators whose effect is suppressed by q^2/Λ^2. Using this approach, the analysis of the L3 and OPAL collaborations lead to the PDG 95% CL limit[1]:

$$-0.052 < a_\gamma < 0.058 \quad (32)$$

As can be seen from eq. (31) our result, coming mainly from LEP2, is about one order of magnitude better than the ones obtained from the radiative Z-decay.

The standard model prediction for the weak magnetic moment a_Z, which was computed in [11], is far below the present bound.

The above bounds are completely model independent and no assumption has been made on the relative size of couplings α_B and α_W in the

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effective Lagrangian (4). For the sake of comparison with published data [4] we present now the limits that can be found by considering separately only operator $\mathcal{O}_B$ or only operator $\mathcal{O}_W$ in the Lagrangian (4). Consider that only $\mathcal{O}_B$ is present, as in Ref. [4], is equivalent to impose the relation $\epsilon_Z = -s_W^2 \epsilon_\gamma$ or equivalently $c_W a_Z = -s_W a_\gamma$. Thus, from fig. 1, it is straightforward to obtain that the bounds on the anomalous magnetic moment (at 2σ) are reduced to $-0.004 < a_\gamma < 0.003$, while little change is found on the weak-magnetic moment $-0.0019 < a_Z < 0.0024$.

Universality tests in W decays do not provide any interesting constraint on a_Z and a_γ. In fact, the straight lines coming from the direct 1σ bound from universality tests in W decays lie well outside the figure. However, because of the relationship (8), the LEP1-SLD and LEP2 constraints on ϵ_Z and ϵ_γ can be translated into constraints on ϵ_W (or κ^W defined in eq. (11)), the weak magnetic moment couplings of the W-gauge-boson to taus and neutrinos. One obtains the 95% CL limits

$$ -0.003 < \kappa^W < 0.004 . \tag{33} $$

We have shown that the use of all available data at the highest available energies (LEP1, SLD, LEP2, D0, CDF) leads to strong constraints in all the magnetic moments (γ, W, Z) of the tau lepton without making any assumption about naturalness or fine tuning. The obtained bounds (eq. (31) and eq. (33)) are, to our knowledge, the best bounds that one can find in published data.

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