

# The $K^-d \rightarrow \pi\Sigma n$ reaction revisited

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The appearance of some papers dealing with the  $K^-d \rightarrow \pi\Sigma n$  reaction, with some discrepancies in the results and a proposal to measure the reaction at forward  $n$  angles at J-PARC justifies to retake the theoretical study with high precision to make accurate predictions for the experiment and extract from there the relevant physical information. We do this in the present paper showing results using the Watson approach and the truncated Faddeev approach. We argue that the Watson approach is more suitable to study the reaction because it takes into account the potential energy of the nucleons forming the deuteron, which is neglected in the truncated Faddeev approach. Predictions for the experiment are done as well as spectra with the integrated neutron angle.

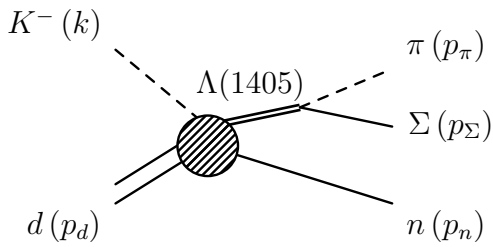
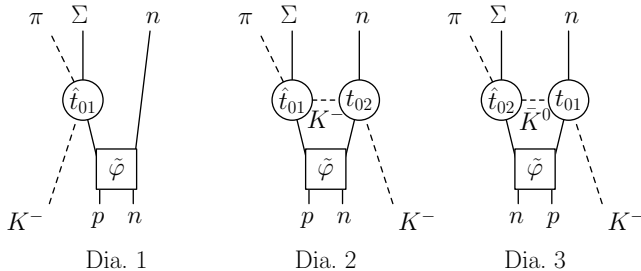
## I. INTRODUCTION

The unexpected peak in the invariant mass of the  $\pi\Sigma$  system found in [1] in the  $K^-d \rightarrow \pi\Sigma n$  reaction at energies around 1420 MeV was an interesting surprise which gives support to the theory of two  $\Lambda(1405)$  states [2] as was discussed in [3]. In this paper it was found that for kaons in flight the single scattering peak and the double scattering were well separated, such that the double scattering showed a clear peak due to the excitation of the  $\Lambda(1405)$ . The process can only occur in nuclei. Indeed, the single scattering  $K^-p \rightarrow \pi\Sigma$  occurs for invariant masses above the  $K^-p$  threshold and does not show the resonance shape of the  $\Lambda(1405)$  since this one occurs below threshold. However, in the deuteron, the initial kaon can collide with the neutron, give energy to this neutron, hence losing energy such that in a rescattering with the proton it can produce the  $\Lambda(1405)$ . Since the production is done with a proton, one predicts that the  $\Lambda(1405)$  produced is the one that appears at energies around 1420 MeV and narrow in the theoretical framework of [2], which is also supported by all chiral dynamical works on the issue (see [4] for a recent update). The work of [3] was extended in [5] to study the  $\Lambda(1405)$  production at the lower energies of DAFNE, where the experiment was still predicted to be successful if forward neutrons in coincidence were measured. Based on the experimental observation and the calculations of [3, 5], a proposal has been done at J-PARC [6] looking for neu-

trons in the forward direction.

In between, two more theoretical papers on the issue have appeared with different results [7, 8]. The first one uses a three body approach for the final state, in the line of Faddeev equations but truncated to second order. The authors claim that the peak observed corresponds to a threshold effect. The second one considers full Faddeev equations for the final three particles but only shapes and not absolute cross sections are presented. In this latter case it is also suggested to divide the cross section by a predicted background in order to visualize better the signal of the resonance that in the calculation shows up clearly.

In the present work we present two studies based on the Watson expansion and Faddeev equations, truncated to second order, and we observe that the Watson expansion is more realistic than the truncated Faddeev approach, since it considers the potential energy of the nucleons in the deuteron, which is neglected in [7]. We show that the Watson expansion is equivalent to the one used in [3], thus reconfirming the results of that work. In addition we make predictions for forward neutrons, apart from the angle integrated cross sections, which should be useful in the planning of the J-PARC experiment.

FIG. 1. Kinematics of the  $K^-d \rightarrow \pi\Sigma n$ .FIG. 2. Diagrams for the calculation of the  $K^-d \rightarrow \pi\Sigma n$  reaction.

## II. FORMULATION

We consider the  $K^-d \rightarrow \pi\Sigma n$  reaction, in which the  $\Lambda(1405)$  resonance is produced by the  $\bar{K}N$  channel in the intermediate state and decays to  $\pi\Sigma$  being observed in the final state (see Fig. 1). Because the  $\Lambda(1405)$  is located below the threshold of  $\bar{K}N$ , in order to create the  $\Lambda(1405)$  by the  $\bar{K}N$  channel one needs nuclear targets. Here we take a deuteron target, which is the simplest nucleus. In this reaction, since the strangeness is brought into the system by the incident kaon from the outside and the flow of the strangeness is traceable, one can confirm that the  $\Lambda(1405)$  resonance is produced selectively by the  $\bar{K}N$  channel. This is an advantage of the kaonic production over photo and pionic production in which the strangeness should be created inside of the system and the  $\Lambda(1405)$  can be produced by both  $\bar{K}N$  and  $\pi\Sigma$  channels. Here we consider in-flight incident kaons in order to avoid the  $\bar{K}N$  threshold contribution which contaminates the  $\Lambda(1405)$  spectrum (see Ref. [5] for details).

In Ref. [3, 5], the cross section of this process was calculated by considering the Feynman diagram shown in Fig. 2. The total transition amplitude is given by summing up these three contributions:

$$\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3. \quad (1)$$

The left diagram of Fig. 2 corresponds to the impulse approximation in which the  $\Lambda(1405)$  is produced by the incident kaon and the bound proton in the deuteron. The transition amplitude calculated in the rest frame of the deuteron target was obtained in Ref. [3] as

$$\mathcal{T}_1 = \hat{t}_{01}(M_{\pi\Sigma})\tilde{\varphi}(\vec{p}_n) \quad (2)$$

with  $\hat{t}_{01}$  the scattering amplitude of the  $K^-p \rightarrow \pi\Sigma$ ,  $M_{\pi\Sigma}$  the invariant mass of  $\pi\Sigma$  and  $\tilde{\varphi}$  the deuteron wave function in momentum space. The kinematical variables are defined in Fig. 1. The middle and right diagrams shown in Fig. 2 are for the double scattering contributions. There the kaon which scatters with one of the nucleons of the deuteron rescatters with the other nucleon and creates the  $\Lambda(1405)$ . In fact, it has turned out that these double scattering processes dominate the  $\Lambda(1405)$  production for in-flight kaons because the energetic incident kaons can lose the energy by kicking out one of the nucleons and create the  $\Lambda(1405)$  below the  $\bar{K}N$  threshold. The transition amplitudes were calculated in Ref. [3] as

$$\mathcal{T}_2 = \hat{t}_{01}(M_{\pi\Sigma}) \int \frac{d^3q}{(2\pi)^3} \frac{\tilde{\varphi}(\vec{q} + \vec{p}_n - \vec{k})}{q^2 - m_K^2 + i\epsilon} t_{02}(W_1), \quad (3)$$

$$\mathcal{T}_3 = -\hat{t}_{02}(M_{\pi\Sigma}) \int \frac{d^3q}{(2\pi)^3} \frac{\tilde{\varphi}(\vec{q} + \vec{p}_n - \vec{k})}{q^2 - m_K^2 + i\epsilon} t_{01}(W_1), \quad (4)$$

where  $t_{01}$ ,  $t_{02}$ ,  $\hat{t}_{01}$ , and  $\hat{t}_{02}$  are the two-body scattering amplitudes of  $K^-p \rightarrow \bar{K}^0n$ ,  $K^-n \rightarrow K^-n$ ,  $K^-p \rightarrow \pi\Sigma$ , and  $\bar{K}^0n \rightarrow \pi\Sigma$ , respectively, and the energies  $q^0$  and  $W_1$  are given by

$$q^0 = M_N + k^0 - p_n^0, \quad (5)$$

$$W_1 = \sqrt{(q^0 + p_n^0)^2 - (\vec{q} + \vec{p}_n)^2}. \quad (6)$$

The minus sign appearing in Eq. (4) takes account of the isospin configuration of the nucleons in the deuteron. The details of the derivation of the transition amplitudes and the approximations done were shown in Refs. [3, 5]. In the calculation given in Refs. [3, 5] the two-body scattering amplitudes are calculated by purely two-body dynamics based on the chiral unitary approach, and they depend only on the invariant mass carried by the interacting pair.

The issue raised in Ref. [7] is how one should calculate the energy of the exchange kaon,  $q^0$ , in the loop of the double scattering diagram with bound particles. Here it is not our intention to derive an exact formulation which takes into account all the contributions but to find an efficient approximation to treat the bound nucleons by considering a few diagrams. One has the Faddeev approach as one of the exact treatments of this reaction by considering the  $K^-d$  scattering as a three-body dynamical problem. Therefore solving the equation following the Faddeev approach to all orders with given two-body dynamics one would obtain an exact solution of the  $K^-d \rightarrow \pi\Sigma n$ .

In fact, the prescription given by Eq. (5) is based on the Watson formalism [9] for reactions with bound particles. Here we compare the Watson and Faddeev approaches based on Ref. [10]. To make the formulation simpler let us consider a  $K^-d \rightarrow \Lambda(1405)n$  transition in which the deuteron is a bound state of a proton and a neutron and the  $\Lambda(1405)$  is a bound state of  $\bar{K}N$ . The coupled channels effect of  $\bar{K}N$  and  $\pi\Sigma$  is irrelevant to the present discussion and can be implemented into the two-body

dynamics straightforwardly in certain ways. We assume that there exist only two-body forces between  $p$ ,  $n$  and  $\bar{K}$ , and we label  $\bar{K}$ ,  $p$  and  $n$  in the initial state as particle 0, 1 and 2, respectively. The system is described by the total Hamiltonian of the three-body system

$$H = K_0 + K_1 + K_2 + v_{01} + v_{02} + v_{12} , \quad (7)$$

where  $v_{ij}$  represents the potential between the particle  $i$  and  $j$  and  $K_i$  is the kinetic energy operator for particle  $i$ . We take non-relativistic kinematics for simplicity and just the  $K^-pn$  channel. Later on we shall generalize it to have  $K^-p \rightarrow \pi\Sigma$  in the last step. For the Watson formalism we define also the Hamiltonian of the unperturbed systems for the deuteron as

$$H_d = K_0 + (K_1 + K_2 + v_{12}) = K_0 + H_{12} . \quad (8)$$

The transition operator for the  $K^-d \rightarrow K^-pn$  process can be written based on the Watson equation for the deuteron target with  $A = 2$  as

$$T = T_{01}^d + T_{02}^d \quad (9)$$

with the coupled equations

$$T_{01}^d = \tau_{01} + \tau_{01}G_dT_{02}^d , \quad (10)$$

$$T_{02}^d = \tau_{02} + \tau_{02}G_dT_{01}^d . \quad (11)$$

The operator  $\tau_{0i}$  satisfies

$$\tau_{0i} = v_{0i} + v_{0i}G_d\tau_{0i} = v_{0i} + \tau_{0i}G_dv_{0i} \quad (12)$$

with the resolvent of  $H_d$

$$G_d \equiv [E - H_d + i\epsilon]^{-1} . \quad (13)$$

Note that the  $\tau_{0i}$  is obtained with the unperturbed resolvent  $G_d$  for the deuteron in the Watson approach instead

of the free three-body Green's function in the Faddeev approach.

The transition operator can be also written in the multiple scattering structure

$$T = \tau_{01} + \tau_{02} + \tau_{01}G_d\tau_{02} + \tau_{02}G_d\tau_{01} + \dots \quad (14)$$

Taking the deuteron wave function and the  $K^-$  plain wave in the initial state and the plain waves for the three particles in the final state, we find that the first term of Eq. (14) corresponds to the amplitude  $\mathcal{T}_1$  and the third and fourth terms do to  $\mathcal{T}_2$  and  $\mathcal{T}_3$  after approximating  $\tau_{0i}$  as the free two-body scattering operator  $t_{0i}$  given by the two-body scattering equation

$$t_{ij} = v_{ij} + v_{ij}G_0t_{ij} = v_{ij} + t_{ij}G_0v_{ij} \quad (15)$$

for  $i \neq j$  with the free Green's function  $G_0 = [E - H_0 + i\epsilon]^{-1}$ . Here it is important to note that in the Watson formalism the double scattering process is calculated with the Green's function  $G_d = [E - K_0 - K_1 - K_2 - v_{12} + i\epsilon]^{-1}$  in which the potential energy for the nucleons also appears together with the kinetic energies. Generalizing the approach to have  $\pi\Sigma$  in the final state, the  $\mathcal{T}_2$  amplitude can be obtained in the Watson formulation as

$$\mathcal{T}_2 = \langle \pi\Sigma n | t_{01}G_d t_{02} | K^-d \rangle \quad (16)$$

$$= \int \frac{d^3q}{(2\pi)^3} \langle \pi(p_\pi)\Sigma(p_\Sigma) | t_{01} | K^-(q)p(p_1) \rangle \\ \times \langle K^-pn | G_d | K^-pn \rangle \\ \times \langle K^-(q)n(p_n) | t_{02} | K^-(k)n(p_2) \rangle \varphi(\vec{p}_2) \quad (17)$$

where the matrix element of the Green's function operator can be calculated as

$$\langle K^-pn | G_d | K^-pn \rangle = \\ = \langle K^-pn | \frac{1}{E_{\text{tot}} - K_0 - K_1 - K_2 - V_{12} + i\epsilon} | K^-pn \rangle \quad (18)$$

$$= \langle K^-pn | \frac{1}{E_{\text{tot}} - K_0 - (K_1 + \frac{1}{2}V_{12}) - (K_2 + \frac{1}{2}V_{12}) + i\epsilon} | K^-pn \rangle \quad (19)$$

$$= \frac{1}{M_d + k^0 - \omega_K - (M_N + \frac{\vec{p}_1^2}{2M_N} + \frac{1}{2}V_{NN}) - p_n^0 + i\epsilon} \quad (20)$$

$$\equiv \frac{1}{q^0 - \omega_K + i\epsilon} \quad (21)$$

where  $\omega_K = \sqrt{m_K^2 + \vec{q}^2}$  and

$$q^0 = M_d + k^0 - \left( M_N + \frac{\vec{p}_1^2}{2M_N} + \frac{1}{2}V_{NN} \right) - p_n^0. \quad (22)$$

In Eq. (20) we have considered that  $K_2 + \frac{1}{2}V_{12}$  should be the total energy of the outgoing neutron. Recalling that the sum of averages of the kinetic energy and potential of the nucleon in the bound state is given by minus a half of the binding energy and neglecting the

small deuteron binding energy, we find that  $q^0$  is given by Eq. (5). Therefore the prescription (5) is based on the Watson formalism.

The equivalent transition operators to Eq. (9) can be obtained in terms of  $t_{ij}$  given in Eq. (15), instead of  $\tau_{ij}$  calculated with  $G_d$ , in the following way [10]: Let us define

$$\tilde{T}_{ij}^d \equiv v_{ij} G G_d^{-1} \quad (23)$$

for  $i \neq j$  with the full Green's function  $G = [E - H + i\epsilon]^{-1}$ . The operator  $\tilde{T}_{01}^d$  and  $\tilde{T}_{02}^d$  satisfy the coupled equations (10) and (11) with the help of the resolvent identity

$$G = G_d + G_d(v_{01} + v_{02})G. \quad (24)$$

Therefore we find  $\tilde{T}_{0i}^d = T_{0i}^d$ . Hereafter we use  $T_{ij}^d$  instead of  $\tilde{T}_{ij}^d$ . The full propagator can be also written as

$$G = G_0 + G_0(v_{01} + v_{02} + v_{12})G. \quad (25)$$

Inserting Eq. (25) into Eq. (23), we have

$$T_{01}^d = v_{01} G_0 G_d^{-1} + v_{01} G_0 (T_{01}^d + T_{02}^d + T_{12}^d). \quad (26)$$

Multiplying  $(1 + t_{01} G_0)$  to both sides of Eq. (26) from the left and using Eq. (15), we obtain

$$T_{01}^d = t_{01} G_0 G_d^{-1} + t_{01} G_0 (T_{02}^d + T_{12}^d). \quad (27)$$

Similarly we have

$$T_{02}^d = t_{02} G_0 G_d^{-1} + t_{02} G_0 (T_{01}^d + T_{12}^d), \quad (28)$$

$$T_{12}^d = t_{12} G_0 G_d^{-1} + t_{12} G_0 (T_{01}^d + T_{02}^d). \quad (29)$$

Equations (27), (28) and (29) are one of the expressions of the Faddeev equation [11, 12]. As seen in these equations, the three-body transition operator in the Faddeev approach can be written as the free three-body Green's function  $G_0$  and the free two-body scattering operator  $t_{ij}$  given in Eq. (15).

If one makes the multiple scattering expansion of the transition operator given in the Faddeev approach, one finds

$$T = t_{01} + t_{02} + t_{01} G_0 t_{02} + t_{02} G_0 t_{01} + \dots \quad (30)$$

where we have used  $G_0 G_d^{-1} = 1 - G_0 v_{12}$ . If we compare the multiple scattering expansions obtained in the Watson and Faddeev approaches, (14) and (30), we find that each term in the expansions is different in these two formulations, and especially, the scattering process in the Faddeev approach is calculated with the free Green's function.

Let us write down the Watson amplitude (14) in terms of  $t_{ij}$  and  $G_0$ , which are the building blocks of the Faddeev approach. Considering Eq. (15) and  $G_d = G_0 + G_0 v_{12} G_d$ , we have the identity

$$G_d = G_0 + G_0 t_{12} G_0. \quad (31)$$

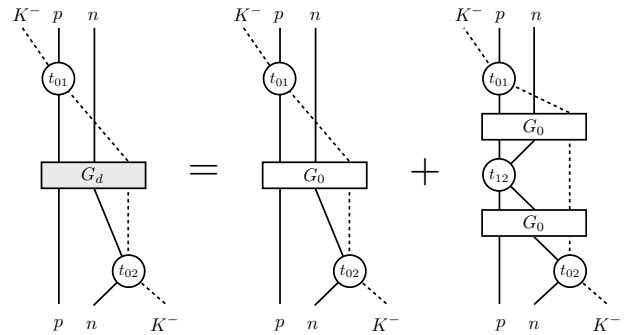


FIG. 3. Schematic diagrams for Eq. (34). The solid and dashed lines denote nucleons (proton and neutron) and kaon, respectively.  $G_d$  and  $G_0$  stand for the Green's function of the unperturbed deuteron system and the free three body, respectively, and  $t_{ij}$  means the two-body transition amplitude. The double scattering contribution in the Faddeev approach contains only the middle diagram, while the double scattering in the Watson formulation have both middle and right diagrams.

Inserting this identity into Eq. (12) and recalling that  $t_{0i} = v_{0i}/(1 - v_{0i} G_0)$  we find

$$\tau_{0i} = t_{0i} + t_{0i} G_0 t_{12} G_0 \tau_{0i} \quad (32)$$

$$= t_{0i} + t_{0i} G_0 t_{12} G_0 t_{0i} + t_{0i} G_0 t_{12} G_0 t_{0i} G_0 t_{12} G_0 t_{0i} + \dots \quad (33)$$

This implies that the single scattering term in the Watson formalism includes the multiple scattering terms in the Faddeev approach. Also for the double scattering term in the Watson approach, say  $\tau_{02} G_d \tau_{01}$ , using Eqs. (31) and (32), we find that it includes more terms than the double scattering term in Faddeev approach. Especially, in our approximation the double scattering terms are calculated with Eq. (31) as

$$t_{01} G_d t_{02} = t_{01} G_0 t_{02} + t_{01} G_0 t_{12} G_0 t_{02} \quad (34)$$

where the first term corresponds to the double scattering term obtained in the Faddeev approach. Obviously, the Watson approach is better because it takes into account the interaction between the nucleons, as shown in Fig. 3. Finally we emphasize again that if one considers all of the terms of the multiple scattering expansion, namely if one solves the equations without truncation, one should get completely identical solutions from both approaches.

### III. NUMERICAL RESULTS

In the previous section we have discussed two approaches to describe the  $K^- d \rightarrow \pi \Sigma n$  reaction from the viewpoint of a three-body dynamical problem. One is the Watson approach and the other is the Faddeev approach. They are equivalent to each other if one takes into account all orders of the multiple scatterings, but they are different from each other if one truncates the

multiple scatterings at some finite order. Especially the double scattering process in the Watson approach has more terms than that in the Faddeev approach. Generally, each term of the multiple scattering expansion has different contributions in both approaches, but in practice it could give very similar contributions in some systems with certain kinematical conditions.

In this section, we compare the double scattering terms obtained by the Faddeev and Watson approaches by taking the  $K^-d \rightarrow \pi\Sigma n$  reaction as an example. In the present study we employ the chiral unitary approach to evaluate the two-body meson-baryon dynamics and use physical masses for ground-state hadrons, which slightly breaks the isospin symmetry. We consider scattering amplitudes for the  $K^-d \rightarrow \pi\Sigma n$  reaction up to the double scattering terms both in the Watson and Faddeev approaches. An important difference in the two approaches is the treatment of the Green's function in the intermediate states. The single scattering term in our approximation is exactly the same as that obtained in the Faddeev formulation.

In the Watson approach one uses the Green's function for the deuteron  $G_d$  in the intermediate states, therefore in the Watson approach the energy of the exchanged kaon in the double scattering are expressed as,

$$q_{(A)}^0 = M_N + k^0 - p_n^0, \quad (35)$$

to which we refer as case A. This is the exchanged kaon energy in double scattering which has been used in Refs. [3, 5].

In contrast, in the Faddeev approach one uses the free Green's function  $G_0$  in the intermediate state, hence the on-shell nucleons, which go for the second scattering, can appear in the intermediate state in the double scattering. In the case for the double scattering, we have the expression for  $q^0$  (case B) as,

$$q_{(B)}^0 = M_N + k^0 - p_n^0 - \frac{|\vec{q} + \vec{p}_n - \vec{k}|^2}{2M_N}. \quad (36)$$

This is the exchanged kaon energy in double scattering which the authors in Ref. [7] have used.

The difference of two expressions for the exchanged kaon energy  $q^0$  can be interpreted as how to implement the binding effect on two nucleons in a deuteron. Namely, in the Watson approach the potential between proton and neutron is taken into account and hence the secondary scattering nucleon in the intermediate state keeps off-shell in the double scattering. Equation (35) can be understood so that the kinetic energy of the secondary scattering nucleon is cancelled almost by the potential energy owing to the small binding energy. In contrast, in the Faddeev approach the effect of the bound nucleons is not considered and the secondary scattering nucleon in the intermediate state goes on-shell, meaning that the potential energy is neglected in the double scattering and the binding of nucleons would be accounted separately in other terms of the multiple scattering expansion.

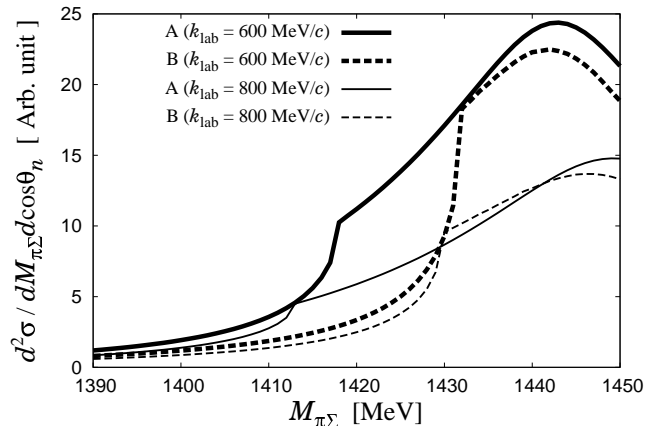


FIG. 4. Differential cross section  $d^2\sigma/dM_{\pi\Sigma}d\cos\theta_{\text{cm}}^n$  coming from diagram 2 in Fig. 2 for the reaction  $K^-d \rightarrow \pi^-\Sigma^+n$  with a constant  $\bar{K}N \rightarrow \pi\Sigma$  amplitude (38). Here we take two initial kaon momenta,  $k_{\text{lab}} = 600$  and  $800$  MeV/c, and consider two intermediate kaon energy  $q^0$  [A (35) and B (36)] coming from the Watson and Faddeev approaches, respectively.

Now let us perform numerical calculations of the single plus double scattering for the  $K^-d \rightarrow \pi\Sigma n$  reaction by using  $q^0$  of the above prescriptions A and B. For simplicity we approximate  $W_1$  in Eq. (6) as,

$$W_1 \approx \sqrt{(M_N + k^0)^2 - |\vec{k}|^2}, \quad (37)$$

bearing in mind that the first scattering amplitude,  $t_{01}$  and  $t_{02}$  [see Fig. 2 and Eqs. (3) and (4)], do not make particular structures in the cross sections.

First of all, we consider the differential cross section for the center-of-mass neutron scattering angle  $\theta_{\text{cm}}^n = 0^\circ$ , on which the authors in Ref. [7] concentrated. Here, in order to see the structure created by the underlying kinematical features of the amplitudes rather than by the shape of the  $\Lambda(1405)$ , we take the  $\bar{K}N \rightarrow \pi\Sigma$  scattering amplitude appearing in the second scatterings as,

$$\hat{t}_{01} = \hat{t}_{02} = \text{const.} \quad (38)$$

The result of the differential cross section coming from diagram 2 of Fig. 2 for the  $K^-d \rightarrow \pi^-\Sigma^+n$  reaction with a constant  $\bar{K}N \rightarrow \pi\Sigma$  amplitude is plotted in Fig. 4, which corresponds to Fig. 9 of Ref. [7]. The initial kaon momentum  $k_{\text{lab}}$  is fixed as  $k_{\text{lab}} = 600$  and  $800$  MeV/c. From Fig. 4, we find a cusp structure in both cases of A and B. The cusp structure comes from the three-body unitarity cut for the intermediate  $K^-pn$  system as pointed out in Ref. [7]. It is important noting that the cusp position depends on the prescription of the intermediate kaon energy,  $q_{(A)}$  or  $q_{(B)}$ . While the cross section rapidly rises around  $M_{\pi\Sigma} \sim 1430$  MeV in case B with initial kaon momentum  $k_{\text{lab}} = 600$  MeV/c, this effect becomes moderate in case A with the same  $k_{\text{lab}}$ . Bearing in mind that

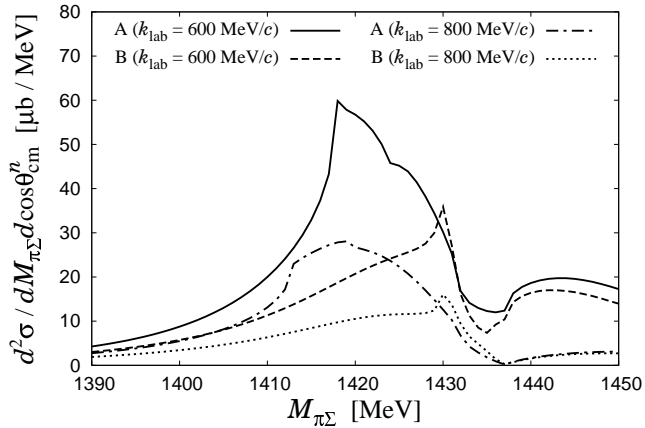


FIG. 5. Differential cross section  $d^2\sigma/dM_{\pi\Sigma}d\cos\theta_{\text{cm}}^n$  of the reaction  $K^-d \rightarrow \pi^-\Sigma^+n$  with  $\bar{K}N \rightarrow \pi\Sigma$  amplitude in chiral dynamics. Here we take two initial kaon momenta,  $k_{\text{lab}} = 600$  and  $800$  MeV/ $c$ , and consider two intermediate kaon energy  $q^0$  [A (35) and B (36)] coming from the Watson and Faddeev approaches, respectively.

the Watson approach contains more terms compared to the Faddeev approach up to the double scattering, we expect that, even if the cusp from the three-body unitary cut appears in the cross section, it will be more moderate than predicted in Ref. [7]. This moderation can be interpreted to be caused by the fact that a more accurate energy sharing of two bound nucleons in the deuteron is accomplished with the nonperturbative Green's function for the deuteron.

From the results of the initial kaon momentum  $k_{\text{lab}} = 800$  MeV/ $c$  with constant  $\bar{K}N \rightarrow \pi\Sigma$  amplitude, we see that more moderate cusp structures are obtained for higher initial kaon momentum, and the position shifts to lower energies for higher momentum. We have also checked the angle  $\theta_{\text{cm}}^n$  dependence for the cusps and have found that the cusp position moves to lower energies, which will be important when we integrate the angle to obtain the mass spectrum for the  $\Lambda(1405)$ .

Next let us calculate the differential cross section  $d^2\sigma/dM_{\pi\Sigma}d\cos\theta_{\text{cm}}^n$  at  $\theta_{\text{cm}}^n = 0^\circ$  with the actual  $\bar{K}N \rightarrow \pi\Sigma$  amplitude for  $\hat{t}_{01}$  and  $\hat{t}_{02}$  in chiral dynamics summing up all the three diagrams. The results are plotted Fig. 5 for the initial kaon momenta of  $600$  MeV/ $c$  and  $800$  MeV/ $c$ . The reason that we have two cusps in each line is that these come from the unitary cuts of the  $\bar{K}^0nn$  and  $K^-pn$  intermediate states of diagram 2 and 3, respectively. In Fig. 5 we see that the  $\Lambda(1405)$  peak appears in case A in both momentum cases. In spite of the cusps coming from the unitary cuts in  $d^2\sigma/dM_{\pi\Sigma}d\cos\theta_{\text{cm}}^n$  at around  $M_{\pi\Sigma} \sim 1420$  MeV, in case A this does not spoil the  $\Lambda(1405)$  spectrum. The cusps in the differential cross section become moderate in the higher momentum, as expected from Fig. 4. In case B, on the other hand, the

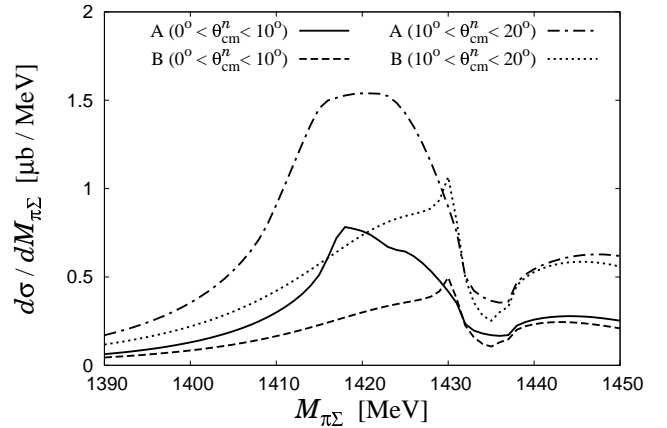


FIG. 6. Invariant mass spectrum  $d\sigma/dM_{\pi\Sigma}$  of the reaction  $K^-d \rightarrow \pi^-\Sigma^+n$  with the initial kaon momentum  $k_{\text{lab}} = 600$  MeV/ $c$ . Here the neutron scattering angle is integrated for two forward ranges,  $0^\circ < \theta_{\text{cm}}^n < 10^\circ$  and  $10^\circ < \theta_{\text{cm}}^n < 20^\circ$ , and consider two intermediate kaon energies  $q^0$  [A (35) and B (36)] coming from the Watson and Faddeev approaches, respectively.

rapid rise plotted in Fig. 4 around  $1430$  MeV would spoil the  $\Lambda(1405)$  peak in the differential cross section. However, as discussed before, the rapid rise plotted in Fig. 4 and hence the spoiled  $\Lambda(1405)$  structure in Fig. 5 originate from the use of the free Green's function in the intermediate state. Therefore, by taking more terms as the Watson approach does, one can obtain the  $\Lambda(1405)$  peak in the differential cross section. Note, however, that even if the shape of the  $\Lambda(1405)$  is not well reproduced in case B, the presence of the resonance has had an effect on the peaks, shifting the strength of Fig. 4 from around  $1430$  MeV– $1450$  MeV to  $1420$  MeV– $1430$  MeV in Fig. 5.

Up to now we have considered only the limited forward angle of the emitted neutron  $\theta_{\text{cm}}^n = 0^\circ$ . This is not realistic, because in actual experiments one will observe the neutron in finite angles. Let us see the finite angle contribution. We calculate the invariant mass spectra by integrating the angular dependence. Since we are especially interested in the forward scattering with small  $\theta_{\text{cm}}^n$ , where the  $\Lambda(1405)$  production will be large due to the double scattering processes [3, 5], we plot in Fig. 6 the invariant mass spectrum at  $k_{\text{lab}} = 600$  MeV/ $c$  for angles  $0^\circ < \theta_{\text{cm}}^n < 10^\circ$  and  $10^\circ < \theta_{\text{cm}}^n < 20^\circ$ . From the figure, in case A, even in the scattering angle with  $0^\circ < \theta_{\text{cm}}^n < 10^\circ$  the cusps at the  $\Lambda(1405)$  peak are smeared due to the angular dependence for the cusps, and with  $10^\circ < \theta_{\text{cm}}^n < 20^\circ$  the cusps are already negligible. Since  $\theta_{\text{cm}}^n = 10^\circ$  in our condition corresponds to the neutron scattering angle  $\theta_{\text{lab}}^n \sim 5^\circ$  in the laboratory frame, this result indicates that the cusps structure does not contaminate the  $\Lambda(1405)$  peak structure if the neutron detector in experiments is located in the forward

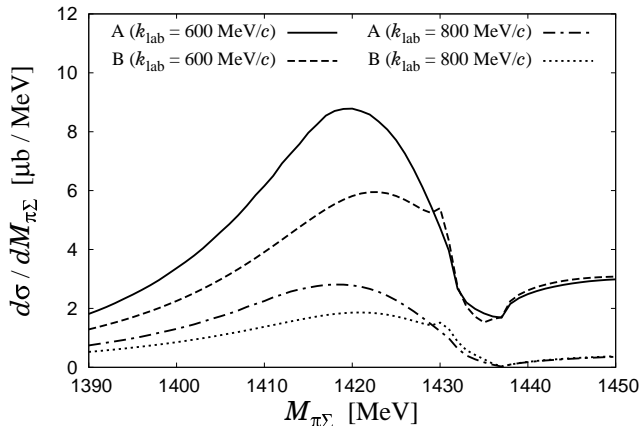


FIG. 7. Invariant mass spectrum  $d\sigma/dM_{\pi\Sigma}$  of the reaction  $K^-d \rightarrow \pi^-\Sigma^+n$  integrated for the whole neutron scattering angle. Here we take two initial kaon momenta,  $k_{\text{lab}} = 600$  and  $800$  MeV/c, and consider two intermediate kaon energy  $q^0$  [A (35) and B (36)] coming from the Watson and Faddeev approaches, respectively.

angle of the reaction with  $\theta_{\text{lab}}^n \gtrsim 5^\circ$ . For comparison we also show the result for case B, which indicates that in the Faddeev approach up to the double scattering the cusps depend slightly on the scattering angle.

Finally we show the invariant mass spectrum integrated for the whole angle in Fig. 7. In this case the cusp structures for case A having appeared in previous figures completely disappear due to the angular dependence for the three-body unitary cut, and we observe the  $\Lambda(1405)$  peak structure also in case B.

With respect to the paper of [8] a few comments are in order. The input to construct the two body amplitudes is taken from [13], where an energy independent separable potential is used. This neglects the important energy dependence in the interaction provided by chiral theories. In any case, in the paper it is unclear how the initial state  $K^-d$  is produced and how does it couple to the final state. It would be important to clarify this in view of our arguments that the potential energy of the nucleons of the deuteron plays a role in the energy denominators that appear in the final formulas. The paper adds a new interesting observation, realizing that the kinematic peaks appear because of threshold of the reaction, and then suggest to divide the cross section by another one that has only background and no dynamical amplitudes. It is shown there that after division by this new spectra the cross sections reflects the properties of the resonance clearly. There is only one problem to it. This procedure is model dependent. Within a certain model the kinematical spikes disappear with this procedure, but the spikes depend on the model that one is using. If one wishes to apply the method to a given experimental spectrum one is forced to choose some model

and the procedure is bound to create model dependent spikes rather than eliminate them. Hence, the method is not advisable for an experimental analysis.

#### IV. CONCLUSIONS

In this paper we have analyzed the  $K^-d \rightarrow \pi\Sigma n$  reaction for kaons in flight, looking for the cross section for forward neutron angles for which a proposal is prepared for J-PARC. At the same time we take advantage to introduce two new papers on the issue recently done and discuss the meaning of their approach to the light of two expansions, the Watson expansion of multiple scattering and the truncated Faddeev approach. We realized that in the truncated Faddeev approach the potential energy of the nucleons in the deuteron is neglected. On the other hand, the Watson approach of multiple scattering, which was used in [3], takes into account this information and represents a more suitable approach to the problem. Because in the Faddeev formalism the three particles are treated democratically, it is insufficient to consider only the kaon double scattering contributions, in which nucleon-nucleon interactions are not taken into account. Thus, when one considers a bound particle of two particles in the initial state in the Faddeev approach, one must consider contributions beyond the kaon double scattering contributions in order to take into account the  $NN$  interaction properly. In the Watson formulation, the bound particle is treated separately, and thus one has an efficient multiple scattering expansion scheme. In any case we also showed that the peak observed in [7], related there to threshold effects, is actually determined by the excitation of the  $\Lambda(1405)$  but somewhat distorted. This we could see by changing the input and removing the  $\Lambda(1405)$  in the amplitudes that we use, and then we realize a substantial shift of the peak in all the approaches. We also observe that in all approaches, when the neutron angle is integrated, the threshold peaks are washed away and a clear signal of the  $\Lambda(1405)$  resonance shows up, though in the Watson approach the curves are smoother and the predictions are deemed more accurate.

The present results clarify the situation and present a clear case for the experimental investigation of this reaction following the lines of the successful work of [1].

#### ACKNOWLEDGMENTS

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