Charged lepton induced one kaon production off the nucleon

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Abstract

We study single kaon production off the nucleon induced by electrons (positrons) i.e. $e^-(e^+) + N \rightarrow \nu_e(\bar{\nu}_e) + \bar{K}(K) + N'$ at low energies. The possibility of observing these processes with the high luminosity beams available at TJNAF and Mainz is discussed, taking into account that the strangeness conserving electromagnetic reactions have a higher energy threshold for $\bar{K}(K)$ production. The calculations are done using a microscopic model that starts from the SU(3) chiral Lagrangians, includes background terms and the resonant mechanisms associated to the lowest lying resonance $\Sigma^*(1385)$.

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I. INTRODUCTION

Recently the importance of the study of kaon production induced by real and virtual photons on nucleons and nuclei has been emphasized due to the development of accelerators like MAMI, JLAB, LNS, ELSA, SPring-8, GRAAL, etc. [1–11]. In particular, the availability of very high luminosity beams has provided the opportunity to study the electromagnetic associated strangeness production [12–22] of a strange and an anti-strange particle.

The cross section for weak associated strangeness production is obviously much smaller than that of the electromagnetic one. However, weak interaction allows for processes where only one strange/anti-strange particle is produced, ($\Delta S = \pm 1$) and these reactions could have a substantially lower threshold. For instance, the threshold for electron induced weak K^- production on a proton is around 600 MeV whereas it is 1.5 GeV for electromagnetic production, as an additional kaon is required.

The study of these reactions could provide valuable information on the coupling constants D and F that govern the interaction of the SU(3) lightest baryon octet with the pseudoscalar mesons and also their β decays. More specifically, the $g_A(= D + F)$ combination, related to the neutron β decay, is very well known, but the knowledge of the D and F values is less precise [23]. Also, one may investigate the Q^2 dependence of the weak axial form factors of nucleons and hyperons. In this paper, we explore the possibility of doing such experiments and present a quantitative analysis of the charged current reaction in which a kaon/antikaon is produced without conserving ΔS. This study is based on our earlier works on neutrino/antineutrino induced single kaon production [24–26] and the same formalism is applied here to study one kaon production off the nucleon obtained from electron as well as positron beams.

We proceed by introducing the formalism in brief in Sec. II. Results and discussions are presented in Sec. III.

II. FORMALISM

The processes considered here are the charged lepton induced weak $|\Delta S| = 1$ $K(\bar{K})$ production. The single antikaon production channels induced by electrons are

$$e^- + n \to \nu_e + K^- + n, \;\; e^- + p \to \nu_e + \bar{K^0} + n, \;\; e^- + p \to \nu_e + K^- + p, \tag{1}$$

and the corresponding positron induced channels are

$$e^+ + n \to \bar{\nu}_e + K^+ + n, \;\; e^+ + n \to \bar{\nu}_e + K^0 + p, \;\; e^+ + p \to \bar{\nu}_e + K^+ + p \tag{2}$$

The expression for the differential cross section in the laboratory frame for the above processes is given by

$$d^9\sigma \;=\; \frac{1}{4ME_e(2\pi)^5}\frac{d\vec{k}'}{(2E_\nu)}\frac{d\vec{p}\,'}{(2E'_p)}\frac{d\vec{p}_k}{(2E_k)}\delta^4(k+p-k'-p'-p_k)\bar{\Sigma}\Sigma|\mathcal{M}|^2, \tag{3}$$

where $k(k')$ is the momentum of the incoming(outgoing) lepton with energy $E_e(E_\nu)$, $p(p')$ is the momentum of the incoming(outgoing) nucleon with mass M. The kaon 3-momentum is $\vec{p}_k$ having energy E_k, $\bar{\Sigma}\Sigma|\mathcal{M}|^2$ is the square of the transition amplitude averaged(summed) over the spins of the initial(final) state. The transition amplitude may be written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, j_\mu J^\mu = \frac{g}{2\sqrt{2}} j_\mu \frac{1}{M_W^2}\frac{g}{2\sqrt{2}} J^\mu, \tag{4}$$

where j_μ and J^μ are the leptonic and hadronic currents respectively, $G_F = \sqrt{2}\frac{g^2}{8M_W^2}$ is the Fermi coupling constant, g is the gauge coupling and M_W is the mass of the W-boson.

First, we shall discuss the leptonic current, the hadronic current and the transition amplitude corresponding to the reactions shown in Eq. 1. The leptonic current is obtained from the Standard Model Lagrangian coupling of the W-boson to the leptons

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left[W_\mu^- \bar{l}\gamma^\mu(1-\gamma_5)\nu_l + W_\mu^+ \bar{\nu}_l\gamma^\mu(1-\gamma_5)l\right] = -\frac{g}{2\sqrt{2}}\left[j_{(L)}^\mu W_\mu^- + h.c.\right]. \tag{5}$$

The Feynman diagrams that contribute to the hadronic current are depicted in Fig. 1. There is a meson (πP,ηP) exchange term, a contact term (CT) and a kaon pole (KP) term. For the electron induced reactions we also have the s-channel diagrams with Σ, Λ(SC) and Σ^*(SCR) as intermediate states. In the case of positron induced reactions the s-channel diagrams (SC and SCR) do not contribute, but we must include the u-channel (UC) one. The contributions to the hadronic current coming from different terms are written using the Lagrangian obtained from chiral perturbation theory(χPT). The lowest-order SU(3) chiral Lagrangian describing the interaction of pseudoscalar mesons in the presence of an external weak current is written as [28]:

$$\mathcal{L}_M^{(2)} = \frac{f_\pi^2}{4}\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger] + \frac{f_\pi^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger), \tag{6}$$

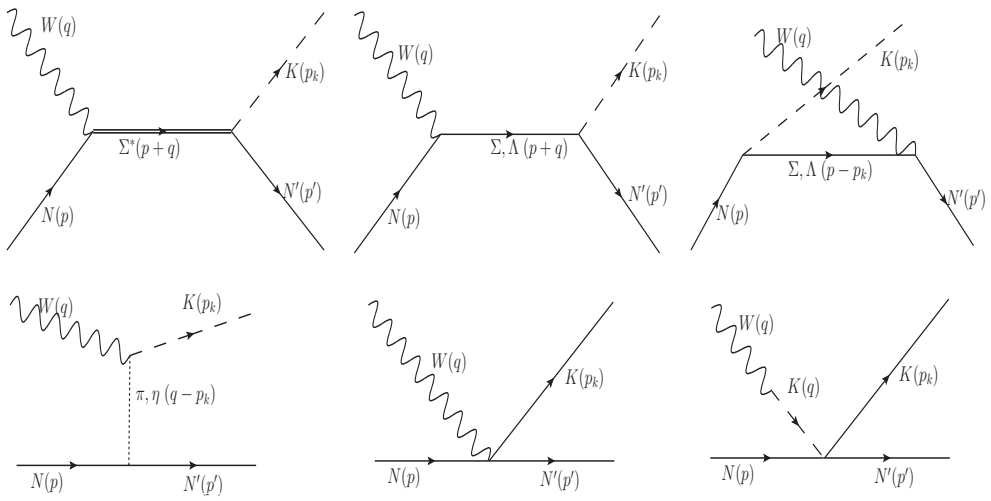

FIG. 1: Feynman diagrams for the processes $e^- N \to \nu_e N' \bar{K}$ and $e^+ N \to \bar{\nu}_e N' K$. Here $\bar{K}$ stands for a K^- or $\bar{K}^0$ obtained in an electron induced process and K stands for K^+ or K^0 obtained in a positron induced process. First row from left to right: s-channel Σ^* resonance term (labeled SCR in the text) , s-channel (SC) and u-channel (UC) Σ, Λ propagator; second row: Pion/Eta meson ($\pi P/\eta P$) exchange terms, Contact term (CT) and finally kaon pole term (KP).

where f_π(=92.4MeV) is the pion decay constant, $U(x) = \exp\left(i\frac{\phi(x)}{f_\pi}\right)$ is the SU(3) representation of the pseudoscalar meson octet fields $\phi(x)$ and $D_\mu U$ is its covariant derivative given by:

$$D_\mu U \equiv \partial_\mu U - i r_\mu U + i U l_\mu \,. \tag{7}$$

Here, l_μ and r_μ correspond to left and right handed currents, that for the charged current(CC) case are given by

$$r_\mu = 0, \quad l_\mu = -\frac{g}{\sqrt{2}}(W_\mu^+ T_+ + W_\mu^- T_-), \tag{8}$$

with $W^\pm$, the W boson fields. The elements of the Cabibbo-Kobayashi-Maskawa matrix V_{ij} (for light quarks) can be written as

$$T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad T_- = \begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix},$$

The lowest order baryon-meson interaction Lagrangian coupling to the W-boson field is given by:

$$\mathcal{L}^{(1)}_{MB} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M\right)B\right] - \frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_{\mu}, B\}\right) - \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5[u_{\mu}, B]\right), \tag{9}$$

where M denotes the mass of the baryon octet B. For the coupling constants we take the values $D = 0.80$ and $F = 0.46$ which have been determined from the baryon semileptonic decays [23].

The covariant derivative of B is given by

$$D_{\mu}B = \partial_{\mu}B + [\Gamma_{\mu}, B], \tag{10}$$

with

$$\Gamma_{\mu} = \frac{1}{2}\left[u^{\dagger}(\partial_{\mu} - ir_{\mu})u + u(\partial_{\mu} - il_{\mu})u^{\dagger}\right], \tag{11}$$

where we have introduced a new variable u, $u^2 = U$. Finally,

$$u_{\mu} = i\left[u^{\dagger}(\partial_{\mu} - ir_{\mu})u - u(\partial_{\mu} - il_{\mu})u^{\dagger}\right]. \tag{12}$$

We have also included the contribution of terms with the $\Sigma^*(1385)$ resonance belonging to the SU(3) baryon decuplet, which is near the threshold of the NK system. This is suggested by the dominant role played by the $\Delta(1232)$ in pion production reactions. For the weak excitation of the $\Sigma^*(1385)$ resonance and its subsequent decay in NK, the lowest order SU(3) Lagrangian coupling the pseudoscalar mesons with decuplet-octet baryons in presence of external weak current is given by:

$$\mathcal{L}_{dec} = \mathcal{C}\left(\epsilon^{abc}\bar{T}^{\mu}_{ade}u^{d}_{\mu,b}B^{e}_{c} + h.c.\right), \tag{13}$$

where T^{μ} is the SU(3) representation of the decuplet fields, $a - e$ are flavor indices [1].

The parameter $\mathcal{C} \simeq 1$ has been fitted to the $\Delta(1232)$ decay-width. The spin 3/2 propagator for Σ^* is given by

$$G^{\mu\nu}(P) = \frac{P^{\mu\nu}_{RS}(P)}{P^2 - M^2_{\Sigma^*} + iM_{\Sigma^*}\Gamma_{\Sigma^*}}, \tag{14}$$

[1] The physical states of the decuplet are: $T_{111} = \Delta^{++}, T_{112} = \frac{\Delta^+}{\sqrt{3}}, T_{122} = \frac{\Delta^0}{\sqrt{3}}, T_{222} = \Delta^-, T_{113} = \frac{\Sigma^{*+}}{\sqrt{3}}, T_{123} = \frac{\Sigma^{*0}}{\sqrt{6}}, T_{223} = \frac{\Sigma^{*-}}{\sqrt{3}}, T_{113} = \frac{\Xi^+}{\sqrt{3}}, T_{133} = \frac{\Xi^0}{\sqrt{3}}, T_{333} = \Omega^-$.

where $P = p + q$ is the momentum carried by the resonance, $q = k - k'$ and $P^{\mu\nu}_{RS}$ is the projection operator

$$P^{\mu\nu}_{RS}(P) = \sum_{spins} \psi^{\mu}\bar{\psi}^{\nu} = -(\not{P} + M_{\Sigma^*}) \left[g^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu} - \frac{2}{3}\frac{P^{\mu}P^{\nu}}{M^2_{\Sigma^*}} + \frac{1}{3}\frac{P^{\mu}\gamma^{\nu} - P^{\nu}\gamma^{\mu}}{M_{\Sigma^*}} \right], \quad (15)$$

with M_{Σ^*} the resonance mass and ψ^{μ} the Rarita-Schwinger spinor. The Σ^* width obtained using the Lagrangian of Eq. 13 can be written as

$$\Gamma_{\Sigma^*} \; = \; \Gamma_{\Sigma^* \to \Lambda\pi} + \Gamma_{\Sigma^* \to \Sigma\pi} + \Gamma_{\Sigma^* \to N\bar{K}} \; , \quad (16)$$

where

$$\Gamma_{\Sigma^* \to Y, meson} \; = \; \frac{C_Y}{192\pi} \left(\frac{\mathcal{C}}{f_{\pi}} \right)^2 \frac{(W + M_Y)^2 - m^2}{W^5} \lambda^{3/2}(W^2, M_Y^2, m^2)\, \Theta(W - M_Y - m). \quad (17)$$

Here, m, M_Y are the masses of the emitted meson and baryon. $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and Θ is the unit step function. The factor C_Y is 1 for Λ and $\frac{2}{3}$ for N and Σ.

Using symmetry arguments, the most general $W^- N \to \Sigma^*$ vertex can be written in terms of a vector and an axial-vector part as,

$$\begin{aligned} \langle \Sigma^*; P = p + q \,|V^{\mu}|N; p\rangle \; &= \; V_{us}\bar{\psi}_{\alpha}(\vec{P})\Gamma^{\alpha\mu}_{V}(p, q)\, u(\vec{p}), \\ \langle \Sigma^*; P = p + q \,|A^{\mu}|N; p\rangle \; &= \; V_{us}\bar{\psi}_{\alpha}(\vec{P})\Gamma^{\alpha\mu}_{A}(p, q)\, u(\vec{p}), \end{aligned} \quad (18)$$

where

$$\begin{aligned} \Gamma^{\alpha\mu}_{V}(p, q) \; &= \; \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \frac{C_4^V}{M^2}(g^{\alpha\mu} q\cdot P - q^{\alpha}P^{\mu}) + \frac{C_5^V}{M^2}(g^{\alpha\mu} q\cdot p - q^{\alpha}p^{\mu}) + C_6^V g^{\mu\alpha} \right]\gamma_5 \\ \Gamma^{\alpha\mu}_{A}(p, q) \; &= \; \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \frac{C_4^A}{M^2}(g^{\alpha\mu} q\cdot P - q^{\alpha}P^{\mu}) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^{\mu}q^{\alpha} \right]. \end{aligned} \quad (19)$$

The details of the C_i N-Σ^* transition form factors are given in Ref. [25, 27]. For all background terms, we adopt a global dipole form factor $F(q^2) = 1/(1 - q^2/M_F^2)^2$, with a mass $M_F \simeq 1.05$ GeV that multiplies the hadronic currents. Its effect, for energies of electron presently available at the accelerators will be discussed.

The final expressions of the hadronic currents j^{μ} for the electron as well as positron induced processes are given in the Appendix- A and the various parameters of the currents are shown in Table I.

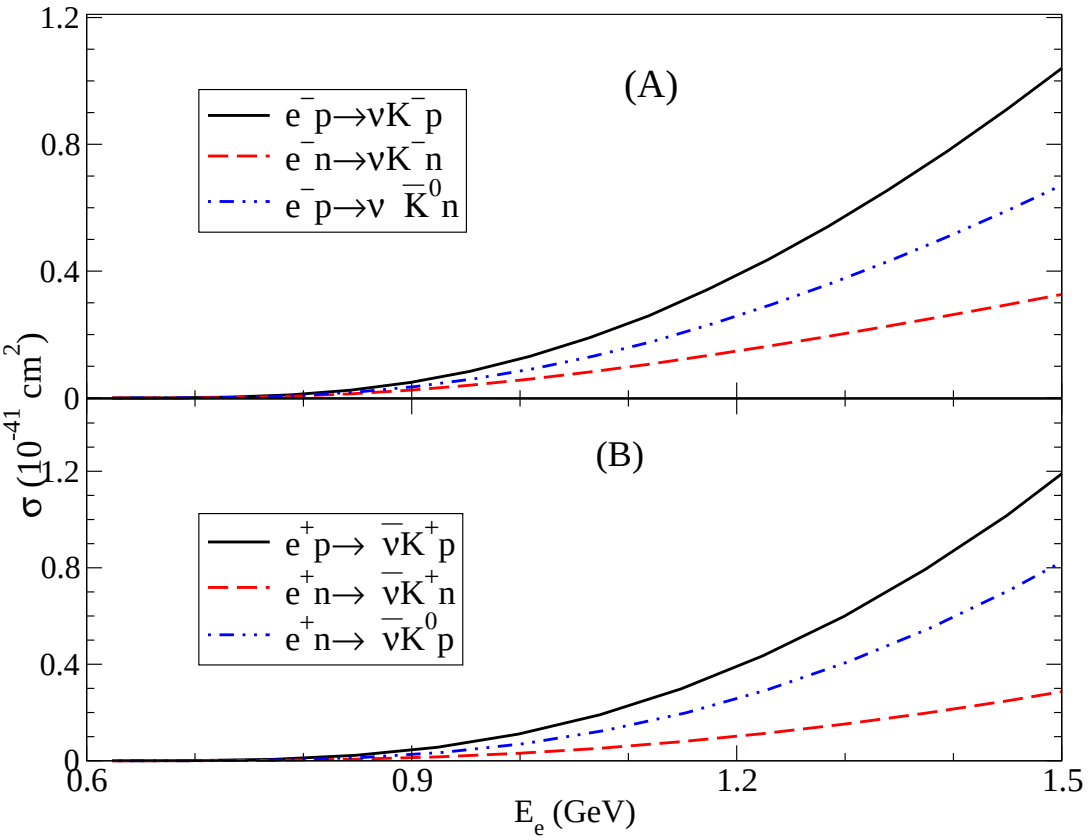

FIG. 2: Cross section σ vs electron(positron) energy E_e for the $\bar{K}(K)$ production

III. RESULTS AND DISCUSSION

Firstly, we present in Fig. 2 the results for the total scattering cross section σ for the reactions given in Eqs. 1 and 2.

We find that $e^-(e^+) + p \to \nu_e(\bar{\nu}_e) + K^-(K^+) + p$ has the largest cross section followed by $e^-(e^+) + p(n) \to \nu_e(\bar{\nu}_e) + \bar{K^0}(K^0) + n(p)$ and $e^-(e^+) + n \to \nu_e(\bar{\nu}_e) + K^-(K^+) + n$. Furthermore, we find that the cross sections for the positron induced processes are larger than for the corresponding electron induced process. This is basically due to the different interference between the s-channel and contact terms, as can be seen in Fig. 3, where we explicitly show the contribution of the individual terms of the hadronic current for two channels[2].

[2] Certainly, these individual contributions are not observable and they are shown here to help explaining the sensitivity of the physical processes to the various parameters.

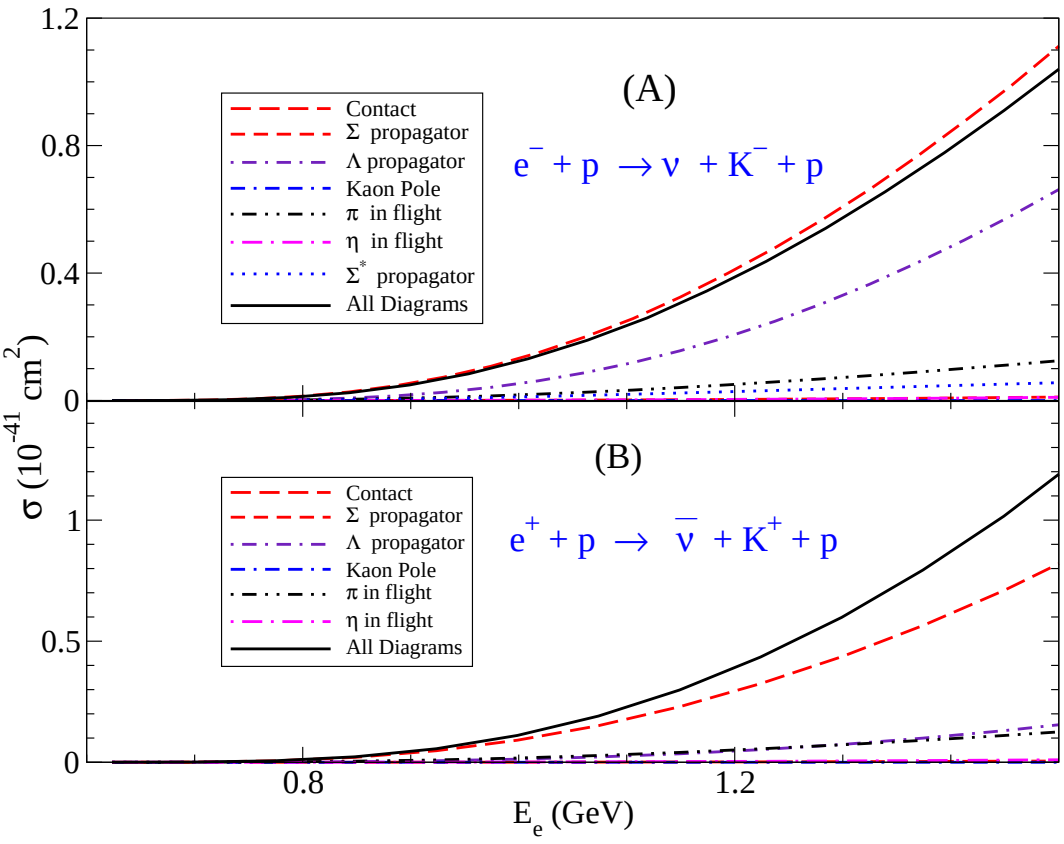

FIG. 3: Contribution of the different terms to the total cross section σ vs electron(positron) energy E_e for the $\bar{K}(K)$ production

We find that the contribution of the contact term is the largest followed by the mechanism with a Λ in the intermediate state, the π pole term, etc. The contribution due to the Σ^* in cross section for the discussed energies is quite small, for example it is around 10% at E_e =1 GeV and 5% at E_e =1.5 GeV of the total cross section. Therefore, these reactions are not suitable to learn about the $\Sigma^*(1385)$ resonance properties. The contributions of Λ intermediate states both in UC and SC are larger than those corresponding to the Σ hyperon, which can be easily understood by the corresponding Clebsch-Gordan coefficients. Similar is the trend for the other channels not shown in the figure.

Given the smallness of the resulting cross sections, it is important to consider the feasibility of their experimental measurement. Here, we will only discuss electron processes. Let us remark that at energies below 1.5 GeV and in electron induced reactions, the presence of an antikaon in the final state fully defines the process, in the sense that it is necessarily a

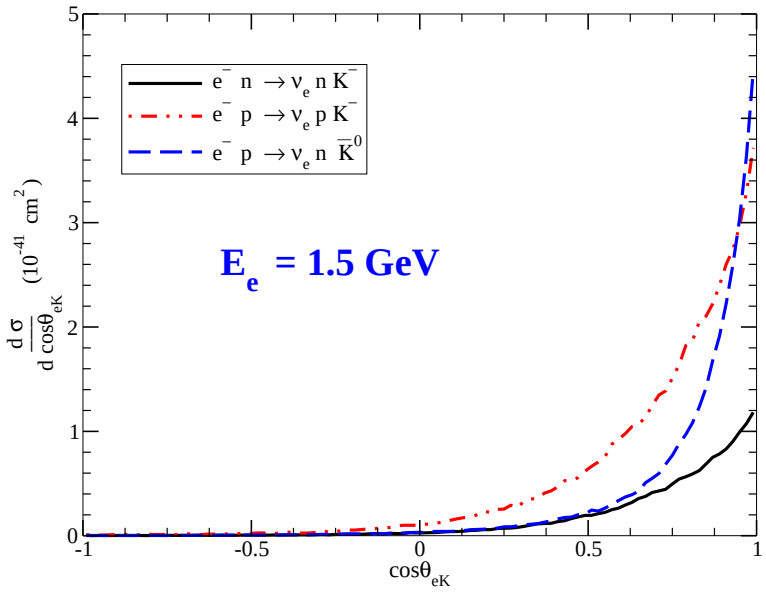

FIG. 4: Kaon angle distributions at electron energy $E_e = 1.5$ GeV

charge exchange weak production process and there is no other additional strange particle in the final state[3]. This is so because of the higher energy threshold of any other mechanism that could produce antikaons. Therefore, there is no need to measure other particles in coincidence.

To estimate the number of events for single kaon production we have considered a luminosity of $5 \times 10^{37}\, s^{-1}$ cm^{-2} for MAMI, that corresponds to a 10 cm liquid hydrogen target at an electron beam current of 20 μA as described in [29]. For TJNAF, we take a luminosity of $5 \times 10^{38}\, s^{-1}$ cm^{-2} that corresponds to a current of 100 μA and a larger liquid hydrogen target [30] that has been used on the measurement of parity violating electron proton scattering. Under these conditions and for 1.5 GeV electrons, we would have some 480 events per day for the reaction $e^- + p \to \nu_e + K^- + p$ at TJNAF (48 at MAMI). For the $e^- + p \to \nu_e + \bar{K^0} + n$ reaction, we would get 320 events per day at TJNAF (32 at MAMI). Certainly, the numbers could be changed by using different targets and/or current but equally important is the efficiency in the kaon detection, that depends on the kaon kinematics and the detector.

The kaon angle and momentum distributions for the electron induced processes are shown

[3] Processes with additional particles, such as a pion, are also expected to be much smaller because of their phase space.

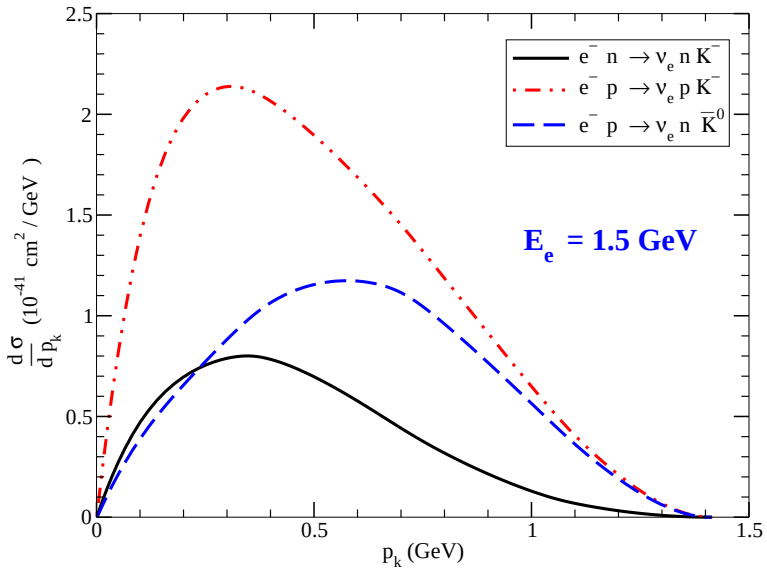

FIG. 5: Kaon momentum distributions at electron energy $E_e = 1.5$ GeV

in Figs. 4 and 5 at an electron energy of $E_e = 1.5$ GeV, that could be appropriate for both the TJNAF and MAMI facilities. As shown in Fig. 4, the three channels under study are forward peaked, specially for the $\bar{K}^0$ production. The momentum distributions peak around 0.3 GeV for $e^- n \to \nu_e K^- n$, and $e^- p \to \nu_e K^- p$ while for $e^- p \to \nu_e \bar{K}^0 n$ it peaks around 0.6 GeV. This is because the contact term, which has dominant contribution for K^- production channels, peaks at low p_k. For $\bar{K}^0$ production, there is a significant contribution from s-channel Λ term, which flattens for a wide range of p_k. Its interference with the contact term shifts the peak for the kaon momentum distribution.

As an example, we have applied in our calculation some cuts corresponding to the KAOS spectrometer at MAMI. Following Ref. [1], the kaon momentum has been restricted to the range $400 - 700$ MeV/c and the kaon angle to the range $21 - 43°$. For electron energies of 1.5 GeV, these cuts would reduce the signal by a factor ≈ 6. Additionally, taking into account the kaons survival probability in KAOS [2] would further reduce the number of events by a similar factor. Thus, the measurement of these cross section at currently existing facilities, with their luminosities and detectors would require quite long runs. We have also investigated how the Q^2 dependence of the weak form factors would affect our predictions. As mentioned above, very little is known for this dependence given that the existing experimental information comes from beta decay that occurs for very low Q^2 values. In this

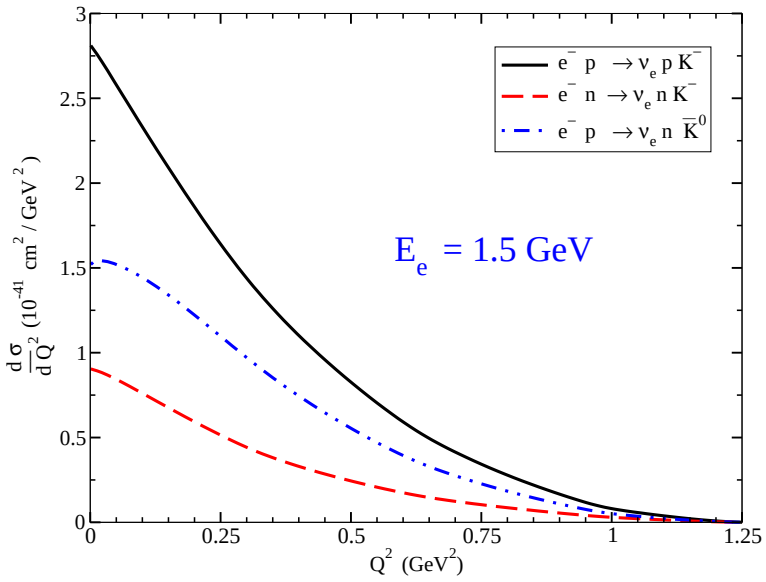

FIG. 6: Q^2 distribution at electron energy $E_e = 1.5$ GeV

calculation, we are assuming a simple dipole dependence and the same form factor for all background channels. Thus, we can only obtain some idea on the uncertainty/sensitivity of our results to the form factors. As can be seen in Fig. 6, the processes have a small contribution from large Q^2 values. Thus the size of the cross section depends moderately on the form factors. For instance, by changing the dipole mass($M_F = 1.05$ GeV), a 20% up/down, one gets changes of about 10% for the neutron channel and about 28% for the two proton channels at $E_e = 1.5$ GeV. We have also studied the sensitivity of the electron induced cross sections to the D and F parameters, that govern the hyperon beta decays. For that, we have modified the D value by a 5% while keeping g_A(=D + F)=1.26 constant. Our results show cross sections that grow around 5% for the proton processes and decrease by a similar factor for the neutron case. This implies that some ratios, such as $\sigma_{\bar{K}^0}/\sigma_{K^-}$ on deuteron could be a sensitive probe for these parameters.

The measurement of the Q^2 dependence would require the detection of the final nucleon in addition to the kaon. However, also some purely kaonic observables show some sensitivity to the form factors. For instance, Fig. 7 shows how a larger dipole mass pushes the kaon momentum towards larger values.

In summary, we have developed a microscopical model for single kaon production off nucleons induced by electrons/positrons. This model is based on the SU(3) chiral Lagrangians.

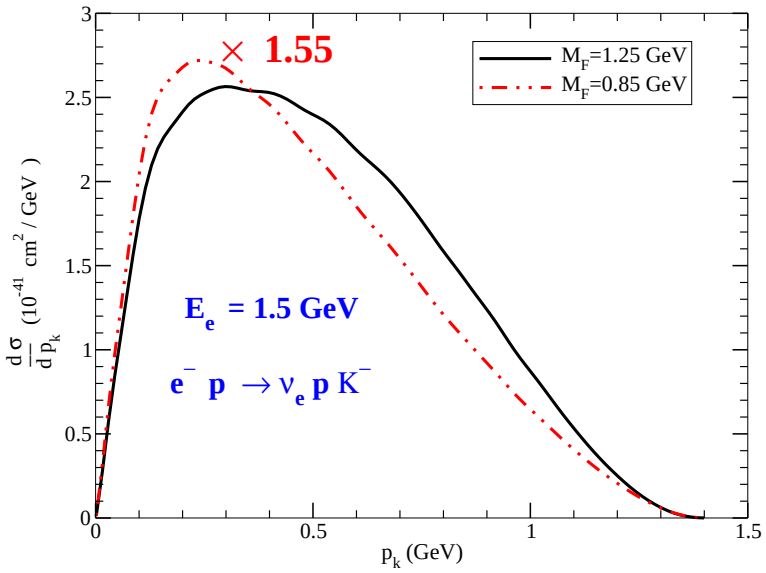

FIG. 7: Kaon momentum distribution at electron energy $E_e = 1.5$ GeV for the $e^- + p \to \nu_e(\bar{\nu}_e) + K^- + p$ channel for a dipole mass $M_F = 1.25$ and $M_F = 0.85$ GeV. The second curve ($M_F = 0.85$ GeV) has been scaled to get the same area.

The calculations are performed up to an electron/positron energy of 1.5 GeV where we expect this model to be quite reliable. The parameters of the model for the background terms are f_π, the pion decay constant, Cabibbo's angle, the proton and neutron magnetic moments and the axial vector coupling constants for the baryons octet, the D and F couplings. All of them are relatively well known. To account for the Q^2 dependence we have taken a global dipole form factor as in Ref. [25]. For the electron induced process, we have also considered the contribution from the $\Sigma^*(1385)$ resonance term, the weak couplings for which has been obtained using SU(3) symmetry from those of the $\Delta(1232)$ resonance. We predict cross sections that, although small, could be measured at current experimental facilities. Furthermore, our results could facilitate the study of the hyperon/nucleon weak coupling constants and their form factors.

We should remark that for some of the studied channels and for the considered energies, there are no electromagnetic competing processes that could hinder their investigation, and therefore, the present study of the single kaon production cross section induced by electrons/positrons opens an interesting window for research for strangeness physics in the weak interaction sector.

APPENDIX

Appendix A: Hadronic Currents

Process	B_{CT}	A_{CT}	A_Σ	A_Λ	$A_{Cr\Sigma}$	$A_{Cr\Lambda}$	A_{KP}	A_π	A_η	A_{Σ^*}
$e^- n \to \nu K^- n$	D-F	1	-1	0	0	0	-1	1	1	2
$e^- p \to \nu K^- p$	-F	2	$-\frac{1}{2}$	1	0	0	-2	-1	1	1
$e^- p \to \nu \bar{K}^0 n$	-D-F	1	$\frac{1}{2}$	1	0	0	-1	-2	0	-1
$e^+ n \to \bar{\nu} K^+ n$	D-F	-1	0	0	-1	0	-1	-1	-1	0
$e^+ p \to \bar{\nu} K^+ p$	-F	-2	0	0	$-\frac{1}{2}$	1	-2	1	-1	0
$e^+ n \to \bar{\nu} K^0 n$	-D-F	-1	0	0	$\frac{1}{2}$	1	-1	2	0	0

TABLE I: Constant factors appearing in the hadronic current

The contributions to the hadronic current are

$$
\begin{aligned}
J^\mu|_{CT} &= iA_{CT}V_{us}\frac{\sqrt{2}}{2f_\pi}\bar{N}(p')\left(\gamma^\mu + B_{CT}\,\gamma^\mu\gamma_5\right)N(p) \\
j^\mu|_{Cr\Sigma} &= iA_{Cr\Sigma}V_{us}\frac{\sqrt{2}}{2f_\pi}\bar{N}(p')\left(\gamma^\mu + i\frac{\mu_p+2\mu_n}{2M}\sigma^{\mu\nu}q_\nu + (D-F)(\gamma^\mu - \frac{q^\mu}{q^2-M_k^2}\not{q})\gamma^5\right) \\
&\quad \times\frac{\not{p}-\not{p}_k+M_\Sigma}{(p-p_k)^2-M_\Sigma^2}\not{p}_k\gamma^5 N(p), \\
j^\mu|_{Cr\Lambda} &= iA_{Cr\Lambda}V_{us}\frac{\sqrt{2}}{4f_\pi}\bar{N}(p')\left(\gamma^\mu + i\frac{\mu_p}{2M}\sigma^{\mu\nu}q_\nu - \frac{D+3F}{3}(\gamma^\mu - \frac{q^\mu}{q^2-M_k^2}\not{q})\gamma^5\right) \\
&\quad \times\frac{\not{p}-\not{p}_k+M_\Lambda}{(p-p_k)^2-M_\Lambda^2}\not{p}_k\gamma^5 N(p), \\
J^\mu|_{\Sigma} &= iA_{\Sigma}(D-F)V_{us}\frac{\sqrt{2}}{2f_\pi}\bar{N}(p')\not{p}_k\gamma_5\frac{\not{p}+\not{q}+M_\Sigma}{(p+q)^2-M_\Sigma^2}\left(\gamma^\mu + i\frac{(\mu_p+2\mu_n)}{2M}\sigma^{\mu\nu}q_\nu\right. \\
&+ \left.(D-F)\left\{\gamma^\mu - \frac{q^\mu}{q^2-{M_k}^2}\not{q}\right\}\gamma^5\right)N(p) \\
J^\mu|_{\Lambda} &= iA_{\Lambda}V_{us}(D+3F)\frac{1}{2\sqrt{2}f_\pi}\bar{N}(p')\not{p}_k\gamma^5\frac{\not{p}+\not{q}+M_\Lambda}{(p+q)^2-M_\Lambda^2}\left(\gamma^\mu + i\frac{\mu_p}{2M}\sigma^{\mu\nu}q_\nu\right. \\
&- \left.\frac{(D+3F)}{3}\left\{\gamma^\mu - \frac{q^\mu}{q^2-{M_k}^2}\not{q}\right\}\gamma^5\right)N(p) \\
J^\mu|_{KP} &= iA_{KP}V_{us}\frac{\sqrt{2}}{2f_\pi}\bar{N}(p')\not{q}\;N(p)\frac{q^\mu}{q^2-M_k^2}
\end{aligned}
$$

$$
\begin{aligned}
J^\mu|_\pi &= iA_\pi \frac{M\sqrt{2}}{2f_\pi} V_{us}(D+F) \frac{2{p_k}^\mu - q^\mu}{(q-p_k)^2 - {m_\pi}^2} \bar{N}(p')\gamma_5 N(p) \\
J^\mu|_\eta &= iA_\eta \frac{M\sqrt{2}}{2f_\pi} V_{us}(D-3F) \frac{2{p_k}^\mu - q^\mu}{(q-p_k)^2 - {m_\eta}^2} \bar{N}(p')\gamma_5 N(p) \\
J^\mu|_{\Sigma^*} &= -iA_{\Sigma^*} \frac{\mathcal{C}}{f_\pi} \frac{1}{\sqrt{6}}\, V_{us}\, \frac{p_k^\lambda}{P^2 - M_{\Sigma^*}^2 + i\Gamma_{\Sigma^*} M_{\Sigma^*}}\, \bar{N}(p') P_{RS_{\lambda\rho}} (\Gamma_V^{\rho\mu} + \Gamma_A^{\rho\mu}) N(p)
\end{aligned}
$$

In $\Gamma_V^{\rho\mu} + \Gamma_A^{\rho\mu}$, the form factors are taken as for the Δ^+ case in Ref. [25]. The extra factors for each of the Σ^* channels are given by A_{Σ^*} in Tab. I.

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