Vector current conservation and neutrino emission from singlet-paired baryons in neutron stars.

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Abstract

Neutrino emission caused by singlet Cooper pairing of baryons in neutron stars is recalculated by accurately taking into account for conservation of the vector weak currents. The neutrino emissivity via the vector weak currents is found to be several orders of magnitude smaller than that obtained before by different authors. This makes unimportant the neutrino radiation from singlet pairing of protons or hyperons.

One of the mechanisms leading the neutron star cooling, specially for temperatures near the critical one T_c, consists on the recombination of thermally excited baryon BCS pairs into the condensate. This process has received the attention of many authors, and is currently thought to be dominant, for some ranges of the temperature and/or matter density (modulo the uncertainties arising from the incomplete knowledge of the gap value). A better understanding of this process is, therefore, of great importance for the secular evolution of such objects.

Under the description of nuclear matter in the nonrelativistic limit, the simplest case for baryon pairing corresponds to two particles bounded in the $^1S_0$ state. The neutrino emission for recombination into this state was first calculated by Flowers et al. [1] and reproduced by other authors [2], [3]. The neutrino energy losses due to pairing of hyperons [4], [5] are also discussed in the literature as possible cooling mechanisms for superdense baryonic matter in neutron stars. Nowadays, these ideas are widely accepted and used in numerical simulations of neutron star evolution [6], [7], [8].

In the case of singlet pairing, the averaged weak axial current vanishes, and the emissivity is directly mediated by the weak vector current. As it is well known, the vector current possesses the property of being conserved by nuclear and electroweak interactions. Of course, this fundamental property has to be accounted for in any calculation of neutrino emission processes. As we show below, however, previous calculations did not pay attention to this particular topic. This translates into a dramatic overestimation of the energy production from the process under consideration.

Let us recall shortly the main steps in the above calculations. The low-energy Lagrangian of the weak interaction may be described by a point-like current-current approach. For interactions mediated by neutral weak currents, it can be written as[1]

$$\mathcal{L}_{vac} = \frac{G_F}{2\sqrt{2}} J_B^{\mu} l_{\mu}. \tag{1}$$

Here G_F is the Fermi coupling constant, and the neutrino weak current is given by $l_\mu = \bar{\nu}\gamma_\mu (1-\gamma_5)\,\nu$. The vacuum weak current of the baryon is of the form $J_\mu = \bar{\psi}\,(C_V\gamma_\mu - C_A\gamma_\mu\gamma_5)\,\psi$, where ψ represents the baryon field, and the weak vertex includes the vector and axial-vector terms with the corresponding coupling constants C_V and C_A.

As mentioned above, in the case of singlet pairing only the vector current contributes. The nonrelativistic limit for this current is taken as $\bar{\psi}_B\gamma^0\psi_B \to \hat{\Psi}_B^{+}\hat{\Psi}_B$, all others being zero. Here $\hat{\Psi}_B$ is the second-quantized nonrelativistic spinor wave function. The process is kinematically allowed due to the existence of a superfluid energy gap Δ, which admits the transition with time-like momentum transfer $K = (\omega, \mathbf{k})$, with $\omega = \omega_1 + \omega_2$ and $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ being the energy and momentum carried out by the freely escaping neutrino pair. We have $\omega > 2\Delta$ and $\omega > k$.

The relevant input for this calculation is the recombination matrix element between the baryon state, which has a pair of quasi-particle excitations of momentum-spin labels $(\mathbf{p}, \mathrm{up}; \mathbf{p}', \mathrm{down})$, and the same state but with these excitations restored to the condensate. To the leading (zero) order in $k \ll p_F$, this matrix element is usually estimated as [1]

$$\left|\mathcal{M}_B\right|^2 = \frac{\Delta^2}{\epsilon_p^2} \tag{2}$$

where ϵ_p is the quasi-particle energy, as given by Eq. (6). As a result, the neutrino energy losses at temperature $T < T_c$ are found to be:

$$Q_{\mathrm{FRS}} = \frac{4G_F^2 p_F M^* C_V^2}{15\pi^5} \mathcal{N}_\nu T^7 y^2 \int_0^\infty \frac{z^4 dx}{\left(e^z+1\right)^2}, \tag{3}$$

where M^* is the effective nucleon mass, $y = \Delta/T$, $z = \sqrt{x^2+y^2}$, and $\mathcal{N}_\nu = 3$ is the number of neutrino flavors.

The naive estimate (2) is inconsistent with the hypothesis of conservation of the vector current in weak interactions. Indeed, a longitudinal vector current of quasi-particles consisting only on a temporal component can not satisfy the continuity equation. It is well known, however, that the Bardeen-Cooper-Schrieffer

[1] In what follows we use the Standard Model of weak interactions, the system of units $\hbar = c = 1$ and the Boltzmann constant $k_B = 1$. The fine-structure constant is $\alpha = e^2/4\pi = 1/137$.

theory of superconductivity is gauge invariant [9] and that the current conservation can be restored if the interaction among quasi-particles is incorporated in the coupling vertex to the same degree of approximation as the self-energy effect is included in the quasi-particle [10], [11]. In the present paper we recalculate the neutrino energy losses with allowance for conservation of the weak vector current.

It is convenient to use the Nambu-Gorkov formalism, where the quasi-particle fields are represented by two-component objects

$$\Psi\left(p\right)=\left(\begin{array}{c}\psi_{\mathbf{1}}\left(p\right)\\ \psi_{2}^{\dagger}\left(-p\right)\end{array}\right). \tag{4}$$

Here $\psi_{\mathbf{1}}\left(p\right)$ is the the quasi-particle component of the excitation with momentum $\mathbf{p}$ and spin σ, and $\psi_{2}^{\dagger}\left(-p\right)$ is the hole component of the same excitation, which can be interpreted as the absence of a particle with momentum $-\mathbf{p}$ and spin $-\sigma$. The two-component fields (4) obey the standard fermion commutation relations

$$\left\{\Psi_{p,\sigma},\Psi_{p',\sigma'}\right\}=\delta_{\sigma,\sigma'}\delta_{p,p'}.$$

With the aid of the 2×2 Pauli matrices $\hat{\tau}_{i}=\left(\hat{\tau}_{1},\hat{\tau}_{2},\hat{\tau}_{3}\right)$ operating in the particle-hole space, the Hamiltonian of the system of quasi-particles can be recast as [11]

$$H=H_{0}+H_{1},$$

where

$$H_{0}=\sum_{p}\Psi_{p}^{\dagger}\left(\xi_{\mathbf{p}}\hat{\tau}_{3}+\Delta\hat{\tau}_{1}\right)\Psi_{p} \tag{5}$$

is the BCS reduced Hamiltonian, the nonrelativistic energy is measured relatively to the Fermi level

$$\xi_{\mathbf{p}}\equiv\frac{p^{2}}{2M^{*}}-\mu,$$

and μ is the Fermi energy. The BCS reduced Hamiltonian (5) bears a resemblance to the one describing the Dirac equation. It has eigenvalues $p_{0}=\pm\epsilon_{p}$ with

$$\epsilon_{p}=\sqrt{\xi_{\mathbf{p}}^{2}+\Delta^{2}}, \tag{6}$$

which correspond to excited states in the particle-hole picture, while the ground state (vacuum) is the state where all negative energy "quasi-particles" ($\epsilon<0$) are occupied and no positive energy particles exist. The positive and negative states are separated by an energy gap 2Δ.

The Hamiltonian of residual interaction among quasi-particles has the following form

$$H_{1}=\frac{1}{2}\sum_{p',q}V_{pp'}\left(q\right)\left(\Psi_{p+q}^{\dagger}\hat{\tau}_{3}\Psi_{p}\right)\left(\Psi_{p'-q}^{\dagger}\hat{\tau}_{3}\Psi_{p'}\right).$$

As follows from the Hamiltonian (5), the inverse of the quasi-particle propagator can be written as [11]:

$$\text{Ģ}^{-1} \; = \; p_0 - \xi_{\mathbf{p}} \hat{\tau}_3 - \Delta \hat{\tau}_1, \tag{7}$$

The self-energies are, in general, complex numbers due to the instability of single particles. However, to the extent that the single-particle picture makes some physical sense, we will ignore the small imaginary part of the self-energies, and describe the quasi-particles with the aid of wave-functions. The states of quasi-particles obey the equation

$$\text{Ģ}^{-1} \Psi_{\mathbf{p}} = 0. \tag{8}$$

The solution to this equation corresponding to the energy $p_0 = \epsilon_{\mathbf{p}}$ and spin state χ_σ has the following form

$$\Psi_{\mathbf{p},\sigma} = \begin{pmatrix} u_{\mathbf{p}} \chi_\sigma \\ v_{\mathbf{p}} \chi_{-\sigma} \end{pmatrix} e^{i\mathbf{pr} - i\epsilon_{\mathbf{p}} t} \tag{9}$$

with

$$u_{\mathbf{p}} = \sqrt{\frac{\epsilon_p + \xi_p}{2\epsilon_p}}, \qquad v_{\mathbf{p}} = \sqrt{\frac{\epsilon_p - \xi_p}{2\epsilon_p}}.$$

There is also a solution of negative frequency $p_0 = -\epsilon_{\mathbf{p}}$

$$\Psi_{-\mathbf{p},-\sigma} = \begin{pmatrix} -v_{\mathbf{p}} \chi_{-\sigma} \\ u_{\mathbf{p}} \chi_\sigma \end{pmatrix} e^{-i\mathbf{pr} + i\epsilon_{\mathbf{p}} t}, \tag{10}$$

which corresponds to the same excitation energy. This solution is connected to the hole state by the particle-antiparticle conjugation

$$C: \; \Psi^C = C\Psi^\dagger = \hat{\tau}_2 \Psi^\dagger.$$

which changes quasi-particles of energy-momentum $(p_0, \mathbf{p})$ into holes of energy-momentum $(-p_0, -\mathbf{p})$.

The components of the bare vertex

$$\gamma^\mu = \begin{cases} \hat{\tau}_3 & \text{if } \mu = 0, \\ \frac{1}{M^*}\mathbf{p} & \text{if } \mu = i = 1,2,3 \end{cases} . \tag{11}$$

are 2×2 matrices in the Nambu-Gorkov space. As already mentioned, the longitudinal current corresponding to the bare vertex does not satisfy the continuity equation. To restore the current conservation, one must consider the modification of the vertex γ^μ to the same order as the modification of the propagator is done. The relation between the modified vertex Γ^μ and the quasi-particle propagator (7) is given by the Ward identity [12]

$$K_\mu \Gamma^\mu \left(p', p\right) = \hat{\tau}_3 \text{Ģ}^{-1} \left(p\right) - \text{Ģ}^{-1} \left(p'\right) \hat{\tau}_3, \tag{12}$$

where $K = (\omega, \mathbf{k})$ is the transferred momentum. The plane wave solutions

$$u_{\mathbf{p},\alpha} \exp\left(i\mathbf{pr} - i\epsilon_{\mathbf{p}}t\right), \qquad u^*_{\mathbf{p}',\alpha'} \exp\left(-i\mathbf{pr} + i\epsilon_{\mathbf{p}}t\right)$$

for Ψ and Ψ^+ obey the equations $\text{Ç}^{-1}(p)\, u_{\mathbf{p},\alpha} = 0$, and $u^*_{\mathbf{p}',\alpha'}\text{Ç}^{-1}(p') = 0$. Therefore, the Ward identity implies conservation of the vector current on the energy shell of the quasi-particles. Following the prescriptions of quantum electrodynamics, an approximation which satisfies the Ward identity (and hence the continuity equation) is the sum of ladder diagrams.

Consider first the case of electrically neutral baryons. Then the corrected vertex can be found from the following Dyson equation

$$\begin{aligned} \Gamma^\mu(p-K,p) &= \hat{\gamma}^\mu(p-K,p) \\ &\quad + i\int \frac{d^4p'}{(2\pi)^4}\, \hat{\tau}_3 \text{Ç}(p'-K)\, \Gamma^\mu(p'-K,p')\, \text{Ç}(p')\, \hat{\tau}_3 V_{pp'}, \end{aligned} \tag{13}$$

where the "dressed" particles interact with the same primary interaction $V_{pp'}$ which produces the self-energy of the quasi-particle.

In the limit $K = (\omega, \mathbf{0})$, the Ward identity gives[2]

$$\Gamma^0(p-K,p) = \hat{\tau}_3 - \frac{2}{\omega} i\hat{\tau}_2 \Delta$$

The poles of the vertex function correspond to collective eigen-modes of the system. Therefore, the pole which appears at $\omega \to 0$, $k = 0$ implies the existence of a collective mode, which plays an important role in the conservation of the vector current. The corresponding nonperturbative solution to Eq. (13) has been found by Nambu [11] (see also [13]):

$$\Gamma_0(p-K,p) = \hat{\tau}_3 - 2i\hat{\tau}_2\Delta \frac{\omega}{\omega^2 - a^2k^2} \tag{14}$$

$$\mathbf{\Gamma} = \frac{\mathbf{p}}{M} - 2i\hat{\tau}_2\Delta \frac{a^2\mathbf{k}}{\omega^2 - a^2k^2}, \tag{15}$$

The poles in this vertex correspond to the collective motion of the condensate, with the dispersion relation $\omega = ak$, where $a^2 = V_F^2/3$.

The effective vertex satisfies the Ward identity (12), and thus the continuity equation on the energy shell

$$\omega\Gamma_0 - \mathbf{k}\mathbf{\Gamma} \simeq 0 \tag{16}$$

We are now in a position to evaluate the matrix element of the vector weak current. In the particle-hole picture, the creation and recombination of two quasi-particles is described by the off-diagonal matrix elements of the Hamiltonian, which corresponds to quasi-particle transitions into a hole (and a correlated pair). Thus, we calculate the matrix element of the current between the

[2]To obtain the weak vector current this vertex should be multiplied by the weak coupling constant C_V.

initial (positive-frequency) state of a quasi-particle with momentum $\mathbf{p}$ and the final (negative-frequency) state with the same momentum $\mathbf{p}$.

$$\mathcal{M}_\mu = \left\langle \Psi^\dagger_{-\mathbf{p},-\sigma} \left|\Gamma_\mu\right| \Psi_{\mathbf{p},\sigma} \right\rangle$$

Let us consider separately the contributions from the bare vertex, given by the first term in Eq. (14), and the collective part, given by the second term, so that $\mathcal{M}_\mu = \mathcal{M}_\mu^{\mathrm{bare}} + \mathcal{M}_\mu^{\mathrm{coll}}$.

Making use of the wave functions described by Eqs. (9), (10) for $\mu = 0$, we find

$$\mathcal{M}_0^{\mathrm{bare}} = -\left(u_p v_{p'} + v_p u_{p'}\right) \simeq -\frac{\Delta_{\check{\mathbf{p}}}}{\epsilon_p}, \qquad k \ll p \simeq p_F \tag{17}$$

$$\mathcal{M}_0^{\mathrm{coll}} = 2\Delta_{\check{\mathbf{p}}} \frac{\omega}{\omega^2 - a^2 k^2}\left(u_p u_{p'} + v_p v_{p'}\right) \simeq 2\Delta_{\check{\mathbf{p}}} \frac{\omega}{\omega^2 - a^2 k^2}.$$

with $\epsilon_p + \epsilon_{p'} = \omega$ and $\mathbf{p} + \mathbf{p'} = \mathbf{k}$.

The velocity of the collective mode $a^2 = V_F^2/3$ is small in the nonrelativistic system. Therefore, we expand the collective contribution in this parameter to obtain

$$\mathcal{M}_0^{\mathrm{coll}} \simeq \frac{\Delta_{\check{\mathbf{p}}}}{\epsilon_p}\left(1 + \frac{1}{3} V_F^2 \frac{k^2}{\omega^2}\right). \tag{18}$$

The contribution of the bare vertex $\mathcal{M}_0^{\mathrm{bare}}$ reproduces the matrix element (2) derived by Flowers et al. [1] and Yakovlev et al. [3]. However, the collective correction modifies this crucially. In the sum of the two contributions, the leading terms mutually cancel, yielding the matrix element

$$\mathcal{M}_0 = \mathcal{M}_0^{\mathrm{bare}} + \mathcal{M}_0^{\mathrm{coll}} \simeq \frac{1}{3} V_F^2 \frac{k^2}{\omega^2} \frac{\Delta_{\check{\mathbf{p}}}}{\epsilon_p}$$

which is at least $\sim V_F^2$ times smaller than the bare result.

The spatial component of the longitudinal (with respect to $\mathbf{k}$) component of the matrix element can be obtained from Eq. (16). Since $\check{\mathbf{k}}\boldsymbol{\Gamma} = (\omega/k)\,\Gamma_0$ we have

$$\mathcal{M}_{\|} \simeq \frac{1}{3} V_F^2 \frac{k}{\omega} \frac{\Delta_{\check{\mathbf{p}}}}{\epsilon_p}.$$

In the above, $\check{\mathbf{k}} = \mathbf{k}/k$ is a unit vector directed along the transferred momentum.

Since the collective interaction modifies only the longitudinal part of the vertex, the transverse part of the matrix element can be evaluated directly from the bare vertex (11). This yields

$$\mathcal{M}_\perp \simeq \left(v_p u_{p'} - u_p v_{p'}\right) \frac{\mathbf{p}_\perp}{M^*} \simeq -\frac{1}{2} V_F^2 \frac{k \Delta_{\check{\mathbf{p}}}}{\epsilon_{\mathbf{p}}^2} \left(\check{\mathbf{k}}\check{\mathbf{p}}\right) \check{\mathbf{p}}_\perp$$

with $\check{\mathbf{p}} = \mathbf{p}/p$.

The rate of the process is proportional to the square of the matrix element. This means that the vector current contribution to the neutrino energy losses is

V_F^4 times smaller than estimated before. The corresponding neutrino emissivity in the vector channel can be evaluated with the aid of Fermi's golden rule:

$$\begin{aligned} Q_V &= \left(\frac{G_F}{2\sqrt{2}}\right)^2 \frac{C_V^2}{(2\pi)^8}\mathcal{N}_\nu \int d^3p d^3p' f(\epsilon_{\mathbf{p}}) f(\epsilon_{\mathbf{p}'}) \\ &\times \int \frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\omega \mathrm{Tr}\left(l_\mu l_\nu^*\right)\mathcal{M}^\mu\mathcal{M}^\nu \delta\left(\mathbf{p}+\mathbf{p}'-\mathbf{k}\right)\delta\left(\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}'}-\omega\right). \end{aligned}$$

One can simplify this equation by inserting $\int d^4K\ \delta^{(4)}\left(K-k_1-k_2\right)=1$. Then, the phase-space integrals for neutrinos are readily done with the aid of Lenard's formula

$$\begin{aligned} &\int \frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\delta^{(4)}\left(K-k_1-k_2\right)\mathrm{Tr}\left(l^\mu l^{\nu *}\right) \\ = &\frac{4\pi}{3}\left(K_\mu K_\nu - K^2 g_{\mu\nu}\right)\Theta\left(K^2\right)\Theta\left(\omega\right), \end{aligned}$$

where $\Theta(x)$ is the Heaviside step function.

For $k \ll p_F$ we obtain

$$\begin{aligned} Q_V &= \frac{4\pi}{3}\left(\frac{G_F}{2\sqrt{2}}\right)^2 \frac{C_V^2}{(2\pi)^8}\mathcal{N}_\nu \int d^3p f^2(\epsilon_{\mathbf{p}}) \int_0^\infty d\omega\omega \int_0^\omega dk k^2 d\Omega_k \\ &\times \left(\left(K_\mu \mathcal{M}^\mu\right)^2 - K^2\mathcal{M}^\mu\mathcal{M}_\mu\right)\delta\left(2\epsilon_{\mathbf{p}}-\omega\right). \end{aligned}$$

The next integrations are trivial. We get

$$Q_V = \frac{592}{42\,525\pi^5} V_F^4 G_F^2 C_V^2 p_F M^* T^7 y^2 \int_0^\infty \frac{z^4 dx}{\left(e^z+1\right)^2}$$

This is to be compared with Eq. (3). We see that

$$\frac{Q_V}{Q_{\mathrm{FRS}}\left({}_1S^0\right)} = \frac{148}{2835}V_F^4$$

i.e. the neutrino radiation from ${}^1S_0$ pairing in the nonrelativistic system ($V_F \ll 1$) is suppressed by several orders of magnitude with respect to the predictions of [1] and [3].

Consider now the case when quasi-particles carry an electric charge. Including the long-range Coulomb interaction $V_C(k) = e^2/k^2$ implies that the vertex part is multiplied by a string of closed loops, which represents the polarization of the surrounding medium. In this case, the new vertex $\tilde{\Gamma}^\mu$ can be found as the solution of the Dyson equation, according to the diagram of Fig. 1 or, analytically

$$\begin{aligned} \tilde{\Gamma}^\mu(p-K,p) &= \Gamma^\mu(p-K,p) - \Gamma_0(p-K,p)V_C(k) \\ &\times i\int \frac{d^4p'}{(2\pi)^4}\, Tr\left[\hat{\tau}_3 \mathcal{G}(p'-K)\tilde{\Gamma}^\mu(p'-K,p')\mathcal{G}(p')\right] \end{aligned} \quad (19)$$

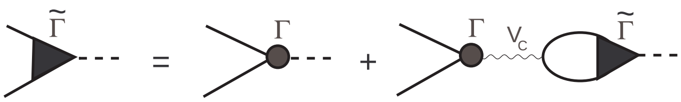

Figure 1: Dyson equation for the vertex correction for charged quasi-particles. The shaded areas represent the modified effective vertex, and the wavy line stands for the Coulomb interaction.

This equation can be readily solved yielding

$$\tilde{\Gamma}^{\mu}\left(p-K,p\right)=\Gamma^{\mu}\left(p-K,p\right)\left(1-\frac{V_C\left(k\right)\Pi^{0\mu}\left(K\right)}{1+V_C\left(k\right)\Pi^{00}\left(K\right)}\right).$$

with

$$\Pi^{0\mu}\left(K\right)\equiv i\int\frac{d^4p'}{\left(2\pi\right)^4}\;Tr\left[\hat{\tau}_3\text{Ģ}\left(p'-K\right)\Gamma^{\mu}\left(p'-K,p'\right)\text{Ģ}\left(p'\right)\right].$$

In particular, for Γ^0 we arrive to[3]

$$\tilde{\Gamma}^{0}\left(p-K,p\right)=\frac{\Gamma^{0}\left(p-K,p\right)}{1+V_C\left(k\right)\Pi^{00}\left(K\right)}. \tag{20}$$

The polarization function $\Pi^{00}\left(K\right)$ can be readily calculated with the help of Γ^0 given by Eq. (14). By neglecting the small dependence of the energy gap on the transferred momentum $\mathbf{k}$, we have

$$\begin{aligned}\Pi^{00}\left(K\right) &= i\int\frac{d^4p'}{\left(2\pi\right)^4}\;Tr\left[\hat{\tau}_3\text{Ģ}\left(p'-K\right)\hat{\tau}_3\text{Ģ}\left(p'\right)\right]\\ &\quad-\frac{2\omega}{\omega^2-a^2k^2}i\int\frac{d^4p'}{\left(2\pi\right)^4}\;Tr\left[\hat{\tau}_3\text{Ģ}\left(p'-K\right)i\hat{\tau}_2\Delta\left(p'\right)\text{Ģ}\left(p'\right)\right].\end{aligned}$$

Here, the quasi-particle propagator follows from Eq. (7):

$$\text{Ģ}\left(p\right)=\frac{i}{p_0^2-\epsilon_p^2}\left(p_0+\xi_p\hat{\tau}_3+\hat{\tau}_1\Delta\right). \tag{21}$$

We are interested in the regime defined by $k<\omega,\quad \omega>2\Delta\gg\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p-k}}\simeq kV_F$. In this case we obtain, after some simplifications

$$\tilde{\Gamma}^{0}\left(p-K,p\right)=\frac{\Gamma^{0}\left(p-K,p\right)}{1+\chi\left(K\right)}, \tag{22}$$

[3]The solution to the equation $1+V_C\left(k\right)\Pi^{00}\left(K\right)=0$ determines the new dispersion law $\omega=\omega\left(k\right)$ for the collective excitations, which represents plasma waves [11].

with

$$\chi(K) = \frac{e^2}{8\pi^3}\left[\frac{a^2}{\omega^2 - a^2k^2}\int d^3p\,\frac{\Delta^2}{\epsilon_p\left(\epsilon_p^2 - a^2k^2/4\right)} + \int d^3p\,\frac{\Delta^2}{\epsilon_p\left(\omega^2 - 4\epsilon_p^2\right)}\left(\frac{p^2}{3M^2\epsilon_p^2} - \frac{a^2}{\epsilon_p^2 - a^2k^2/4}\right)\right]. \quad (23)$$

Since $a \ll 1$ and $p \simeq p_F$, the second integral in Eq. (23) may be dropped. By neglecting also the small contributions from $a^2k^2 \ll \epsilon_p^2, \omega^2$ we get

$$\chi(K) = e^2\frac{a^2}{\omega^2}\int \frac{\Delta^2}{\epsilon_p^3}\frac{d^3p}{(2\pi)^3} = \frac{\omega_p^2}{\omega^2}$$

with $\omega_p^2 = e^2 n/M^*$ (n is the number of baryons per unit volume). This agrees with the plasma frequency for a free gas of charged particles.

The energy exchange in the medium goes naturally as the temperature scale. Therefore, the energy transferred to the radiated neutrino-pair is $\omega \sim T \leq T_c$, while the plasma frequency ω_p is typically much larger than the critical temperature for Cooper pairing. For instance, for a number density n of the order of the nuclear saturation density $n_0 \simeq 0.17$ fm and the effective mass of the baryon M^* of the order of the bare nucleon mass, we obtain $\omega_p \sim 10\ MeV$, while the critical temperature for baryon pairing is about 1 MeV or less. Under these conditions, we obtain

$$\tilde{\Gamma}^0(p-K,p) \simeq \frac{\omega^2}{\omega_p^2}\Gamma^0(p-K,p) \sim \frac{T_c^2}{\omega_p^2}\Gamma^0(p-K,p)$$

Thus, in superconductors, the vector current contribution to the neutrino radiation is suppressed additionally by a factor $\left(T_c^2/\omega_p^2\right)^2$: this is the *plasma screening effect*.

We have considered the problem of conservation of the vector weak current in the theory of neutrino-pair radiation from Cooper pairing in neutron stars. The correction to the vector weak vertex is calculated within the same order of approximation as the quasi-particle propagator is modified by the pairing interaction in the system. This correction restores the conservation of the vector weak current in the quasi-particle transition into a paired state. As a result, in the nonrelativistic baryon system, the matrix element of the vector current is V_F^2 times smaller than previous estimations. This means that the vector weak current contribution to neutrino radiation caused by Cooper paring is V_F^4 times smaller than it was thought before. The vector weak current contribution from pairing of charged baryons is suppressed additionally by a factor $\sim \left(T_c^2/\omega_p^2\right)^2$ due to plasma screening. The total suppression factor due to both the current conservation and the plasma effects is of the order

$$\left(T_c^2/\omega_p^2\right)^2 V_F^4 \lesssim 10^{-6}.$$

Thus the neutrino energy losses due to singlet-state pairing of baryons can, in practice, be neglected in simulations of neutron star cooling. This makes unimportant the neutrino radiation from pairing of protons or hyperons.

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