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Invariants and Flavour in the General Two-Higgs Doublet Model

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Abstract

The flavour structure of the general Two Higgs Doublet Model (2HDM) is analysed and a detailed study of the parameter space is presented, showing that flavour mixing in the 2HDM can be parametrized by various unitary matrices which arise from the misalignment in flavour space between pairs of various Hermitian flavour matrices which can be constructed within the model. This is entirely analogous to the generation of the CKM matrix in the Standard Model (SM). We construct weak basis invariants which can give insight into the physical implications of any flavour model, written in an arbitrary weak basis (WB) in the context of 2HDM. We apply this technique to two special cases, models with MFV and models with NNI structures. In both cases non-trivial CP-odd WB invariants arise in a mass power order much smaller than what one encounters in the SM, which can have important implications for baryogenesis in the framework of the general 2HDM.

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1 Introduction

The two Higgs doublet model (2HDM) [1] is one of the simplest extensions of the Standard Model (SM) and arises in many models beyond the SM, including supersymmetric (SUSY) ones. The 2HDM was first introduced by Lee [2] with the aim of achieving spontaneous CP violation [3] in the context of the SM, at a time when only two incomplete fermion generations were known. No extra symmetries are introduced in Lee's model and, as a result, the model has flavour-changing-neutral currents (FCNC) of arbitrary strength at tree level. In order to avoid FCNC at tree level in the 2HDM, a discrete Z_2 symmetry can be introduced [4], which guarantees Natural Flavour Conservation (NFC) in the scalar sector. It was pointed out that in this case neither spontaneous [5] nor hard CP violation [6] in the Higgs sector can be achieved, unless one introduces a third Higgs doublet. An alternative scenario is to break this Z_2 symmetry softly [7]. In this paper we study the flavour content of the general 2HDM and construct weak basis (WB) invariants which can give insight into the physical implications of any flavour model written in an arbitrary WB. It should be stressed that even in models where each charge quark sector receives mass contributions from only one Higgs, as it is the case in SUSY models, "wrong" couplings are generated at higher orders [8], [9], [10]. Therefore, the present analysis maybe relevant also for models with NFC at tree level.

At this stage it is worth recalling that in the presence of a flavour symmetry or an Ansatz, the Yukawa couplings may contain texture zeros which arise only in a specific basis. In another WB the Yukawa coupling matrices change, the texture zeros may no longer be present but the physical content of the model does not change. The great advantage of the WB invariants stems from the fact that they can be evaluated in any WB. Furthermore, we point out that the flavour structure in the 2HDM can be parametrized by various unitary matrices which are entirely analogous to the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the SM. All the unitary flavour mixing matrices of the 2HDM arise from the misalignment in flavour space of various Hermitian matrices constructed in the framework of the 2HDM. In order to illustrate the usefulness of these WB invariants, we apply them to the analysis of 2HDM which have Higgs mediated FCNC at tree level (HFCNC), but with their structure entirely defined [11] , [12] in terms of V_{CKM}. It has been pointed out that some of these models satisfy the hypotheses of Minimal Flavour Violation [13] (see also [14], [15], [16]) . The paper is organized as follows: in the next section, we settle the notation and analyse the flavour parameter space of the general 2HDM, explaining how the various unitary flavour mixing matrices are generated. In section 3 we display how the various Yukawa couplings transform under WB transformations, construct various WB invariants and analyse their physical meaning. Next we illustrate how WB invariants can be used to analyse specific flavour models, based on 2HDM. As examples we use the class of models named BGL [11], where there are FCNC at tree level but with their flavour structure controlled by V_{CKM}, and a model with nearest-neighbour-interaction (NNI) pattern for the quark mass matrices in the framework of a 2HDM [17]. Finally in section 4 we present our Conclusions.

2 The Two Higgs Doublet Parameter Space

We consider the extension of the SM consisting of the addition of two Higgs doublets (2HDM) with no additional symmetries. This implies that each of the doublets Φ_1, Φ_2 contributes to both up and down quark mass matrices, through the Yukawa couplings:

$$\mathcal{L}_Y \;=\; -\,\overline{Q_L^0}\;\Gamma_1\Phi_1 d_R^0 - \overline{Q_L^0}\;\Gamma_2\Phi_2 d_R^0 - \overline{Q_L^0}\;\Delta_1\tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\;\Delta_2\tilde{\Phi}_2 u_R^0 + \text{h.c.} \quad (1)$$

where we have used standard notation. The interactions of the neutral Higgs with the quarks, obtained from Eq.(1) are given by:

$$\begin{aligned} \mathcal{L}_Y(\text{neutral}) \;&=\; -\overline{d_L^0}\frac{1}{v}\left[M_d H^0 + N_d^0 R + i N_d^0 I\right] d_R^0 + \\ &\quad -\; \overline{u_L^0}\frac{1}{v}\left[M_u H^0 + N_u^0 R - i N_u^0 I\right] u_R^0 + \text{h.c.}\;, \end{aligned} \quad (2)$$

with $v \equiv \sqrt{v_1^2 + v_2^2}$, and H^0, R orthogonal combinations of the fields ρ_j, given by $\phi_j^0 = \frac{e^{i\theta_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j)$, where H^0 is defined so that its couplings are proportional to the mass matrices. In an analogous way, I is a linear combination of η_j orthogonal to the neutral Goldstone boson. The quark mass matrices M_d and M_u and the matrices N_d^0 and N_u^0 are given by:

$$\begin{aligned} M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\theta}\Gamma_2)\;, &\qquad M_u = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2 e^{-i\theta}\Delta_2)\;, \\ N_d^0 = \frac{v_2}{\sqrt{2}}\Gamma_1 - \frac{v_1}{\sqrt{2}}e^{i\theta}\Gamma_2\;, &\qquad N_u^0 = \frac{v_2}{\sqrt{2}}\Delta_1 - \frac{v_1}{\sqrt{2}}e^{-i\theta}\Delta_2\;, \end{aligned} \quad (3)$$

where θ denotes the relative phase of the vevs of the neutral components of Φ_i. The four matrices of Eq.(3) are written in an arbitrary weak-basis (WB). It is well known that one can make a WB transformation defined by:

$$d^0_L = W_L \; {d^0_L}', \qquad d^0_R = W^d_R \; {d^0_R}', \qquad u^0_L = W_L \; {u^0_L}', \qquad u^0_R = W^u_R \; {u^0_R}' \quad (4)$$

without physical implications. Under these WB transformations, the matrices of Eq. (3) transform as:

$$\begin{aligned} M_d \to M'_d = W_L^\dagger M_d W^d_R, \qquad M_u \to M'_u = W_L^\dagger M_u W^u_R, \\ N^0_d \to {N^0_d}' = W_L^\dagger N^0_d W^d_R, \qquad N^0_u \to {N^0_u}' = W_L^\dagger N^0_u W^u_R \end{aligned} \quad (5)$$

In order to analyse the physical content of the above four matrices, one may choose, without loss of generality, a weak-basis where M_u is diagonal real, while M_d is a Hermitian matrix with only one rephasing invariant phase given by $\varphi = \arg[(M_d)_{12}(M_d)_{23}(M_d)_{31}]$. The six real parameters in M_d, together with φ and the up quark masses m_u, m_c, m_t, total the ten parameters contained in the flavour sector of the SM, seen in a weak basis. In the quark mass eigenstate basis these appear as the six quark masses and the four parameters characterizing V_{CKM}. In the above described WB, the matrices N^0_d, N^0_u are in general complex arbitrary 3×3 matrices, each one containing nine physical phases. Note that we have considered the general 2HDM with no flavour symmetries introduced.

In the presence of flavour symmetries and/or texture zeros, the number of parameters in N^0_d, N^0_u can be drastically reduced. Flavour symmetries (FS) are introduced in a specific WB, with the choice dictated by the FS representation assumed for the fermions and Higgs doublets. Similarly, texture zeros imply the choice of a particular WB. In view of this freedom of choice of WB, it is very useful to express the physical content of M_d, M_u, N^0_d, N^0_u in terms of WB invariants.

We shall construct these invariants and analyse their physical content in section 3.

It is useful to see how the parameters of N^0_d, N^0_u appear when one parametrizes N^0_d, N^0_u through unitary matrices. It can be readily seen that, without loss of generality, one can write:

$$N^0_d = K_L \; \hat{V}^{N_d}_L \; D^{N_d} \; \overline{K} \; (\hat{V}^{N_d}_R)^\dagger \; K^\dagger_R \quad (6)$$

where K_L, K_R are diagonal unitary matrices of the form:

$$K_{L,R} = \mathrm{diag}[1, \exp(i\varphi_{1L,R}), \exp(i\varphi_{2L,R})] \quad (7)$$

while $\hat{V}_{L,R}^{N_d}$ are unitary matrices with one physical non factorizable phase each, analogous to V_{CKM}. Finally one has:

$$\overline{K} = \text{diag}[\exp(i\sigma_1), \exp(i\sigma_2), \exp(i\sigma_3)] \tag{8}$$

and D^{N_d} stands for a real diagonal matrix. The explicit counting of parameters is:

$$\begin{array}{ll} \text{phases:} & 2(K_L) + 2(K_R) + 1(\hat{V}_L^{N_d}) + 1(\hat{V}_R^{N_d}) + 3(\overline{K}) = 9 \\ \text{real parameters} & 3(\hat{V}_L^{N_d}) + 3(\hat{V}_R^{N_d}) + 3(D^{N_d}) = 9 \end{array}$$

for each one of the matrices N_d^0, N_u^0.

3 Weak Basis Invariants

3.1 The General Case

The four matrices, M_d, M_u, N_d^0, N_u^0 fully characterize the flavour sector of the 2HDM in the sense that they encode the breaking of the large flavour symmetry present in the gauge sector of the theory. The above four flavour matrices contain a large redundancy of parameters which results from the fact that under a WB transformation M_d, M_u, N_d^0, N_u^0 change transforming as indicated by Eq. (5) without altering their physical content. Different Lagrangians related to each other by WB transformations describe the same physics. In view of the above redundancy, it is of great interest to construct WB invariants which can be very useful in the analysis of the physical content of the flavour sector of a given model. For example, in the context of the SM, it has been shown [18] that from the four WB invariants $\text{tr}(H_u\, H_d)$, $\text{tr}(H_u\, H_d^2)$, $\text{tr}(H_u^2\, H_d)$, $\text{tr}(H_u^2\, H_d^2)$, where $H_{d,u} \equiv (M_{d,u} M_{d,u}^\dagger)$, one can construct the full V_{CKM}, with only a two-fold ambiguity in the sign of $\text{Im}Q$, where Q stands for a rephasing invariant quartet of V_{CKM}, defined by $Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$ ($\alpha \neq \beta$, $i \neq j$). WB invariants are also very useful in the study of CP violation. In the context of the SM, it has been derived from first principles [19] that the necessary and sufficient condition for CP invariance is the vanishing of the WB invariant:

$$\begin{aligned} I_1^{CP} \equiv\ & \text{tr}\, [H_u, H_d]^3 = 6i(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \times \\ & \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \text{Im} Q_{uscb} \end{aligned} \tag{9}$$

for three generations I_1^{CP} is proportional to $\det[H_u, H_d]^3$, introduced in Ref. [20].

In this section we use WB invariants to analyse the flavour structure and CP violation in the general 2HDM. We shall apply here the same technique that was introduced in [19] to the study of CP violation in the SM. This technique was later generalized to many different scenarios, in particular to the study of explicit CP violation in the scalar sector of multi-HDM prior to gauge symmetry breaking [21] as well as CP violation in the scalar sector after this breaking [22] and also taking into account both the scalar and the fermionic sector [23]. In Ref. [24] CP violation in the supersymmetric case is analysed. WB invariants can also be built to study other important features of flavour models such as alignment and the pattern of fermion masses and mixing [25]. One can check the predictions of a flavour model by comparing invariant quantities with their corresponding experimental values. In Ref. [26], the authors classified all the invariants that can be built in a given theory, using the ring of polynomials that are invariant under the action of a group.

From the transformation properties of the flavour matrices M_d, M_u, N_d^0, N_u^0 given in Eq. (3), it is clear that one can build new WB invariants, which do not arise in the SM, by evaluating traces of blocks of matrices involving the up and down quark sector, like for example $M_\gamma N_\gamma^{0\dagger}$ or $N_\gamma^0 N_\gamma^{0\dagger}$. We shall analyse the lowest WB invariants and indicate some of the physical aspects of the 2HDM probed by each one of these invariants. For definiteness let us consider the WB invariant $\mathrm{tr}(M_d N_d^{0\dagger})$ and note that its physical significance becomes transparent in the WB where M_d is diagonal, real, since in this basis the matrix N_d^0 already coincides with the couplings to the physical quarks. In this basis one has:

$$I_1 \equiv \mathrm{tr}(M_d N_d^{0\dagger}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33} \tag{10}$$

We denote N_d, the matrix N_d^0 in the basis where it couples to the physical quarks. This invariant is not sensitive to Higgs-mediated FCNC, but $\mathrm{Im}(I_1)$ is specially important, since it probes the phases of $(N_d)_{jj}$ which contribute to the electric dipole moment of down-type quarks. Obviously, one can construct an analogous invariant for the up-quark sector, namely $\mathrm{tr}(M_u N_u^{0\dagger})$. Let us now consider a WB invariant which is sensitive to the off-diagonal elements of N_d, namely:

$$I_2 \equiv \mathrm{tr}\left[M_d N_d^{0\dagger}, M_d M_d^\dagger\right]^2 = -2 m_d m_s (m_s^2 - m_d^2)^2 (N_d^*)_{12} (N_d^*)_{21} -$$
$$-2 m_d m_b (m_b^2 - m_d^2)^2 (N_d^*)_{13} (N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2)^2 (N_d^*)_{23} (N_d^*)_{32}, \tag{11}$$

where we have kept the notation used in Eq. (10), having evaluated I_2 in the WB where M_d is real and diagonal. It is well known that I_1^{CP} given in Eq. (9) measures the strength of CP violation arising from weak charged currents with the appearance of a non-trivial quark mixing matrix $V_{CKM} \equiv U_{uL}^\dagger U_{dL}$ reflecting the fact that $U_{dL} \neq U_{uL}$, i.e. the misalignment of the matrices H_d, H_u in flavour space. In an entirely analogous way, one can construct the invariant:

$$I_2^{CP} \equiv \ \text{tr}\left[H_u, H_{N_d^0}\right]^3 = 6i\Delta_u\Delta_{N_d}\text{Im}Q_2 \tag{12}$$

where Q_2 is a rephasing invariant quartet of $V_2 \equiv U_{uL}^\dagger U_{N_d^0 L}$, $\Delta_u \equiv (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$ and Δ_{N_d} is defined in analogy to Δ_u but refering to the eigenvalues of $H_{N_d^0} \equiv N_d^0 N_d^{0\dagger}$. It is clear that V_2 reflects the misalignment of the matrices H_u, $H_{N_d^0}$ in flavour space. Similarly, one has the invariant :

$$I_3^{CP} \equiv \ \text{tr}\left[H_d, H_{N_d^0}\right]^3 = 6i\Delta_d\Delta_{N_d}\text{Im}Q_3 \tag{13}$$

where Q_3 is a rephasing invariant quartet of $V_3 \equiv U_{dL}^\dagger U_{N_d^0 L}$. In an entirely analogous way, one can also construct the invariants:

$$I_4^{CP} \equiv \text{tr}\left[H_u, H_{N_u^0}\right]^3; \quad I_5^{CP} \equiv \text{tr}\left[H_d, H_{N_u^0}\right]^3; \quad I_6^{CP} \equiv \text{tr}\left[H_{N_d^0}, H_{N_u^0}\right]^3 \tag{14}$$

which are proportional to the imaginary parts of the invariant quartets of $U_{uL}^\dagger U_{N_u^0 L}$, $U_{dL}^\dagger U_{N_u^0 L}$ and $U_{N_d^0 L}^\dagger U_{N_u^0 L}$ respectively. So far, we have only considered invariants which are sensitive to left-handed mixings. One can construct analogous invariants which are sensitive to right-handed mixings, like:

$$I_7^{CP} \equiv \text{tr}\left[H'_d, H'_{N_d^0}\right]^3 = 6i\Delta_d\Delta_{N_d}\text{Im}Q_7 \tag{15}$$

where $H'_d \equiv M_d^\dagger M_d$, $H'_{N_d^0} \equiv N_d^{0\dagger} N_d^0$ and Q_7 is a rephasing invariant quartet of $U_{dR} U_{N_d^0 R}^\dagger$. Obviously, one can construct analogous invariants with the up sector, namely $I_8^{CP} \equiv \text{tr}\left[H'_u, H'_{N_u^0}\right]^3$.

3.2 The Minimal Flavour Violation Case

The invariants considered in the general 2HDM can obviously be applied to any flavour model where the matrices Γ_1, Γ_2, Δ_1 and Δ_2 have specific flavour structures (e.g. texture zeros) resulting, for example, from a flavour symmetry introduced in the Lagrangian. As we have seen, in the general 2HDM, the flavour structure of N_d^0, N_u^0 is arbitrary, which may lead to dangerous Higgs mediated FCNC, unless some natural suppression mechanism is found. Some time ago a class of models was constructed by Branco, Grimus and Lavoura (BGL) [11] where HFCNC are present at tree level with their structure entirely controlled by V_{CKM} with no other new flavour parameters. The class of models considered in Ref. [11] are entirely natural since their remarkable features result from a symmetry imposed on the Lagrangian. These models were generalized and their MFV character was analysed in Ref.[12]. An extension to the leptonic sector was proposed [27], with the rôle of V_{CKM} replaced by the Pontecorvo-Maki-Nakagawa-Sakata matrix denoted V_{PMNS}. The MFV hypothesis requires that the flavour structure of physics beyond the SM should only depend on V_{CKM} entries, quark masses and, in the case of 2HDM, on the ratio v_1/v_2 of Higgs vevs, with the corresponding analogue for the leptonic sector. The MFV as defined in [13] also requires that the breaking of the flavour symmetry be dominated by the top Yukawa couplings. In the context of the 2HDM this leads to the requirement that the new physics beyond the SM should be suppressed by the third row of V_{CKM} in order to comply with all the criteria introduced in the original paper where the definition of the Minimal Flavour Violation hypothesis was introduced [13].

For definiteness let us consider the Yukawa couplings arising in a specific BGL model, which realizes the MFV hypothesis with HFCNC only in the down sector:

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}; \qquad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} \tag{16}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix}; \qquad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} \tag{17}$$

This BGL model realizes the MFV hypothesis in a natural way. It has been pointed out [11] that there are six BGL models which correspond to interchanges of rows in the matrices given above as well as choosing what sector (up or down) has HFCNC which amounts to interchanging the matrices Γ_i with the matrices Δ_i. As previously emphasized, the specific texture of Eqs. (16), (17), reflects a particular choice of WB. In the sequel, we give

WB-independent necessary and sufficient conditions for a set of Yukawa couplings Γ_i, Δ_i written in an arbitrary WB to be of the BGL type, implying the existence of a WB where these matrices can be cast in the form given above.

Necessary and Sufficient Conditions for BGL

The following relations:

$$\Delta_1^\dagger \Delta_2 = 0; \quad \Delta_1 \Delta_2^\dagger = 0; \quad \Gamma_1^\dagger \Delta_2 = 0; \quad \Gamma_2^\dagger \Delta_1 = 0 \tag{18}$$

are necessary and sufficient conditions for a set of Yukawa matrices Γ_i, Δ_i to be of the BGL type, with Higgs mediated FCNC in the down sector.

Proof

Note that the conditions of Eqs. (18) are WB independent, in the sense that if a set of matrices Γ_i, Δ_i satisfy Eqs. (18) in a given WB, they will satisfy them when written in any other WB. From Eqs. (18) it follows that:

$$\left[\Delta_1 \Delta_1^\dagger, \Delta_2 \Delta_2^\dagger\right] = 0; \quad \left[\Delta_1^\dagger \Delta_1, \Delta_2^\dagger \Delta_2\right] = 0 \tag{19}$$

From Eq. (19) one concludes that one can choose a basis where both Δ_1 and Δ_2 are diagonal, real:

$$\Delta_1 = d_1 \equiv \text{diag.}\left[(d_1)_1, (d_1)_2, (d_1)_3\right]; \quad \Delta_2 = d_2 \equiv \text{diag.}\left[(d_2)_1, (d_2)_2, (d_2)_3\right] \tag{20}$$

This implies that in this case there are no FCNC in the up sector. From the requirement that $\Delta_1 \Delta_2^\dagger = 0$, which is one of the conditions of Eq. (18), one concludes that Eq. (20) leads to the following three solutions for the diagonal matrices d_1, d_2:

$$\text{(up)} \quad d_1 = \text{diag.}\left[\begin{array}{ccc} 0 & \times & \times \end{array}\right]; \qquad d_2 = \left[\begin{array}{ccc} \times & 0 & 0 \end{array}\right] \tag{21a}$$

$$\text{(charm)} \quad d_1 = \text{diag.}\left[\begin{array}{ccc} \times & 0 & \times \end{array}\right]; \qquad d_2 = \left[\begin{array}{ccc} 0 & \times & 0 \end{array}\right] \tag{21b}$$

$$\text{(top)} \quad d_1 = \text{diag.}\left[\begin{array}{ccc} \times & \times & 0 \end{array}\right]; \qquad d_2 = \left[\begin{array}{ccc} 0 & 0 & \times \end{array}\right] \tag{21c}$$

We have not included above, solutions corresponding to the interchange of d_1, d_2. Without loss of generality, we shall concentrate on one of the models, namely the "top model". This is the variant of the BGL models which satisfies all the constraints of the MFV hypothesis. It is also the most interesting version, from the phenomenological point of view, with strong natural suppression of FCNC in $\Delta S = 2$ transitions. In the top model, the Δ_i matrices have the following form in any arbitrary WB:

$$\Delta_1 = W_L^\dagger \begin{bmatrix} (d_1)_1 & 0 & 0 \\ 0 & (d_1)_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_R^u \tag{22}$$

$$\Delta_2 = W_L^\dagger \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (d_1)_3 \end{bmatrix} W_R^u \tag{23}$$

This implies that there are indeed WBs where these matrices can be cast in the form given by Eq. (17). This is obtained by choosing unitary matrices W_L and W_R^u of the block form:

$$W_L = \begin{bmatrix} (W_L)_{2\times 2} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}; \quad W_R^u = \begin{bmatrix} (W_R^u)_{2\times 2} & 0 \\ 0 & e^{i\beta} \end{bmatrix} \tag{24}$$

leading to:

$$\Delta_1 = W_L^\dagger \begin{bmatrix} (d_1)_1 & 0 & 0 \\ 0 & (d_1)_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_R^u = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix}; \tag{25}$$

$$\Delta_2 = W_L^\dagger \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (d_1)_3 \end{bmatrix} W_R^u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} \tag{26}$$

Let us now see how the other conditions restrict the form of Γ_1 in this WB. In the top model, the condition $\Gamma_1^\dagger \Delta_2 = 0$, leads to:

$$\Gamma_1^\dagger \Delta_2 = \begin{bmatrix} \times & \times & a \\ \times & \times & b \\ \times & \times & c \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} = 0 \tag{27}$$

From Eq.(27) one obtains $a = b = c = 0$, so Γ_1 has the form

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \tag{28}$$

in the basis where Δ_2 has the form of Eq. (26). The other condition in Eqs. (18) requires $\Gamma_2^\dagger \Delta_1 = 0$ which leads to:

$$\Gamma_2^\dagger \Delta_1 = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \tag{29}$$

From this equation one obtains:

$$\begin{aligned} \alpha_1 A + \alpha_2 C = 0 \\ \alpha_1 B + \alpha_2 D = 0 \end{aligned} \tag{30}$$

Note that since, in the chosen WB, the up and charm quark only receive mass from Δ_1 the non-vanishing of m_u and m_c imply $AD - BC \neq 0$ which together with Eqs. (30) leads to:

$$\alpha_1 = \alpha_2 = 0 \tag{31}$$

In an entirely analogous manner, one can show that β_1, β_2 and γ_1, γ_2 vanish. One concludes then that Γ_2 has the form:

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} \tag{32}$$

This completes the proof that the relations of Eq. (18) are necessary and sufficient conditions to have a BGL type model, with HFCNC in the down quark sector.

Similarly, using the up and charm solutions of Eqs. (21a) and (21b) one obtains the other two BGL models with FCNC in the down sector. The necessary and suffient conditions for BGL models with FCNC in the up sector can be written like those of Eq. (18) with the rôle of the Yukawa matrices Γ_i and Δ_i interchanged. These conditions are WB independent and therefore they allow one to identify BGL type models when written in an arbitrary WB where the zero texture patterns of the WB chosen by the symmetry are not present.

The lowest invariants in the MFV framework and CP violation

It is instructive to evaluate the lowest non-trivial invariants in the case of BGL models. In the general 2HDM one has:

$$M_d N_d^{0\dagger} = \frac{1}{2}\left[v_1 v_2 \left(\Gamma_1\Gamma_1^\dagger - \Gamma_2\Gamma_2^\dagger\right) + \left(v_2^2\Gamma_2\Gamma_1^\dagger - v_1^2\Gamma_1\Gamma_2^\dagger\right)\cos\theta + \right.$$

$$\left. + \, i\left(v_2^2\Gamma_2\Gamma_1^\dagger + v_1^2\Gamma_1\Gamma_2^\dagger\right)\sin\theta\right] \qquad (33)$$

It can be readily verified that in BGL models one has:

$$\mathrm{tr}\left[\Gamma_1\Gamma_2^\dagger\right] = 0 \qquad (34)$$

so that we obtain from Eq. (33):

$$I_1 \equiv \mathrm{tr}(M_d N_d^{0\dagger}) = \frac{1}{2}\mathrm{tr}\left[v_1 v_2 \left(\Gamma_1\Gamma_1^\dagger - \Gamma_2\Gamma_2^\dagger\right)\right] \qquad (35)$$

The important point is that in BGL models $M_d N_d^{0\dagger}$ is an Hermitian matrix and thus:

$$\mathrm{Im}\ \mathrm{tr}(M_d N_d^{0\dagger}) = 0 \qquad (36)$$

From Eqs. (10) and (36), it follows that in this class of models $\mathrm{Im}(N_d)_{jj} = 0$, thus avoiding too large e.d.m. for down-type quarks.

In order to extend the discussion of BGL type models to higher order WB invariants it is instructive to review the formulation of these models in a more generic way. Here we present some relations which greatly simplify the explicit computation of higher order invariants in terms of physical quantities. In Ref. [12] the special characteristics of BGL type models were analysed and generalized. It was pointed out that the particular BGL example given explicitly at the beginning of this section, corresponds to a class of models where N_d^0 and N_u^0 can be writen as:

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_i^\gamma\ M_d \qquad (37)$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^\beta\ M_u \qquad (38)$$

where $\mathcal{P}_i^\alpha$ are the projection operators defined [28] by

$$\mathcal{P}_i^\alpha = U_{\alpha L} P_i U_{\alpha L}^\dagger \qquad (39)$$

$$(P_i)_{lk} = \delta_{il}\delta_{ik} \qquad (40)$$

and α, β, γ denote u (up) or d (down). BGL models have $\gamma = \beta$ and therefore lead to HFCNC in one sector only. In BGL models we also have $i = j$. For $\gamma = \beta = u$ there are HFCNC only in the down sector and vice versa for $\gamma = \beta = d$. The example given at the beginning of this section corresponds to $\gamma = \beta = u$ and $i = j = 3$ and was presented in a particular weak basis.That weak basis was chosen by the symmetry imposed on the Lagrangian. Notice that the formulation presented here corresponds to the generalization of the model to any weak basis. The choice $i = 3$ together with $\gamma = \beta = u$ insures that the HFCNC are suppressed by the third row of V_{CKM}. In the WB where M_d is real and diagonal this particular example corresponds to:

$$V_{CKM} \equiv U_{uL}^\dagger U_{dL} = U_{uL}^\dagger \tag{41}$$

which leads to:

$$M_d = D_d, \qquad M_u = V_{CKM}^\dagger D_u U_{uR}^\dagger \tag{42}$$

$$N_d^0 \equiv N_d = \frac{v_2}{v_1} D_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) V_{CKM}^\dagger P_3 V_{CKM}\ D_d \tag{43}$$

$$N_u^0 = \frac{v_2}{v_1} V_{CKM}^\dagger D_u U_{uR}^\dagger - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) V_{CKM}^\dagger P_3 D_u U_{uR}^\dagger \tag{44}$$

Eqs. (37) – (40) together with the definition of V_{CKM} enable us to express all WB invariants in terms of physical quantities. All six cases with $\gamma = \beta$ and $i = j$ can be obtained as the result of a discrete symmetry [11].

In Ref. [12] a MFV expansion for N_d^0, N_u^0 with proper transformation properties under a WB transformation, corresponding to a generalization of Eqs. (37) and (38) is given by:

$$N_d^0 = \lambda_1\ M_d + \lambda_{2i}\ U_{dL} P_i U_{dL}^\dagger\ M_d + \lambda_{3i}\ U_{uL} P_i U_{uL}^\dagger\ M_d + ... \tag{45}$$

$$N_u^0 = \tau_1\ M_u + \tau_{2i}\ U_{uL} P_i U_{uL}^\dagger\ M_u + \tau_{3i}\ U_{dL} P_i U_{dL}^\dagger\ M_u + ... \tag{46}$$

In the quark mass eigenstate basis N_d^0, N_u^0 become:

$$N_d = \lambda_1\ D_d + \lambda_{2i}\ P_i\ D_d + \lambda_{3i}\ (V_{CKM})^\dagger\ P_i\ V_{CKM}\ D_d + ... \tag{47}$$

$$N_u = \tau_1\ D_u + \tau_{2i}\ P_i\ D_u + \tau_{3i}\ V_{CKM}\ P_i\ (V_{CKM})^\dagger\ D_u + ... \tag{48}$$

conforming explicitly with the requirement of depending only on the V_{CKM} matrix. This expansion contains as particular cases the six BGL models mentioned above. Only these six models can be obtained by means of an

Abelian symmetry of the Lagrangian [29], [27]. The symmetry also fixes the coefficients of the expansion in the form given by Eqs. (37) and (38). The expansion given by Eqs. (45) and (46) differs from the usual one considered in the literature [13] by splitting each component of $M_d M_d^\dagger$ and $M_u M_u^\dagger$ into [28]:

$$H_\alpha = \sum_i m_{\alpha i}^2 \mathcal{P}_i^\alpha \tag{49}$$

and allowing for different coefficients for each term of the expansion in $\mathcal{P}_i^\alpha$ with a different index i. In this sense the expansion given here is more general and contains the one used in the literature by many authors as a special case.

It is well known that in the SM the lowest order WB invariant sensitive to CP violation is given by Eq. (9) and has dimension twelve in powers of mass. Obviously, this invariant is also relevant for BGL type models. However, in BGL type models we have a richer flavour structure parametrized in terms of the four matrices M_d, M_u, N_d^0 and N_u^0 rather than the two mass matrices of the SM. As a result, in this case the lowest order invariant sensitive to CP violation is of lower order, namely:

$$I_9^{CP} \equiv \text{Im tr}\left[M_d N_d^{0\dagger} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger\right] \tag{50}$$

In BGL models invariants that see CP violation must contain flavour matrices both from the up and down sector. In fact the sector that has HFCNC has couplings that are proportional to only one row of V_{CKM} and it is always possible to choose a parametrization where any single row of V_{CKM} is real. This invariant can be readily evaluated using Eqs. (42), (43), which correspond to the specific BGL model given at the beginning of this section with $\gamma = u$ and $i = 3$, and one obtains:

$$\begin{aligned} I_9^{CP}(\gamma = u, i = 3) \qquad &= -\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \times \\ &\times (m_c^2 - m_u^2)\text{Im}\,(V_{22}^* V_{32} V_{33}^* V_{23}) \end{aligned} \tag{51}$$

This result is in agreement with the MFV character of BGL models namely, all flavour changing and CP violation are controlled by V_{CKM}, therefore this CP violating quantity must be proportional to the imaginary part of rephasing invariant quartets of V_{CKM} as in the SM [3]. Another important result is that $I_9^{CP}(\gamma = u, i = 3)$ is different from zero even if $m_t = m_c$ or $m_t = m_u$. In fact the discrete symmetry leading to this specific BGL model singles out the top quark [11]. It is important to emphazise that this invariant is defined in such a way that the trace involves the sum over all quarks, therefore it can be related to the baryon asymmetry generated at the electroweak phase transition [30], [31], [32], [33].

In the BGL model defined by $\gamma = d$, $i = 1$, where:

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_1^d \; M_d \tag{52}$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_1^d \; M_u \tag{53}$$

we can get an enhancement in the CP violating contribution to the baryon asymmetry of the order:

$$\frac{I_9^{CP}(\gamma = d, i = 1)}{I_1^{CP}} \; \frac{E^{12}}{E^8} \simeq \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \frac{E^4}{m_b^2 m_s^2} \tag{54}$$

where E is the scale relevant for baryogenesis at the electroweak phase transition. For $E \sim 100\text{GeV}$ we get an enhancement of about 10^{10}. This enhancement can be traced to the fact that this model singles out the d quark in such a way that the only mass difference involving down quarks appearing in $I_9^{CP}(\gamma = d, i = 1)$ is the suppression term $(m_b^2 - m_s^2)$ unlike in I_1^{CP} where the three different down square mass differences appear, so that the ratio of this invariant by I_1^{CP} is larger by a factor of the order $E^4/(m_b^2 - m_d^2)(m_s^2 - m_d^2)$

It is instructive to make use of Eqs. (42), (43) to compute I_2 which is real in this case:

$$\begin{aligned} I_2 = & -2 m_d^2 m_s^2 (m_s^2 - m_d^2)^2 \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)^2 |V_{31}|^2 |V_{32}|^2 - \\ & -2 m_d^2 m_b^2 (m_b^2 - m_d^2)^2 \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)^2 |V_{33}|^2 |V_{31}|^2 - \\ & -2 m_s^2 m_b^2 (m_b^2 - m_s^2)^2 \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)^2 |V_{33}|^2 |V_{32}|^2 \end{aligned} \tag{55}$$

the dominant term is the last one.

3.3 Two Higgs doublets with the NNI texture

Some time ago [34] it has been shown that, in the three generation SM, starting with arbitrary Yukawa couplings, one can always make a WB transformation such that the quark mass matrices M_d, M_u get the form:

$$M_d = \begin{bmatrix} 0 & a_d & 0 \\ a'_d & 0 & b_d \\ 0 & b'_d & c_d \end{bmatrix}; \qquad M_u = \begin{bmatrix} 0 & a_u & 0 \\ a'_u & 0 & b_u \\ 0 & b'_u & c_u \end{bmatrix} \tag{56}$$

this form, usually denoted nearest-neighbour-interaction (NNI) basis has no physical implications in the context of the SM with one Higgs doublet. If one further assumes that M_d, M_u are Hermitian in the NNI basis (i.e., $a'_{d(u)} = a^*_{d(u)}$, $b'_{d(u)} = b^*_{d(u)}$) then one obtains the Fritzsch Ansatz [35] which does have physical implications, correctly predicting $|V_{us}|$ but making a wrong prediction for $|V_{cb}|$, taking into account that $m_t \gg m_c$. This implies that the original Fritzsch Ansatz has been ruled out. Recently, it has been shown that one can reproduce all the current data on quark masses and mixing, by allowing deviations of Hermiticity of about 20% in the NNI form. It was also shown [17] that, in the context of 2HDM, one can obtain the NNI form for the quark mass matrices, through the introduction of a Z_4 symmetry in the Lagrangian, which leads to:

$$\frac{v_1}{\sqrt{2}}\Gamma_1 = \begin{bmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{bmatrix}; \qquad \frac{v_2 e^{i\theta}}{\sqrt{2}}\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{bmatrix} \tag{57}$$

$$\frac{v_1}{\sqrt{2}}\Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b'_u & 0 \end{bmatrix}; \qquad \frac{v_2 e^{-i\theta}}{\sqrt{2}}\Delta_2 = \begin{bmatrix} 0 & a_u & 0 \\ a'_u & 0 & 0 \\ 0 & 0 & c_u \end{bmatrix} \tag{58}$$

It is clear that the couplings of Eqs. (57), (58) lead to HFCNC in both the up and down sectors. In this section, we evaluate some of the previously defined WB invariants, illustrating their usefulness in the analysis of HFCNC and CP violating effects.

Let us consider I_1 again. It can be easily checked that this invariant is real in the NNI case.:

$$I_1 \equiv \mathrm{tr}(M_d N_d^{0\dagger}) = \frac{v_2}{v_1}(a_d a_d^* + a'_d {a'_d}^*) - \frac{v_1}{v_2}(b_d b_d^* + b'_d {b'_d}^*) + \frac{v_2}{v_1} c_d c_d^* \tag{59}$$

The same is true for I_2 which in this case is given by:

$$I_2 \equiv \text{tr}\left[M_d N_d^{0\dagger}, M_d M_d^\dagger\right]^2 = \left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)^2 \; 2\; c_d c_d^* \left[c_d c_d^* b_d b_d^* b'_d {b'_d}^* + \right.$$
$$\left. +(b'_d {b'_d}^*)^2 a_d a_d^* + (b_d b_d^*)^2 a'_d {a'_d}^* - a'_d {a'_d}^* b_d b_d^* b'_d {b'_d}^* - a_d a_d^* b_d b_d^* b'_d {b'_d}^*\right] \qquad (60)$$

In order to compare this result to the one obtained in the MFV case given by Eq. (55) we rewrite the coefficients of the NNI mass matrices in terms of quark masses using the approximate relations of Ref. [17]:

$$c_d c_d^* \sim m_b^2, \quad |a_d| \sim |a'_d| \sim \sqrt{m_d m_s}, \quad |b_d| \sim |b'_d| \sim \sqrt{m_s m_b} \qquad (61)$$

which lead to:

$$I_2 \sim 2\left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)^2 m_b^6 m_s^2 \qquad (62)$$

There are similarities between the dominant term in the MFV case and the NNI case, but in the NNI case there is no suppression factor given by the V_{CKM} matrix elements. Therefore HFCNC are potentially more dangerous in NNI models than in the MFV case. Another important point is the fact that in the NNI case the lowest invariants in powers of masses, sensitive to CP violation, are much lower than I_9^{CP}. One such example is:

$$\text{Im tr}\left[M_d N_d^{0\dagger} M_u M_u^\dagger\right] \sim \left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right) m_c^{\frac{1}{2}} m_t^{\frac{3}{2}} m_s^{\frac{1}{2}} m_b^{\frac{3}{2}} \sin\beta \qquad (63)$$

where the angle β is one of the two factorizable phases that cannot be removed from the mass matrices by rephasing of the quark fields. Note that in the NNI case it is possible to choose a WB where M_d (or else M_u) is real by rephasing quark fields. In this WB N_d^0 (or else N_u^0) is also real. Further rephasing on the righthanded side allows to remove three phases from the other mass matrix and also, at the same time, from the corresponding N^0 matrix, so that we are left with only two meaningful factorizable phases in the other mass matrix coinciding with the two phases left in the corresponding N^0 matrix. In Ref. [17] these two phases are evaluated, their sine is roughly of order one. Implications for the baryon asymmetry of the Universe are also important in this case.

4 Conclusions

We have presented a discussion of various flavour aspects of the general 2HDM. In particular, we have shown that flavour mixing in the 2HDM can be parametrized by a set of unitary matrices which arise from the misalignment in flavour space of various pairs of Hermitian matrices constructed from the Yukawa couplings of the 2HDM. These unitary mixing matrices are entirely analogous to the CKM matrix which arises in the SM from the misalignment of $H_d \equiv M_d M_d^\dagger$ and $H_u \equiv M_u M_u^\dagger$. Some of the CP violating phases are entirely analogous to the CKM phase, reflecting the non-vanishing of the imaginary parts of the various invariant quartets of the above unitary flavour matrices which arise in the 2HDM. We construct various WB invariants which can play a crucial rôle in the analysis of both CP violation and FCNC. Apart from a general analysis, we also applied the WB invariants to the study of specific flavour models, in the framework of 2HDM, such as MFV models of BGL type and models with a NNI structure. It is likely that the flavour structure of the 2HDM is not generic, reflecting on the contrary, the presence of some flavour symmetry. The WB invariants which we have constructed can be very useful in the study of new sources of CP violation in 2HDM constrained by some flavour symmetry. In particular, these WB invariants can be applied to the study of Higgs mediated FCNC in flavoured 2HDM. We also point out that in the 2HDM with MFV as well as in NNI models, CP-odd WB invariants arise in terms of much lower powers of masses than the CP-odd invariant of the SM, a feature which can have important implications for baryogenesis. The recent discovery of a Higgs boson at the LHC is an important step towards understanding the electroweak symmetry breaking sector. The LHC and, in the future, a linear collider will play an important rôle in putting further constraints on different two Higgs doublet model scenarios [1], [36], [37] taking into account, in particular, the distinguishing features between models with NFC and with MFV [38], [39], [40], [41], [42], [43].

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