Double parton correlations in constituent quark models

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Double parton correlations, having effects on the double parton scattering processes occurring in high-energy hadron-hadron collisions, for example at the LHC, are studied in the valence quark region, by means of constituent quark models. In this framework, two particle correlations are present without any additional prescription, at variance with what happens, for example, in independent particle models, such as the MIT bag model in its simplest version. From the present analysis, conclusions similar to the ones obtained recently in a modified version of the bag model can be drawn: correlations in the longitudinal momenta of the active quarks are found to be sizable, while those in transverse momentum are much smaller. However, the used framework allows to understand clearly the dynamical origin of the correlations. In particular, it is shown that the small size of the correlations in transverse momentum is a model dependent result, which would not occur if models with sizable quark orbital angular momentum were used to describe the proton. Our analysis permits therefore to clarify the dynamical origin of the double parton correlations and to establish which, among the features of the results, are model independent. The possibility to test experimentally the studied effects is discussed.

I. INTRODUCTION

Long time ago, the relevance of multiple hard partonic collisions taking place within a single hadronic scattering, the so called multiple parton interactions (MPI), has been addressed and studied [1]. Although MPI are higher twist, i.e. they are suppressed by a power of $\Lambda^2_{\rm QCD}/Q^2$ with respect to single parton interactions, Q being the partonic center-of-mass energy in the collision, experimental evidence of these kind of processes has been obtained, already some years ago [2]. MPI are of great relevance for LHC Physics, where they represent a background for the search of new Physics. It is not surprising therefore that a strong debate around MPI has been arising in

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recent years, when several dedicated Workshops have been organized, starting from that illustrated in Ref. [3]. In this work we concentrate on double parton scattering (DPS) processes. It is now understood that DPS contributes, e.g., to same-sign WW and same-sign dilepton production [4–7] and represents a background for Higgs studies in the channel $pp \to WH \to \ell\nu b\bar{b}$ [8–11]. Evidence of the occurrence of DPS at LHC has been already established [12]. Two comprehensive papers reporting the status of the art of theoretical knowledge for these processes have recently appeared [13, 14]. New signatures of DPS producing double Drell-Yan processes have been studied recently [15]. The dynamical origin of double parton correlations, having potential effects in DPS, in semi-inclusive deep inelastic scattering and in hard exclusive processes, has been the subject of recent comprehensive studies [16].

In the original work [1], the DPS cross section was written in terms of double parton distribution functions (dPDF) $F_{ij}(x_1, x_2, \vec{z}_\perp)$, describing the joint probability of having two partons with flavors $i, j = q, \bar{q}, g$, longitudinal momentum fractions x_1, x_2 and transverse separation $\vec{z}_\perp$ inside a hadron:

$$\begin{aligned} \mathrm{d}\sigma = \frac{1}{S} \sum_{i,j,k,l} \int \mathrm{d}^2\vec{z}_\perp\, F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) F_{kl}(x_3, x_4, \vec{z}_\perp, \mu) \\ \times\, \hat{\sigma}_{ik}(x_1 x_3 \sqrt{s}, \mu) \hat{\sigma}_{jl}(x_2 x_4 \sqrt{s}, \mu)\,. \end{aligned} \tag{1}$$

The partonic cross sections $\hat{\sigma}$ represent the hard, short-distance processes, while S is a symmetry factor arising if identical particles are present in the final state. In Eq. (1), contributions due to diparton flavor, spin and color correlations are neglected, as well as parton-exchange interference contributions [13, 14, 17, 18]. These correlations are present in QCD and could be phenomenologically relevant. Positivity bounds on dPDFs have been recently derived for polarized dPDFs [19].

Two main assumptions are usually made in DPS analyses:

i) the dependence on the transverse separation and on the momentum fractions or parton flavors are not correlated:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)\,; \tag{2}$$

ii) a factorized form is chosen also for the x_1, x_2 dependence:

$$\begin{aligned} & F_{ij}(x_1, x_2, \mu) \\ & = q_i(x_1, \mu) q_j(x_2, \mu)\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n\,, \end{aligned} \tag{3}$$

where q is the usual parton distribution function (PDF). $\theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$ introduces the kinematic constraint $x_1 + x_2 \le 1$, and $n > 0$ is a parameter to be fixed phenomenologically. dPDFs, describing soft physics, are nonperturbative functions.

dPDFs cannot be evaluated in QCD but, as it happens for the usual PDFs, they can be at least estimated at a low scale, $Q_0 \sim \Lambda_{\rm QCD}$, the so called hadronic scale, using quark models. In order to compare them with data taken at a momentum scale $Q > Q_0$, the results of model calculations should be evolved using perturbative QCD (pQCD). This evolution of dPDFs, namely the way they change from Q_0 to $Q > Q_0$, is currently a matter of debate [13, 14, 18, 20–26]. The resolution of this debate will be instrumental to relate not only data from different experiments but also model calculations at the hadronic scale to LHC data. In this way, the analysis of data involving DPS can be guided.

A first model calculation is the one recently presented in Ref. [27], where a bag model framework for the proton [28] has been considered to evaluate dPDFs in the valence region, at the hadronic scale Q_0, without evolution to $Q > Q_0$. As in any model where the valence quarks carry all the momentum, in the bag model the scale Q_0 has to be taken quite low. If the bag were assumed to be rigid, as in early calculations, corresponding to the so-called cavity approximation [29], the quarks would be independent and none of the relevant correlations described by dPDFs would be found. In Ref. [27], therefore, a prescription to recover momentum conservation, leading to quark correlations in the bag, already applied in model calculations of parton distributions, has been used [30–32]. The main outcome of Ref. [27] is that, in the modified MIT bag model, the approximation Eq. (2) holds reasonably well, but Eq. (3) is strongly violated. Problems with Eq. (3) had already been pointed out in Ref. [33–35].

The analysis of Ref. [27] is retaken here in a constituent quark model (CQM) framework. CQM calculations of parton distributions have been proven to be able to predict the gross features of PDFs [36, 37], generalized parton distributions (GPDs) [38] and transverse momentum dependent parton distributions (TMDs) [39]. Similar expectations motivated the present analysis. With respect to the approach of Ref. [27], the non relativistic (NR) dynamics used here includes from the very beginning correlations into the scheme, so that, for example, the dynamical origin of the breaking of the approximation Eq. (2) is clarified. As will be seen, this analysis helps to understand which among the conclusions of Ref. [27] are model dependent. It is important to stress from the very beginning that this kind of analysis, as the one in Ref. [27], is valid at the low hadronic scale corresponding to the model and only in the valence quark region. For this study to be directly used in the LHC data analysis, having for the moment high statistics only for very small values of x, far therefore from the valence region, and corresponding to high momentum transfer, additional studies, in particular the QCD evolution of the model results, are necessary. This investigation, like the one in Ref. [27], is therefore a first exploratory estimate, thought to understand the dynamical origin of the different momentum correlations and to guide experimental measurements accordingly.

The paper is structured as follows. In the next section, the formalism to evaluate the dPDFs in a CQM is clarified and the main equations presented. In the third one, results are illustrated and discussed. Eventually, conclusions are drawn in the last section.

II. CONSTITUENT QUARK MODEL DESCRIPTION OF DOUBLE PARTON DISTRIBUTION FUNCTIONS

As already stated in the Introduction, we are going to use a CQM framework and therefore only correlations for two valence quarks of flavor i, j can be studied. Let us work in momentum space, in terms of the Fourier-transformed distribution $F_{ij}(x_1, x_2, \vec{k}_\perp)$, defined as follows:

$$F_{ij}(x_1, x_2, \vec{k}_\perp) \; = \int d\vec{z}_\perp \, e^{i\vec{z}_\perp \cdot \vec{k}_\perp} F_{ij}(x_1, x_2, \vec{z}_\perp) \; . \tag{4}$$

The formalism introduced in Ref. [14] will be used. Color-correlated and interference dPDFs will not be considered, being Sudakov suppressed at high energies [14, 40].

In a NR quark model, the color-summed dPDF for two valence quarks, one with flavor q_1 and longitudinal momentum x_1, the other with flavor q_2 and longitudinal momentum x_2, with polarization s_1, s_2 in the proton, respectively, is:

$$\begin{aligned} F_{q_1 q_2}(x_1, x_2, \vec{k}_\perp) \; &= \int d\vec{k}_1 d\vec{k}_2 \int d\vec{k}'_1 d\vec{k}'_2 \, n_{q_1 q_2}(\vec{k}_1, s_1; \vec{k}_2, s_2; \vec{k}'_1, s_1; \vec{k}'_2, s_2) \\ &\times \; \delta\left(\vec{k}'_1 - \vec{k}_1 - \vec{k}_\perp\right) \delta\left(\vec{k}'_2 - \vec{k}_2 + \vec{k}_\perp\right) \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) \;, \end{aligned} \tag{5}$$

where light cone components, given by $a^\pm = p_0 \pm p_3$ for a generic 4-vector a^μ have been used for the quark and proton momenta and the two quark, spin dependent, off diagonal momentum distribution

$$\begin{aligned} n_{q_1 q_2}(\vec{k}_1, s_1; \vec{k}_2, s2; \vec{k}'_1, s_1; \vec{k}'_2, s2) \; &= \; 3 \sum_{s_3} \int d\vec{k}_3 \, \Phi^* \left(\vec{k}_1, s_1; \vec{k}_2, s_2; \vec{k}_3, s_3\right) \hat{P}_{q_1}(1) \hat{P}_{q_2}(2) \hat{P}_{s_1}(1) \hat{P}_{s_2}(2) \\ &\times \; \Phi\left(\vec{k}'_1, s_1; \vec{k}'_2, s_2; \vec{k}_3, s_3\right) \end{aligned} \tag{6}$$

has been introduced, using momentum space wave functions. In Eq. (6), the flavor and spin projectors, given by

$$\hat{P}_{u(d)}(i) = \frac{1 \pm \tau_3(i)}{2} \tag{7}$$

and

$$\hat{P}_{\uparrow(\downarrow)}(i) = \frac{1 \pm \sigma_3(i)}{2} \ , \tag{8}$$

respectively, for the particles 1, 2 appear. The factor 3 in Eq. (6) represents the number of valence quark pairs in the proton. It arises from the fact that any pair is equivalent to each other, since the proton ground state wave function is symmetric, once the color degrees of freedom are factorized out.

By taking into account properly the corresponding flavor and spin projectors, any kind of spin and flavor correlations can be easily evaluated. For the sake of definiteness, in this paper we will concentrate on spin averaged dPDFs, which we call, for brevity, distributions $q_1 q_2$. In a CQM framework, using translational invariance, they have therefore, according to Eqs. (5) and (6), the following form:

$$\begin{aligned} q_1 q_2(x_1, x_2, k_\perp) \ &= \ 3 \int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \, \psi^* \left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2}, \vec{k}_3 \right) \hat{P}_{q_1}(1) \hat{P}_{q_2}(2) \\ &\times \ \psi \left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2}, \vec{k}_3 \right) \delta \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 \right) \delta \left(x_1 - \frac{k_1^-}{P^-} \right) \delta \left(x_2 - \frac{k_2^-}{P^-} \right) \\ &= \ 3 \int d\vec{k}_1 d\vec{k}_2 \, \psi^* \left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2} \right) \hat{P}_{q_1}(1) \hat{P}_{q_2}(2) \\ &\times \ \psi \left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2} \right) \delta \left(x_1 - \frac{k_1^-}{P^-} \right) \delta \left(x_2 - \frac{k_2^-}{P^-} \right) \ , \end{aligned} \tag{9}$$

where the intrinsic wave function $\psi(\vec{k}_1, \vec{k}_2)$ has been introduced and $k_\perp = |\vec{k}_\perp|$. A similar expression was presented already in Ref. [14], obtained as the NR limit of the general definition of the dPDFs, given in terms of light-cone quantized operators and states. This equation allows one to evaluate $q_1 q_2$ in any CQM, by using the corresponding intrinsic momentum space wave functions.

In our presentation, we will use two different CQM:

i) a simple harmonic oscillator (HO) potential model, assuming an SU(6) spin-flavor structure, with the proton representing the ground state. In this case, the intrinsic wave function for the nucleon N can be written:

$$|N^2 S_{1/2}\rangle_S = \psi^S(\vec{k}_1, \vec{k}_2) = \frac{\sqrt{3}^{3/2}}{\pi^{3/2}\alpha^3} e^{-\frac{(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)}{\alpha^2}} \times SU(6) \tag{10}$$

where the spectroscopic notation $|^{2S+1}X_J\rangle_t$, with $t = A, M, S$ being the symmetry type, has been used and $SU(6)$ is the usual SU(6) spin-flavor proton wave function (χ refers to spin, Φ to flavor):

$$SU(6) = \frac{1}{\sqrt{2}} (\chi_{MS} \Phi_{MS} + \chi_{MA} \Phi_{MA}) \ , \tag{11}$$

whose explicit expression is given in textbooks and not repeated here. The parameter $\alpha^2 = m\omega$ of the HO potential is fixed to the value 1.35 fm^{-2}, in order to reproduce the slope of the proton charge form factor at zero momentum transfer. Despite its simplicity, it is perfectly clear that in this model dynamical correlations between the quarks are present from the very beginning, due to the $\vec{k}_1 \cdot \vec{k}_2$ term in the wave function, preventing it from being a simple product of two HO ground state wave functions. It is remarkable that this does not happen in the MIT bag model, whose simplest realization describes just independent, uncorrelated particles. In Ref. [27], correlations are introduced by properly modifying the simplest bag model scenario. In a potential model, we have them even in the simplest case. Therefore it is very appropriate to study double parton correlations in this framework.

ii) the CQM of Isgur and Karl (IK) [41]. In this model, the proton wave function is obtained by adding a one gluon exchange contribution to a confining HO potential, along the line addressed already in Ref. [42]; including contributions up to the $2\hbar\omega$ shell, neglecting a very small $\mathcal{D}$ wave contribution, the proton $|N\rangle$ is given by the following admixture of states

$$|N\rangle = \psi(\vec{k}_1, \vec{k}_2, s_1, s_2, s_3) = a_{\mathcal{S}}|N^2 S_{1/2}\rangle_S + a_{\mathcal{S}'}|N^2 S'_{1/2}\rangle_S + a_{\mathcal{M}}|N^2 S_{1/2}\rangle_M \ . \qquad (12)$$

The coefficients were determined by spectroscopic properties to be $a_{\mathcal{S}} = 0.931$, $a_{\mathcal{S}'} = -0.274$, $a_{\mathcal{M}} = -0.233$. [43]. In this case, the parameter $\alpha^2 = m\omega$ of the HO potential is fixed to the value 1.23 fm^{-2}, in order to reproduce the slope of the proton charge form factor at zero momentum transfer [43]. If $a_{\mathcal{S}} = 1$ and $a_{\mathcal{S}'} = a_{\mathcal{M}} = 0$ the simple HO model is recovered.

The formal expressions of the wave functions appearing in Eq. (12), yielding the IK model, can be found in Ref. [43, 44], given in terms of the following sets of conjugated intrinsic coordinates

$$\begin{aligned}
\vec{R} = \frac{1}{\sqrt{3}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \ &\leftrightarrow \ \vec{K} = \frac{1}{\sqrt{3}}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \ , \\
\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \ &\leftrightarrow \ \vec{k}_\rho = \frac{1}{\sqrt{2}}(\vec{k}_1 - \vec{k}_2) \ , \\
\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \ &\leftrightarrow \ \vec{k}_\lambda = \frac{1}{\sqrt{6}}(\vec{k}_1 + \vec{k}_2 - 2\vec{k}_3) \ . \qquad (13)
\end{aligned}$$

There are many good reasons to use the IK model to estimate dPDFs. First of all, the IK is a typical CQM, successful in reproducing the low energy properties of the nucleon, such as the spectrum and the electromagnetic form factors at small momentum transfer [41, 43]. In studies of DIS phenomenology, IK based model calculations were able to describe the gross features of PDFs [36, 37], generalized parton distributions (GPDs) [38] and transverse momentum dependent parton distributions (TMDs) [39].

From now on, for the sake of definiteness, we will consider $q_1 = u_1$, $q_2 = u_2$, i.e., the correlations between a quark u with longitudinal momentum fraction x_1 and the other quark u with longitudinal momentum x_2. Since this situation cannot be distinguished from that in which the momenta are exchanged between the two u quarks, we will actually show and discuss our results for the distribution

$$uu(x_1, x_2, k_\perp) = u_1 u_2(x_1, x_2, k_\perp) + u_2 u_1(x_1, x_2, k_\perp) = 2u_1 u_2(x_1, x_2, k_\perp) \ . \tag{14}$$

This quantity is the one usually discussed in the literature, e.g., in Refs. [27, 34], and this choice of presentation allows therefore an easy comparison of our results with those of other authors. The normalization of the dPDF Eq. (14), built according to Eqs. (9) - (11), is found to be

$$\int dx_1 dx_2 uu(x_1, x_2, k_\perp = 0) = N_u(N_u - 1) \ , \tag{15}$$

N_u being the number of u quarks in the system under investigation. For the proton discussed here, $N_u = N_u(N_u - 1) = 2$. In this case, the dPDF represents a number density, its norm yielding just the number of u quarks, with any momentum, in the proton.

By inserting the wave function Eq. (10) in Eq. (9), one gets, in the HO model:

$$uu^{HO}(x_1, x_2, k_\perp) = Ce^{-\frac{k_\perp^2}{2\alpha^2}} \int d\vec{k}_1 d\vec{k}_2 e^{-f(\vec{k}_1, \vec{k}_2, \alpha)} \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) \ , \tag{16}$$

where $C = 2(\sqrt{3}/(\pi\alpha^2))^3$ and $f(\vec{k}_1, \vec{k}_2, \alpha) = 2(k_1^2 + k_2^2 + \vec{k}_1 \cdot \vec{k}_2)/\alpha^2$. The delta function is worked out considering the nucleon with mass M at rest and the quark with mass $m \simeq M/3$ on mass shell, with energy $k_0 = \sqrt{m^2 + k^2}$. In this way one has, for $i = 1, 2$:

$$x_i = \frac{k_i^-}{P^-} = \frac{\sqrt{m^2 + k_i^2} - k_{iz}}{M} \ . \tag{17}$$

From this assumption, a support violation arises, e.g., a small tail of the longitudinal momentum distribution is found for $x > 1$. From Eq. (16), it is clear that uu depends on $k_\perp$, only in the exponent of the Gaussian function outside the integral. By inserting the wave function Eq. (12) in Eq. (9), one gets instead, in the IK model:

$$uu^{IK}(x_1, x_2, k_\perp) = a_{\mathcal{S}}^2 \, uu_{\mathcal{S}} + a_{\mathcal{S}'}^2 \, uu_{\mathcal{S}'} + a_{\mathcal{S}} a_{\mathcal{S}'} \, uu_{\mathcal{S}\mathcal{S}'} + a_{\mathcal{M}}^2 \, uu_{\mathcal{M}}, \tag{18}$$

where $uu_{\mathcal{S}} = uu^{HO}$ and

$$\begin{aligned} uu_{\mathcal{S}'}(x_1, x_2, k_\perp) &= \frac{C}{3} e^{-\frac{k_\perp^2}{2\alpha^2}} \int d\vec{k}_1 d\vec{k}_2 e^{-f(\vec{k}_1, \vec{k}_2, \alpha)} \{[f(\vec{k}_1, \vec{k}_2, \alpha) + k_\perp^2/(2\alpha^2)]^2 \\ &- [\vec{k}_\perp \cdot (\vec{k}_1 - \vec{k}_2)/\alpha^2]^2 - 3[2f(\vec{k}_1, \vec{k}_2, \alpha) + k_\perp^2/\alpha^2] + 9\} \\ &\times \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) \ , \end{aligned} \tag{19}$$

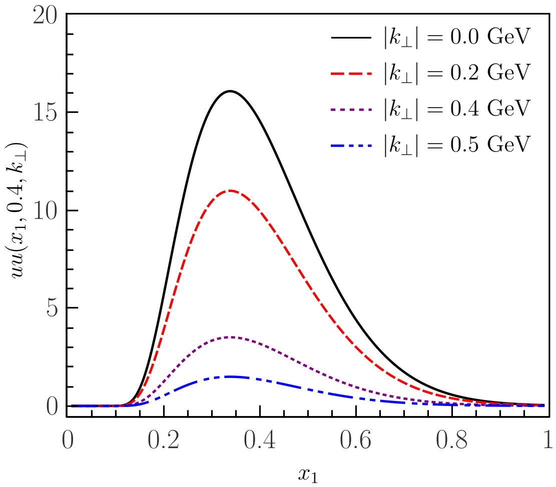

FIG. 1: The dPDF $uu(x_1, x_2, k_\perp)$ in the HO model, Eq. (16), for $x_2 = 0.4$ at four values of $k_\perp$.

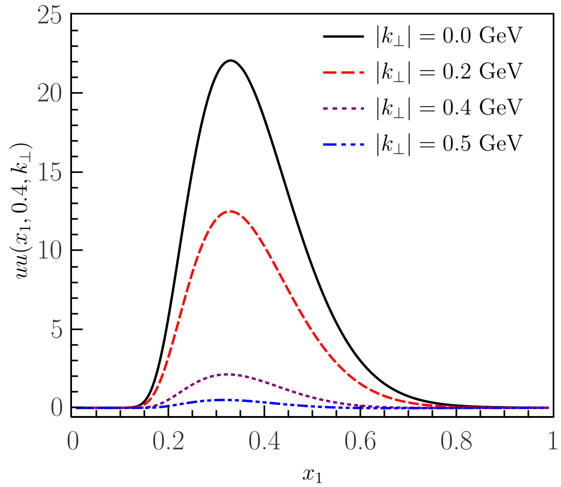

FIG. 2: The same as in Fig. 1, but in the IK model, Eq. (18).

$$uu_{\mathcal{S}\mathcal{S}'}(x_1, x_2, k_\perp) = \frac{C}{\sqrt{3}} e^{-\frac{k_\perp^2}{2\alpha^2}} \int d\vec{k}_1 d\vec{k}_2 e^{-f(\vec{k}_1, \vec{k}_2, \alpha)} \{2f(\vec{k}_1, \vec{k}_2, \alpha) + k_\perp^2/\alpha^2 - 6\} \times \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) , \quad (20)$$

$$uu_{\mathcal{M}}(x_1, x_2, k_\perp) = \frac{C}{3\alpha^4} e^{-\frac{k_\perp^2}{2\alpha^2}} \int d\vec{k}_1 d\vec{k}_2 e^{-f(\vec{k}_1, \vec{k}_2, \alpha)} \left\{ \frac{1}{2} \left[k_1^2 + k_2^2 + 4\vec{k}_1 \cdot \vec{k}_2 - \frac{k_\perp^2}{2} \right]^2 - \frac{1}{2} \left[\vec{k}_\perp \cdot (\vec{k}_1 - \vec{k}_2) \right]^2 + \frac{32}{3}\pi^2 \left\{ (k_1^2 - k_2^2)^2 - \left[\vec{k}_\perp \cdot (\vec{k}_1 + \vec{k}_2) \right]^2 \right\} g(\vec{k}_1, \vec{k}_2, \vec{k}_\perp) \right\} \times \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) . \quad (21)$$

where

$$g(\vec{k}_1, \vec{k}_2, \vec{k}_\perp) = \sum_{m_1} \langle 1\ m_1\ 1\ -m_1 \mid 0\ 0 \rangle Y_{1m_1}^*(\Omega_\rho) Y_{1-m_1}^*(\Omega_\lambda) \times \sum_{m_2} \langle 1\ m_2\ 1\ -m_2 \mid 0\ 0 \rangle Y_{1m_2}(\Omega_{\rho'}) Y_{1-m_2}(\Omega_\lambda) = \frac{3}{16\pi^2} \left[\cos\theta_{\rho'} \cos\theta_\lambda + \sin\theta_{\rho'} \sin\theta_\lambda \cos(\phi_\lambda - \phi_{\rho'}) \right] \times \left[\cos\theta_\rho \cos\theta_\lambda + \sin\theta_\rho \sin\theta_\lambda \cos(\phi_\lambda - \phi_\rho) \right] . \quad (22)$$

In Eq. (22), the solid angles Ω_λ, Ω_ρ, $\Omega_{\rho'}$ are the ones defined by the vectors $\vec{k}_{\rho'} = (\vec{k}_1 - \vec{k}_2 + \vec{k}_\perp)/\sqrt{2}$, $\vec{k}_\rho = (\vec{k}_1 - \vec{k}_2 - \vec{k}_\perp)/\sqrt{2}$ and $\vec{k}_\lambda = \sqrt{3}(\vec{k}_1 + \vec{k}_2)/\sqrt{2}$, so that the angles $\theta_{\rho(\rho')}, \phi_{\rho(\rho')}, \theta_\lambda, \phi_\lambda$ appearing in Eq. (22) can be written therefore through the components of $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_\perp$ according to the transformation laws Eq. (13).

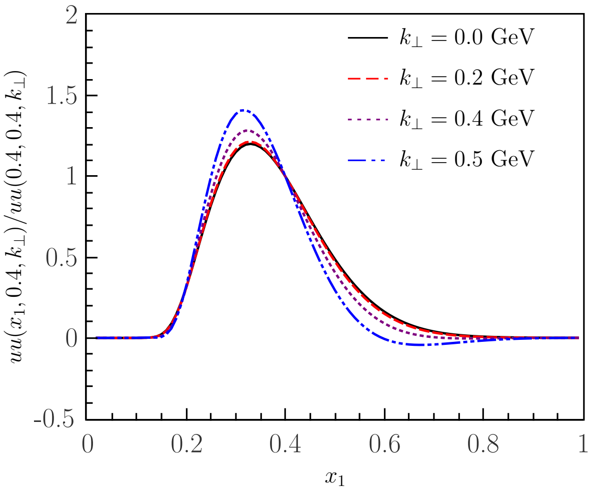

FIG. 3: The ratio Eq. (23) in the IK model, for four values of $k_\perp$.

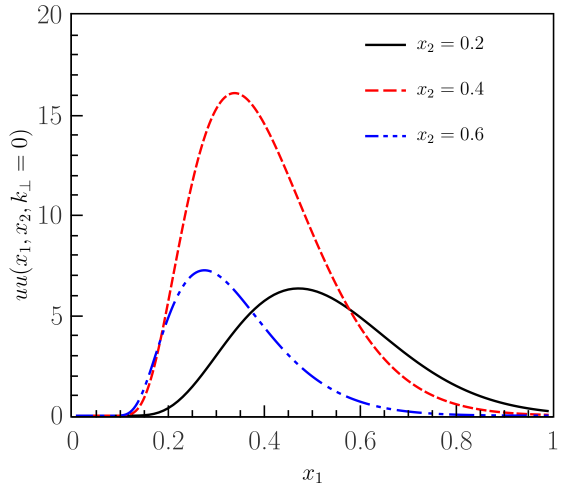

FIG. 4: The function $uu(x_1, x_2, k_\perp = 0)$ evaluated in the HO model for three different values of x_2.

It is clear from Eqs. (19-22) that in the IK model there is an additional dependence on $k_\perp$, aside from the Gaussian dependence found in the HO case. In particular, in Eq. (21), a peculiar $k_\perp$ dependence is introduced by the presence of orbital angular momentum of the quarks. Anyway, since the correlations under scrutiny are for unpolarized protons, no dependence on the orientation of $\vec{k}_\perp$ remains. One can easily realize this result mathematically by rewriting the above expressions in terms of scalar products.

III. RESULTS AND DISCUSSION

The results of the calculation are collected in Figs. 1-7. We have decided to show basically the same quantities presented in Ref. [27], because they describe all the features we want to emphasize and moreover they allow an easy comparison of the results of the different models.

The dPDF $uu(x_1, x_2, k_\perp)$ evaluated in the HO model, Eq. (16), for $x_2 = 0.4$ at four values of $k_\perp$, is shown in Fig. 1. These results are qualitatively and quantitatively similar to those in Ref. [27] once correlations are added to the simplest bag model scheme. As already stated several times, in the present scheme correlations are present from the very beginning. The $k_\perp$ dependence found here is Gaussian, as a consequence of the HO wave functions used. Also in the model of Ref. [27] a similar Gaussian behavior was found. In Fig. 2 , the same is shown in the IK model, using Eq. (18). Although the results are qualitatively similar and even quantitatively not far from the HO ones, relevant differences are found, as illustrated in the next two figures. As a matter of facts, in

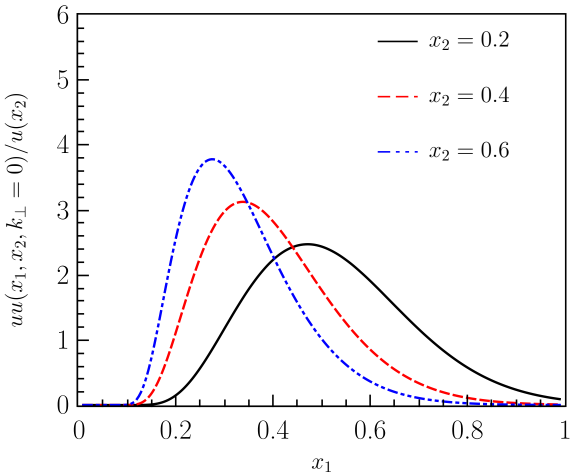

FIG. 5: The ratio Eq. (24) in the HO model, for three different values of x_2.

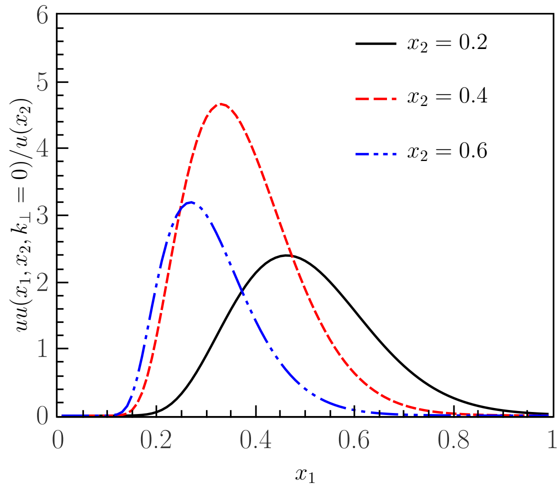

FIG. 6: The same as in Fig. 5, but in the IK model.

the HO calculation, the approximation Eq. (2) perfectly holds: the $k_\perp$ dependence is only in the exponent of the Gaussian function outside the integral in Eq. (16). In the IK model, it is not the case, as it is seen in Figs. 3, where the following ratio is shown

$$r_1(x_1, k_\perp) = \frac{uu(x_1, x_2 = 0.4, k_\perp)}{uu(x_1 = 0.4, x_2 = 0.4, k_\perp)} \ , \tag{23}$$

which should be $k_\perp$ independent if the approximation Eq. (2) were correct. Actually, a small violation, comparable in size to the one found in Ref. [27] is obtained. In the present calculation it is perfectly clear that the size of the violation of the approximation Eq. (2) is model dependent: it holds exactly if the ground state of the HO is taken, it is violated if mixture of states, with given OAM, are added. The fact that it is violated weakly is, again, a model dependent feature. In a model with stronger SU(6) breaking with respect to the IK one, strong violations, related to the OAM content of the proton, would arise.

The extent to which the other approximation, Eq. (3), is violated, is shown in Figs. 4-7. In particular, the ratio

$$r_2(x_1, x_2) = \frac{uu(x_1, x_2, k_\perp = 0)}{u(x_2)} \ , \tag{24}$$

where $u(x_2)$ is the standard PDF, shown in Fig. 5 (6) in the HO (IK) model, should not depend on the choice of x_2, if the approximation Eq. (3) were valid. Results in Fig. 4 are qualitatively similar to the ones in Ref. [27]: for example the maxima move towards lower x_1 values by increasing x_2. In Fig. 5, in which r_2 is plotted for the HO model, this trend is confirmed but the maxima become bigger when x_2 gets bigger, a behavior which is opposite to the one found in the corresponding figure of Ref. [27]. In Fig. 6, in which r_2 is plotted for the IK model, the trend of the HO model is basically confirmed, although the curve at $x_2 = 0.4$ has a peculiar behavior, confirming the model dependence of this ratio. This can be understood thinking to the different shape of the single particle PDFs appearing in the denominator. For example, the difference with the bag results is understood realizing that, in the latter framework, PDFs are not vanishing at $x = 0$ and present a sizable tail at $x < 0$ (see, e.g. Ref. [29]), while the same does not happen using constituent quark model, where a few percent support violation is found only at $x > 1$.

The issue of support violation in the present dPDFs calculation deserves further discussion. As a matter of fact, since dPDFs involve two quarks momenta, x_1 and x_2, with $x_1 + x_2 < 1$, in their evaluation the total support violation turns out to be greater than in the case of the PDFs. For example, looking at Fig. 4, it is clear that, while the curve corresponding to $x_2 = 0.6$ should go to zero already at $x_1 = 0.4$, a rather sizable tail is obtained, in the HO calculation, in the non-physical region. The same Figure shows that the situation gets worse by increasing x_2. Besides, comparing Figs. 1 and 2, the IK model seems to work slightly better than the HO one. Quantitatively, the amount of support violation, evaluated as the fraction of quark momentum in the non-physical region, is, at $k_\perp = 0$, in the HO (IK) model, 5 % (4 %) at $x_2 = 0.2$, 9 % (7 %) at $x_2 = 0.4$, 18 % (15 %) at $x_2 = 0.6$. The IK model is confirmed to work a little better than the HO one. However, since a severe support violation could originate serious problems when pQCD evolution is applied to the model results, a proper framework to evaluate dPDFs could be one where the problem does not arise, like, e.g., light-front dynamics [45, 46]. We are presently investigating this scenario.

The strong violation of Eq. (3) is also reported in Fig. 6. This figure has been drawn thinking that the HO framework is indeed a strongly correlated one. By defining dPDFs uu and PDFs u introducing a parameter β, as follows

$$uu_\beta(x_1, x_2, k_\perp = 0) = 2\frac{(4-\beta)^{3/2}}{\pi^3\alpha^6} \int d\vec{k}_1 d\vec{k}_2 e^{-2(k_1^2+k_2^2+\beta\vec{k}_1\cdot\vec{k}_2)/\alpha^2} \delta\left(x_1 - \frac{k_1^-}{P^-}\right) \delta\left(x_2 - \frac{k_2^-}{P^-}\right) , \quad (25)$$

$$u_\beta(x_i) = 2\frac{(4-\beta)^{3/2}}{\pi^3\alpha^6} \int d\vec{k}_1 d\vec{k}_2 e^{-2(k_1^2+k_2^2+\beta\vec{k}_1\cdot\vec{k}_2)/\alpha^2} \delta\left(x_i - \frac{k_i^-}{P^-}\right) , \quad (26)$$

it is easily seen that the HO model, a very strongly correlated one due to the HO potential, is recovered for $\beta = 1$. $\beta = 0$ represents instead an uncorrelated model, like, e.g., the genuine bag in the cavity approximation. Intermediate values of β may be similar to the modified bag model shown in Ref. [27]. This behavior is found indeed, as demonstrated by Fig. 7, where the ratio

$$r_\beta(x_1, x_2) = \frac{2uu_\beta(x_1, x_2, k_\perp = 0)}{u_\beta(x_1)u_\beta(x_2)} \quad (27)$$

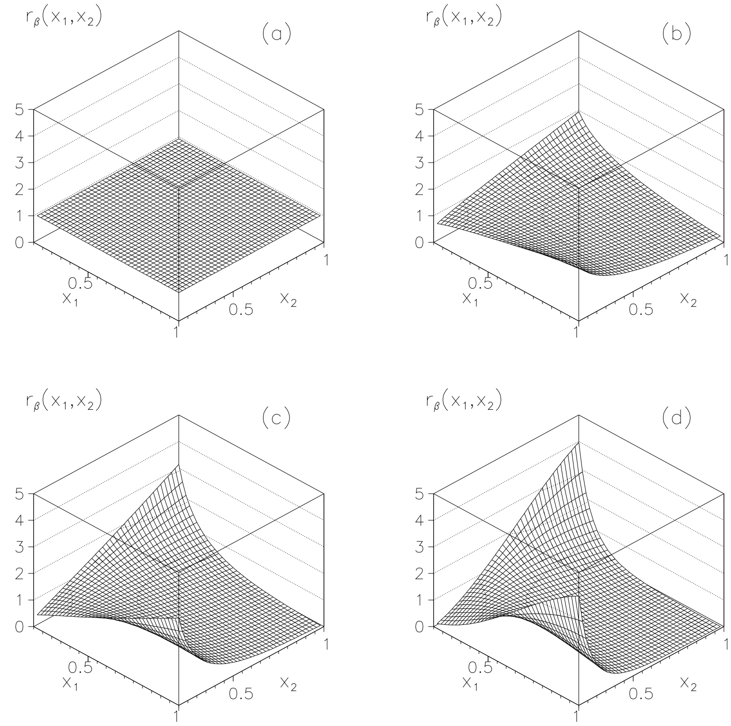

FIG. 7: The ratio Eq. (27), for: a) $\beta = 0$, i.e., in an uncorrelated scenario; b) $\beta = 0.25$; c) $\beta = 0.5$; d) $\beta = 1$, i.e., in the correlated HO framework. In the figure, for presentation convenience, the variables x_1 and x_2 range between 0.2 and 1.

is shown. The ratio we obtain is actually rather flat and approximately zero for $x_1 \simeq x_2 \simeq 1$, i.e. in the extreme non-physical region, as expected. In the same region, due possibly to momentum non-conservation in the bag model, some structures are seen in the calculation of Ref. [27].

We remind that the distribution u has norm 2, i.e., the same of uu (cf. Eqs. (15) and (26)). This is in agreement with the number density interpretation of both distributions, and with the number sum rule in the valence sector derived in Ref. [34]. One should notice that, for presentation convenience, in the numerator of Eq. (27) the uu distribution has been multiplied by a factor of two. In this way, a ratio equal to one is obtained when the x_1, x_2 factorization holds.

In closing this section, an important caveat is in order. We reiterate that the present estimate of quark correlations, as the one of Ref. [27], showing important violations of the often assumed approximations Eqs. (2) and (3), is expected to be reliable only in the valence quark region, i.e. for $x > 0.1$, and at the low energy scale described by the CQM. Actual data from LHC are dominated by the very low x region ($x < 10^{-3}$), at a very high scale of momentum transfer. Any comparison of this kind of results with data is instructive only once pQCD effects are implemented by using

QCD evolution to the experimental scale. Certainly, due to the coupling of quarks and gluons, correlations will be generated even if exact factorization is assumed at the low energy scale of the model.

IV. CONCLUSIONS

In many phenomenological applications, relevant, e.g., for the LHC experimental program, parton correlations are neglected in modelling double parton distribution functions. The latter quantities are therefore factorized in two terms, one depending on the longitudinal momentum fractions of the two quarks, x_1 and x_2, and another on the transverse momentum separation, $\vec{k}_\perp$, e.g. a sort of $x - \vec{k}_\perp$ factorization is assumed. In addition to that, also an $x_1 - x_2$ factorization assumption is usually made, e.g., the distribution on the longitudinal momenta x_1, x_2 is taken to be uncorrelated. This study aims at establishing to what extent constituent quark models support the two types of factorization, comparing the results with those of an important previous analysis, performed in a properly modified version of the simplest bag model. In this way, model-dependent features can be distinguished from model-independent ones. An analysis of double quark correlations has been therefore performed in the valence quark region, for spin averaged double parton distribution functions. Two different constituent quark models have been used, i.e., a simple non relativistic SU(6) symmetric model and its generalization, the model of Isgur and Karl, corresponding to a mild SU(6) breaking due to the exchange of one gluon. For the aim of studying correlations and of understanding their dynamical origin, the framework used here, correlated from the very beginning through the interquark potential, is to be preferred with respect to models where correlations are added through prescriptions modifying independent particle models, such as the simplest version of MIT bag model. The conclusions of our work are as follows. In both constituent quark models, the $x_1 - x_2$ factorization is strongly violated, confirming a similar conclusion obtained already in the bag model framework. This feature seems therefore a model-independent one. In the SU(6) symmetric model, where the proton is described by a pure symmetric $\mathcal{S}$ wave, exact $x - k_\perp$ factorization is obtained. This features is mildly spoiled in the Isgur and Karl model, as it happens in the bag model framework. With respect to the latter, the present approach allows to understand more clearly the dynamical origin of the breaking of the $x - k_\perp$ factorization, and its relation to the orbital angular momentum of the quarks. It is also found that the amount of violation of the $x - k_\perp$ factorization is a model dependent feature. Certainly this effect is a small one in the scenario discussed here, where the symmetric $\mathcal{S}$ wave is dominating. Models with more sizable contributions from components of the wave function with higher values of the orbital angular momentum would certainly show a stronger violation. In closing, it is important to stress that, for the results of model calculations of double parton distributions, obtained either in a bag model or in a constituent quark one, to be phenomenologically relevant to the LHC Physics programme, the evaluation of their QCD evolution is crucial. This will be the subject of further studies, together with the evaluation of the same observables using relativistic, Light-Front approaches. In this framework, successfully applied in Hadronic Physics in general (see, e.g., [45]) and for the calculation of PDFs in particular (see, e.g., [46]), important contributions from high values of the relative orbital angular momentum naturally arise.

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