

Horizon Geometry, Duality and Fixed scalars in Six Dimensions*

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Abstract

We consider the problem of extremizing the tension for BPS strings in $D = 6$ supergravities with different number of supersymmetries. General formulae for fixed scalars and a discussion of degenerate directions is given. Quantized moduli, according to recent analysis, are supposed to be related to conformal field theories which are the boundary of three dimensional anti-de Sitter space time.

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1 Introduction

Recently possible connections of brane dynamics of the world volume theory and supergravity in the ambient space of near horizon geometry have been proposed and investigated [1, 6, 3, 4, 7, 5, 2]. These attempts heavily rely on the correspondence of supergravity on an anti-de Sitter bulk and special properties of “topological” singleton representations confined on anti-de Sitter boundaries [8].

Interestingly enough, as pointed out by Maldacena [1], this correspondence can only occur at some special points of the moduli space of the theory. Indeed this observation is based on previous recent analysis which shows that certain moduli fields satisfy, in a given black brane background, flow equations [9] with an attractor point at the horizon where the geometry becomes anti-de Sitter [11].

Fixed scalars (attractors) have been until recently mainly investigated in connection to black holes in $D = 4$ and black holes and black strings in $D = 5$.

In $D = 5$ strings are dual (magnetic) to black holes so fixed scalars in these cases are easily obtained for both strings and black holes by using dual formulae replacing the BPS mass of a black hole with the BPS tension of a string [28]. However, the recent developments on possible dualities between anti-de Sitter spacetimes and brane dynamics has led to focus on string horizon geometries at $D = 6$ because of their relation to two dimensional conformal field theories [1, 3, 5, 29].

In the present paper we generalize the analysis of fixed scalars to $D = 6$ BPS strings, a case which has not been thoroughly treated in previous investigations. We will examine cases with 8, 16 and 32 supersymmetries and explicitly compute fixed scalars for all these cases. A discussion of particular theories to which these results apply will also be given.

2 Central charges and BPS tension in $D = 6$ supergravity theories

We recall here some properties of the moduli dependence of BPS string tension in six dimensional theories.

At $D = 6$, 3-form field strengths can be self-dual or antiself-dual. Let us denote by P, Q the number of self-dual and antiself-dual field strengths

respectively.

For all $D = 6$ theories we can use a unified formalism based on an underlying moduli space relevant to the problem, locally of the form $O(P, Q)/O(P) \times O(Q)$, where the range of P, Q will depend on the theory [22, 23]. More specifically for theories with (1,0) supersymmetry $P = 1, Q = n$. For (1,1) supersymmetry $P = Q = 1$, for (2,0) supersymmetry $P = 5, Q = n$ and for (2,2) theories $P = Q = 5$. Moreover, since (1,0) and (2,0) theories are chiral, n is constrained by the anomaly cancellation to be $n = 21$ in (2,0) theories and $n = (273 + n_V - n_H)/29$ in (1,0) theory (n_H, n_V being the number of hyper and vector multiplets respectively)².

The coset space $O(P, Q)/O(P) \times O(Q)$ can be defined by coset representatives $X_{r\Lambda}, X_{I\Lambda}, r = 1, \dots, P, I = 1, \dots, Q, \Lambda = 1, \dots, P + Q$ satisfying the conditions

$$X_{r\Lambda}X_{r\Sigma} - X_{I\Lambda}X_{I\Sigma} = \eta_{\Lambda\Sigma}. \quad (1)$$

Explicitly, X_{rI} are PQ coordinates and:

$$X_{rs} = \sqrt{\delta_{rs} + X_{rI}X_{sI}}; \quad X_{IJ} = \sqrt{\delta_{IJ} + X_{rI}X_{rJ}}. \quad (2)$$

To define BPS string tension through the BPS condition, we must introduce a ‘‘metric’’ $\mathcal{N}_{\Lambda\Sigma}$ for the self-dual and antiself-dual 3-forms H^Λ . The metric $\mathcal{N}_{\Lambda\Sigma}$ is defined as follows [12]

$$\mathcal{N}_{\Lambda\Sigma} = X_{r\Lambda}X_{r\Sigma} + X_{I\Lambda}X_{I\Sigma} = 2X_{r\Lambda}X_{r\Sigma} - \eta_{\Lambda\Sigma}. \quad (3)$$

with the property:

$$\mathcal{N}\eta\mathcal{N}\eta = \mathbb{1}, \quad \text{i.e.} \quad \mathcal{N}^{-1} = \eta\mathcal{N}\eta \quad (4)$$

The self-duality condition of the field strengths becomes:

$$\mathcal{N}_{\Lambda\Sigma} *H^\Sigma = \eta_{\Lambda\Sigma}H^\Sigma \quad (5)$$

which therefore implies, by integrating Eq. 5 on a 3-sphere

$$q_\Lambda = \eta_{\Lambda\Sigma}m^\Sigma \quad (6)$$

q_Λ, m^Λ being the electric and magnetic charges respectively.

² The construction of D=6 chiral theories has been recently completed in [24]

Therefore, P forms have $q = m$ and Q have $q = -m$ as appropriate for self-dual (antiself-dual) forms. Note that the Dirac-Zwanzinger-Schwinger quantization condition for dyonic odd p-forms is [12][13]

$$q_\Lambda m'^\Lambda + m^\Lambda q'_\Lambda = 2\pi k \quad (7)$$

Using the equation above we get the manifestly $O(P, Q)$ invariant condition

$$q_\Lambda \eta^{\Lambda\Sigma} q'_\Sigma = \pi k \quad (8)$$

We note that chiral theories, with $P \neq Q$, are non-lagrangian because of a net number of self- (antiself-) dual tensors [22],[23]. However for non-chiral theories, with $P = Q$, Eq. 5 can be replaced by P unconstrained 2-form fields for which an action exists. In an appendix we give the relation between the lagrangian couplings of non-chiral theories and the matrix $\mathcal{N}_{\Lambda\Sigma}$ appearing in Eq. 5.

From the quantized set of charges q we must construct “dressed” charges, appropriate to discuss BPS states. They are defined as follows,

$$Z_r = X_r^\Lambda q_\Lambda, \quad Z_I = X_I^\Lambda q_\Lambda \quad (9)$$

with the property

$$W = Z_r Z_r + Z_I Z_I = q^\Lambda \mathcal{N}_{\Lambda\Sigma} q^\Sigma, \quad Z_r Z_r - Z_I Z_I = q^\Lambda \eta_{\Lambda\Sigma} q^\Sigma \quad (10)$$

Note that the first of the above equations defines an analogous of the Weinhold potential [14] for $D = 6$, BPS strings. The physical meaning of Z_r and Z_I is that they provide the 3-form charge eigenstates which enter in the self-dual tensor of the gravitational sector and the antiselfdual matter tensor of (1,0) and (2,0) theories respectively. For the (1,1) and (2,2) theories Z_r and Z_I all belong to the gravity multiplet. Let us note that Z_r (and not Z_I) is a “central charge” which occurs in the chiral supersymmetry algebra, while in the non chiral case both Z_r and Z_I are central charges.

For (1,0) theory we have one Z ³ while for (2,0) theories Z_r is a 5 of $USp(4)$. This means that in both cases there is only one central charge eigenvalue and only one type of BPS state (1/2 BPS).

³In the (1,1) theory $Z^2 = Z_L^2 = Z_R^2 + em$ where Z_L (Z_R) is the central charge of the (1,0) ((0,1)) subalgebra and e, m denote the electric and magnetic charges of the BPS string. Both 1/2 and 1/4 BPS strings exist in this case.

For the (2,2) theory, Z_r and Z_I are in the 5's of two different $\text{USp}(4)$. Therefore, there are two independent central charge eigenvalues and 1/4 BPS states are those for which $Z_r^2 \neq Z_I^2$ ⁴ while 1/2 BPS correspond to light-like charges $q^\Lambda \eta_{\Lambda\Sigma} q^\Sigma = 0$ [15].

We also remark that the (1,0) geometry of tensor multiplets is entirely analogous to the D=5 “very special” geometry [16] for vector multiplets in respect to the fact that to each tensor multiplet corresponds one real scalar as for vectors in D=5. It is not then surprising that the central charge and the moduli geometry carries a resemblance from that case.

First observe that the potential is now

$$W = Z^2 + Z_I^2, \quad Z = X^\Lambda q_\Lambda \quad (11)$$

where

$$Z_I = P_I^i \partial_i Z \quad (12)$$

and P_I^i is the inverse Vielbein of $\text{O}(1, n)/\text{O}(n)$. Then it follows that

$$Z_I^2 = G^{ij} \partial_i Z \partial_j Z \quad (13)$$

and in particular the condition $Z_I = 0$ is equivalent to $\partial_i Z = 0$.

From the previous equations we also note the relation

$$G_{ij} = \partial_i X^\Lambda \mathcal{N}_{\Lambda\Sigma} \partial_j X^\Sigma, \quad (X^\Lambda \eta_{\Lambda\Sigma} X^\Sigma = 1) \quad (14)$$

which follows from the Maurer–Cartan equations:

$$\partial_i X_\Lambda = X_{\Lambda I} P_i^I \quad (15)$$

and the relation:

$$\mathcal{N}^{\Lambda\Sigma} X_{\Sigma I} = X_I^\Lambda \quad (16)$$

The explicit expressions of G_{ij} and $\mathcal{N}_{\Lambda\Sigma}$ are

$$G_{ij} = \delta_{ij} - \frac{x_i x_j}{1 + x_i^2}, \quad x_0 = \sqrt{1 + x_i^2}, \quad x^i = x_i \quad (17)$$

$$\mathcal{N}_{\Lambda\Sigma} = 2X_\Lambda X_\Sigma - \eta_{\Lambda\Sigma} \quad (18)$$

where Eq. 18 follows from:

$$\begin{aligned} X_\Lambda X_\Sigma - X_{\Lambda I} X_{\Sigma I} &= \eta_{\Lambda\Sigma} \\ X_\Lambda X_\Sigma + X_{\Lambda I} X_{\Sigma I} &= \mathcal{N}_{\Lambda\Sigma} \end{aligned} \quad (19)$$

⁴ By Z_r^2 we mean $Z_r Z_r$

3 Fixed scalars and Horizon geometry

From general considerations based on the relation of brane dynamics to asymptotic horizon geometry we expect [12] that the flow of moduli toward the horizon implies an “attractor” condition equivalent to the extremization of W as defined by Eq. 10.

This condition signals “supersymmetry enhancement” in the horizon geometry [10, 14]. It is remarkable that such “phase transition” occurs at “special points” of the “asymptotic” moduli space of the theory.

In D=4,5 black holes and strings the asymptotic geometry is $Ad_2 \times S_2$ and $Ad_3 \times S_2$ respectively [11].

Supersymmetry enhancement of the former, the so called Bertotti–Robinson geometry, is due to the fact that the background is superconformal invariant in a 4-dimensional sense [17]. For D=5 the geometry of the string corresponds to a (4,0) two dimensional conformal field theory with the supersymmetry enhancement in the corresponding superalgebra.

In these theories the “fixed moduli” are the vector multiplets while the hypermultiplets are not fixed by the horizon geometry [9, 10].

In the D=6 case, according to the analysis of Ref. [12], fixed scalars correspond to partners of antiselfdual tensors, i.e. the moduli of (1,0) tensor multiplets. In this case the horizon geometry is $Ad_3 \times S_3$.

For theories with higher supersymmetry (i.e. with P=5) the degenerate directions should correspond to D=6 (1,0) hypermultiplets, in analogy with the phenomenon in D=4,5 for theories with $N > 2$ supersymmetry [18]. Fixed scalars can be obtained following the analysis of Ref.[18] for D=4,5 theories.

The basic point are the Maurer–Cartan equations, satisfied by the one form differentials of the BPS central charges [12].

For a $O(P, Q)/O(P) \times O(Q)$ moduli geometry these equations read

$$\nabla Z_r = P_r^I Z_I \tag{20}$$

$$\nabla Z_I = P_I^r Z_r \tag{21}$$

where P_{rI} is the Vielbein one-form ($P_r^I = \delta^{IJ} P_{rI}, P_I^r = \delta^{rs} P_{sI}$). From the first equation we get that the “central charge” Z_r is extremized by $Z_I = 0$, which therefore solves $\delta W = 0$. It is important to notice that “fixed scalars” for which $Z_I = 0$ give a “regular” horizon since in this case the analogous of

the Bekenstein–Hawking entropy

$$S = \frac{A^{D-2}}{G_N} \quad (22)$$

takes the value

$$W|_{Z_I=0} = q^\Lambda \eta_{\Lambda\Sigma} q^\Sigma \quad (23)$$

which therefore requires the charge vector q to satisfy $q^\Lambda \eta_{\Lambda\Sigma} q^\Sigma = q^2 > 0$ when $P < Q$. For $P = Q$, the solution with $q^2 < 0$ is isomorphic to the previous one by interchanging Z_r with Z_I .

We now consider the details of the general remarks to the four particular theories with 8,16,32 supersymmetries ⁵.

In the (1,0) theory the “attractor equation”:

$$\partial_i Z = 0 \quad (24)$$

has the unique solution

$$X_\Lambda = \frac{q_\Lambda}{\sqrt{q^2}} \quad (25)$$

The string tension at this point is

$$Z|_{extr} = \sqrt{q_\Lambda q^\Lambda} = \sqrt{q_0^2 - q_i^2} \quad (26)$$

Note that the hypermultiplet scalars (which must be present in the theory because of anomaly cancellation) do not enter in the discussion so the 3-dimensional anti-de Sitter geometry cannot depend on them [8].

For $q^2 < 0$, the equation $\delta W = 0$ has a different solution which occurs for tensionless strings, i.e. at the point $Z = 0$. This is not an extremum of the string tension, but rather at this point:

$$W|_{Z=0} = |Z_I|^2 = -q^2 \quad (27)$$

A particular solution of this equation is:

$$X_i^{(0)} = \frac{q_i}{\sqrt{q_i^2}} \frac{q_0}{\sqrt{q_i^2 - q_0^2}} \quad (28)$$

⁵The (1,1) theory, having one tensor field, is like the (1,0) theory for $n = 1$. However the string is 1/4 BPS when it is dyonic [30], while it is 1/2 BPS when it is purely electric or magnetic.

Let us now consider the (2,0) theory with $P = 5, Q = 21$. From the form of W and Eq. 20 we see that

$$\partial_i W = 0 \quad (29)$$

occurs at $Z_I = 0$ if $q^2 > 0$.

Since in this case i takes $5Q$ values and $Z_I = 0$ are Q conditions, it means that there are precisely $4Q$ (84 for the case at hand) moduli directions not fixed by Eq. 29. These are precisely the Q hypermultiplets in the (1,0) decomposition of the (2,0) tensor multiplets.

Note that, according to the results of Ref. [19], the relevant solvable algebra decomposition of the coset reads

$$\text{solv}\left(\frac{\text{O}(5, Q)}{\text{O}(5) \times \text{O}(Q)}\right) = \text{solv}\left(\frac{\text{O}(1, Q)}{\text{O}(Q)}\right) + \text{solv}\left(\frac{\text{O}(4, Q)}{\text{O}(4) \times \text{O}(Q)}\right) \quad (30)$$

where the two factors correspond to the (1,0) tensor- and hyper-multiplets respectively.

Eq. 29 implies that the hypermultiplets do not occur in the solution so $4Q$ directions are undetermined by the attractor condition and will not occur at the extremum of W .

It is straightforward to give a particular solution for the fixed scalars which solves Eq. 29. This is given by

$$X_{r\Lambda}^{(0)} = \frac{q_r q_\Lambda}{\sqrt{q_r^2} \sqrt{q_r^2 - q_I^2}} \quad (31)$$

It can be easily shown that at $X_{rI} = X_{rI}^{(0)}$

$$Z_r = \frac{q_r \sqrt{q_r^2 - q_I^2}}{\sqrt{q_r^2}} \quad (32)$$

which implies

$$W|_{X=X^{(0)}} = Z_r^2 = q_r^2 - q_I^2 \quad (33)$$

Note that $X^{(0)}$ is a particular solution of $Z_I = 0$ since it does not determine the moduli directions \widehat{X} orthogonal to Z_r

$$\widehat{X}_{rI} = X_{rI} - \frac{Z_r X_{sI} Z_s}{Z_s^2} \quad (34)$$

Using Eq. 18 it is also possible to give a general formula for the $\mathcal{N}_{\Lambda\Sigma}$ metric at the attractor point. It reads

$$\mathcal{N}_{\Lambda\Sigma}|_{X=X^{(0)}} = \frac{2q_\Lambda q_\Sigma}{q^2} - \eta_{\Lambda\Sigma} \quad (35)$$

which is of course consistent with Eq. 33.

Let us now consider the case $q^2 < 0$. Similar to the (1,0) case, $\delta W = 0$ occurs in this case at $Z_r = 0$ at which point:

$$W|_{Z_r=0} = Z_I^2 = -q^2 \quad (36)$$

This equation implies that tensionless strings are not extrema of the BPS tension. A particular solution of the moduli at this point is:

$$X_{rI}^{(0)} = \frac{q_I}{\sqrt{q_I^2}} \frac{q_r}{\sqrt{q_I^2 - q_r^2}} \quad (37)$$

This solution leaves $5(Q - 1) = 100$ directions undetermined as it appears from the moduli directions orthogonal to Z_I :

$$\widehat{X}_{rI} = X_{rI} - \frac{Z_I X_{rJ} Z_J}{Z_J^2} \quad (38)$$

We finally come to the (2,2) theory with R-symmetry given by $O(5) \times O(5) \sim \text{USp}(4)_L \times \text{USp}(4)_R$. In this case the moduli space is locally $O(5, 5)/O(5) \times O(5)$ and the attractor condition

$$Z_I = 0 \quad (39)$$

implies that the lower eigenvalue of the central charge matrix vanishes, as in the $D = 4, 5$ cases [10, 18]. For charges $q^\Lambda q_\Lambda \lesseqgtr 0$, $Z_r^2 \lesseqgtr Z_I^2$, and the ‘‘attractor point’’ gives a 1/4 BPS state with Weinhold potential given by

$$W|_{Z_I=0} = q^2 \quad \text{if} \quad q^2 > 0 \quad (40)$$

$$W|_{Z_r=0} = -q^2 \quad \text{if} \quad q^2 < 0 \quad (41)$$

This situation corresponds to the BPS orbit $O(5, 5)/O(4, 5)$ according to the analysis of Ref. [15, 20].

One half BPS strings require, on the other hand, two coinciding eigenvalues for the central charge and this requires a light-like orbit $q^\Lambda q_\Lambda = 0$. The

BPS orbit is in this case $O(5, 5)/IO(4, 4)$ and implies (for all values of the moduli) the identity

$$Z_r^2 = Z_I^2 \tag{42}$$

Note that in the previous cases of chiral theories the 1/2 BPS orbits fall into three categories, i.e. $O(P, Q)/O(P-1) \times O(Q)$ for $q^2 > 0$, $O(P, Q)/IO(P-1, Q-1)$ for $q^2 = 0$ and $O(P, Q)/O(P) \times O(Q-1)$ for $q^2 < 0$. Light-like orbits do not correspond to supersymmetry enhancement in this case. This is similar to 1/8 BPS orbits with vanishing entropy for $D=4$ black holes [15, 21]. Space-like orbits ($q^2 < 0$) correspond to the occurrence of tensionless strings [27]. The moduli at which the string becomes tensionless do not correspond to an extremum of the string tension.

4 Concluding remarks

In this paper we have considered in some details BPS strings in different theories at $D = 6$ and studied the extremization of the string tension in the moduli space.

Fixed scalars correspond to particular quantized values in terms of the string charges.

In (1,0) theories the fixed scalars are in tensor multiplets, in (1,1) theory the fixed scalar is in the gravity multiplet (dilaton) and in (2,0), (2,2) theories the fixed scalars correspond to the tensor multiplet scalars resulting by decomposing the theories in (1,0) representations. The other scalar fields are not fixed by the horizon geometry, therefore 84 and 20 moduli are at their initial value in the (2,0) and the (2,2) theories respectively.

The four kinds of $6D$ supergravity theories considered here correspond, in string theory language, to different compactifications of heterotic and Type II strings on K_3 and T_4 . (1,0) theories with more than one tensor multiplet naturally arise in open string constructions [26] and F-theory compactifications [25]. The fixed scalars correspond to special isolated points in the K_3 and T_4 moduli spaces. These points should play an important role because it is precisely at these values that the horizon geometry $Ad_3 \times S_3$ is defined.

3-d anti-de Sitter supergravity has been recently reconsidered on the new light of possible duality relations between anti-de Sitter physics and the brane (string in this case) conformal dynamics [3, 5, 29].

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A Appendix

In this appendix we consider the relation of the “coupling” $\mathcal{N}_{\Lambda\Sigma}$ in Eq. 5 and the lagrangian of P unconstrained 2-form potentials.

The basic lagrangian is:

$$\frac{1}{2}g_{\Lambda\Sigma}H^\Lambda \wedge *H^\Sigma + \frac{1}{2}b_{\Lambda\Sigma}H^\Lambda \wedge H^\Sigma \quad (43)$$

where $g_{\Lambda\Sigma}$ is the real symmetric kinetic coupling and $b_{\Lambda\Sigma}$ is the real antisymmetric “axionic” coupling.

Decomposing :

$$H^\Lambda = H^{+\Lambda} + H^{-\Lambda} \quad (44)$$

in self-dual and antiself-dual parts we get:

$$\frac{1}{2}\mathcal{N}_{+\Lambda\Sigma}H^{+\Lambda} \wedge H^{-\Sigma} + \frac{1}{2}\mathcal{N}_{-\Lambda\Sigma}H^{-\Lambda} \wedge H^{+\Sigma} \quad (45)$$

with:

$$\mathcal{N}_{\pm\Lambda\Sigma} = \mp g_{\Lambda\Sigma} + b_{\Lambda\Sigma} \quad (46)$$

and therefore:

$$\mathcal{N}_+ = -\mathcal{N}_-^T \quad (47)$$

\mathcal{N}_\pm transforms with a fractional transformation under the $O(P, P)$ action, i.e.:

$$\mathcal{N}'_\pm = (C + D\mathcal{N}_\pm)(A + B\mathcal{N}_\pm)^{-1} \quad (48)$$

where:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in O(P, P) : \quad A^T C + C^T A = B^T D + D^T B = 0 \quad , \quad A^T D + C^T B = 1 \quad (49)$$

The relation between \mathcal{N}_\pm as given by Eq. 46 and the matrix \mathcal{N} as given in Eq. 5 turns out to be:

$$\begin{aligned} \mathcal{M} \equiv C^T \mathcal{N} C &= \begin{pmatrix} 2(\mathcal{N}_-^{-1} - \mathcal{N}_+^{-1})^{-1} & (\mathcal{N}_- + \mathcal{N}_+)(\mathcal{N}_- - \mathcal{N}_+)^{-1} \\ -(\mathcal{N}_- - \mathcal{N}_+)^{-1}(\mathcal{N}_- + \mathcal{N}_+) & 2(\mathcal{N}_- - \mathcal{N}_+)^{-1} \end{pmatrix} \\ &= \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -b & 1 \end{pmatrix} \end{aligned} \quad (50)$$

where:

$$C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \quad (51)$$

The matrix given by Eq. 50 satisfies the property:

$$\mathcal{M} \hat{\eta} \mathcal{M} \hat{\eta} = \mathbb{1} \quad (52)$$

with respect to the off-diagonal metric:

$$\hat{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C^T \eta C \quad (53)$$

The above equation, Eq. 50, is obtained using the results of ref. [31] for the parametrization of the coset representative X of $O(P, P)/[O(P) \times O(P)]$ as explained in ref. [12].

In other words, the $2P \times 2P$ matrix \mathcal{N} transforms as a $O(P, P)$ tensor when \mathcal{N}_\pm undergo the projective transformation given by Eq. 48.

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