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Born-Infeld Corrections to D3 Brane Action in $\mathbf{AdS_5 \times S_5}$ and N=4, d=4 Primary Superfields. *

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Abstract

We consider certain supersymmetric Born-Infeld couplings to the D3 brane action and show that they give rise to massless and massive KK excitations of type IIB on $AdS_5 \times S_5$, in terms of primary singleton Yang-Mills superfields.

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1 Introduction

Recently the proposal of an AdS/CFT correspondence [1, 2, 3], originated by the study of p-brane solitons as interpolating solutions between maximally symmetric vacua of string and M-theories [4], has received considerable attention, particularly in relation to non perturbative aspects of supersymmetric field theories [5].

In this context, AdS supergravity theories allow to compute correlation functions of operators of the super Yang-Mills theory, supposedly exact in some large n limit of the U(n) color gauge group.

This recent analysis is related in an essential way to special properties of AdS geometries and their boundary at infinity [6]. Indeed, while typically the graviton in AdS_{d+1} can be regarded as a unitary irreducible representation of O(d,2), it can also be represented as a composite boundary excitation of singleton conformal fields [7] on the boundary at infinity $\widetilde{M}_d$, a conformal completion of Minkowski space .

The extension of this correspondence to the interacting singleton theory is the essence of the proposal of the authors in [1, 2, 3] on the AdS/CFT correspondence which amounts to compute correlation functions of conformal composite singleton operators in terms of supergravity Green functions.

For the case of Type IIB string theory in $AdS_5 \times S_5$ [8], related to D3-brane horizon geometry [1, 2, 3, 4], an infinite sequence of N=4 primary conformal superfields [9] has recently been identified with the Kaluza-Klein excitations of type IIB supergravity compactified on $AdS_5 \times S_5$ [10, 11].

Further analysis have shown that the D3-brane coupled to supergravity background fields enable to correctly reproduce the correlators of marginal conformal composite operators [12, 13].

This analysis has been extended to include also some irrelevant operators which couple to other background supergravity fields when Born-Infeld corrections [14, 15] to the D3-brane action are taken into account [16, 17].

It is worth mentioning that the concept of marginal operator is not invariant under supersymmetry since in a given supermultiplet relevant, marginal and irrelevant operators are mixed by supersymmetry.

What is more relevant in the present context is to disentangle some operators as components of N=4 primary supermultiplets and to look at their supersymmetric counterpart.

It is the aim of the present paper to extend the analysis of Born-Infeld corrections to other composite operators and to investigate some supersymmetric aspects of the non-linear couplings of the D3-brane Born-Infeld action in the $AdS_5 \times S_5$ background.

The paper is organized as follows. In Section 2 we will show how the N=4 primary superfields, corresponding to the short representations of SU(2,2/4), have a natural interpretation in terms of the Kaluza-Klein spectrum of Type IIB supergravity on $AdS_5 \times S_5$. In Section 3 the N=1 decomposition of N=4 primaries is performed. This allows to analyze Born-Infeld couplings in terms of N=1 supersymmetry. In Section 4 the Born-Infeld corrections to the D3-brane action and their supersymmetric properties are studied. An explicit analysis of all the SU(4) singlet operators occurring in the N=4 primary superfields is performed and the analysis for a class of non-singlet operators is outlined. Section 5 ends with some concluding remarks.

2 Primary N=4, d=4 Superfields and Type IIB supergravity in $\mathbf{AdS_5 \times S_5}$

The Kaluza-Klein spectrum of Type IIB supergravity compactified on $AdS_5 \times S_5$ was found by Kim, Romans and Van Nieuwenhuizen [10] using Kaluza-Klein techniques and by Gunaydin and Marcus [11] using the oscillator method to construct unitary irreducible representations of the SU(2,2/4) superalgebra.

More recently, the same spectrum was analyzed in terms of composite operators of N=4 superconformal SU(n) Yang-Mills theory on the boundary and a precise correspondence between the Kaluza-Klein states and the components of "twisted chiral" primary superfields was obtained [9]. In this section we will first make some remarks on the generic structure of the order p primary superfields in relation to the Kaluza-Klein modes coming from higher harmonics on the five sphere.

Let us first recall that the N=4 primaries [18] are obtained as suitable composite operators obtained as polynomials of the singleton superfield $W_{[AB]}, AB = 1, \ldots, 4$, satisfying the constraints [19]

$$W_{[AB]} = \frac{1}{2}\epsilon_{ABCD}\bar{W}^{[CD]} \tag{1}$$

$$\mathcal{D}_{\alpha A}W_{[BC]} = \mathcal{D}_{\alpha[A}W_{BC]} \tag{2}$$

An order p primary superfield $A_p = W^p$ is obtained by taking the trace of the order p polynomial of the SU(n) algebra valued singleton superfield projected on the (0,p,0) SU(4) representation. The A_p superfield corresponds to a short representation of the SU(2,2/4) superalgebra, including $256 \times \frac{1}{12}p^2(p^2-1)$ states where the factor $\frac{1}{12}p^2(p^2-1)$ is the dimension of the (0, p-2,0) SU(4) representation to which the highest spin (two) belongs. The A_p superfield has the property that its lowest component is

$$A_p|_{\theta=0} = \mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_p\}}) - \text{traces}, \qquad [AB] = l = 1, \ldots, 6 \tag{3}$$

which is the irreducible p-symmetric tensor representation (0, p, 0) of SO(6)≈ SU(4). Such polynomial has conformal dimension $E_0 = p$. We here denote the dimension of the conformal operator with the corresponding energy level E_0 of the field representation in AdS_5. In the D3-brane interpretation of the N=4 super Yang-mills theory the scalar field sextet ϕ_l plays the role of the coordinates transverse to the brane $\phi_l = x_l^T$ (at least in the case where ϕ_l belongs to the Cartan subalgebra of U(N)). By setting $r = \sqrt{\mathrm{Tr}\phi_l\phi_l}$ we can define generalized spherical harmonics

$$A_p|_{\theta=0} = r^p Y_p(\hat{\phi}) \tag{4}$$

where $Y_p(\hat{\phi})$ is in the (0,p,0) representation of SU(4). Indeed $Y_p(\hat{\phi})$ reduces to an ordinary spherical harmonic on S_5 in the case of an U(1) color gauge group.

In this way we find that any order p primary superfield contains the following $D(E_0, J_1, J_2)$ O(2,4) representations

$$D(p,0,0) \qquad \text{in} \quad (0,p,0) \tag{5}$$

$$D(p+1,1,0) + D(p+1,0,1) \qquad \text{in} \quad (0,p-1,0) \tag{6}$$

$$D(p+2,0,0) \qquad \text{in} \quad (0,p-2,0) \tag{7}$$

$$D(p+2,1,1) \qquad \text{in} \quad (0,p-2,0) \tag{8}$$

$$D(p+3,1,0) + D(p+3,0,1) \qquad \text{in} \quad (0,p-3,0) \tag{9}$$

$$D(p+4,0,0) \qquad \text{in} \quad (0,p-4,0) \tag{10}$$

These are the only states in A_p which are in a (0,p,0) SU(4) representation and which therefore survive when fermions are neglected and only constant values of the bosonic singleton fields $\phi_l, F_{\mu\nu}$ are retained.

In terms of the singleton fields $\phi_l, F_{\alpha\beta}, F_{\dot\alpha\dot\beta,}$ $(F_{\alpha\beta} = \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu}, F_{\dot\alpha\dot\beta} = \bar F_{\alpha\beta} = \sigma^{\mu\nu}_{\dot\alpha\dot\beta} F_{\mu\nu})$ the states given by (6) to (10) correspond to the following conformal operators

$$\mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_{p-1}\}} F_{\alpha\beta}) - \text{traces} \tag{11}$$

$$\mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_{p-2}\}} F_{\alpha\beta} F^{\alpha\beta}) - \text{traces} \tag{12}$$

$$\mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_{p-2}\}} F_{\alpha\beta} F_{\dot\alpha\dot\beta}) - \text{traces} \tag{13}$$

$$\mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_{p-3}\}} F_{\alpha\beta} F^{\alpha\beta} F_{\dot\alpha\dot\beta}) - \text{traces} \tag{14}$$

$$\mathrm{Tr}(\phi_{\{l_1} \cdots \phi_{l_{p-4}\}} F_{\alpha\beta} F^{\alpha\beta} F_{\dot\alpha\dot\beta} F^{\dot\alpha\dot\beta}) - \text{traces} \tag{15}$$

We observe that in the abelian case (n=1 D3-brane) the previous operators (11) to (15) would reduce to

$$\begin{array}{c} r^{p-1} Y_{p-1}(\hat\phi) F, \qquad r^{p-2} Y_{p-2}(\hat\phi) F^2, \qquad r^{p-2} Y_{p-2}(\hat\phi) F \bar F, \\ r^{p-3} Y_{p-3}(\hat\phi) F^2 \bar F, \quad r^{p-4} Y_{p-4}(\hat\phi) F^2 \bar F^2 \end{array} \tag{16}$$

These operators, as we will show in the following, are precisely those who couple to a particular spherical harmonic on the S^5 sphere corresponding to a p-wave background field. In this context a state or operator in the (0,p,0) SO(6) representation will be called a p-wave in connection to its interpretation with respect to harmonic analysis on a the five-sphere.

From the above formulae we observe that a given unitary irreducible representation of SU(2,2/4), corresponding to a p-primary, for $p \geq 4$ mixes p-wave states with $\Delta p = 4$ as a consequence of the fact that the $\theta, \bar\theta$ expansion of the generic twisted chiral superfield has components up to $\theta^4 \bar\theta^4$. For $p < 4$ the representation, as already shown in Ref.[11] is not generic and in fact we have $\Delta p = 3, \Delta p = 2$ for the $p = 3$, $p = 2$ primaries respectively. From (12) to (15) we can see that the only primaries which contain SO(6) singlet states (s-wave) are $p = 2$, $p = 3$ and $p = 4$, giving rise respectively to a spin 2, a complex scalar, an antisymmetric tensor and a real scalar operator.

From the above analysis all non singlet components which occur in the $p > 4$ primaries in (12) to (15) have the interpretation of operators which couple to the KK recurrences of the lowest singlet states which occur for $p \leq 4$.

From the explicit analysis of Ref. [10] the real scalar state of (4) corresponds to the p wave of the internal components $a_{\alpha\beta\gamma\delta}$ of the self-dual four form, the real scalar (15) corresponds to the $p-4$ wave of the internal part h^{α}_{α} of the metric, the complex scalar (12) corresponds to the $p-2$ wave of the dilaton-axion complex B field, the spin 2 state (13) corresponds to the $p-2$ wave of the metric $h_{\mu\nu}$ in AdS_5 and finally the antisymmetric tensor states (11) and (14) correspond to the $p-1$ and $p-3$ waves of the NS-NS and R-R antisymmetric tensors $B_{\mu\nu}$.

As shown in Ref.[10] all the states correspond to the same spherical harmonic Y^l on S^5. From the above consideration it then appears explicit the fact that when the Yang-Mills D3-brane action (or its Born-Infeld extension) is coupled to the $AdS_5 \times S^5$ background the very same term which originate the coupling $h^{\mu\nu}T_{\mu\nu}$ will also originate couplings $h^{\mu\nu}_{(p-2)}T^{(p-2)}_{\mu\nu}$ where $h^{\mu\nu}_{(p-2)}$ is the $p-2$ wave background field and the $T^{(p-2)}_{\mu\nu}$ is the spin 2 operator (13) contained in the p primary N=4 superfield. The same considerations apply to the other operators and other background fields which are in fact related to the previous one by N=4 supersymmetry.

If we limit ourselves to read the s-wave terms from the Born Infeld action, they do occur in the $p = 2$ primary for $h_{\mu\nu}$ and B, in the $p = 3$ primary for $B_{\mu\nu}$ and in the $p = 4$ primary for h^{μ}_{μ}. Interestingly enough, N=4 supersymmetry predicts the non linear corrections to the quadratic Yang Mills actions from the structure of the N=4 primary superfields. It should be also noted that $B_{\mu\nu}$ also couples linearly to the Yang-Mills field strength as shown in (11), but this operator comes in the l=1 wave as component of the $p = 2$ primary supercurrent multiplet [18, 20, 21, 24].

Finally, we remark that the above analysis is complete as far as only constants $\phi_l, F_{\alpha\beta}$ occur. The generalization to all other states, to include fermions as well as non constant ϕ_l is in principle implemented by N=4 conformal symmetry. It will be shown in the following sections that the Born-Infeld couplings of the D3-brane action in the $AdS_5 \times S^5$ background exactly produce the terms discussed in the body of this section.

3 N=1 decomposition of N=4 primary superfields

In order to examine the supersymmetric structure of the Born-Infeld non linear action, it is useful to decompose the N=4 primary superfields in N=1 components since the analysis of the four dimensional Born-Infeld action is mostly known from an N=1 perspective [20, 22, 21].

From a group theoretical point of view, the N=1 decomposition of N=4 superfields amounts to decompose the SU(2,2/4) superalgebra and its representations under the U(2,2/1)×SU(3)×U(1). The decomposition of the N=4 Yang-Mills superfield strength in N=1 parts is [23]

$$W_{[AB]} = (S_i, W_\alpha) \tag{17}$$

where S_i is a triplet of chiral N=1 superfields and W_α is the chiral N=1 field strength. These superfields have U(1) charge 1 and $\frac{3}{2}$ respectively. All these superfields are supposed to be U(n) Lie algebra valued. It is now straightforward to obtain the N=1 decomposition of the N=4 twisted chiral superfields. It suffices to decompose the (0,p,0) SU(4) representation into SU(3) representations, using the fact that $6 \mapsto 3 + \bar{3}$ so that the real scalar sextet ϕ_l goes into $S_i, \bar{S}_i$ while W_α is SU(3) inert.

Let us first consider the $p = 2$ primary corresponding to the graviton multiplet in AdS_5. Its N=1 superfield content is

$$S_i S_j, \quad \bar{S}_i \bar{S}_j, \quad S_i \bar{S}^j - \frac{1}{3}\delta_i^j S_k S^k \tag{18}$$

$$S_i W_\alpha, \quad \bar{S}_i \bar{W}_{\dot{\alpha}} \tag{19}$$

$$S_i \bar{W}_{\dot{\alpha}}, \quad \bar{S}_i \; W_\alpha \tag{20}$$

$$W_\alpha^2, \quad \bar{W}_{\dot{\alpha}}^2 \tag{21}$$

$$W_\alpha \bar{W}_{\dot{\alpha}} \tag{22}$$

Note that the 20 scalars of (4) are in (18) since under SU(4)↦SU(3) $20 \mapsto 6 + \bar{6} + 8$. The fifteen SU(4) currents are in (18), (20), (22) since $15 \mapsto 8 + 3 + \bar{3} + 1$. The 8 spin 3/2 supercurrents are in (20),(22) since $4 + \bar{4} \mapsto 3 + \bar{3} + 1 + \bar{1}$. The 12 antisymmetric tensors are in (19),(20) since $6_c \mapsto (3 + \bar{3})_c$ and finally the stress tensor singlet is in (22).

The above results apply for constant values of the $S_i|_{\theta=0}$ component and vanishing fermions. The stress tensor in (22) would otherwise receive an extra contribution.

At this point we notice that the conformal supergravity multiplet is contragradient to the supercurrent multiplet [19, 9, 24]. Then we learn that its SU(4) singlet part reduces to the N=1 superconformal gravity potential $H_{\alpha\dot{\alpha}}$ [24] and to an extra chiral superfield S.

We now move to the $p = 3$ primary superfield. The 50 scalars in the (0,3,0) SU(4) representation correspond to

$$S_i S_j S_k, \quad S_i \bar{S}_j \bar{S}_k - \text{traces}, \bar{S}_i S_j S_k - \text{traces}, \quad \bar{S}_i \bar{S}_j \bar{S}_k \tag{23}$$

This correspond to the SU(3) decomposition $50 \mapsto 10 + \bar{1}0 + 15 + \bar{1}5$. The other N=1 component superfields are

$$S_i W_\alpha^2, \quad \bar{S}_i W_\alpha^2, \quad \bar{S}_i \bar{W}_{\dot{\alpha}}^2, \quad S_i \bar{W}_{\dot{\alpha}}^2 \tag{24}$$

$$S_i W_\alpha \bar{W}_{\dot{\alpha}}, \bar{S}_i W_\alpha \bar{W}_{\dot{\alpha}} \tag{25}$$

$$W_\alpha^2 \bar{W}_{\dot{\alpha}}, \quad \bar{W}_{\dot{\alpha}}^2 W_\alpha \tag{26}$$

$$S_i S_j W_\alpha, \quad (S_i \bar{S}^j - \frac{1}{3}\delta_i^j \bar{S}_k \bar{S}^k) W_\alpha, \quad \bar{S}_i \bar{S}_j W_\alpha \tag{27}$$

$$S_i S_j \bar{W}_{\dot{\alpha}}, \quad (S_i \bar{S}^j - \frac{1}{3}\delta_i^j \bar{S}_k \bar{S}^k) \bar{W}_{\dot{\alpha}}, \quad \bar{S}_i \bar{S}_j \bar{W}_{\dot{\alpha}} \tag{28}$$

We can proceed further. For the $p = 4$ primary superfield the (0,4,0) SU(4) representation decomposes under SU(3) as follows

$$105 \mapsto 15 + \bar{1}5 + 24 + \bar{2}4 + 27 \tag{29}$$

This corresponds to the N=1 polynomials

$$\begin{array}{ccc} S_i S_j S_k S_l, & S_i S_j S_k \bar{S}_l - \text{traces}, & S_i S_j \bar{S}_k \bar{S}_l - \text{traces}, \\ S_i \bar{S}_j \bar{S}_k \bar{S}_l - \text{traces}, & \bar{S}_i \bar{S}_j \bar{S}_k \bar{S}_l & \end{array} \tag{30}$$

Superfield multiplication with the W_α gives the rest of the superfields as before. For instance, the stress tensor recurrence sits in the N=1 superfields

$$S_i S_j W_\alpha \bar{W}_{\dot{\alpha}}, \quad \bar{S}_i \bar{S}_j W_\alpha \bar{W}_{\dot{\alpha}}, (S_i \bar{S}^j - \frac{1}{3}\delta_i^j S_k \bar{S}^k) W_\alpha \bar{W}_{\dot{\alpha}} \tag{31}$$

Note that the new superfield $W_\alpha^2 \bar{W}_{\dot{\alpha}}^2$ appears.

The above superfields give the supersymmetric completion of the component fields given by (11) to (15). We see that p waves superfields correspond to a dependence on the S_i multiplet. If we want to confine only to s wave superfields we can se $S_i = 0$ and then only $p = 2, 3, 4$ primaries remain with N=1 components given by

$$W_\alpha^2, W_\alpha \bar{W}_{\dot\alpha}, W_\alpha^2 \bar{W}_{\dot\alpha}, W_\alpha^2 \bar{W}_{\dot\alpha}^2 \tag{32}$$

The bosonic components of these singlets operators are

$$W_\alpha^2 \mapsto O_2 = \frac{1}{2}(F^2 \pm F\tilde{F}) \qquad (\tilde{F}_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}) \tag{33}$$

$$W_\alpha \bar{W}_{\dot\alpha} \mapsto T_{\mu\nu} = F_{\mu\nu}F_{\nu\rho} - \frac{1}{4}\eta_{\mu\nu}(F_{\sigma\rho})^2 \tag{34}$$

$$W_\alpha^2 \bar{W}_{\dot\alpha} \mapsto O_3 = T_{\mu\rho}F_{\rho\nu} = F_{\mu\sigma}F_{\rho\sigma}F_{\rho\nu} - \frac{1}{4}(F_{\sigma\rho})^2 F_{\mu\nu} \tag{35}$$

$$W_\alpha^2 \bar{W}_{\dot\alpha}^2 \mapsto O_4 = \frac{1}{4}[(F^2)^2 - (F\tilde{F})^2] = F_{\mu\rho}F_{\nu\rho}F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}(F^2)^2 \tag{36}$$

The above expression strictly apply for a U(1) gauge theory. Suitable symmetrizations must been understood when a non abelian trace is taken [15].

All other higher order primaries are accordingly obtained by suitable chiral and antichiral multiplication of the superfields S_i, W_α.

Note that these multiplications only generate field components with spin ≤ 2, as follows from a general N=1 superfield with at most one external vector index.

In particular we note that an N=4 twisted chiral superfield is not chiral when reduced to N=1 or N=2 supersymmetry. Therefore the KK towers identified in [3] only reproduce part of the entire KK spectrum on $AdS_5 \times S_5$.

4 The AdS/CFT correspondence

The relation between $N = 4$ SYM and type IIB on $AdS_5 \times S^5$ predicts that there is a one-to-one correspondence between the supergravity KK modes and the SYM composite operators belonging to short multiplets. The $N = 4$ covariant description of this correspondence was discussed in [9]. Given the mass and the $SU(4)$ quantum number of the supergravity mode, the corresponding SYM operator can be uniquely identified as a component of one of the superfields $A_p = \mathrm{Tr}\ W^p$ as defined in (3).

It has been proposed in [17] that the SYM operator corresponding to a given supergravity mode can be also be identified by expanding the Born-Infeld lagrangian around the anti-de-Sitter background. We now give evidence that there is a complete agreement between the two methods.

The Born-Infeld lagrangian can be generalized to include the couplings to the type IIB supergravity fields. Considering only the bosonic terms, the coupling to the NS-NS fields (ϕ, B^{NS}) can be written as

$$L_{\mathrm{BI}} = \sqrt{-\det\left(g_{\mu\nu} + e^{-\phi/2}\mathcal{F}_{\mu\nu}\right)} \tag{37}$$

where $\mathcal{F} = F - B^{\mathrm{NS}}$ [26], while the coupling to the RR fields $(\tilde{\phi}, B^{\mathrm{RR}}, A^{(4)})$ requires the introduction of a Wess-Zumino term [27],

$$L_{\mathrm{WZ}} = A^{(4)} + B^{\mathrm{RR}} \wedge \mathcal{F} + \tilde{\phi}\mathcal{F} \wedge \mathcal{F} \tag{38}$$

We will focus on the states discussed in Section 2, which can be constructed in terms of the $N = 4$ gauge fields and constant scalars. The generalization of the composite operators, which will be identified in this section, to include the fermions and derivatives of the scalars is dictated by the $N = 4$ supersymmetry.

We first discuss the three $SU(4)$ singlets (besides the graviton) contained in the KK tower and given by (33),(35) and (36). The corresponding bosonic operators are a complex scalar, an antisymmetric tensor and a real scalar.

The dimension E_0 of a scalar SYM operator is related to the mass of the corresponding supergravity mode by the formula [6, 7]

$$m^2 = E_0(E_0 - 4) \tag{39}$$

while for an antisymmetric tensor we have [9]

$$m^2 = (E_0 - 2)^2 \tag{40}$$

$A_2 = \mathrm{Tr}W^2$ gives the two scalars in (33) with dimension $E_0 = 4$. They couple respectively, to the type IIB dilaton via the Born-Infeld lagrangian, and the RR scalar via the Wess-Zumino term. The corresponding N=1 background superfield was called S in the previous section. The full multiplet of currents corresponds indeed to the graviton multiplet in AdS_5 [7, 9], which contains a massless complex singlet scalar (the dilaton and the RR scalar), in agreement with eq. (39).

The explicit form of the singlet operators in $A_3 = \mathrm{Tr}W^3$ and $A_4 = \mathrm{Tr}W^4$ is given by (32) (using an $N = 1$ superfield formalism). Their bosonic components, as given by (35) and (36), have dimension $E_0 = 6, 8$ respectively.

Let us start with the latter. Using formula (39), we see that it corresponds to a scalar supergravity mode with squared mass 32. The full KK spectrum of type IIB on $AdS_5 \times S^5$ was computed in [10] and a singlet scalar with squared mass 32 was identified (see fig. 2 and table III in [10]). The corresponding type IIB field is the dilatational mode of the internal (S^5) metric h^α_α. In the higher harmonic modes in this KK tower, h^α_α mixes with the scalar obtained by reducing the type IIB four-form along S^5. However, the lower state, which corresponds to the singlet with squared mass 32, is not coupled to the four-form. The only other non-zero type IIB field is the Weyl mode of the space-time metric $g_{\mu\mu}$. The coupled equations are (eq. (2.21) and (2.40) in [10])

$$(\partial^2 - 32)h^\alpha_\alpha = 0 \qquad g_{\mu\mu} - \frac{8}{3}h^\alpha_\alpha = 0 \tag{41}$$

According to the proposal in [17], we should now expand the Born-Infeld lagrangian in the AdS background, perturbed with non-trivial $g_{\mu\mu}, h^\alpha_\alpha$, satisfying eq. (41). Let us call $\pi = g_{\mu\mu} = (8/3)h^\alpha_\alpha$. h^α_α enters in the Born-Infeld action when we compute the pullback of the type IIB metric; it is coupled to terms with SYM scalar derivatives and fermions that we put, for simplicity, to zero. $\pi = g_{\mu\mu}$ is the only field that couples to $F_{\mu\nu}$. It is the first component of a chiral superfield which we denote by H.

The expansion of L_{BI} in (37) up to quartic order in the super Yang-Mills operators and in flat background reads

$$L_{BI} = \frac{1}{2}F^2 - \frac{1}{8}O_4 + \cdots \tag{42}$$

where the higher dimensional operators have been supressed. The term F^2 is clearly conformal invariant. Therefore the leading operator that couples to the Weyl mode π is exactly the operator O_4, in agreement with the expectations. The expansion has been done, strictly speaking, for an abelian theory. Here and in the following, a suitable symmetrization in color space must be understood in the non-abelian case.

The N=1 supersymmetric generalization of (42) in N=1 superbackground with $S, H_{\alpha\dot\alpha}, H$ turned on is

$$SW_\alpha^2|_F + H^{\alpha\dot\alpha}W_\alpha \bar W_{\dot\alpha}|_D + HW_\alpha^2\bar W_{\dot\alpha}^2|_D + \cdots \tag{43}$$

Note that the first term of the above expression also contains the last term of (38).

Let us now consider the operator O_3 with dimension 6. It should correspond (using equation (40)) to a singlet antisymmetric tensor in AdS_5 with squared mass 16. It can be found in fig.3 and table III in [10]; it comes from a combination of the modes of the NS-NS and R-R type IIB antisymmetric tensors which are constant along the five-sphere. The expansion of the Born-Infeld lagrangian in the AdS_5 background perturbed by non-zero type IIB antisymmetric tensors was performed in [28, 17], where only part of the operator O_3 was found. That the full expression in (35) must appear is predicted by supersymmetry and it is easily derived from the lagrangian (42). Ignoring SYM scalars and fermions, the only modification due to the non-trivial background amounts to substitute F with $F - B$ in eq. (42). Here B is the NS-NS type IIB antisymmetric tensor. The R-R tensor is determined by the equation of motion to be $B^{\mathrm{RR}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\tau\rho}B^{\tau\rho}$ [10, 17]. The RR tensor appears in the D3 action via the Wess-Zumino term, and gives couplings of the form $F_{\mu\nu}B^{\mu\nu}$ which are subleading in the large n limit. The Wess-Zumino term is however crucial, as notice in [17], in cancelling dimension four operators, constructed with fermions, which couple to B in the Born-Infeld lagrangian.

Substituting F with F-B in the Born-Infeld lagrangian (42), expanding at the first order in B and neglecting terms which are subleading for large N, we explicitely find the coupling BO_3.

The $N = 1$ supersymmetric generalization of the operator O_3 is easily obtained by promoting $B_{\mu\nu}$ to a chiral superfield L_α [25] and by shifting $W_\alpha \mapsto W_\alpha - L_\alpha$ in (43). From the quartic term in the Born-Infeld lagrangian we then obtain the coupling

$$\bar W_{\dot\alpha}^2 W_\alpha L^\alpha + \mathrm{h.c} \tag{44}$$

It should be noted that, as observed in [17], it is crucial to expand the lagrangian in the AdS_5 background. This means that the metric along the D3 brane is flat, but depends on the fifth coordinate r of AdS as $g_{\mu\nu} = \frac{r}{R}\eta_{\mu\nu}$, where R is the AdS radius. The D3 brane is assumed to live at a fixed value of r. In the expansion around this background, all the coupling between supergravity fields and SYM operators are dressed by powers of r/R.

Having identified the three SU(4) singlets in the Born-Infeld expansion, we can turn to the higher harmonics. Consider first the abelian case. The scalar field sextet ϕ_l describe the position of the D3 brane in the transverse space. Our prescription will be to expand the action of a single D3 brane in the AdS_5 background, keeping the position r fixed and indentifying the coordinates on S^5 (which appear in a p-wave operator as the spherical function Y_p) with the scalar fields $\hat{\phi}_l = \phi_l/r$. With this identification, the harmonic of degree p in the KK expansion of the dilaton, for example, gives rise to a coupling to the operator $O_2 \times Y_p(\hat{\phi})$, where Y_p is the spherical harmonic of degree p.

Let us now outline the identification of all the other states in (0,p,0) representations of SU(4). According to the analysis in Ref. [10], the real scalar state of (4) corresponds to the p wave of the internal components $a_{\alpha\beta\gamma\delta}$ of the self-dual four form. It can indeed be obtained using the first term in eq. (38), because, as noticed in Ref. [10], the components of the four-form along the D3 brane are determined in terms of the internal ones by the equations of motion. The expansion of the Born-Infeld lagrangian in the Weyl mode of the metric gives the real scalars (15) corresponding to the higher harmonics of the internal part h^α_α of the metric. The equations of motion found in Ref. [10] imply indeed the relation $g_{\mu\nu} = \frac{16}{15} h^\alpha_\alpha$. Finally, the two antisymmetric tensor states (11) and (14), corresponding to modes of the NS-NS and R-R antisymmetric tensors, can be obtained by expanding both the Born-Infeld lagrangian and the Wess-Zumino term around the solution of the equation of motion for $B^{\rm NS}$ and $B^{\rm R}$. We obtain contributions of the form,

$$L_{\rm BI} = B^{\rm NS} Y_p(\hat{\phi})(F + O_3), \qquad L_{\rm WZ} = B^{\rm RR} Y_p(\hat{\phi}) \wedge F \tag{45}$$

Notice that the terms BFY_p are no more subleading in the large n limit. According to the equations of motion discussed in Ref.[10], the p-wave components of the antisymmetric tensors, when $p \geq 1$, give two different modes, satisfying $B^{\rm RR} = \pm * B^{\rm NS}$, with squared mass p^2 and $(p+4)^2$. This splitting in the masses can be understood by observing that eq.(45) gives BFY_p for one of the modes, while for the other there is a cancellation between $L_{\rm BI}$ and $L_{\rm WZ}$, and the first non-trivial coupling occurs for an higher dimensional operator, BO_3Y_p [2].

[2]The same splitting was noticed in the context of the absorption of supergravity modes by the D3 branes in [29].

Notice that, as discussed in Ref. [10], the $p = 0, 1$ modes for $a_{\alpha\beta\gamma\delta}$ and one of the $p = 0$ modes for $B_{\mu\nu}$ are zero or can be gauged away, in agreement with the fact that Trϕ_l and Tr$F_{\mu\nu}$ do not belong to the series of composite operators in A_p.

It is easy to check, in all the cases discussed above, that the dimensions of the operators $O_i Y_p$ correspond, via the eqs.(39) and (40), to the masses computed in Ref.[10].

It should be possible to extend the present analysis to include all the remaining states in the superfields A_p in SU(4) representations other than (0,p,0). In general, they involves fermions and scalar derivatives. The structure of the 15 currents of SU(4), associated with the massless gauge fields in the graviton multiplet in AdS_5, for example, is easily seen to emerge from the coupling of the D3 brane action to the mixed components of the metric $g_{\mu\alpha}$ along the S^5 Killing vectors. The KK recurrences of the SU(4) adjoint currents are states in the (1, p-2, 1), (1, p-3, 1) and (1, p-4, 1) SU(4) representations. They correspond to the $D(p + 1, 1/2, 1/2), D(p + 2, 1/2, 1/2)$ and $D(p + 3, 1/2, 1/2)$ O(2,4) conformal fields respectively. It is likely that, also for all these states, the composite operators, which can be determined by expanding the Born-Infeld action around the corresponding supergravity modes, will agree with what is predicted by superconformal invariance, in terms of components of some superfield A_p.

The previous analysis is in agreement with the discussion in section 2 in the abelian case. As discussed in details in section 2, the composite SYM operators, that we expect to be associated to these supergravity modes in the non-abelian case, are the non-abelian generalizations of the operators $O_i \times Y_p(\hat{\phi})$ which are contained in the superfields A_p. It is likely that the color structure will emerge naturally from a full $N = 4$ non-abelian generalization of the Born-infeld action and from a better understanding of how the coordinates of n D3 branes can be promoted to SU(n) matrices [15]. From the present analysis, it should be possible to extend the Born-Infeld action to include the dependence on the non singlet fields S_i. In this case the KK graviton recurrences should couple to the conformal operators as given in (31) while the dilaton-axion recurrences should be given by operators like (24). It is clear that these operators must come from the N=4 completion of (43).

5 Conclusions

In this paper, we expanded the Born-Infeld action around the $AdS_5 \times S^5$ background and we found that certain KK supergravity modes exactly couple to the composite operators predicted by the SU(2,2/4) algebra, and studied in [9]. It is likely that this correspondence, with a correct description of how the D3 brane is embedded in the non-trivial AdS geometry and a better understanding of the non-abelian and $N = 4$ supersymmetric structure of the Born-infeld action, can be extended to the entire KK spectrum. Information about the composite operators associated to KK modes can be obtained also by studying the absorption of the supergravity fields by the D3 branes [30, 16, 29]. This method makes use of the full D3 brane metric and not only of the near-horizon geometry, and it would be interesting to understand the relation with the approach considered in this paper.

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