Model analysis of the world data on the pion transition form factor

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We discuss the impact of recent Belle data on our description of the pion transition form factor based on the assumption that a perturbative formalism and a nonperturbative one can be matched in a physically acceptable manner at a certain hadronic scale Q_0. We discuss the implications of the different parameters of the model in comparing with world data and conclude that within experimental errors our description remains valid. Thus we can assert that the low Q^2 nonperturbative description together with an additional $1/Q^2$ term at the matching scale have a strong influence on the Q^2 behavior up to very high values of Q^2 .

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New data of the pion transition form factor (πTFF) from the Belle collaboration have just appeared [1]. These data, above 10 GeV^2, are smaller in magnitude than the previous BABAR data [2], which generated considerable excitement. The question to unveil is the scale of asymptotia. BABAR data, taken at face value, implied that asymptotic QCD behavior lies at much higher Q^2 than initially expected [3, 4]. Belle data seem to lower that scale. We show here that our scheme can accomodate easily all data without changing the physical input.

At the time of the BABAR data we developed a formalism to calculate the πTFF [5], which consists of three ingredients: *i*) a low energy description of the πTFF; *ii*) a high energy description of the πTFF; *iii*) a matching condition between the two descriptions at a scale Q_0 characterizing the separation between the two regimes. For the low energy description we took a parametrization of the low energy data to avoid model dependence at Q_0. The high energy description of the πTFF, defined by the pion Distribution Amplitude (πDA), contains Quantum Chromodynamic (QCD) evolution from Q_0 to any higher Q, a mass cut-off to make the formalism finite, and an additional $1/Q^2$ term which leads to modifications of the matching condition.

Let us recall some aspects of the formalism. The high energy description, to lowest order in perturbative QCD, for the transition form factor in the process $\pi^0 \to \gamma\,\gamma^*$ in terms of the pion distribution amplitude (πDA), is given by

$$Q^2 F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 \frac{dx}{x + \frac{M^2}{Q^2}} \phi_\pi(x, Q^2). \tag{1}$$

We follow the proposal of Polyakov [6] and Radyushkin [7] and introduce a cutoff mass M to make the expression finite. $Q^2 = -q^2$, q_μ is the momentum of the virtual photon, $\phi_\pi\left(x, Q^2\right)$ is πDA at the Q^2 scale and $f_\pi = 0.131$ GeV. In this expression, the Q^2 dependence appears through the QCD evolution of the πDA.

Despite the fact that several models reproduce the low energy data, in order to have a model independent expression for the form factor at low virtualities, we adopted a monopole parametrization of the

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πTFF in the low energy region as

$$F^{LE}\left(Q^2\right) = \frac{F\left(0\right)}{1 + a\frac{Q^2}{m^2_{\pi^0}}} \,. \qquad (2)$$

with $F\left(0\right) = 0.273(10)$ GeV^{-1} and $a = 0.032\left(4\right)$ [8], determined from the experimental study of $\pi^0 \to \gamma\, e^+\, e^-$ [9].

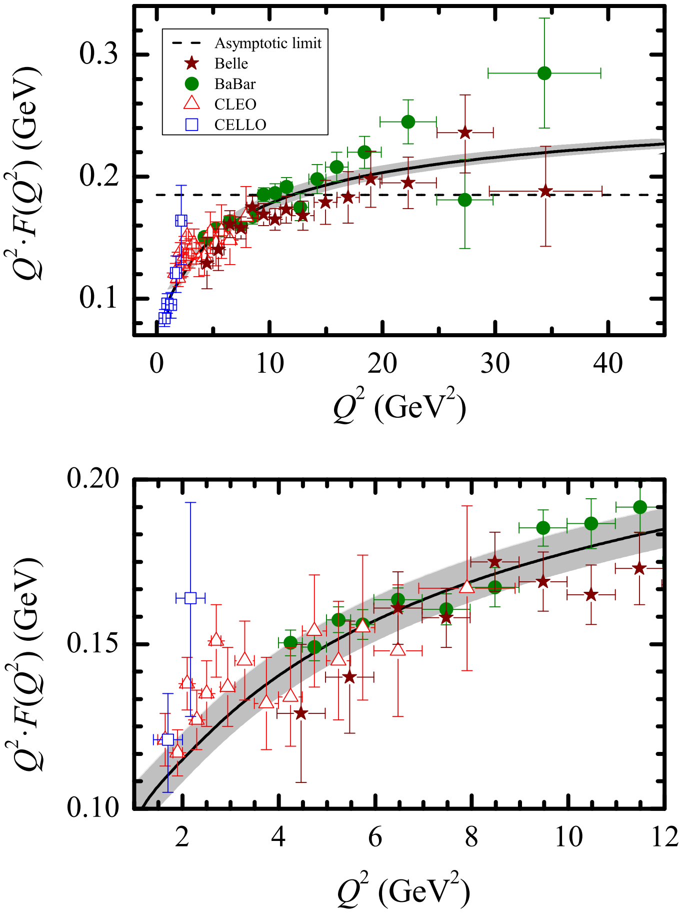

FIG. 1: We show the result for the transition form factor in our formalism for $M = 0.690$ GeV, $a = 0.032$ and $C_3 = 2.98\, 10^{-2}$ GeV^3 and defining the matching point at $Q_0 = 1$ GeV (solid line). The band region results from the indeterminacy in $\Delta a = \pm 0.004$. The lower plot shows the detailed behavior for low virtuality. Data are taken from CELLO [10], CLEO [11], BABAR [2] and Belle [1].

Additional power corrections can be introduced in Eq. 1 by adding to the lowest order calculation a

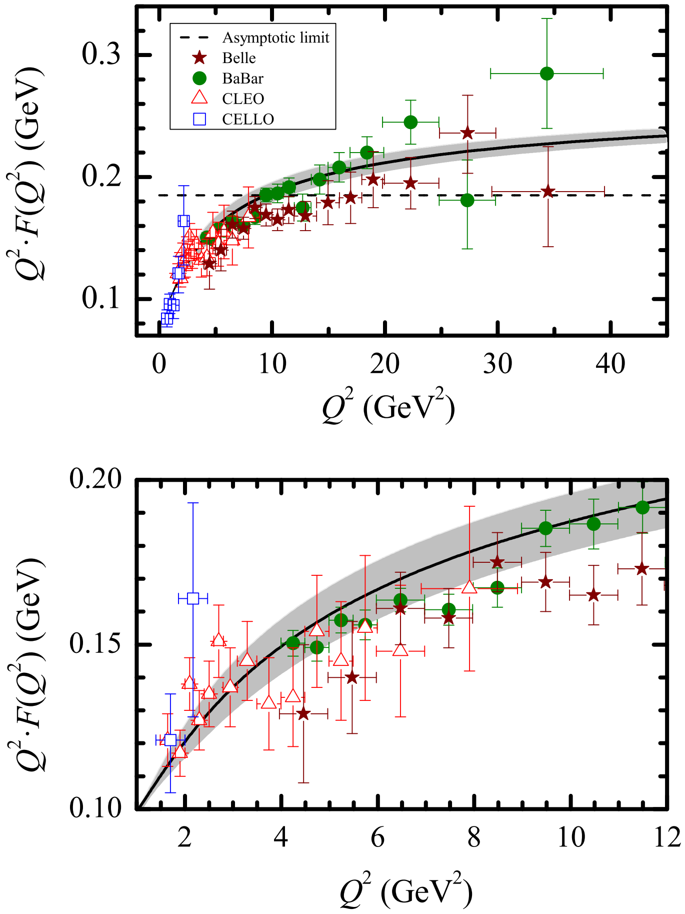

FIG. 2: We show the result for the transition form factor in our formalism for $M = 0.620$ GeV, $a = 0.032$ and the value of $C_3 = 1.98\,10^{-2}$ GeV^3 corresponding to 20% of the contribution at the matching point at $Q_0 = 1$ GeV (solid line). The band region gives the variation of the results due in ±10% in the contribution of higher twist. The lower plot shows the detailed behavior for low virtuality. Data are taken from CELLO [10], CLEO [11], BABAR [2] and Belle [1].

term proportional to Q^{-2},

$$Q^2F(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 \frac{dx}{x+\frac{M^2}{Q^2}}\phi_\pi(x,Q^2) + \frac{C_3}{Q^2}. \tag{3}$$

Using a constant π DA the matching condition becomes [5],

$$\frac{\sqrt{2}f_\pi}{3} ln\frac{Q_0^2+M^2}{M^2}+\frac{C_3}{Q_0^2}=\frac{F\left(0\right)\,Q_0^2}{1+a\frac{Q_0^2}{m_{\pi^0}^2}}, \tag{4}$$

with $Q_0 = 1$ GeV. This equation allows to determine M, once we have fixed the value of C_3.

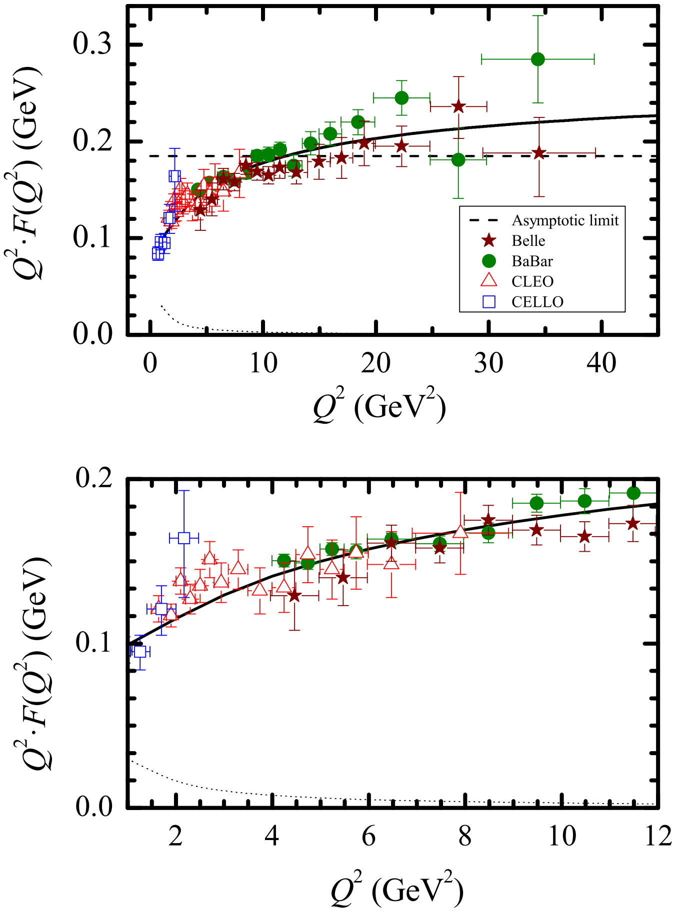

FIG. 3: We show the result for the transition form factor in our formalism for $M = 0.690$ GeV, $a = 0.032$ and the value of $C_3 = 2.98\,10^{-2}$ GeV^3 corresponding to 30% of the contribution at the matching point at $Q_0 = 1$ GeV (solid line).The lower plot shows the detailed behavior for low virtuality.The dotted curve represents the higher twist contribution.Data are taken from CELLO [10], CLEO [11], BABAR [2] and Belle [1].

We analyze here the sensitivity of the data to the various parameters involved. We keep as close as possible to our previous fit analyzing the data with respect to small variations in the low virtuality parameter a and in the higher twist parameter C_3. In Fig.1 we show the effect of the precision in the determination of the monopole parametrization. We see that as a increases from 0.032 to 0.036, i.e. within the error bars, the πTFF decreases. The sensitivity to C_3 is shown in Fig. 2 and we note that as the value of C_3 increases from $C_3 = 0.99\,10^{-2}\mathrm{GeV}^3$, which corresponds to a 10% contribution to the form factor at Q_0, to 2.98 10^{-2} GeV^3 , which corresponds to a 30% contribution, again the value of the πTFF decreases. Thus a small increase in a and C_3 moves our result toward the Belle data. Finally, in Fig. 3 we plot the better fit ($\chi^2/dof = 1.21$) taking into account all the world data which corresponds to $a = 0.032$ with the C_3 term at the 30% value. We stress that there is no strong correlation between a and C_3 as long as a is kept within its experimental error bars. Thus the fit is quite stable with respect to the parameters of the low energy model.

The fit to the data is excellent with a very small variation of the $1/Q^2$ contribution at Q_0 from previous fit, i.e. from 20% to 30%. It must be said, before entering the discussion of this fit, that in our previous work [5] we pointed out that the average value of the highest energy data points of BABAR were too large, a conclusion reached also by other analyses [12, 13]. In Fig. 3 we show not only the fit for 30% contribution of C_3/Q^2 at Q_0, but its behavior for higher values of Q^2. As can be seen, also stressed in our previous work, this contribution is small in size. However, and this an important outcome of our analysis, it is instrumental in fixing the initial slope at the matching point, which determines, after evolution, the high energy behavior of the form factor.

In our opinion the Belle data confirm the BABAR result that the πTFF crosses the asymptotic QCD limit. This limit is well founded under QCD assumptions, but nothing is known of how this limit is reached, if from above or from below. BABAR and Belle data suggest that the limit is exceeded around $10 - 15$ GeV. Our calculation is consistent with this result. The necessary growth of the πTFF between $5 - 10$ GeV to achieve this crossing is in our case an indication of nonperturbative behavior and C_3/Q^2 contribution at low virtuality. The determination of the crossing point is a challenge for any theoretical model and therefore, the precise experimental determination of it is of relevance. Many models fail to achieve this crossing because their pion DA is defined close to its asymptotic form.

The pion DA can be expressed as a series in the Gegenbauer polynomials,

$$\phi_\pi\left(x, Q^2\right) = 6x\left(1-x\right)\left(1 + \sum_{n(even)=2}^{\infty} a_n\left(Q^2\right)\, C_n^{3/2}\left(2x-1\right)\right) \tag{5}$$

We can compare different models by looking at the values of the coefficients of the expansion $a_n\left(Q^2\right)$. In our case, at $Q^2 = 1$ GeV^2 many a_n coefficients are significant, but we focus our attention in a few terms: $a_2 = 0.389$, $a_4 = 0.244$ and $a_6 = 0.179$. At $Q^2 = 4$ GeV^2 we obtain the values $a_2 = 0.307$, $a_4 = 0.173$ and $a_6 = 0.118$, which are close to those obtained by Polyakov [6]. Consistently, our result for the πTFF is similar to that obtained in ref. [6]. At $Q^2 = 5.76$ GeV^2 we obtain $a_2 = 0.292$, $a_4 = 0.161$ and $a_6 = 0.108$, which are very different from those of ref. [14]. These author use for their fit BABAR data for the ηTFF [15], together with the pion data. It is therefore not a surprise that these authors come to a different conclusion, namely, that the Belle and the BABAR data cannot be reproduced to the same level of accuracy within the Light Cone Sum Rules approach [16]. However, in an extension of the ideas developed in the present paper to the η case studied in ref. [17] looking at the state $|q\rangle = \frac{1}{2}\left(|u\,\bar{u}\rangle + \left|d\,\bar{d}\right\rangle\right)$ a very different structure of the a_n coefficients to that of the pion arises. At $Q^2 = 1$ GeV^2, the values of the coefficients are $a_2 = 0.134$ and $a_4 = 0.352$ or, equivalently, at $Q^2 = 5.76$ GeV^2 we have $a_2 = 0.101$ and $a_4 = 0.232$. Therefore, that study does not supports the combined use of both data sets

We have developed a formalism to describe the πTFF on all experimentally accessible range, and hopefully beyond. The formalism is based on a two energy scale description. The formulation in the low energy scale is nonperturbative, while that of the high energy scale is based on perturbative QCD. The two descriptions are matched at an energy scale Q_0 called hadronic scale [18, 19]. We stress the crucial role played by the nonperturbative input at the level of the low energy description. It is an important outcome of this calculation the role played by the $1/Q^2$ power correction term in determining the slope of the data at high Q^2, despite the fact that they do almost not contribute to the value of the πTFF .

We have used a flat π DA, i.e. a constant value for all x [6, 7], which with our normalization becomes $\phi(x) = 1$. Our choice has been motivated by chiral symmetry [5]. Model calculations, Nambu-Jona-Lasinio (NJL) [20–23] and the "spectral" quark model [24], give a constant π DA. The πTFF calculated in these models, however, overshoots the data [25], emphasizing the importance of QCD evolution.

The calculation shown proves that the BABAR and Belle results can be accommodated in our scheme, which only uses standard QCD ingredients and low energy data. Moreover, at the light of our results, we confirm that at 40 GeV^2 we have not yet reached the asymptotic regime which will happen at higher energies.

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