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## Conformal and Nonconformal Symmetries in 2D Dilaton Gravity \*

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### Abstract

We study finite-dimensional extra symmetries of generic 2D dilaton gravity models. Using a non-linear sigma model formulation we show that the unique theories admitting an extra (conformal) symmetry are the models with an exponential potential  $V \propto e^{\beta\phi}$  ( $S = \frac{1}{2\pi} \int d^2x \sqrt{-g} [R\phi + 4\lambda^2 e^{\beta\phi}]$ ), which include the model of Callan, Giddings, Harvey and Strominger (CGHS) as a particular though limiting ( $\beta = 0$ ) case. These models give rise to black hole solutions with

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a mass-dependent temperature. The underlying extra symmetry can be maintained in a natural way in the one-loop effective action, thus implying the exact solubility of the semiclassical theory including back-reaction. Moreover, we also introduce three different classes of (non-conformal) transformations which are extra symmetries for generic 2D dilaton gravity models. Special linear combinations of these transformations turn out to be the (conformal) symmetries of the CGHS and  $V \propto e^{\beta\phi}$  models. We show that one of the non-conformal extra symmetries can be converted into a conformal one by means of adequate field redefinitions involving the metric and the derivatives of the dilaton. Finally, by expressing the Polyakov-Liouville effective action in terms of an invariant metric, we are able to provide semiclassical models which are also invariant. This generalizes the solvable semiclassical model of Bose, Parker and Peleg (BPP) for a generic 2D dilaton gravity model.

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# 1 Introduction

The physics of black holes provides an excellent arena on the interphase between General Relativity and Quantum Mechanics. The fate of a black hole and the possible loss of quantum coherence are problems the final resolution of which requires to quantize the gravitational degrees of freedom. However, a consistent formulation of a quantum theory of gravity is still lacking, and the semiclassical theory, that could serve as a first step to the full quantized theory, is very difficult to solve.

A natural approach to bypass this situation is to consider simplified models that keep the fundamental features of the four dimensional theory but makes the analytic question more tractable. General Relativity is trivial—up to global degrees of freedom—in lower dimensions than four. However, some models derived as the S-wave sector of the four dimensional theories are dynamically non-trivial and contain formation of black holes by gravitational collapse. These models involve the addition of a dilaton field to lower dimensional General Relativity. The string-inspired model introduced by Callan, Giddings, Harvey and Strominger (CGHS) [1] is one of the simplest toy models that describe the formation of a black hole by gravitational collapse of massless scalar fields. The quantum back-reaction is included at the one-loop level by adding the Polyakov-Liouville term [2] to the classical action. In this simplified context, a special modification of the original semiclassical CGHS model was introduced—the so-called RST model [3]—making it possible to solve the semiclassical equations exactly (see also [4]-[6]).

The key point in the formulation of the RST model is the addition of a kinetic counterterm to the one-loop semiclassical CGHS action in such a way that an extra symmetry is maintained at the quantum level thus implying exact solubility. More recently, a new quantum corrected version of the CGHS model was introduced (the BPP model [7]), which is also exactly solvable in the semiclassical approximation including back-reaction (see also [8] for the one-parameter class of models interpolating between the RST and BPP models). Unlike the RST model, in the BPP model the symmetry transformation is exactly the same as in the classical one, and describes an evaporating black hole with a non-flat end-state geometry. Although both the RST and BPP mod-

els (among other models (see [9]) can be converted, through field redefinitions, into a Liouville model possessing the infinite-dimensional conformal symmetry (which guarantees the background independence in the sigma-model formulation) the aim of this work is to explore the existence of finite-dimensional extra symmetries for generic models of dilaton gravity. The main consequence of the extra symmetries is that, as we will see, allow to define in a natural way a related semiclassical theory invariant under the extra symmetry.

The 2D dilaton models would, nonetheless, be a highly more useful tool if (at least part of) the developments which have been made with the CGHS theory would also be possible with more general models. In particular, for spherically symmetric Einstein gravity, which is one of the most realistic models of 2D dilaton gravity. In this paper this goal is pursued in two different though complementary directions. Firstly, we look for models which, alike the CGHS model, are invariant under an extra (conformal) symmetry. We shall show that, in the approach of the non-linear sigma models [10] these theories, when the kinetic term has been dropped out by an appropriate field redefinition, are restricted to have a potential of exponential form  $V = 4\lambda^2 e^{\beta\phi}$  where  $\phi$  is the dilaton field. Secondly, we shall present three (model-dependent) transformations which are symmetries for generic 2D dilaton gravity models with arbitrary potential  $V = V(\phi)$ . (This includes spherically symmetric gravity, for which the potential is  $V(\phi) \propto \frac{1}{\sqrt{\phi}}$ ). These symmetry transformations generalize that of the CGHS (BPP and RST) model in the sense that, although they are non-conformal for a generic potential and involve the space-time derivatives of the fields, a linear combination of them is conformal and equals the extra conformal symmetry of the CGHS model in the particular case in which the potential is constant. Therefore, by following parallel lines to those which produce the semiclassical BPP model from the classical CGHS model, we are able to provide semiclassical models for a generic 2D dilaton gravity model which maintain an extra symmetry.

The organization of the paper is as follows. In Sec. 2 we briefly review the semiclassical theory of the CGHS model, and present a simple procedure to construct a family of quantum corrected actions maintaining the classical free field equation. In Sec. 3 we find out the exponential models as the only models

with an extra (conformal) symmetry. Sec. 4 is devoted to analyze the main semiclassical aspects of these models. In Sec. 5 we present the main goal of the paper. We introduce three new extra symmetries for generic 2D dilaton gravity. We work out an invariant metric for a particular (non-conformal) symmetry and construct an invariant semiclassical action for a generic theory. In Sec. 6 we state our conclusions.

## 2 Semiclassical theory of the CGHS model

A particularly simple model for black hole physics [1] is given by the classical action

$$S_0 = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} (R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right], \quad (2.1)$$

where  $\phi$  is the dilaton field and  $f_i$  are a set of  $N$  massless scalar fields. The solubility of the theory is based on the existence of a free field. In conformal gauge (i.e,  $ds^2 = -e^{2\rho} dx^+ dx^-$ ), one has the free field equation

$$\partial_+ \partial_- (\rho - \phi) = 0. \quad (2.2)$$

This is a consequence of the following symmetry

$$\delta\phi = \delta\rho = \epsilon e^{2\phi}. \quad (2.3)$$

A simple way to see the invariance of the classical action under the above transformations is to perform the following redefinition of variables

$$\begin{aligned} \bar{g}_{\mu\nu} &= g_{\mu\nu} e^{-2\phi}, \\ \bar{\phi} &= e^{-2\phi}. \end{aligned} \quad (2.4)$$

In terms of  $\bar{g}_{\mu\nu}$  and  $\bar{\phi}$ , the classical CGHS action (2.1) takes the reduced form

$$S_0(\bar{g}, \bar{\phi}) = \frac{1}{2\pi} \int d^2x \sqrt{-\bar{g}} \left[ \bar{R} \bar{\phi} + 4\lambda^2 - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right]. \quad (2.5)$$

The transformation (2.3), when written in terms of the new fields, adopts the trivial form

$$\begin{aligned} \delta\bar{g}_{\mu\nu} &= 0, \\ \delta\bar{\phi} &= \epsilon, \end{aligned} \quad (2.6)$$

and the action (2.5) is invariant because  $\sqrt{-g}R$  is a total derivative in two dimensions. To account for the back-reaction effect one has to add the one-loop Polyakov-Liouville term

$$S_P = -\frac{N}{96\pi} \int d^2x \sqrt{-g} R \square^{-1} R. \quad (2.7)$$

This term breaks the symmetry (2.3) although the RST counterterm

$$S_{RST} = -\frac{N}{96\pi} \int d^2x \sqrt{-g} 2\phi R, \quad (2.8)$$

reestablishes it with the following correction:

$$\delta\phi = \delta\rho = \epsilon \frac{e^{2\phi}}{\left(1 - \frac{k}{4}e^{2\phi}\right)}. \quad (2.9)$$

Therefore, the conserved current  $\partial_+\partial_-(\rho - \phi)$  is maintained at one-loop level.

However, the easiest way to construct a semiclassical action, invariant under the transformations (2.3), is to consider the Polyakov-Liouville term with respect to the invariant metric  $\bar{g}_{\mu\nu}$ . Going back to the fields  $(g_{\mu\nu}, \phi)$ , the effective action decomposes into

$$S_P(\bar{g}) = S_P(g) + S_{BPP}(g, \phi), \quad (2.10)$$

where the local term  $S_{BPP}(g, \phi)$  turns out to be the counterterm of the Bose-Parker-Peleg model [7]:

$$S_{BPP}(g, \phi) = \frac{N}{24\pi} \int d^2x \sqrt{-g} \left( (\nabla\phi)^2 - \phi R \right). \quad (2.11)$$

This also provides a way to select a particular metric in the functional integral of conformal matter fields. The metric is chosen to be invariant with respect to the symmetry of the classical dilaton-gravity action.

At this point it is interesting to comment that, despite of the fact that the classical equations imply the vanishing of the scalar curvature  $\bar{R}$ , and therefore the trace anomaly, the black holes do radiate. Indeed, the BPP model describes the formation and evaporation of a black hole producing a non-trivial remnant geometry. Moreover, the BPP model has also emerged as the semiclassical limit of the non-perturbative approach of Ref. [11].

It is easy to see now that one can construct a general class of one-loop models preserving the current conservation equation  $\partial_+\partial_-\bar{\rho} \equiv \partial_+\partial_-(\rho - \phi) = 0$ . If we

introduce a new field (where  $G$  is an arbitrary function)

$$\tilde{\phi} = \bar{\phi} + \frac{N}{12}G(\bar{\phi}), \quad (2.12)$$

and modify the classical action (2.5) by

$$S_0(\bar{g}, \bar{\phi}) \longrightarrow S_0\left(\bar{g}, \bar{\phi} + \frac{N}{12}G(\bar{\phi})\right), \quad (2.13)$$

it is clear that the new action is invariant under the transformation

$$\begin{aligned} \delta\bar{g}_{\mu\nu} &= 0, \\ \delta\tilde{\phi} &= \epsilon. \end{aligned} \quad (2.14)$$

Returning now to the primitive fields  $(g_{\mu\nu}, \phi)$ , the action

$$S_0(\bar{g}, \tilde{\phi}) + S_P(\bar{g}) \quad (2.15)$$

turns out to be

$$\begin{aligned} &S_0(g, \phi) + S_P(g) \\ &+ \frac{N}{24\pi} \int d^2x \sqrt{-g} \left[ (\nabla\phi)^2 - 2g^{\mu\nu} \nabla_\mu F(\phi) \nabla_\nu \phi + F(\phi)R - \phi R \right], \end{aligned} \quad (2.16)$$

where  $F(\phi) = G(\bar{\phi}(\phi))$ , and the transformations (2.14) take the form

$$\delta g_{\mu\nu} = \epsilon \frac{2e^{2\phi}}{1 - \frac{N}{24}F'(\phi)e^{2\phi}} g_{\mu\nu}, \quad (2.17)$$

$$\delta\phi = \epsilon \frac{e^{2\phi}}{1 - \frac{N}{24}F'(\phi)e^{2\phi}}. \quad (2.18)$$

The above family of 2D dilaton gravity models, parametrized by the arbitrary function  $F$ , was obtained in Ref. [8] (see also [12]) by suitable field redefinitions from the classical CGHS action corrected with a (non-covariant) Polyakov-type term, invariant under Weyl transformations [13] [14]. The RST model can be recovered with the choice  $F(\phi) = \frac{1}{2}\phi$ . In general, the field redefinitions

$$\begin{aligned} \Omega &= \sqrt{k}F(\phi) + \frac{e^{-2\phi}}{\sqrt{k}}, \\ \chi &= \sqrt{k}\rho + \sqrt{k}(F(\phi) - \phi) + \frac{e^{-2\phi}}{\sqrt{k}}, \end{aligned} \quad (2.19)$$

convert the models (2.16) into a Liouville theory

$$\frac{1}{\pi} \int d^2x \left[ -\partial_+ \chi \partial_- \chi + \partial_+ \Omega \partial_- \Omega + \lambda^2 e^{\frac{2}{\sqrt{k}}(\chi - \Omega)} + \frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_- f_i \right]. \quad (2.20)$$

We have to mention now that the Polyakov-Liouville term coupled to the invariant metric seems to be the unique semiclassical term which, for a generic theory (see for instance the exponential model of section 4), maintains the extra symmetry.

### 3 Extra (conformal) symmetries in 2D dilaton gravity

A natural way to analyze the existence of extra symmetries in generic dilaton gravity theories, generalizing the powerful symmetry of the CGHS model, is based on the non-linear sigma model formulation of these theories . Let us now review briefly the results of Ref. [9].

The expression of a non-linear sigma model action associated with a wide class of 2D dilaton gravity models is

$$S = \int d^2x \sqrt{-\hat{g}} \left[ \hat{g}^{\mu\nu} \partial_\mu X^i G_{ij}(X) \partial_\nu X^j + Q(X) \hat{R} + \Lambda e^{W(X)} \right], \quad (3.1)$$

where  $\Lambda$  is a constant,  $\hat{g}_{\mu\nu}$  is a reference metric, and  $G_{ij}$  is a metric depending on the target space coordinates  $X^i(x)$  ( $i = 1, 2$ ) ( $X^1$  and  $X^2$  stand for the dilaton field and the conformal factor respectively). If we consider now a flat reference metric  $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$ , (3.1) reduces to

$$S_{flat} = \int d^2x \left[ \partial_\mu X^i G_{ij} \partial^\mu X^j + \Lambda e^W \right]. \quad (3.2)$$

The change of the Lagrangian under a general variation  $\delta X^i$  is

$$\begin{aligned} \delta \mathcal{L} &= 2\partial^\mu \left( \partial_\mu X^i G_{ij} \delta X^j \right) \\ &\quad - 2 \left( \square X^k + \Gamma_{ij}^k \partial_\mu X^i \partial^\mu X^j \right) G_{kl} \delta X^l + \Lambda W_{,k} \delta X^k e^W, \end{aligned} \quad (3.3)$$

where  $\Gamma_{ij}^k$  is referred to the metric  $G_{ij}$ ,  $\square$  is the world sheet Laplacian and  $W_{,k}$  denotes derivative with respect to the target space coordinate  $X^k$ . It is easy to check that there exists a symmetry if the variation  $\delta X^i$  satisfies

$$W_{,k} \delta X^k = 0, \quad (3.4)$$

$$\nabla_i \delta X_j = \partial_i \delta X_j - \Gamma_{ij}^k \delta X_k = 0. \quad (3.5)$$

Equation (3.4) can be solved to give

$$\delta X^k = \frac{\epsilon^{kl}}{\sqrt{-|G|}} W_{,l}, \quad (3.6)$$



being  $|G| = \det G_{ij}$ , and  $\epsilon^{kl}$  is the Levi-Civita symbol. Substitution of (3.6) into equation (3.5) yields the condition

$$\nabla_i \nabla_j W = 0 . \quad (3.7)$$

An immediate consequence of this symmetry is that it allows to construct a free field  $F(x)$ .  $F(x)$  is associated with the corresponding Noether current  $j^\mu$ :

$$j_\mu = \partial_\mu X^i G_{ij} \frac{\epsilon^{jk}}{\sqrt{-|G|}} W_{,k} \quad (3.8)$$

by the simple relation

$$j_\mu = -\partial_\mu F , \quad (3.9)$$

where the condition (3.7) guarantees the existence of the function  $F(x)$ . Moreover, the field  $F(x)$  is orthogonal to the field  $W(x)$ , which satisfies the Liouville equation.

Let us return now to the generally covariant expression of a generic 2D dilaton gravity model,

$$S(g, \phi) = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ D(\phi)R + \frac{1}{2} (\nabla\phi)^2 + V(\phi) \right] , \quad (3.10)$$

where  $D(\phi)$  and  $V(\phi)$  are arbitrary functions of the dilaton field. It is not difficult to see that one can eliminate the kinetic term for the dilaton by a conformal redefinition of the fields [15] [16]. Therefore, one can study a generic 2D dilaton gravity model from the action

$$S(g, \phi) = \frac{1}{2\pi} \int d^2x \sqrt{-g} (R\phi + V(\phi)) , \quad (3.11)$$

For the CGHS model we have  $V = 4\lambda^2$ , and the Jackiw-Teitelboim model [17] and spherically symmetric gravity [5] correspond to  $V = \Lambda\phi$  and  $V = \lambda^2/\sqrt{2\phi}$ , respectively.

Our aim now is to classify the extra (conformal) symmetries of the 2D dilaton gravity models according to the criteria (3.7). This can be achieved immediately if one consider the reduced form (3.11) of generic dilaton gravity in conformal gauge,

$$S = \frac{1}{2\pi} \int d^2x \left( -4\partial_+ \phi \partial_- \rho + \frac{1}{2} V(\phi) e^{2\rho} \right) . \quad (3.12)$$

This action fits the general expression (3.2), where

$$G_{ij} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

and

$$W(X) = 2\rho + \frac{1}{2} \log V(\phi). \quad (3.13)$$

As the components of  $G_{ij}$  are constants, the condition (3.7) becomes

$$\frac{d^2 \log V(\phi)}{d\phi^2} = 0. \quad (3.14)$$

Thus the general solution is  $V(\phi) = 4\lambda^2 e^{\beta\phi}$ , where  $\lambda$  and  $\beta$  are constants. The CGHS model is recovered for  $\beta = 0$ . In this case the conformal symmetry  $\delta X^i$  is (2.6) and the free field  $F = \rho$ .

## 4 The $V \propto e^{\beta\phi}$ model

The CGHS model has provided an useful arena to describe back-reaction effects in the black hole evaporation process. The crucial property of the model is the existence of a powerful symmetry that can be maintained at the one-loop level and, therefore, guarantees the solubility of the semiclassical theory. The models with an exponential potential  $V = 4\lambda^2 e^{\beta\phi}$  possesses also a similar conformal symmetry which can be exploited to produce an exactly solvable semiclassical model.

### 4.1 Classical equations

In conformal gauge, the equations of motion of the model

$$S_\beta = \frac{1}{2\pi} \int \sqrt{-g} \left[ R\phi + 4\lambda^2 e^{\beta\phi} - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right] \quad (4.1)$$

are equivalent to

$$\partial_+ \partial_- \left( \rho - \frac{\beta}{2} \phi \right) = 0, \quad (4.2)$$

$$\partial_+ \partial_- \left( \rho + \frac{\beta}{2} \phi \right) = -\lambda^2 \beta e^{2(\rho + \frac{\beta}{2} \phi)}, \quad (4.3)$$

$$-\partial_\pm^2 \phi + 2\partial_\pm \rho \partial_\pm \phi = T_{\pm\pm}^f. \quad (4.4)$$

The field  $\rho - \frac{\beta}{2}\phi$  is a free field and  $\rho + \frac{\beta}{2}\phi$  verifies the Liouville equation. In Kruskal gauge ( $\rho - \frac{\beta}{2}\phi = 0$ ), the equation (4.3) is

$$\partial_+\partial_-(2\rho) = -\lambda^2\beta e^{4\rho}, \quad (4.5)$$

the general solution of which can be written as

$$\rho = \frac{1}{4} \log \frac{\partial_+ F \partial_- G}{(1 + \lambda^2 \beta F G)^2}, \quad (4.6)$$

with  $F$  ( $G$ ) an arbitrary function of  $x^+$  ( $x^-$ ). Using the above expression, the constrained equations (4.4) become Riccati differential equations with respect to  $\log \partial_+ F$  and  $\log \partial_- G$ :

$$\frac{1}{4} (\partial_+ \log \partial_+ F)^2 - \frac{1}{2} \partial_+^2 \log \partial_+ F = \beta T_{++}^f, \quad (4.7)$$

$$\frac{1}{4} (\partial_- \log \partial_- G)^2 - \frac{1}{2} \partial_-^2 \log \partial_- G = \beta T_{--}^f. \quad (4.8)$$

In the absence of matter, the general solution is

$$e^{2\rho} = \frac{1}{Cx^+x^- + Ax^+ + Bx^- + D}, \quad (4.9)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are arbitrary constants with the restriction

$$AB - CD = -\lambda^2\beta. \quad (4.10)$$

If  $C < 0$ , the solution (4.9) is similar to the CGHS black hole solution. The event horizon is located at  $x^- = -A/C$ , and the curvature singularity is space-like for  $\lambda^2\beta < 0$ . For a positive value of  $\lambda^2\beta$  we have a naked singularity. On the other hand, if  $C > 0$ , the curvature singularity is also naked for  $\lambda^2\beta > 0$ , but the metric has not the appropriate signature. When  $C = 0$ , an elementary change of coordinates brings the metric to the form

$$ds^2 = \frac{1}{\sqrt{|\lambda^2\beta|}} \frac{-dt^2 + dx^2}{2x}. \quad (4.11)$$

Although this space-time is geodesically complete, one can check by solving (4.7)(4.8) for a collapsing shock-wave of matter (or by matching static solutions) that it produces a naked singularity for  $\lambda^2\beta > 0$ . Therefore, assuming a cosmic censorship conjecture we shall restrict the model to  $\lambda^2\beta < 0$  and consider only the elementary solutions with  $C < 0$ .

In analogy with the CGHS model, one can introduce asymptotically flat coordinates  $\{\sigma^\pm\}$  by

$$\sqrt{|C|}x^\pm = \pm e^\pm \sqrt{|C|}\sigma^\pm. \quad (4.12)$$

The resulting metric, for  $A = 0 = B$ , is then

$$ds^2 = \frac{-d\sigma^+ d\sigma^-}{1 + \frac{\lambda^2 \beta}{C} e^{-2\sqrt{|C|}\sigma}}. \quad (4.13)$$

At this point, one can use the Euclidean continuation  $t \rightarrow i\tau$  of the black hole metric to obtain the corresponding Hawking temperature. Introducing a new spatial coordinate  $R^2$  defined by (the horizon is located at  $R = 0$ )

$$1 + \frac{\lambda^2 \beta}{C} e^{-2\sqrt{|C|}\sigma} = \frac{1}{R^2 |C|}, \quad (4.14)$$

the Euclidean line element becomes

$$ds_E^2 = R^2 d\left(\sqrt{|C|}\tau\right)^2 + \frac{dR^2}{(1 - |C|R^2)^2}. \quad (4.15)$$

The Hawking temperature, which is the inverse of the period of  $\tau$ , is

$$T_H = \frac{\sqrt{|C|}}{2\pi}. \quad (4.16)$$

The static black hole solution (4.14) has the Killing vector  $k^\mu = \left(\frac{\partial}{\partial t}\right)^\mu = (1, 0)$ . The associated Noether charge  $Q$  is the mass of the black hole and is given by

$$Q = \frac{1}{2\pi} e^{\mu\nu} [2k_\mu \nabla_\nu \phi + \phi \nabla_\mu k_\nu] |_{\sigma=+\infty}. \quad (4.17)$$

In order to calculate  $Q$  it is useful to introduce a new spatial coordinate  $x$  related to  $\sigma$  by

$$1 + \frac{\lambda^2 \beta}{C} e^{-2\sqrt{|C|}\sigma} = \frac{1}{1 - e^{-2\sqrt{|C|}x}}. \quad (4.18)$$

In the coordinates  $(t, x)$  the metric takes the Schwarzschild-type form

$$ds^2 = -\left(1 - e^{-2\sqrt{|C|x}}\right) dt^2 + \frac{1}{1 - e^{-2\sqrt{|C|x}}} dx^2, \quad (4.19)$$

and the dilaton is

$$\phi = -\frac{2}{\beta} \sqrt{|C|x} - \frac{1}{\beta} \log \frac{\lambda^2 \beta}{C}. \quad (4.20)$$

The charge  $Q$  is now easily calculated

$$Q = -\frac{1}{2\pi} (k_0 \nabla_1 \phi - \phi \nabla_1 k_0) |_{x=+\infty} = \frac{2}{\beta\pi} \sqrt{|C|}. \quad (4.21)$$

This result can also be obtained by evaluating the ADM mass of the solution (4.13). (See [15]). Therefore, in sharp contrast with the CGHS model, the Hawking temperature is proportional to the black hole mass

$$T_H = \frac{\beta}{4} M . \quad (4.22)$$

The existence of 2D black holes whose mass is proportional to their temperature was noticed in [18]. The black hole solutions (4.14) also appear, through a different coupling of matter to 2D gravity, in [19].

Finally, we would like to point out that, in parallel with the CGHS model, the present model can also be brought, through a conformal redefinition of the metric, to a form for which the dynamics implies the vanishing of the scalar curvature. In terms of the new metric  $\tilde{g}_{\mu\nu} = e^{-\beta\phi} g_{\mu\nu}$ , the action takes the form

$$S_\beta = \frac{1}{2\pi} \int d^2x \sqrt{-\tilde{g}} \left( \tilde{R}\phi + \beta (\tilde{\nabla}\phi)^2 + 4\lambda^2 e^{2\beta\phi} \right) , \quad (4.23)$$

and the trace of the constrained equations is equivalent to the equation

$$\tilde{R} = 0 . \quad (4.24)$$

## 4.2 Semiclassical theory

We shall now consider the semiclassical theory. The special symmetry of the model (4.1) is

$$g_{\mu\nu} \longrightarrow e^{-\beta\epsilon} g_{\mu\nu} , \quad (4.25)$$

$$\phi \longrightarrow \phi + \epsilon , \quad (4.26)$$

Following the procedure of Section 2 we can construct an one-loop corrected theory invariant under the transformations (4.25) (4.26) by adding to (4.1) a Polyakov-Liouville term  $S_P(\bar{g})$  respect to the invariant metric

$$\bar{g}_{\mu\nu} = e^{\beta\phi} g_{\mu\nu} . \quad (4.27)$$

In terms of the fields  $(\bar{g}_{\mu\nu}, \phi)$ , the semiclassical action is

$$\begin{aligned} & \frac{1}{2\pi} \int d^2x \sqrt{-\bar{g}} \left( \phi R(\bar{g}) - \beta (\bar{\nabla}\phi)^2 + 4\lambda^2 - \frac{1}{2} \sum_{i=1}^N (\bar{\nabla} f_i)^2 \right) \\ & - \frac{N}{96\pi} \int d^2x \sqrt{-\bar{g}} R(\bar{g}) \bar{\square}^{-1} R(\bar{g}) , \end{aligned} \quad (4.28)$$

and, in conformal gauge, the equations of motion are:

$$\partial_+ \partial_- (\bar{\rho} - \beta \phi) = 0, \quad (4.29)$$

$$\partial_+ \partial_- \phi + \lambda^2 e^{2\bar{\rho}} + \frac{N}{12} \partial_+ \partial_- \bar{\rho} = 0, \quad (4.30)$$

$$\begin{aligned} -\partial_{\pm}^2 \phi + 2\partial_{\pm} \bar{\rho} \partial_{\pm} \phi - \beta (\partial_{\pm} \phi)^2 \\ + \frac{N}{12} [(\partial_{\pm} \bar{\rho})^2 - \partial_{\pm}^2 \bar{\rho} + t_{\pm}(x^{\pm})] = T_{\pm\pm}^f. \end{aligned} \quad (4.31)$$

Going back to the physical metric  $\rho = \bar{\rho} - \frac{\beta}{2}\phi$ , and in Kruskal gauge ( $\bar{\rho} - \beta\phi = \rho - \frac{\beta}{2}\phi = 0$ ), the semiclassical equations can be written in the form

$$\partial_+ \partial_- (2\rho) = \frac{-\lambda^2 \beta}{1 + \frac{N\beta}{12}} e^{4\rho}, \quad (4.32)$$

$$e^{2\rho} \partial_{\pm}^2 e^{-2\rho} = \frac{\beta}{1 + \frac{N\beta}{12}} \left( T_{\pm\pm}^f - \frac{N}{12} t_{\pm} \right). \quad (4.33)$$

So, in Kruskal gauge, the one loop corrected equations are the same as the classical ones (4.2)-(4.4), up to a quantum shift for the  $\beta$  parameter

$$\beta \longrightarrow \frac{\beta}{1 + \frac{N\beta}{12}}, \quad (4.34)$$

and the addition of the non-local terms  $t_{\pm}$  coming from the Polyakov-Liouville action.

Let us now consider static black hole solutions. With the choice  $T_{\pm\pm}^f = 0$  and  $t_{\pm} = 0$ , we recover the solutions (4.9)-(4.10) with the quantum corrected  $\beta$  parameter (4.34). In asymptotically flat coordinates  $\{\sigma^{\pm}\}$  (4.12), the energy flux at infinity gives rise to a constant thermal value. Using the anomalous transformation law of  $\langle T_{\pm\pm}^f \rangle$  we arrive at

$$\langle T_{\pm\pm}^f(\sigma^{\pm}) \rangle = -\frac{N}{12} t_{\pm}(\sigma^{\pm}) = \frac{N}{48} |C|, \quad (4.35)$$

corresponding to the Hawking temperature  $T_H = \frac{\sqrt{|C|}}{2\pi}$ . The solution describes then a black hole in thermal equilibrium at temperature  $T_H = \frac{\sqrt{|C|}}{2\pi}$ . Due to the non-static character of the solution with zero mass it is unclear how can we study the evaporation process of a black hole formed by gravitational collapse. It could be considered instead the evolution of a black hole in thermal equilibrium when it absorbs an incoming shock wave. In the CGHS model the black hole remains static since the temperature is independent of the mass. However, for

the exponential model the shift in the mass due to the infalling shock wave increases the temperature and produces evaporation. These questions are out of the aim of the present work and will be considered elsewhere.

## 5 Extra (non-conformal) symmetries in 2D dilaton gravity models

In this section we shall consider the symmetries in 2D dilaton gravity models from a different standpoint. We shall show that suitable modifications of the conformal symmetries we have dealt with up to now produce transformations of the fields which are symmetries of all the models whose action can be brought to the form:

$$\mathcal{L} = \frac{1}{2\pi} \sqrt{-g} [R\phi + V(\phi)] . \quad (5.1)$$

In this way, we shall be able to generalize the procedure described in Section 2—which produces the BPP and RST models from the CGHS model and which requires the construction of a metric which is invariant under the symmetry—to a generic model of 2D dilaton gravity, thus generalizing the BPP and RST models.

To find out these symmetries our strategy consists in finding the generalized conserved currents firstly. Then, by applying a very useful (sort of reciprocal) version of the Noether theorem which is presented next, we shall find out the symmetries these (Noether) currents are associated with.

For any Lagrangian  $\mathcal{L} = \mathcal{L}(\Psi^a)$  and arbitrary transformations of the fields  $\delta\Psi^a$ , we have:

$$\delta\mathcal{L} = (E - L)_a \delta\Psi^a - \nabla_\mu s^\mu , \quad (5.2)$$

where  $(E - L)_a = 0$  are the Euler-Lagrange equations of motion for the fields  $\Psi^a$ , and  $\nabla_\mu s^\mu$  is a total derivative term which appears due to the “integrations by parts” which are generally required to produce the equations of motion.

Let now  $j^\mu$  be a current, which is made from the fields  $\Psi^a$  and which is conserved on-shell:  $\nabla_\mu j^\mu|_{sol} = 0$ . It is easy to see then that a transformation  $\delta\Psi^a$  is the Noether symmetry associated to  $j^\mu$  iff, without using any of the equations of motion, the following equality holds as an identity:

$$(E - L)_a \delta\Psi^a = \nabla_\mu j^\mu . \quad (5.3)$$

In general, and due to semi-invariance, the current  $j^\mu$  will not be equal to  $s^\mu$ .

For the Lagrangian in eq. (5.1) we have:

$$\begin{aligned} \delta\mathcal{L} &= \frac{\sqrt{-g}}{2\pi} \left\{ [R + V'(\phi)] \delta\phi \right. \\ &+ \left[ \nabla_\mu \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} V(\phi) - g_{\mu\nu} \nabla^2 \phi \right] \delta g^{\mu\nu} \\ &+ \nabla_\alpha [-\phi (g^{\mu\nu} \nabla^\alpha \delta g_{\mu\nu} - g^{\alpha\mu} \nabla^\nu \delta g_{\mu\nu}) + \nabla^\alpha \phi g_{\mu\nu} \delta g^{\mu\nu} - \nabla_\nu \phi \delta g^{\nu\alpha}] \left. \right\}. \end{aligned} \quad (5.4)$$

Taking into account that  $G_{\mu\nu} = 0$  implies  $\square\phi = V$  the equations of motion are equivalent to

$$R + V'(\phi) = 0, \quad (5.5)$$

$$\nabla_\mu \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} V(\phi) = 0, \quad (5.6)$$

and then it is not difficult to check that the following currents are conserved:

$$j_1^\mu = \frac{\nabla^\mu \phi}{(\nabla\phi)^2}, \quad (5.7)$$

$$j_2^\mu = j_R^\mu + V \frac{\nabla^\mu \phi}{(\nabla\phi)^2}, \quad (5.8)$$

where  $\nabla_\mu j_R^\mu = R$ .

Now we can make use of the Noether theorem to show that these currents are, in fact, Noether currents associated with symmetry transformations of the theory. The transformations which satisfy (5.3) for the currents  $j_1$  and  $j_2$  are, respectively,

$$\delta_1 \phi = 0 \quad , \quad \delta_1 g_{\mu\nu} = \epsilon_1 \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu \phi \nabla_\nu \phi}{(\nabla\phi)^4} \right), \quad (5.9)$$

$$\delta_2 \phi = \epsilon_2 \quad , \quad \delta_2 g_{\mu\nu} = \epsilon_2 V \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu \phi \nabla_\nu \phi}{(\nabla\phi)^4} \right). \quad (5.10)$$

Though neither of these variations,  $\delta_1$  or  $\delta_2$ , reproduce the conformal symmetry of the CGHS model when the potential  $V$  is constant,  $V = 4\lambda^2$ , it is easy to show that a linear combination of them do:

$$\delta = \delta_2 - 4\lambda^2 \delta_1. \quad (5.11)$$

Therefore the transformation  $\delta$  must be regarded as a symmetry which generalizes that of the CGHS model.



Observe that both  $\delta_1$  and  $\delta_2$  are area-preserving (i.e.,  $\delta_{1,2}\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta_{1,2}g^{\mu\nu} = 0$ ). It is also interesting to remark at this point that when the fields are taken to be on-shell the symmetry transformation  $\delta_2$  can be identified with a diffeomorphism, with infinitesimal space-time vector field  $f^\mu = \frac{\nabla^\mu\phi}{(\nabla\phi)^2}$ .

Moreover the model (5.1) has the following symmetry

$$\delta_E\phi = 0 \quad , \quad \delta_E g_{\mu\nu} = g_{\mu\nu}a_\sigma\nabla^\sigma\phi - \frac{1}{2}(a_\mu\nabla_\nu\phi + a_\nu\nabla_\mu\phi) \quad , \quad (5.12)$$

for arbitrary constant vector  $a_\mu$ . By means of the Noether theorem this symmetry can be easily shown to give rise to the following conserved current

$$J^{\mu\nu} = g^{\mu\nu}E \quad , \quad (5.13)$$

where

$$E = \frac{1}{2} [(\nabla\phi)^2 - J(\phi)] \quad , \quad (5.14)$$

and

$$J'(\phi) = V(\phi) \quad . \quad (5.15)$$

The conservation law for  $J^{\mu\nu}$  implies the space-time independence of the local energy  $E$  [16].

In the present case, as in the CGHS model, the conservation law for the currents  $j_1^\mu, j_2^\mu$  turns out to imply the existence of two free fields. It is not difficult to check that  $j_1^\mu$  and  $j_2^\mu - j_R^\mu$  satisfy the integrability condition. The corresponding free-fields equations are:

$$\begin{aligned} \square j_1 &= 0 \quad , \\ R + \square j_2 &= 0 \quad , \end{aligned} \quad (5.16)$$

where

$$j_1 = \int^\phi \frac{d\tau}{2E + J(\tau)} \quad , \quad (5.17)$$

and

$$j_2 = \log(2E + J) = \log(\nabla\phi)^2 \quad . \quad (5.18)$$

We have to note that in the integral of (5.17)  $E$  should be considered as a constant. At this point it is clear that we have generalized the conformal symmetry of the CGHS model in the sense that it could be recovered as a special linear combination of  $\delta_1$  and  $\delta_2$ . In section 3 we also showed that the CGHS model

can be seen as a particular case ( $\beta = 0$ ) of a family of models (with potential  $V = 4\lambda^2 e^{\beta\phi}$ ) having a conformal symmetry  $\delta_\beta$ . However, it is easy to see that, for non-constant potentials, no non-trivial linear combination of  $\delta_1$  and  $\delta_2$  is conformal. This suggests that it may exist another symmetry  $\delta_3$ , which must be independent from  $\delta_1$  and  $\delta_2$ , such that a linear combination of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  gives rise to a conformal symmetry, at least in the particular case in which the potential is an exponential of the dilaton.

To find out this symmetry we note that, for the exponential models, the Noether current associated to the conformal symmetry  $\delta_\beta$  can be written as:

$$j_\beta^\mu = j_2^\mu + 2\beta E j_1^\mu . \quad (5.19)$$

However, since  $E$  is constant, this current is also conserved for a generic 2D dilaton gravity model. Our task, therefore, is to work out, for a generic model, the Noether symmetry  $\delta_3$  which is associated with the conserved current  $E j_1$ . A straightforward application of the Noether theorem yields

$$\delta_3\phi = 0 \quad , \quad \delta_3 g_{\mu\nu} = -\frac{\epsilon_3}{2} \left[ g_{\mu\nu} + J \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu\phi\nabla_\nu\phi}{(\nabla\phi)^4} \right) \right] . \quad (5.20)$$

Hence the conformal symmetry of the exponential models, for which  $V = \beta J$ , is given by

$$\delta_\beta = \delta_2 + 2\beta\delta_3 . \quad (5.21)$$

Therefore, the CGHS model and the models with an exponential potential are special only in the sense that

a) One of the symmetries  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  is a linear combination of the other two.

b) A linear combination of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  is conformal and does not involve the space-time derivatives of the fields.

Finally, we mention that the three symmetries close down to a non-abelian Lie algebra. The symmetry  $\delta_2$  is a central generator, but  $\delta_1$  and  $\delta_3$  generate the affine algebra:  $[\delta_1, \delta_3] = \frac{1}{2} \delta_1$ .

## Construction of an invariant semiclassical action

To complete the program which introduced the present section, we must consider now the construction of an invariant metric  $\bar{g}_{\mu\nu}$ . Although for a generic

2D dilaton gravity model the invariant metric will not be unique, in the present paper, and for the sake of clarity, we shall consider the simplest choice.

Here we shall consider the metric  $\bar{g}_{\mu\nu}$  which fulfils the following requirements:

- a) It is invariant under  $\delta = \delta_2 - 4\lambda^2\delta_1$ ,
- b)  $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}$  when  $V = 4\lambda^2$ , and
- c)  $\det g_{\mu\nu} = \det \bar{g}_{\mu\nu}$ .

The requirements a) and b) guarantee that, when  $V = 4\lambda^2$ , our semiclassical model will reduce to the BPP model (or RST model). The requirement c) appears to be a natural one since the symmetry  $\delta$  is area preserving.

Eqs. (5.9) and (5.10) suggest that an invariant metric may be of the form:

$$\bar{g}_{\mu\nu} = \tilde{A}g_{\mu\nu} + \tilde{B}\nabla_\mu\phi\nabla_\nu\phi, \quad (5.22)$$

where  $\tilde{A} = \tilde{A}(g_{\mu\nu}, \phi)$ ,  $\tilde{B} = \tilde{B}(g_{\mu\nu}, \phi)$  are scalar functions to be determined. It can be written in the form

$$\bar{g}_{\mu\nu} = A \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - \frac{\nabla_\mu\phi\nabla_\nu\phi}{(\nabla\phi)^4} \right) + B\nabla_\mu\phi\nabla_\nu\phi, \quad (5.23)$$

where the new scalars  $A = A(g_{\mu\nu}, \phi)$  and  $B = B(g_{\mu\nu}, \phi)$  multiply quantities which are invariant under  $\delta$ . Therefore  $A$  and  $B$  must also be invariant. The simplest scalar which is invariant under  $\delta$  is

$$E_\lambda = \frac{1}{2} \left( (\nabla\phi)^2 - J(\phi) + 4\lambda^2\phi \right) \equiv E + 2\lambda^2\phi. \quad (5.24)$$

Therefore, it appears natural to consider that  $A$  and  $B$  are functions of  $E_\lambda$ .

The condition c) implies  $AB = 1$ . Moreover, since for  $V = 4\lambda^2$  we have  $(\nabla\phi)^2 = 2E_\lambda$ , condition b) requires

$$A = 2E_\lambda. \quad (5.25)$$

Therefore, a metric which fulfils the three requirements above is

$$\bar{g}_{\mu\nu} = \frac{2E_\lambda}{(\nabla\phi)^2}g_{\mu\nu} + \left( \frac{1}{2E_\lambda} - \frac{2E_\lambda}{(\nabla\phi)^4} \right) \nabla_\mu\phi\nabla_\nu\phi. \quad (5.26)$$

The inverse metric is given by

$$\bar{g}^{\mu\nu} = \frac{(\nabla\phi)^2}{2E_\lambda}g^{\mu\nu} + \left( \frac{2E_\lambda}{(\nabla\phi)^4} - \frac{1}{2E_\lambda} \right) \nabla^\mu\phi\nabla^\nu\phi. \quad (5.27)$$

Since  $(\bar{\nabla}\phi)^2 = 2E_\lambda$  the inverse relation of (5.26) takes the form

$$g_{\mu\nu} = \frac{2\bar{E}_\lambda}{(\bar{\nabla}\phi)^2} g_{\mu\nu} + \left( 2\bar{E}_\lambda - \frac{2\bar{E}_\lambda}{(\bar{\nabla}\phi)^4} \right) \nabla_\mu\phi\nabla_\nu\phi, \quad (5.28)$$

where

$$\bar{E}_\lambda = \frac{1}{2} \left( (\bar{\nabla}\phi)^2 + J(\phi) - 4\lambda^2\phi \right). \quad (5.29)$$

Therefore, the inverse transformation is obtained from the direct one by (essentially) changing the sign of the potential.

Once introduced the metric  $\bar{g}_{\mu\nu}$  (5.26), which is invariant under the symmetry (5.11), we can immediately construct a semiclassical action which preserves this symmetry by conformally coupling the matter fields to the metric  $\bar{g}_{\mu\nu}$  and, using the inverse relation (5.28), write the action in terms of the metric  $\bar{g}_{\mu\nu}$ :

$$S = S_{DG} [g(\bar{g}, \phi), \phi] - \frac{1}{2} \sum_{i=1}^N \int d^2x \sqrt{-\bar{g}} \bar{g}^{\mu\nu} \partial_\mu f_i \partial_\nu f_i + S_P(\bar{g}), \quad (5.30)$$

where  $S_{DG}$  is the dilaton-gravity sector of the action  $S_0$ . Such an action will obviously reduce to the BPP model (2.10), (2.11), for  $V = 4\lambda^2$ . In conclusion, we have provided a semiclassical action invariant under the transformation

$$\begin{aligned} \delta\phi &= \epsilon, \\ \delta\bar{g}_{\mu\nu} &= 0. \end{aligned} \quad (5.31)$$

This is the standard expression of the symmetry of the CGHS and exponential model, which allows to reduce the associated sigma model to a Liouville-type theory. We expect that a generalization of the approach of Ref. [9] to a second order non-linear sigma model could imply solubility of the theory, in terms of the invariant metric. This question will be considered elsewhere [20].

## 6 Conclusions and final comments

In this paper we have considered the problem of generalizing the extra symmetry of the CGHS model. This symmetry plays a crucial role in the semiclassical treatment of the CGHS theory and therefore it seems natural to look for some sort of generalization. Our approach has two folds.

First of all, the conformal nature of the CGHS extra symmetry suggests to use a sigma model formalism to classify the possible models possessing a

conformal-type symmetry transformation. Taking into account the fact that a generic 2D dilaton-gravity model can be brought to a form with a vanishing kinetic term, the condition for having an extra conformal symmetry of Ref. [9] can be solved immediately. We have found that the unique models admitting an extra conformal symmetry are those with an exponential potential  $V = 4\lambda^2 e^{\beta\phi}$ . Therefore, any theory obtained from the model with an exponential potential by (conformal) redefinitions of the fields inherits an extra conformal symmetry. For  $\beta = 0$  an elementary change of variables transforms the reduced action into the CGHS standard action, and a further field redefinition of the fields generates also the models of [21]. When  $\beta \neq 0$  the reduced form of the action itself leads to black hole solutions with Hawking temperature proportional to the mass. The analogue of the BPP model can be constructed at once by adding a Polyakov-Liouville term respect to the invariant metric. The semiclassical theory could be exactly solved for especial forms of the energy-momentum tensor  $T_{\pm\pm}^f$  and the boundary conditions  $t_{\pm}$ . In general, the problem reduces to solve Riccati differential equations. If  $T_{\pm\pm}^f = 0$  and  $t_{\pm} = 0$ , the solution describes a black hole in thermal equilibrium and an adequate modification of the boundary conditions, to decrease the incoming thermal flux, can describe an evaporating black hole with a non-constant temperature [22].

Secondly, we have introduced three different extra symmetries for a generic 2D dilaton gravity model. These symmetries, which in general are non-conformal, turn out to be conformal for particular linear combination of them and reduce to the symmetry of the exponential model. This intriguing relation between the symmetries suggests that a special non-conformal symmetry could play the same role as the standard conformal symmetry of the CGHS model for a generic 2D dilaton gravity theory. This could open an avenue to study the semiclassical evolution of Schwarzschild black holes in an analytical setting.

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