Late-time correlators in semiclassical particle-black hole scattering

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We analyse the quantum corrected geometry and radiation in the scattering of extremal black holes by low-energy neutral matter. We point out the fact that the correlators of local observables inside the horizon are the same as those of the vacuum. Outside the horizon the correlators at late times are much bigger than those of the (thermal) case obtained neglecting the backreaction. This suggests that the corrected Hawking radiation could be compatible with unitarity.

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The discovery that black holes emit thermal radiation [1] has raised a serious conflict between quantum mechanics and general relativity. If a black hole is formed from the collapse of matter, initially in a pure quantum state, the subsequent evaporation produces radiation in a mixed quantum state [2]. If the analysis is performed in a fixed background geometry it is very hard to imagine how this conclusion can be avoided. The core of the problem is connected with the black hole causal structure. The information that flows through the horizon is not accessible to the outside observer and therefore one has to trace over the internal (unobserved) states. This generates a density matrix and the information, codified in correlations between internal and external states, is indeed lost in the singularity. There are several posibilities to avoid such a radical conclusion, but the most conservative one suggests that the information is recovered in the corrected Hawking radiation due to large backreaction effects [3–5]. However it is difficult to unravel a detailed mechanism capable to produce information return. Even more, it seems unlikely that unitarity can be preserved within the semiclassical approximation. It is usually stated that unitarity can only be obtained in a pure quantum gravity theory. Since we still do not have such a theory it is useful to consider a particular situation for which the problem can be simplified and, in turn, the backreaction effects can be controlled in a very efficient way. Such a scenario is given by the scattering of low-energy particles by extremal Reissner-Nordström charged black holes.

We now briefly recall the standard picture of the process in a fixed background spacetime approximation. Throwing long-wavelength particles into an extremal black hole results into a non-extremal one which then emits Hawking radiation. The Penrose diagram of such a process is given in Fig.1. There exists radiation flowing to future null infinity I^+ (Hawking radiation) and in general also inside the horizon. The quantum state of radiation is given by:

$$|0\rangle_{in} = \sum_{i,j} c_{i,j} |\psi_i\rangle_{int} \otimes |\psi_j\rangle_{ext} \qquad (1)$$

i.e., a superposition of products of internal and external states of right-moving modes (note that we shall be mostly concerned with right-movers, as in [6], because they are the ones which transmit the Hawking radiation). At late time this state takes the form

$$|0\rangle_{in} = \prod_w \sqrt{1 - e^{-2\pi w/\kappa}} \sum_n e^{-\pi n w/\kappa} |n_w\rangle_{int} \otimes |n_w\rangle_{ext} \qquad (2)$$

where $|n_w\rangle$ is a n-particle state with frequency w. An observer on I^+ will describe his measurements in terms of a reduced thermal density matrix

$$\rho = \prod_w (1 - e^{-2\pi w/\kappa}) \sum_n e^{-2\pi n w/\kappa} |n_w\rangle_{ext}\langle n_w|_{ext}. \qquad (3)$$

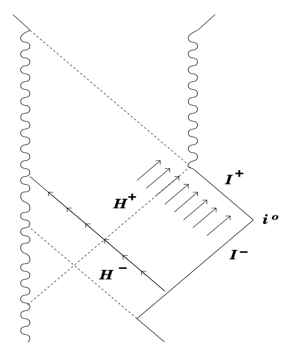

FIG.1.Penrose diagram corresponding to the creation of a near-extremal charged black hole from the extremal one. The ingoing arrow line represents an infalling shock wave.

In this paper we shall analyse how this scenario gets modified when backreaction effects are taken into account. Due to Hawking emission the radiating non-extremal configuration will decay back to the extremal black hole, if charged particles are sufficiently massive. The corresponding Penrose diagram is given in Fig.2. Comparing the diagrams of Figs. 1 and 2 we see that the right singularity, being an artifact of the fixed background approximation, has completely disappeared. It appears very unlikely the preservation of purity if radiation is still present at H (which is part of the future Cauchy horizon), since this would mean that the information is indeed lost in another causally disconnected and asymptotically flat region.

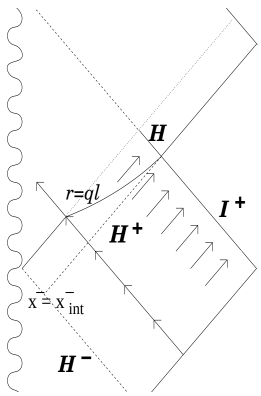

FIG.2. Penrose diagram corresponding to the process of particle capture by an extremal charged black hole followed by Hawking radiation. The end-state geometry is, due to backreaction effects, an extremal black hole. The location of the event horizon H^+ is at $x^- = x^-_{int}$.

We shall exploit the fact that the dominant Hawking emission is carried away in s-waves. Moreover, in the region very close to the initial extremal horizon $r = ql$ (q is the black hole charge and l^2 is Newton's constant), which is the relevant one to study the radiation at H, a scalar matter field f obeys the free equation

$$\partial_t^2 f - \partial_{r^*}^2 f = 0, \tag{4}$$

where r^* is the tortoise coordinate. The dynamics in the region close to $r = ql$ is controlled by the Jackiw-Teitelboim model [7], as it has been explained in [8]. The advantage of this model is that the backreaction effects can be incorporated immediately by adding the Polyakov-Liouville term [9]. In summary, the effective semiclassical model is given by the action

$$I = \int d^2x\sqrt{-g}\left[R\tilde{\phi} + 4\lambda^2\tilde{\phi} - \frac{1}{2}|\nabla f|^2\right] - \frac{\hbar}{96\pi}\int d^2x\sqrt{-g}R\,\square^{-1}R + \frac{\hbar}{12\pi}\int d^2x\sqrt{-g}\lambda^2\,, \tag{5}$$

where the relation between the fields appearing in (5) and the four-dimensional metric is given by

$$ds^2_{(4)} = \frac{ds^2_{(2)}}{\sqrt{\phi}} + 4l^2\phi d\Omega^2, \;\; \phi = \frac{q^2}{4} + \tilde{\phi}, \tag{6}$$

and $\lambda^2 = l^{-2}q^{-3}$. Usually, in order to make physical sense of the semiclassical approximation one considers a huge number, N, of scalar fields. In this way the quantum gravitational corrections can be safely neglected at one-loop order. Here for simplicity we consider $N = 1$, but it is straightforward to generalise our results to arbitrary N. We note that although the model we study is certainly simplified compared to the original 4d one the approximations made are reasonable. Indeed, the Hawking radiation derived from the model (5) has the same form as for 4d scalars in the limits considered, i.e. close to the horizon and I^+ at late times. The initial extremal configuration can be described, near $r = ql$, by the solution

$$ds^2 = -\frac{2l^2q^3dx^+dx^-}{(x^- - x^+)^2}, \quad \tilde{\phi} = \frac{lq^3}{x^- - x^+}. \tag{7}$$

The line $x^- = +\infty$ corresponds to the extremal radius $r = ql$, i.e. $\tilde{\phi} = 0$. This configuration is quantum mechanically stable and it does not produce radiation. If we send a very narrow pulse of classical null matter at $x^+ = x_0^+$ with small energy Δm we create a near-extremal black hole of mass $m = q + \Delta m$. The semiclassical solutions are now more involved, due to the non-locality of the quantum effective action.

We are interested in the Hawking radiation detected by an external observer at I^+. In this region the quantum incoming flux vanishes and therefore the metric can be naturally described in the outgoing Vaidya-type form

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2q^3} - l\tilde{m}(u)\right)du^2 - 2dud\tilde{x}, \tag{8}$$

where $\tilde{x} = l\tilde{\phi}$ and u is a null Eddington-Finkelstein coordinate. The relevant semiclassical equations in conformal gauge, $ds^2 = -e^{2\rho}dx^+dx^-$, are

$$-2\partial_+^2\tilde{\phi} + 4\partial_+\rho\partial_+\tilde{\phi} = -\frac{\hbar}{12\pi}\left[(\partial_+\rho)^2 - \partial_+^2\rho\right] \tag{9}$$

$$-2\partial_-^2\tilde{\phi} + 4\partial_-\rho\partial_-\tilde{\phi} = -\frac{\hbar}{12\pi}\left[(\partial_-\rho)^2 - \partial_-^2\rho\right] - \frac{\hbar}{24\pi}\left(\frac{du}{dx^-}\right)^2\{x^-, u\} \tag{10}$$

where $\{x^-, u\}$ is the Schwarzian derivative proportional to the (late time) Hawking flux. In conformal coordinates, where the metric takes the form (7), the effects of the evaporation are all encoded in the field $\tilde{\phi}$, expressed by means of a single function $G(x^-)$ through

$$\tilde{\phi} = \frac{G(x^-)}{x^+ - x^-} + \frac{1}{2}G'(x^-). \tag{11}$$

The consistency of (8) with the equations (9-10) and (11) implies that $du/dx^- = -lq^3/G(x^-)$ where $G(x^-)$ satisfies the differential equation

$$G''' = -\frac{\hbar}{24\pi}\left(-\frac{G''}{G} + \frac{1}{2}\left(\frac{G'}{G}\right)^2\right). \tag{12}$$

The recovery of the extremal solution at late times ($u \rightarrow +\infty$) requires that $\tilde{m}(u)$ (the mass deviation from extremality) vanishes for $x^- \rightarrow x^-_{int}$ (with $x^- < x^-_{int}$). This implies that in this limit $\{x^-, u\} \rightarrow 0$, i.e. the relation between u and x^- is a Möbius transformation

$$u = \frac{ax^- + b}{cx^- + d}, \tag{13}$$

where a, b, c, d are real parameters verifying the condition $ad - bc = 1$. It is now easy to evaluate the derivative $du/dx^- = 1/(cx^- + d)^2$, and then we face two qualitatively different possibilities: $c \neq 0$ and $c = 0$. We will not consider here the case $c = 0$ as it entails a period of negative Hawking flux (we will give more details in [10]). Therefore the (reasonable) assumption we make in this paper is that the Hawking radiation is always positive.

Let us analyze the case $c \neq 0$. In Fig.3. we numerically generate a solution for $G(x^-)$ with this behaviour

$$G(x^-) \overset{x^- \to x^-_{int}}{\sim} -\frac{1}{2} A (x^- - x^-_{int})^2, \tag{14}$$

where A is a non-vanishing constant. Note that the simplest solution which reproduces the extremal configuration at late times is obtained when $G(x^-)$ becomes a non-zero constant. However this implies $c = 0$. The parabolic behaviour (14) is the only one which allows to recover the extremal solution with c non-zero. Inserting (14) into eq. (11) we get

$$\tilde{\phi} = -\frac{A}{2} \frac{(x^- - x^-_{int})(x^+ - x^-_{int})}{x^+ - x^-} \tag{15}$$

which can be brought to the standard extremal form (7) after the change of coordinates $x^{\pm'} \sim 1/(x^{\pm} - x^-_{int})$. Further, a short manipulation of the differential equation shows that $G^{(n)}(x^-_{int}) = 0$ for $n \geq 3$. This implies that the unique function $G(x^-)$, for $x^- \geq x^-_{int}$, matching with the solution for $x^- \leq x^-_{int}$ is exactly the parabola (14). This is crucial, since it means that inside the horizon H^+, and so along H, we can express the solution in a form similar to (8) with $\tilde{m} = 0$ in terms of a new null coordinate u_H

$$u_H = -\frac{2lq^3}{A(x^- - x^-_{int})}. \tag{16}$$

The correlators of quasi-primary fields Φ_i associated to f at H are given by [11]

$$\langle \Phi_1(u_{H_1}) \ldots \Phi_n(u_{H_n}) \rangle = \tag{17}$$
$$\left(\frac{dx^-}{du_H}\right)^{\lambda_1} (x_1^-) \ldots \left(\frac{dx^-}{du_H}\right)^{\lambda_n} (x_n^-) \left\langle \Phi_1(x_1^-) \ldots \Phi_n(x_n^-) \right\rangle$$

where $\lambda_1, \ldots, \lambda_n$ are the corresponding conformal weights. Since (16) is a Möbius transformation the correlators are the same as those of the vacuum. This means that the state at H is just the restriction of the vacuum to H. Moreover, the range of the coordinate u_H can be prolonged beyond H ($u_H \geq 0$) to cover the whole future Cauchy horizon up to the singularity (i.e. up to $u_H \to +\infty$). This suggests that the state inside the horizon is just the vacuum state (naturally defined by the null time $-\infty < u_H < +\infty$) and, therefore, that the correlators of the Hawking radiation can be obtained from a pure state $|\psi\rangle_{ext}$.

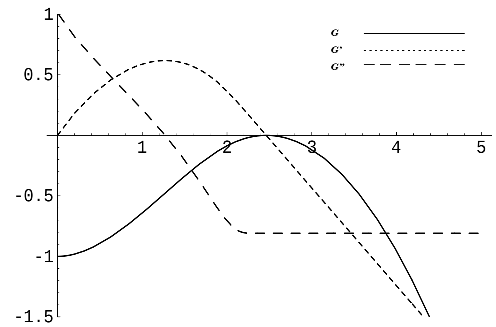

FIG.3.Plot of the function G and its first and second derivatives.We have taken $A \approx 0.808$ and $x^-_{int} \approx 2.463$

To deepen our analysis we shall compare the stress-tensor 2-point correlation $C(x_1^-, x_2^-) \equiv \langle T_{--}(x_1^-) T_{--}(x_2^-) \rangle - \langle T_{--}(x_1^-) \rangle \langle T_{--}(x_2^-) \rangle$ measured by the external observer at late times with and without backreaction. It is well known that neglecting the backreaction the correlation is thermal

$$C_{nb}(u_1, u_2) = \frac{\hbar^2 \kappa^4}{8\pi^2} \frac{e^{2\kappa|u_1 - u_2|}}{(e^{\kappa|u_1 - u_2|} - 1)^4}, \tag{18}$$

where $\kappa = \sqrt{2\Delta m / lq^3}$ is the surface gravity at the event horizon.
In general we have [12]

$$C(u_1, u_2) = \frac{\hbar^2}{8\pi^2} \frac{x^{-'}(u_1)^2 x^{-'}(u_2)^2}{(x^-(u_1) - x^-(u_2))^4}. \tag{19}$$

The expression (18) is obtained using the (no-backreaction) relation $x^- = -e^{-\kappa u}/\kappa$. With backreaction effects included the relation between x^- and u, given by (12), is crucially modified to (up to terms $O(e^{-2Cu})$)

$$x^- = x^-_{int} - \frac{2lq^3}{Au}(1 - \frac{B}{AC} \frac{e^{-Cu}}{u}), \tag{20}$$

where $C = \hbar/(24\pi lq^3)$ and $B/A = (24\pi)^2 lq^3 \Delta m / \hbar^2$. Therefore the two-point correlator at late times becomes

$$C_{wb}(u_1, u_2) = \frac{\hbar^2}{8\pi^2} \frac{1}{(u_1 - u_2)^4} - \frac{\hbar^2}{8\pi^2} \frac{\Delta(u_1, u_2)}{(u_1 - u_2)^4}, \tag{21}$$

with

$$\Delta(u_1, u_2) = \frac{2B}{A}(e^{-Cu_1} + e^{-Cu_2}) + \tag{22}$$
$$\frac{4B}{AC}(\frac{e^{-Cu_1}}{u_1} + \frac{e^{-Cu_2}}{u_2} + \frac{1}{u_1 - u_2}(\frac{u_2}{u_1} e^{-Cu_1} - \frac{u_1}{u_2} e^{-Cu_2}))].$$

In the coincidence limit $u_2 - u_1 = \epsilon \to 0$ the general expression (19) gives

$$C(u_1, u_2) \to \frac{1}{\epsilon^4}[1 - \frac{8\pi}{\hbar} \langle T_{uu} \rangle \epsilon^2]. \tag{23}$$

From eqs. (21) and (22) it is

$$C_{wb} \to \frac{1}{\epsilon^4}[1 - \frac{\epsilon^2}{3}\frac{BC^2}{A}e^{-Cu_1}] \ , \qquad (24)$$

from which one can extract the late time Hawking flux $\langle T_{uu}\rangle = \frac{\hbar}{24\pi}\frac{\Delta m}{lq^3}e^{-\frac{\hbar}{24\pi lq^3}u}$ as computed in [13].

The increase in the correlations when backreaction effects are included can be read off by considering the relative correlator

$$C_{rel} \equiv \frac{C_{wb}(u_1,u_2)}{C_{nb}(u_1,u_2)} = \frac{[1-\Delta(u_1,u_2)]}{(u_1-u_2)^4}\ \frac{(e^{\kappa|u_1-u_2|}-1)^4}{\kappa^4 e^{2\kappa|u_1-u_2|}} \ . \qquad (25)$$

C_{rel} by construction goes to 1 when $u_2 \to u_1$ and is elsewhere always bigger than 1. In particular when $\kappa|u_2-u_1| \gg 1$ it grows exponentially without bound. Therefore backreaction effects restore (fully or partially) the correlations that were lost in the (thermal) fixed background approximation.

Summarizing, we have inspected in detail the process of particle capture by an extremal Reissner-Nordström black hole and its subsequent (Hawking) decay back to extremality. The solvable model (5) has allowed us to determine the quantum corrected evaporation flux as detected by an external asymptotic observer at late times and, by analytic continuation, the quantum corrected geometry along the future Cauchy horizon. We have given arguments indicating that the quantum state of the radiation field in this region is the vacuum (in particular, no radiation is present), thus suggesting that the final state of the Hawking flux is pure (as exemplified by the significant increase of correlations in the emitted radiation). A full understanding of the problem requires to construct the quantum state capable to reproduce the late time correlator (21): the first term is reproduced by the vacuum state and the second one (with (22)) requires a more involved state [10].

To finish we would like to remark that some years ago the particle-hole scattering was widely studied for a dilaton gravity model [14,6]. This raised the hope of finding a possible resolution of the information loss paradox in a simplified context. However additional studies showed that unitarity was not preserved at the one-loop semiclassical level [15] (the emergence of strong correlations has only appeared in the subcritical regime [16] and is crucially related to the presence of negative energy radiation). It was then speculated that only higher-order corrections could restore unitarity [17,18]. We believe that we have provided evidence that, for Reissner-Nordström black holes, the effects of backreaction are stronger than for dilaton black holes, and therefore signals of unitarity already emerge in the semiclassical approximation.

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