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Relativity and constituent quark structure in model calculations of parton distributions

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Abstract

According to recent studies, Parton Distributions Functions (PDFs) and Generalized Parton Distributions (GPDs) can be evaluated in a Constituent Quark Model (CQM) scenario, considering the constituent quarks as composite objects. In here, a fully covariant model for a system of two particles, together with its non relativistic limit, are used to calculate PDFs and GPDs. The analysis permits to realize that by no means the effects of Relativity can be simulated taking into account the structure of the constituent particles, the two effects being independent and necessary for a proper description of available high energy data in terms of CQM.

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1 Introduction

Parton Distributions Functions (PDFs) and Generalized Parton Distributions (GPDs) [1, 2, 3], the latter relating PDFs and electromagnetic Form Factors (FF), encode unique information on the non perturbative hadron structure (for a recent review, see [4]). In principle, any realistic model of hadron structure should be able to estimate them. Among the non perturbative approaches, the Constituent Quark Model (CQM) has a long story of successful predictions in low energy studies of the electromagnetic structure of the nucleon, such as the calculation of FF. In the high energy sector, in order to compare model predictions of PDFs with Deep Inelastic Scattering (DIS) data, one has to evolve, according to perturbative QCD, the leading twist component of the physical structure functions obtained at the low momentum scale associated with the model. Such a procedure, already addressed in [5, 6], has proven successful in describing the gross features of standard PDFs by using different CQM (see, e.g., [7]), and it has been applied also to the calculation of the valence quark contribution to GPDs in Ref. [8]. Anyway, in order to achieve a better agreement with data, such a program has to be improved.

Two main directions have been followed in this respect by different authors. One has been to show that unpolarized and polarized DIS data are consistent with a low energy scenario dominated by composite constituent quarks of the nucleon [9]. The latter are defined through a scheme suggested by Altarelli, Cabibbo, Maiani and Petronzio (ACMP) [10], updated with modern phenomenological information. The idea of complex constituents, as old as the quark-parton model itself [11], used extensively in other frameworks [12], has been recently applied to demonstrate the evidence of complex objects inside the proton, analyzing intermediate energy data of electron scattering off the proton [13]. In Ref.[14], the idea has been successfully applied also to GPDs, in particular allowing for the evaluation of the sea quark contribution, so that GPDs can be calculated in their full range of definition. Such an achievement will permit to estimate the cross-sections which are relevant for actual GPDs measurements, providing us with an important tool for planning future experiments. In any case, one has to realize that the CQM calculations, as developed in [7], even if the structure of the constituent is taken into account as in [9], are affected by the problem of poor support, being the Bjorken variable x_{Bj} not limited between zero and one.

The other direction makes use of light-front dynamics, which allows to estimate relativistic effects in a covariant framework. Another good feature of this program is that, by construction, it is not affected by the problem of poor support. This approach is particularly useful when spin degrees of freedom are considered, and it has been indeed applied to the calculation of polarized, transversity and orbital angular momentum distributions [15]. Recently, it has been also applied to the calculation of the quark contribution to spin independent and spin dependent GPDs [16], using the overlap representation [17]. A relevant contribution to the calculation of GPDs on the light-front has been given by Tiburzi and Miller [18], and some remarks on the use of light-front for calculating GPDs can be found in [19]

A question which naturally arises is whether or not the two approaches described above, the one which takes into account the structure of the constituent quark, or the one which implements relativity in a CQM by a light front approach, are introducing in different effective ways the same physics into the problem. In other words, whether or not the structure of the constituent quark has to be implemented even in a relativistic model, such as the one obtained in a light front approach. These issues are discussed in the present paper. To do so we implement the discussion in a simple model of a bound system of two scalar particles, defined in a quantum field theoretical, explicitly covariant, framework [20]. The model, despite of its simplicity, is a rather general one and allows the exact evaluation of the PDF's and GPD's. Moreover, their Non Relativistic (NR) limit can be calculated. We will show that it is not possible to recover the shape of the initial full covariant PDFs and GPDs by implementing, in the distributions obtained as their NR limit, the structure of the constituent quark. We find therefore that, for the quite general model under scrutiny, the effect of introducing the structure of the constituents describes different physics than that described by implementing Relativity.

The paper is structured as follows. After a short definition of the main quantities of interest, the used model and its NR limit are presented in section II, together with the possible modifications due to the structure of the constituent quark; in section III, the results of the calculations are shown. Conclusions are drawn in the last section.

2 Formalism

PDFs and GPDs are the main quantities of interest in this paper. Here below they are shortly defined together with the conventions used. The GPDs are non-diagonal matrix elements of bi-local field operators. Let us consider a scalar system of mass M, with initial momentum P, final momentum P', and momentum transfer given by $\Delta = P' - P$, made of two scalar particles of mass m. If the momentum of the interacting one is labeled by p and the quantity $\bar{P} = (P + P')/2$ is used, the GPD of such a system is defined by the matrix elements of bi-local scalar field operators [1, 2, 3]:

$$\mathcal{J}^+ \equiv \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P' \right| \Phi^\dagger(0) \overleftrightarrow{\partial}^+ \Phi(z) \left| P \right\rangle \Big|_{z^+=z^\perp=0} = \mathcal{H}(x, \xi, t), \tag{1}$$

In the above equation, ξ is the so-called skewedness parameter, defined as

$$\xi = -\frac{\Delta^+}{2\bar{P}^+} , \tag{2}$$

so that

$$x + \xi = \frac{p^+}{\bar{P}^+} , \tag{3}$$

and $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$ (any four vector v^μ will be denoted $(v^+, v^\perp, v^-)$, where the light cone variables are defined by $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and the transverse part $v^\perp = (v^1, v^2)$). The standard PDF is defined as the forward limit of Eq. (1), when $t \to 0$ an $\xi \to 0$.

The elastic electromagnetic form factor of a system composed of two scalar particles is given by:

$$J^+ \equiv \langle P' | \, \Phi^\dagger (0) \, \overleftrightarrow{\partial}^+ \Phi (0) \, | P \rangle = (P + P')^+ \, F(t). \tag{4}$$

It follows directly from these definitions that integrating the GPD over x gives the form factor,

$$\int \mathcal{H}(x, \xi, t) \, dx = F(t), \tag{5}$$

where the dependence on the skewedness parameter ξ drops out. This result is an important constraint for any model calculation.

Only elastic processes will be considered, so $P^2 = P'^2 = M^2$ and $\Delta^2 = t$. The values of ξ which are possible for a given value of Δ^2 are:

$$0 \leq \xi \leq \sqrt{-\Delta^2} / \sqrt{4M^2 - \Delta^2} \, . \tag{6}$$

In Ref. [20], GPDs have been estimated by a simple model, which allows for a completely analytic solution of the Bethe-Salpeter equation. The model describes a bound state of two distinguishable equal-mass scalar particles bound together by a zero-range interaction. The Lagrangian is,

$$\mathcal{L} = [D_\mu \phi]^\dagger \, [D^\mu \phi] - m^2 \phi^\dagger \phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m^2 \chi^2 - \frac{g}{2} \left(\phi^\dagger \phi \, \chi^2 \right), \tag{7}$$

with $D_\mu = \partial_\mu + ieA_\mu$ so that the electromagnetic charge only couples to the field ϕ. Being the coupling constant g larger than a critical value, bound states are encountered. The corresponding Bethe-Salpeter equation can be trivially solved in the ladder approximation. The theory is renormalizable and a renormalization program for bound states can be defined.

The main advantage of the model defined by Eq. (7) lies in its simplicity, in the fact that one may obtain analytic solutions, avoiding approximations that might destroy physical requirements, symmetries or sum rules. These properties make it a useful playground to perform benchmark calculations, as it was used recently in order to test the viability of certain relativistic quantum mechanics approaches [21].

In this model, the GPD $\mathcal{H}$ is obtained as an integral over Bethe-Salpeter amplitudes. It reads:

$$\mathcal{H}(x, \xi, t) = x \, \frac{C^2}{i} \int \frac{d^4p}{(2\pi)^4} \frac{\delta(x + \xi - p^+ / \bar{P}^+)}{(p^2 - m^2 + i\epsilon) \, [(p + \Delta)^2 - m^2 + i\epsilon] \, [(P - p)^2 - m^2 + i\epsilon]}. \tag{8}$$

Inspecting the pole structure of the integrand for the evaluation of the p^- integral, one realizes that it vanishes unless $-\xi \leq x \leq 1$, i.e., the GPDs have the correct support properties. The integral of Eq. (8) may be calculated analytically and the explicit result is given in Ref. [20], where it is shown that it fulfills the polynomiality condition [22]. The quark distribution function is very simple and is written again here below for the reader's convenience:

$$q(x) \equiv \mathcal{H}(x, 0, 0) = \frac{C^2}{16\pi^2} \frac{x(1 - x)}{m^2 - x(1 - x) M^2}. \tag{9}$$

The normalization integral may be done analytically and determines the normalization constant C; it is clear that crucial parameters of the model are the mass M of the hadron and the mass m of its constituents.

Among the good properties of the model, use will be done here of the fact that it allows for a clear NR limit. This is done by considering a NR approximation of the energies appearing in the denominator of Eq. (8), i.e. by taking, up to order $O(\vec{p}^2/m^2)$:

$$\begin{aligned} p_0 &\simeq m + \frac{\vec{p}^2}{2m} \,, \\ (P-p)_0 &\simeq m + \frac{(\vec{P}-\vec{p})^2}{2m} \,, \\ (p+\Delta)_0 &\simeq m + \frac{(\vec{p}+\vec{\Delta})^2}{2m} \,. \end{aligned} \tag{10}$$

One should realize that the above approximations also imply, as already discussed in [14], $-t \ll m^2$, $\xi^2 \ll 1$. This means that the NR limit in the calculation of GPDs is rougher than in the calculation of standard PDFs, because it implies an additional approximation on the momentum transfer.

Neglecting some terms which are of order $O(\vec{p}^2/m^2)$, one finds:

$$\begin{aligned} \mathcal{H}(x,\xi,t) &= \frac{C^2}{(2\pi)^3}\, x\, \bar{M} \int_{p_{min}(x,t)}^{p_{max}(x,t)} dp\, p \\ &\times \int_0^{2\pi} \frac{d\phi}{2\left(m+\frac{p^2}{2m}\right)\{[\bar{M}(\bar{M}-2(m+\frac{p^2}{2m}))+\frac{t}{4}]^2-d^2\}} \,, \end{aligned} \tag{11}$$

where $\bar{M} = \sqrt{1-t/4}$, $d = \vec{\Delta}_\perp \cdot \vec{p}_\perp - 2\bar{M}\xi\tau p$, $\tau = [\bar{M}(1-x) - m - p^2/(2m)]/p$, $p_{max}(x,t) = m(1+A)$, $p_{min}(x,t) = Max\{m(-1+A), m(1-A)\}$, $A = \sqrt{2(\bar{M}/m)(1-x)-1}$.

The forward limit is again analytical, being given by:

$$q(x) \equiv \mathcal{H}(x,0,0) = \frac{C^2}{2(2\pi)^2}\frac{m^3}{M}\, x \left\{ I(\, p^2_{max}(x,0)\,) - I(\, p^2_{min}(x,0)\,) \right\} \,, \tag{12}$$

where

$$I(y) = -\frac{1}{(y-a)(a-b)} - \frac{1}{(a-b)^2}\ln\left|\frac{y-a}{y-b}\right| \,, \tag{13}$$

with $a = m(M-2m)$ and $b = -2m^2$.

In Ref. [14], a convolution formula has been derived, giving the quantity $\mathcal{H}_q$, i.e. the contribution of the quark of flavor q to the GPD $\mathcal{H}_q$, in terms of a constituent quark off-forward momentum distribution, $\mathcal{H}_{q_0}$, and of a GPD of the constituent quark q_0 itself, $\mathcal{H}_{q_0 q}$. It is assumed that the hard scattering with the virtual photon takes place on a parton of a hadron target, made of complex constituents, in an Impulse Approximation scenario. One parton (*current*) quark, belonging to a given constituent, interacts with the probe and it is afterwords reabsorbed by the same constituent, without further re-scattering with the recoiling system. Details of the approach can be found in Ref. [14, 23].

The convolution formula, valid for low values of t and ξ, can be written in the form [14]:

$$\mathcal{H}_q(x,\xi,t) = \sum_{q_0} \int_x^1 \frac{dz}{z} \mathcal{H}_{q_0}(z,\xi,t) \mathcal{H}_{q_0 q}\left(\frac{x}{z}, \frac{\xi}{z}, t\right) , \tag{14}$$

where $\mathcal{H}_{q_0}$ is the GPD to be evaluated in any CQM, such as the scalar model under scrutiny here, for the flavor q_0, while $\mathcal{H}_{q_0 q}(\frac{x}{z}, \frac{\xi}{z}, t)$ is the constituent quark GPD. One can realize that the GPD defined by Eq. (14) satisfies the polynomiality condition if both the functions $\mathcal{H}$ and $\mathcal{H}_{q_0 q}$ do it. This will be the case for the distributions used in this paper.

The constituent quark GPD $\mathcal{H}_{q_0 q}(\frac{x}{z}, \frac{\xi}{z}, t)$ has also been modelled in [14]. As usual, one can start modelling this quantity thinking first of all to its forward limit, where the constituent quark parton distributions have to be recovered. As already said in the previous section, in a series of papers a simple picture of the constituent quark as a complex system of point-like partons has been proposed [9], re-taking a scenario suggested by Altarelli, Cabibbo, Maiani and Petronzio (ACMP) [10].

According to that idea, the structure of the constituent quark is described by a set of functions $\phi_{q_0 q}(x)$ that specify the number of point-like partons of type q which are present in the constituent of type q_0, with fraction x of its total momentum. These functions will be called, generically, the structure functions of the constituent quark. They are expressed in terms of the independent $\phi_{q_0 q}(x)$ and of the constituent density distributions ($q_0 = u_0, d_0$) as,

$$q(x) = \sum_{q_0} \int_x^1 \frac{dz}{z} q_0(z) \phi_{q_0 q}\left(\frac{x}{z}\right) , \tag{15}$$

where q labels the various partons, i.e., valence quarks (u_v, d_v), sea quarks (u_s, d_s, s), sea antiquarks ($\bar{u}, \bar{d}, \bar{s}$) and gluons g. The different types and functional forms of the structure functions of the constituent quarks are derived from three very natural assumptions, i.e. the point-like partons are QCD degrees of freedom, i.e. quarks, antiquarks and gluons; Regge behavior for $x \to 0$ has to be valid; invariance under charge conjugation and isospin has to be reproduced. The last assumption of the approach relates to the scale at which the constituent quark structure is defined. We choose for it the so called hadronic scale μ_0^2 [7, 24]. This hypothesis fixes *all* but one the parameters of the approach. The only free one is fixed according to the value of F_2 at $x = 0.01$ [10], and its value is chosen again according to [24]. We stress that all these inputs are forced only by the updated phenomenology, through the 2^{nd} moments of PDFs. The values of the parameters obtained are listed in [9].

These considerations define, in the case of the valence quarks, the following structure function

$$\phi_{q_0 q_v}(x) = \frac{\Gamma(A + \frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(A)} \frac{(1-x)^{A-1}}{\sqrt{x}} . \tag{16}$$

The physical arguments leading to this expression are indeed the ones listed above. In fact, first of all, Regge theory fixes the behavior $x^{-1/2}$ for $x \to 0$. Second, the function has to be normalized to 1, because any constituent has to contain a leading valence current quark with the same quantum numbers. This latter fact fixes the constant in front of the function in the above equation. Eventually, the parameter A is fixed to the value 0.435, by imposing that the second moment at the low scale of the model is reproduced. The corresponding structure functions for the sea and gluons, not used here, can be found in [9, 14].

One should realize that the original model was thought for spin 1/2 constituent quarks, while here scalar constituents are discussed. Anyway, since only spin-independent observables will be evaluated, there is no physical reason to change the structure functions of the model.

This scenario has been generalized in Ref. [14] to describe off-forward phenomena. The main steps are reported here. First of all, the forward limit of the GPDs formula, Eq. (14), has to be given by Eq. (15). By taking the forward limit of Eq. (14), one obtains:

$$\begin{aligned} \mathcal{H}_q(x,0,0) &= \sum_{q_0} \int_x^1 \frac{dz}{z} \mathcal{H}_{q_0}(z,0,0) \mathcal{H}_{q_0 q}\left(\frac{x}{z},0,0\right) \\ &= \sum_{q_0} \int_x^1 \frac{dz}{z} q_0(z) \mathcal{H}_{q_0 q}\left(\frac{x}{z},0,0\right) , \end{aligned} \tag{17}$$

so that, in order for the latter to coincide with Eq. (15), one must have $\mathcal{H}_{q_0 q}(x,0,0) \equiv \phi_{q_0 q}(x)$. In such a way, through the ACMP prescription, the forward limit of the unknown constituent quark GPD $\mathcal{H}_{q_0 q}(\frac{x}{z},\frac{\xi}{z},t)$ can be fixed. The off-forward behavior of the Constituent Quark GPDs can be modelled in a natural way by using the "α-Double Distributions" (DD's) language proposed by Radyushkin [25]. DD's, $\Phi(\tilde{x},\alpha,t)$, are a representation of GPDs which automatically guarantees the polynomiality property.

The relation between any GPD $\mathcal{H}$, defined *à la Ji*, for example the one we need, i.e. $\mathcal{H}_{q_0 q}$ for the constituent quark target, is related to the α-DD's, which we call $\tilde{\Phi}_{q_0 q}(\tilde{x},\alpha,t)$ for the constituent quark, in the following way [25]:

$$\mathcal{H}_{q_0 q}(x,\xi,t) = \int_{-1}^{1} d\tilde{x} \int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} \delta(\tilde{x}+\xi\alpha - x) \tilde{\Phi}_{q_0 q}(\tilde{x},\alpha,t) d\alpha \ . \tag{18}$$

With some care, the expression above can be integrated over $\tilde{x}$ and the result is explicitly given in [25]. The DDs fulfill the polynomiality condition [22].

In [25], a factorized ansatz is suggested for the DD's:

$$\tilde{\Phi}_{q_0 q}(\tilde{x},\alpha,t) = h_q(\tilde{x},\alpha,t) \Phi_{q_0 q}(\tilde{x}) F_{q_0}(t) \ , \tag{19}$$

with the α dependent term, $h_q(\tilde{x},\alpha,t)$, which has the character of a mesonic amplitude. Besides, in Eq. (19) $\Phi_{q_0 q}(\tilde{x})$ represents the forward density and, eventually, $F_{q_0}(t)$ the constituent quark form factor. It can be easily verified that the GPD of the constituent quark, Eq. (18), with the factorized form Eq. (19), fulfills the crucial constraints of GPDs, i.e., the forward limit, the first-moment and the polynomiality condition, the latter being automatically verified in the DD's description. In the following the above factorized form will be assumed, so that we need to model the three functions appearing in Eq. (19).

For the amplitude h_q, use will be made of one of the simple normalized forms suggested in [25], on the bases of the symmetry properties of DD's (see [14]).

Besides, since we will identify quarks for $x \geq \xi/2$, pairs for $x \leq |\xi/2|$, antiquarks for $x \leq -\xi/2$, and, since in our approach the forward densities $\Phi_{q_0q}(\tilde{x})$ have to be given by the standard Φ functions of the $ACMP$ approach, one has, for the DD of flavor q of the constituent quark:

$$\tilde{\Phi}_{q_0q}(\tilde{x},\alpha,t) = \begin{cases} (h_q(\tilde{x},\alpha)\Phi_{q_0q_v}(\tilde{x}) + h_q(\tilde{x},\alpha)\Phi_{q_0q_s}(\tilde{x}))F_{q_0}(t) & \text{for } \tilde{x} \geq 0 \\ -h_q(-\tilde{x},\alpha)\Phi_{q_0q_s}(-\tilde{x})F_{q_0}(t) & \text{for } \tilde{x} < 0 \end{cases} \tag{20}$$

Eventually, as a f.f. we will take a monopole form corresponding to a constituent quark size $r_Q \simeq 0.3 fm$:

$$F_{q_0}(t) = \frac{1}{1 - \frac{1}{6} r_Q^2 t} \ , \tag{21}$$

a scenario strongly supported by the analysis of [13].

By using such a f.f., the amplitude h_q and the standard ACMP Φ's, in Eq. (20), and inserting the obtained $\Phi_{q_0q}(\tilde{x},\alpha,t)$ into Eq. (18), the constituent quark GPD in the ACMP scenario can be eventually calculated.

All the ingredients of the calculation have therefore been introduced. In the next section, results will be shown for the GPD $\mathcal{H}$, calculated in the scalar model according to Eq. (8) , for its NR limit, evaluated by means of Eq. (11), and for their forward limit, also considering the structure of the constituent described above.

3 Results and discussion

In this sections we show the results of our calculation in a series of figures and discuss their implication in physical terms.

In Fig. 1, the PDF obtained with the scalar model, Eq. (9), is shown together with its NR limit, Eq. (12). The mass of the constituent is taken to be $m = 0.240$ MeV, while the mass of the bound system is fixed to $M = 0.432$ MeV. This corresponds to a binding energy of 48 MeV, i.e. 20 % of the constituent mass. The system defined in this way is therefore quite relativistic. It is seen that the NR limit does not reproduce the high momentum tail of the exact distribution. Moreover, it is has poor support. This defect can be quantified by measuring the second moment, once the first has been fixed to 1. The second moment of the exact PDF gives 0.5, as it should. Instead, the NR distribution gives 0.444, so that a violation of the order of 10%, due to the poor support, is found.

Fig. 2 shows the same results but for a hadron mass of $M = 0.475$ MeV, i.e. a system with a binding energy 2 % the mass of the constituent, and weakly bound, essentially a NR one. One should recall that a system like this is still more bound than any atomic nucleus, by a factor of two, approximately. In this case the amount of support violation is only of the order of 1% since the the second moment sum rule gives 0.488. This observation supports the use of the Impulse Approximation and of NR wave functions for the estimates of the nuclear parton distributions. In any case, it is evident that also in this case the NR approximation is not able to reproduce the high momentum tail of the quark distribution, the problem being anyway less serious than for the relativistic situation shown in Fig. 1.

From Figs 1 and 2 it is evident that, in order to describe more relativistic distributions by means of the scalar model under scrutiny, it is enough to increase the binding of the system, i.e., the mass of the system has to be reduced keeping fixed the mass of the constituents.

In Fig. 3, we show the effect of considering the structure of the constituents, in the deeply bound scenario of Fig. 1. Eq. (14) has been evaluated in the forward limit, by using Eq. (16) for the structure function of the constituents, $\mathcal{H}_{q_0 q}$, and using Eq. (9) and Eq. (12) as $\mathcal{H}_{q_0}(x, 0, 0)$. The two curves are shown together with the result obtained without considering the structure of the constituents, given simply by Eqs. (9) and (12). It is seen that the effect of inserting some structure for the constituents in the NR model produces does not help to reproduce the high momentum components dropped in the while performing the NR limit. Quite on the contrary, the ACMP structure increases the number of low-x current quarks. One should also notice that, even changing the parameters of the constituent structure functions, it is not possible to simulate the relativistic result, unless the physical arguments used to build Eq. (16) are obviated. We stress that these arguments are quite general based on pQCD and Regge theory arguments. To drop them would be equivalent to consider questionable Regge theory or the capability of pQCD to predict the evolution of the second moment of the valence quark distribution. All the curves shown in Fig.3 are multiplied by x, for the sake of clarity.

Fig. 4 is just an illustration of the full procedure required to describe DIS data starting from CQM. First of all, the parton distribution is to be calculated in a (relativistic) model. Relativity is necessary especially if one wants to evaluate GPDs at large t and ξ. After that, some structure for the constituents, which fixes the scale of the model calculations (here it turns out to be $\mu_0^2 = 0.34$ GeV^2), should be considered. Finally, pQCD evolution of the model result up to the experimental scale should be performed. At this point, the model predictions can be compared with data. In Fig. 4, the valence quark distribution of the pion, extracted from data at Q^2= 4 GeV^2 [26], is compared with the result of the scalar model calculation, once the structure is taken into account and the evolution is performed. The mass of the constituent is taken to be $m = 0.240$ MeV, as always, while the mass of the hadron is chosen to be $M = 0.140$ MeV, close to the physical pion mass. Considering that the scalar model is basically used here as toy model, the exercise produces an unexpected good agreement with data. We do not claim that a system like the pion can be described by a model like the one under scrutiny. However, from the agreement found draw some conclusions: i) the model used, despite its simplicity, has many good features which makes its use to study hadron structure physically meaningful; ii) the structure of the constituent, given by Eq. (16), kept unchanged throughout our investigations

[9, 14], seems quite general and useful in varied situations; iii) if one starts from a CQM, relativity and structure of the constituents have to be considered simultaneously to be able to describe the data. To emphasize the latter point, recall that relativity strongly changes the large x region, while the structure of the constituents mainly affects the small x region. The issue of a detailed study of the pion DIS structure function, starting from a more realistic CQM, as has been done by several groups [27, 28], is beyond the scope of the present paper and will be discussed elsewhere.

Results in the non forward case, at low values of t and ξ, are shown in Figs. 5 and 6.

In Fig. 5, the GPD in the scalar model, obtained from Eq. (8), is compared with its NR limit, Eq. (11) again in the deeply bound scenario of Fig. 1. It is seen that the poor support becomes more serious with respect to the forward case.

In Fig. 6, the same is shown once the structure has been taken into account, according to Eq. (14). The same conclusions as for the forward case, i.e. the different nature of the effects due to relativity and constituent quark structure, can be drawn.

4 Conclusions

In this paper, a fully covariant model for a scalar system of two scalar particles is used as a physically meaningful toy model to calculate Parton Distributions Functions and Generalized Parton Distributions. The analysis permits to check the conclusions of recent studies, according to which parton distributions can be evaluated in a Constituent Quark Model scenario, considering the constituent quarks as composite objects, developing an idea which dates back to the seventies. The NR limits of the corresponding distributions are also evaluated. The analysis shows that the effects of Relativity cannot be simulated by the structure proposed for the constituent particles, which is based on quite general physical arguments. The two effects are found to be independent and both necessary for a proper description of available high energy data in terms of CQM.

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References

[1] D. Muller, D. Robaschik, B. Geyer, F.M. Dittes, J. Horejsi, Fortsch. Phys. 42 (1994) 101; hep-ph/9812448.

[2] A. Radyushkin, Phys. Lett. B 385 (1996) 333; Phys. Rev D 56 (1997) 5524.

[3] X. Ji, Phys. Rev. Lett. 78 (1997) 610.

[4] M. Diehl, Phys. Rept. 388, 41 (2003).

[5] G. Parisi, R. Petronzio, Phys. Lett. B 62 (1976) 331.

[6] R.L. Jaffe, G.G. Ross, Phys. Lett. B 93 (1980) 313.

[7] M. Traini, V. Vento, A. Mair and A. Zambarda, Nucl. Phys. A 614 (1997) 472.

[8] S. Scopetta and V. Vento, Eur. Phys. J. A 16 (2003) 527.

[9] S. Scopetta, V. Vento and M. Traini, Phys. Lett. B 421 (1998) 64; Phys. Lett. B 442 (1998) 28.

[10] G. Altarelli, N. Cabibbo, L. Maiani and R. Petronzio, Nucl. Phys. B 69 (1974) 531.

[11] G. Morpurgo, Physics 2 (1965) 95.

[12] R.C. Hwa, Phys. Rev. D 22, 759 (1980); R.C. Hwa and C.B. Yang, Phys. Rev. C 66, 025204 (2002).

[13] R. Petronzio, S. Simula, and G. Ricco, Phys. Rev. D 67, 094004 (2003).

[14] S. Scopetta and V. Vento, Phys. Rev. D 69, 094004 (2004).

[15] P. Faccioli, M. Traini, V. Vento, Nucl.Phys. A 656, 400 (1999); F. Cano, P. Faccioli, S. Scopetta, M. Traini, Phys.Rev. D 62, 054023 (2000); F. Cano, P. Faccioli, M. Traini, Phys.Rev. D62 (2000) 094018.

[16] S. Boffi, B. Pasquini, M. Traini Nucl. Phys.B 649, 243 (2003); Nucl. Phys. B 680, 147 (2004).

[17] By M. Diehl, T. Feldmann, R. Jakob, P. Kroll Nucl. Phys. B596 (2001) 33.

[18] B.C. Tiburzi, G.A. Miller Phys. Rev. C64 (2001) 065204; Phys.Rev.D65 (2002) 074009; Phys. Rev. D67, 113004 (2003).

[19] S. Simula, hep-ph/0406074.

[20] S. Noguera, L. Theussl, V. Vento, Eur. Phys. J. A 20 483 (2004).

[21] B. Desplanques, L. Theußl, and S. Noguera, Phys. Rev. C 65, 038202 (2002); A. Amghar, B. Desplanques, and L. Theußl, Nucl. Phys. A714, 213 (2003).

[22] X. Ji, J. Phys. G24 (1998) 1181.

[23] S. Scopetta, Phys. Rev. C 70, 015205 (2004).

[24] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C 5, 461 (1998) and references therein.

[25] A.V. Radyushkin, Phys. Rev. D 59 (1999) 014030.

[26] P.J. Sutton, A.D. Martin, R.G. Roberts, W.J. Stirling, Phys. Rev. D 45, 2349 (1992).

[27] R.M. Davidson, E. Ruiz Arriola, Phys. Lett. B348, 163 (1995).

[28] G. Altarelli, S. Petrarca and F. Rapuano, Phys. Lett. B 373, 200 (1996).

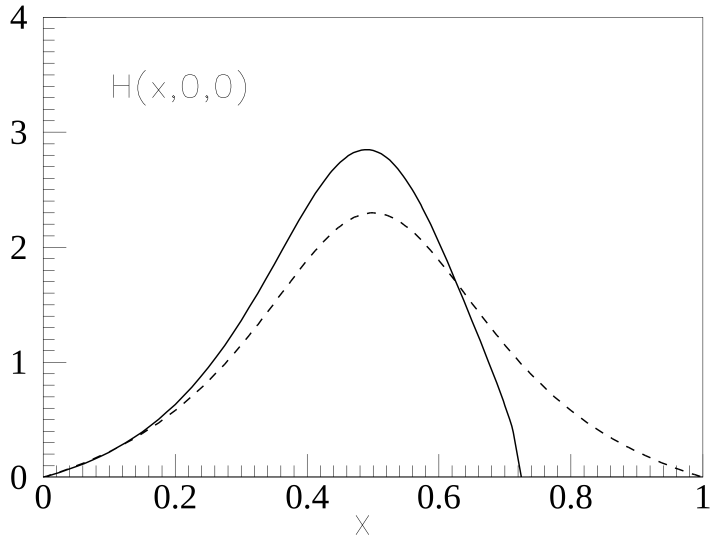

Figure 1: The PDF obtained with the scalar model, Eq. (9) (dashed), is shown together with its NR limit, Eq. (11) (full). The mass of the constituent is taken to be $m = 0.240$ MeV, while the mass of the bound system is fixed to $M = 0.432$ MeV.

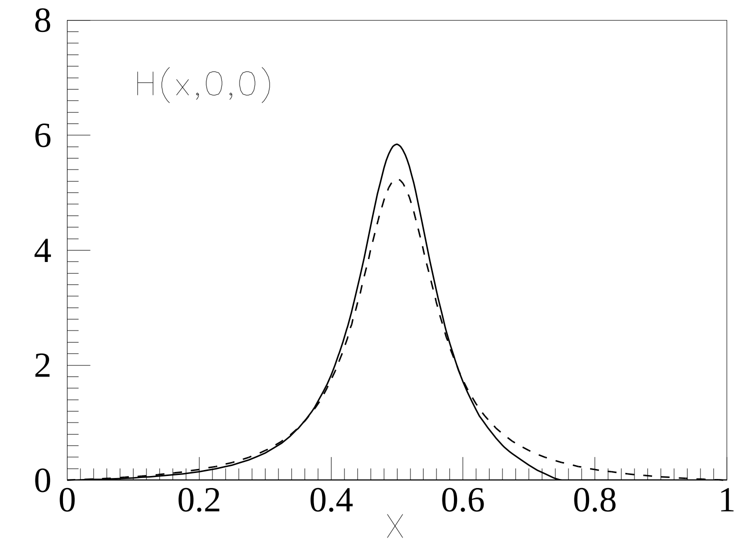

Figure 2: The same as in Fig. 1 but for constituents of mass $m = 0.240$ MeV and a bound system of mass $M = 0.475$ MeV.

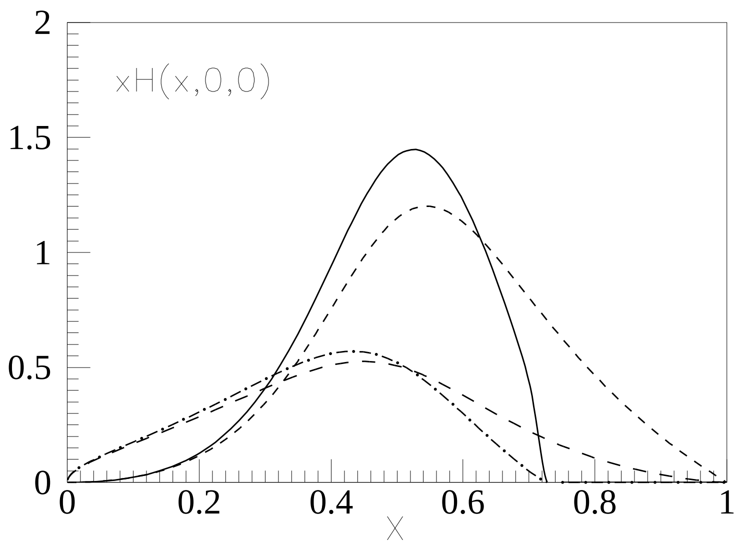

Figure 3: The long-dashed curve represents Eq. (14), evaluated in the forward limit, using Eq. (16) for the structure function of the constituent, $\mathcal{H}_{q_0 q}$, and Eq. (9) for $\mathcal{H}_{q_0}(x, 0, 0)$. The dot-dashed curve represents Eq. (14) evaluated in the forward limit, using Eq. (16) for the structure function of the constituent, $\mathcal{H}_{q_0 q}$, and Eq. (12) for $\mathcal{H}_{q_0}(x, 0, 0)$. Dashed curve is given by Eq. (9), the full curve by Eq. (12). All the Equations have been multiplied by x to give the curves which are shown. The mass of the constituent is $m = 0.240$ MeV, while the mass of the bound system is $M = 0.432$ MeV.

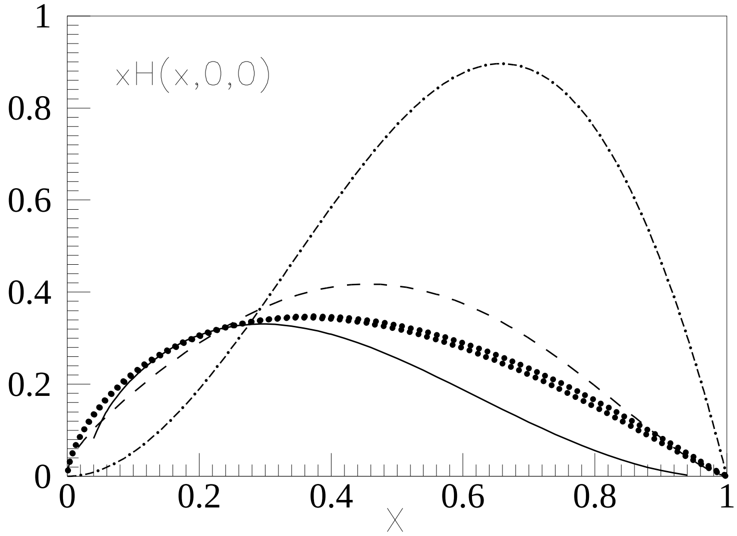

Figure 4: The dot-dashed line is the result of the scalar model, in the forward limit, Eq. (9), for $M = 0.140$ MeV and $m = 0.240$ MeV; the dashed line is obtained by considering the structure of the constituent, i.e. inserting the Eq. (16), together with Eq. (9), into Eq. (14). The full line represents the evolution, up to $Q^2 = 4$ GeV^2, of the dashed curve, the latter assumed to be valid at a scale of $\mu_0^2 = 0.34$ GeV^2. All the Equations have been multiplied by x, to give the curves which are shown. The dots represent the data for the valence quark distribution in the pion at $Q^2 = 4$ GeV^2, multiplied by x, as it is parameterized in [26], with their uncertainties.

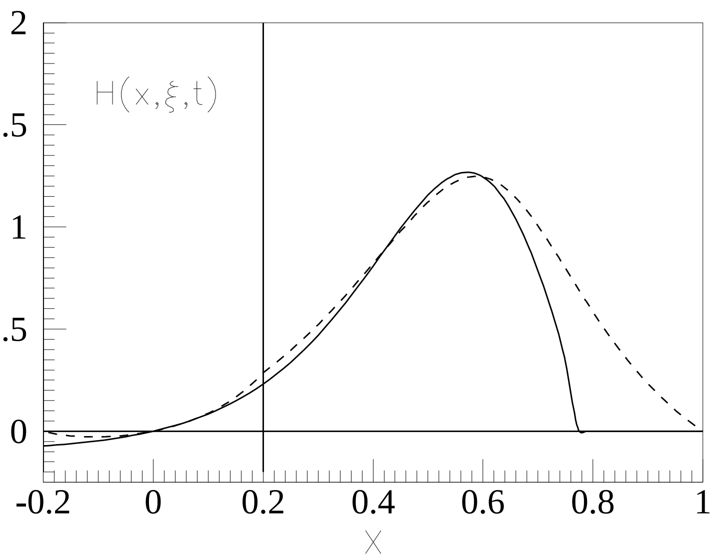

Figure 5: The GPD in the scalar model, obtained from Eq. (8) at $t = -0.3$ GeV^2 and $\xi = 0.2$ (dashed), compared with its NR limit, Eq. (11) (full). The mass of the constituent is $m = 0.240$ MeV, while the mass of the bound system is $M = 0.432$ MeV.

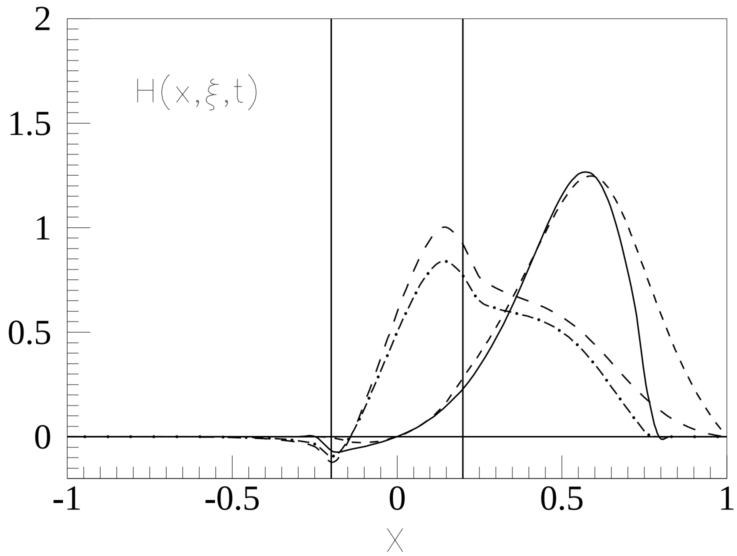

Figure 6: The GPD in the scalar model, obtained from Eq. (8) at $t = -0.3$ GeV^2 and $\xi = 0.2$ (dashed), compared with its NR limit, Eq. (11) (full). The long-dashed and dot-dashed lines are obtained from the two previous ones, respectively, taking into account the structure of the constituent quark, according to Eq. (14). The mass of the constituent is $m = 0.240$ MeV, while the mass of the bound system is $M = 0.432$ MeV.