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Quantum evolution of near-extremal Reissner-Nordström black holes

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Abstract

We study the near-horizon $\mathrm{AdS}_2 \times \mathrm{S}^2$ geometry of evaporating near-extremal Reissner-Nordström black holes interacting with null matter. The non-local (boundary) terms $t_\pm$, coming from the effective theory corrected with the quantum Polyakov-Liouville action, are treated as dynamical variables. We describe analytically the evaporation process which turns out to be compatible with the third law of thermodynamics, i.e., an infinite amount of time is required for the black hole to decay to extremality. Finally we comment briefly on the implications of our results for the information loss problem.

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1 Introduction

Since Hawking discovered that black holes decay by emission of thermal radiation [1], a lot of work has been done on quantum aspects of black holes aiming to improve our understanding of the quantization of the gravitational field. Hawking's result provided a physical meaning to the formal analogy between the classical laws of black hole dynamics and those of thermodynamics. But, in turn, it has been the origin of some intriguing puzzles. Indeed, it was Hawking himself the first who pointed out that the information can be lost as a pure quantum state collapses gravitationally into a black hole which then evaporates into a mixed state [2]. An opposite scheme has been suggested by 't Hooft [3] to maintain quantum coherence. Since then, several alternatives have been proposed (see the reviews [4]) in order to avoid the information loss although the problem is still unsolved. A final answer requires a complete and consistent theory of quantum gravity. Despite the recent achievements of String Theory to explain the Bekenstein-Hawking formula for extremal and near-extremal black holes [5, 6, 7], this theory is still far to describe the evaporation process of a black hole. However, restricting the situation to the scattering of low-energy particles with zero angular momentum off extremal black holes one can get a more tractable problem. This was the main reason to analyse dilatonic black holes [8, 9]. The problem can be reduced then to study a two-dimensional effective theory which turns out to be solvable [10, 11, 12, 13, 14, 15, 16]. However, the dilatonic black holes have very special properties which make it hard to extrapolate the physical picture of their evaporation process. Near extremality they have a constant temperature, so the decay of near-extremal black holes is governed by a Hawking radiation with a mass independent temperature. Although this makes the mathematical treatment of the non-locality of the effective action easier, the general situation is more involved. In fact, for near-extremal Reissner-Nordström (RN) black holes, the temperature depends on the mass and this makes elusive an exact analytical framework. The aim of this paper is to show that one can get exact quantum results of the dynamical evolution of near-extremal RN black holes in a region very close to the horizon.

The paper is organised as follows. In section 2 we shall show that near-extremal RN black holes, in the S-wave approximation and near the horizon, can be described by the Jackiw-Teitelboim (JT) model [17]. In section 3 we shall consider the formation of a near-extremal black hole by throwing a low-energy neutral particle into an extremal one and, in section 4, we shall describe the Hawking radiation within the near-horizon scheme. In section 5 we shall analyse the back reaction effects obtained when the classical equations are modified with terms proportional to $\hbar$ coming from the effective Liouville-Polyakov action. We find that a near-extremal black hole evaporates within a finite proper time and after the end-point, the evaporating solution exactly and smoothly matches with the extremal black hole geometry. Therefore one could expect that extremal RN black holes are really stable end-points of Hawking evaporation. However a further quantum correction changes this picture and suggests that only an exact treatment of back reaction can produce a reliable result. In section 6 we shall provide the exact results, outlined in [18], coming from equations admitting a series expansion in powers of $\hbar$. We find that the evaporation process requires an infinite amount of time, in agreement with previous results based on the adiabatic approximation [19]. Finally, in section 7, we shall state our conclusions and comment on the implications of our results for the information loss problem.

2 Reissner-Nordström black holes near extremality and Jackiw-Teitelboim theory

The RN black hole has been widely studied in literature. Recently, its interest has increased as an example of AdS_2 arising as a near-horizon geometry. In this section we shall review some of these features and show the close connection with Jackiw-Teitelboim (JT) theory. The RN metric is given by

$$d\bar{s}^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega^2 \,, \tag{2.1}$$

where

$$U(r) = \frac{(r-r_+)(r-r_-)}{r^2} = 1 - \frac{2l^2 m}{r} + \frac{l^2 q^2}{r^2} \,, \tag{2.2}$$

l^2 is Newton's constant and

$$r_\pm = l^2 m \pm l\sqrt{l^2 m^2 - q^2} \,, \tag{2.3}$$

are the two roots of $U(r)$. Taking into account (2.3), there are three cases to consider: for $lm < |q|$ there is no horizon and the singularity at $r = 0$ is naked[1]. For $lm > |q|$, $r_\pm$ are the respective inner (r_-) and outer (r_+) horizons and the thermodynamical variables, entropy and temperature, associated to the outer horizon are given by

$$S = \frac{\pi r_+^2}{l^2} \,, \qquad T_H = \frac{\kappa_+}{2\pi} \,, \tag{2.4}$$

where

$$\kappa_+ = \frac{r_+ - r_-}{2r_+^2} \,, \tag{2.5}$$

is the surface gravity on the outer (event) horizon. Finally, for the critical value of the mass $m = m_0 = l^{-1}|q|$, both inner and outer horizons merge to $r_0 = l^2 m_0 = l|q|$. This is the extremal black hole with a vanishing temperature.

[1] This case is similar to the one $m < 0$ of the Schwarzschild black hole.

Now we consider small perturbations near extremality $m = m_0 + \Delta m$, keeping the charge q unchanged. The inner and outer horizons (2.3), to leading order in $\sqrt{\Delta m}$ read

$$r_\pm = lq \pm l\sqrt{2ql\Delta m}\,, \tag{2.6}$$

where from now on we assume $q = |q|$. The extremal black hole is recovered for $\Delta m = 0$. Near extremality the entropy deviation $\Delta S = S_H - S_0$ and temperature are given by

$$\Delta S = 4\pi\sqrt{\frac{lq^3\Delta m}{2}}\,, \tag{2.7}$$

$$T_H = \frac{1}{2\pi}\sqrt{\frac{2\Delta m}{lq^3}}\,. \tag{2.8}$$

The RN metric (2.1) comes from the Einstein-Maxwell action

$$I = \frac{1}{16\pi l^2}\int d^4x\sqrt{-\bar{g}^{(4)}}(\bar{R}^{(4)} - l^2(\bar{F}^{(4)})^2)\,. \tag{2.9}$$

Assuming spherical symmetry, dimensional reduction leads to the following effective two-dimensional theory

$$I = \int d^2x\sqrt{-g}(R\phi + l^{-2}V(\phi))\,. \tag{2.10}$$

To get the above expression we have performed a conformal reparametrization of the metric

$$ds^2 = \sqrt{\phi}d\bar{s}^2\,, \tag{2.11}$$

where

$$\phi = \frac{r^2}{4l^2}\,, \tag{2.12}$$

and

$$V(\phi) = (4\phi)^{-\frac{1}{2}} - q^2(4\phi)^{-\frac{3}{2}}\,. \tag{2.13}$$

The extremal configuration is recovered for the zero of the potential $V(\phi_0) = 0$. Moreover expanding (2.10) around $\phi_0 = \frac{q^2}{4}$

$$\phi = \phi_0 + \tilde{\phi}\,, \tag{2.14}$$

$$m = m_0 + \Delta m\,, \tag{2.15}$$

we get

$$I = \int d^2x\sqrt{-g}(R\tilde{\phi} + l^{-2}\tilde{V}(\tilde{\phi})) + \mathcal{O}(\tilde{\phi}^2)\,, \tag{2.16}$$

where $\tilde{V}(\tilde{\phi})$ is given by

$$\tilde{V}(\tilde{\phi}) = V'(\phi_0)\tilde{\phi} = \frac{4}{q^3}\tilde{\phi}\,, \tag{2.17}$$

and the leading order term is just the JT theory. We can also get this result studying the behavior of the metric (2.11) near extremality. The general solution of (2.10) is [20]

$$ds^2 = -(J(\phi) - lm)dt^2 + (J(\phi) - lm)^{-1}dx^2\,, \tag{2.18}$$
$$\tilde{\phi} = \frac{x}{l}\,, \tag{2.19}$$

where $J(\phi) = \int^{\phi} d\bar{\phi} V(\bar{\phi})$. The thermodynamical variables, in terms of the two-dimensional effective theory, read [21]

$$S = 4\pi\phi_+\,, \tag{2.20}$$
$$T_H = \frac{1}{4\pi}\left|\frac{dU(r)}{dr}\right|_{r_+} = \frac{1}{2\pi}\frac{V(\phi_+)}{2l}\,, \tag{2.21}$$

where $J(\phi_\pm) - l\Delta m = 0$ and we take into account that

$$\sqrt{\phi}U(r(\phi)) = J(\phi) - lm\,. \tag{2.22}$$

Expanding (2.18), (2.19) around ϕ_0 we get[2]

$$ds^2 = -(\tilde{J}(\tilde{\phi}) - l\Delta m)dt^2 + (\tilde{J}(\tilde{\phi}) - l\Delta m)^{-1}d\tilde{x}^2 + \mathcal{O}(\tilde{\phi}^3)\,, \tag{2.23}$$
$$\tilde{\phi} = \frac{\tilde{x}}{l}\,, \tag{2.24}$$

where

$$\tilde{J}(\tilde{\phi}) = \frac{2}{q^3}\tilde{\phi}^2\,. \tag{2.25}$$

The leading order terms in the above expansion are AdS_2 geometries, which are the solutions of the JT theory. Moreover, the mass deviation Δm is just the conserved parameter of JT theory

$$l\Delta m = \tilde{J}(\tilde{\phi}) - l^2|\nabla\tilde{\phi}|^2\,. \tag{2.26}$$

Therefore the JT theory describes both extremal ($\Delta m = 0$) and near-extremal RN black holes.

Let us now see how the deviations from extremality of the thermodynamical variables are given in terms of JT magnitudes. After the near-horizon approximation (2.14), (2.15), we get

$$\Delta S = 4\pi\tilde{\phi}_+\,, \tag{2.27}$$
$$T_H = \frac{1}{2\pi}\frac{\tilde{V}(\tilde{\phi})}{2l}\,, \tag{2.28}$$

[2] A slightly different near-horizon approach, in which the expansion is taken around the outer horizon r_+, has been considered in [22, 23]

where $\tilde{\phi}_+$ is the positive root of $J(\tilde{\phi}) - l\Delta m = 0$

$$\tilde{\phi}_+ = \sqrt{\frac{ql\Delta m}{2}}\,, \tag{2.29}$$

and for the above value, (2.27), (2.28) coincide with the near-extremal values (2.7), (2.8). This is nothing but JT thermodynamics[3] All these features can be resumed in the following table

Near-extremal black hole	Jackiw-Teitelboim theory
radius deviation $r - r_0$	$\frac{2l}{q}\tilde{\phi}$
mass deviation Δm	JT mass $\tilde{m}$
inner and outer horizons	AdS_2 horizons
entropy deviation ΔS	S_{JT}
T_H	T_{JT}

To finish this section we review some aspects of the matter-coupled theory given by the action

$$\int d^2x\sqrt{-g}\left(R\tilde{\phi} + 4\lambda^2\tilde{\phi} - \frac{1}{2}|\nabla f|^2\right)\,, \tag{2.30}$$

where the field f models the matter degrees of freedom. Note that the field f propagates freely as it happens in the original 4d theory in a region very close to the horizon. In conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$, the equations of motion derived from the above action are

$$\begin{aligned}
2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} &= 0\,, & (2.31)\\
\partial_+\partial_-\tilde{\phi} + \lambda^2\tilde{\phi}e^{2\rho} &= 0\,, & (2.32)\\
\partial_+\partial_- f &= 0\,, & (2.33)\\
-2\partial_\pm^2\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi} - T^f_{\pm\pm} &= 0\,, & (2.34)
\end{aligned}$$

where $T^f_{\pm\pm} = (\partial_\pm f)^2$ is the stress tensor of matter fields. The general solution can be written in terms of four chiral functions $A_\pm(x^\pm)$, $a_\pm(x^\pm)$ [26] [27]

$$\begin{aligned}
ds^2 &= -\frac{\partial_+A_+\partial_-A_-}{(1+\frac{\lambda^2}{2}A_+A_-)^2}dx^+dx^-\,, & (2.35)\\
\tilde{\phi} &= -\frac{1}{2}\left(\frac{\partial_+a_+}{\partial_+A_+} + \frac{\partial_-a_-}{\partial_-A_-}\right) + \frac{\lambda^2}{2}\frac{A_+a_- + A_-a_+}{1+\frac{\lambda^2}{2}A_+A_-}\,, & (2.36)
\end{aligned}$$

[3] A realisation of the AdS_2/CFT_1 correspondence in the JT theory accounts for the deviation from extremality of the Bekenstein-Hawking entropy of RN black holes [24, 25].

obeying the constrain equations

$$\partial_\pm^2 \left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm} \right) - \frac{\partial_\pm^2 A_\pm}{\partial_\pm A_\pm} \, \partial_\pm \left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm} \right) = T^f_{\pm\pm} \, . \tag{2.37}$$

In the absence of matter fields, these solutions are parametrized by a diffeomorphism invariant quantity, related with the mass, which for this case reads (see (2.26))

$$\tilde{m} = \frac{2}{lq^3} \tilde{\phi}^2 - l^2 |\nabla \tilde{\phi}|^2 \, . \tag{2.38}$$

When $T^f_{--} = 0$, $\tilde{m}$ is a chiral function ($\partial_- \tilde{m}$=0) having the physical meaning of local mass [28]

$$\tilde{m}(x^+) = \tilde{m}_0 - 2l^2 \int dx^+ e^{-2\rho} \partial_- \tilde{\phi} \, T^f_{++} \, , \tag{2.39}$$

where $\tilde{m}_0$ is the primordial mass of the black hole in the absence of matter fields.

Finally, we would like to stress the fact that the four chiral functions $A_\pm(x^\pm)$, $a_\pm(x^\pm)$ define two free fields with a quadratic stress-tensor equals to the left hand side of (2.37) [26]. Moreover the transformation from the gravity variables to the free fields is a canonical transformation which makes the underlying conformal symmetry of the theory more transparent.

3 Making a near-extremal black hole

In this section we shall study the process which makes a black hole leave extremality due to infalling null neutral matter. This can be done analytically by means of a Vaidya-type metric

$$d\bar{s}^2 = - \left(1 - \frac{2l^2 m(v)}{r} + \frac{l^2 q^2}{r^2} \right) dv^2 + 2drdv + r^2 d\Omega^2 \, , \tag{3.1}$$

generated by the following stress tensor for the infalling matter

$$\bar{T}^{(4)}_{vv} = \frac{\partial_v m(v)}{4\pi r^2} \, . \tag{3.2}$$

It is possible to match an extremal black hole solution with a near-extremal one by means of a shock wave along the line $v = v_0$. The corresponding mass function is given by

$$m(v) = l^{-1} q + \Delta m \Theta(v - v_0) \, , \tag{3.3}$$

where Θ is the step function and the stress tensor is

$$\bar{T}^{(4)}_{vv} = \frac{\Delta m}{4\pi r^2} \delta(v - v_0) \, . \tag{3.4}$$

In terms of the two-dimensional effective theory (2.16), where $\lambda^2 = l^{-2}q^{-3}$, the stress tensor of matter fields read

$$T^f_{\mu\nu} = 4\pi r^2 \bar{T}^{(4)}_{\mu\nu} \,, \tag{3.5}$$

and the shock wave turns into

$$T^f_{vv} = \Delta m \delta(v - v_0) \,. \tag{3.6}$$

Now we go back to the metric (3.1) and make use of the near-horizon approximation considered in previous section. After the conformal reparametrization (2.11) and performing the expansion (2.14), (2.15), the metric (3.1) becomes

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2 q^3} - l\Delta m \Theta(v - v_0)\right) dv^2 + 2d\tilde{x}dv \,, \tag{3.7}$$

where $\tilde{x} = l\tilde{\phi}$. The Penrose diagram of this process is represented in Fig.1

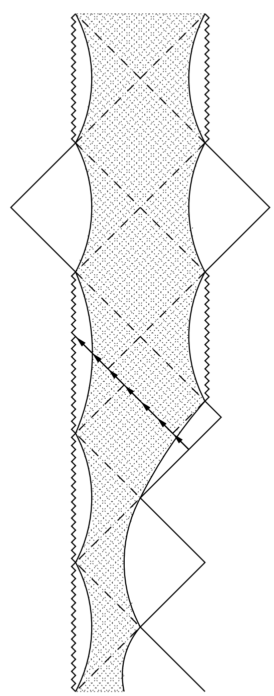

Fig.1. Penrose diagram corresponding to the creation of a near-extremal RN black hole from the extremal one. Dashed lines are the black hole horizons and the shaded strip is the near-horizon AdS_2 geometry. The arrow line represents the infalling shock wave. The shaded region between the AdS_2 boundaries is described by Kruskal coordinates in Fig.3.

In terms of the coordinates $x^{\pm}$ defined by

$$x^+ = v\,, \tag{3.8}$$

$$x^- = v + \frac{l^2 q^3}{\tilde{x}}\,, \tag{3.9}$$

for $v < v_0$, and

$$v = x_0^+ + \sqrt{\frac{2lq^3}{\Delta m}}\,\mathrm{arctanh}\sqrt{\frac{\Delta m}{2lq^3}}(x^+ - x_0^+)\,, \tag{3.10}$$

$$\tilde{x} = lq^3 \frac{1 - \frac{\Delta m}{2lq^3}(x^+ - x_0^+)(x^- - x_0^+)}{x^- - x^+}\,, \tag{3.11}$$

for $v > v_0$ (where $x_0^+ = v_0$) the metric (3.7) turns out

$$ds^2 = -\frac{2l^2 q^3 dx^+ dx^-}{(x^- - x^+)^2}\,, \tag{3.12}$$

which corresponds to the solution (2.35) of JT theory with the following gauge fixing

$$A_+ = x^+\,, \tag{3.13}$$

$$A_- = \frac{-2}{\lambda^2 x^-}\,, \tag{3.14}$$

where $\lambda^2 = l^{-2} q^{-3}$. The corresponding dilaton functions are

$$\tilde{\phi} = \frac{lq^3}{x^- - x^+}\,, \qquad v < v_0\,, \tag{3.15}$$

$$\tilde{\phi} = lq^3 \frac{1 - \frac{\Delta m}{2lq^3}(x^+ - x_0^+)(x^- - x_0^+)}{x^- - x^+}\,, \qquad v > v_0\,, \tag{3.16}$$

which are respectively recovered from (2.36) when

$$a_+ = -lq^3\,, \qquad a_- = 0\,, \tag{3.17}$$

for $v < v_0$ and

$$a_+ = -\frac{1}{2} x_0^+ \Delta m (x^+ + \Delta^+)\,, \qquad a_- = -\frac{1}{2} x_0^+ \Delta m (\frac{-2}{\lambda^2 x^-} + \Delta^-)\,, \tag{3.18}$$

for $v > v_0$, where

$$\Delta^+ = -x_0^+ + \frac{2}{l \Delta m \lambda^2 x_0^+}\,, \tag{3.19}$$

$$\Delta^- = \frac{2}{\lambda^2 x_0^+}\,. \tag{3.20}$$

We see explicitly how the JT theory describes both extremal and near-extremal geometries, being the horizon structure described by the dilatonic function $\tilde{\phi}$. It is worth to remark that the near-extremal configuration (3.16) leads to the extremal one (3.15) in the limit $\Delta m = 0$ and that both dilatonic functions match continuously along $x^+ = x_0^+$. So matching the discontinuity of $-2\partial_+^2\tilde{\phi} + 4\partial_+\rho\partial_+\tilde{\phi}$ along x_0^+, we recover the shock wave (3.6).

Finally, let us consider the description of the near-extremal black hole in the near-horizon approximation. The outer and inner horizons $r_\pm$ are given in $x^\pm$ coordinates by the curves

$$\tilde{\phi} = \tilde{\phi}_\pm = \pm\sqrt{\frac{q^3 l \Delta m}{2}}\,, \tag{3.21}$$

which is equivalent to the standard condition $\partial_+\tilde{\phi} = 0$ of two-dimensional dilaton gravity. We get

$$x^- = x_0^+ \pm a\,, \tag{3.22}$$

where

$$a = \sqrt{\frac{2lq^3}{\Delta m}}\,. \tag{3.23}$$

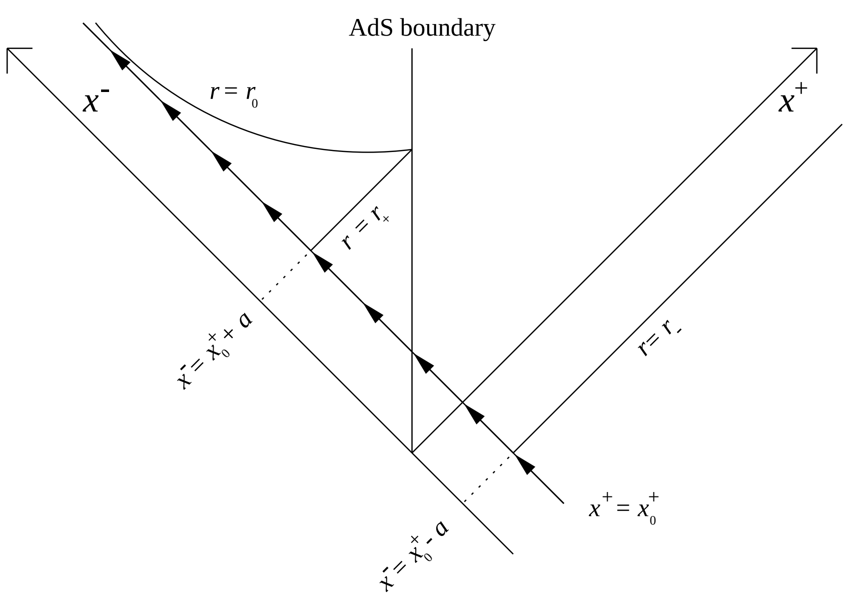

Fig.2. Kruskal diagram of near-extremal RN black hole. The two timelike boundaries of near-horizon geometry AdS_2 are represented by the vertical line $x^- = x^+$. The infalling shock wave emerges from one boundary (left side of $x^- = x^+$ line) and, crossing the outer and inner horizons, reaches the other boundary (right side of $x^- = x^+$ line).

Moreover we are also interested on the curve $r = r_0$ (the old horizon of the extremal black hole which is no longer a horizon) when the near-extremal black hole is created. In $x^{\pm}$ coordinates, this curve is given by $\tilde{\phi} = 0$ (see (2.14))

$$(x^{+} - x_0^{+})(x^{-} - x_0^{+}) = a^2 \,. \tag{3.24}$$

The corresponding Kruskal diagram is given in Fig.2. As illustrated in this figure, the two timelike boundaries of AdS_2 are representated in coordinates $x^{\pm}$ by the same curve $x^{-} = x^{+}$.

4 Hawking radiation

We shall focus now on the presence of Hawking's radiation in this model. To obtain the Hawking radiation we start writing the metric (3.1) in light-cone coordinates (v, u)

$$d\bar{s}^2 = -U(r)dudv + r^2 d\Omega^2 \,, \tag{4.1}$$

where $U(r)$ is given by (2.2) and

$$\frac{v+u}{2} = t \,, \tag{4.2}$$

$$\frac{v-u}{2} = r^* \,, \tag{4.3}$$

where r^* is the tortoise coordinate

$$r^* = \int \frac{dr}{U(r)} = r + \frac{1}{2\kappa_+} \ln|r - r_+| - \frac{1}{2\kappa_-} \ln|r - r_-| \,. \tag{4.4}$$

After the conformal reparametrization (2.11) we get

$$ds^2 = -(J(\phi) - lm)dudv \,, \tag{4.5}$$

where now

$$u = v - 2l \int \frac{d\phi}{J(\phi) - lm} \,. \tag{4.6}$$

Performing the 'near-horizon' approximation (2.14), (2.15), the above expression turns out

$$u = v - 2 \int \frac{d\tilde{x}}{2\lambda^2 \tilde{x}^2 - l\Delta m \Theta(v - v_0)} \,. \tag{4.7}$$

The extremal black hole before the shock wave ($v < v_0$) is given by

$$ds^2 = -2\lambda^2 \tilde{x}^2 dudv \,, \tag{4.8}$$

and the near-extremal black hole created after the shock wave ($v > v_0$)

$$ds^2 = -(2\lambda^2 \tilde{x}^2 - l\Delta m) d\bar{u}dv \,. \tag{4.9}$$

Integrating (4.7) for both cases we get the following relations for the coordinate u before and after the shock wave

$$u = v + \frac{1}{\lambda^2 \tilde{x}}\,, \tag{4.10}$$

$$\bar{u} = v + \sqrt{\frac{2}{l\Delta m \lambda^2}}\,\text{arctanh}\sqrt{\frac{2\lambda^2}{l\Delta m}}\tilde{x}\,. \tag{4.11}$$

Imposing the continuity of solutions (4.8) and (4.9) along $v = v_0$ (and taking also into account $\tilde{x}(v_0, u) = \tilde{x}(v_0, \bar{u})$) we obtain the relation between the coordinate u before and after the shock wave

$$u = v_0 + a\,\text{cotanh}\frac{\bar{u} - v_0}{a}\,. \tag{4.12}$$

From this relation we can work out immediately the outgoing energy flux measured by an observer near the horizon in terms of the Schwarzian derivative between the coordinates u and $\bar{u}$

$$\langle T^f_{\bar{u}\bar{u}} \rangle = -\frac{\hbar}{24\pi}\{u, \bar{u}\} = \frac{\hbar}{12\pi a^2}\,. \tag{4.13}$$

We observe that this flux is constant and coincides with thermal value of Hawking flux for near-extremal RN black holes

$$\langle T^f_{\bar{u}\bar{u}} \rangle = \frac{\pi}{12} T_H^2 = \frac{\hbar \Delta m}{24\pi l q^3}\,, \tag{4.14}$$

where T_H is Hawking's temperature (2.8). This fact can be understood easily since the $\text{AdS}_2 \times \text{S}^2$ geometries associated to (4.8) and (4.9) represent indeed the near-horizon geometry of the RN geometries (3.1) due to the shock wave (3.6). So the constant thermal flux for every value of $\bar{u}$ corresponds to the flux measured by an inertial observer at future null infinity approaching the event horizon of the RN black hole.

5 Back reaction to leading order in $\hbar$

In this section we shall start our analysis of the back reaction of near-extremal black holes in the near-horizon effective theory. The starting point must be the matter-coupled classical action (2.16) corrected with the Polyakov-Liouville term [29]

$$\begin{aligned} \tilde{I}_{\text{eff}} &= \int d^2x\sqrt{-g}\left(R\tilde{\phi} + 4\lambda^2\tilde{\phi} - \frac{1}{2}\sum_{i=1}^{N}|\nabla f_i|^2\right) - \frac{N\hbar}{96\pi}\int\sqrt{-g}R\,\square^{-1}R \\ &+ \xi\frac{N\hbar}{12\pi}\int d^2x\sqrt{-g}\lambda^2\,, \end{aligned} \tag{5.1}$$

where we have considered the presence of N scalar fields f_i in order to define the semi-classical theory in the large N limit. Nevertheless we have to point out the important fact that, for the JT theory, the exact quantum effective action is locally equivalent to the one-loop corrected theory [30] and, therefore, we could maintain $N = 1$.

The Polyakov-Liouville action in (5.1) has a cosmological term [29] and we shall fix it in such a way that the extremal black hole solution remains a solution of the quantum theory. This is, in some sense, analogous to the manner in which the local counterterm of the RST model [10, 11] is introduced. This argument implies that $\xi = 1$. Therefore the unconstrained equations remain as the classical ones

$$2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} = 0\,, \tag{5.2}$$

$$\partial_+\partial_-\tilde{\phi} + \lambda^2\tilde{\phi}e^{2\rho} = 0\,, \tag{5.3}$$

$$\partial_+\partial_- f_i = 0\,, \tag{5.4}$$

so the solutions (2.35), (2.36) are not modified. In contrast, the constrain equations (2.34) get modified according to

$$-2\partial^2_\pm\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi} = T^f_{\pm\pm} - \frac{N\hbar}{12\pi}\left((\partial_\pm\rho)^2 - \partial^2_\pm\rho + t_\pm\right)\,, \tag{5.5}$$

where the functions $t_\pm(x^\pm)$ are related with the boundary conditions of the theory and depend on the quantum state of the system. They transform according the Schwarzian derivative to make covariant the equation (5.5). In terms of the four chiral functions appearing in (2.35), (2.36), the above constraints read

$$\partial^2_\pm\left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm}\right) - \frac{\partial^2_\pm A_\pm}{\partial_\pm A_\pm}\,\partial_\pm\left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm}\right) = T^f_{\pm\pm} + \frac{N\hbar}{12\pi}\left(\frac{1}{2}\{A_\pm, x^\pm\} - t_\pm\right)\,. \tag{5.6}$$

The crucial point to go on the analysis is to choose the adequate functions $t_\pm(x^\pm)$. For the CGHS theory [10, 11, 12] the correct choice is $t_\pm(x^\pm) = \frac{1}{4(x^\pm)^2}$, where $x^\pm$ are Kruskal coordinates, since this corresponds to vanishing $t_\pm(\sigma^\pm)$ in null Minkowskian coordinates $\sigma^\pm$. In our case we should choose $t_v(v) = t_u(u) = 0$ and, according to (3.8) and (3.9), we have[4]

$$t_+(x^+) = \frac{a^2\Theta(x^+ - x^+_0)}{(a^2 - (x^+ - x^+_0)^2)^2}\,. \tag{5.7}$$

Moreover, the coordinate x^- always coincides with the null coordinate u as a consequence of the matching condition for the metric and dilaton with the gauge fixing conditions (3.13) and (3.14).

$$t_-(x^-) = 0\,. \tag{5.8}$$

With the above choice, the constraints (5.6) for the evaporating solution ($x^+ > x^+_0$) become

$$\partial^3_+ a_+ = -\frac{N\hbar}{12\pi}\frac{a^2}{(a^2 - (x^+ - x^+_0)^2)^2}\,, \tag{5.9}$$

$$3\partial_- a_- + 4x^-\partial^2_- a_- + (x^-)^2\partial^3_- a_- = 0\,, \tag{5.10}$$

[4]This type of behaviour for the functions $t_\pm$ was pointed out in [31].

equations which can easily integrated to get the following solutions

$$a_+ = C(x^+ + \Delta^+) + \zeta_+(x^+)^2 - \frac{N\hbar}{\pi}P(x^+)\,, \tag{5.11}$$

$$a_- = C(\frac{-2}{\lambda^2 x^-} + \Delta^-) + \zeta_-(x^-)^{-2}\,, \tag{5.12}$$

where C, $\zeta_\pm$ and $\Delta^\pm$ are integration constants and the function $P(x^+)$ is

$$P(x^+) = \frac{x^+ - x_0^+}{48} - \frac{a^2 - (x^+ - x_0^+)^2}{48a}\mathrm{arctanh}\frac{x^+ - x_0^+}{a}\,. \tag{5.13}$$

It is interesting to point out that the above function vanishes at x_0^+ and the quantum solution is the classical one (3.18) plus the correction introduced through $P(x^+)$. This is so for $C = -\frac{1}{2}x_0^+\Delta m$, $\zeta_\pm = 0$ and $\Delta^\pm$ given by (3.19) and (3.20). The dilaton function for $x^+ > x_0^+$ is then

$$\tilde{\phi} = lq^3 \frac{1 - \frac{(x^+ - x_0^+)(x^- - x_0^+)}{a^2}}{x^- - x^+} + \frac{N\hbar}{2\pi}\frac{(x^- - x^+)P'(x^+) + 2P(x^+)}{x^- - x^+}\,, \tag{5.14}$$

which matches along $x^+ = x_0^+$ with the extremal one, which continues being solution at the semiclassical level

$$\tilde{\phi} = \frac{lq^3}{x^- - x^+}\,. \tag{5.15}$$

A remarkable property of these solutions is that the dynamical evolution can be followed analytically. As before, the curve $\tilde{\phi} = 0$ represents the location of the 'extremal radius'

$$x^- = \frac{lq^3 + \frac{lq^3 x_0^+}{a^2}(x^+ - x_0^+) - \frac{N\hbar}{2\pi}x^+P' + \frac{N\hbar}{\pi}P}{\frac{lq^3}{a^2}(x^+ - x_0^+) - \frac{N\hbar}{2\pi}P'}\,. \tag{5.16}$$

And for the apparent horizon $\partial_+\tilde{\phi} = 0$ in the spacetime of the evaporating black hole, we get the following equation

$$lq^3\left(1 - \frac{(x^- - x_0^+)^2}{a^2}\right) + \frac{N\hbar}{2\pi}\left(\frac{1}{2}(x^- - x^+)^2P'' + (x^- - x^+)P' + P\right) = 0\,. \tag{5.17}$$

The main property of both above curves is that, unlike the classical case and unexpectedly, they intersect before reaching the AdS boundary at a finite advanced time $x^+ = x_{\rm int}^+$ given implicitly by the below relation obtained by substituting (5.16) in (5.17)

$$\mathrm{arctanh}\frac{x_{\rm int}^+ - x_0^+}{a} = \frac{48lq^3}{Na} - \frac{x_{\rm int}^+ - x_0^+}{\sqrt{a^2 - (x_{\rm int}^+ - x_0^+)^2}}\,. \tag{5.18}$$

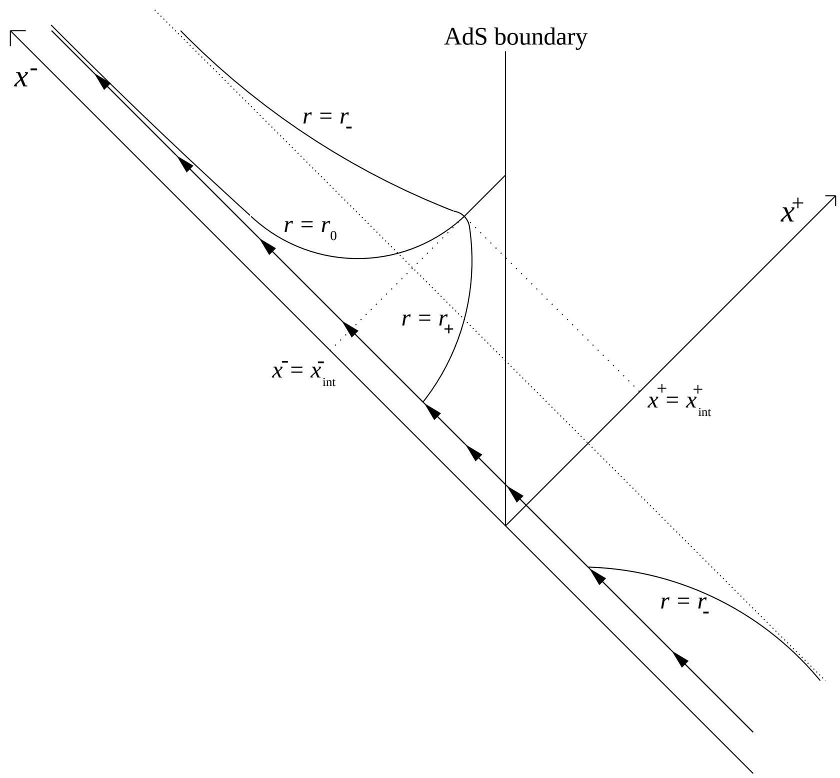

Fig.3. Kruskal diagram of semiclassical evolution of RN black hole. The outer apparent horizon shrinks until meets both inner horizon and extremal radius curve at a finite advanced time. This is the end-point of evaporation and a static solution can be matched so that the extremal radius curve becomes the horizon of extremal black hole remnant.

A graphic description of this process is represented in Fig.3. At this point the extremal radius curve $\tilde{\phi} = 0$ is null and both outer and inner horizons meet. This means that we have arrived at the end-point of the evaporation and, since we are dealing with analytic expressions, one can check explicitly that the evaporating solution (5.14) matches smoothly along $x^+ = x^+_{\rm int}$ with a static solution for $x^+ > x^+_{\rm int}$

$$\tilde{\phi} = -k\frac{(x^+ - x^-_{\rm int})(x^- - x^-_{\rm int})}{x^- - x^+}\,, \tag{5.19}$$

where

$$k = \frac{\frac{lq^3}{a^2}(x^+_{\rm int} - x^+_0) - \frac{N\hbar}{2\pi}P'(x^+_{\rm int})}{x^+_{\rm int} - x^-_{\rm int}}\,, \tag{5.20}$$

which turns out to be just the extremal black hole. A conformal coordinate transformation brings the metric and $\tilde{\phi}$ into the form (4.8) of the extremal solution

$$v = \frac{1}{\lambda^2 k(x^+ - x^-_{\rm int})}\,, \tag{5.21}$$

$$u \;=\; \frac{1}{\lambda^2 k(x^- - x^-_{\rm int})}\,. \qquad (5.22)$$

We must note that, in contrast with the studies of dilatonic black holes [10, 11, 12], the matching of the evaporating solution here is along the line $x^+ = x^+_{\rm int}$ and not $x^- = x^-_{\rm int}$. There is a physical reason for this, the line $x^+ = x^+_{\rm int}$ in our effective theory represents, as we have stressed before, points very near to the apparent horizon of the evaporating RN black hole. Moreover, the matching is smooth and T^f_{++} vanishes at $x^+_{\rm int}$ in contrast with the dilatonic black holes, for which there is an emanating thunderpop of negative energy [10, 11].

Concerning the mass curve for the evaporating solution (5.14) in $x^+_0 < x^+ < x^+_{\rm int}$, it corresponds to a chiral energy distribution which accounts for the neutral mass lost by the near-extremal black hole during its evaporation. Taking into account (2.38) we get

$$\begin{aligned}\tilde{m}(x^+) \;&=\; \Delta m - \frac{N\hbar}{12\pi a}{\rm arctanh}\frac{x^+ - x^+_0}{a} - \\ &\frac{N^2\hbar^2}{1152\pi^2 l q^3}\left[\frac{(x^+ - x^+_0)^2}{a^2 - (x^+ - x^+_0)^2} - {\rm arctanh}^2\frac{x^+ - x^+_0}{a}\right]\,, \qquad (5.23)\end{aligned}$$

that just vanishes at the end-point $x^+ = x^+_{\rm int}$ as it is showed in Fig.4.

To end this section we would like to comment on the boundary condition (5.7). It was derived through the Schwarzian derivative of the classical relations (3.8)), (3.9). This should be considered as a first approximation since the evaporating solution modifies the classical relation (3.9). Plugging (5.23) and (5.14) into (3.7) we get the following relation to leading order in $\hbar$

$$v = x^+_0 + a\,{\rm arctanh}\frac{x^+ - x^+_0}{a} - \frac{a^2 N\hbar}{96 l q^3}\left(\frac{a^2}{a^2 - (x^+ - x^+_0)^2} - {\rm arctanh}^2\frac{x^+ - x^+_0}{a}\right)\,. \qquad (5.24)$$

As a consequence of this, the function $t_+(x^+)$ get modified

$$t_{x^+} = \frac{a^2}{(a^2 - (x^+ - x^+_0)^2)^2} - \frac{a^4 N\hbar}{24 l q^3}\frac{x^+ - x^+_0}{(a^2 - (x^+ - x^+_0)^2)^3}\,. \qquad (5.25)$$

We should remark that this quantum correction for the function $t_+(x^+)$ does not happen in the CGHS model, due to the fact that the temperature is independent of the mass. In our case the back reaction is more involved and produces this type of effect. Solving the equations (5.2-5.5) again in terms of the quantum corrected function $t_+(x^+)$ ($t_-(x^-)$ remains zero) and repeating the process described in this section we arrive to a new dynamical function $\tilde{\phi}$ and evaporating mass $\tilde{m}(x^+)$ describing the evaporation. But what one surprisingly finds is an unphysical evolution with periods of antievaporation. This suggests that only an exact treatment of the boundary functions $t_+(x^+)$ can produce a correct result. This will be considered in the next section.

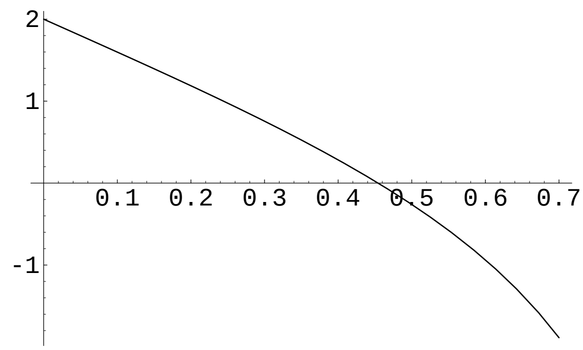

Fig.4. Dynamical evolution of the mass deviation from extremality of the evaporating near-extremal black hole. We set $x_0^+ = 0$, $\Delta m = 2$, $l = q = 1$ and $N = 48$ in (5.23). It vanishes at $x^+_{\rm int}$ indicating the end-point of the evaporation. Even though the evaporation could seem to continue for negative values of the mass, the static extremal solution can be matched at this point.

6 Back reaction for dynamical $t_\pm$ functions

We shall now modify the scheme used in the previous section to work out the gravitational back reaction. The crucial point is to consider the functions $t_\pm(x^\pm)$, appearing in the equations (5.5), as dynamical objects to be determined at the end together with the solutions for the dilaton and the metric. We should do it requiring the physical consistence of the procedure. To start with, it is clear that $t_-(x^-)$ does not get any quantum correction since the coordinate x^- remains equal to the u coordinate of the extremal solution. Therefore we have

$$t_-(x^-) = 0\,. \tag{6.1}$$

The advantage of (6.1) is that, irrespective to the form of $t_+(x^+)$, the general solutions for the metric (even in the presence of arbitrary incoming classical matter T^f_{++}) is given by [32]

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2q^3} - l\tilde{m}(v)\right)dv^2 + 2d\tilde{x}dv\,, \tag{6.2}$$

where $\tilde{m}(v) = m - m_0$ is the deviation of evaporating mass from extremality. This metric can be brought into the gauge-fixed form (3.12) by means of the transformation

$$v = v(x^+)\,, \tag{6.3}$$

$$\tilde{x} = l\tilde{\phi}(x^\pm)\,, \tag{6.4}$$

where

$$\frac{dv}{dx^+} = \frac{-lq^3}{\partial_-\tilde{\phi}(x^- - x^+)^2}\,. \tag{6.5}$$

This requires that $\partial_- \tilde{\phi}(x^- - x^+)^2$ is a chiral function of x^+ and this follows from the constrain equation

$$-2\partial_-^2 \tilde{\phi} + 4\partial_- \rho \partial_- \tilde{\phi} = 0\,. \tag{6.6}$$

Since in the advanced time coordinate v the function $t_v(v)$ vanishes, in the coordinate x^+ it is

$$t_+(x^+) = \frac{1}{2}\{v, x^+\}\,. \tag{6.7}$$

Now we find convenient to introduce the function $F(x^+)$ such that

$$\frac{dv}{dx^+} = \frac{lq^3}{F(x^+)}\,. \tag{6.8}$$

In gauge (3.13), (3.14), the equations (5.2-5.5) can be integrated leading to

$$\tilde{\phi} = \frac{F(x^+)}{x^- - x^+} + \frac{1}{2}F'(x^+)\,, \tag{6.9}$$

where the function $F(x^+)$ satisfies the following differential equation

$$F''' = \frac{N\hbar}{24\pi}\left(-\frac{F''}{F} + \frac{1}{2}\left(\frac{F'}{F}\right)^2\right)\,. \tag{6.10}$$

Physical considerations concerning the matching along $x^+ = x_0^+$ between the extremal and the evaporating solutions provide the boundary conditions for the above differential equation. Namely, from the continuity of the function $\tilde{\phi}$ (6.9) along x_0^+ we get

$$F(x_0^+) = lq^3\,, \qquad F'(x_0^+) = 0\,, \tag{6.11}$$

whereas, from the discontinuity of the shock-wave stress tensor $T^f_{++} = \Delta m \delta(x^+ - x_0^+)$, we obtain

$$F''(x_0^+) = -\Delta m\,. \tag{6.12}$$

It is not difficult to relate the deviation of the evaporating mass $\tilde{m}(x^+)$ with the function $F(x^+)$

$$\tilde{m}(x^+) = \frac{24\pi}{lq^3 N\hbar}F^2 F'''\,. \tag{6.13}$$

Moreover the boundary function $t_+(x^+)$ (6.7) can be written as

$$t_+(x^+) = \frac{lq^3 \tilde{m}(x^+)}{2F^2}\,, \tag{6.14}$$

which makes clear the dynamical content of the function $t_+(x^+)$. To compute the evaporating mass $\tilde{m}(x^+)$ we need to solve equation (6.10) but, fortunately, in terms of the advanced time coordinate the problem is simpler. Derivating (6.13) with respect to x^+, one can readily arrive to

$$\partial_+ \tilde{m}(x^+) = -\frac{N\hbar}{24\pi l q^3}\frac{\tilde{m}(x^+)}{F}\,, \tag{6.15}$$

which can be readily integrated in coordinate v

$$\tilde{m}(v) = \Delta m e^{-\frac{N\hbar}{24\pi l q^3}(v-v_0)}\,, \tag{6.16}$$

leading to an infinite evaporation time.

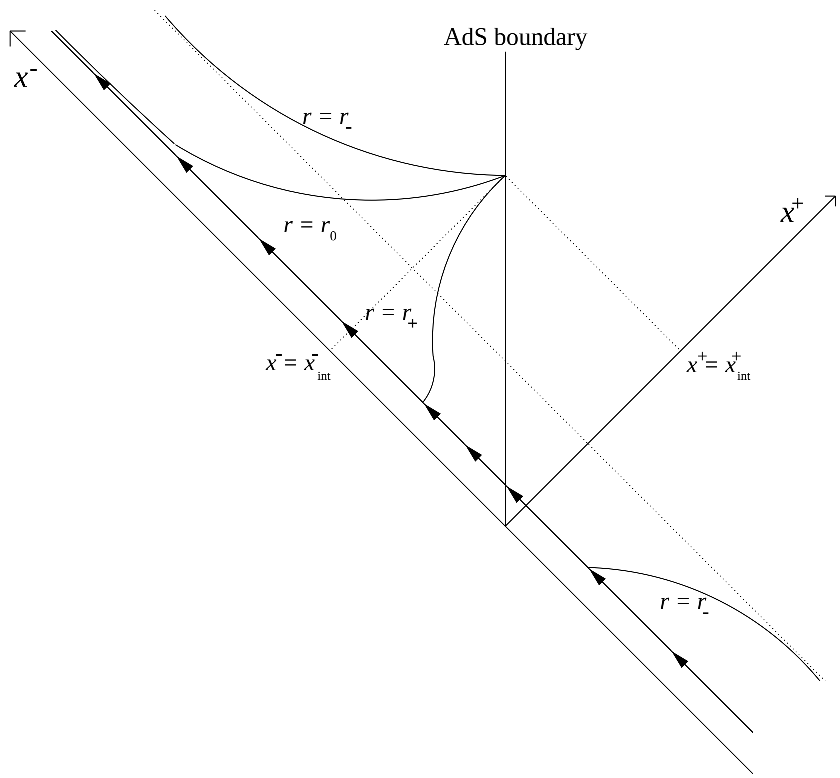

Fig.5. Kruskal diagram of semiclassical evolution of RN black hole. The outer apparent horizon shrinks until it meets both inner horizon and extremal radius curve at the AdS boundary.

Now let us go back to the $x^{\pm}$ frame. As seen in last section, the physical information about the dynamical evolution of the near-extremal black hole is encoded in the function (6.9). The evolution of the 'extremal radius' is represented by the curve $\tilde{\phi} = 0$

$$x^- = x^+ - \frac{2F}{F'}\,, \tag{6.17}$$

and the curve $\partial_+\tilde{\phi} = 0$ accounts for the inner and outer apparent horizons

$$x^- = x^+ - \frac{F'}{F''} \pm \frac{\sqrt{F'^2 - 2FF''}}{F''}\,. \tag{6.18}$$

These three curves meet only when

$$F'(x^+_{int})^2 - 2F(x^+_{int})F''(x^+_{int}) = 2lq^3 m(x^+_{int}) = 0\,, \tag{6.19}$$

and it takes place at a finite value x^+_{int} at the end of the evaporation. Since $v \to +\infty$ as $x^+ \to x^+_{int}$, taking into account (6.8) we also get that $F(x^+_{int}) = 0$. And from this feature it follows immediately, see (6.17) and (6.18), that $x^+_{int} = x^-_{int}$ and then the intersection point belongs to the AdS boundary so that it gives an infinite amount of proper time in accordance with (6.16). We can also show that $F'''(x^+_{int}) = F'(x^+_{int}) = 0$ and since for all of these three curves (6.17), (6.18), we have

$$\frac{dx^-}{dx^+} \stackrel{x^+ \to x^+_{int}}{\longrightarrow} 0\,, \tag{6.20}$$

one can conclude that the three curves meet at the end-point becoming a null line. The complete physical process is represented in Fig.5.

To finish this analysis we consider the numerical solution to the differential equation (6.10). From the numerical plot of the function $F(x^+)$ and its derivatives (see Fig.6a), apart from the properties found before ($F(x^+_{int}) = F'(x^+_{int}) = F'''(x^+_{int}) = 0$), it also follows that that $F''(x^+_{int})$ is nonzero while further derivatives vanish. Thus locally close to the intersection point $F(x^+)$ behaves as a parabola with exponentially suppressed corrections. The numerical plot of $\tilde{\phi} = 0$ and $\partial_+\tilde{\phi} = 0$ coincides exactly with that of Fig.5. The saddle point in the outer apparent horizon curve r_+ signals the transition from the strong to the weak back-reaction regimes as discussed in [33]. At the end-point the curves $\tilde{\phi} = 0$ and $\partial_+\tilde{\phi} = 0$ are null and the dilaton function is well represented asymptotically by the extremal solution

$$\tilde{\phi} = \frac{F''(x^+_{int})}{2}\frac{(x^+ - x^+_{\rm int})(x^- - x^+_{\rm int})}{x^- - x^+} + \mathcal{O}(e)\,, \tag{6.21}$$

where $\mathcal{O}(e)$ are exponentially small high order terms. So the evaporating black hole approaches asymptotically to the extremal configuration without actually coming back to the extremal state in a finite continuous process. This result appears to be well motivated from a thermodynamical point of view, in particular from Stefan's law $\frac{dm}{dt} \sim -4\pi r_+^2 T_H^4$ which predicts that a near-extremal evaporating black hole does never come back to the extremal state [34]. This feature is closely related to Nernst's version of the third law of thermodynamics which states that the temperature of a system cannot be reduced to zero in a finite number of operations. Israel [35] showed that the same conclusions must be true in the case of the RN black hole provided that the stress energy tensor of the infalling matter satisfies the weak energy condition (WEC). It is well known that in the Hawking process the WEC is violated close to the horizon; nevertheless our exact results do not violate the third law. We think that this is an encouraging sign towards generalising its validity to more general (quantum) frameworks.

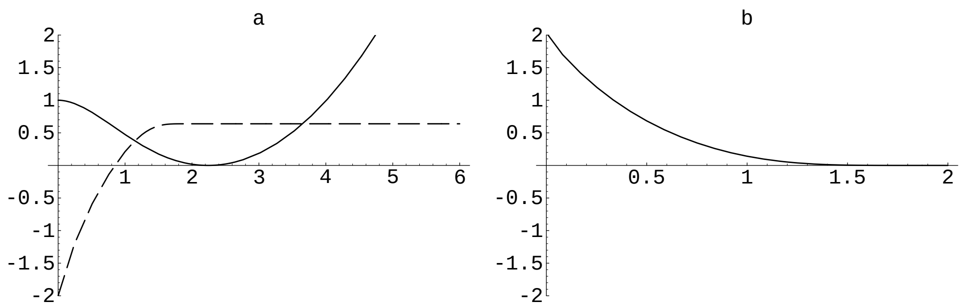

Fig.6. Numerical plots for some solutions to the differential equation (6.10), $x_0^+ = 0$, $\Delta m = 2$, $l = q = 1$ and $N = 48$. (a) Plot of the function $F(x^+)$ (continuous line) and its second derivative (dashed line). The zero of F corresponds to the value x_{int}^+ while the second derivative is non-zero at this point. (b) Plot of the dynamical evolution of the mass deviation. It approaches smoothly to zero as it reaches x_{int}^+.

7 Conclusions and final comments

In this paper we have studied an effective model which describes the physics of near-extremal RN black holes in a region very close to the horizon. We have focused our analysis on the evaporation process produced when a low-energy shock wave excites the extremal black hole and the non-extremal configuration decays via Hawking emission. We have shown that a self-consistent treatment of the gravitational back reaction requires to consider the non-local contributions $t_+(x^+)$ of the effective action as dynamical variables. So doing this we find a physical picture of the evaporation process compatible with the third law of thermodynamics. An infinite amount of time is necessary for the black hole to go back to extremality and this 'happens' in the AdS_2 boundary of the near horizon geometry. We have to remark that we have obtained a rather accurate description near the horizon. For the in-falling observer the function $t_+(x^+)$ (6.14) is proportional to the flux of negative energy across the horizon and, therefore, responsible of the black hole radiation. However our scheme do not describe directly the effects of the back reaction on the radiation measured by an asymptotic observer (not very close to the horizon). This makes our results compatible with the principle of complementarity [3, 36, 37].

Finally we would like to comment on the implications of considering a continuous distribution of incoming matter T^f_{vv}. The equation (6.15) in terms of the v coordinate is then

$$\partial_v \tilde{m}(v) = -\frac{N\hbar}{24\pi l q^3}\tilde{m}(v) + T^f_{vv}\,. \tag{7.1}$$

We observe that the evaporating mass $\tilde{m}(v)$, and hence the function $t_+(x^+)$, contains detailed information of the classical matter. In other words, the information of the stress tensor T^f_{vv} is also codified in the quantum incoming flux[5]

$$-\frac{N\hbar}{12\pi}t_+(x^+)\,. \tag{7.2}$$

Note that this is true because all the higher-order quantum corrections to $t_+(x^+)$ have been taken into account, otherwise we could not get (7.1). In the approximation of section 5 the information is lost. The functions $t_+(x^+)$, at leading order, does not see the details of T^f_{vv}. Moreover the full solution, in contrast with the models [10, 11, 12], seems to depend on all higher-order momenta of the classical stress tensor. All this means that the information might not be lost. However to get definite conclusions we should be able to describe the outgoing radiation at infinity and this is out of the reach of the near-horizon scheme of this paper. Nevertheless energy conservation suggests that an analogous mechanism to that storing the information in $t_+(x^+)$ across the horizon should radiate the information out to infinity.

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