Low-energy scattering of extremal black holes by neutral matter

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Abstract

We investigate the decay of a spherically symmetric near-extremal charged black hole, including back-reaction effects, in the near-horizon region. The non-locality of the effective action controlling this process allows and also forces us to introduce a complementary set of boundary conditions which permit to determine the asymptotic late time Hawking flux. The evaporation rate goes down exponentially and admits an infinite series expansion in Planck's constant. At leading order it is proportional to the total mass and the higher order terms involve higher order momenta of the classical stress-tensor. Moreover we use this late time behaviour to go beyond the near-horizon approximation and comment on the implications for the information loss paradox.

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1 Introduction

The discovery that black holes emit thermal radiation [1] has been considered a sign that the evaporation process implies a loss of quantum coherence [2]. However, it has been stressed [3] that gravitational back-reaction effects could change the standard picture of black hole decay. In particular, 't Hooft [3] suggested that the interaction between the infalling matter and the outgoing radiation could preserve the unitarity of the process through non-local effects.

One can consider a simplified scenario analysing the scattering of an extremal Reissner-Nördstrom (RN) black hole by low-energy massless neutral particles. The existence of a stable ground state (extremal configuration, with vanishing Hawking temperature T_H) avoids to encounter the problem of the singularity at the large stages of the evaporation (indeed, in the case of the Schwarzschild black holes $T_H \sim 1/M$ grows without bound) . It is this feature which makes the process more tractable than the evaporation of uncharged black holes. Moreover, restricting the problem to spherically symmetric configurations, one can maintain the main physical ingredients of the problem while at the same time simplifying the mathematics involved. The resulting model, with back-reaction effects included, was studied by Strominger and Trivedi [4] in the adiabatic approximation, and numerically by Lowe and O'Loughlin [5] (see also [6]).

If we also restrict the analysis to a region very close to the horizon we can describe the physical process by an effective theory which turns out to be equivalent to a solvable two-dimensional model. The effective model remains solvable also at the one-loop quantum level and it has been studied in [7, 8]. We shall summarize its main ingredients in section 2. Since the effective action is non-local, a crucial point to properly define the quantum theory is to select the appropriate boundary conditions associated to the non-local terms of the semiclassical equations of motion. In references [7, 8] we chose the boundary conditions in such a way that they naturally describe the evaporation of the black hole from the point of view of an infalling observer very close to the horizon (see also [9]). In section 3 we shall consider an alternative set of boundary conditions which turns out to be very relevant from the physical point of view because it corresponds to an asymptotic observer at late retarded times. This is just the part of future null infinity which can still be described by our model. These two sets of boundary conditions are not compatible (up to the extremal, static configuration). This fact can be connected with the principle of complementarity [3, 10, 11, 12], which states that the simultaneous measurements made by an external observer and those made by an infalling observer crossing the horizon are forbidden. The solution we get for the new boundary conditions will be given in section 4 and it is very different in form from the original one (they indeed provide two different descriptions of the evaporation process). However we will crucially impose that they match at the end-point of the evaporation since then both solutions become extremal. This matching condition allows us to determine the asymptotic late time Hawking flux, including back-reaction effects. In contrast with the standard picture, the Hawking flux goes down exponentially at late times and it is not proportional to the total mass of the classical incoming matter. Instead, we find that it is proportional to a parameter which admits an infinite series expansion in Planck's constant. At leading order this parameter is the total mass and the higher order terms involve higher order momenta of the classical stress-tensor. One can go beyond the near-horizon approximation to evaluate the Hawking flux by requiring energy conservation. We shall do it in section 5 for the simplest case obtained by perturbing the extremal black hole by means of a shock wave. All these results have, potentially, far reaching consequences for the information loss problem and we shall comment on it in the final section.

2 The near-horizon model

Imposing spherical symmetry to the Einstein-Maxwell theory

$$ds^2_{(4)} = d\bar{s}^2_{(2)} + 4l^2\phi d\Omega^2 \,, \tag{1}$$

where l^2 is Newton's constant, the corresponding dimensional reduction leads to a two-dimensional theory. If one rescales the metric by

$$ds^2_{(2)} = \sqrt{\phi} d\bar{s}^2_{(2)} \,, \tag{2}$$

the two-dimensional action turns out to be

$$I = \int dx^2 \sqrt{-g} \left(R\phi + l^{-2} V(\phi) \right) \,, \tag{3}$$

where

$$V(\phi) = (4\phi)^{-\frac{1}{2}} - q^2 (4\phi)^{-\frac{3}{2}} \,. \tag{4}$$

The extremal black hole radius $r_0^2 = 4l^2\phi_0$ is recovered when $V(\phi_0) = 0$, and expanding ϕ around $\phi_0 = \frac{q^2}{4}$ ($\phi = \phi_0 + \tilde{\phi}$) the action (3) leads to the Jackiw-Teitelboim model [13]

$$I = \int d^2x \sqrt{-g} \left[(R + \frac{4}{l^2 q^3})\tilde{\phi} - \frac{1}{2} |\nabla f|^2 \right] \,, \tag{5}$$

where we have added a matter field f representing a four-dimensional spherically symmetric scalar field which propagates freely in the region close to the horizon. To properly account for back-reaction effects we have to consider the corresponding one-loop effective theory. Therefore we have to correct (5) by adding the Polyakov-Liouville term [14]

$$\begin{aligned} I &= \int d^2x \sqrt{-g} \left(R\tilde{\phi} + 4\lambda^2 \tilde{\phi} - \frac{1}{2} \sum_{i=1}^{N} |\nabla f_i|^2 \right) \\ &- \frac{N\hbar}{96\pi} \int d^2x \sqrt{-g} R \,\square^{-1} R + \frac{N\hbar}{12\pi} \int d^2x \sqrt{-g} \lambda^2 \,, \end{aligned} \tag{6}$$

where we have considered the presence of N scalar fields to enforce that the above effective action captures the proper quantum theory in the large N limit (keeping $N\hbar$ constant). In this limit the fluctuations of the gravity degrees of freedom can be neglected [15]. Note that the Polyakov-Liouville action has a cosmological constant term which has been fixed ($\lambda^2 = l^{-2}q^{-3}$) to ensure that the extremal configuration remains a solution of the quantum theory. In conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$ the equations of motion derived from (6) are

$$\begin{aligned} 2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} &= 0\,, && (7)\\ \partial_+\partial_-\tilde{\phi} + \lambda^2\tilde{\phi}e^{2\rho} &= 0\,, && (8)\\ \partial_+\partial_- f_i &= 0\,, && (9)\\ -2\partial_\pm^2\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi} &= T^f_{\pm\pm} - \frac{N\hbar}{12\pi}t_\pm - && (10)\\ & \quad \frac{N\hbar}{12\pi}\left((\partial_\pm\rho)^2 - \partial_\pm^2\rho\right)\,, \end{aligned}$$

where the chiral functions $t_\pm(x^\pm)$, coming from the non-locality of the Polyakov-Liouville action, are related with the boundary conditions of the theory associated with the corresponding observers. The equation (7) is the Liouville equation with a negative cosmological constant. It has a unique solution up to conformal coordinate transformations. It is very convenient to choose the following form of the metric

$$ds^2 = -\frac{2l^2q^3dx^+dx^-}{(x^- - x^+)^2}\,, \tag{11}$$

which, in turn, is a way to fix the conformal coordinates $x^\pm$, up to Möbius transformations. In these coordinates only the $t_\pm$ terms survive in the quantum part of the constraints (10), i.e. the semiclassical stress tensor is just

$$\langle T_{\pm\pm}\rangle = -\frac{N\hbar}{12\pi}t_\pm\,, \tag{12}$$

and the relevant information of the solutions is therefore encoded in the field $\tilde{\phi}$.

In the gauge defined by the metric (11) the solution to the equations of motion is

$$\tilde{\phi} = \frac{1}{2}\partial_+F(x^+) + \frac{F(x^+)}{x^- - x^+} + \frac{1}{2}\partial_-G(x^-) + \frac{G(x^-)}{x^+ - x^-}\,, \tag{13}$$

where the chiral functions $F(x^+)$, $G(x^-)$ are related to the boundary functions $t_\pm(x^\pm)$

$$\begin{aligned} -\partial_+^3F &= -\frac{N\hbar}{12\pi}t_+(x^+) + T^f_{++}\,, && (14)\\ -\partial_-^3G &= -\frac{N\hbar}{12\pi}t_-(x^-)\,. && (15) \end{aligned}$$

The crucial point is then to choose the suitable functions $t_\pm(x^\pm)$.

3 Boundary conditions

The choice of the functions $t_\pm(x^\pm)$ should be done on the basis of physical considerations. The extremal black hole can be described by the solution (up to Möbius transformations)

$$\tilde{\phi} = \frac{lq^3}{x^- - x^+}\,, \tag{16}$$

where the coordinates x^-, x^+ can be identified with the classical Eddington-Finkelstein coordinates u, v. To match the extremal solution with a near-extremal one necessarily requires the vanishing of $\partial_-^3 G$ and therefore

$$t_-(x^-) = 0\,, \tag{17}$$

thus implying that

$$\langle T_{--}\rangle = 0\,. \tag{18}$$

The point now is to choose the function $t_+(x^+)$. Due to (18) we can also write a generic metric obeying the equations of motion in the ingoing Vaidya-type gauge

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2q^3} - l\tilde{m}(v)\right)dv^2 + 2dvd\tilde{x}\,, \tag{19}$$

where $\tilde{x} = l\tilde{\phi}$ and

$$\partial_v \tilde{m}(v) = T^f_{vv} - \frac{N\hbar}{12\pi}t_v(v)\,. \tag{20}$$

If the incoming classical matter T^f_{vv} starts at v_i and is turned off at some advanced time v_f we have

$$ds^2 = -\frac{2\tilde{x}^2}{l^2q^3}dv^2 + 2dvd\tilde{x}\,, \tag{21}$$

before v_i. This solution can be brought into the the form (11) with the coordinate change

$$x^+ \;=\; v\,, \tag{22}$$

$$x^- \;=\; v + \frac{l^2q^3}{\tilde{x}}\,. \tag{23}$$

However, for $v > v_f$ the analysis is more involved. Let us first simplify the problem and consider that (19) is the classical solution. Therefore we have (for $v > v_f$)

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2q^3} - l\tilde{m}_{cl}(v_f)\right)dv^2 + 2dvd\tilde{x}\,. \tag{24}$$

This solution can also be transformed into (11) with the coordinate change

$$v \;=\; x_0^+ + \sqrt{\frac{2lq^3}{\tilde{m}_{cl}(v_f)}}\,\mathrm{arctanh}\sqrt{\frac{\tilde{m}_{cl}(v_f)}{2lq^3}}(x^+ - x_0^+)\,, \tag{25}$$

$$\tilde{x} \;=\; lq^3\frac{1 - \frac{\tilde{m}_{cl}(v_f)}{2lq^3}(x^+ - x_0^+)(x^- - x_0^+)}{x^- - x^+}\,, \tag{26}$$

where x_0^+ is an integration constant.

Since the incoming classical and quantum fluxes vanish before v_i we should have

$$t_v(v) = 0\,, \tag{27}$$

and therefore, according to (22) and (25) and the transformation law for the t's functions

$$t_+(x^+) = \left(\frac{dv}{dx^+}\right)^2 t_v(v) + \frac{1}{2}\{v, x^+\}\,, \tag{28}$$

we get

$$t_+(x^+) = \frac{2lq^3}{\tilde{m}_{cl}(v_f)} \frac{1}{\left(\frac{2lq^3}{\tilde{m}_{cl}(v_f)} - (x^+ - x_0^+)^2\right)^2}\,, \tag{29}$$

for $x^+ > x_f^+$.

The above boundary condition has the following drawbacks

- It has been calculated according to the classical solution (24). So the back-reaction effects have not been included.
- It requires that $x^+ > x_f^+$ and it is unclear how to match with the condition $t_+(x^+) = 0$ for $x^+ < x_i^+$.

We can solve these problems just considering

$$t_+(x^+) = \frac{1}{2}\{v, x^+\}\,, \tag{30}$$

where the Eddington-Finkelstein type coordinate v is the one appearing in the evaporating metric (19). It is worth remarking that the relation $x^+ = x^+(v, \hbar)$ is no longer given by the classical expression (25), but rather it will be determined once we solve the semiclassical equations of motion. Therefore (30) incorporates the back-reaction effects in a self-consistent way, in contrast with the choice (29).

The above discussion may appear rather surprising since the equation (18) means that there is not Hawking radiation at all. The evaporation is due to the negative incoming flux given by

$$\langle T_{++}\rangle = -\frac{N\hbar}{24\pi}\{v, x^+\}\,, \tag{31}$$

as measured by a free falling observer. However for an outside observer the black hole shrinks due to the Hawking radiation. In fact, with a fixed classical background, it is given by the expression

$$\langle T_{uu}\rangle = -\frac{N\hbar}{24\pi}\{u_{in}, u\}\,, \tag{32}$$

where u_{in} is the outgoing null coordinate of the extremal solution. But we know that $u_{in} = x^-$ and this implies that the Hawking flux is proportional to the classical incoming mass $m_{cl}(v_f)$ (see the appendix)

$$\langle T_{uu} \rangle = \frac{N\hbar}{24\pi l q^3} m_{cl}(v_f) \,. \tag{33}$$

This corresponds to the constant thermal flux of near-extremal Reissner-Nördstrom black holes measured by the asymptotic observer at future null infinity at late times. Moreover (33) also reflects the fact that the late time behaviour of the Hawking radiation depends only on the total classical mass of the matter forming the near-extremal black hole. There is not dependence on the details of the incoming matter.

Now we have arrived at an apparent contradiction. The quantum equations, which incorporate back-reaction effects, imply that $\langle T_{--} \rangle = 0$, but our last argument shows that we have indeed Hawking radiation. This puzzle is solved by invoking the principle of complementarity [3]. According to it we cannot have a detailed description of the physics given by an infalling observer and, simultaneously, by an asymptotic one. Therefore, with this idea and the above discussion in mind, it seems natural to consider the following boundary condition

$$t_+(x^+) = 0 \,, \tag{34}$$

meaning that for the outside observer ($v >> v_f$) there is not incoming quantum flux. This boundary condition allows us to introduce a generic metric satisfying the equations of motion in the outgoing Vaidya-type gauge

$$ds^2 = -(\frac{2\tilde{x}^2}{l^2 q^3} - l\tilde{m}(u))du^2 - 2dud\tilde{x} \,. \tag{35}$$

Then for the function $t_-(x^-)$ we have to choose

$$t_-(x^-) = -\frac{1}{2}\{u, x^-\} \,, \tag{36}$$

since it reproduces the Hawking-type flux

$$\langle T_{uu} \rangle = \left(\frac{dx^-}{du}\right)^2 \langle T_{--} \rangle = \frac{N\hbar}{24\pi} \left(\frac{dx^-}{du}\right)^2 \{u, x^-\} = -\frac{N\hbar}{24\pi}\{x^-, u\} \,. \tag{37}$$

As before, the relation $x^- = x^-(u, \hbar)$ is dynamical and it can only be determined once we solve the complete set of equations. In the limit $\hbar \to 0$ we reproduce the coordinate change obtained from the classical solutions, but in general we will have an infinite series expansion in $\hbar$ expressing the large quantum effects of back-reaction.

We want to finish our discussion on the boundary conditions by stressing again that this alternative sets of conditions fits with the idea of complementarity. The conditions (17), (30) are the natural ones to describe the evaporation process for an infalling observer very close to the horizon and correspond to a negative influx of radiation crossing the apparent horizon and no outgoing flux. Alternatively, one can provide a description of the evaporation process from the point of view of an outside observer. The conditions (34), (36) give a positive outflux of radiation and vanishing incoming flux. It is worth to remark the important fact that we cannot impose simultaneously these conditions. Obviously, there is an exception and it corresponds to the solution with $t_+(x^+) = 0 = t_-(x^-)$, but it is just the extremal configuration.

In summary, our scheme excludes the fact of having simultaneously Hawking radiation and an ingoing quantum flux. The Hawking radiation does exist in the boundary conditions (34), (36), although there one does not see the negative ingoing flux. On the other hand, the infalling observer does not see outgoing radiation and the evaporation is due to the ingoing radiation.

4 Solutions and Hawking radiation

The suitable boundary conditions for the infalling observer

$$\begin{aligned} t_+(x^+) &= \frac{1}{2}\{v, x^+\}\,, && (38)\\ t_-(x^-) &= 0\,, && (39)\end{aligned}$$

imply that the solution can be written as

$$\tilde{\phi} = \frac{F(x^+)}{x^- - x^+} + \frac{1}{2}F'(x^+)\,, \tag{40}$$

where the function $F(x^+)$ satisfies the differential equation

$$F''' = \frac{N\hbar}{24\pi}\left(-\frac{F''}{F} + \frac{1}{2}(\frac{F'}{F})^2\right) - T^f_{++}(x^+)\,. \tag{41}$$

The function $F(x^+)$ relates the coordinates x^+ and v

$$\frac{dv}{dx^+} = \frac{lq^3}{F}\,, \tag{42}$$

and in terms of the mass function $\tilde{m}(v)$ the differential equation for F turns out to be

$$\partial_v \tilde{m}(v) = -\frac{N\hbar}{24\pi l q^3}\tilde{m}(v) + T^f_{vv}(v)\,. \tag{43}$$

If the incoming classical matter is turned off at some advanced time v_f then the evaporating solution approaches asymptotically the extremal configuration (up to exponentially small corrections) [7, 8]

$$\tilde{\phi} = \frac{F''(x^+_{\rm int})}{2} \frac{(x^+ - x^-_{\rm int})(x^- - x^-_{\rm int})}{x^- - x^+} \,, \tag{44}$$

where $(x^{\pm}_{\rm int})$ represent the end-point coordinates that belong to the AdS$_2$ boundary $(x^+_{\rm int} = x^-_{\rm int})$.

In the alternative description of the evaporation process, suitable for the outside observer, the boundary conditions are

$$t_+(x+) \;=\; 0 \,, \tag{45}$$

$$t_-(x-) \;=\; -\frac{1}{2}\{u, x^-\} \,. \tag{46}$$

The solution can then be written as

$$\tilde{\phi} = \frac{G(x^-)}{x^+ - x^-} + \frac{1}{2}G'(x^-) \,, \tag{47}$$

where the function $G(x^-)$ verifies the differential equation

$$G''' = -\frac{N\hbar}{24\pi}\left(-\frac{G''}{G} + \frac{1}{2}(\frac{G'}{G})^2\right) \,, \tag{48}$$

and serves to relate the coordinates x^- and u

$$\frac{du}{dx^-} = -\frac{lq^3}{G(x^-)} \,. \tag{49}$$

The evaporating mass function $\tilde{m}(u)$ obeys now the equation

$$\partial_u \tilde{m}(u) = -\frac{N\hbar}{24\pi l q^3}\tilde{m}(u) \,, \tag{50}$$

which implies that

$$\tilde{m}(u) = \tilde{m}_0 \, e^{-\frac{N\hbar}{24\pi l q^3}u} \,. \tag{51}$$

The point now is how to determine the integration constant $\tilde{m}_0$, but this is related to the choice of the ”initial” conditions for the differential equation (48). Since the alternative pair of boundary conditions are compatible in the extremal configuration we shall impose that the two solutions (40), (47) match at the end-point (x^+_{int}, x^-_{int}), where both solutions approach the extremal one. It is worth noting that once we move away from it the corrections to eq.(44) will of course be different in the two cases and this agrees with the idea of complementarity. Moreover, such a requirement is certainly nonlocal (and this reminds the sort of nonlocal effects advocated by 't Hooft) because it implies that the form of the function $G(x^-)$ for $x^- < x^-_{int}$ (and therefore $\langle T_{uu}\rangle$ for finite u) depends on the precise form of the solution at the end-point (where $\langle T_{uu}\rangle = 0$). Expanding $G(x^-)$ around x^-_{int} and imposing that (47) be exactly (44) for $x^- \to x^-_{\rm int}$ we obtain

$$\begin{aligned} G(x^-_{\rm int}) &= F(x^+_{\rm int}) = 0\,, & (52)\\ G'(x^-_{\rm int}) &= F'(x^+_{\rm int}) = 0\,, & (53)\\ G''(x^-_{int}) &= -F''(x^+_{int}) < 0\,. & (54) \end{aligned}$$

So F and G are solutions of the differential equations (41) and (48), which in the region where $T^f_{vv} = 0$ differ just for an overall sign in their r.h.s. Moreover both solutions have similar boundary conditions, again up to a sign, in $F''(x^+_{\rm int}) = -G''(x^-_{\rm int})$ where $x^+_{\rm int} = x^-_{\rm int}$. Therefore $G(x^-)$ is functionally equal to $-F(x^+)$ after exchanging x^+ with x^-. $F''(x^+_{\rm int})$ uniquely fixes $\tilde{m}(v_f)$ and so (54) implies that $\tilde{m}_0 = \tilde{m}(v_f)\, e^{\frac{N\hbar}{24\pi l q^3} v_f}$. Therefore the Hawking flux is

$$\langle T_{uu}(u)\rangle = \frac{N\hbar}{24\pi l q^3}\tilde{m}(u) = \frac{N\hbar}{24\pi l q^3}\tilde{m}(v_f) e^{-\frac{N\hbar}{24\pi l q^3}(u - v_f)}\,, \tag{55}$$

where the explicit expression for $\tilde{m}(v_f)$ is given by the formal solution to the equation (43)

$$\begin{aligned} \tilde{m}(v_f) &= \sum_{n=0}^{\infty}\left(-\frac{N\hbar}{24\pi l q^3}\right)^n \int_{-\infty}^{v_f} dv_1 \int_{-\infty}^{v_1} dv_2 \\ & \quad .... \int_{-\infty}^{v_n} dv_{n+1} T^f_{vv}(v_{n+1})\,. \end{aligned} \tag{56}$$

It is important to point out the fact that $\tilde{m}(v_f)$ depends on the details of the collapsing matter through all the higher-order momenta of the classical stress tensor. We observe that for $\hbar \to 0$ $\tilde{m}(v_f)$ is the total classical mass of the collapsing matter and (55) recovers the constant thermal value of a static near-extremal black hole (33). So when back-reaction effects are neglected we loose the information of the initial state.

5 Beyond the near-horizon approximation

The solvability of the near-horizon model studied in the previous sections has allowed us to work out the late time behaviour of the Hawking flux. Of course, one would like to know $\langle T_{uu}\rangle$ for every u, but this is out of the reach of our model. However, if the incoming matter has the form of a spherical null shell

$$T^f_{uu} = \Delta m \delta(v - v_0)\,, \tag{57}$$

general physical requirements for $\langle T_{uu}\rangle$ are so strong as to determine it completely. At leading order in $\hbar$ we impose that (from now on we set $l=1$)

$$\langle T^f_{uu}\rangle = \langle T^f_{uu}\rangle_{NBR} + \mathcal{O}(\hbar^2)\,, \tag{58}$$

where $\langle T^f_{uu}\rangle_{NBR}$ is the Hawking flux computed in the classical background (no back-reaction) defined by the matching of the extremal black hole of mass q (for $v<v_0$)

$$ds^2 = -\left(1-\frac{q}{r}\right)^2 du_{in}dv\,, \tag{59}$$

and the near-extremal one of mass $q+\Delta m$ as $v>v_0$

$$ds^2 = -\frac{(r-r_+)(r-r_-)}{r^2}dudv\,. \tag{60}$$

The relation between u and u_{in} is given by

$$\frac{du}{du_{in}} = \frac{(r-q)^2}{(r-r_+)(r-r_-)}\,, \tag{61}$$

and the Hawking flux without back-reaction $\langle T^f_{uu}\rangle_{NBR}$ is

$$\langle T^f_{uu}\rangle_{NBR} = -\frac{N\hbar}{24\pi}\{u_{in},u\}\,, \tag{62}$$

which turns out to be

$$\begin{aligned}\langle T^f_{uu}\rangle_{NBR}(u,m,q) &= \frac{N\hbar}{24\pi}[\frac{(m-q)(r-r_+)(r-r_-)(r^2+rq-q^2)}{r^5(r-q)^2} \\ &+ \frac{1}{2}\frac{(m-q)^2(r+q)^2}{r^4(r-q)^2}]\,,\end{aligned} \tag{63}$$

where

$$r_\pm = m\pm\sqrt{m^2-q^2}\,, \tag{64}$$
$$m = q+\Delta m\,, \tag{65}$$

and

$$\frac{v_0-u}{2} = r + \frac{1}{r_+-r_-}\left[r_+^2\ln\left|\frac{r_-r_+}{r_+}\right| - r_-^2\ln\left|\frac{r_-r_-}{r_-}\right|\right]\,. \tag{66}$$

Moreover, energy conservation implies that

$$\int_{-\infty}^{+\infty}\langle T_{uu}\rangle du = \Delta m\,. \tag{67}$$

In addition, $\langle T_{uu}\rangle$ should verify, as $u\to-\infty$,

$$\langle T_{uu}\rangle \sim \langle T_{uu}\rangle_{NBR} = \frac{N\hbar\Delta m}{3\pi|u|^3}\,, \tag{68}$$

because at early times the back-reaction can be ignored[1]. These conditions are satisfied automatically if $\langle T_{uu}\rangle(u)$ is just the r.h.s. of the differential equation

$$-\frac{dm}{du} = \frac{N\hbar}{24\pi}\langle T^f_{uu}\rangle_{NBR}(u, m(u), q)\,, \tag{69}$$

where $m = m(u)$ and $r_\pm = r_\pm(u)$, $r = r(m)$ are given by expressions similar to (64)-(66), and $m(u)$ verifies the initial condition

$$m(u = -\infty) = q + \Delta m\,. \tag{70}$$

It is easy to see that this proposal for $\langle T_{uu}\rangle(u)$ fulfills the conditions (58), (67), (68), and it is very difficult to imagine an alternative solution. Moreover the late time behaviour of $\langle T_{uu}\rangle$ is also of the form

$$\langle T_{uu}\rangle \sim \frac{N\hbar}{24\pi q^3}\tilde{m}_0 e^{-\frac{N\hbar}{24\pi l q^3}u}\,, \tag{71}$$

as $u \to +\infty$, where $\tilde{m}_0$ is an integration constant. It can be shown [17] numerically that $\tilde{m}_0$ agrees with the expression obtained in section 4 ($\tilde{m}_0 = \Delta m e^{\frac{N\hbar}{24\pi l q^3}v_0}$) thus providing a self-consistency test of our approach.

6 Conclusions

In this paper we have studied the near-horizon effective theory controlling the decay of a near-extremal charged black hole. We have focused on the delicate point of how to choose the integration functions $t_\pm$ coming from the non-locality of the Polyakov-Liouville action. We have stressed the fact that it is not possible to choose, simultaneously, non-vanishing functions $t_\pm(x^\pm)$. Since they are proportional, in a particular coordinate system $\{x^\pm\}$, to the quantum fluxes this seems to lead to inconsistencies. The vanishing of t_- implies the absence of Hawking radiation, and the black hole shrinks due to the negative incoming quantum radiation produced by t_+. We have interpreted this apparently disturbing situation in terms of a complementarity between the physical descriptions given by an infalling observer and by an asymptotic one. Our model, which captures the (near-horizon) quantum back-reaction in the large N limit, dictates that there is not an unique choice for $t_\pm$, up to the extremal configuration, and it seems natural to choose $t_-(x^-) = 0$ and $t_+(x^+) = \frac{1}{2}\{v, x^+\}$ for the infalling observer and $t_+(x^+) = 0$ and $t_-(x^-) = -\frac{1}{2}\{u, x^-\}$ for the outside observer.

We would like to stress that eq. (55) is the first calculation of the Hawking radiation flux for RN black holes at late times, which takes into account consistently

[1] See the essay [16] for a comparison of this problem to the Planck problem of black body radiation.

back-reaction effects, and in the large N limit it is exact. Our result is highly non-trivial because two different expansions in $N\hbar$ are implicit in (55), one being associated to the exponential $e^{-\frac{N\hbar}{24\pi l q^3}(u-v_f)}$ and the other inside $\tilde{m}(v_f)$, see (56). While the first expansion is of no surprise, the second one is completely unexpected on physical grounds. Actually it implies that the information carried by the classical incoming matter can be read from the late time Hawking radiation, which admits an infinite series expansion in $N\hbar$ and where each term involves different momenta of the classical stress tensor. This is in contrast with the predictions based on fixed background calculations. When the back-reaction is ignored the late-time Hawking flux goes to the constant thermal value (33) . To deepen our result we can mention that the relation between the coordinates u_{in} and u_{out} before and after the classical influx of matter T^f_{vv} is given by ($u_{out} \to +\infty$)

$$\frac{du_{out}}{du_{in}} \sim u^2_{out}(A - Be^{-Cu_{out}})\,, \tag{72}$$

where A, B and C are positive integration constants depending on $m(v_f)$. This also implies that the radiation is quite different from the standard late-time thermal radiation coming from the relation ($u_{out} \to +\infty$)

$$\frac{du_{out}}{du_{in}} \sim e^{2\pi T_H u_{out}}\,, \tag{73}$$

where T_H is the Hawking temperature (for similar results see also [18]).

Nevertheless we have to remark that this result does not necessarily means that the ”quantum information”, in addition to the classical one given by T^f_{vv}, is also encoded in the late time radiation. In the standard picture of black hole evaporation the ”quantum information” is encoded in the correlation between outgoing and incoming particle-antiparticle pairs. The outgoing (Hawking) radiation is uncorrelated and therefore represents a mixed state. In our scheme we have either ingoing or outgoing radiation, according to the observer. So, this suggests that for the asymptotic observer the outgoing radiation can only be correlated with itself. This opens the interesting possibility, using the proposal of section 5, of studying the correlation functions between the outgoing radiation at early and late times to see whether or not it corresponds to a pure state. This must be done numerically [17].

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Appendix

We can determine the Hawking flux without back-reaction by matching static solutions. If $T^f_{vv} = m_1\delta(v - v_1)$ we have

$$ds^2 = -\left(\frac{2\tilde{x}^2}{l^2q^3} - l\tilde{m}_{cl}(v)\right)dv^2 + 2dvd\tilde{x}\,, \tag{74}$$

where $\tilde{m}_{cl}(v) = m_1\Theta(v - v_1)$. In conformal gauge we get

$$ds^2 = -\frac{2\tilde{x}}{l^2q^3}du_{in}dv\,, \tag{75}$$

for $v < v_0$, and

$$ds^2 = -\left(\frac{2\tilde{x}}{l^2q^3} - lm_1\right)du_{out}dv\,, \tag{76}$$

for $v > v_0$. The matching at $v = v_1$ implies that

$$u_{in} = v_0 + \sqrt{\frac{2lq^3}{m_1}}\mathrm{cotanh}\sqrt{\frac{m_1}{2lq^3}}(u_{out} - v_1)\,. \tag{77}$$

The Hawking flux is given by

$$\langle T_{u_{out}u_{out}}\rangle = -\frac{N\hbar}{24\pi}\{u_{in}, u_{out}\} = \frac{N\hbar}{24\pi lq^3}m_1\,. \tag{78}$$

In the case of two shock waves $T^f_{vv} = m_1\delta(v - v_1) + m_2\delta(v - v_2)$ one obtains

$$\begin{aligned} u_{in} \;&=\; v_0 + \sqrt{\frac{2lq^3}{m_1}}\mathrm{cotanh}[\sqrt{\frac{m_1}{2lq^3}}(v_1 - v_0) + \\ &\quad \mathrm{arctanh}\sqrt{\frac{m_1}{m_1+m_2}}\tanh\sqrt{\frac{m_1+m_2}{2lq^3}}(u_{out} - v_2)]\,, \end{aligned} \tag{79}$$

and then

$$\langle T_{u_{out}u_{out}}\rangle = \frac{N\hbar}{24\pi lq^3}(m_1 + m_2)\,. \tag{80}$$

The argument can be repeated so on for an arbitrary finite set of shock waves $T^f_{vv} = \sum_{i=1}\delta(v - v_i)$ and the result is

$$\langle T_{uu}\rangle = \frac{N\hbar}{24\pi lq^3}\tilde{m}_{cl}(v_f)\,, \tag{81}$$

where $\tilde{m}_{cl}(v_f) = \sum_i m_i$ is the total mass of the incoming matter and $v_f = v_N$. It is interesting to remark that the above Hawking flux does not see the details of the incoming matter . It is only sensitive to the total incoming mass. The information carried out by the classical stress tensor is lost if one neglects the back-reaction effects in the Hawking flux.

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