$D^0 - \overline{D^0}$ mixing in the weak–gauged
$U(4)_L \otimes U(4)_R$ Chiral Lagrangian Model

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Abstract

The D^0–$\overline{D^0}$ mixing is analyzed in a weak gauged $U(4)_L \otimes U(4)_R$ chiral lagrangian model where the electroweak interaction is introduced as a gauge theory over the meson degrees of freedom. This model allows a particular realization of the G.I.M. mechanism and then could be useful in the study of processes where G.I.M. suppression is effective. As a test of the model we have also analyzed the K^0–$\overline{K^0}$ mixing. We find Δm_K in good agreement with the experimental result and we show that the D^0–$\overline{D^0}$ mixing is very much suppressed in agreement with previous estimates in the Heavy Quark expansion framework.

1 Introduction

Higher order effects in the perturbative treatment of the electroweak Standard Model (SM) play an active rôle both in testing some of its fundamental ingredients and in uncovering phenomena not explained by the Standard framework. One of the peculiarities of the SM is the absence (at leading order in the perturbative expansion) of flavour changing neutral currents (FCNC). This characteristic feature strongly affects flavour mixing and rare meson decays.

Neglecting the tiny CP–violating amplitudes (as we do in this Letter) the perturbative contributions to P^0–$\overline{P^0}$ mixing are straightforward to evaluate in the SM. However the numerical quantification is affected by our rather poor knowledge of some of the relevant elements of the CKM matrix and of the quark masses. In any case more dubious is the treatment of long–distance contributions. These have two main sources: a) the evaluation of the matrix element of the four–quark $\Delta F = 2$ operator in the effective hamiltonian (F is short for flavour), and b) the handling of mesonic intermediate states contributing to the dispersive amplitude. Both features are characterized by a manifestly non–perturbative origin.

The relative importance of short vs. long–distance contributions depends crucially of the flavour involved. This is so because as the flavour is heavier the transitions involve higher transfer of momenta and therefore non–perturbative corrections are less relevant. As a consequence B_s^0–$\overline{B_s^0}$ and B_d^0–$\overline{B_d^0}$ are expected to be dominated by the perturbative regime. This is not so clear for K^0–$\overline{K^0}$ and definitely not the case of D^0–$\overline{D^0}$ (due to the possible contribution of resonances in the highly populated kinematical region of charmed mesons).

In this Letter we address the issue of non–perturbative corrections to the mixing and therefore we will be concerned with these last two cases.

At hadronic level meson mixing with $\Delta F = 2$ transitions has ushered permanent interest because its implications : tests of FCNC, close relation with CP–violation, prospects of new physics beyond the SM, etc. (see [1, 2, 3, 4] and references therein). Mixing occurs through radiative corrections in the SM. The mass eigenstates are

$$|P_{\frac{2}{1}}\rangle = \frac{1}{\sqrt{2}} \left[|P^0\rangle \pm |\overline{P}^0\rangle \right] \ , \tag{1}$$

and the mixing produces $P^0 \leftrightarrow \overline{P^0}$ oscillations of amplitude $\exp(i\Delta M t)$ with $\Delta M = m_2 - m_1 - i(\Gamma_2 - \Gamma_1)/2$. The parameter controlling the oscillation is $x \equiv \Delta m/\Gamma$ [1] and consequently the mass difference between the eigenstates. With our normalization this mass difference is related with the $\Delta F = 2$ transition through

$$\Delta m_P = \frac{1}{m_P} Re\langle P^0|\mathcal{H}|\overline{P}^0\rangle \ , \tag{2}$$

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[1] We call $x_K \equiv \Delta m_K/\Gamma_S$ and $x_D \equiv \Delta m_D/\Gamma_D$.

1

where m_P is short for the relevant pseudoscalar mass.

K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixings have notable different features in the SM. Both mixings have a complementary interest because test FCNC in the upper and lower sector of the families, respectively. Experimentally [5] $x_K \simeq 0.5$ and $|x_D| < 0.09$ and, as we will shortly see, the perturbative evaluation in the SM is able to point out these different behaviours because it predicts $\Delta m_K \sim m_c$ and $\Delta m_D \sim m_s$. Moreover the perturbative evaluation gives the right order for the measured K^0–$\overline{K^0}$ mixing while it gives a very small D^0–$\overline{D^0}$ mixing. This is the case because GIM mechanism [6] is much more effective in the lower sector of the families than in the upper one due to the quark mass differences.

Dispersive amplitudes due to intermediate mesonic states could give relevant non–perturbative contributions. These have been considered by several authors [2, 7, 8, 9, 10, 11] and the conclusion is that they indeed are important. In D^0–$\overline{D^0}$ could even give a result several orders of magnitude bigger than the perturbative one. This might be due to the fact that the expected GIM suppression could be spoiled by the bad $SU(3)$ breaking observed in several decays of charmed mesons [9, 12].

We have proposed a weak-gauged chiral model up to and including charmed mesons in order to implement at hadronic level all the structure of symmetries of the SM, specifically including the GIM mechanism [13]. Therefore, the $SU(3)_L \otimes SU(3)_R$ chiral symmetry of the strong interactions must be extended to a $U(4)_L \otimes U(4)_R$ symmetry in order to implement the weak $SU(2)_L$ representations. Nevertheless it must be clear that our model is not pretended to be a model for the strong interaction. Of course chiral symmetry is badly broken in the charm sector. We expect that this model may provide a realistic approach to those processes where relevant features of the flavour sector of the SM like GIM play a significant rôle. In this Letter we evaluate in our model [14], the leading order result for the K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixings.

In Section 2 we remind briefly the perturbative evaluations of the mixings in the SM. An outline of the basic ingredients of our model is sketched in Section 3. Then in Sections 4 and 5 we specify our results for K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$, respectively. We end with our conclusions in Section 6.

2   The perturbative regime

The short–distance contribution to meson mixing in the SM is represented by the usual box diagrams. In order to carry out this calculation one starts with the weak effective hamiltonian $\mathcal{H}_{eff}^{\Delta F=2}$ in terms of four–quark operators. We explain both K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixings in turn.

2.1 K^0–$\overline{K^0}$

The effective hamiltonian (we are not interested in CP-violation) including the next-to-leading logarithmic QCD corrections, is given by [15, 16]

$$\mathcal{H}_{eff}^{\Delta S=2} \ = \ \frac{1}{16\pi^2}\, G_F^2\, m_c^2\, \lambda_c^2\, \eta_1\, S\left(\frac{m_c^2}{M_W^2}\right)\, \alpha_s(\mu^2)^{(-2/9)}\, \mathcal{O}^{\Delta S=2} \ , \tag{3}$$

where

$$\mathcal{O}^{\Delta S=2} \ = \ (\overline{s}_L^\alpha \gamma_\mu d_L^\alpha)(\overline{s}_L^\beta \gamma^\mu d_L^\beta) \quad , \tag{4}$$

$\lambda_c = V_{cd}V_{cs}^*$ with V_{ij} the CKM matrix, α and β are colour indices and

$$S(x) \ = \ \left[1 + \frac{9}{1-x} - \frac{6}{(1-x)^2} - \frac{6x^2}{(1-x)^3}\ln x\right] \quad . \tag{5}$$

In Eq. (3) η_1 carries the information of strong QCD corrections [17].

A part of the perturbative uncertainties in order to get a firm prediction we need to do the evaluation of the matrix element of the four–quark operator in Eq. (4) between the asymptotic kaon states. This, of course, involves the unsolved problem of hadronization and our ignorance is expressed by the B parameter defined as

$$\langle \overline{K}^0 | \mathcal{O}^{\Delta S=2} | K^0 \rangle \ = \ \frac{4}{3}\, f_K^2 m_K^2\, B(\mu) \quad , \tag{6}$$

where, in the leading logarithmic approximation in the strong coupling,

$$B(\mu) \ = \ B_K\, \alpha_s(\mu^2)^{+2/9} \quad . \tag{7}$$

A simple factorization approach and vacuum insertion [1] gives $B(\mu) = 1$. The non-perturbative parameter B_K has been calculated using lattice gauge theories [18], $1/N_c$ expansion [19] and QCD sum rules [20] leading to $B_K = 0.7 \pm 0.2$. The QCD hadron duality approach [21] favours lower values $B_K = 0.4 \pm 0.1$.

2.2 D^0–$\overline{D^0}$

The effective hamiltonian (neglecting the contribution of the b quark due to the suppression of the relevant CKM matrix elements) is given by [3, 9]

$$\mathcal{H}_{eff}^{\Delta C=2} \ = \ -\frac{1}{2\pi^2}\, G_F^2\, \widetilde{\lambda}_s \widetilde{\lambda}_d\, \frac{(m_s^2 - m_d^2)^2}{m_c^2}\left(\mathcal{O}_1^{\Delta C=2} + 2\,\mathcal{O}_2^{\Delta C=2}\right) \ , \tag{8}$$

where $\widetilde{\lambda}_i = V_{ci}^* V_{ui}$ and the four-quark operators are defined as

$$\mathcal{O}_1^{\Delta C=2} \ = \ (\overline{u}_L^\alpha \gamma_\mu c_L^\alpha)(\overline{u}_L^\beta \gamma^\mu c_L^\beta) \ ,$$

$$\mathcal{O}_2^{\Delta C=2} \ = \ (\overline{u}_L^\alpha c_R^\alpha)(\overline{u}_L^\beta c_R^\beta) \ . \tag{9}$$

This last operator has no analogous in the K^0–$\overline{K^0}$ case and it is due to the non–negligible charm quark mass m_c carried by two of the four external legs of the box diagram.

Analogously to the K^0–$\overline{K^0}$ mixing the evaluation of the matrix elements of the two four–quark operators involves non–perturbative aspects that, this time, are parameterized by two quantities B_D and B'_D for the two operators in Eq. (9) respectively, and defined to be the unity if factorization and vacuum insertion approximations are used. Corrections to this value are potentially large but are not expected to change the order of magnitude.

At any rate from Eq. (8) we can conclude that D^0–$\overline{D^0}$ mixing is suppressed typically by a factor $\sim m_s^4/m_c^4$ over the K^0–$\overline{K^0}$ case.

3 The model

Our lagrangian is [13] :

$$\mathcal{L} = \mathcal{L}_{mesons} + \mathcal{L}_{Higgs} + \mathcal{L}_{HM} + \dots \quad , \tag{10}$$

where the dots are short for pure gauge boson terms. Here $\mathcal{L}_{mesons}$ contains the strong interaction between mesons and the couplings of mesons to the gauge bosons. We have a set of 16 scalar and 16 pseudoscalar fields which we assign to the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation of the chiral $U(4)_L \otimes U(4)_R$ group. We denote the meson matrix by $U = \Sigma + i\Pi$, where Σ is the scalar and Π the pseudoscalar matrices of fields. The explicit expression for the pseudoscalar matrix is

$$\Pi = \begin{pmatrix} \frac{1}{2}\eta_\circ + \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{12}}\eta_{15} & \pi^+ & K^+ & \overline{D^0} \\ \pi^- & \frac{1}{2}\eta_\circ - \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{12}}\eta_{15} & K^0 & D^- \\ K^- & \overline{K^0} & \frac{1}{2}\eta_\circ - \frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{12}}\eta_{15} & D_S^- \\ D^0 & D^+ & D_S^+ & \frac{1}{2}\eta_\circ - \frac{3}{\sqrt{12}}\eta_{15} \end{pmatrix} . \tag{11}$$

A similar matrix can be written for the scalar mesons. Our notation for scalar mesons is $\sigma_\circ, \sigma_8, \sigma_{15}, \sigma^+, \sigma_3, \kappa, \delta, \delta_S$ instead of $\eta_\circ, \eta_8, \eta_{15}, \pi^+, \pi^0, D$ and D_S respectively.

With these definitions $\mathcal{L}_{mesons}$ is

$$\mathcal{L}_{mesons} = \frac{1}{2}Tr[(D^\mu U')^\dagger(D_\mu U')] - V_{chiral}(U) \quad , \tag{12}$$

4

where V_{chiral} is the chiral potential

$$V_{chiral}(U) = -\mu_\circ^2 \, Tr(U^\dagger U) +$$
$$\mu_\circ^2 \left[a \, Tr(U^\dagger U)^2 + b \, (Tr(U^\dagger U))^2 + c \, (detU + detU^\dagger) \right] , \quad (13)$$

with $\mu_\circ^2 > 0$ in order to develop spontaneous breaking of chiral symmetry. The covariant derivative is:

$$D_\mu U' = \partial_\mu U' - ig\overrightarrow{T} \cdot \overrightarrow{W_\mu} U' - ig'Y_L B_\mu U' + ig'U'Y_R B_\mu , \quad (14)$$

with

$$U' = SUS^\dagger . \quad (15)$$

Here $\overrightarrow{W_\mu}$ and B_μ are the gauge bosons related with the $SU(2)_L$ and the $U(1)_Y$ groups. The $\overrightarrow{T}$ matrices are the $SU(2)$ generators and Y_L and Y_R are the left and right hypercharges. The matrix S will be the Cabibbo rotation once the $SU(2)_L \otimes U(1)_Y$ symmetry gets spontaneously broken. With our definitions for U we have

$$\overrightarrow{T} = \frac{1}{2} \begin{pmatrix} \overrightarrow{\tau} & 0 \\ 0 & \tau^1\overrightarrow{\tau}\tau^1 \end{pmatrix} , \qquad Y_L = \frac{1}{6}I_{4\times4} ,$$

$$Y_R = \frac{1}{3} \begin{pmatrix} 2 & & & \\ & -1 & & \\ & & -1 & \\ & & & 2 \end{pmatrix} , \quad S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_c & \sin\theta_c & 0 \\ 0 & -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \quad (16)$$

with $\overrightarrow{\tau}$ the usual Pauli matrices and θ_c the Cabibbo angle. The charge operator is $Q = Y_R = Y_L + T_3$.

In Eq. (10) $\mathcal{L}_{Higgs}$ is the usual lagrangian for the minimal model of Higgs of the Standard Theory. $\mathcal{L}_{HM}$, finally, is a Higgs–mesons coupling term which will give masses to the mesons after the spontaneous symmetry breaking of the weak symmetry. In order to write in a compact form this term we introduce the usual Higgs doublet as a 4×4 matrix:

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}}(\psi - i\chi) & s^+ & 0 & 0 \\ -s^- & \frac{1}{\sqrt{2}}(\psi + i\chi) & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}(\psi + i\chi) & -s^- \\ 0 & 0 & s^+ & \frac{1}{\sqrt{2}}(\psi - i\chi) \end{pmatrix} . \quad (17)$$

We consider only the simplest local gauge invariant term

$$\mathcal{L}_{HM} = Tr(AS^\dagger H^\dagger SU + h.c.) , \quad (18)$$

where S is given in Eq. (16) and the most general form of A assuming isospin symmetry is

$$A = \begin{pmatrix} \alpha & & & \\ & \alpha & & \\ & & \gamma & \\ & & & \delta \end{pmatrix} \; . \tag{19}$$

$\mathcal{L}_{HM}$ breaks explicitly the $SU(4) \otimes SU(4)$ chiral symmetry. Through the spontaneous breaking of the chiral and weak symmetry the matrices of mesons and Higgs get a non–zero vacuum expectation value

$$\langle \circ|U|\circ \rangle \equiv F = \frac{1}{\sqrt{2}} \begin{pmatrix} f_\alpha & & & \\ & f_\alpha & & \\ & & f_\gamma & \\ & & & f_\delta \end{pmatrix} , \qquad \langle \circ|H|\circ \rangle \equiv \frac{1}{\sqrt{2}} \phi_\circ I_{4 \times 4} \; . \tag{20}$$

Therefore from Eq. (18) we get

$$\mathcal{L}_{HM} = \frac{\phi_\circ}{\sqrt{2}} Tr(A(U + U^\dagger)) + Tr(AS^\dagger \tilde{H}^\dagger SU + h.c.) \; , \tag{21}$$

where

$$\tilde{H} = H - \langle \circ|H|\circ \rangle \; . \tag{22}$$

We note that the lagrangian in Eq. (18) transforms under chiral $SU(4) \otimes SU(4)$ as the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation and, therefore, the first term in Eq. (21) is similar to the usual explicit breaking of chiral symmetry.

Some aspects of this model are worth to emphasize:

a) The starting point of our ideas is to believe that the symmetries of the Standard Theory are essential to describe the weak processes of hadrons, and any model for them has to support not only its symmetries but its symmetry breaking patterns too. Our realization with two complete families allows to implement rigorously the structure of the SM at hadronic level.

b) The GIM mechanism is naturally implemented in our scheme. Therefore we have not flavour changing neutral currents at leading order in the perturbative expansion and in this way the model might presumably be trusted in the study of processes where GIM plays a significant rôle.

c) The model has been tested satisfactorily in the study of non–leptonic D decays into two pseudoscalars [22], non–resonant $D \to PPP$ [23] and also, at one–loop level, in radiative rare kaon decays ($K^+ \to \pi^+ \ell^+ \ell^-$, $K_S \to \pi^\circ \ell^+ \ell^-$) [13] with satisfactory results.

Hence we think it is worth to use the model just described in order to study K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixings. We have already seen in Section 2 that the short–distance contribution to K^0–$\overline{K^0}$ mixing is dominated by the charm degree of freedom while the strange quark dominates D^0–$\overline{D^0}$ mixing. Our model provides these ingredients and, moreover, allows the evaluation of long–distance dispersive contributions mediated by the pseudoscalar (and maybe scalar) mesons.

Therefore we proceed to calculate them at $\mathcal{O}(G_F^2)$ in the weak perturbation expansion. Due to our ignorance on the scalar masses and in order to present simplified analytical expressions, we first analyze the case in which $m_{scalar} \gg m_{pseudoscalar}$, that is $\mu_\circ^2 \to \infty$ in Eq. (13) but with $\mu_\circ^2 c \equiv c'$ constant in order to keep the $\eta_\circ$ mass finite. In this case F in Eq. (20) is

$$F = \frac{1}{\sqrt{2}} f_\circ \left[I_{4\times 4} + \mathcal{O}\left(\frac{m_{Pseudoscalar}^2}{\mu_\circ^2}\right) \right] \quad , \tag{23}$$

with

$$f_\circ^2 = \frac{1}{a + 4b} \quad . \tag{24}$$

With our normalization $f_\circ = f_\pi \sim 90 MeV$ in the chiral limit.

Using this expansion, at leading order, we get a relation between the pseudoscalar masses,

$$m_{D_S}^2 - m_D^2 = m_K^2 - m_\pi^2 \quad . \tag{25}$$

Eq. (23) is a crude approximation for the charm sector, we must remember here that $U(4)_L \otimes U(4)_R$ is not a symmetry for the strong interaction in nature. Therefore we have also considered the case of finite scalar masses in both calculations where one goes beyond the rough approximation represented by Eq. (23) and (25).

We use the renormalizable R_ξ gauge [24] where there is no mixing meson–W and instead we have a meson–charged Higgs and meson mixings as shown in Fig. 1.

At this order the two different topologies of Feynman diagram contributions to the mixing are those in Fig. 1. Both diagrams are separately finite but only the sum of the two is gauge independent.

4 K^0–$\overline{K^0}$ mixing

In order to test the model we can calculate Δm_K. The problem of evaluating long–distance contributions to the K^0–$\overline{K^0}$ mixing has been addressed extensively in the bibliography [7, 10, 25, 26]. These are due to intermediate hadronic states in the oscillation and are presumably dominated by the octet of pseudoscalars: $K^0 \leftrightarrow (\pi^0, \eta) \leftrightarrow \overline{K^0}$, $K^0 \leftrightarrow \pi\pi \leftrightarrow \overline{K^0}$, etc. An estimate of these contributions gives results comparable with the experimental mass difference. However the evaluation has significant uncertainties.
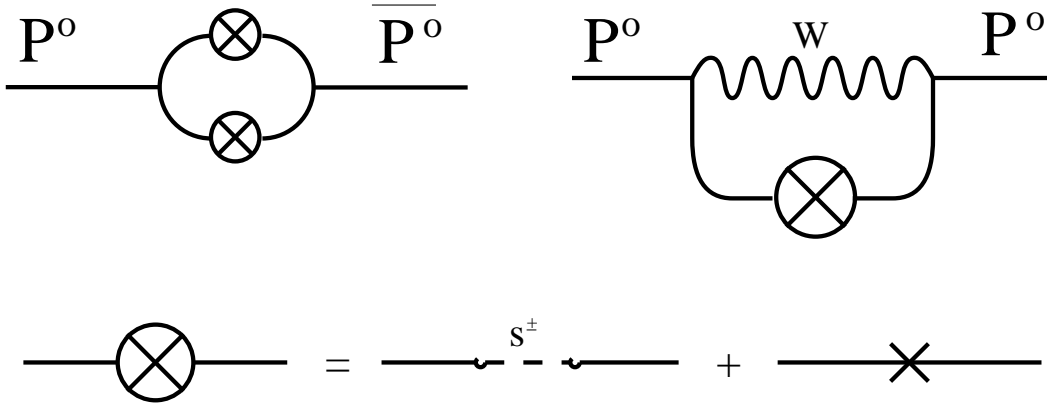
Figure 1: The loop diagrams are the contribution of the model to the K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixing. The crossed propagator is the sum of the Higgs $s^\pm$ contribution plus the meson mixing.

The computation of the K^0–$\overline{K^0}$ mixing in our model through the Feynman diagrams of Fig. 1 gives for the difference of masses:

$$
\begin{aligned}
\Delta m_K \;=\;& \frac{G_F^2 \sin^2\theta \cos^2\theta f_\pi^2}{8\pi^2 m_K} \Big[(m_D^2 - m_\pi^2)^2 + (m_D^2 - m_\pi^2)(m_{D_S}^2 \ln m_{D_S}^2 - m_K^2 \ln m_K^2) \\
& + m_D^2(m_D^2 + m_K^2) \left(\frac{m_{D_S}^2}{m_D^2 - m_{D_S}^2} \ln \frac{m_D^2}{m_{D_S}^2} - \frac{m_K^2}{m_D^2 - m_K^2} \ln \frac{m_D^2}{m_K^2} \right) \\
& + m_\pi^2(m_\pi^2 + m_K^2) \left(\frac{m_K^2}{m_\pi^2 - m_K^2} \ln \frac{m_\pi^2}{m_K^2} - \frac{m_{D_S}^2}{m_\pi^2 - m_{D_S}^2} \ln \frac{m_\pi^2}{m_{D_S}^2} \right) \\
& - \frac{1}{2}(m_K^2 - m_{D_S}^2)^2 P(m_{D_S}^2, m_{D_S}^2) - \frac{1}{2}(m_K^2 - m_D^2)^2 P(m_D^2, m_D^2) \\
& - \frac{1}{2}(m_K^2 - m_\pi^2)^2 P(m_\pi^2, m_\pi^2) + (m_K^2 - m_D^2)(m_K^2 - m_\pi^2) P(m_D^2, m_\pi^2) \Big],
\end{aligned}
$$

(26)

where $P(m_1^2, m_2^2)$ is defined through

$$
I_2(m_1^2, m_2^2) \;=\; -\frac{i}{16\pi^2} \left\{ \frac{2}{D-4} + \gamma - \ln(4\pi) - 2 + P(m_1^2, m_2^2) \right\} \;,
\qquad (27)
$$

and $I_2(m_1^2, m_2^2)$ the scalar two–point function.

Several comments are in order about our result Eq. (26):

- Our result is finite and only depends on the masses of the pseudoscalar mesons and the meson decay constants once the mass expansion induced by the assumption $m_{scalar} \gg m_{pseudoscalar}$ is carried out.

- We can easily see the effect of the GIM cancellation (in the $SU(4)_F$ limit $m_u = m_c$) by inputting $m_D = m_\pi$ and $m_K = m_{D_S}$ in Eq. (26) that gives evidently $\Delta m_K = 0$.

8

- From the observation that the short–distance contribution to Δm_K scales with m_c^2 in Eq. (3) one should expect in our model that our result for Δm_K might scale like m_D^4. However using the mass relation Eq. (25) in Eq. (26) we see that the term in m_D^4 cancels. This fact provides a further suppression beyond GIM mechanism.

Using the mass relation in Eq. (25) and expanding the logarithms in turn the result in Eq. (26) can be written as

$$\Delta m_K = \frac{1}{4\pi^2} G_F^2 \, \sin^2\theta \, \cos^2\theta \, f_\pi^2 \, \frac{19}{12} \, (1 + \omega_K) \, m_D^2 \, m_K \quad . \tag{28}$$

where $\omega_K = \mathcal{O}\left(\dfrac{m_{K,\pi}^2}{m_D^2}\right)$ and our result gives $\omega_K \simeq -0.09$ providing a tiny 10% correction to the leading term. Using the values of the decay constants and masses of [5] we get

$$\Delta m_K = 3.45 \times 10^{-15} \, GeV \quad . \tag{29}$$

Inputting the experimental result for Γ_S [5] we predict $x_K = 0.49$. Our results are in rather good agreement with the experimental figure [5]

$$\Delta m_K^{exp} = (3.491 \pm 0.009) \times 10^{-15} \, GeV \quad . \tag{30}$$

This fact can be understood because the dispersive part of the amplitude is dominated by the $\Delta I = 3/2$ transitions and, therefore, through the $\langle \pi^+\pi^\circ | \mathcal{H}_W | K^+ \rangle$ amplitude [25] that is well recovered in our model.

One could also consider the corrections coming from finite scalar masses. These happen to depend only on two parameters : f_γ / f_α defined in Eq. (20) and c' related to the masses of the mesons η and η'. This dependence is showed in Fig. 2. Inside reasonable values of these parameters our prediction Δm_K in Eq. (29) could only be modified in a factor 2. We can conclude that the model gives reasonable values for Δm_K, in the range $(1.8 - 3.5) \times 10^{-15} \, GeV$.

5 D^0–$\overline{D^0}$ mixing

As has been noted previously in Section 2 the short–distance contribution to the D^0–$\overline{D^0}$ mixing is very much suppressed relatively to the K^0–$\overline{K^0}$ mixing, roughly a factor m_s^4/m_c^4. A careful evaluation shows that the suppression is not that much but in any case the perturbative contribution gives

$$\Delta m_D^{box} \simeq 10^{-2} \, \Delta m_K^{box} \quad . \tag{31}$$

The study of long–distance contributions to D^0–$\overline{D^0}$ mixing has been considered previously [8, 9, 11] with very different conclusions. While Wolfenstein [8], Donoghue et al. [9] and

Kaeding [12] conclude that the dispersive non–perturbative contribution to D^0–$\overline{D^0}$ mixing might be huge at the level of providing $\Delta m_D \simeq \Delta m_K$ due to the fact that there is a strong $SU(3)$ breaking that overcomes the GIM suppression, Georgi [11] using arguments of a Heavy Quark expansion concludes that this is unlikely, and a careful study in this framework [27] seems to confirm this assessment.

The evaluation of Δm_D at leading order in our model goes again through the corresponding Feynman diagrams in Fig. 1. By expanding in $m_{Pseudoscalar}/m_{Scalar}$ and keeping the leading term we get

$$
\begin{aligned}
\Delta m_D &= \frac{G_F^2 \sin^2 \theta \cos^2 \theta f_\pi^2}{8\pi^2 m_D} \Big[(m_K^2 - m_\pi^2)^2 + (m_K^2 - m_\pi^2)(m_{D_S}^2 \ln m_{D_S}^2 - m_D^2 \ln m_D^2) \\
&\quad + m_K^2 (m_K^2 + m_D^2) \left(\frac{m_{D_S}^2}{m_K^2 - m_{D_S}^2} \ln \frac{m_K^2}{m_{D_S}^2} - \frac{m_D^2}{m_K^2 - m_D^2} \ln \frac{m_K^2}{m_D^2} \right) \\
&\quad + m_\pi^2 (m_\pi^2 + m_D^2) \left(\frac{m_D^2}{m_\pi^2 - m_D^2} \ln \frac{m_\pi^2}{m_D^2} - \frac{m_{D_S}^2}{m_\pi^2 - m_{D_S}^2} \ln \frac{m_\pi^2}{m_{D_S}^2} \right) \\
&\quad - \frac{1}{2}(m_D^2 - m_{D_S}^2)^2 P(m_{D_S}^2, m_{D_S}^2) - \frac{1}{2}(m_D^2 - m_K^2)^2 P(m_K^2, m_K^2) \\
&\quad - \frac{1}{2}(m_D^2 - m_\pi^2)^2 P(m_\pi^2, m_\pi^2) + (m_D^2 - m_K^2)(m_D^2 - m_\pi^2) P(m_K^2, m_\pi^2) \Big],
\end{aligned}
\tag{32}
$$

where $P(m_1^2, m_2^2)$ has been defined in Eq. (27). Again we find a finite result in terms of the pseudoscalar masses and the meson decay constants. Moreover GIM suppression (for exact $SU(3)_F$ symmetry, $m_d = m_s$) is translated now into the meson language as $m_K = m_\pi$ and $m_{D_S} = m_D$ that gives $\Delta m_D = 0$.

Using the mass relation in Eq. (25) and expanding over the charmed meson mass we have:

$$
\Delta m_D = \frac{1}{4\pi^2} G_F^2 \sin^2 \theta \cos^2 \theta f_\pi^2 \left(1 - \frac{\pi}{4\sqrt{3}} \right) (1 + \omega_D) \frac{m_K^4}{m_D} \quad , \tag{33}
$$

with $\omega_D \simeq 0.05$ carrying the contribution of higher order in $m_{K,\pi}^2/m_D^2$. As we see the Δm_D scales with the mass of the kaon to the fourth over the mass of the charmed meson. We notice that our result implies a suppression

$$
\Delta m_D \sim \frac{m_K^3}{m_D^3} \Delta m_K \ . \tag{34}
$$

From Eq. (33) we predict

$$
\Delta m_D = 2.21 \times 10^{-17} \, GeV \quad , \tag{35}
$$

and with the experimental Γ_D [5] we have $x_D \simeq 1.4 \times 10^{-5}$. There is only an experimental upper limit on Δm_D [5]

$$
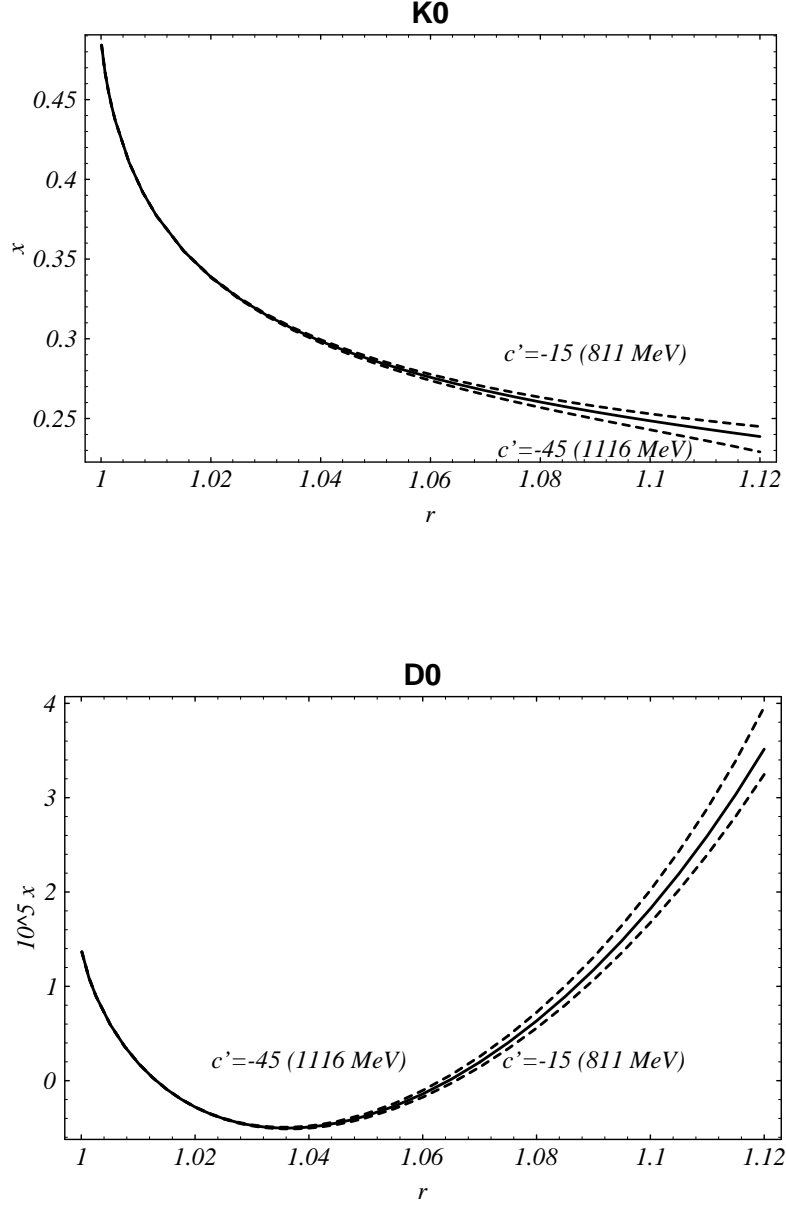|\Delta m_D^{exp}| < 1.38 \times 10^{-13} \, GeV \qquad (90\%CL) \quad , \tag{36}
$$

10

Figure 2: $x = \Delta m_P / \Gamma$ as function of $r = f_\gamma / f_\alpha$. This ratio is related to the mass of the scalar mesons. The left-hand side ($r = 1$) of the plots correspond to the limit $m_{scalars} \longrightarrow \infty$ and the right-hand side to $m_{scalars} \sim 2\ GeV$. The parameter $c' = -15, -30, -45$ is related to the mass of the η' meson; we indicate the corresponding mass in brackets.

that is four orders of magnitude bigger than our result.

However, there are many dynamical effects in the energy region of $1 - 2\,GeV$ which could influence the final states in D decays and then modify our prediction. Analogously to the K^0–$\overline{K^0}$ mixing we can give an estimate of the dependence of Δm_D on the masses of the scalar mesons (Fig. 2). In this case the dependence is bigger than in Δm_K but for reasonable values of f_γ/f_α and c' there is no change in the order of magnitude estimate. Moreover we notice that the sign of Δm_D could be changed due to the contribution of the scalar mesons. We think that, in the SM, a bigger order of magnitude for the D^0–$\overline{D^0}$ mixing than our result implied by Eq. (35) could only be expected through a large $SU(3)_F$ breaking possibly induced by final–state interactions generated through vector meson resonances. This effect we know it is important in non–leptonic decays of charmed mesons [28] and therefore could be relevant in the D^0–$\overline{D^0}$ mixing. However, it can be expected a cancellation in the total contribution of the vector mesons due to the GIM mechanism. Actually we expect, from these new intermediate channels, contributions without changing noticeably the order of magnitude. Once incorporated all the contributions, the final–state interactions correspond to a unitarity transformation and then it is not expected a modification in the mixing.

A result for Δm_D as given by our prediction in Eq. (35), if true, is out of the experimental reach in the foreseen future. However an experimental result at level of $\Delta m_D > 10^{-16}\,GeV$ could be a signal of new effects outside of the SM. An overview of the experimental techniques and problems has been given in [29]. The recent experimental determination of the doubly Cabibbo suppressed channel $D^0 \to K^+\pi^-$ [5], even still with a big error to reduce [2], represents a good step forward in this direction.

6   Conclusion

We have studied the K^0–$\overline{K^0}$ and D^0–$\overline{D^0}$ mixings in a weak gauged $U(4)_L \otimes U(4)_R$ chiral lagrangian model that incorporates the GIM mechanism at hadronic level. We have got a result for Δm_K in rather good agreement with the experimental result and a prediction for Δm_D that shows a strong GIM cancellation and provides x_D in the range $(4, -1) \times 10^{-5}$. Our result is in good agreement with previous estimates in the framework of a Heavy Quark expansion [11, 27]. Other calculations giving largest Δm_D [9] are not really in contradiction with our result. Reevaluating the estimate in [9] with the actual experimental values for the widths we obtain more cancellation than the originally assumed in [9]. Using theoretical results for the widths from [22] and Eq. (32) we can also observe that the phenomenological expression used in [9] must be corrected, giving

---

[2]In [22] we predicted, with the model used here, this width and our result $\Gamma(D^0 \to K^+\pi^-) = 3.3 \times 10^{-16}GeV$ is in very good agreement with the experimental figure $\Gamma(D^0 \to K^+\pi^-) = (4.6 \pm 2.2) \times 10^{-16}GeV$.

additional cancellations. From the present analysis and previous works on the subject we can safely conclude that a value of $|x_D|$ over 10^{-4} would be a clear signal of new physics in the charm sector.

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