A (p/E) CALCULATION OF STRONG PIONIC DECAYS OF BARYONS

F. Cano[1], P. González[1], B. Desplanques[2] and S. Noguera[1]

[1] Departamento de Física Teórica and IFIC,
Centro Mixto Universidad de Valencia-CSIC
46100 Burjassot (Valencia), Spain

[2]Institut des Sciences Nucléaires, F-38026
Grenoble Cedex, France

Abstract

Strong pionic decays of baryons are studied in a non–relativistic quark model framework via a convergent (p/E) expansion of the transition operator. Results are compared to the ones obtained within a more conventional (p/m) expansion.

PACS: 12.39.Jh, 14.20.Gk, 13.30.Eg
Keywords: Non-relativistic quark models, strong baryonic decays

1 Introduction

There is a current experimental and theoretical interest in the study of the baryon decays (see for instance [1]). Measurements which are planned at different experimental facilities, TJNAF (CEBAF), MAMI, ELSA, GRAAL, aim to answer open questions about the baryon resonances, in particular having to do with the position of the Roper resonance in the baryon spectrum and with the absence of strong experimental evidence for many resonances around 1.8-2 GeV. Both subjects are closely related to the study of the baryon decay amplitudes. Some authors have proposed that small couplings in the $N\pi$ production channel can provide an explanation for the missing states [2]. On the other hand the analysis of the corresponding decays is essential to any attempt to establish the true nature of the Roper resonance for which several alternatives have been proposed (hybrid three quark+one gluon state, first radial nucleon excitation...) Hence, the study of the $N\pi$ decay amplitudes becomes of great interest, as a test for the proposed models and in order to have a guide-line for future experiments. In this article we shall center our attention at the lowest energy decays for reasons that will become clear later on.

In a previous paper [3] we put the emphasis in the construction of the effective transition operator for $B \rightarrow B'\pi$ decays, making clear not only the very relevant known role played by the pion structure (otherwise said, by the $qqq\ q\bar{q}$ effective component of the baryon wave function) but also the need of taking into account relativistic effects. Here we go further in this analysis and show that 'better accounted' relativistic corrections through a convergent (p/E) (instead of (p/m)) expansion does not spoil our results but instead confirm in a more sound way our previous conclusions.

For the sake of completeness we briefly review next the quark models used in ref. [3] to calculate the baryon wave functions which we shall employ for estimating baryon decay transition amplitudes. There are many other competitive models available in the literature. However for our purpose here, the comparison of a (p/E) expansion calculation against a (p/m) one, we do not feel our particular choice to suppose any serious limitation.

2 The Quark Models.

A precise description of the octet and decuplet baryonic spectrum (with and without strangeness) including the second excitation energy have been consistently obtained by solving the Schrödinger equation with a potential containing apart from the 'minimal ingredients' (confinement + (coulomb + spin–spin) one gluon exchange interactions) a three quark phenomenological force [4]. Specifically the expression used for the potential is:

$$V = V^{(2)} + V^{(3)} \tag{1}$$

$$V^{(2)} = \sum_{i<j} \frac{1}{2} \left[\frac{r_{ij}}{a^2} - \frac{\kappa}{r_{ij}} + \frac{\kappa}{6 m_i m_j} \frac{\exp(-r_{ij}/r_0)}{r_0^2 r_{ij}} \vec{\sigma}_i \vec{\sigma}_j - D \right] \tag{2}$$

$$V^{(3)} = \sum_{i \neq j \neq k \neq i} \frac{1}{2} \frac{V_0}{m_i m_j m_k} \frac{e^{-m_0 r_{ij}}}{m_0 r_{ij}} \frac{e^{-m_0 r_{ik}}}{m_0 r_{ik}} \tag{3}$$

where a, κ, r_0, V_0, m_0 are free parameters fixed from the spectrum and the the quark masses are chosen to get the baryon magnetic moments (see table 1). r_{ij} is the interquark distance and the σ's are the Pauli matrices. D is a constant to fix the absolute value of the nucleon mass to its experimental value.

Such a model is adequate to solve altogether some endemic problems concerning the baryon spectrum, say, a unified description of the positive and negative parity states, the correct position of the Roper resonances (first radial excitations) and the appearance of extra Δ negative parity states at a relatively low energy ($\Delta(1/2^+)$ at 1900 MeV and $\Delta(5/2^+)$ at 1930 MeV). Moreover, when combined with an improved transition operator, the $B \to B'\pi$ decay widths for the lowest energy non-strange resonances are in very good agreement (to the standard in the field) with the experimental data [3].

As a side effect, the splitting between the two lowest $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states, is rather small for all the baryons studied (related to the absence of one gluon exchange tensor hyperfine or spin–orbit forces) and an unobserved proliferation of states would come out if we were to push the model further up the second excitation energy (maybe denoting the energy limit of the model description or maybe indicating the presence of missing states).

We shall compare the results from the $(V^{(2)} + V^{(3)})$ potential model with the ones obtained with a two-body $V^{(2)}$ potential model with parameters fitted to get an overall fit to the meson and baryon spectrum [5], the aim being to try to extract general features associated to 'two–body' models and to make clear, if so, some bias of the results from our 'three–body' approach. The small core size typical in spectroscopic models ($\langle r^2 \rangle^{1/2} \approx 0.47$ fm for $V^{(2)}$) makes the speed of the quarks be close to 1. As a matter of fact $(p/m) \gtrsim 1$. In this sense a (p/E) expansion is more founded and incorporates automatically some relativistic corrections.

Our results are expressed in terms of the width for each process. In all cases, the amplitudes have been calculated with the model wave functions, but we have used the physical masses for the kinematical factors. This is important for the $V^{(2)}$ potential model due to the discrepancies of its predictions with the experimental masses.

Several mechanisms have been proposed to study the $B \to B'\pi$ decays.

3 Decay Mechanisms and Results.

The **elementary emission model** considers the point-like pion emission by one of the quarks of the baryon. The transition operator is obtained via the non–relativistic reduction of the $qq\pi$ interaction for which a pseudovector form is used. The (p/m) order results have been published elsewhere [3]. The predicted widths of some low-lying non–strange baryons in the elementary emission model are summarised in tables 2a and 2b, showing a general disagreement with experiment. Furthermore, the calculation of the quark matrix element $\bar{u}(p)\gamma_\mu\gamma_5 u(p)$ at the $(p/m)^2$ order has been worked out [7] (we do not use the recalculated value of $f_{qq\pi}$ in order not to have an even bigger contribution), making clear the relevance of the corrections and the lack of convergence hence raising serious doubts about the adequacy to proceed to such an expansion.

These problems can be partially palliated by proceeding to a (p/E) expansion ($E = \sqrt{p^2+m^2}$) of the transition operator (for obvious reasons we shall work in momentum space). It is very instructive to do it order by order so that we can check the convergence of the (p/E) expansion and compare it to the (p/m) one. Thus the amplitude at (p/E) order, can be written as:

$$\langle B'(J'_z=\lambda),\pi^\alpha|H|B(J_z=\lambda)\rangle = \frac{1}{(2\pi)^{3/2}}\delta(\vec{P}_B-\vec{P}_{B'}-\vec{k})A_\lambda \tag{4}$$

$$\begin{aligned} A_\lambda &= -\frac{3i}{(2\omega_\pi)^{1/2}}\frac{f_{qq\pi}}{m_\pi}\int d\vec{p}_{\xi_1}\, d\vec{p}_{\xi_2} \qquad (5)\\ &\left[\Psi_{B'}(\vec{p}_{\xi_1},\vec{p}_{\xi_2}+\sqrt{\frac{2}{3}}\vec{k})\Phi_{B'}(M',M'_S;I',M'_I)\right]^*_{J'\lambda}\\ &(\tau^{\alpha\,(3)})^\dagger\vec{\sigma}^{(3)}\left[\vec{k}\left(1+\frac{\omega_\pi}{6E'}\right)\right.\\ &\left.+\frac{\omega_\pi}{2}\sqrt{\frac{2}{3}}\left(\frac{\vec{p}_{\xi_2}}{E}+\frac{\vec{p}_{\xi_2}+\sqrt{\frac{2}{3}}\vec{k}}{E'}\right)\right].\\ &[\Psi_B(\vec{p}_{\xi_1},\vec{p}_{\xi_2})\Phi_B(S,M_S;I,M_I)]_{J\lambda} \qquad (6)\end{aligned}$$

where J,J' are the total baryon angular momenta, $\vec{P}_{B,B'}$ and $\vec{k}$ the three-momenta of the baryons and pion respectively, ω_π (m_π) is the pion energy (mass), E (E') refers to the energy of the incoming (outgoing) quark in the emission, Ψ (Φ) stands for the momentum space (spin–isospin) wave function, $\vec{\tau}^{(3)}$ ($\vec{\sigma}^{(3)}$) is the isospin (spin) Pauli matrix acting on the emitting quark, and $f_{qq\pi}$ is the coupling constant in the $qq\pi$ vertex and $\vec{p}_{\xi_1}$, $\vec{p}_{\xi_2}$ are the momenta associated to quark Jacobi coordinates.

The first term in the intermediate square bracket appearing in (5), arises (except for the $\frac{\omega_\pi}{6E'}$ contribution) from the spatial components while all the other terms, proportional to ω_π, arise from the time component of the current which is coupled to the derivative of the pion field. It is easy to verify that eq. (5) has well defined properties (change in sign) when exchanging the role of the final and the initial baryon. Let's also note that the pseudo-scalar πqq coupling leads to an expression similar to (5) where ω_π is replaced by the difference of the kinetic energies of quarks in the initial and final states, which is nothing else than the corresponding contribution to ω_π.

Beyond the p/E order, typical terms like $\vec{p}/(E+m)$, that appear in the Dirac spinors will be expanded as

$$\frac{\vec{p}}{E+m} = \frac{\vec{p}}{2E}(1 + \frac{p^2}{4E^2} + ...), \tag{7}$$

whereas a p/m expansion would give

$$\frac{\vec{p}}{E+m} = \frac{\vec{p}}{2m}(1 - \frac{p^2}{4m^2} + ...), \tag{8}$$

As usual, we shall work in the center of mass system of the decaying baryon, i.e. $\vec{P}_B = 0$. The only free parameter $f_{qq\pi}$ is fixed from the $NN\pi$ form factor at zero momentum transfer ($|\vec{k}| = 0$). Additionally, the $NN\pi$ form factor is predicted at $|\vec{k}| \neq 0$ (both models do very well for low $|\vec{k}|$). $f_{qq\pi}$ is determined to the order of the calculation from $f_{NN\pi}(0)$.

The results for the same set of widths are shown in tables 2a and 2b. By comparing columns four, five and six (this one corresponding to the complete calculation for the pseudovector form) with column two and three, the better convergence of the (p/E) calculation becomes obvious for both quark models. In the p/E expansion, the $(p/E)^2$ correction make the result to tend smoother to the final one, while in the p/m expansion the $(p/m)^2$ correction either go in the wrong direction or make the result to overshoot the final one. Strictly speaking only for some cases: $\Delta(1232) \to N\pi$, $N(1520) \to N\pi$, $N(1535) \to \Delta\pi$, $N(1650) \to \Delta\pi$ and $N(1700) \to N\pi$, a first order (p/E) treatment is justified whereas for $N(1440) \to \Delta\pi$ and $N(1520) \to \Delta\pi$ it should be taken with caution. The situation becomes more dramatic when considering higher excitations for which we do not find a justification neither to a (p/m) nor to a (p/E) expansion. Let us note that for $N(1440) \to N\pi$, $\Delta(1600) \to \Delta\pi$ decay amplitudes, the first order is suppressed by the orthogonality of the baryonic radial wave functions. The corrections $(p/m)^2$ obviously break the orthogonality argument, hence the effect is large for these transitions. Instead, for corrections $(p/E)^2$, which are close to the unit operator, the orthogonality argument seems to work again, hence the smaller effects.

Differences with the data require to look back at the reaction mechanism; in particular one can wonder whether the pion structure may play or not a significant role.

The **$^3P_0$ quark pair creation model** considers the creation of a $q\bar{q}$ pair in the hadronic medium that by later recombination gives rise to the outgoing pion. The effective transition operator is [8, 9]:

$$T = -\sum_{i,j} \int d\vec{p}_q d\vec{p}_{\bar{q}} \left[3\gamma\delta(\vec{p}_q + \vec{p}_{\bar{q}}) \sum_m (110|m,-m) \mathcal{Y}_1^m(\vec{p}_q - \vec{p}_{\bar{q}}) \mathcal{Z}_{i,j}^{-m} \right] \cdot b_i^\dagger(\vec{p}_q) d_j^\dagger(\vec{p}_{\bar{q}}) \tag{9}$$

where $b_i^\dagger$ ($d_j^\dagger$) are the i-quark (j-antiquark) creation operators, $\vec{p}_q$ ($\vec{p}_{\bar{q}}$) is the three-momentum of the quark (antiquark) of the pair, $\mathcal{Y}_1^m$ is the solid harmonic polynomial, $\mathcal{Z}$ is the color–spin–isospin wave function of the pair, (110|m,-m) is the SU(2) Clebsch–Gordan coefficient and γ is the coupling constant at the vacuum–$q\bar{q}$ vertex. Following our criterium given above, γ is extracted from $f_{NN\pi}(0)$.

Then the transition matrix element reads:

$$\langle B'\pi|T|B\rangle = -3\gamma \sum_m (110|m,-m) I_m \tag{10}$$

where

$$I_m = \delta(\vec{P}_B - \vec{P}_{B'} - \vec{k}) \int d\vec{p}_{\xi_1}\, d\vec{p}_{\xi_2} \; \mathcal{Y}_1^m \left[-\frac{4}{3}\vec{k} - \sqrt{\frac{2}{3}} \left(\vec{p}_{\xi_2} + (\vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k}) \right) + \frac{2}{3}\vec{P}_B \right] \Phi_{\rm Pair}^{-m} \cdot \left[\Psi_{B'}(\vec{p}_{\xi_1}, \vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k}) \Phi_{B'} \right]^*_{J'\lambda} \left[\Psi_\pi(-\sqrt{\frac{2}{3}}\vec{p}_{\xi_1} + \frac{\vec{P}_B}{3} - \frac{\vec{k}}{2}) \Phi_\pi \right]^* \cdot \left[\Psi_B(\vec{p}_{\xi_1}, \vec{p}_{\xi_2}) \Phi_B \right]_{J\lambda} \tag{11}$$

where $\Phi_{\rm Pair}^{-m}$ is the spin-isospin wave function of the pair and Ψ_π (Φ_π) refers to the momentum space (spin-isospin) wave function of the pion. We choose for Ψ_π a gaussian form with the parameter $R_A^2 = 8$ GeV^{-2} in order to reproduce the root mean square radius of the pion.

The decay widths, calculated elsewhere [3], are compiled in tables 3a and 3b. A comparison to the second column of tables 2a and 2b shows clearly the relevance of the pion structure. However, the predicted values differ very much from the experimental data. It is very illuminating to take the point-like pion limit of the transition matrix element $< B'\pi|T|B >$. In ref. [3] this was done in configuration space and compared to the (p/m) order expression suggesting a way to introduce relativistic–like corrections in the $^3P_0$ scheme. The results obtained in this manner (we shall call it modified $^3P_0$ model (M$^3P_0$)) shown in tables 3a and 3b, represent a rather amazing improvement of the predictions. Here we repeat the procedure in momentum space in order to get a 'more convergent' (p/E) version of the modified $^3P_0$ model.

In the point-like limit we get:

$$\begin{aligned}\langle B'\pi|T|B\rangle \;\; \rightarrow \;\; & \frac{1}{(2\pi)^{3/2}}(-\gamma 3\sqrt{3}\pi)\int d\vec{p}_{\xi_1}\, d\vec{p}_{\xi_2} \\ & \left[\frac{4}{3}\vec{k} + \sqrt{\frac{2}{3}}\left[\vec{p}_{\xi_2} + (\vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k})\right]\right]\vec{\Phi}_{\rm Pair}\,\cdot \\ & \left[\Psi_{B'}(\vec{p}_{\xi_1}, \vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k})\Phi_{B'}\right]^*_{J'\lambda} \Phi^*_M \left[\Psi_B(\vec{p}_{\xi_1}, \vec{p}_{\xi_2})\Phi_B\right]_{J\lambda} \quad (12)\end{aligned}$$

This expression reproduces (5) under the formal substitutions:

$$3\sqrt{3}\frac{\pi}{4}\gamma \;\; \rightarrow \;\; \frac{3if_{qq\pi}}{(2\omega_\pi)^{1/2}m_\pi} \qquad (13.a)$$

$$\frac{4}{3} \;\; \rightarrow \;\; \left(1 + \frac{\omega_\pi}{6E'}\right) \qquad (13.b)$$

$$\vec{p}_{\xi_2} + (\vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k}) \;\; \rightarrow \;\; \frac{\omega_\pi}{2}\left[\frac{\vec{p}_{\xi_2}}{E} + \frac{(\vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k})}{E'}\right] \qquad (13.c)$$

Eq. (13.a) points out an energy dependence (through the factor $\omega_\pi^{1/2}$) of the coupling constant γ that can be associated to the boost from the rest frame of the pion to the decaying baryon rest frame. Moreover the expressions on the right hand side of (13.b) and (13.c) reduce to the left hand side in the very non–relativistic limit $E \sim E' \sim m$, $(\omega_\pi)_{^3P_0} \sim 2m$.

Hence we can go the other way around and use the right hand side of eq. (13) as the relativistic expressions of the left hand side, to modify the $^3P_0$ effective transition operator. Technically the substitutions translate in the modification of the argument of the spherical harmonic plus an energy dependent factor:

$$\begin{aligned}& \mathcal{Y}_1^m\left[-\tfrac{4}{3}\vec{k} - \sqrt{\tfrac{2}{3}}\left(\vec{p}_{\xi_2} + (\vec{p}_{\xi_2} + \sqrt{\tfrac{2}{3}}\vec{k})\right)\right] \\ \rightarrow \;\; & \sqrt{\tfrac{m_\pi}{\omega_\pi}}\mathcal{Y}_1^m\left[-\vec{k}\left(1 + \tfrac{\omega_\pi}{6E'}\right) + \tfrac{\omega_\pi}{2}\left(-\sqrt{\tfrac{2}{3}}\right)\left(\tfrac{\vec{p}_{\xi_2}}{E} + \tfrac{\vec{p}_{\xi_2} + \sqrt{\frac{2}{3}}\vec{k}}{E'}\right)\right]\end{aligned} \qquad (14)$$

Thus, the transition operator evidences symmetry properties expected from the elementary coupling of the derivative of the pion field to a current involving baryons. This 'relativized' $^3P_0$ model that we shall call R$^3P_0$ from now on confirm the good predictions the M$^3P_0$ version gave (the $NN\pi$ form factor is very similar to the one obtained with the usual $^3P_0$ model). It is worthwhile to mention the excellent fit we get for the radial decays $N(1440) \to N\pi$, $\Delta(1600) \to \Delta\pi$, due entirely to the incorporation of the pion structure not having any need to consider any exotic nature for them. To this respect, it should be mentioned that the $(p/E)^2$ and higher order corrections to them, although they were not small in the elementary emission model, are not very relevant compared to those arising from the pion structure. We get a good fit (exceptions are the $N(1535) \to \Delta\pi$ and $N(1650) \to \Delta\pi$) even for some non-justified first-order (p/E) treated cases maybe indicating the reabsortion of some relativistic effects through the only parameter.

The $N(1535) \to N\pi$ and $N(1650) \to N\pi$ decay widths are very badly described. For this several reasons can be pointed out. Higher order corrections are very significant as can be easily inferred from the tables. Besides, for these decays one should be aware of the presence of threshold effects [10] (the coupling for the $N\eta$ channel is important). Additionally the lack of a tensor interaction at our model prevents mixing effects that could be relevant. To this respect we have evaluated the mixing that a tensor potential with the OGE parameter we have used would introduce. Though the mixing angle ($\vartheta \approx -40^{\rm O}$ is similar to the obtained with other models it does not translate in ours in a general improvement of all the $N(1535)$ and $N(1650)$ decays.

In all cases, the effect of the emitted baryon width [11] and the final state interactions which have been neglected might play some role as well. Furthermore an extended pion-quark vertex could be considered through a form factor multiplying $f_{qq\pi}$ (or γ). An estimation of its importance for a standard dipole type form factor [12] gives corrections to the widths of a 20 % at most. It is also worthwhile to realize the better predictions on the average that three–quark potential provide. They support the results for the baryonic spectrum and may indicate the convenience of the presence of high momentum components in the baryon wave function at low energies, though no strong conclusion should be derived from our restricted calculation.

Other treatments of strong decays can be found in the literature within a $^3P_0$ scheme [13] getting a reasonable overall fit to a much wider range of data than considered here. However the adopted philosophy is quite a different one; we put the emphasis in the (p/E) expansion that allows the study of better relativistically controlled processes and show that a precise simultaneous description of the spectrum and the decays is feasible in some cases.

4 Summary.

We have performed a detailed analysis of strong pionic decay widths from a non–relativistic scheme, first following a (p/m) expansion approach, second using for the energy its relativistic expression ($E = \sqrt{m^2 + p^2}$) instead of the $E \approx m$ non-relativistic limit. A clear improvement of the convergence is obtained and a justification of a (p/E) treatment is obtained for some cases. The strong remaining discrepancies between our theoretical predictions and experiment has led us to analyse through a ${}^3P_0$ pair creation model the influence of the $q\bar{q}$ content of the pion in the decay width and the need of introducing relativistic–like corrections into the effective transition operator. These relativistic corrections have basically a kinematical origin An improvement of the (p/m) results for the examined cases is achieved in good agreement with experimental data even for some non–rapidly convergent processes. Let us mention in particular that the sizeable transitions between the N and Δ ground states and their radial excitations originate from the pion structure and not from the $(p/m)^2$ terms in the operator as one may naively expect from the fact that the orthogonality of the states is of no relevance in this case. The validity of the argument is restored when the full series of higher order terms is considered. Being conscious of possible minor improvements that could be considered, as a general conclusion we might say that in order to do better for the pionic decay widths a more complete treatment of relativistic effects and the $q\bar{q}$ structure altogether in a consistent scheme seems to be unavoidable

This work has been partially supported by DGES under grants PB95-1096, PB94-0080 and by EC-TMR network HaPHEEP under contract ERBTMRX CT96-0008. F.C. acknowledges the Ministerio de Educación y Ciencia for a FPI fellowship.

References

[1] Proc. of Baryons' 95 Conference, Santa Fe, 1995, ed. B.F. Gibson et al., (World Scientific, 1996).

[2] S. Capstick and W. Roberts, Phys Rev D47 (1993) 1994.

[3] F. Cano et al., Nucl. Phys A603 (1996) 257.

[4] B. Desplanques et al., Z. Phys. A343 (1992) 331.

[5] B. Silvestre–Brac and C. Gignoux, Phys. Rev. D37 (1985) 74.
R.K. Bhaduri et al., Nuovo Cim. A65 (1981) 376.

[6] Particle Data Group, Phys. Rev. D50 (1994) 1173.

[7] F. Cano, Master Thesis, Univ. of Valencia (1994).

[8] A. Le Yaouanc et al., Phys. Rev. D8 (1973) 2223; D11 (1975) 1272.

[9] W. Roberts and B. Silvestre–Brac, Few Body Sys. 11 (1992) 171.

[10] B. Silvestre–Brac and C. Gignoux, Phys. Rev. D43 (1991) 3699.

[11] S. Capstick and W. Roberts, Phys. Rev. D49 (1994) 4570.

[12] K. Itonaga et al., Nucl. Phys. A609 (1996) 422.

[13] Fl. Stancu and P. Stassart, Phys. Rev. D38 (1988) 233.

Table captions

Table 1 Fitted values of the parameters of the two–body and two+three body potentials.

Table 2a Pionic decay widths and the $f_{qq\pi}$ coupling constant obtained with the elementary emission model (EEM) for the two–body quark potential model. Experimental data from [6].

Table 2b Pionic decay widths and the $f_{qq\pi}$ coupling constant obtained with the elementary emission model (EEM) for the two+three body quark potential model.

Table 3a Pionic decay widths and the γ constant obtained with several versions of the $^3P_0$ model for the two–body quark potential model.

Table 3b Pionic decay widths and the γ constant obtained with several versions of the $^3P_0$ model for the two+three body quark potential model.

		$V^{(2)}$	$V^{(2)}+V^{(3)}$
$m_u = m_d$ (GeV)		0.337	0.355
$V^{(\mathrm{COUL})}$	κ (GeV fm)	0.1027	0.289
$V^{(\vec\sigma\vec\sigma)}$	κ_σ (GeV fm) r_0 (fm)	0.1027 0.4545	0.049 0.40
$V^{(\mathrm{CONF})}$	a^2 (GeV^{-1} fm)	1.063	4.570
$V^{(3)}$	V_0 (GeV^{-2} fm^{-6}) m_0 (GeV)	– –	-61.63 0.25

Table 1

$V^{(2)}$	EEM (p/m)	EEM $(p/m)^2$	EEM (p/E)	EEM $(p/E)^2$	EEM All (p/E) orders	$\Gamma_{\rm Exp}$ (MeV)
$\Delta(1232) \to N\pi$	79.6	25.1	74.3	75.0	75.8	115–125
$N(1440) \to N\pi$	3.4	177	0.007	4.91	13.1	210–245
$N(1440) \to \Delta\pi$	7.1	13.3	5.4	7.82	9.30	70–105
$\Delta(1600) \to N\pi$	20.1	94.1	31.0	15.4	7.98	35–88
$\Delta(1600) \to \Delta\pi$	2.85	56.4	0.72	5.08	9.30	140–245
$N(1520) \to N\pi$	61.8	21.5	62.3	60.6	57.7	60–72
$N(1520) \to \Delta\pi$	78.0	95.3	24.3	39.2	45.1	18–30
$N(1535) \to N\pi$	240	494	24.8	76.5	101	53–83
$N(1535) \to \Delta\pi$	9.7	4.25	9.93	10.4	10.3	$<$1.5
$N(1650) \to N\pi$	47.9	135	2.49	14.5	20.9	90–120
$N(1650) \to \Delta\pi$	12.4	5.49	12.6	13.0	12.8	4–11
$N(1700) \to N\pi$	4.07	1.22	3.65	3.43	3.24	5–15
$N(1700) \to \Delta\pi$	383	612	112	190	222	81–393
$f_{qq\pi}$	0.602	0.602	0.619	0.743	0.782	

Table 2a

$V^{(2)} + V^{(3)}$	EEM (p/m)	EEM $(p/m)^2$	EEM (p/E)	EEM $(p/E)^2$	EEM All (p/E) orders	Γ_{Exp} (MeV)
$\Delta(1232) \to N\pi$	72.1	4.18	67.4	71.7	75.7	115–125
$N(1440) \to N\pi$	0.17	690	0.04	4.01	15.3	210–245
$N(1440) \to \Delta\pi$	17.6	43.1	16.7	24.5	30.9	70–105
$\Delta(1600) \to N\pi$	94.1	226	84.9	64.1	50.4	35–88
$\Delta(1600) \to \Delta\pi$	0.10	70.4	0.006	2.08	5.89	140–245
$N(1520) \to N\pi$	22.3	15.3	25.9	29.3	31.5	60–72
$N(1520) \to \Delta\pi$	56.1	67.4	20.4	36.2	44.8	18–30
$N(1535) \to N\pi$	149	259	20.7	60.7	82.8	53–83
$N(1535) \to \Delta\pi$	8.3	5.78	9.25	11.3	12.6	<1.5
$N(1650) \to N\pi$	24.9	60.3	2.54	11.0	15.8	90–120
$N(1650) \to \Delta\pi$	9.95	6.35	10.4	12.3	13.6	4–11
$N(1700) \to N\pi$	1.43	0.75	1.29	1.41	1.52	5–15
$N(1700) \to \Delta\pi$	220	347	79.7	148	185	81-393
$f_{qq\pi}$	0.604	0.604	0.624	0.780	0.852	

Table 2b

$V^{(2)}$	$^3P_0$	M$^3P_0$	R$^3P_0$	$\Gamma_{\rm Exp}$ (MeV)
$\Delta(1232) \to N\pi$	167	88.6	83.9	115–125
$N(1440) \to N\pi$	452	114	73.5	210–245
$N(1440) \to \Delta\pi$	66.5	27.6	20.7	70–105
$\Delta(1600) \to N\pi$	19.8	2.1	0.27	35–88
$\Delta(1600) \to \Delta\pi$	255	62.0	41.9	140–245
$N(1520) \to N\pi$	268	95.1	92.0	60–72
$N(1520) \to \Delta\pi$	532	45.9	17.1	18–30
$N(1535) \to N\pi$	429	49.2	0.18	53–83
$N(1535) \to \Delta\pi$	28.1	15.3	15.7	<1.5
$N(1650) \to N\pi$	49.1	6.14	1.13	90–120
$N(1650) \to \Delta\pi$	49.3	20.9	20.9	4–11
$N(1700) \to N\pi$	11.5	3.49	3.15	5–15
$N(1700) \to \Delta\pi$	1643	228	114	81-393
$-i\gamma$	7.02	8.20	8.29	

Table 3a

$V^{(2)}+V^{(3)}$	$^3P_0$	M$^3P_0$	R$^3P_0$	$\Gamma_{\rm Exp}$ (MeV)
$\Delta(1232)\to N\pi$	210	112	106	115–125
$N(1440)\to N\pi$	1076	307	236	210–245
$N(1440)\to \Delta\pi$	228	116	106	70–105
$\Delta(1600)\to N\pi$	0.53	2.5	8.82	35–88
$\Delta(1600)\to \Delta\pi$	498	121	93.5	140–245
$N(1520)\to N\pi$	319	105	100	60–72
$N(1520)\to \Delta\pi$	999	75.2	34.4	18–30
$N(1535)\to N\pi$	464	44.1	0.52	53–83
$N(1535)\to \Delta\pi$	74.0	36.3	37.7	<1.5
$N(1650)\to N\pi$	44.2	4.01	0.77	90–120
$N(1650)\to \Delta\pi$	109	45.7	43.1	4–11
$N(1700)\to N\pi$	11.2	3.25	2.83	5–15
$N(1700)\to \Delta\pi$	2417	323	217	81-393
$-i\gamma$	9.78	11.4	11.6	

Table 3b