FTUV/95-17
IFIC/95-17
Imperial-TP/94-95/31
hep-th/9505139

Weyl Invariance and Black Hole Evaporation*

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January 15, 2014

Abstract

We consider the semiclassical dynamics of CGHS black holes with a Weyl-invariant effective action for conformal matter. The trace anomaly of Polyakov effective action is converted into the Virasoro anomaly thus leading to the same flux of Hawking radiation. The covariance of semiclassical equations can be restored through a non-local redefinition of the metric-dilaton fields. The resulting theory turns out to be equivalent to the RST model. This provides a mechanism to solve semiclassical equations of 2D dilaton gravity coupled to conformal matter for classically soluble models.

* Work partially supported by the *Comisión Interministerial de Ciencia y Tecnología* and *DGICYT*.

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1 Introduction

Since the pioneer work of Hawking [1], the formation and subsequent evaporation of a black hole has provided an excellent scenario to study the interplay between gravity and quantum mechanics. In recent years it has been a revival of interest in the subject, due to the emergence of simplified (two-dimensional) models sharing basic features with the four-dimensional theory. The model proposed by Callan, Giddings, Harvey and Strominger [2] (CGHS-model), involving gravity coupled to a dilaton and N massless scalar fields f_i, $i = 1, \ldots, N$, describes, at the classical level, the formation of a black hole by incoming conformal matter. Hawking radiation in the classical background geometry can be computed from the trace anomaly of the matter fields [3]

$$< T^{f\alpha}{}_{\alpha} >= \frac{N}{24} R \,, \tag{1}$$

and back-reaction can be incorporated by adding to the classical action the Polyakov effective action [4]

$$S_P = -\frac{N}{96\pi} \int d^2x \sqrt{-g} R \Box^{-1} R \,. \tag{2}$$

The new equations of motion have the quantum stress tensor of matter as the source for the classical gravity dilaton fields. Although the semiclassical equations have not been solved in closed form, a special modification of the model [5] allows to construct exact solutions and, therefore, to study the evolution of a quantum black hole analytically.

The trace anomaly equation (1) is a direct consequence of the breaking of Weyl symmetry in the definition of the functional measure for the conformal matter fields,

$$||\delta f_i||^2 = \int d^2x \sqrt{-g} \delta f_i \delta f_i \,; \tag{3}$$

the above definition respects diffeomorphism invariance of the classical theory but sacrifices Weyl symmetry. The former symmetry leads to the standard covariant conservation of the quantum energy-momentum tensor

$$\nabla_{\mu} < T^{f\mu\nu} >= 0 \,, \tag{4}$$

and the effective action capturing the equations (1) and (4) is given by the induced gravity action (2).

Recently, it has been advocated an alternative definition of the measure (3) that preserves Weyl invariance [6, 7]. Diffeomorphism invariance is partially lost and only area-preserving diffeomorphisms are maintained. In this note we shall develop further this alternative approach and explore the relationship between the diffeomorphism and Weyl invariant schemes in the context of the semiclassical theory of 2D gravity.

2 The Weyl-invariant effective action and the Virasoro anomaly

The Weyl-invariant effective action proposed in [6, 7] can be obtained from the Polyakov action replacing $g^{\mu\nu}$ by $\sqrt{-g}g^{\mu\nu}$:

$$S_W = -\frac{N}{96\pi}\int d^2x R(\sqrt{-g}g^{\mu\nu})(\sqrt{-g}\Box)^{-1}R(\sqrt{-g}g^{\mu\nu})\,, \tag{5}$$

where $\Box = (\sqrt{-g})^{-1}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$. To study the relation between the action (5) and (2) in a simple way it is convenient to introduce an auxiliary field Φ verifying the equation

$$\Box\Phi = R\,. \tag{6}$$

In terms of $g^{\mu\nu}$ and Φ the Polyakov action can be rewritten as (see, for instance, [8])

$$S_P = -\frac{N}{96\pi}\int d^2x\sqrt{-g}(-\Phi\Box\Phi + 2R\Phi)\,. \tag{7}$$

The equation (6) follows from the action (7) and inserting (6) into (7) we recover (2). The metric-dependent Weyl transformation $g^{\mu\nu} \to \sqrt{-g}g^{\mu\nu}$ induces the following transformation for the scalar curvature and the auxiliary field Φ

$$\begin{aligned} R &\to (R + \Box\log\sqrt{-g})\sqrt{-g}\,, && (8)\\ \Phi &\to \Phi + \log\sqrt{-g}\,. && (9) \end{aligned}$$

Using (8) and (9) it is easy to find that the Weyl-invariant action S_W is given by

$$\begin{aligned} S_W = -\frac{N}{96\pi}\int d^2x\Big[&\sqrt{-g}(-\Phi\Box\Phi + 2R\Phi)\\ &+ \partial_\mu(\sqrt{-g}(\log\sqrt{-g}\overleftrightarrow{\partial^\mu}\Phi))\\ &+ \sqrt{-g}(\log\sqrt{-g}\Box\log\sqrt{-g} + 2R\log\sqrt{-g})\Big]\,. \end{aligned} \tag{10}$$

Therefore, S_W and S_P differ, up to total derivative terms, by a local action

$$S_W = S_P - \frac{N}{96\pi}\int d^2x\sqrt{-g}(\log\sqrt{-g}\Box\log\sqrt{-g} + 2R\log\sqrt{-g})\,. \tag{11}$$

The energy-momentum tensor $T^W_{\mu\nu} = -\frac{2\pi}{\sqrt{-g}}\frac{\delta S_W}{\delta g^{\mu\nu}}$ coming from (11) can be easily computed. It admits the following decomposition

$$\begin{aligned} T^W_{\mu\nu} = &(T^P_{\mu\nu} - \frac{N}{48}g_{\mu\nu}R)\\ &-\frac{N}{48}\left[\partial_\mu\log\sqrt{-g}\partial_\nu\log\sqrt{-g} - \frac{1}{2}g_{\mu\nu}\partial_\alpha\log\sqrt{-g}\partial^\alpha\log\sqrt{-g}\right]\\ &-\frac{N}{24}\left[\nabla_\mu\nabla_\nu\log\sqrt{-g} - \frac{1}{2}g_{\mu\nu}\Box\log\sqrt{-g}\right]\,, \end{aligned} \tag{12}$$

where $T^P_{\mu\nu}$ is the energy-momentum associated with the Polyakov effective action

$$\begin{aligned} T^P_{\mu\nu} &= \frac{N}{48}\left[\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi\right] \\ &\quad -\frac{N}{24}\left[\nabla_\mu\nabla_\nu\Phi - \frac{1}{2}g_{\mu\nu}\Box\Phi\right] \\ &\quad +\frac{N}{48}g_{\mu\nu}R\,. \end{aligned} \tag{13}$$

Note that ${T^{P\mu}}_\mu = \frac{N}{24}R$ is consistent with (1).

Due to the breakdown of reparametrization invariance $T^W_{\mu\nu}$ is not covariantly conserved, in general. Instead, we have

$$\nabla^\mu T^W_{\mu\nu} = -\frac{N}{48}\left[\partial_\nu(R+\Box\log\sqrt{-g}) + (R+\Box\log\sqrt{-g})\partial_\nu\log\sqrt{-g}\right]\,, \tag{14}$$

and taking into account that

$$R+\Box\log\sqrt{-g} = \frac{1}{\sqrt{-g}}R(\sqrt{-g}g^{\mu\nu})\,, \tag{15}$$

we can rewrite (14) as

$$\nabla^\mu T^W_{\mu\nu} = -\frac{N}{48}\frac{1}{\sqrt{-g}}\partial_\nu R(\sqrt{-g}g^{\mu\nu})\,. \tag{16}$$

This formula has been obtained in [7] in a different way. However, in special gauges the stress tensor $T^W_{\mu\nu}$ can be conserved. For metrics of the form

$$ds^2 = -e^{2\rho}\left(dx^+dx^- + c\,(x^-)^2(dx^+)^2\right)\,, \tag{17}$$

where c is a constant, the r.h.s. of (16) vanishes. In the conformal gauge the conservation equations take the simple form

$$\nabla^+ T^W_{++} = 0 = \nabla^- T^W_{--}\,, \tag{18}$$

which implies that

$$T^W_{++} = T^W_{++}(x^+)\,, \tag{19}$$

$$T^W_{--} = T^W_{--}(x^-)\,. \tag{20}$$

Despite of the covariant conservation equations (18) the transformation law of $T^W_{\pm\pm}$ is anomalous. Under conformal coordinate transformations $x^\pm \to y^\pm(x^\pm)$, $T^W_{\mu\nu}$ transforms as

$$T^W_{x^\pm x^\pm} = \left(\frac{dy^\pm}{dx^\pm}\right)^2 T^W_{y^\pm y^\pm} - \frac{N}{24}\left\{y^\pm, x^\pm\right\}\,, \tag{21}$$

where

$$\{y,x\} = \frac{\frac{\partial^3 y}{\partial x^3}}{\frac{\partial y}{\partial x}} - \frac{3}{2}\frac{\left(\frac{\partial^2 y}{\partial x^2}\right)^2}{\left(\frac{\partial y}{\partial x}\right)^2} \tag{22}$$

is the Schwartzian derivative. Note that the expression (21) coincides with the well-known transformation law of the normal-ordered energy-momentum tensor of a conformal field theory. Therefore we can conclude that the local counterterm in (11) converts the trace anomaly into the Virasoro anomaly.

3 Semiclassical 2D black holes. Field redefinitions and covariance

The action of the CGHS model is [2]

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right] . \tag{23}$$

The classical solutions describe the formation of a black hole by gravitational collapse. In conformal gauge, $ds^2 = -e^{2\rho}dx^+dx^-$ ($x^\pm = x^0 \pm x^1$), and in Kruskal coordinates, $\rho = \phi$, the black hole formation from the vacuum by left moving incoming matter $f_i = f_i^+(x^+)$ is described by the solution

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \frac{1}{\lambda^2}P(x^+)) + \frac{M(x^+)}{\lambda} , \tag{24}$$

where

$$M(x^+) = \lambda\int_0^{x^+} d\tilde{x}^+\tilde{x}^+T^f_{++} , \tag{25}$$

$$P(x^+) = \int_0^{x^+} d\tilde{x}^+T^f_{++} , \tag{26}$$

and

$$T^f_{++} = \frac{1}{2}\sum_{i=1}^{N}(\partial_+ f_i^+)^2 . \tag{27}$$

This solution corresponds to a black hole of mass $M = M(x^+ \to \infty)$ with an event horizon located at $x^- = -\frac{1}{\lambda^2}P(x^+ \to \infty)$. In the semiclassical approximation the Hawking radiation at future null infinity can be obtained from the trace anomaly [2], and the back-reaction is incorporated [2] by adding to the classical action S the Polyakov term (2).

Let us consider an f shock wave travelling in the x^--direction, described by the stress tensor

$$\frac{1}{2}\partial_+ f\partial_+ f = a\,\delta(x^+ - x_0^+) . \tag{28}$$

For $x^+ < x_0^+$ the classical solution is the linear dilaton vacuum (LDV). In this region the natural coordinate system is the Minkowskian one. We assume, as boundary condition of the gravitational collapse, that the outgoing energy flux measured by the Minkowskian observer $\sigma^\pm$ vanishes,

$$T^W_{\sigma^-\sigma^-} = 0 . \tag{29}$$

After the collapse, $x^+ > x_0^+$, the classical solution describes a black hole of mass $M = ax_0^+\lambda$ with a horizon at $x^- = -\frac{a}{\lambda^2}(= -\frac{P}{\lambda^2})$. But then the natural coordinates are the Schwarzschild type coordinates $\tilde{\sigma}^\pm$. The coordinate transformation

$$\tilde{\sigma}^+ = \sigma^+ , \tag{30}$$

$$\tilde{\sigma}^- = -\frac{1}{\lambda}\log\left(e^{-\lambda\sigma^-} - \frac{a}{\lambda}\right) , \tag{31}$$

allows to evaluate the energy flux measured by the Schwarzschild-type observer $(\tilde{\sigma}^{\pm})$. It is given by

$$T^W_{\tilde{\sigma}^-\tilde{\sigma}^-} = -\frac{N}{24}\{\sigma^-, \tilde{\sigma}^-\} \ , \tag{32}$$

$$T^W_{\tilde{\sigma}^+\tilde{\sigma}^+} = 0 \, , \tag{33}$$

and hence

$$T^W_{\tilde{\sigma}^-\tilde{\sigma}^-} = \frac{\lambda^2 N}{48}\left(1 - \frac{1}{(1+\frac{1}{\lambda}Pe^{\lambda\tilde{\sigma}^-})^2}\right) \ , \tag{34}$$

in agreement with the result predicted by the trace anomaly. The reason of this is that the difference between $T^W_{\mu\nu}$ and $T^P_{\mu\nu}$ vanishes at infinity in asymptotically flat coordinates. So both T^W and T^P yield to the same flux of Hawking radiation.

Let us briefly analyze a more involved model: spherically symmetric gravity coupled to 2D conformal matter. After appropriate reduction this model is described by the two-dimensional action (see, for instance, [9])

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}(R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right] \ , \tag{35}$$

where the 4D spherically symmetric metric is related to the 2D metric and dilaton fields by ${}^{(4)}ds^2 = ds^2 + e^{-2\phi}/\lambda^2 d\Omega^2$. This model is classically soluble and the solutions are the Vaidya space-times. If we consider the collapse of a null shell of matter, the stress tensor and two-dimensional metric take the form

$$\begin{aligned} T^f_{vv} &= m\delta(v - v_0) \, , \\ ds^2 &= -\left(1 - \frac{2m\theta(v - v_0)}{r}\right)dv^2 + 2drdv \, . \end{aligned} \tag{36}$$

In conformal coordinates matching the discontinuity across $v = v_0$ we obtain for the metric

$$ds^2 = -\left[\theta(v_0 - v) + \theta(v - v_0)\left(1 - \frac{2m}{r}\right)\left(1 - \frac{4m}{v_0 - u}\right)^{-1}\right]dvdu \, , \tag{37}$$

where r is defined implicitly by

$$u - 4m\log\left(\frac{v_0 - u}{4m} - 1\right) - v = -2\left(r + 2m\log\left(\frac{r}{2m} - 1\right)\right) \ , \tag{38}$$

and for the dilaton

$$\frac{e^{-\phi}}{\lambda} = \frac{v - u}{2}\theta(v_0 - v) + r\theta(v - v_0) \, . \tag{39}$$

The coordinates (v, u) are Minkowskian inside the shell $(v < v_0)$, whereas the asymptotically flat conformal coordinates $(\tilde{v}, \tilde{u})$ are given by the relations

$$\begin{aligned} \tilde{v} &= v \, , \\ \tilde{u} &= u - 4m\log\left(\frac{v_0 - u}{4m} - 1\right) \, . \end{aligned} \tag{40}$$

Therefore, assuming that $T^W_{uu} = 0$, the stress tensor evaluated by the Schwarzschild observer is given by

$$T^W_{\tilde{u}\tilde{u}} = -\frac{m(u - v_0 + 3m)}{3(u - v_0)^4} \,. \quad (41)$$

As the horizon is approached $u \to v_0 - 4m$, $T^W_{\tilde{u}\tilde{u}}$ builds up to the value $(3 \cdot 2^8 m^2)^{-1}$, that corresponds to the Hawking temperature $T = \frac{1}{8\pi m}$.

Our aim now is to study the semiclassical back-reaction, for the CGHS model, defined by the Weyl-invariant effective action (11), in the large N limit. Due to Weyl invariance the ρ, ϕ classical equations are unmodified,

$$\begin{aligned} e^{-2\phi}\left(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2\rho}\right) &= 0 \,, & (42) \\ e^{-2(\phi+\rho)}\left(-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2 e^{2\rho}\right) &= 0 \,. & (43) \end{aligned}$$

However, the constraint equations are modified according to

$$e^{-2\phi}\left(4\partial_\pm\rho\partial_\pm\phi - 2\partial^2_\pm\phi\right) + \frac{1}{2}\sum_{i=1}^{N}(\partial_\pm f_i)^2 + T^W_{\pm\pm} = 0 \,. \quad (44)$$

The components $T^W_{\pm\pm}$ of the non-local effective stress tensor are ρ-independent and have to be adjusted by boundary conditions.

At this point we should note that, although the Virasoro anomaly of $T^W_{\pm\pm}$ accounts for the Hawking radiation, it destroys the covariance of the one-loop equations (42-44). Moreover, if we select a particular coordinate system ($\sigma^\pm$ before the collapse and $\tilde{\sigma}^\pm$ after it) the solution is not continuous at $x^+ = x^+_0$; and coordinates that match continuously $\sigma^\pm$ and $\tilde{\sigma}^\pm$ do no adjust to the gauge (17). Then, how can we define, in this scheme, a consistent semiclassical theory?. To gain insights, let us first analyse the question of the vacuum stability.

In conformal gauge, the vacuum solution to equations (42-44) is

$$\begin{aligned} e^{-2\rho} &= e^{-\omega}e^{-2\phi} \,, \\ e^{-2\phi} &= u - h_+h_- \,, \end{aligned} \quad (45)$$

where $\omega = \omega_+(y^+) + \omega_-(y^-)$, $u = u_+ + u_-$,

$$\begin{aligned} u_\pm &= -\int^{y^\pm} e^{\omega_\pm}\int e^{-\omega_\pm}T^W_{\pm\pm} \,, \\ h_\pm &= \lambda\int^{y^\pm} e^{\omega_\pm} \,. \end{aligned} \quad (46)$$

According to (21), $T^W_{y^\pm y^\pm}$ is given by the Schwartzian derivative with respect to the Minkowskian coordinates $\sigma^\pm$. Hence the general solution to the semiclassical equations in vacuum is found to be

$$\begin{aligned} e^{-2\rho} = e^{-2(\rho_{cl}-\phi_{cl})}\Big[& e^{-2\phi_{cl}} + \frac{N}{24}(2\rho_{cl} - \phi_{cl}) \\ & + \frac{N}{12}\int^{y^+} e^{2(\rho_{cl}-\phi_{cl})}\int e^{-2(\rho_{cl}-\phi_{cl})}[\partial_{y^+}(\rho_{cl} - \phi_{cl})]^2 \\ & + \frac{N}{12}\int^{y^-} e^{2(\rho_{cl}-\phi_{cl})}\int e^{-2(\rho_{cl}-\phi_{cl})}[\partial_{y^-}(\rho_{cl} - \phi_{cl})]^2\Big], \end{aligned} \quad (47)$$

where ρ_{cl} and ϕ_{cl} represent the classical vacuum solutions

$$\begin{aligned} \rho_{cl} &= \frac{1}{2}\log\frac{dy^+}{d\sigma^+}\frac{dy^-}{d\sigma^-}\,, \\ \phi_{cl} &= -\frac{\lambda}{2}\left(\sigma^+(y^+) - \sigma^-(y^-)\right)\,. \end{aligned} \tag{48}$$

In the light of the above expressions we can conclude that, due to the term $T^W_{\pm\pm}(y^\pm)$, the solutions do no transform covariantly, thus producing some sort of vacuum instability that makes problematic a Weyl-invariant semiclassical theory.

A way to construct a sensible semiclassical theory is suggested by the non-covariant vacuum solutions themselves (47). The non-local field redefinition $\rho \to \hat{\rho}$, $\phi \to \hat{\phi}$, defined by

$$\begin{aligned} \rho - \phi &= \hat{\rho} - \hat{\phi} \\ e^{-2\phi} &= \Big[e^{-2\hat{\phi}} + \frac{N}{24}(2\hat{\rho} - \hat{\phi}) \\ &\quad + \frac{N}{12}\int^{y^+} e^{2(\hat{\rho}-\hat{\phi})}\int e^{-2(\hat{\rho}-\hat{\phi})}[\partial_{y^+}(\hat{\rho}-\hat{\phi})]^2 \\ &\quad + \frac{N}{12}\int^{y^-} e^{2(\hat{\rho}-\hat{\phi})}\int e^{-2(\hat{\rho}-\hat{\phi})}[\partial_{y^-}(\hat{\rho}-\hat{\phi})]^2\Big]\,, \end{aligned} \tag{49}$$

recovers the classical vacuum solution as a solution of the semiclassical vacuum equations: $\hat{\rho} = \rho_{cl}$, $\hat{\phi} = \phi_{cl}$, and reestablishes the general covariance as well.

After the redefinitions (49), the constraint equations (44) become

$$\left(e^{-2\hat{\phi}} + \frac{N}{48}\right)\left(4\partial_\pm\hat{\rho}\partial_\pm\hat{\phi} - 2\partial^2_\pm\hat{\phi}\right) - \frac{N}{12}\left(\partial_\pm\hat{\rho}\partial_\pm\hat{\rho} - \partial^2_\pm\hat{\rho}\right) + \frac{1}{2}\partial_\pm f_i\partial_\pm f_i + T^W_{\pm\pm} = 0\,, \tag{50}$$

and the equations (42-43) can be rewritten as

$$\partial_+\partial_-\left(\hat{\rho} - \hat{\phi}\right) = 0\,, \tag{51}$$

$$\partial_+\partial_-\left(e^{-2\hat{\phi}} + \frac{N}{24}\hat{\rho}\right) = -\lambda^2 e^{2(\hat{\rho}-\hat{\phi})}\,. \tag{52}$$

Remarkably these equations turn out to be equivalent to the equations of motion of the RST model [5]. Note that in Kruskal coordinates $\hat{\rho} = \hat{\phi}$ the transformation (49) is local,

$$e^{-2\rho} = e^{-2\hat{\rho}} + \frac{N}{24}\hat{\rho}\,, \tag{53}$$

and coincides with the field redefinitions of [5] that maps (50) into a Liouville theory (see also [10]), where the original variable $e^{-2\phi}$ is the analogue of the $\sqrt{\frac{N}{12}}\Omega$ field.

This intriguing relation between the RST model and the Weyl-invariant one-loop CGHS model offers a new explanation of the solubility of the former.

In the Weyl-invariant scheme the semiclassical equations can be solved in a straightforward way by replacing the classical stress tensor $T^f_{\pm\pm}$ by $T^f_{\pm\pm}+T^W_{\pm\pm}$ in the expression of the classical solutions. In other words, the classical solubility of the model together with the field redefinitions that ensure the vacuum stability allows to solve a related covariant semiclassical model.

4 Final comments

We have exhibit in a simple way the relationship between diffeomorphism and Weyl-invariant effective actions of conformal matter coupled to 2D gravity. They differ in a local term that converts the trace anomaly of the effective stress tensor into the Virasoro anomaly thus yielding to the same flux of Hawking radiation. In special gauges the semiclassical equations of the Weyl-invariant effective action are consistent with Bianchi identities although the solutions do not transform covariantly. A consequence of this is the impossibility of defining a covariant vacuum solution. To reestablish the LDV as the semiclassical vacuum solution of the CGHS model, we are forced to perform specific, non-local field redefinitions of the metric and dilaton fields. The field redefinitions yields to new covariant semiclassical equations, which turn out to be the equations of the RST model. The solubility of the classical model and the Weyl invariance of the semiclassical correction implies exact solubility of the related semiclassical covariant model. This mechanism can be applied to solve the semiclassical theory of more realistic models, as 4D-spherically symmetric gravity coupled to 2D conformal matter [11]. Moreover, the insights gained in unravelling the relationship between the Weyl and diffeomorphism invariant schemes could be of great interest for the non-perturbative canonical approach. There are some arguments [7] indicating that the Weyl invariant scheme reflects more appropriately the exact canonical quantization.

Acknowledgements

M. Navarro acknowledges to the *MEC* for a Postdoctoral fellowship. C. F. Talavera is grateful to the *Generalitat Valenciana* for a FPI grant.

References

[1] S. W. Hawking, *Commun. Math. Phys.* 43 (1975) 199.

[2] C. G. Callan, S. B. Giddings, J. A. Harvey and A. Strominger, *Phys. Rev.* D45 (1992) 1005.

[3] S. M. Christensen and S. A. Fulling, *Phys. Rev.* D15 (1977) 2088.

[4] A. M. Polyakov, *Phys. Lett.* B103 (1981) 207.

[5] J. G. Russo, L. Susskind and L. Thorlacius, *Phys. Rev.* D46 (1993) 3444; *Phys. Rev.* D47 (1993) 533.

[6] D. R. Karakhanyan, R. P. Manvelyan and R. Mkrtchyan, *Phys. Lett.* B329 (1994) 185.

[7] R. Jackiw, *Another view on Massless Matter-Gravity Fields in Two Dimensions*, hep-th/9501016.

[8] J. Navarro-Salas, M. Navarro and V. Aldaya, *Nucl. Phys.* B403 (1993) 291.

[9] A. Strominger, *Les Houches Lectures on Black Holes*, hep-th/9501071.

[10] A. Bilal and C. Callan, *Nucl. Phys.* B394 (1993) 73.
S. de Alwis, *Phys. Lett.* B289 (1992) 278.

[11] J. Navarro-Salas, M. Navarro and C. F. Talavera, work in progress.