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Free Fields via Canonical Transformations of Matter-coupled 2D Dilaton Gravity Models *

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Abstract

It is shown that the 1+1-dimensional matter-coupled Jackiw-Teitelboim model and the model with an exponential potential can be converted by means of appropriate canonical transformations into a bosonic string theory propagating on a flat target space with an indefinite signature. This makes it possible to consistently quantize these models in the functional Schrödinger representation thus generalizing recent results on the CGHS theory.

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1 Introduction

In recent years there has been a lot of interest in two-dimensional dilaton-gravity theories. The main motivation is that they posses most of the interesting physical features of the four-dimensional theory such as the formation of black holes and their subsequent evaporation, while the technical difficulties are reduced. The simplest theory describing the formation and evaporation of 2D black holes is the model introduced by Callan, Giddings, Harvey and Strominger (CGHS) [1]. The semiclassical approach as well as the canonical quantization of the model have been considered from different viewpoints [2]. In a recent work [3] it was shown that the CGHS model can be converted, by means of a canonical transformation, into a bosonic string theory propagating on a Minkowskian target space. A further canonical transformation brings the constraints into the form of a parametrized theory [4]. At the quantum level the commutators of the constraint operators produce the Virasoro anomaly and the theory cannot be quantized without modification. However the value of the anomaly depends on the choice of the vacuum used to normal order the constraints. The one associated with the Schrödinger representation produces a cancellation between the anomaly of the two free fields of the pure gravity theory [3]. Therefore it is possible to find solutions to the quantum Dirac constraint algebra [5, 3]. For the matter-coupled theory a consistent quantum theory can be constructed on a cylinder by an appropriate modification of the quantum constraints which removes the anomaly [6]. This mechanism is based on an embedding-dependent factor ordering of the constraints [4]. The extension of the anomaly-free Dirac quantization of matter-coupled CGHS theory to an open two-dimensional space-time has been given in [7] (see also [8]). This quantization procedure yields to a physical spectrum in accordance with the degrees of freedom of the classical theory [6, 7]. The BRST quantization gives an inequivalent physical result [6]. The aim of this paper is to generalize the results of [6, 7] concerning the CGHS model to other models of two-dimensional dilaton gravity.

Up to conformal redefinitions of the fields the action of a generic (first-order) model of 2D dilaton gravity can be written in the form [5]

$$S = \int d^2x \sqrt{-g} \left[R\phi + V(\phi) - \frac{1}{2} (\nabla f)^2 \right] . \tag{1.1}$$

where $V(\phi)$ is an arbitrary potential term. The CGHS model can be recovered by choosing a constant potential $V(\phi) = 4\lambda^2$. If we parametrize the two-dimensional metric as

$$g_{\mu\nu} = e^{2\rho} \begin{pmatrix} v^2 - u^2 & v \\ v & 1 \end{pmatrix} , \tag{1.2}$$

where the functions u and v are related to the shift and lapse functions, the hamiltonian form of the action becomes

$$S = \int d^2x \left(\pi_\rho \dot{\rho} + \pi_\phi \dot{\phi} + \pi_f \dot{f} - uH - vP \right) , \tag{1.3}$$

where the constraint functions are given by

$$H = -\frac{1}{2} \pi_\rho \pi_\phi + 2 (\phi'' - \phi' \rho') - e^{2\rho} V(\phi) + \frac{1}{2} \left(\pi_f^2 + f'^2 \right) , \tag{1.4}$$

$$P = \rho' \pi_\rho - \pi'_\rho + \phi' \pi_\phi + \pi_f f' . \tag{1.5}$$

In [3] it has been shown that when the potential $V(\phi)$ is constant, a canonical transformation converts the constraints H and P into those a bosonic string theory propagating on a 3-dimensional Minkowski space

$$H = \frac{1}{2} \left(\pi_0^2 + \left(r^{0\prime} \right)^2 \right) - \frac{1}{2} \left(\pi_1^2 + \left(r^{1\prime} \right)^2 \right) + \frac{1}{2} \left(\pi_f^2 + f'^2 \right) , \tag{1.6}$$

$$P = \pi_0 r^{0\prime} + \pi_1 r^{1\prime} + \pi_f f' . \tag{1.7}$$

In this paper we shall show that the Jackiw-Teitelboim model $\left(V(\phi) = 4\lambda^2 \phi\right)$ and the model with an exponential potential $\left(V(\phi) = 4\lambda^2 e^{\beta\phi}\right)$, which includes the CGHS model as a limiting case ($\beta = 0$), can also be converted through a canonical transformation into a bosonic string theory propagating on a Minkowskian target space. In conformal gauge the exponential model possesses a free field η and a Liouville one φ. It is well known that a Liouville field φ can be mapped into a free field ψ through a canonical transformation. However the energy momentum tensor of the two free fields η and ψ contains an "improvement" term which does not allow to implement the Schrödinger quantization approach [6]. In fact the improvement terms are naturally related to the BRST quantization scheme. Therefore a further canonical transformation mixing the two free fields is required to transform the exponential model into a bosonic string theory with a Minkowskian target-space. In section 2 we shall construct this canonical transformation directly from the ρ and ϕ fields. As a by-product we shall also show how to recover the canonical transformation of the CGHS theory [7] as a limiting case $\beta = 0$ of our approach. Moreover we shall carry out the Dirac constraint quantization of the model following the lines of [6]. In section 3 we shall analyze the Jackiw-Teitelboim model. This model is more involved because it is described by a Liouville field and a field propagating in a De Sitter space with a curvature term. Both fields can be also combined, through a canonical transformation, to produce two free fields without any improvement term and with opposite contributions to the hamiltonian constraint. Therefore this theory is also equivalent to a bosonic string theory with a Minkowskian target space. These results make it possible to consistently quantize these theories.

2 The Exponential Model

2.1 Canonical Transformation

Lets us consider the model (1.1) with $V(\phi) = 4\lambda^2 e^{\beta\phi}$. This model includes the CGHS theory as a particular case $\beta = 0$. In the absence of matter fields the above model possesses static black hole solutions similar to the ones of the CGHS model, but in contrast with them these black holes have a Hawking temperature proportional to their mass [9]. Due to the existence of an extra symmetry, it is possible, in analogy with the CGHS theory, to construct a solvable semiclassical theory [10, 11]. The semiclassical analysis indicates that these black holes never disappear completely. The equations of motion of the model, in conformal gauge ($ds^2 = -e^{-2\rho}dx^+dx^-$), are

$$\partial_+\partial_-\left(2\rho - \beta\phi\right) = 0\,, \tag{2.1}$$

$$\partial_+\partial_-\left(2\rho + \beta\phi\right) = -2\lambda^2\beta e^{2\rho+\beta\phi}\,, \tag{2.2}$$

$$\partial_{\pm}^2\phi - 2\partial_{\pm}\rho\partial_{\pm}\phi = T^f_{\pm\pm} = \frac{1}{2}\left(\partial_{\pm}f\right)^2 \ , \tag{2.3}$$

$$\partial_+\partial_- f = 0\,. \tag{2.4}$$

The unconstrained equations (2.1-2.2) are equivalent to a free field equation and a Liouville equation respectively. The general solution to these equations suggests the following transformation in terms of a set of new variables $A_{\pm}, a_{\pm}$

$$\rho = \frac{1}{2}\log\frac{-A'_+A'_-}{1+\lambda^2\beta A_+A_-} - \frac{\beta}{2}\left(a_+ + a_-\right) \ , \tag{2.5}$$

$$\pi_\rho = 2\lambda^2\frac{\left(A'_+A_- - A_+A'_-\right)}{1+\lambda^2\beta A_+A_-} - 2\left(a'_+ - a'_-\right) \ , \tag{2.6}$$

$$\phi = -\frac{1}{\beta}\log\left(1+\lambda^2\beta A_+A_-\right) + a_+ + a_- \ , \tag{2.7}$$

$$\pi_\phi = -\left(\frac{A''_+}{A'_+} - \frac{A''_-}{A'_-}\right) + \lambda^2\beta\frac{\left(A'_+A_- - A_+A'_-\right)}{1+\lambda^2\beta A_+A_-} + \beta\left(a'_+ - a'_-\right) \ . \tag{2.8}$$

The canonical structure of the theory can be equivalently described by the 2-form ω defined by

$$\omega = \int dx \left(\delta\rho\wedge\delta\pi_\rho + \delta\phi\wedge\delta\pi_\phi + \delta f\wedge\delta\pi_f\right) \ . \tag{2.9}$$

In terms of the fields $A_{\pm}, a_{\pm}$ this 2-form turns out to be

$$\begin{aligned}\omega \ &= \ \int dx \left[\delta a_+\wedge\delta\left(-2\frac{A''_+}{A'_+} + 2\beta a'_+ + 2\frac{a''_+}{a'_+}\right) + \right.\\ &\left.\delta a_-\wedge\delta\left(2\frac{A''_-}{A'_-} - 2\beta a'_- - 2\frac{a''_-}{a'_-}\right) + \delta f\wedge\delta\pi_f\right] + \omega_b \ ,\end{aligned} \tag{2.10}$$

where ω_b is just a boundary term (from now on the exterior product will be omitted)

$$\begin{aligned}\omega_b \ &= \ \int d\left[-\frac{\delta A'_+}{A'_+}\delta a_+ + \frac{\delta A_-}{A'_-}\delta a_- + \delta a_+\frac{\delta A'_-}{A'_-} - \delta a_-\frac{\delta A'_+}{A'_+}\right.\\ &-2\beta\delta a_+\delta a_- + 2\frac{\delta a'_+}{a'_+}\delta a_+ - 2\frac{\delta a'_-}{a'_-}\delta a_-\\ &+\lambda^2\left(\frac{A_-}{A'_+}\delta A_+\delta A'_+ - \frac{A_+}{A'_-}\delta A_-\delta A'_- + \frac{A_+}{A'_+}\delta A_-\delta A'_+ + \frac{A_-}{A'_-}\delta A_+\delta A'_-\right.\\ &+2\delta A_+\delta A_- - \frac{1}{2}\frac{\delta\left(A_+A_-\right)}{1+\lambda^2\beta A_+A_-}\left(\frac{\delta A'_+}{A'_+} - \frac{\delta A'_-}{A'_-}\right)\\ &\left.\left.-\frac{\delta A_+\delta A_-}{1+\lambda^2\beta A_+A_-}\right)\right] \ ,\end{aligned} \tag{2.11}$$

and the constraints $C_\pm = \pm\frac{1}{2}(H \pm P)$ become

$$C_\pm = \mp 2a'_\pm \left(\frac{A''_\pm}{A'_\pm} - \beta a'_\pm - \frac{a''_\pm}{a'_\pm} \right) \pm \frac{1}{4}\left(\pi_f \pm f'\right)^2 \ . \tag{2.12}$$

At this point it is clear that the following additional transformation $(A_\pm, a_\pm) \to (X^\pm, \Pi_\pm)$

$$X^\pm = a_\pm \ , \tag{2.13}$$

$$\Pi_\pm = \mp 2 \left(\frac{A''_\pm}{A'_\pm} - \beta a'_\pm - \frac{a''_\pm}{a'_\pm} \right) \ . \tag{2.14}$$

implies that

$$\omega = \int dx \left(\delta X^+ \delta \Pi_+ + \delta X^- \delta \Pi_- + \delta f \delta \pi_f \right) + \omega_b \ , \tag{2.15}$$

and brings the constraints into the form of a parametrized scalar field theory on a flat background [12]

$$C_\pm = \Pi_\pm X^{\pm\prime} \pm \frac{1}{4}\left(\pi_f \pm f'\right)^2 \ , \tag{2.16}$$

The composition of (2.5-2.8) with the inverse of (2.13-2.14)

$$a_\pm = X^\pm \ , \tag{2.17}$$

$$A_\pm = \pm \int^x \exp \int^x \mp\frac{1}{2}\Pi_\pm + \beta X^{\pm\prime} + \left(\log X^{\pm\prime}\right)' \ , \tag{2.18}$$

defines, up to the boundary term, a canonical transformation. However if we restrict the analysis to a closed spatial section ($x \in [0, 2\pi]$) the boundary contribution to the 2-form ω

$$\omega_b = -2\delta \left(\log \frac{A'_+ A'_-}{X^{+\prime} X'} - \beta \left(X^+ + X^-\right) \right)(0)\, \delta\left(X^+(2\pi) - X^+(0)\right) \ , \tag{2.19}$$

vanishes fixing the monodromy of the fields $X^\pm$ as follows (in a parallel way to the CGHS theory [6])

$$X^\pm(2\pi) - X^\pm(0) = \pm 2\pi \ . \tag{2.20}$$

These conditions are consistent with the requirement $X^{\pm\prime} \neq 0$ needed to have a non-singular transformation. A further transformation in the gravitational sector [4, 3, 6]

$$2\Pi_\pm = -\left(\pi_0 + \pi_1\right) \mp \left(r^{0\prime} - r^{1\prime}\right) \ , \tag{2.21}$$

$$2X^{\pm\prime} = \mp\left(\pi_0 - \pi_1\right) - \left(r^{0\prime} + r^{1\prime}\right) , \tag{2.22}$$

casts the constraints of the matter-coupled gravity theory into those of a bosonic string in a 3-dimensional Minkowskian target-space (1.6-1.7).

At this point it is interesting to consider the case $\beta = 0$ (i.e, the CGHS model). When $\beta = 0$ we can alternatively rewrite the two-form ω as

$$\omega = \int dx \left[-\delta A_+ \delta \left(\frac{a_+'}{A_+'} \right)' + \delta A_- \delta \left(\frac{a_-'}{A_-'} \right)' + \delta f \delta \pi_f \right] + \tilde{\omega}_b , \tag{2.23}$$

where $\tilde{\omega}_b$ is a new boundary term and a factor -2 has been absorbed in the arbitrary functions $a_\pm$. Defining now the canonical variables as

$$X^\pm = \mp A_\pm , \tag{2.24}$$

$$\Pi_\pm = \left(\frac{a_\pm'}{A_\pm'} \right)' , \tag{2.25}$$

the constraints take the standard form (2.16). Composing now (2.5-2.8) with the inverse of (2.24-2.25) we recover immediately the canonical transformation proposed in [7]. In the general case ($\beta \neq 0$) it is no longer possible to choose the embedding fields $A_\pm$ as a commuting set of canonical variables and the natural choice is (2.13-2.14).

Having found the canonical transformation, we are now exactly in the situation described in [6], so it is possible to perform the same analysis from this point on, and we shall sketch it here for completeness. At the quantum level the constraints (2.16) close down an anomalous algebra

$$\begin{aligned} \left[C_\pm(x), C_\pm(\tilde{x})\right] = i \left(C_\pm(x) + C_\pm(\tilde{x}) \pm \tfrac{1}{24\pi} \right) \delta'(x - \tilde{x}) \\ \mp \tfrac{i}{24\pi} \delta'''(x - \tilde{x}) , \end{aligned} \tag{2.26}$$

$$\left[C_+(x), C_-(\tilde{x})\right] = 0 , \tag{2.27}$$

and the theory can not be quantized without modification. Remarkably, the addition of a term depending on the coordinate fields $X^\pm$ [3, 4] cancels the anomaly and the new constraints

$$C_\pm(x) \pm \frac{1}{48\pi} \left[\log \pm X^{\pm\prime}\right]'' \mp \frac{1}{48\pi} , \tag{2.28}$$

satisfy the algebra (2.26-2.27) without centre. To solve the new Dirac quantization condition one can construct a quantum canonical transformation based on an expansion of the matter fields in terms of "gravitationally dressed" mode operators

$$a_n^{\pm} \equiv \frac{1}{2\sqrt{\pi}} \int_0^{2\pi} dx\ e^{inX^{\pm}} \left(\pi_f \pm f'\right) . \tag{2.29}$$

If we order the fields with respect to the mode operators (2.29) the constraint algebra is modified but a new modification of the constraints

$$\bar{C}_{\pm} = C_{\pm} \mp \frac{1}{48\pi} X^{\pm\prime} \left[X'_{\pm} + \left(\frac{1}{X^{\pm\prime}} \right)'' \right] , \tag{2.30}$$

leads the algebra (2.26-2.27) without the central terms. Moreover the transformation

$$\bar{X}^{\pm\prime} \bar{\Pi}_{\pm} = \bar{C}_{\pm} , \tag{2.31}$$

$$\bar{X}^{\pm} = X^{\pm} , \tag{2.32}$$

is a quantum canonical transformation and brings the quantum constraints to the simple form (2.16). The physical states are constructed by acting with the creation operators $a^{\pm}_{-|n|}$ on the zero-mode states $|p>$ defined by

$$a_0^- |p> \equiv a_0^+ |p> = p|p> , \tag{2.33}$$

$$a_n^{\pm} |p> = 0 \quad n > 0 , \tag{2.34}$$

and verify the level-matching condition as in the CGHS theory [6]. This restriction comes from the integral condition

$$\int_0^{2\pi} dx \, (\Pi_+ - \Pi_-) = 0 , \tag{2.35}$$

as can be seen immediately from inspection of (2.14) and (2.20).

2.2 Relation with Liouville theory

To finish this section we would like to discuss the relation of our approach to the standard one of Liouville theory. In terms of a new metric $\tilde{g}_{\mu\nu} = e^{-\beta\phi} g_{\mu\nu}$ the action (1.1) in the absence of matter fields takes the form

$$\int d^2x \sqrt{-\tilde{g}} (\tilde{R}\phi + \beta \left(\tilde{\nabla}\phi\right)^2 + 4\lambda^2 e^{2\beta\phi}) \quad . \tag{2.36}$$

The Liouville lagrangian can be recovered from (2.36) by making use of one of the generally covariant equations of motion, $\tilde{R}=0$, and fixing the gauge with the choice of the flat metric $d\tilde{s}^2=-dx^+dx^-$. The resulting theory has been shown to be canonically equivalent to a free field one (see, for instance [13, 14, 15]). We shall now explain why this result does not give directly the above discussed string theory formulation of the exponential model, as might appear to be the case because this model is described in the conformal gauge by a Liouville field $\varphi=2\rho+\beta\phi$ and a free one $\eta=2\tilde{\rho}=2\rho-\beta\phi$, (see (2.1), (2.2)) with certain constraints. To this end let us first write H and P (1.4-1.5) in terms of φ and η (we omit the matter fields f for simplicity)

$$H = -(\beta\pi_\varphi^2+\frac{1}{4\beta}\varphi'^2-4\lambda^2e^\varphi-\frac{1}{\beta}\varphi'')+\beta\pi_\eta^2+\frac{1}{4\beta}\eta'^2-\frac{1}{\beta}\eta'' \, , \quad (2.37)$$

$$P = \pi_\varphi\varphi'+\pi_\eta\eta'-2\pi'_\varphi-2\pi'_\eta \quad . \quad (2.38)$$

Note that neither the Liouville nor the free field parts of H and P correspond to the canonical energy momentum tensor due to the presence of the spatial derivative terms $\varphi'',\eta'',\pi'_\varphi,\pi'_\eta$. Rather, they correspond to the improved one, which can be obtained (as shown in [16]) by varying the metric $\tilde{g}$ in (2.36) (or, in terms of H and P, by simply substituting φ and η for ϕ and ρ in (1.4) and (1.5)). A canonical transformation that relates the Liouville field to a free one is the following (see also [14])

$$\varphi = \psi-2\log(1+\lambda^2\beta A_+A_-)\, , \quad (2.39)$$

$$\pi_\varphi = \pi_\psi-\lambda^2\frac{A'_+A_--A_+A'_-}{(1+\lambda^2\beta A_+A_-)} \quad , \quad (2.40)$$

where

$$A_+=\int^x \exp\int^x(\frac{\psi'}{2}+\beta\pi_\psi) \quad , \quad A_-=-\int^x \exp\int^x(\frac{\psi'}{2}-\beta\pi_\psi) \quad , \quad (2.41)$$

and the boundary term vanishes as can be checked by writing ψ,π_ψ in terms of $X^\pm,\pi_\pm$ and then using (2.20). The constraints are given in terms of the new variables as

$$H = -(\beta\pi_\psi^2+\frac{1}{4\beta}\psi'^2-\frac{1}{\beta}\psi'')+\beta\pi_\eta^2+\frac{1}{4\beta}\eta'^2-\frac{1}{\beta}\eta'' \quad , \quad (2.42)$$

$$P = \pi_\psi\psi'+\pi_\eta\eta'-2\pi'_\psi-2\pi'_\eta \quad , \quad (2.43)$$

which correspond to the difference of two free *improved* energy momentum tensors. ¿From the point of view of Liouville theory this is enough. However, the goal here was to connect the generally covariant theory given by (1.1) with string theory in order to examine its Schrödinger quantization. This demands that the improvement terms must disappear due to the canonical transformation, which is not the case here. Earlier we were able to obtain such a transformation, so it is immediate to write down a new one that removes the improvement terms from the ψ and η pieces of H and P in (2.42-2.43)

$$\psi' = \frac{1}{2}(r^{0\prime} - r^{1\prime}) - \beta(r^{0\prime} + r^{1\prime}) + \left(\log[(r^0 + r^1)^{\prime 2} - (\pi_0 - \pi_1)^2]\right)' , \qquad (2.44)$$

$$\pi_\psi = -\frac{1}{2}(\pi_0 - \pi_1) + \frac{1}{4\beta}(\pi_0 + \pi_1) + \frac{1}{2\beta}\left(\log\left[\frac{(r^0 + r^1)' + (\pi_0 - \pi_1)}{(r^0 + r^1)' - (\pi_0 - \pi_1)}\right]\right)' , \qquad (2.45)$$

$$\eta' = \frac{1}{2}(r^{0\prime} - r^{1\prime}) + \beta(r^{0\prime} + r^{1\prime}) + \left(\log[(r^0 + r^1)^{\prime 2} - (\pi_0 - \pi_1)^2]\right)' , \qquad (2.46)$$

$$\pi_\eta = -\frac{1}{2}(\pi_0 - \pi_1) - \frac{1}{4\beta}(\pi_0 + \pi_1) - \frac{1}{2\beta}\left(\log\left[\frac{(r^0 + r^1)' + (\pi_0 - \pi_1)}{(r^0 + r^1)' - (\pi_0 - \pi_1)}\right]\right)' . \qquad (2.47)$$

After this transformation H reads

$$H = \frac{1}{2}(\pi_0^2 + (r^{0\prime})^2) - \frac{1}{2}(\pi_1^2 + (r^{1\prime})^2) . \qquad (2.48)$$

Note that this transformation mixes the ψ and η fields up, so it cannot be used to achieve the same for ψ alone.

3 The Jackiw-Teitelboim model

The Jackiw-Teitelboim model [17] coupled to conformal matter is given by the action

$$S = \int d^2x\sqrt{-g}\left[R\phi + 4\lambda^2\phi - \frac{1}{2}(\nabla f)^2\right] , \qquad (3.1)$$

and it is also one of the most relevant models of 2D dilaton-gravity. The equations of motion, in conformal gauge, become

$$2e^{-2\rho}\partial_+\partial_-\rho + \lambda^2 = 0 , \qquad (3.2)$$

$$e^{-2\rho}\partial_+\partial_-\phi + \lambda^2\phi = 0 , \qquad (3.3)$$

plus the constrained and matter equations (2.3-2.4), which are model independent. Equation (3.2) implies that ρ is a Liouville field so the general solution for ρ is

$$\rho = \frac{1}{2} \log \frac{\partial_+ A_+ \partial_- A_-}{\left(1 + \frac{\lambda^2}{2} A_+ A_-\right)^2} , \tag{3.4}$$

where $A_\pm$ is an arbitrary function depending on the $x^\pm$ coordinate. Taking into account (3.4) and the constrained equations it is possible to find the following solution to the equation (3.3)

$$\phi = -\frac{1}{2}\left(\frac{\partial_+ a_+}{\partial_+ A_+} - \frac{\partial_- a_-}{\partial_- A_-}\right) + \frac{\lambda^2}{2} \frac{(a_+ A_- - a_- A_+)}{1 + \frac{\lambda^2}{2} A_+ A_-} . \tag{3.5}$$

with $a_\pm$ an arbitrary function of the $x^\pm$ coordinate. Therefore the general solution is parametrized by four arbitrary chiral functions. Two of them are simply the two gauge fixing functions associated with conformal coordinate transformations and the other two account for the two chiral sectors of the matter field. The general solution (3.4-3.5) suggests the following transformation to the new variables $A_\pm, a_\pm$ (from now on $A_\pm$ and $a_\pm$ are not required to be chiral functions)

$$\rho = \frac{1}{2} \log \frac{-A'_+ A'_-}{\left(1 + \frac{\lambda^2}{2} A_+ A_-\right)^2} , \tag{3.6}$$

$$\begin{aligned} \pi_\rho &= \left(\frac{a'_+}{A'_+}\right)' + \left(\frac{a'_-}{A'_-}\right)' + \frac{\lambda^4}{2} \frac{(A'_+ A_- - A_+ A'_-)}{\left(1 + \frac{\lambda^2}{2} A_+ A_-\right)^2} (a_+ A_- - a_- A_+) \\ &\quad -\lambda^2 \frac{(a_+ A_- - a_- A_+)'}{1 + \frac{\lambda^2}{2} A_+ A_-} , \end{aligned} \tag{3.7}$$

$$\phi = -\frac{1}{2}\left(\frac{a'_+}{A'_+} - \frac{a'_-}{A'_-}\right) + \frac{\lambda^2}{2} \frac{(a_+ A_- - a_- A_+)}{1 + \frac{\lambda^2}{2} A_+ A_-} , \tag{3.8}$$

$$\pi_\phi = -\left(\frac{A''_+}{A'_+} - \frac{A''_-}{A'_-}\right) - 2\lambda^2 \frac{(A'_+ A_- - A_+ A'_-)}{1 + \frac{\lambda^2}{2} A_+ A_-} , \tag{3.9}$$

The 2-form (2.9) becomes now

$$\begin{aligned} \omega &= \int dx \left[\delta \log A_+ \delta \left(\frac{a'_+}{A'_+} - \frac{A_+}{A'_+} \left(\frac{a'_+}{A'_+} \right)' \right)' \right. \\ &\quad \left. + \delta \log A_- \delta \left(\frac{a'_-}{A'_-} - \frac{A_-}{A'_-} \left(\frac{a'_-}{A'_-} \right)' \right)' + \delta f \delta \pi_f \right] + \omega_b , \end{aligned} \tag{3.10}$$

where the boundary term ω_b is

$$\begin{aligned}
\omega_b &= \int d\left[-\frac{1}{2}\left(\frac{\delta A'_+}{A'_+}-\frac{\delta A'_-}{A'_-}\right)\left(\delta\left(\frac{a'_+}{A'_+}\right)-\delta\left(\frac{a'_-}{A'_-}\right)\right)\right. \\
&+\delta A_+\delta\left(\frac{a''_+}{A'^2_+}-\frac{a'_+A''_+}{A'^3_+}\right)+\delta A_-\delta\left(\frac{a''_-}{A'^2_-}-\frac{a'_-A''_-}{A'^3_-}\right) \\
&+\frac{1}{2}\frac{\lambda^2}{1+\frac{\lambda^2}{2}A_+A_-}\left[\left(\frac{\delta A'_+}{A'_+}-\frac{\delta A'_-}{A'_-}\right)\delta\left(a_+A_- - A_-a_+\right)\right. \\
&\left.+\delta\left(\frac{a'_+}{A_+{}'}+\frac{a'_-}{A'_-}\right)\delta\left(A_+A_-\right)-2\left(\delta a_+\delta A_- + \delta a_-\delta A_+\right)\right] \\
&-\frac{1}{4}\frac{\lambda^4}{\left(1+\frac{\lambda^2}{2}A_+A_-\right)^2}\left[\left(a_+A_- - a_-A_+\right)\left(\frac{\delta A'_+}{A'_+}-\frac{\delta A'_-}{A'_-}\right)\delta\left(A_+A_-\right)\right. \\
&\left.\left.-2\left(a_+A_- - a_-A_+\right)\delta A_+\delta A_-\right]\right] . \qquad (3.11)
\end{aligned}$$

Performing now the transformation

$$X^{\pm} = \log A_{\pm} , \qquad (3.12)$$

$$\Pi_{\pm} = \left(\frac{a'_{\pm}}{A'_{\pm}} - \frac{A_{\pm}}{A'_{\pm}}\left(\frac{a'_{\pm}}{A'_{\pm}}\right)'\right)' , \qquad (3.13)$$

the 2-form (3.10) converts into (2.15) and the constraints adopts the form of a parametrized field theory (2.16). The composition of (3.6-3.9) with the inverse of (3.12-3.13)

$$A_{\pm} = e^{X^{\pm}} , \qquad (3.14)$$

$$a_{\pm} = -\int^x e^{X^{\pm}} X^{\pm\prime} \int^x e^{X^{\pm}} X^{\pm\prime} \int^x e^{-X^{\pm}} \Pi_{\pm} , \qquad (3.15)$$

defines, up to boundary terms, a canonical transformation. If we consider the case of a closed spatial section and impose the condition (2.20) then the boundary contribution ω_b

$$\omega_b = -\delta\left[\frac{a'_+}{A'_+} - \frac{A_+}{A'_+}\left(\frac{a'_+}{A'_+}\right)' - \frac{a'_-}{A'_-} + \frac{A_-}{A'_-}\left(\frac{a'_-}{A'_-}\right)'\right](0)\,\delta\left(X^+(2\pi) - X^+(0)\right) , \qquad (3.16)$$

vanishes. As in the CGHS and exponential models, the additional transformation (2.21-2.22) maps the theory into a bosonic string theory propagating on a 3-dimensional Minkowskian target space. Therefore the quantum analysis can be carried out along the lines of [6, 7] as explained in the previous section, although now the integral conditions are stronger

$$\int_0^{2\pi} dx \; \Pi_\pm = 0 \, . \tag{3.17}$$

It follows immediately from (3.13) and the monodromy properties of the fields $A_\pm, a_\pm$.

The fact that a canonical transformation that relates the dilaton gravity to a string theory can be found for several different cases seems to indicate that there might be more models with this property, perhaps including the spherically symmetric Einstein gravity (i.e, $V \propto \frac{1}{\sqrt{\phi}}$).

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