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AdS_2/CFT_1 correspondence and near-extremal black hole entropy

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Abstract

We provide a realization of the AdS_2/CFT_1 correspondence in terms of asymptotic symmetries of the $AdS_2 \times S^1$ and $AdS_2 \times S^2$ geometries arising in near-extremal BTZ and Reissner-Nordström black holes. Cardy's formula exactly accounts for the deviation of the Bekenstein-Hawking entropy from extremality. We also argue that this result can be extended to more general black holes near extremality.

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1 Introduction

Since the discovery of the thermodynamical properties of black holes a crucial open problem has been to find a microscopical structure responsible for the Bekenstein-Hawking entropy. In the last few years this question has started to receive some answers. The discovery of D-branes led to an explicit statistical derivation of the black hole entropy for extremal [1] and near-extremal [2] black holes (see also the reviews [3, 4]). In a different context, Strominger has proposed [5] a unified way to account for the Bekenstein-Hawking entropy of black holes whose near-horizon geometries are locally similar to the BTZ black holes [6]. The idea of the approach of [5] is that a conformal symmetry of the gravity theory can control the asymptotic density of states, irrespective of the details of quantum gravity theory, thus providing a statistical explanation to the area formula for the entropy and, in turn, a sort of universality. Strominger's argument is based in the holographic relation, first discovered by Brown and Henneaux [7], between gravity on AdS_3 and a two-dimensional conformal field theory on the boundary. AdS_3 gravity possesses a set of asymptotic symmetries closing down two copies of the Virasoro algebra with central charge $c = \frac{3\ell}{2G}$, where G is Newton's constant and $-1/\ell^2$ is the cosmological constant. Using Cardy's formula [8] for the boundary CFT_2 one reproduces the expected entropy. However, the validity of Cardy's formula requires that the lowest eigenvalues of the Virasoro operators L_0 and $\bar{L}_0$ vanish. As it has been pointed out in [9], this is not the case of the boundary theory of AdS_3 gravity, because it is, up to global issues, Liouville theory [10]. The asymptotic level density of states is then controlled by the effective central charge [11] which, for Liouville theory, turns out to be equal to one and therefore cannot properly account for the entropy. However, the fact that the entropy fits Cardy's formula with the ordinary central charge seems to indicate that gravity theory itself can provide relevant information about the microscopic theory, but apparently not enough to characterise it completely. Interesting attempts to avoid the restrictions of 2+1 dimensions to explain the Bekenstein-Hawking entropy by means of symmetry principles has been given in [12, 13, 14] by considering the horizon as a boundary.

Among the family of AdS_D/CFT_{D-1} dualities [15], the pure gravity case AdS_3/CFT_2 is the best understood. In contrast, the AdS/CFT correspondence in two space-time dimensions is quite enigmatic. Some progress has been made in [16, 17, 18, 19]. One of the aims of this paper is to further investigate the AdS_2/CFT_1 correspondence in terms of asymptotic symmetries. In section 2 we shall analyse the relation between the first sub-leading terms in the asymptotic expansion of the metric field, obeying suitable AdS_2 boundary conditions, and the stress tensor of the boundary theory, as happen in higher dimensional situations [20, 21]. Following a similar line of reasoning as in [5] we shall show that the application of Cardy's formula to the unique copy of the Virasoro algebra emerging as an asymptotic symmetry, yields to the entropy of spinless BTZ black holes. But more interestingly, the AdS_2/CFT_1 correspondence, implemented via asymptotic symmetries, can be used to correctly account for the deviation of the Bekenstein-Hawking entropy from extremality in the near-horizon approximation. On general grounds, two-dimensional Anti-de Sitter space naturally arises in the near-horizon limit around the degenerate radius of coincident horizons [22]. Therefore, a way to study Maldacena's duality in D=2 and its implication for black holes is to consider gravity theories having black hole solutions with degenerate horizons. In section 3 we consider near-extremal BTZ black holes and in section 4 four-dimensional Reissner-Nordström black holes near extremality. Finally, in section 5 we show that the above results can be extended to any black hole with degenerate horizons that can be properly described by a two-dimensional effective theory.

2 Dimensional reductions of AdS_3 gravity and the AdS_2/CFT_1 correspondence

Einstein gravity on AdS_3 is described by the action

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}(R + \frac{2}{\ell^2}) \tag{2.1}$$

and we can dimensionally reduce the theory [23, 24] via a decomposition of the metric of the form

$$ds^2_{(3)} = g_{\mu\nu}dx^\mu dx^\nu + \ell^2\phi^2(x)(d\theta + A_\mu(x)dx^\mu)^2\,, \qquad \mu,\nu = 0,1 \tag{2.2}$$

where $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ is a two-dimensional metric, ϕ a scalar (dilaton) field and A_μ a Kaluza-Klein $\mathcal{U}(1)$ gauge field. The two-dimensional effective theory is governed by the action

$$\frac{\ell}{8G}\int d^2x\sqrt{-g}\phi(R + \frac{2}{\ell^2} - \frac{\ell^2}{4}\phi^2F_{\mu\nu}F^{\mu\nu})\,. \tag{2.3}$$

The equations of motion of the gauge field imply that

$$2\frac{\ell^3\phi^3}{\sqrt{-g}}F_{+-} = constant\,, \tag{2.4}$$

where $x^{\pm} = x^0 \pm x^1$ and $F_{+-} = \partial_+ A_- - \partial_- A_+$. Moreover, by varying the dilaton one obtains

$$R + \frac{2}{\ell^2} - \frac{3}{4}\ell^2\phi^2 F^2 = 0\,, \tag{2.5}$$

and using (2.4) one gets

$$R = -\frac{2}{\ell^2} - \frac{3}{2}\frac{J^2}{\ell^2\phi^4}\,, \tag{2.6}$$

where J is related with the integration constant of (2.4). The action (2.3) turns out to be then

$$\frac{\ell}{8G}\int d^2x\sqrt{-g}(\phi R + V(\phi))\,, \tag{2.7}$$

where

$$V(\phi) = \phi\left(\frac{2}{\ell^2} - \frac{J^2}{2\ell^2\phi^4}\right)\,. \tag{2.8}$$

The most general solution of (2.7) with a linear dilaton corresponds to the dimensional reduction of the BTZ black hole which, in the Schwarzschild gauge, takes the form (with $A_t = -\frac{4GJ}{r^2}$)

$$ds^2 = -\left(\frac{r^2}{\ell^2} - 8GM + \frac{16G^2J^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} - 8GM + \frac{16G^2J^2}{r^2}\right)}\,, \tag{2.9}$$

$$\phi = \frac{r}{\ell}\,. \tag{2.10}$$

The two event horizons are located at

$$r_\pm^2 = 4GM\ell^2\left(1 \pm \sqrt{1 - \left(\frac{J}{M\ell}\right)^2}\right) \tag{2.11}$$

and the outer horizon give rise to the entropy [24]

$$S = \frac{2\pi r_+}{4G} = \pi\sqrt{\frac{\ell(\ell M + J)}{2G}} + \pi\sqrt{\frac{\ell(\ell M - J)}{2G}}\,, \tag{2.12}$$

which reproduce as expected the entropy of the original three-dimensional theory. To get a two-dimensional AdS geometry from (2.6) we have two different ways. We can restrict the theory to the spinless sector $J = 0$ with

$$R = -\frac{2}{\ell^2}\,, \tag{2.13}$$

or we can fix the value of the dilaton in a way consistent with the equation of motion [22]

$$\Box\phi = V(\phi)\,, \tag{2.14}$$

where $V(\phi)$ is given by (2.8). This implies that the value of the dilaton $\phi = \phi_0$ should be a zero of the potential

$$V(\phi_0) = \phi_0 \left(\frac{2}{\ell^2} - \frac{J^2}{2\ell^2\phi_0^4}\right) = 0\,, \tag{2.15}$$

and then

$$R = -V'(\phi_0) = -\frac{8}{\ell^2}\,. \tag{2.16}$$

In the remaining part of this section we shall study the first possibility, which is equivalent to consider the Jackiw-Teitelboim model of two-dimensional gravity [25]. The second way to get an AdS_2 geometry ($AdS_2 \times S^1$ from the three-dimensional point of view) will be widely analysed in the next section, although the basic features of the AdS_2/CFT_1 correspondence considered in our approach will be presented here.

The reduced theory with $J = 0$ coincides with the Jackiw-Teitelboim model

$$S = \frac{\ell}{8G}\int d^2x\sqrt{-g}\,\phi(R + \frac{2}{\ell^2})\,, \tag{2.17}$$

whose solutions are of the form

$$\begin{aligned} ds^2 &= -(\frac{x^2}{\ell^2} - a^2)dt^2 + (\frac{x^2}{\ell^2} - a^2)^{-1}dx^2\,, & (2.18)\\ \phi &= \frac{x}{\ell}\,, & (2.19)\end{aligned}$$

with $a^2 = 8GM$. The metrics (2.18) are locally AdS_2 and in order to define a quantum theory we have to specify boundary conditions for the fields at infinity. Mimicking the analysis of three-dimensional gravity [20] we shall assume the following asymptotic behaviour of the metric ‡

$$g_{tt} = -\frac{x^2}{\ell^2} + \gamma_{tt}(t) + \mathcal{O}(\frac{1}{x^2}) , \tag{2.20}$$

$$g_{tx} = \frac{\gamma_{tx}}{x^3} + \mathcal{O}(\frac{1}{x^5}) , \tag{2.21}$$

$$g_{xx} = \frac{\ell^2}{x^2} + \frac{\gamma_{xx}}{x^4} + \mathcal{O}(\frac{1}{x^6}) , \tag{2.22}$$

where we have now introduced the first sub-leading terms in the expansion aiming to relate them with a conformal field on the boundary.

The infinitesimal diffeomorphisms $\zeta^a(x,t)$ preserving the above boundary conditions are

$$\zeta^t = \epsilon(t) - \frac{\ell^4}{2x^2}\epsilon''(t) + \mathcal{O}(\frac{1}{x^4}) , \tag{2.23}$$

$$\zeta^x = -x\epsilon'(t) + \mathcal{O}(\frac{1}{x}) . \tag{2.24}$$

Using the "gauge" diffeomorphisms

$$\zeta^t = \frac{\alpha^t(t)}{x^4} + \mathcal{O}(\frac{1}{x^5}) , \tag{2.25}$$

$$\zeta^x = \frac{\alpha^x(t)}{x} + \mathcal{O}(\frac{1}{x^2}) , \tag{2.26}$$

where α^t and α^x are arbitrary functions, one can easily show that the only gauge invariant quantity is

$$\Theta_{tt} = \kappa(\gamma_{tt} - \frac{\gamma_{xx}}{2\ell^4}) , \tag{2.27}$$

where κ is a constant coefficient.

The action of the infinitesimal diffeomorphism (2.23-2.24) on the metric induces the following transformation for the function Θ_{tt}:

$$\delta_\epsilon \Theta_{tt} = \epsilon(t)\Theta'_{tt} + 2\Theta_{tt}\epsilon'(t) - \kappa\ell^2\epsilon'''(t) . \tag{2.28}$$

So, the quantity Θ_{tt} behaves as the (chiral component of the) stress tensor of a conformal field theory. To evaluate the central charge we need to know the

‡These boundary conditions where first introduced in [18].

coefficient κ. To this end we have to work out the Noether charges associated to the above asymptotic symmetries. Using the decomposition of the metric

$$ds^2 = -N^2 dt^2 + \sigma^2 (dx + N^x dt)^2 \,, \tag{2.29}$$

the bulk Hamiltonian of the theory is given by

$$H_0 = \int dx (N\mathcal{H} + N^x \mathcal{H}_x) \,, \tag{2.30}$$

where the constraints are

$$\mathcal{H} = -\Pi_\phi \Pi_\sigma + (\frac{\phi'}{\sigma})' - \frac{\sigma\phi}{\ell^2} \,, \tag{2.31}$$

$$\mathcal{H}_x = \Pi_\phi \phi' - \sigma \pi'_\sigma \,, \tag{2.32}$$

and the momenta

$$\Pi_\phi = N^{-1}(\sigma^{-1} + (N^x \sigma)') \,, \tag{2.33}$$

$$\Pi_\sigma = N^{-1}(-\dot{\phi} + N^x \phi') \,. \tag{2.34}$$

The full Hamiltonian is given by

$$H = H_0 + K \,, \tag{2.35}$$

where K is a boundary term necessary to have well-defined variational derivatives. Assuming the boundary condition for the dilaton

$$\phi = \frac{x}{\ell} + \frac{\ell}{2x}\gamma_{\phi\phi}(t) + \mathcal{O}(\frac{1}{x^2}) \tag{2.36}$$

and imposing that K vanishes for $a^2 = 0$, the boundary term K can be worked out [18]

$$K(\epsilon) = \frac{\ell}{4G} \lim_{x\to\infty} \left\{ -\frac{x}{\ell}\zeta^\perp (\phi' - \frac{1}{\ell}) + \frac{x}{\ell}\frac{\partial \zeta^\perp}{\partial x}(\phi - \frac{x}{\ell}) + \frac{x^3}{2\ell^4}\zeta^\perp (g_{xx} - \frac{\ell^2}{x^2}) + \frac{\ell}{x}\zeta^\parallel \Pi_\sigma \right\} \,. \tag{2.37}$$

Using the asymptotic expansion for the metric and the dilaton we obtain

$$K(\epsilon) = \frac{\epsilon}{4G}\left(\frac{1}{2\ell^4}\gamma_{xx} + \gamma_{\phi\phi} \right) \,. \tag{2.38}$$

Moreover, the equation for the dilaton

$$\Box\phi = \frac{2}{\ell^2}\phi \tag{2.39}$$

allows to relate $\gamma_{\phi\phi}$ with the remaining quantities

$$\gamma_{\phi\phi} = \left(\gamma_{tt} - \frac{\gamma_{xx}}{\ell^4}\right) , \tag{2.40}$$

and then $K(\epsilon)$ can be written in terms of the unique gauge invariant quantity

$$K(\epsilon) = \epsilon\frac{1}{4G}\left(\gamma_{tt} - \frac{1}{2\ell^4}\gamma_{xx}\right) . \tag{2.41}$$

The standard identification of $K(\epsilon)$ in terms of the stress tensor [26]

$$K(\epsilon) = \epsilon\Theta_{tt} \tag{2.42}$$

allows us to know the coefficient κ, which turns out to be

$$\kappa = \frac{1}{4G} . \tag{2.43}$$

We still have to compute the central charge. Defining the Fourier components L_n^R of Θ_{tt} as

$$L_n^R = \frac{1}{2\pi\ell}\int_0^{2\pi\ell} dt\Theta_{tt}\ell e^{int/\ell} , \tag{2.44}$$

where we assume periodicity of t in the interval $0 \leq t < 2\pi\ell$, the Poisson algebra can be expressed as follows

$$\{L_n^R, L_m^R\} = \delta_{\epsilon_m} L_n^R , \tag{2.45}$$

where $\epsilon_m = \ell e^{imt/\ell}$. Using (2.28) it is easy to get the following Virasoro algebra

$$i\{L_n^R, L_m^R\} = (n-m)L_{n+m}^R + \frac{c}{12}n^3\delta_{n,-m} \tag{2.46}$$

with central charge

$$c = 12\kappa\ell = \frac{3\ell}{G} . \tag{2.47}$$

For the black holes (2.18) we have a constant value of Θ_{tt}

$$\Theta_{tt} = \frac{\kappa}{2}a^2 = \frac{1}{8G}a^2 \tag{2.48}$$

and, in terms of the mass $a^2 = 8GM$, we have

$$L_0^R = \ell\Theta_{tt} = M\ell \tag{2.49}$$

Observe that to have a Virasoro algebra of the Neveu-Schwartz form we must shift the Ramond-type generator: $L_0^R \rightarrow L_0^{NS} = L_0^R + \frac{c}{24}$.

The results (2.47) and (2.49) allows us to compute the asymptotic density of states using Cardy's formula

$$\log \rho(\Delta) \sim 2\pi\sqrt{\frac{c\Delta}{6}}\,, \tag{2.50}$$

where Δ is the eigenvalue of the Virasoro generator L_0^{NS}. In our case $c = \frac{3\ell}{G}$ and $\Delta = M\ell + \frac{\ell}{8G}$. For large mass $\Delta \gg c$ we get the following statistical entropy

$$S = 2\pi\sqrt{\frac{M\ell^2}{2G}}\,, \tag{2.51}$$

which coincides with the thermodynamical formula (2.12) with $J = 0$. We should stress the fact that, in contrast with the analysis of [18], which uses the convention $4G = \ell$, we have found an exact agreement between the statistical entropy of the two-dimensional black hole (2.18) and the corresponding 2D Bekenstein-Hawking formula. The discrepancy comes from the evaluation of the central charge. Our result is $c = \frac{3\ell}{G}$ and the authors of [18] claim that $c = 24(\frac{\ell}{4G})$.

We must stress now the important fact that the statistical entropy is independent of the length of the interval of the compactified parameter t. If we choose a different periodicity for t: $0 < t < 2\pi\beta$, the central charge shift $c \rightarrow c\frac{\ell}{\beta}$, but L_0^R get modified $L_0^R \rightarrow \frac{\beta}{\ell}L_0^R$ in such a way that cL_0^R, and hence the entropy, is not sensitive to the compactification scale.

3 Near-extremal BTZ black holes

The second possibility to get a AdS_2 geometry in AdS_3 gravity is by means of a constant dilaton solution. This can be obtained performing a perturbation around the degenerate radius of the extremal solutions, keeping the angular momentum $\mid \frac{J}{\ell} \mid = M_0$ fixed, (see [22] for the general case):

$$M = \mid J/\ell \mid (1 + k\alpha^2)\,, \tag{3.1}$$

where α is an infinitesimal parameter $0 < \alpha \ll 1$ and k is an arbitrary positive constant. Introducing the coordinates $(\tilde{t}, \tilde{x})$ defined by

$$t = \frac{\tilde{t}}{\alpha}\,, \qquad r = r_0 + \alpha\tilde{x}\,, \tag{3.2}$$

where $r_0 = r_+ = r_- = 2\ell\sqrt{GM_0}$, the solutions (2.9-2.10) have a well-defined limit when $\alpha \to 0$:

$$\begin{aligned} ds^2 &= -\left(\frac{4\tilde{x}^2}{\ell^2} - a^2 + \mathcal{O}(\alpha)\right) d\tilde{t}^2 + \left(\frac{4\tilde{x}^2}{\ell^2} - a^2 + \mathcal{O}(\alpha)\right)^{-1} d\tilde{x}^2\,, && (3.3)\\ \phi &= \frac{r_0}{\ell} + \frac{\alpha}{\ell}\tilde{x}\,, && (3.4) \end{aligned}$$

with $a^2 = 8M_0Gk$. This way we recover, in the $\alpha \to 0$ limit, an AdS_2 geometry with curvature $R = -\frac{8}{\ell^2}$. Arguing now as in the previous section and assuming analogous boundary conditions for the asymptotic expansion of the two-dimensional AdS_2 metric

$$\begin{aligned} g_{\tilde{t}\tilde{t}} &= -\frac{4\tilde{x}^2}{\ell^2} + \gamma_{\tilde{t}\tilde{t}} + \mathcal{O}(\frac{1}{\tilde{x}^2})\,, && (3.5)\\ g_{\tilde{t}\tilde{x}} &= \frac{\gamma_{\tilde{t}\tilde{x}}}{\tilde{x}^3} + \mathcal{O}(\frac{1}{\tilde{x}^5})\,, && (3.6)\\ g_{\tilde{x}\tilde{x}} &= \frac{\ell^2}{4\tilde{x}^2} + \frac{\gamma_{\tilde{x}\tilde{x}}}{\tilde{x}^4} + \mathcal{O}(\frac{1}{\tilde{x}^6})\,, && (3.7) \end{aligned}$$

we find that

$$\Theta_{\tilde{t}\tilde{t}} = \kappa(\gamma_{\tilde{t}\tilde{t}} - \frac{\gamma_{\tilde{x}\tilde{x}}}{2(\ell/2)^4}) \tag{3.8}$$

transforms as

$$\delta_\epsilon \Theta_{\tilde{t}\tilde{t}} = \epsilon(t)\Theta'_{\tilde{t}\tilde{t}} + 2\Theta_{\tilde{t}\tilde{t}}\epsilon'(t) - \frac{\kappa\ell^2}{4}\epsilon'''(t)\,. \tag{3.9}$$

Observe that the modification of the above expression with respect to the case $J = 0$ is due to the shift in the two-dimensional curvature, $R = -\frac{8}{\ell^2}$ instead of $R = -\frac{2}{\ell^2}$ for $J = 0$.

Since the AdS_2 codifies in some sense the black hole geometry in the near-extremal situation

$$\frac{M - M_0}{M_0} = k\alpha^2 \ll 1\,, \tag{3.10}$$

the idea now is to exploit this feature to explain the near-extremal entropy in terms of the asymptotic symmetries of AdS_2. To evaluate the central charge when we approach to the extremal black hole we have to work out the Noether charges. The calculation is similar to that given in section 2, since only the derivative term in the action (2.7) is relevant, and it coincides with that of Jackiw-Teitelboim theory. Therefore, the Noether charges, to leading order in α, are

$$K(\epsilon) = \epsilon(\tilde{t})\frac{\alpha\ell}{4G}\frac{1}{\ell}\left(\gamma_{\tilde{t}\tilde{t}} - \frac{\gamma_{\tilde{x}\tilde{x}}}{2(\ell/2)^4}\right), \tag{3.11}$$

and this implies that the coefficient κ is

$$\kappa = \frac{\alpha}{4G}. \tag{3.12}$$

Assuming $\tilde{t}$ varies in the interval $0 < \tilde{t} < 2\pi(R/2)^{-1/2} = \pi\ell$, the central charge is

$$c = 6\kappa\ell = \frac{3\ell}{2G}\alpha. \tag{3.13}$$

We also want to compute the value of L_0^R in the near-extremal black hole solutions. Since $L_0^R = \frac{\ell}{2}K \quad (\epsilon(\tilde{t}) = 1)$ we find that

$$L_0^R = \frac{\ell}{2}\frac{\alpha}{4G}\frac{1}{2}a^2 = \frac{1}{2}M_0 k\alpha\ell. \tag{3.14}$$

It is interesting to observe that, with respect to the time $t = \frac{\tilde{t}}{\alpha}$, the generators $L^R_{m(\tilde{t})}$ shift into $L^R_{m(t)} = \alpha L^R_{m(\tilde{t})}$, and then $L^R_{0(t)}$ is equal to one half of the deviation of the mass from the extremal case:

$$L^R_{0(t)} = \frac{1}{2}M_0\ell k\alpha^2 = \frac{1}{2}(M - M_0)\ell. \tag{3.15}$$

From (3.13) and (3.14) we can evaluate, via Cardy's formula, the degeneracy of states if $M - M_0$ is large in the microscopic sense

$$2\pi\sqrt{\frac{cL_0^R}{6}} = \pi\sqrt{\frac{\ell^2(M - M_0)}{2G}}, \tag{3.16}$$

which turns out to be just the difference between the entropy of a nearly extremal black hole and a extremal one

$$\Delta S = S - S_e = \pi\sqrt{\frac{\ell^2(M - M_0)}{2G}}. \tag{3.17}$$

Therefore, the statistical entropy (3.16) just account for microscopic excitations from the extremal macroscopic state.

4 Near-extremal Reissner-Nordström black holes

Let us start with the Einstein-Hilbert action in 3+1 dimensions

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g^{(4)}}(R^{(4)} - G(F^{(4)})^2)\,. \tag{4.1}$$

Imposing spherical symmetry on the electromagnetic field and on the metric

$$ds^2_{(4)} = g^{RN}_{\mu\nu}dx^\mu dx^\nu + \frac{1}{2}\ell^2\bar{\phi}^2(x)d\Omega^2\,, \tag{4.2}$$

where $d\Omega^2$ is the metric on the two-sphere and ℓ is the Planck length ($\ell^2 = G$), the action (4.1) reduces to

$$\int d^2x\sqrt{-g}\left[\frac{1}{2}\left(\frac{\bar{\phi}^2}{4}R + \frac{1}{2}\mid\nabla\bar{\phi}\mid^2 + \frac{1}{\ell^2}\right) - \frac{\ell^2}{8}\bar{\phi}^2F^{\mu\nu}F_{\mu\nu}\right]. \tag{4.3}$$

To perform a similar analysis to that of section 3 we need to reparametrise the fields to eliminate the kinetic term in (4.3) and bring the action to the more reduced form (2.3). To this end we introduce the new fields

$$\phi = \frac{\bar{\phi}^2}{4}\,, \tag{4.4}$$

$$g_{\mu\nu} = \sqrt{2\phi}\,g^{RN}_{\mu\nu}\,. \tag{4.5}$$

The two-dimensional effective action becomes

$$\frac{1}{2}\int d^2x\sqrt{-g}\phi\left(R + \frac{1}{\sqrt{2}\ell^2\phi^{3/2}} - \sqrt{2}\phi^{1/2}\ell^2F^{\mu\nu}F_{\mu\nu}\right) \tag{4.6}$$

and the equations of motion of the electromagnetic field yield to

$$\frac{4\sqrt{2}\ell\phi^{3/2}}{\sqrt{-g}}F_{+-} = Q\,, \tag{4.7}$$

where Q is an integration constant. Plugging (4.7) into (4.6) we get

$$\frac{1}{2}\int d^2x\sqrt{-g}(\phi R + V(\phi))\,, \tag{4.8}$$

where

$$V(\phi) = \frac{1}{\ell^2}\left(\frac{1}{\sqrt{2\phi}} - \frac{\ell^2Q^2}{(2\phi)^{3/2}}\right). \tag{4.9}$$

The general solution with a non-constant dilaton is

$$ds^2 = -(\sqrt{2\phi} + \frac{\ell^2 Q^2}{\sqrt{2\phi}} - 2M\ell)dt^2 + (\sqrt{2\phi} + \frac{\ell^2 Q^2}{\sqrt{2\phi}} - 2M\ell)^{-1}dx^2 \quad (4.10)$$

$$\phi = \frac{x}{\ell}\,. \quad (4.11)$$

Note that the rescaling (4.4-4.5) map the above solutions into the standard form

$$(ds^2)^{RN} = -\left(1 - \frac{2GM}{r} + \frac{Q^2G^2}{r^2}\right)dt^2 \quad (4.12)$$

$$+ \left(1 - \frac{2GM}{r} + \frac{Q^2G^2}{r^2}\right)^{-1}dr^2 \quad (4.13)$$

$$\bar{\phi} = \sqrt{2}\frac{r}{\ell}\,. \quad (4.14)$$

As is well known, there are two event horizons, located at

$$\sqrt{2\phi} = \ell(M \pm \sqrt{M^2 - Q^2})\,. \quad (4.15)$$

Perturbing the solution (4.10) around the degenerate radius $x_0 = \frac{1}{2}\ell^3 M_0^2$ of the extremal solution $M_0 =\mid Q \mid$

$$M = M_0(1 + k\alpha^2)\,, \quad (4.16)$$

$$t = \frac{\tilde{t}}{\alpha}\,, \qquad x = x_0 + \alpha\tilde{x}\,, \quad (4.17)$$

we get in the near-horizon limit $\alpha \to 0$

$$ds^2 = -\left(\frac{1}{\ell^5 \mid Q \mid^3}\tilde{x}^2 - 2 \mid Q \mid k\ell\right)d\tilde{t}^2 + \left(\frac{1}{\ell^5 \mid Q \mid^3}\tilde{x}^2 - 2 \mid Q \mid k\ell\right)^{-1}d\tilde{x}^2\,. \quad (4.18)$$

So, the curvature is $R_0 = \frac{2}{\ell^5 M_0^3}$ and $a^2 = 4M_0\ell$. Note that for the metric $g_{\mu\nu}^{RN}$ the curvature is $\frac{2}{\ell^4 M_0^2}$, which corresponds to that of the Robinson-Bertotti geometry.

Proceeding in a parallel way as in the case of near-extremal BTZ black holes, we find here that the Noether charges are

$$K(\epsilon) = \epsilon(\tilde{t})\frac{\alpha}{\ell}\left(\gamma_{\tilde{t}\tilde{t}} - \frac{1}{2\ell^{10}Q^6}\gamma_{\tilde{x}\tilde{x}}\right)\,, \quad (4.19)$$

and these yield to the central charge

$$c = 12 \mid Q \mid^3 \frac{\ell^4 \alpha}{\beta} \tag{4.20}$$

if $\tilde{t} \in [0, 2\pi\beta]$. The value of L_0^R near extremality is

$$L_0^R = \mid Q \mid k\alpha\beta \tag{4.21}$$

and using Cardy's formula we obtain

$$\Delta S = 2\pi\sqrt{\frac{cL_0^R}{6}} = 2\pi\sqrt{2Q^3\ell^4\Delta M}\,, \tag{4.22}$$

where

$$\Delta M = \mid Q \mid k\alpha^2 = M - M_0\,. \tag{4.23}$$

It is now easy to see that the statistical expression (4.22) exactly agrees with the deviation of the Bekenstein-Hawking entropy of near-extremal black holes from the extremal case $S_e = \pi Q^2 \ell^2$

$$S = \pi\ell^2(\mid Q \mid + \Delta M + \sqrt{2 \mid Q \mid \Delta M + (\Delta M)^2})^2 = S_e + \Delta S + \mathcal{O}((\Delta M)^{3/2})\,. \tag{4.24}$$

5 AdS$_2$/CFT$_1$ correspondence and near-extremal black holes

In this section we shall generalise the argument leading to the statistical explanation of the near-extremal Bekenstein-Hawking entropy of BTZ and Reissner-Nordström black holes to a wider family of black holes. We shall consider a generic black hole solution, in an arbitrary dimension n, which can be described by the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \frac{1}{2}\ell^2\phi^2 d\Omega^{n-2} \tag{5.1}$$

where ℓ is the Planck length of the theory. By dimensional reduction and integrating the equations of motion of any abelian gauge field we can arrive at an effective two-dimensional theory. An additional conformal rescaling of the metric and a redefinition of the dilaton field yield into an action of the form [28, 29]

$$\frac{1}{2G}\int d^2x\sqrt{-g}(R\phi+\ell^{-2}V(\phi))\,, \tag{5.2}$$

where $V(\phi)$ is a potential function parametrising the original theory and G is a dimensionless constant playing the role of Newton constant in two-dimensions. The solutions for the 2D effective metric are

$$\begin{aligned} ds^2 &= -(J(\phi)-2M\ell)dt^2+(J(\phi)-2M\ell)^{-1}dr^2\,, && (5.3)\\ \phi &= \frac{r}{\ell}\,, && (5.4)\end{aligned}$$

where $V'(\phi)=J(\phi)$. The horizons are the solutions of the equation $J(\phi)=2M\ell$ and we have a degeneration at the zeros of the potential

$$V(\phi_0)=J'(\phi_0)=0\,. \tag{5.5}$$

If we perturb around the degenerate radius of coincident horizons

$$\begin{aligned} M &= M_0(1+k\alpha^2)\,, && (5.6)\\ t &= \frac{\tilde{t}}{\alpha}\,, && (5.7)\\ r &= r_0+\alpha\tilde{x}\,, && (5.8)\end{aligned}$$

the two-dimensional metric transforms into

$$ds^2=-(-\frac{R_0}{2}\tilde{x}^2-2kM_0\ell+\mathcal{O}(\alpha))d\tilde{t}^2+(-\frac{R_0}{2}\tilde{x}^2-2kM_0\ell+\mathcal{O}(\alpha))^{-1}d\tilde{x}^2\,, \tag{5.9}$$

where

$$R_0=\frac{J''(\phi_0)}{\ell^2}\,. \tag{5.10}$$

Imposing boundary conditions of the form

$$\begin{aligned} g_{\tilde{t}\tilde{t}} &= \frac{R_0}{2}\tilde{x}^2+\gamma_{\tilde{t}\tilde{t}}+\ldots\,, && (5.11)\\ g_{\tilde{t}\tilde{x}} &= \frac{\gamma_{\tilde{t}\tilde{x}}}{\tilde{x}^3}\,, && (5.12)\\ g_{\tilde{x}\tilde{x}} &= -\frac{2}{R_0}\frac{1}{\tilde{x}^2}+\frac{\gamma_{\tilde{x}\tilde{x}}}{\tilde{x}^4}+\ldots\,, && (5.13)\\ & && (5.14)\end{aligned}$$

and working in the gauge $\gamma_{\tilde{t}\tilde{x}} = 0$, the Noether charges can be worked out without difficulty because of the simple form of the two-dimensional effective action (5.2).

$$K(\epsilon) = \epsilon(\tilde{t})\Theta_{\tilde{t}\tilde{t}} \,, \tag{5.15}$$

where $\Theta_{\tilde{t}\tilde{t}}$ is the stress tensor

$$\Theta_{\tilde{t}\tilde{t}} = \frac{\alpha}{\ell G}\left(\gamma_{\tilde{t}\tilde{t}} - \frac{1}{2}(\frac{R_0}{2})^2\gamma_{\tilde{x}\tilde{x}}\right) \,. \tag{5.16}$$

Assuming a periodicity of $2\pi\beta$ in $\tilde{t}$, we obtain

$$c = \frac{24\alpha}{\ell G R_0 \beta} \,, \tag{5.17}$$

$$L_0^R = \frac{M_0 k \alpha \beta}{G} \,. \tag{5.18}$$

Applying now Cardy's formula we get

$$\Delta S = 2\pi\sqrt{\frac{4M_0 \ell k \alpha^2}{R_0 \ell^2 G^2}} \tag{5.19}$$

and, taking into account (5.10)and that

$$M_0 \ell k \alpha^2 = (M - M_0)\ell = \frac{1}{2}(J(\phi_h) - J(\phi_0)) \,, \tag{5.20}$$

where ϕ_h is the value of the dilaton at the outer horizon, we can rewrite (5.19) as

$$\Delta S = \frac{2\pi}{G}\sqrt{\frac{2M_0(J(\phi_h) - J(\phi_0))}{J''(\phi_0)}} \,. \tag{5.21}$$

On the other hand, the Bekenstein-Hawking entropy for the two-dimensional effective theory is given by the simple expression [29]

$$S = \frac{2\pi}{G}\phi_h \,, \tag{5.22}$$

and therefore,

$$\Delta S = \frac{2\pi}{G}(\phi_h - \phi_0) \,. \tag{5.23}$$

Expanding $J(\phi)$ around the extremal situation ($J'(\phi_0) = 0$)

$$J(\phi_h) = J(\phi_0) + \frac{1}{2} J''(\phi_0)(\phi_h - \phi_0)^2 + ... \tag{5.24}$$

and, in the near-extremal approximation, we have

$$\frac{2(J(\phi_h) - J(\phi_0))}{J''(\phi_0)} = (\phi_h - \phi_0)^2 \,, \tag{5.25}$$

implying the equality between the statistical expression (5.21) and the thermodynamical one (5.23).

6 Conclusions and final remarks

We have shown that the asymptotic symmetries of BTZ and Reissner-Nordström extremal black holes, whose near-horizon geometry is $AdS_2 \times S^n$ (n=1,2 respectively) are powerful enough to control the deviation of the Bekenstein-Hawking entropy of nearly extremal black holes from the extremal situation. We have also argued that the above results can be generalised for arbitrary black holes near extremality if they can be described by an effective two-dimensional dilaton gravity theory.

Our approach is based on a realization of the boundary conformal field theory in terms of the sub-leading terms in the asymptotic expansion of the metric field. The evaluation of the Noether charges associated with the asymptotic symmetries near extremality allows to compute the central charge and the value of L_0^R. These values depend on an arbitrary parameter β in such a way that cL_0^R, and hence the statistical entropy, has an absolute meaning [§]. However, in the present context the physical excitations are associated to the "would-be gauge" diffeomorphisms characterised by the functions $\epsilon(\tilde{t})$ and these degrees of freedom have an effective central charge $c_{eff} = 1$ (see [27]). Therefore, it could appear natural to choose β in such a way that $c = 1$ and bypass the question of the discrepancy between c and c_{eff}. This type of argument was put forward in 2+1 gravity in [30, 31] and could have some unexpected consequences.

[§] A similar situation appears in [12, 14]

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