

Neutrino Propagation and Spin Zero Sound in Hot Neutron Matter with Skyrme Interactions

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Abstract

We present microscopic calculations of neutrino propagation in hot neutron matter above nuclear density within the framework of the Random Phase Approximation. Calculations are performed for non-degenerate neutrinos using various Skyrme effective interactions. We find that for densities just above nuclear density, spin zero sound is present at zero temperature for all Skyrme forces considered. However it disappears rapidly with increasing temperature due to a strong Landau damping. As a result the mean-free path is given, to a good approximation, by the mean field value. Because of the renormalization of the bare mass in the mean field, the medium is more transparent as compared to the free case. We find, in contrast, that at several times nuclear density, a new type of behavior sets in due to the vicinity of a magnetic instability. It produces a strong reduction of the mean free path. The corresponding transition density however occurs in a region where inputs from more realistic calculations are necessary for the construction of a reliable Skyrme type parametrization.

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I. INTRODUCTION

The gravitational collapse of massive stars at the end of their thermonuclear burning is believed to produce a core of hot dense matter in which most of the initial gravitational binding energy is stored into neutrinos. Numerical simulations of the subsequent evolution, leading to the formation of a neutron star, require the knowledge of the equation of state of hot dense matter as well as a reliable description of neutrino transport phenomena. Relevant quantities are the specific heat of the medium and the neutrino mean free path.

The mean free path of a neutrino due to scattering inside neutron matter at temperature T is proportional to the optical potential. It can be expressed in the case of non-degenerate neutrinos as [1]

$$\frac{1}{\lambda(\mathbf{k}_i, T)} = \frac{G_F^2}{32\pi^3(\hbar c)^4} \int d\mathbf{k}_f \left[(1 + \cos\theta) S^{(0)}(\omega, \mathbf{q}, T) + g_A^2 (3 - \cos\theta) S^{(1)}(\omega, \mathbf{q}, T) \right] \quad (1)$$

where G_F is the Fermi constant, g_A the axial coupling constant, \mathbf{k}_i and \mathbf{k}_f are the initial and final neutrino momenta, \mathbf{q} is the transferred momentum $\mathbf{k}_i - \mathbf{k}_f$, ω is the transferred energy $|\mathbf{k}_i| - |\mathbf{k}_f|$, and $\cos\theta = \hat{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_f$. The functions $S^{(S)}(\omega, \mathbf{q}, T)$ represent the dynamical structure factors in the spin symmetric ($S=0$) or spin antisymmetric ($S=1$) channels. They are given by the imaginary part of the respective response function $\chi^{(S)}(\omega, \mathbf{q})$ and contain the relevant information on the medium. The vector (axial) part of the neutral current gives rise to density (spin-density) fluctuations, corresponding to the $S=0$ ($S=1$) channel. Although we shall consider only nondegenerate neutrinos, we note that the previous formula should be corrected for additional Pauli blocking factors in the degenerate case.

Several authors have evaluated the neutrino mean free path using various approximation schemes and various models of the trapping environment. In their early calculations Tubbs and Schramm [2] omitted the effect of nucleon-nucleon interactions, whose importance was later on emphasized by Sawyer [3] showing that for standard equations of state, the neutrino opacity due to coherent scattering off density fluctuations can be significantly larger than in a neutron Fermi gas. Iwamoto and Pethick [1] investigated the effect of nucleon-nucleon interaction within the framework of Landau theory of Fermi liquids, considering both density and spin-density fluctuations according to Eq. (1). The effective interaction was taken into account by the monopolar Landau parameters F_0 and G_0 derived earlier by Bäckman et al. [4]. The main conclusion of their work is that for degenerate neutrinos, there is a reduction of in-medium scattering cross sections by a factor of 2-3 due to nucleon-nucleon interactions, and a subsequent increase of the neutrino mean free path. Calculations have also been performed using the ring approximation [6,7], in which the response function $\chi^{(S)}(\omega, \mathbf{q})$ of the interacting system is expressed in terms of the bare response function $\chi_0(\omega, \mathbf{q})$ as

$$\chi^{(S)}(\omega, \mathbf{q}) = \frac{\chi_0(\omega, \mathbf{q})}{1 - V^{(S)}(\mathbf{q})\chi_0(\omega, \mathbf{q})}, \quad (2)$$

where $V^{(S)}(\mathbf{q})$ is the interaction in momentum space in spin channel S .

The case of non-degenerate neutrinos was examined by Haensel and Jerzak [5], who computed both the scattering and the absorption rates within the framework of Landau theory. They used a quasiparticle interaction derived from the Reid soft core nucleon-nucleon

potential. Their results indicate that the higher the density of the medium, the larger the effect of the interaction. More recently, Reddy *et al* [6] considered degenerate neutrinos immersed in an environment of nucleons interacting via a Skyrme two-body interaction out of which the monopole Landau parameters F_0 and G_0 are constructed. These authors consider the Skyrme parametrizations SGII and SkM* and simulate the momentum dependence of the force through the direct matrix element of the meson exchange potential.

In this paper we extend preliminary results presented in Ref. [8] and report systematic, fully self-consistent calculations based on effective interactions of the Skyrme type, including some of the most recent ones. Such interactions have been very successful at describing properties of finite nuclei and nuclear as well as neutron matter. An attractive feature of Skyrme interactions is that response functions can be calculated in closed form [9–11] in the Random Phase Approximation (RPA), which generalizes the simple ring approximation (2) based on Hartree-like dynamics, to the more realistic case of Hartree-Fock mean field dynamics. An advantage of such a framework is that it also avoids splitting structure factors into single particle and collective contributions based on different approximation schemes. It further avoids postulating a factorization of the temperature dependence of the structure factors, which are in conflict with the fact that zero sound disappears at high temperature [12]. Moreover, sum rules hold in the case of fully self-consistent calculations, thus allowing the possibility of consistency checks.

In the following we focus on the evaluation of the mean free path of neutrinos interacting with neutrons through neutral currents only. This coupling is important mainly after deleptonization has taken place and it affects the late behaviour of the neutrino burst, where neutrino diffusion is driven by the temperature gradient. Indeed, in the earlier stages, just after the formation of the protoneutron star, charged current reactions leading to neutrino absorption are dominant, due to the fact that the high temperatures involved prevent efficient inhibition of charged reactions by the energy-momentum conservation constraints. We consider neutron matter above nuclear density $\rho_0=0.17$ nucleons/fm³. The case of high densities is of special interest because medium effects are expected to play an important role. In contrast, below nuclear density propagation is dominated by coherent scattering off nuclei, a case for which reliable estimates of the opacities are available.

This paper is organized as follows. In Sec. II we briefly summarize the formalism for the computation of the response of the neutron liquid and analyze the main results in Sec. III. Our conclusions and perspectives are presented in Sec. IV.

II. RESPONSE FUNCTION OF HOT NEUTRON MATTER

In the present work we use the techniques developed in Refs. [10,11] to obtain RPA susceptibilities at finite temperature, in the particular case of Skyrme-type effective interactions. The RPA dynamical susceptibility $\chi^{(S)}(q, \omega, T)$ in spin channel S of a Fermi system in thermal equilibrium at temperature T is given by the the polarizability to an infinitesimal external field

$$\hat{V}_{\text{ext}} = \varepsilon \hat{\mathbf{O}} \exp(i\omega t - i\mathbf{q}\cdot\mathbf{x}). \quad (3)$$

In this equation ω contains a small imaginary part to ensure an adiabatic switching of the external field starting at time $t = -\infty$ when the medium is in its equilibrium state at

temperature T . The operator $\hat{\mathbf{O}}$ is equal to the unit matrix in spin space for spin zero ($S=0$) and to the third Pauli matrix σ_z for $S=1$. At late enough times (and small enough ε) the expectation value of the operator $\hat{\mathbf{O}}$ has the same space and time dependence as the external field. The dynamical susceptibility is defined by

$$\chi^{(S)} = \lim_{\varepsilon \rightarrow 0} \frac{\langle \hat{\mathbf{O}} \rangle}{\varepsilon \exp(i\omega t - i\mathbf{q} \cdot \mathbf{x})}, \quad (4)$$

and it can be shown to be given by the algebraic expression

$$\chi^{(S)} = \chi_0 + G_0 \tilde{V}_{ph}^{(S)} G_0, \quad (5)$$

where χ_0 is the response of the free system, G_0 is the free particle-hole (ph) propagator at temperature T and $\tilde{V}_{ph}^{(S)}$ is the RPA induced interaction, related to the effective ph one $V_{ph}^{(S)}$ by the integral equation

$$\tilde{V}_{ph}^{(S)} = V_{ph}^{(S)} + V_{ph}^{(S)} G_0 \tilde{V}_{ph}^{(S)}. \quad (6)$$

In Ref. [9], it has been shown that for the general class of ph interactions of the following form containing a monopole and a dipole term

$$V_{ph}^{(S)}(q_1, q_2; q) = W_1^{(S)}(q) + W_2^{(S)}(q)(\mathbf{q}_1 - \mathbf{q}_2)^2, \quad (7)$$

to which the Skyrme ones belong, the solution of the integral equation can be algebraically obtained in a closed form, in terms of generalized Lindhard functions. This result was derived for the zero temperature case. However the generalization to a thermally excited liquid is straightforward. The effect of temperature appears implicitly in the RPA equations through the Fermi-Dirac occupation numbers contained in the free ph propagator. The algebraic structure of both the RPA equation and its solution is preserved [10,11] and the dynamical susceptibility in channel (S) has the form [9]

$$\chi^{(S)}(q, \omega, T) = 2 \frac{\chi_0(q, \omega, T)}{D^{(S)}(q, \omega, T)}, \quad (8)$$

where the factor 2 takes into account the spin degeneracy and

$$\begin{aligned} D^{(S)}(q, \omega, T) = & 1 - W_1^{(S)}(q) \chi_0 - 2W_2^{(S)} \left[\frac{q^2}{4} - \frac{\omega m^{*2}}{q} \frac{1}{1 - \frac{m^* k_F^3}{3\pi^2} W_2^{(S)}} \right] \chi_0 \\ & + 2W_2^{(S)} \left(\frac{q^2}{2} \chi_0 - k_F^2 \chi_2 \right) \\ & + [W_2^{(S)} k_F^2]^2 \left[\chi_2^2 - \chi_0 \chi_4 + \left(\frac{\omega m^*}{k_F^2} \right)^2 \chi_0^2 - \frac{m^*}{6\pi^2 k_F} q^2 \chi_0 \right]. \end{aligned} \quad (9)$$

In this expression k_F is the neutron Fermi momentum while m^* denotes the nucleon effective mass in the mean field.

Since in pure neutron matter the isospin exchange operator P_τ reduces identically to the unit matrix, only particular combinations of the Skyrme interaction parameters t_i and x_i , namely

$$\begin{aligned} s_0 &= t_0(1 - x_0), & s_1 &= t_1(1 - x_1) \\ s_2 &= t_2(1 + x_2), & s_3 &= t_3(1 - x_3), \end{aligned} \quad (10)$$

are required. Thus, one obtains the following expressions for the neutron effective mass

$$\frac{\hbar^2}{2m^*} = \frac{\hbar^2}{2m} + \frac{1}{8}(s_1 + 3s_2)\rho, \quad (11)$$

where ρ is the density of neutron matter, and for the coefficients $W_{1,2}^{(S)}$ of the effective ph interaction

$$\begin{aligned} W_1^{(0)} &= s_0 + \frac{(\gamma + 1)(\gamma + 2)}{12}s_3\rho^\gamma + \frac{1}{4}[s_1 - 3s_2]q^2 \\ W_2^{(0)} &= \frac{1}{4}[s_1 + 3s_2] \\ W_1^{(1)} &= -s_0 - \frac{1}{6}s_3\rho^\gamma - \frac{1}{4}[s_1 + s_2]q^2 \\ W_2^{(1)} &= \frac{1}{4}[-s_1 + s_2], \end{aligned} \quad (12)$$

with γ being the exponent of the density dependent term in the Skyrme force

$$\frac{t_3}{6}\rho^\gamma(\mathbf{r}_{12})(1 + x_3 P_{12}^\sigma)\delta(\mathbf{r}_{12}). \quad (13)$$

In the special case of the Skyrme interaction SIII we have considered it as a three-body force rather than a density dependent one, i.e. we have used a value $s_3 = 0$ in the previous equations.

The functions $\chi_{2i}(\omega, q)$ ($i=0, 1, 2$) are generalized susceptibilities per unit volume Ω , defined as [9]

$$\begin{aligned} \chi_{2i} &= \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{1}{2} \left[\left(\frac{\mathbf{k}^2}{k_F^2} \right)^i + \left(\frac{(\mathbf{k} + \mathbf{q})^2}{k_F^2} \right)^i \right] (1 - n(\mathbf{k} + \mathbf{q})) \\ & n(\mathbf{k}) \left(\frac{1}{\omega - \omega(\mathbf{k}, \mathbf{q}) + i\eta} - \frac{1}{\omega + \omega(\mathbf{k}, \mathbf{q}) + i\eta} \right), \end{aligned} \quad (14)$$

with

$$n(\mathbf{k}) = \frac{1}{1 + \exp(\varepsilon(\mathbf{k}) - \mu)/T} \quad (15)$$

the Fermi-Dirac occupation probability, μ being the chemical potential at the given temperature. Here

$$\omega(\mathbf{k}, \mathbf{q}) = \varepsilon(\mathbf{k} + \mathbf{q}) - \varepsilon(\mathbf{k}), \quad (16)$$

with

$$\varepsilon(\mathbf{k}) = \hbar^2 k^2 / 2m^*, \quad (17)$$

the single-particle energy. The dynamical structure factor is obtained, at both positive and negative energies ω , from the detailed balance relationship

$$S^{(S)}(q, \omega, T) = -\frac{1}{\pi} \frac{\Im m \chi^{(S)}(q, \omega, T)}{1 - e^{-\omega/T}}. \quad (18)$$

It is worthwhile stressing that opposite to the ring approximation defined by equation (2), which involves only the Linhard function χ_0 , new terms involving the generalized susceptibility χ_2 and χ_4 arise in the full RPA approximation.

Another important point about our self-consistent approach is that it preserves sum rules. In particular one has the following energy weighted sum rule in the spin one case

$$\int_{-\infty}^{+\infty} S^{(S=1)}(q, \omega, T) \omega d\omega = \left\{ \frac{\hbar^2}{2m^*} + \frac{1}{8} (s_1 - s_2) \rho \right\} \rho q^2. \quad (19)$$

In the spin zero case the right hand side is simply $\hbar^2 q^2 \rho / 2m$.

In the following discussions regarding the appearance of zero sound modes it will turn out to be convenient to have the explicit expressions of the real and imaginary parts of the bare response function. Its imaginary part is given by

$$\Im m \chi_0(\omega, q) = -\frac{m^2}{2\pi q} \frac{kT}{1 - e^{-\beta\omega}} \log \frac{1 + e^{\beta(A+\omega/2)}}{1 + e^{\beta(A-\omega/2)}}. \quad (20)$$

In this equation $\beta = 1/kT$ and the quantity A is given by

$$A = \mu - \frac{m\omega^2}{2q^2} - \frac{q^2}{8m^2}. \quad (21)$$

The expression of the real part is a little more difficult to construct. In the zero temperature case it reads [17]

$$\Re e \chi_0(\omega, q, k_F, T = 0) = \frac{1}{4\pi^2} \frac{mk_F^2}{q\hbar^2} \left\{ -\frac{2q}{k_F} + \varphi(x_+) - \varphi(x_-) \right\}. \quad (22)$$

In this equation the quantities x_{\pm} are defined by

$$x_{\pm} = \frac{m\omega}{\hbar k_F q} \pm \frac{q}{2k_F}, \quad (23)$$

and the function φ reads

$$\varphi(x) = (1 - x^2) \log \left| \frac{x - 1}{x + 1} \right|. \quad (24)$$

For a non zero temperature the real part of the response function is obtained by performing an average of the zero temperature results obtained for various values k of the Fermi momentum with a weight factor equal to the occupation number $n(k)$:

$$\Re e \chi_0(\omega, q, k_F, T) = - \int_0^{\infty} \Re e \chi_0(\omega, q, k_F = k, T = 0) dn(k). \quad (25)$$

III. RESULTS

We have considered two standard Skyrme parametrizations, namely SIII [13] and SkM* [14], as well as parametrizations SLy230a and SLy230b recently adjusted [15] to reproduce the latest equation of state for pure neutron matter calculated by Wiringa *et al* [16] within the Fermi Hypernetted Chain scheme, using the Urbana V14 two-body interaction plus the Urbana three-body interaction. The energies per particle given by parametrizations SLy230a,b and by the realistic force are very close to each other up to densities of about 1 nucleon/fm³. In this section we analyze in detail the evolution of the mean free path with neutron density, its dependence upon temperature and neutrino momentum and the effect of residual interactions, as follows.

A. Evolution of the mean-free path with density

In Fig. 1 we show the evolution of the mean free path with density for an incoming neutrino momentum $k_i = 5$ MeV/c and a temperature $T = 5$ MeV. Results are displayed for the four effective interactions mentioned above. It can be noticed that for interactions SLy230a and SLy230b the mean free path exhibits a slow variation with density. This behavior is similar to the one obtained by Haensel and Jerzak [5] who find mean free paths $\lambda = 2, 1.5$ and 1km for densities 0.2, 0.36 and 0.6 fm⁻³ respectively, for $k_i c = T = 5$ MeV. In contrast the results obtained with interactions SIII and SkM* show a substantially different behavior, both exhibiting a rapid monotonic decrease with density. This decay is the consequence of a pronounced increase in the magnetic strength, that reflects in the vicinity of an instability in the channel S=1. The magnetic instability can be located by examining the Landau parameters of neutron matter, in particular, the Landau coefficient G_0 related to the spin asymmetry coefficient a_σ in neutron matter through the relation

$$a_\sigma = \frac{\hbar^2 k_F^2}{6m^*} (1 + G_0). \quad (26)$$

In terms of the force parameters, the asymmetry coefficient is given by the following expression

$$a_\sigma = \frac{\hbar^2 k_F^2}{6m^*} - (s_0 + \frac{s_3}{6} \rho^\gamma) \frac{\rho}{4} + \frac{5}{24} (s_2 - s_1) \tau. \quad (27)$$

where $\tau = 3/5 \rho k_F^2$ is the kinetic energy density. For interaction SIII the last term dominates at high values of the neutron density and as a result the Landau parameter takes the value -1 at a critical density around 0.317 fm⁻³. This critical density ρ_c is (in fm⁻³) 0.1967 for SkM*, 0.5411 for SLy230a and 0.5928 for SLy230b. These are precisely the abscissae at which the neutrino mean free path drops to a very small value in Fig. 1. Actually, the mean-free path at the minimum is not exactly zero, since even though the monopole contribution vanishes there remains a small contribution from the dipole term.

B. Spin zero-sound with Skyrme forces

To illustrate this point, in Fig. 2 we display the dynamical structure factors computed with interaction SIII, for spin channels $S=0$ and 1 (left and right parts of the figure, respectively), as functions of energy ω , for transferred momenta q equal to 5 and 100 MeV/c (dashed and solid lines, respectively) and a temperature of 5 MeV. The upper and lower parts in this figure correspond to densities 0.1 fm^{-3} and ρ_c . The zero sound mode is visible in the $S=1$, $\rho=0.1 \text{ fm}^{-3}$ case although it is smeared by Landau damping at a temperature $T=5$ MeV. We also clearly appreciate the significant increase and change of scale in $S^{(1)}(\omega, q)$ as the density reaches the transition point, which contrasts with the rather moderate variation undergone by $S^{(0)}$ with density. The numerical data indicate a scale factor of three orders of magnitude in $S^{(1)}$ as the density reaches ρ_c . For $S=0$ one also observes a peak which corresponds to the maximum in the imaginary part of the bare response. This peak is reduced and shifted towards lower energies in the RPA response because of the contribution of the real part.

In order to have a qualitative criterion for the existence of a zero sound mode it is convenient to calculate the response in the monopole approximation. In this approximation one evaluates the particle-hole interaction strength at the Fermi surface and one ignores its angular dependence i.e. one sets $q_1^2 = q_2^2 = k_F^2$ and $\mathbf{q}_1 \cdot \mathbf{q}_2 = 0$ in Eq. (7). One thus uses Eq. (2) replacing $V^{(S)}(q)$ by $W_1 + 2W_2 k_F^2$ i.e.

$$V_0^{(S=1)} = -s_0 - \frac{1}{6}s_3\rho^\gamma + \frac{1}{2}(s_2 - s_1)k_F^2 - \frac{1}{4}(s_1 + s_2)q^2, \quad (28)$$

in the spin one case, and by

$$V_0^{(S=0)} = s_0 + \frac{(\gamma + 1)(\gamma + 2)}{12}s_3\rho^\gamma + \frac{1}{2}(s_1 + 3s_2)k_F^2 + \frac{1}{2}(s_1 - 3s_2)q^2, \quad (29)$$

in the spin zero case. From the expression for the structure function

$$S(\omega, \mathbf{q}, T) = -\frac{1}{\pi} \frac{2\Im m \chi_0(\omega, \mathbf{q})}{(1 - V_0^{(S=1)})\Re e \chi_0)^2 + (V_0^{(S=1)})\Im m \chi_0)^2} \frac{1}{1 - e^{-\omega/T}}, \quad (30)$$

we see that, for a given value of the momentum \mathbf{q} , zero sound will be present whenever there exists a solution ω_R of the equation

$$1 - V_0^{(S)}\Re e \chi_0(\omega_R, \mathbf{q}) = 0, \quad (31)$$

in the energy domain where the imaginary part of the bare response function is small (or zero). At zero temperature this domain corresponds to (see equation(20))

$$\omega_R \geq \omega_+ = \frac{\hbar^2 q}{2m}(2k_F + q). \quad (32)$$

At this point the real part of the response function is given by equation(22) i.e.

$$\Re e \chi_0(\omega_+) = \frac{m^*}{4\pi^2 \hbar^2} \left\{ (2k_F + q) \log\left(\frac{2k_F + q}{q}\right) - 2k_F \right\}. \quad (33)$$

Since the real part of the response function decreases with energy in the domain of interest we find that the condition for zero sound at zero temperature is

$$V_0^{(S)} \Re \chi_0(\omega_+, \mathbf{q}) \geq 1. \quad (34)$$

For interaction SIII one finds that for a momentum q of the order of 5 MeV/c and a density of 0.1 neutron per fm³ one has

$$V_0^{(S=1)} = 112.9 \text{MeV} \times \text{fm}^3 \quad \text{and} \quad V_0^{(S=0)} = -560.8 \text{MeV} \times \text{fm}^3 \quad (35)$$

and

$$\Re \chi_0(\omega_+, \mathbf{q}) = 6.2 \times 10^{-3} \text{MeV}^{-1} \times \text{fm}^{-3}. \quad (36)$$

These values show that the residual particle-hole interaction in the S=0 channel has the wrong sign to produce a zero sound mode. In contrast in the S=1 case it is repulsive and strong enough to give a zero sound mode. It is however not as strong as in the case of liquid helium-3 [17] where zero sound occurs far in the quadratically decreasing tail of the real part of the response function. Here in contrast, the energy of the zero sound mode is close to the maximum of the real part of the response function, $\omega_{max} = qk_F/m$. Note that this value belongs to the integration domain of equation (1), namely

$$|\omega| \leq q \leq 2E_\nu - \omega \quad \omega \leq E_\nu, \quad (37)$$

thus allowing, in principle, the zero-sound mode to contribute to the neutrino mean-free path. It is worthwhile noting that for a strong particle-hole interaction the magnetic strength would be driven outside the integration domain which would significantly increase the mean-free path.

C. Energy density near the magnetic transition

At the critical density the symmetric Hartree-Fock ground state becomes unstable. This can be visualized looking at the expression of the energy density of spin polarized neutron matter:

$$\mathcal{H} = \frac{\hbar^2}{2m^*} (\tau_\uparrow + \tau_\downarrow) + (s_0 + \frac{1}{6} s_3 \rho^\gamma) \rho_\uparrow \rho_\downarrow + \frac{1}{8} (-s_1 + s_2) (\rho_\uparrow - \rho_\downarrow) (\tau_\uparrow - \tau_\downarrow). \quad (38)$$

from which one can construct the energy per neutron as a function of the spin polarization parameter

$$x = \frac{\rho_\uparrow - \rho_\downarrow}{\rho}. \quad (39)$$

The minimum energy per particle occurs at $x=0$ for densities smaller than the critical one and at nonvanishing asymmetries for larger densities. It actually reaches rapidly the value $x=1$ which corresponds to a fully polarized medium.

In the vicinity of the magnetic transition, the response function in the S=1 channel behaves, according to equation (4), as the inverse of the spin asymmetry coefficient a_σ in

the limit $\omega=0$ and $q=0$. This can be checked by noting that in the limit of a small momentum transfer one has

$$\Re\chi_0(\omega, q = 0, k_F, T = 0) = -\frac{m^* k_F}{\pi^2 \hbar^2}, \quad (40)$$

and a negligible value of the imaginary part of the bare response function. The RPA response thus becomes [10]

$$\chi^{(S=1)}(\omega = 0, \mathbf{q} = 0) = -\frac{\rho}{2a_\sigma}. \quad (41)$$

This formula accounts for the presence of the peak which develops near zero energy in figure 2 when the density becomes close to its critical value. It also shows that in the limit of a vanishing momentum transfer the mean-free path would exactly vanish at the transition point. This result is not in conflict with the sum rule mentioned previously (see eq. (19)) because a peak at zero energy gives no contribution to the energy weighted sum. We have not attempted to push our RPA calculations beyond the critical point, since more work is needed to produce a reliable Skyrme type parametrization in this region.

D. Momentum and temperature dependence

In Fig. 3, the mean free paths obtained with the interaction SLy230b are displayed as functions of the density for six combinations of k_i and T . It is worthwhile noticing that the effect of increasing momentum and/or temperature is similar and mostly consists of an overall reduction of the scale. Scaling properties were already discussed by Haensel and Jerzak [5]. As a result of their approximation scheme these authors found that scaling the neutrino energy and the temperature by a factor α produces a scaling of the mean free path by a factor $1/\alpha^3$. Comparing the curves $k_i c = T = 5$ MeV and $k_i c = T = 10$ MeV we see that this scaling law holds to a reasonable accuracy in our RPA calculations, although the curvature of the uppermost line is somewhat too high.

E. Effect of residual interactions

An important issue regarding neutrino propagation is the effect of the residual neutron-neutron interaction. Various answers to this question are found in the literature. Indeed while Iwamoto and Pethick [1] conclude that Fermi liquid effects increase the mean free path to some extent, Haensel and Jerzak [5] find that near normal density, the overall effect of the nucleon-nucleon interaction on the mean free path is rather weak. In contrast, the recent paper by Reddy *et. al.* [6] reports that many-body effects suggest considerable reductions in the opacities compared to the free gas estimates.

Our results are displayed for interactions SIII and SLy230b in the cases $T = k_i c = 5$ MeV and $T = k_i c = 10$ MeV in Fig. 4. This figure shows the relative opacities $\lambda^{-1}/\lambda_{free}^{-1}(m^*)$ where $\lambda_{free}^{-1}(m^*)$ corresponds to a gas of free neutrons with effective mass m^* , rather than m , i.e. the scale is furnished by the opacity in the mean field. In the vicinity of the saturation density of nuclear matter it can be seen that this relative opacity is close to unity. This means

that for such densities, the major role of the interaction is to dress the bare mass giving rise to the mean field density dependent effective one m^* . If the reference for the relative opacity is however chosen as $\lambda_{free}(m)$, then one finds that interactions do make the medium more transparent. Indeed at normal density one has $m^*/m = 0.695$ for interaction SLy230b and the smaller the mass, the smaller the opacity. In particular, at zero temperature

$$\frac{1}{\lambda_{free}} = \frac{G_F^2 k_F^2}{24\pi^3} \left(\frac{m^* k_i}{m^* + k_F} \right)^3. \quad (42)$$

Since the effective mass becomes small at high density, large reductions can occur. Nevertheless, at large enough densities the dominant effect in Fig. 4 is the reduction of the mean free due to the vicinity of the magnetic instability.

The reason for having RPA values of the mean-free path very close to the mean field ones (at least in the region just above nuclear density), in spite of the presence of a zero sound mode at zero temperature, is that this mode is rapidly smeared as temperature increases because of the strong Landau damping, clearly visible on Fig. 2. Moreover when temperature rises there is a strong reduction of the real part of the response function, which may correspond to a factor 2-3 for a temperature of the order of 5 MeV (see e.g. figure 1 of reference [18]). As a result one finds that the term $V_0^{(S=1)}\chi$ in the expression of the RPA response function is substantially smaller than unity, which implies that RPA and mean field response functions are nearly identical in the density temperature range considered here.

IV. DISCUSSION

In the present work we have calculated neutrino propagation in neutron matter within a microscopic and consistent framework able to describe both the nuclear equation of state and dynamical structure factors. The approach adopted here is based on Skyrme type effective interactions and linear response (RPA) theory. As already mentioned, an advantage of the present procedure is the treatment of single particle and collective contributions to the structure factors on an equal footing at any temperature, making unnecessary to postulate a factorization of the temperature dependence of the structure factors. Such factorizations, which are in conflict with the fact that spin zero sound disappears at temperatures of a few MeV [12], overestimate the contribution of collective modes. Our calculation indicates that magnetic zero sound modes appear at zero temperature for all Skyrme parametrizations here considered, even at the largest momenta (i.e., $q = 100$ MeV/c) up to density ρ_c ; however, at finite temperatures they become subject to Landau damping on the ph continuum, as shown in Fig. 2 and discussed in section III.

In the domain where the RPA is reliable (stable symmetric ground state in the mean field picture) we have found that the most important effects of the interaction are the renormalization of the mass of the neutrons into their effective mass m^* in the response function. This makes the medium more transparent, thus giving rise to shorter deleptonization time scales [6].

We have found that at high density instabilities occur, which signal a breakdown of the Skyrme like parametrization. The possible existence of a magnetic transition in neutron matter, as well as its origin, have been investigated by several authors [19–25]. In general,

one should expect that the short-range repulsion of the nuclear interaction originates spin alignment at sufficiently high densities; this has been verified in various model calculations extending from relativistic approaches [23,24] to HF description with finite range effective interactions [25]. The effect of a short range repulsion in neutron matter is most conveniently illustrated in the case of a Skyrme force without velocity dependent terms. In this case a transition takes place provided the density dependent repulsion grows rapidly enough. Indeed, a zero range interaction does not contribute to the potential energy in fully polarized neutron matter, because the Pauli principle forbids configurations of two neutrons with the same spin at the same point. As a result, when the repulsion in spin symmetric neutron matter exceeds $(2^{1/3} - 1)$ times the kinetic energy, the cost in kinetic energy requested to turn all spins up becomes balanced by the gain in potential energy, thus making the fully polarized state more favorable. The previous analysis also shows how to work out interactions without magnetic transitions. One needs a weak density dependent repulsion, more realistic to reproduce the monopole resonance, in order to reduce the gain in potential energy. To explore the results which would be obtained with such a force we have slightly modified the interaction SLy230b so as to increase the value of the critical density up to a factor 2, without impairing the equation of state. The results are indicated in Fig. 3 as SLy* for $T = 5$ MeV (solid line), $T = 10$ MeV (dashed line), and $k_{ic} = 5$ MeV in both cases. We find that the mean free path remains roughly constant up to densities around 0.9fm^{-3} .

Further studies are obviously needed in the vicinity of the magnetic transition. Indeed, it occurs in a region where inputs from more realistic calculations are necessary for the determination of a reliable Skyrme type parametrization. Furthermore, other types of phase transitions, such as strangeness production or a quark-hadron transition, are likely to occur before one reaches this region.

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FIGURES

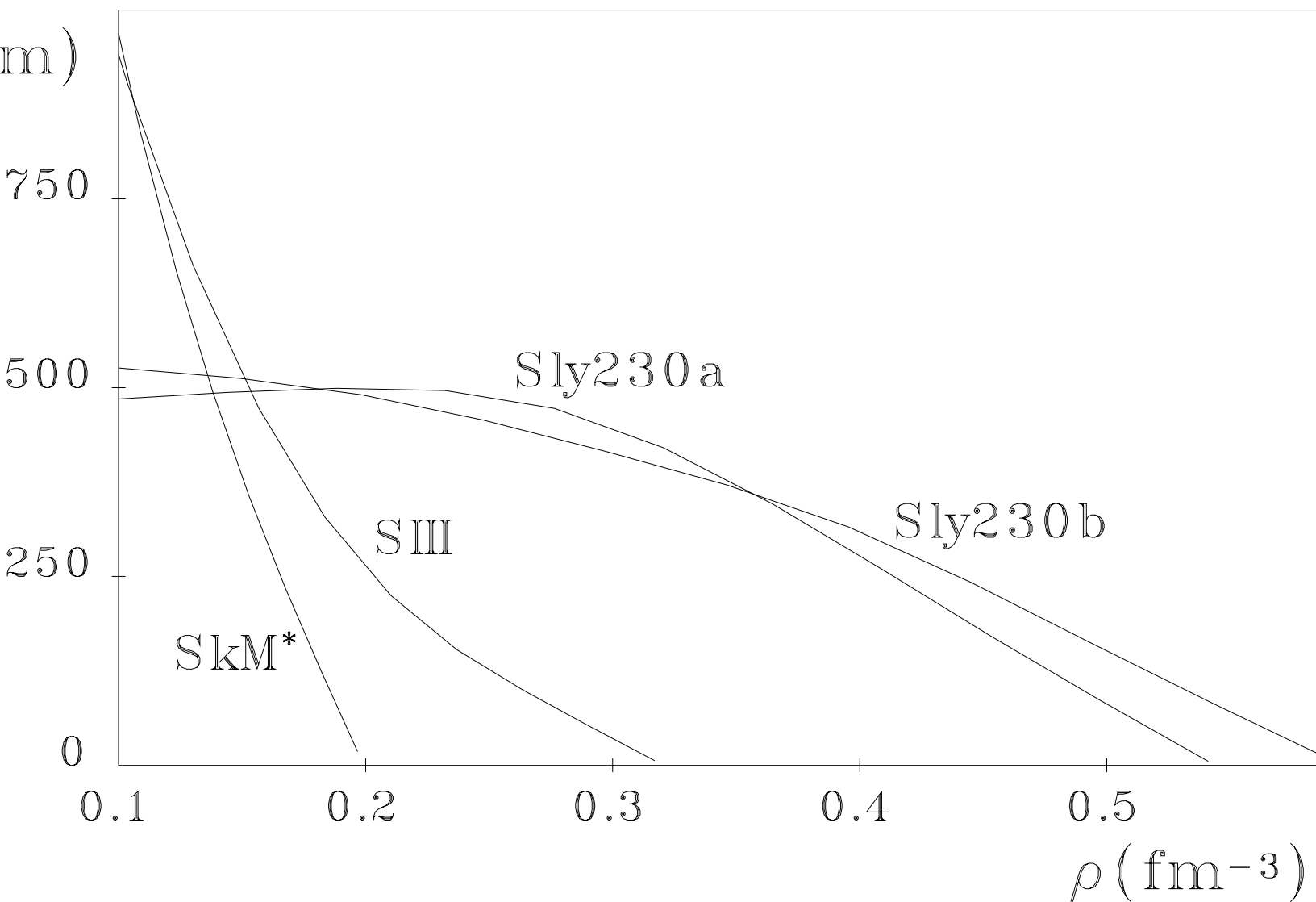


FIG. 1. The scattering mean free path for the neutrino as a function of neutron density, for various Skyrme force parametrizations.

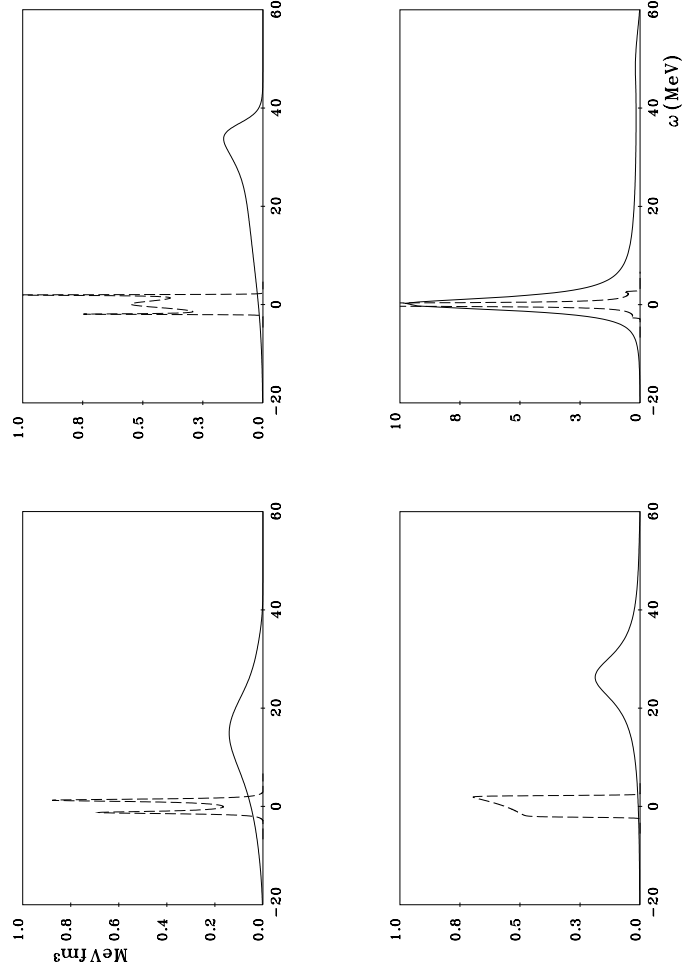


FIG. 2. Dynamical structure factors $S^{(S)}(\omega, q)$ for $S=0$ and 1 channels (left and right parts, respectively), as functions of energy for interaction SIII at a temperature $T = 5$ MeV. Upper and lower parts respectively correspond to densities 0.1 fm^{-3} and ρ_c . Dashed and solid lines respectively correspond to transferred momenta $q=5$ and $100 \text{ MeV}/c$.

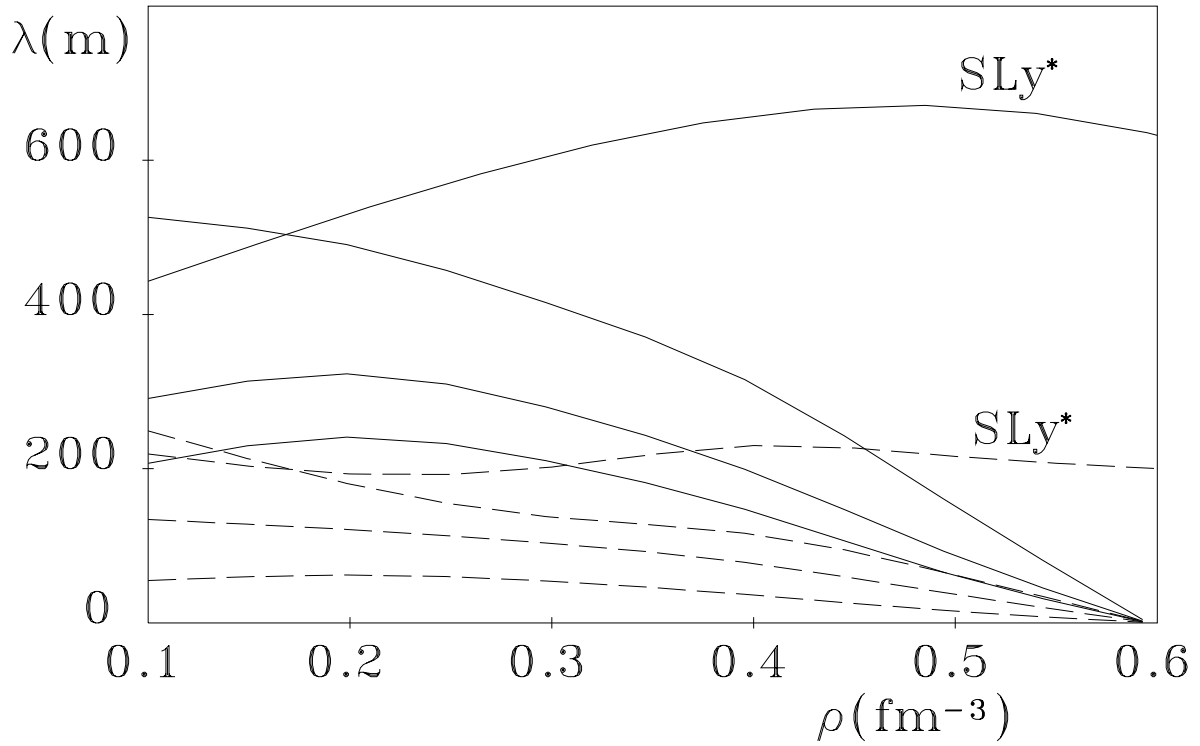


FIG. 3. The scattering mean free path for the neutrino as a function of neutron density for the parametrization SLy230b and its modified version SLy*. Full and dashed lines respectively correspond to temperatures 5 and 10 Mev. In each group, from above to below the lines correspond to incoming momenta $k_i = 5, 10$ and $3 T$ (in MeV/c).

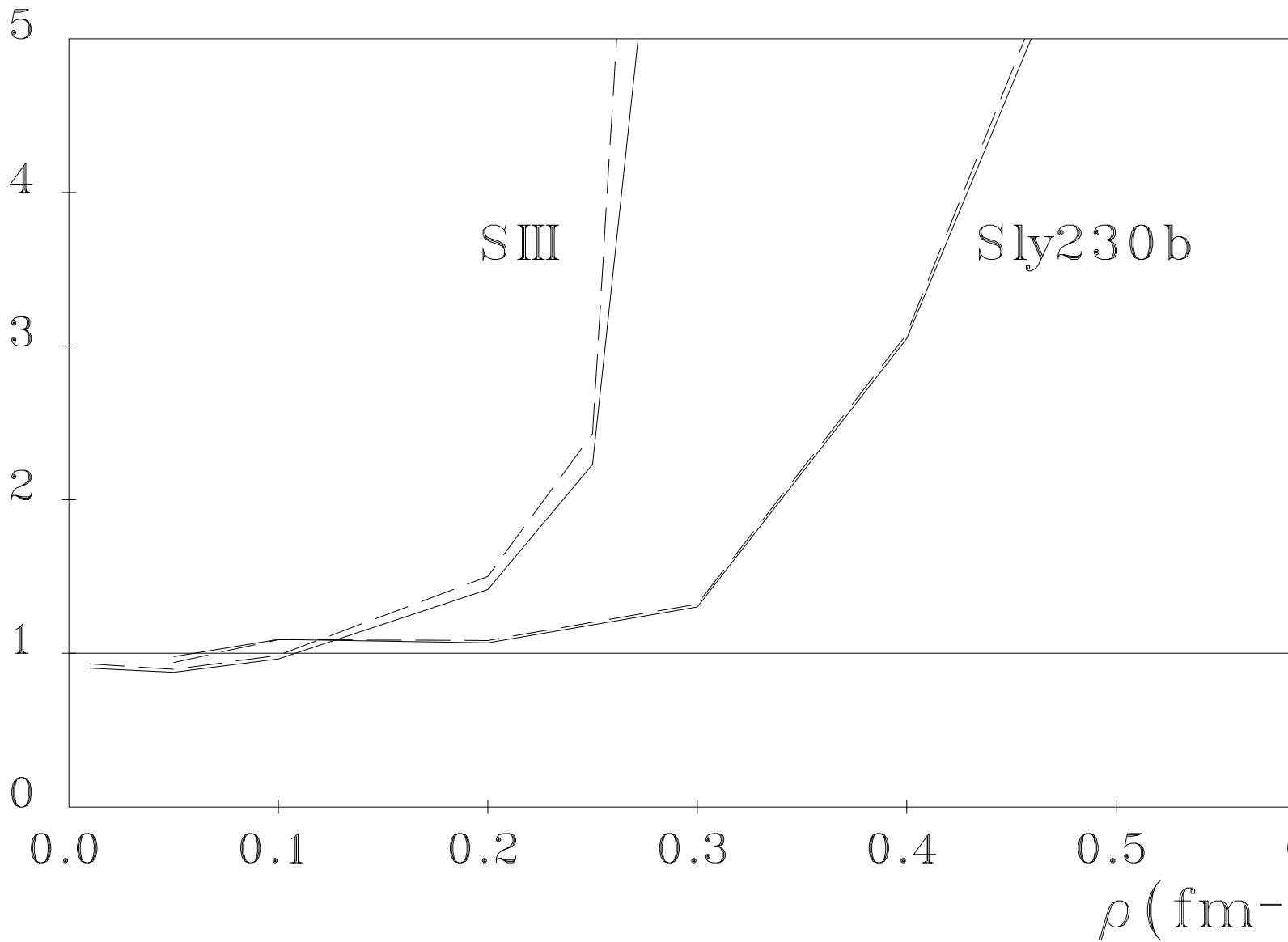


FIG. 4. The relative opacity $\lambda^{-1}/\lambda_{free}^{-1}$ as a function of the density for interactions SIII and SLy230b. Full and dashed lines respectively correspond to $T = k_{iC} = 5$ and 10 MeV. A horizontal line corresponding to relative opacity equal to unity has been drawn for easier comparison.