Departament d'Anàlisi Econòmica

A game theory approach to conflicting preferences, service search and peer review problems

TESI DOCTORAL

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A ma mare.

Agraïments

En primer lloc, vull agraïr el temps, el treball i el recolçament que m'ha brindat Penélope Hernández, la meua directora de tesi, durant tot el període que he dedicat a formar-me com a investigador. El que sé en aquesta matèria li ho dec a ella.

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Introducció

Aquest treball està composat per dos estudis teòrics i un experimental. En tots s'utilitza la teoria de jocs per a modelar la interacció estratègica dels agents econòmics que prenen part en ells i analitzar i caracteritzar les estratègies d'equilibri en els jocs, i en dos d'ells s'utilitzen estructures de xarxa per a especificar les seues inter-relacions. A més, en un s'utilitza la Economia Experimental per obtenir dades empíriques sobre el comportament de les persones a qui se'ls presenta el joc.

Resum

Capítol 1: Conflict and segregation in networks: An experiment on the interplay between individual preferences and social influence

El primer treball de la tesi 'Conflict and segregation in networks: An experiment on the interplay between individual preferences and social influence' utilitza la metodologia que ofereix l'Economia Experimental per testar els resultats exposats en un article teòric previ sobre jocs en xarxa amb heterogeneïtat de tipus. En ell, els jugadors guanyen punts per cada coordinació amb els seus veïns en l'acció que decideixen. Cada tipus de jugador prefereix una acció en el sentit que guanya més per cada coordinació en la seua acció preferida que en l'altra, factor que crea un conflicte de preferències. Una analogia d'aquest joc al món real pot observar-se en el camp dels ordinadors, on cada persona té una preferència individual (per exemple, Mac versus Windows) però, si molt poca gent o ningú del seu entorn utilitza el seu sistema preferit, pot arribar a ignorar-la en pro d'una major coordinació. En aquest article afegim una primera etapa on els subjectes trien amb qui volen connectar. Executem tres tractaments diferent variant el nivell d'heterogeneïtat en la xarxa: cap, baixa i alta. Trobem que, sense importar aquest nivell, els subjectes únicament es connecten amb aquells qui tenen les mateixes preferències que ells. Les conseqüències d'aquest comportament són la ineficiència, tant des del punt de vista individual com el social, i la completa segregació dels jugadors segons el seu tipus.

Capítol 2: Analysis of Strategies to Promote Cooperation in Distributed Service Discovery

El segón estudi de la tesi 'Analysis of Strategies to Promote Cooperation in Distributed Service Discovery' utilitza la teoria de jocs per a caracteritzar els perfils d'estratègies d'equilibri que sostenen la cooperació en processos de cerca de serveis en una xarxa de proveïdors. Açò és una forma novedosa d'enfrontar aquest tipus de problemes, àmpliament estudiats en el camp de la Inteligència Artificial i sistemes multi-agent, sobretot mitjançant simulacions. Dissenyem un joc repetit i calculem analíticament les condicions que ha de satisfer el sistema de recompenses a la cooperació per a que el prefil d'estratègies on tots els agents reenvien les peticions que no poden resoldre per una de les seues connexions elegida aleatòriament siga equilibri del joc. L'anàlisi d'aquesta estratègia es basa en la dinàmica del procés de recorregut de xarxes conegut com a *Random Walk*, de la qual ens servim per a caracteritzar les condicions que fan que cap agent tinga incentius econòmics a decidir unilateralment no cooperar. A més, trobem que la estructura de la xarxa té un paper clau en la tasa d'èxit del procés i la quantia dels pagaments que obtenen, de mitjana, els agents. Per contrastar els resultats exposats i estudiar diferents estructures de xarxa realitzem simulacions amb agents virtuals. Les dades que obtenim experimentalment van en la mateixa direcció que els analítics, i ens permet fer rànquings d'estructures en quant a rendiment. En ells, les xarxes *Scale-free* emergeixen com les número u en tots els experiments.

Capítol 3: Bayesian model of peer review problem

El tercer i últim capítol d'esta tesi 'Bayesian model of peer review problem' novament fa servir un enfocament de teoria de jocs per modelar el procés de revisió d'un article científic. El sistema d'avaluació conegut com a *peer review* consisteix en sotmetre un treball a judici per part d'un o més experts en l'àrea de coneixement del mateix. En l'àmbit acadèmic és un sistema àmpliament utilitzat per les revistes científiques per decidir quins articles publicar, i qui millor per prendre aquest tipus de decisions que un comitè d'experts en la matèria. Però açò pot comportar conflictes d'interessos i problemes de risc moral, ja que els autors habitualment són competència directa dels revisors en diversos assumptes (beques, finançament de projectes, places laborals, etc.) i pot fer que no valoren el treball de manera justa. Moltes branques d'investigació dins de l'economia han estudiat aquest tipus de problemes, però hi ha escassetat de treballs que s'hagen centrat en el *peer review problem*. Nosaltres afrontem el problema des d'un punt de vista econòmic, assumint agents racionals maximitzadors d'utilitat individual. Hem modelat el sistema com un joc, on els autors tenen una 'qualitat' o 'nivell' a l'hora de crear articles i els avaluadors poden ser de tipus distints segons el seu comportament a l'hora de valorar els treballs. Un autor no sap amb exactitud quina classe d'avaluador va a revisar el seu article, únicament té unes creences sobre el tema. Usem el concepte d'equilibri bayesià per trobar les estratègies d'estabilitat, caracteritzant l'equilibri *pooling* quan les funcions de cost són constants i iguals per tots els agents, i l'equilibri separador en la resta de casos (entenent el cost com la inversió realitzada en el procés de creació o avaluació). Adicionalment realitzem estàtica comparativa, on observem que, generalment, a major proporció d'avaluadors no-confiables o tramposos, la qualitat dels treballs i de les valoracions rebudes pot decréixer.

Chapter 1

Conflict and segregation in networks: An experiment on the interplay between individual preferences and social influence

Abstract:

We examine the interplay between a person's individual preference and the social influence others exert. We provide a model of network relationships with conflicting preferences, where individuals are better off coordinating with those around them, but not all prefer the same ac-

tion. We test our model in an experiment, varying the level of conflicting preferences between individuals. Our findings suggest that preferences are more salient than social influence, under conflicting preferences: subjects relate mainly with others who prefer the same. This leads to two undesirable outcomes: net- work segregation and social inefficiency. The same force that helps people individually hurts society.

1.1 Introduction

The interplay between what we prefer to choose and the influence those around us exert on our choices is at the core of our social and economic life. Both individual preferences and social influence guide our behavior and whether to establish relationships with others or not (Tajfel and Turner, 1979; Lazarsfeld et al., 1954; McPherson et al., 2001). For instance, when choosing our friends (Marsden, 1990) or neighbors (Schelling, 1978) individual preferences are a strong determinant of how we make such decisions. But also, the social influence peers exercise on human behavior is enormous (Jackson, 2009), affecting whether people act in alignment or not with those they relate to (Morris, 2000; López-Pintado, 2006). Examples of social influence range from which products we buy or languages we learn (Galeotti et al., 2010), whether we engage or not in criminal activities (Ballester et al., 2006), to our participation in collective action (Granovetter, 1978). Arguably, by addressing the interplay between individual preferences and social influence we can understand the forces motivating how people decide what relationships to form and how to behave with others, that is the aim of this paper.

One of the most prominent theoretical tools to study the effect individual preferences have on the way people behave is *identity theory* (Tajfel and Turner, 1979; Akerlof and Kranton, 2000). From the perspective of identity theory a person's sense of self, her identity, is composed by three elements. First, *categorization*, putting ourselves and others into social categories (i.e., being a Christian orthodox, a female, a police man). Second, *identification*, the process we use to associate ourselves with certain groups. The group we identify with, say because we share a common identity with its members, is the *in-group*. Conversely, the group we do not identify with, for we do not share the identity of its members, is the *out-group*. Third, *comparison*, the process we use to compare our in-group and the out-group, most likely favoring one over the other. Identity theory has highlighted how the social categories people identify with are associated with particular behaviors prescribed for them. We refer to this prescribed behavior as a person's individual preference. Thus, what people care about and how much they care about it greatly depends on their identity. For example, in latin cultures, when dancing salsa or tango, males are meant to lead and females are supposed to follow; such is the behavior associated to each category. In this direction, identity theory stresses that a person obtains greater benefit from behaving as indicated by her identity than doing otherwise. When people are doing what is in accordance to their individual preferences they are happy, they get more out of it, and those who are not living up to the norms set by their social categories are unhappy, so they tend to change their decisions to meet their standards (Akerlof and Kranton, 2010).

On the other hand, a leading research program studying how the structure of social relationships influences behavior is that of *strategic interaction in networks* (i.e., network games). Work on network interactions gives account of the way we make our decisions influenced by the decisions of our neighbors. For instance, if a person is choosing a technological product and wants it to be compatible with her co-workers or friends, her choice can change depending on how many of them are using the same technology or a different one (Vives, 1990, 2005). These interactions are known as coordination games with strategic complementarities, where a person's incentives to choose a given product or adopt a given behavior increase as more of those around her make the same choice. The underlying mechanism from social influence is that people perceive coordinating with the behavior of others as beneficial for them. As a result, this line of research has highlighted that people are more likely to adopt a given behavior or not depending on who they are related with, even if such a behavior is not the one prescribed for their identity (Hernández et al., 2013).

The existing research on these two lines of work has illustrated ways in which identities *or* social influence affect our relationships and our behavior. However, it leaves open the very fundamental aspect of how these elements relate to each other and work together. The current paper aims to address this gap and give account of the interplay between individual preferences and social relationships. To do so, we elaborate and analyze a formal model where actors choose with whom to interact and which behavior to adopt (i.e, network games), and experimentally tests the model by varying the way identities and social influence take place. Our model moves beyond the existing work in its combination of three features. First, our model introduces identities as part of the strategic considerations actors have by allowing for heterogeneity in social categories. In our case there are two social categories and an actor either belongs to one or the other. Second, to assess the effect of identities on the establishment of relationships, actors in our model form a social network by making decisions about whom to link with and whom to leave out. Particularly, the choice of forming connections is made after actors are informed of their own identity and the identity of the other actors in the population. Third, to understand how social influence affects actors choices, we model the adoption of behavior as a choice that is made once the structure of relationships has been formed. There is one behavior prescribed to each social category, so that the preference of an individual is to adopt the behavior that corresponds to her identity but there is a benefit in behaving the way those around us do. In this way, our theoretical model considers the essentials of identity theory and social influence in network relationships to unravel the way these two determinants of our decision-making process relate to each other.

A key aspect of the relationships we model is that they portray strategic complementarities. This means that actors are better off aligning their behavior to that of those around them (i.e., their network relationships). But, by introducing identities actors are in conflict about the behavior each prefers to adopt. Depending on the social category they belong to, some actors prefer one behavior and others prefer a different, yet they rather coordinate with as many others as possible. Thus, we model the interplay between identities and social influence in a context of *conflicting preferences*. In this setting we design an experiment in which subjects choose with whom to connect and how to behave playing a game derived from our theoretic model. In our experimental design subjects are artificially assigned an identity and they know the identities of others. Our focus is to consider different conditions where the relative size of the social categories vary. In this way we are able to control the social context and therefore the intensity of the conflict in preferences between social categories. On one hand, social influence points to the idea that subjects rather convey to the pressure of the strongest category (i.e., the majority) and be better off by it. On the other, identities showed that subjects will require different levels of pressure to adopt the behavior that does not correspond to their individual preference, for people have a strong inclination to behave accordingly to the prescription for their social category. In this way, our theoretical and experimental work contribute to the understanding of how the interplay between identities and social influence determine what relationships are formed and what behaviors are adopted in a network environment.

The remainder of this paper builds as follows: In section 1.2 we describe the theoretical framework of our modeling and experimental design in relation to previous research on identity theory and on social influence. Our game theoretic model is presented in Section 3.2. Section 1.4 analyzes the network structures that emerge from the interactions of actors belonging to different social categories, and the conditions under which either identities or social influence are stronger determinants of behavior and of the resulting network architectures. In section 1.5 we describe the experimental study, the design, procedures and methods used. Section 3.3 presents the main results of our study guided by hypotheses derived from our theoretical model. We conclude with a discussion of the implications and limitations of the study in Section 1.7.

1.2 Theoretical framework: Identities and social influence

Our theoretical framework builds on two lines of work examining how relationships and behavior emerge from actors' individual preferences and the influence from those around them: *identity theory* (originating from psychology but recently increasingly adopted in economics; see (Akerlof and Kranton, 2010) and *strategic interaction in networks* (from economics). While elaborations of these lines of work differ in the extent to which actors are modeled as perfectly or imperfectly rational and strategic (i.e., myopic or farsighted), both identity theory and the theory of strategic interaction in networks start from the assumption that individual actors strive to obtain optimal outcomes for themselves. Our study integrates both lines of research for the particular case of interactions with *strategic complementarities.*

Research on the theory of identities was initiated in psychology (Abrams and Hogg, 2012; Tajfel, 1978; Tajfel and Turner, 1979; Turner et al., 1987), mainly focusing on the effects that the social context has on group processes and inter-group relations. The aim, to understand how different inter-group interactions could be explained and whether groups of people who share/differ in certain traits were more likely to integrate or discriminate each other (Tajfel and Turner, 1979). A consistent finding in identity theory is that people favor their in-group relative to out-groups, because people desire a positive and secure self-concept, which leads them to think of their groups as good groups. The argument of in-group bias has been widely supported by experimental research on identities (Billig and Tajfel, 1973).

To assess the effect of identities on inter-group relations, experimental studies on identities are characterized for their use of a methodology called the *minimal group paradigm*. In these experiments researchers sought minimal conditions that would create group identification. To do so, subjects were assigned to groups using arbitrary criteria (i.e., the toss of a coin). After informing subjects of their group membership (i.e., their identity), they were asked to allocate points to members of their own group (the in-group) and to members of the other group (the out-group). Minimal group experiments have typically shown a tendency to allocate more points to in-group members than to out-group members (Brewer, 1979; Mullen et al., 1992). This tendency of maximum differentiation between in-group and out-group has even occurred when it means sacrificing absolute in-group benefit. Psychological research on identities has illustrated the strong tendencies that group identification generate on our individual preferences. Nonetheless, this findings do not come without shortcomings. An important limitation is that this approach has no strategic considerations about the way people behave given the behavior of others. In general, participants in these experiments could not benefit or lose in any way from their point allocation strategy, and even in some experiments points did not carry any value at all (Turner, 1978). Therefore, the importance of identities for the understanding of rational behavior was not clear from the existing research in psychology. The interaction of identity considerations and individual incentives had not been directly addressed theoretically or experimentally, leaving an important gap for the development of rational choice theory.

Goerge Akerlof and Rachel Kranton initiated research on identities in economics by developing a model in which identities are introduced in the utility function of the actors (Akerlof and Kranton, 2000). By doing this, they were able to characterize ways in which identities are included as part of the process of maximization when rational actors choose how to behave. For instance, an action may increase monetary benefits but decrease identity utility, such as complying with social pressure to behave as opposed to the prescription of our social category. The application of their model has been found useful to explain gender discrimination (Akerlof and Kranton, 2000), education (Akerlof and Kranton, 2002), and contract theory (Akerlof and Kranton, 2005). A set of experimental work has also included identity as part of the analysis, addressing the limitation that the psychological approach has in the perspective of the behavior of a rational actor, by taking into account monetary stakes (Bernhard et al., 2006; Goette et al., 2006; Tanaka and Camerer, 2009; Eckel and Grossman, 2005; Charness et al., 2007; McLeish and Oxoby, 2007; Chen and Li, 2009). Particularly, (Chen and Li, 2009) have adopted the minimal group paradigm and showed that group divisions matter even when monetary stakes are involved. Subjects gave more points to members of their in-group, and in cases where punishment was possible they punished out-group members more. While the existing modeling of identities in economics provides insight into broad patterns of social behavior, it does not incorporate the micro-details of who interacts with whom (i.e., social networks). The inclusion of network relations in the analysis is a matter of great importance because networks have a profound effect on our decision-making process, and have proven to be necessary for our understanding of the way others influence our behavior.

Research on network interactions has introduced the strategic behavior of people into the analysis of social influence by modeling the interaction as a game (for surveys of the literature see (Goyal, 2007; Jackson, 2009; Vega-Redondo, 2007). Network games model the way individuals behave as a function of the actions of their neighbors. For the case of coordination games with strategic complementarities; settings where individuals are better off the more of their neighbors in the network behave as they do but there are at least two possible behaviors, network research has captured individual behavior through thresholds (Granovetter, 1978;

Galeotti et al., 2010). Such thresholds are the representation of the social influence a person requires from her neighbors to adopt a given behavior. For instance, when a person is deciding whether to acquire a specific technology or not, if more than a given number of her neighbors (i.e., the threshold) have that same technology, this person would acquire it as well, otherwise she would acquire a different one. A main interest in this line of research has been to understand equilibrium selection, for there are multiple equilibria and it is not clear which outcome is more likely to occur. It is possible that all actors choose one of the available options, the same for all, or some acquire one technology and some acquire the other. Work following this aim are (Ellison, 1993), (Kandori et al., 1993), (Young, 1993), (Morris, 2000), and (López-Pintado, 2006). A persistent finding in the theoretical modeling of social influence in games with strategic complementarities is that the most likely outcome is the risk-dominant equilibrium. This means that instead of aiming to get the highest payoffs by choosing a risky option, actors are more likely to focus on the less risky behavior at the expense of payoffs. Two main aspects of this research that need attention are: (i) relationships are given exogenously, so that people do not have the choice of selecting with whom they want to interact, and (ii) actors have been assumed to be identical so that identities are not part of the analysis.

The first aspect of these limitations has received a great deal attention by modeling social relationships as endogenous decisions actors make (Jackson and Wolinsky, 1996). This block of research aims to understand which network structures will emerge when rational actors have the discretion to create and severe their connections. Papers following this aim are (Jackson and Wolinsky, 1996), (Bala and Goyal, 2000), (Jackson and Watts, 2002) and (Muñoz-Herrera et al., 2013). A main finding that endogenous formation brings to network games is that the risk-dominant equilibrium is not the most salient equilibrium anymore. So that if actors can choose with whom they want to affiliate, other outcomes are likely. The possibility actors have to select their partners reduces risk and the payoff dominant equilibrium becomes salient (Jackson and Watts, 2002). Social influence has therefore a strong impact on behavior given the network of relationships. But, the possibility people have to influence the relationships they form proves to have a strong impact on the outcomes that occur. The idea is that people act strategically when deciding with whom to form social relationships. We choose our relationships because they are beneficial to us, and if an existing relationship with someone is not beneficial anymore, it is very likely to terminate it (Jackson and Wolinsky, 1996). Thus, a particularity of social influence is that its strength can vary depending on whether we are able to adapt our behavior to respond to what others around us are doing or to adapt our relationships with others given what we are interested in choosing.

The second aspect of these limitations, the inclusion of identities, has not received much attention until now. A study of conflicting preferences, closely linked to ours, is the work by (Hernández et al., 2013). In their model the authors address the effect of heterogeneity in identities in network games. However, their analysis is restricted to a particular set of exogenously given networks (i.e., Erdös-Renyi networks), so that actors have no choice regarding whom they relate to. Our model extends (Hernández et al., 2013) into a two stage game in which actors endogenously decide over their connections in the first stage and then play a coordination game with strategic complements in the second stage. Our extension is motivated by the pervasive empirical findings showing how actors' identities influence who they connect with in their networks. For instance, many social networks portray homophily (Jackson, 2009) and show that is it more likely to have friends of the same race (Marsden, 1990) or gender (Verbrugge, 1977). By modeling both stages we can study how the level of conflicting preferences influences the way networks are strategically formed, given the interplay between individual preferences and social influence.

1.3 The model

In this section we present our model of network interactions taking into account identities and social influence. Identities are associated to two social categories, each giving a behavioral prescription for the players, and the utility from the adopted behavior will depend on the identity of the players. This means that in our network game players have identities, each identity is associated with a behavior that gives it higher payoffs than the other, and the identities and behavior need not be the same for all players. Thus, conflicting preferences can be present as part of the social interaction.

Consider the set of players $N = \{1, \ldots, n\}$, with cardinality $n \geq 2$, who interact in a network game denoted by Γ. In Γ there are two social categories expressed by the set $\Theta = \{0, 1\}$. Every player $i \in N$ is *ex-ante* and exogenously endowed with an identity corresponding to one of the two social categories, $\theta_i \in \{0, 1\}$. Prior to the start of the game, players are informed about the size of the network and the identity of all players, including theirs. The network game Γ has two stages: affiliation and behavior adoption.

In the first stage, *affiliation*, players decide with whom they want to interact in the game. To do so, players create undirected connections between them. These connections are only created if both players mutually agree on their formation. Therefore, the action set of player i is a vector in $\{0,1\}^N$. We denote by $\mathbf{p^i}$ the vector of connections proposed by player i at stage 1 where $p^i_j = 1$ means that player i proposes a link to j, and $p^i_j = 0$ otherwise. We suppose that $p^i_i = 0$. Only if $p^i_j = p^j_i = 1$, we say there is a link between i and j. The profile of vectors $\mathbf{p} = (\mathbf{p^1}, \mathbf{p^2}, \ldots, \mathbf{p^n})$ represents the network by the set of links, g. Notice that, the set of potential connections is the complete network, g^N, and any network configuration is part of the set $G = \{g : g \subset g^N\}$. In the network, if a pair of players i and j are connected by a link, it is denoted as $g_{ij} = g_{ji} = 1$, and if there is no link between them, we say $g_{ij} = 0$. The set of neighbors a player i has is $k_i(g) = \{j : g_{ij} = 1\}, \ \forall \ j \neq i$. For simplicity we assume that $ii \notin g$, so that all neighbors in $k_i(g)$ are different from i. The cardinality of $k_i(g)$ is k_i, the degree of node i in the network.

In the second stage of the game: *behavior adoption*, players choose an action from the binary set $X = \{0,1\}$, once the network has been formed. The action chosen by i, $x_i \in X$, is the same for all neighbors she plays with. We construct identity-based preferences given the existing social categories. A player i who has identity 1 (0) *prefers* action 1 over 0 (0 over 1). This is a behavioral prescription expressed in the payoff function below. We denote $x_{k_i}(g)$ as the vector of actions taken by i's neighbors. The game is expressed through a linear payoff function, u_i, that strategically depends on the choices made by connected players (i.e., those that can influence i's behavior), their identities and proposed links in the first stage, as follows:

$$u_i(\theta_i, \mathbf{p}, x_i, x_{k_i}(g)) = \lambda_{x_i}^{\theta_i} \left(1 + \sum_{j=1}^{k_i} I_{\{x_j = x_i\}}\right) - c \sum_{j=1}^{n} p_j^i, \qquad (1.1)$$

where $I_{\{x_j=x_i\}}$ is the indicator function of those neighbors choosing the same action as player i. The parameter λ is defined by $\lambda_{x_i}^{\theta_i} = \alpha$ when a player chooses what she likes ($x_i = \theta_i$), the action prescribed for her identity, and $\lambda_{x_i}^{\theta_i} = \beta$ otherwise ($x_i \neq \theta_i$). The cost of proposing a link is $c > 0$, and the relation between the parameters in the model is $0 < c < \beta < \alpha$. Note that the cost of proposing a link, c, is paid independently of whether a connection is formed or not.[1]

The main feature of our utility specification is that it captures heterogeneity in several strategic scenarios in a simple way. As a result, we can observe how a player's payoff is affected by the choices of others (i.e., social influence) given her identity. This is motivated by our desire to develop an understanding of how the conflict of preferences, the scenario in which players want to coordinate with others but the preferred choice is not the same for all, interacts in a network game. As discussed in the

[1]We assume c to be lower than β. Otherwise, the only outcome is the empty network because the benefit of coordinating one's behavior to that of a neighbor would not be enough to cover the cost of affiliation. Note that if $\beta < c < \alpha$, this is the case only when choosing the disliked option.

Introduction, by incorporating players' identities and social influence in the analysis we are extending the applicability of network models to situations in which the preferences of different players may not be aligned.

In order to study the equilibrium of the sequential game, we fix a network configuration $\{g\}$ generated by the profile $\mathbf{p}$. In the second stage of the game, players decide on an action from the binary choice set X. This is a formal game, represented by $\Gamma = \{N, \{g\}_{i,j \in N}, X, \{\theta_i\}_{i \in N}, \{u_i\}_{i \in N}\}$, and the proper equilibrium concept is the Nash equilibrium. Hence, fix $\{g\}$, a unilateral deviation by player i changes her choice x_i to choice x'_i, where $x_i \neq x'_i$. When no player has incentives to deviate from an action profile $(x^*_1, \ldots, x^*_n)$, it is a Nash equilibrium. Formally:

$$u_i(\theta_i, \mathbf{p}, x^*_1, \ldots, x^*_i, \ldots x^*_n) \geq u_i(\theta_i, \mathbf{p}, x^*_1, \ldots, x'_i, \ldots, x^*_n) \quad \forall\ x'_i \neq x^*_i, \quad \forall i \in N.$$

Note that $u_i(\theta_i, \mathbf{p}, x^*_1, \ldots x^*_n) = u_i(\theta_i, \mathbf{p}, x_i, x_{k_i}(g))$, the actions of players that are not i's neighbors do not change her payoff. The next subsection gives an illustration of the particularities of games with strategic complementarities when identities are introduced in the model. This illustration is represented with the 2-person game, in which the link between the two players is already formed. Thus it only relates to the second stage of the game. Since the cost element in the utility function is common, independently of the adopted behavior the link is already there, we can omit it.

1.3.1 The 2-person game

Definition 1. ***Strategic Complements:*** *Let SC be a 2-person game where every player has an identity* $\theta_i \in \{0, 1\}$ *and the finite set of actions* X*. The payoff matrix, where* $2\beta > \alpha > \beta > 0$*, depends on each player's choices and identity as follows:*

		0	
		1	0
1	1	$2\alpha, 2\beta$	α, α
	0	β, β	$2\beta, 2\alpha$

$\theta_1 = 1; \theta_2 = 0$

		1	
		1	0
1	1	$2\alpha, 2\alpha$	α, β
	0	β, α	$2\beta, 2\beta$

$\theta_1 = 1; \theta_2 = 1$

		0	
		1	0
0	1	$2\beta, 2\beta$	β, α
	0	α, β	$2\alpha, 2\alpha$

$\theta_1 = 0; \theta_2 = 0$

Table 1.1: Payoff matrices for SC games with identities.

Each 2×2 coordination game can be played between two players of equal or opposite identities. There are two Nash equilibria in pure strategies and one in mixed strategies.[2] Let us first discuss the pure strategy equilibria. The Nash equilibria (NE) in pure strategies $NE = \{(0, 0), (1, 1)\}$

[2]We consider a payoff structure such that a player prefers to coordinate in the

present conflicting preferences if players have opposite identities, given each likes a different action but both want to coordinate. Thus, it is not possible to Pareto rank them. However, in games between players with the same identity there is no conflict in preferences, because each one likes the same action (i.e., their behavioral prescription is the same). The equilibrium when both choose the action corresponding to their identity is Pareto dominant in payoffs: $(1, 1)$ Pareto dominates $(0, 0)$ if two players with identity 1 are playing, and the opposite for two players with identity 0.

In the mixed strategy equilibrium, the probability of choosing one's favorite action when facing a player with the same identity is given by $\underline{q} = (2\beta - \alpha)/(\alpha + \beta)$. When playing against a player with different identity, the result is $\overline{q} = (2\alpha - \beta)/(\alpha + \beta)$. Following Morris (2000) and (López-Pintado, 2006), these probabilities can be understood as the adoption threshold functions, i.e., the influence required from others, the proportion of neighbors making a given choice, so that a player adopts that same action. The inclusion of identities in the analysis gives a new insight to social influence (i.e, threshold models, see Granovetter, 1978), showing that the q needed varies depending on the identity of the player choosing, but not on the identity of the player(s) she is interacting with. That is, there exist $\underline{q} < \overline{q}$, where $\underline{q}$ is the probability of choosing the

disliked option than staying alone. This payoff structure is observed in the game of the Battle of Sexes (BOS). For an example of the BOS n-person game see (Zhao et al., 2008).

liked action and $\overline{q}$ the disliked action. The intuition of this result relates directly to the threshold functions that define the best responses in the Nash equilibrium configurations of the network games. It is also associated to many social scenarios where the utility of affiliation is based on choices of others (i.e., social influence) and not on other's preferences (i.e., identities), but the utility of the individual is based both on her choice and her identity.

1.4 Equilibrium characterization

In this section we provide the equilibrium characterization for our network game. First we present a categorization of all the possible network configurations that can emerge for any distribution of identities, level of connectivity and action profile chosen. Based on these categories we characterize the set of Nash equilibria for our network game, NE(Γ). That is, the action profile chosen once the network is realized, after the links are proposed. To do this we follow (Hernández et al., 2013), who model network games in fixed networks. We extend their analysis with the characterization of the subgame perfect Nash equilibria of the two stage network game. Finally we conclude with a discussion on equilibrium selection.

Notice that along the analysis we can assume without loss of generality a normalization of the utility function for which the cost of link proposal is equal to zero, given the cost of proposal is independent of the action played in the second stage.[3]

1.4.1 Network categorization

A player in the network game chooses a vector of link proposals and an action from the set $X = \{0, 1\}$, the same for all her formed connections. The action profiles in the network are such that either all players coordinate on one action (*specialized*) or both actions are chosen by different players (*hybrid*). Given the identity of the players, there are two possible categories, depending on whether all players coordinate in choosing the action they prefer (*satisfactory*) or at least one player chooses the disliked action (*frustrated*).[4] Thus, there are four possible configurations: (i) satisfactory specialized (S_S) where all players coordinate on the same action, which is their preferred choice; (ii) frustrated specialized (F_S), where all players coordinate on the same action, but at least one of them is choosing her disliked option; (iii) satisfactory hybrid (S_H), where all

[3]Once the network is realized, for the computation of the best responses for any player, it affects in the same way the cost of links independently of the action chosen: $[u_i(1, p^i, 1, x_{N_i}(g)) - cp^i] - [u_i(1, p^i, 0, x_{N_i}(g)) - cp^i] = u_i(1, 1, x_{N_i}(g)) - u_i(1, 0, x_{N_i}(g))$. Therefore, this cost is cancelled on both sides of the computation.

[4]We differentiate action profiles as satisfactory or frustrated following the arguments in Akerlof and Kranton (2000). When a player adopts the behavior prescribed for her identity, this reinforces who she is. However, anyone who chooses the non-prescribed behavior suffers a loss in her identity, entailing a reduction in her utility. That is the reason why $\alpha > \beta$.

players choose the action they prefer but there is at least one player with a different identity from the rest, so that both actions are present; and (iv) frustrated hybrid (F_H) which portray both actions and at least one player chooses her disliked option. Figure 1.1 illustrates these categories.

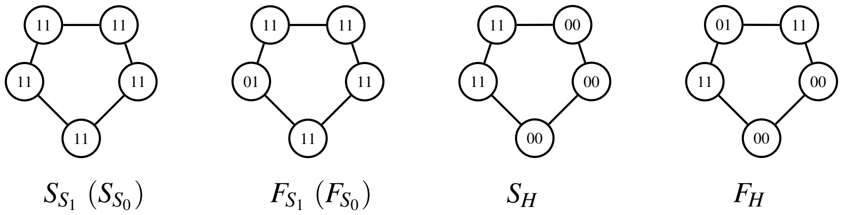

Figure 1.1: Categories of network configurations. The first digit of a node refers to the identity of a player and the second to her adopted behavior.

1.4.2 Nash equilibrium

As mentioned above, given the cost of link proposal is omitted from the best response characterization, once the network is realized, the results in (Hernández et al., 2013) for fixed networks are applicable to our case. In summary, their findings show that there are two threshold functions in network games with SC when players have conflicting preferences. These thresholds represent the social influence a player's neighbors must exert for her to choose one behavior or another. A function $\underline{\tau}(k_i)$ that represents the minimum number of i's neighbors choosing the action she likes, for her to choose her favorite action as a best response. The threshold function $\overline{\tau}(k_i)$ is the maximum number of neighbors choosing the non-favorite action so that i's best response is still to adopt the behavior she likes, so that if one more of her neighbors chooses the non-favorite action, player i's best response is to change and adopt her disliked option. The results are presented in Proposition 1, where the number of i's neighbors choosing action 1 is χ_i and the number of her neighbors choosing action 0 is $k_i - \chi_i$.[5]

Proposition 1. ***(Hernández et al., 2013)*** *For an SC game, let*

$$\underline{\tau}(k_i) = \lceil \frac{\beta}{\alpha+\beta} k_i - \frac{\alpha-\beta}{\alpha+\beta} \rceil, \quad (1.2)$$

$$\overline{\tau}(k_i) = \lfloor \frac{\alpha}{\alpha+\beta} k_i + \frac{\alpha-\beta}{\alpha+\beta} \rfloor, \quad (1.3)$$

defined for any degree $k_i \in \{1, \dots, n-1\}$*. The best response of player* i *with identity* $\theta_i = 1$ *and degree* k_i*,* x_i^**, is*

$$x_i^* = \begin{cases} 1, & \textit{iff } \chi_i \geq \underline{\tau}(k_i), \\ 0, & \textit{otherwise.} \end{cases} \quad (1.4)$$

The best response of player i *with identity* $\theta_i = 0$ *and degree* k_i*,* x_i^**, is*

$$x_i^* = \begin{cases} 0, & \textit{iff } \chi_i \leq \overline{\tau}(k_i), \\ 1, & \textit{otherwise.} \end{cases} \quad (1.5)$$

[5]Denote by $\lceil \dots \rceil$ and $\lfloor \dots \rfloor$ respectively the maximum lower integer or the minimum higher integer of the real number considered.

The intuition behind Proposition 1 is that in SC a player i wants to coordinate with the highest number of neighbors making the same choice, and prefers coordination on the action prescribed for her identity. Players with identity $\theta_i = 1$ have incentives to choose the action they like when $\chi_i \geq \underline{\tau}(k_i)$. Thus, players with identity $\theta_i = 0$ choose $x_i = 0$ if $\chi_i \leq \overline{\tau}(k_i)$. Clearly $\overline{\tau}(k_i) > \underline{\tau}(k_i)$ for any k_i, so that a player i requires less influence from her social network to choose what she prefers and more social pressure to adopt her disliked behavior, compared to an analysis ignoring identities. For instance, returning to the example of people choosing between two technologies, say two operative systems such as MacOS and Windows, those who prefer Mac over Microsoft need less support from their friends to purchase this operative system. However, they would require more pressure from their friends to buy the Windows system that they dislike. The two tipping points are illustrated in Figure 1.2.

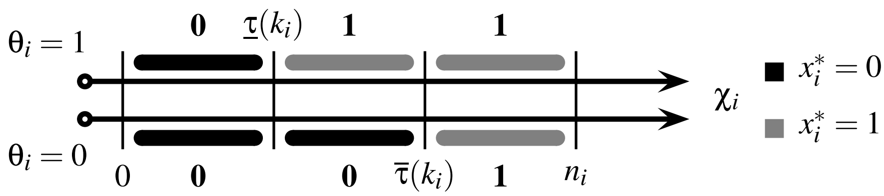

Figure 1.2: SC Adoption Thresholds

From the best responses characterized above, it is clear that there will be very many different equilibria. For an illustration consider the following case: Satisfactory specialized equilibria (S_S) are a very restrictive case in which all players must have the same identity. Assume $\theta_i = 1$ for all $i \in N$. Then, in a S_S equilibrium, all players choose their preferred behavior; action 1. However, if for any reason a player or group of players chose action 0 and for them the condition $\chi_i \geq \underline{\tau}(k_i)$ is not verified, a frustrated Nash equilibrium emerges, i.e., *frustrated hybrid.* This because all players have the same identity but some are choosing the disliked option. In general, when all players share a common identity, if an equilibrium is satisfactory it has to be specialized. There is another manner in which specialized equilibria emerge, namely when the distribution of identities is not homogeneous but both social categories are present (there are 1's and 0's) and either condition $\chi_i \leq \underline{\tau}(k_i)$ or $\chi_i \geq \overline{\tau}(k_i)$ holds for all players. As a consequence, for the same distributions of links and identities, two players with opposite identities can best respond with the same action and vice versa. This points to conditions where social pressure can exert more influence than the individual preference of a player who is interacting with others, when choosing what behavior to adopt. Examples of Nash equilibria are illustrated in Figure 1.3.

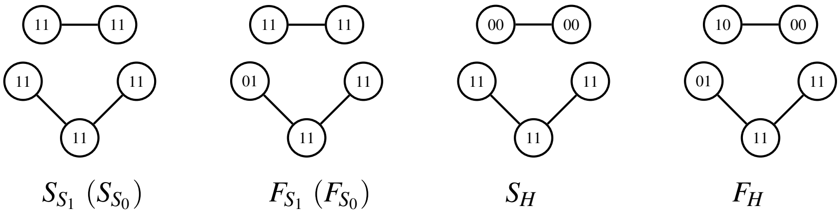

Figure 1.3: Examples of Nash equilibria. The first digit of a node refers to the identity of a player and the second to her adopted behavior.

1.4.3 Subgame perfect Nash equilibrium

Our analysis so far has focused on the best response when players play once the network is formed. We now proceed to the first stage of the network game: *affiliation*. By backward induction analysis we develop a characterization of the subgame perfect Nash equilibria (SPNE). In our case, all players play simultaneously at each stage. Thus, we are interested in knowing which vector of link proposals is part of an equilibrium. Notice that a given network can be generated from different vectors of link proposals. For instance, if player i has k_i neighbors in $\{g\}$, it could be because she proposed a link to only her k_i neighbors, or because she proposed links to those and even more players; who did not proposed a link back to i. The first Lemma states that in a SPNE for a given network $\{g\}$ the best proposed links $\mathbf{p^i}$ for any player i does not exceed the set of her realized neighbors in $\{g\}$.

Lemma 1. *Let $\{g\}$ be a network where player i has k_i neighbors denoted by $\{i_1, i_2, \ldots, i_{k_i}\}$. Consider two vectors of proposals:*

- $\mathbf{p^i}$ *with* $p^i_j = 1$ *if* $j \in \{i_1, i_2, \dots i_{k_i}\}$ *and* $p^i_j = 0$ *if* $j \notin \{i_1, i_2, \dots i_{k_i}\}$
- $\tilde{\mathbf{p}}^\mathbf{i}$ *with* $\tilde{p}^i_j = 1$ *if* $j \in \{i_1, i_2, \dots i_{k_i}, z_i, z_2, \dots z_s\}$ *and* $\tilde{p}^i_j = 0$ *if* $j \notin \{i_1, i_2, \dots i_{k_i}, z_i, z_2, \dots z_s\}$.

For the game Γ *where* $\{g_{ij}\}_{i,j \in N}$ *is realized then*

$$u_i(\theta_i, \mathbf{p^i}, x^*_1, \dots, x^*_i, \dots x^*_n) \geq u_i(\theta_i, \tilde{\mathbf{p}}^\mathbf{i}, x^*_1, \dots, x^*_i, \dots x^*_n)$$

where the set of players $\{z_1, z_2, \dots, z_s\} \cap \{i_1, i_2, \dots i_{k_i}\} = \emptyset$.

Proof: It is straightforward to check that

$$u_i(\theta_i, \tilde{\mathbf{p}}^\mathbf{i}, x^*_1, \dots, x^*_i, \dots x^*_n) = u_i(\theta_i, \mathbf{p^i}, x^*_1, \dots, x^*_i, \dots x^*_n) - c|\{z_1, z_2, \dots, z_s\}|$$

As a consequence of the above Lemma, networks where players proposed the final links will be the survival networks in the backward induction process. In particular, a SPNE is a network that results given that in the *affiliation* stage no link proposal is unreciprocated, and in the *behavior adoption* stage players choose according to Proposition 1. Nonetheless, the analysis of subgame perfection does not permit us to discriminate enough, and there are multiple surviving configurations that satisfy these conditions. In the last part of this section, we will refine the analysis of equilibria by considering some selection criteria that have proven to be essential for network studies.

1.4.4 Equilibrium selection

To model equilibrium selection, we use two different concepts that are commonly applied to network games: Pairwise stability (Jackson and Wolinsky, 1996) and efficiency (i.e., utilitarian welfare). Our aim is to discriminate equilibria in terms of how they dominate in payoffs and how likely is it for players to be satisfied and adopt the behavior prescribed for their identities in the presence of social influence from their neighbors.

We begin by evaluating for which networks, once an action profile is chosen, players have incentives to change their connections (i.e, increase or decrease their degree). Because pairwise stability only takes into account link selection, we fix the set of action profiles, x, to establish what a stable network is. The original concept of pairwise stability, by (Jackson and Wolinsky, 1996), states that a network is pairwise stable with respect to the total value of the network (often the aggregate utility of all nodes) and an allocation rule (how that utility is divided among nodes) if (i) there is no player who is better off by unilaterally cutting one of her existing links and (ii) if there is no pair of unconnected players who would benefit from creating a link between them; if one of them is better off by forming the link then the other is worse off by doing so[6]. We adapt and formally the define the concept for our model as follows:

[6] For different theoretical characterizations of pairwise stability see also (Jackson and Watts, 2001) and (Calvó-Armengol and İlkılıç, 2009).

Definition 2. ***Pairwise stability:*** *Let x be an action profile. A network $\{g\}$ generated by $\mathbf{p}$ is pairwise stable if:*

1. *Suppose $p^i_j = p^j_i = 1$. Consider the network $\tilde{g}$ generated by $\tilde{\mathbf{p}}$ that coincides with g except $\tilde{p}^i_j = \tilde{p}^j_i = 0$, (i.e., players i and j are not connected) then*

$$u_i(\theta_i, \mathbf{p^i}, x) \geq u_i(\theta_i, \tilde{\mathbf{p}}^{\mathbf{i}}, x) \text{ and } u_j(\theta_j, \mathbf{p^j}, x) \geq u_j(\theta_j, \tilde{\mathbf{p}}^{\mathbf{j}}, x)$$

2. *Suppose $p^i_j = p^j_i = 0$. Consider the network $\tilde{g}$ generated by $\tilde{\mathbf{p}}$ that coincides with g except $\tilde{p}^i_j = \tilde{p}^j_i = 1$, (i.e., players i and j are connected) then*

   (a) if $u_i(\theta_i, \mathbf{p^i}, x) < u_i(\theta_i, \tilde{\mathbf{p}}^{\mathbf{i}}, x)$ then $u_j(\theta_j, \mathbf{p^j}, x) > u_j(\theta_j, \tilde{\mathbf{p}}^{\mathbf{j}}, x^*)$ or

   (b) if $u_j(\theta_j, \mathbf{p^j}, x) < u_j(\theta_j, \tilde{\mathbf{p}}^{\mathbf{j}}, x)$ then $u_i(\theta_i, \mathbf{p^i}, x) > u_i(\theta_i, \tilde{\mathbf{p}}^{\mathbf{i}}, x)$

Since in our model players choose both links and actions, we provide now a definition of pairwise stable networks in our game, in order to take into account not only links selection but also the action profile, x, chosen at Stage 2. To do so, we integrate the subgame perfect framework with conditions of an adapted concept of pairwise stability, and evaluate how increasing or decreasing the density of the network affects players' payoffs. Therefore, we select the set of action profiles to those leading to Nash equilibrium in the second stage which correspond to a Nash

Equilibrium profile even when the network configuration changes. The following definition captures this idea:

Definition 3. *The pair* $(\{g\}, x^*)$ *is a pairwise stable Nash equilibrium if*

1. *g is pairwise stable, and*
2. *For the network $\tilde{g}$ generated by $\tilde{\mathbf{p}}$ that coincides with g except $\tilde{p}^i_j = \tilde{p}^j_i = 0$ or $\tilde{p}^i_j = \tilde{p}^j_i = 1$ the action profile x^* is a Nash equilibrium in g and $\tilde{g}$.*

The next lemma characterizes the set of pairs $(\{g\}, x^*)$ which are pairwise stable Nash equilibria in the game Γ. First, it describes the network structures which satisfy the pairwise stability property. Finally, the set of Nash equilibrium action profiles which verify the stability notion.

Lemma 2. *The pair* $(\{g\}, x^*)$ *is a pairwise stable Nash equilibrium of the game* Γ *if it satisfies the following conditions:*

(i) *Every player i is connected to **all** other players in the network who are choosing the same action as her: if $x^*_i = x^*_j$ for any $i, j \in N$, then $g_{ij} = 1$, and*

(ii) *Every player i is connected **only** to players who are choosing the same action as her: if $g_{ij} = 1$ for any $i, j \in N$, then $x^*_i = x^*_j$*

(iii) Let be $\theta_i = 1$, and χ_i the number of neighbors of player i playing action 1 in the network g. Then,

(a) if $x_i^* = 1$ then $\chi_i \geq \underline{\tau}(k_i + 1)$

(b) if $x_i^* = 1$ then $\chi_i - 1 \geq \underline{\tau}(k_i - 1)$

(c) if $x_i^* = 0$ then $\chi_i + 1 < \underline{\tau}(k_i + 1)$

(d) if $x_i^* = 0$ then $\chi_i < \underline{\tau}(k_i - 1)$

The conditions for players with $\theta_i = 0$ are symmetric.

Proof:

From this point on, and abusing notation, we will use $\{g\}$ and $\mathbf{p}$ indistinctively in the utility function, given each $\mathbf{p}$ generates a unique $\{g\}$.

Let us prove that a network structure produces a pairwise stable Nash equilibrium if every player is connected to *all* others coordinating their behavior with her. Consider two networks:

- $\{g\}$ where $x_i^* = 1$ for player i, and there is at least one player j choosing $x_j^* = 1$, such that $g_{ij} = 0$, and
- $\{\tilde{g}\} \supset \{g\}$ in which i and j form a link between them, $\{\tilde{g}\} = \{g\} + g_{ij}$

For the game Γ:

$$u_i(\theta_i, \{\tilde{g}\}, x_1^*, \ldots, x_i^*, \ldots x_n^*) > u_i(\theta_i, \{g\}, x_1^*, \ldots, x_i^*, \ldots x_n^*), \text{and}$$

$$u_j(\theta_j, \{\tilde{g}\}, x_1^*, \ldots, x_j^*, \ldots x_n^*) > u_j(\theta_j, \{g\}, x_1^*, \ldots, x_j^*, \ldots x_n^*)$$

where it is straight forward to check that

$$u_i(\theta_i, \{\tilde{g}\}, x_1, \ldots, x_n) = \alpha(k_i+1) - ck_i > \alpha k_i - c(k_i-1) = u_i(\theta_i, \{g\}, x_1, \ldots, x_n)$$

since $\alpha > c$. From this, it derives that if player i is linked to k neighbors, her utility is increasing in k as long as they choose her same action.

We show now that a network forms a pairwise stable Nash equilibrium if every player is connected *only* to neighbors coordinating their behavior with her. Consider two networks:

- $\{g\}$ where $x_i^* = 1$ for player i, and $\chi_i < k_i$ of i's neighbors play $x_j^* = 1$, while $(k_i - \chi_i) > 0$ play $x_j^* = 0$, and
- $\{\tilde{g}\} \subset \{g\}$ in which i drops any neighbor j whose action is $x_j^* = 0$.

For the game Γ:

$$u_i(\theta_i, \{\tilde{g}\}, x_1^*, \ldots, x_i^*, \ldots x_n^*) > u_i(\theta_i, \{g\}, x_1^*, \ldots, x_i^*, \ldots x_n^*)$$

It is straightforward to check that

$$u_i(\theta_i, \{\tilde{g}\}, x_1^*, \ldots, x_i^*, \ldots x_n^*) = u_i(\theta_i, \{g\}, x_1^*, \ldots, x_i^*, \ldots x_n^*) + cI_{\{x_j \neq x_i\}}$$

Finally, the lemma states the conditions under which the equilibrium action profile x^* is *robust* to the addition or the removal of one link (i.e. it is not profitable for any player to change her action after adding any possible new neighbor or removing an existing one, regardless of the behavior she may adopt). Then the conditions relating χ_i and the threshold functions from (Hernández et al., 2013) must be satisfied when a player's degree is increased or decreased by 1.

Given the utility structure of player i, when two players coordinate in forming a link between them but their adopted behavior is uncoordinated, say i chooses $x_i = 1$ and j chooses $x_j = 0$, there is no positive payoff for any of them from this relationship. On the contrary, there is a negative payoff in terms of the cost of relating, without the complementarities from choosing the same action. Therefore, players prefer networks where everybody in their neighborhood plays the same action they play, and any link to a neighbor who is behaving differently is eliminated. The intuition behind Lemma 2 points to a single argument: for each action profile x^* there is only one network configuration which conforms a pairwise stable Nash equilibrium.

Up until now we have defined and analyzed pairwise stability and pairwise stable Nash equilibria for our network game. For the remaining part of this section we introduce a new concept for equilibrium selection: efficiency.

Let the value of a pair $(\{g\}, x)$ be the aggregate of individual utilities:

$$v(\{g\}, x) = \sum_{i=1}^{n} u_i(\theta_i, \mathbf{p}, x_i, x_{k_i(g)})$$

From this, it follows that a pair $(\{g\}, x)$ is *efficient* if $v(\{g\}, x) \geq v(\{\tilde{g}\}, \tilde{x})$, $\forall \{g\} \neq \{\tilde{g}\}$ and $\forall x \neq \tilde{x}$. The next definition formally expresses the idea:

Definition 4. ***Strong Efficiency:*** *A pair* $(\{g\}, x)$ *is strongly efficient in the game* Γ *if* $(\{g\}, x) = \underset{\{g\}, x}{\text{argmax}}\ v(\{g\}, x)$.

We derive from the concept of pairwise stability that only two kind of network configurations can conform a pairwise stable Nash equilibrium: (1) a completely connected structure if the action profile is *specialized*, and (2) a network with two isolated and completely intra-connected components if the action profile is *hybrid*, where each component is specialized in a different action. For an illustration see Figure 1.4.

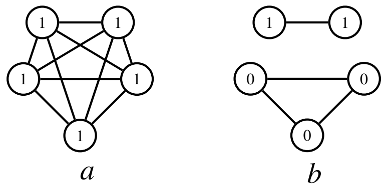

Figure 1.4: Pairwise stable Nash equilibria. Network a portrays the specialized case for action 1. Network b portrays the hybrid case. The digit of a node refers a player's adopted behavior.

Notice that pairwise stability is absent of the inclusion of identities, and just tackles the selection of structures by ranking one configuration over another if both have the same action profile. To take identities into account, we discuss next the equilibrium selection for different compositions of the population of players. As a consequence, this characterization depends on the *a priori* distribution of identities. We will refer to the distribution of identities, the share of players with identity 1 or 0, as the indicator for the *level of conflict in preferences* in the game, denoted by Φ. This will be particularly useful in our experimental study, presented in the next section. We assume there is a proportion of θ_i players with identity θ_i, where $\theta_0 + \theta_1 = 1$. Using the share of players with identity 1 as the reference group, we define the level of conflict in preferences as the binary entropy function of the distribution of identities, where $\Phi \in (0, 1)$. The more homogeneous a population is, the lower the level of conflict.

Thus, if $\theta_1 = 0$ or $\theta_1 = 1$, then $\Phi = 0$. The more heterogeneous the population is the higher the level of conflict. This means that if $\theta_1 = \theta_0$, then $\Phi = 1$. See figure 1.5 for an illustration.

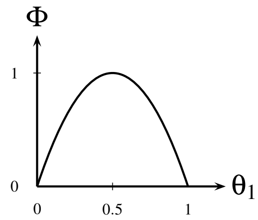

Figure 1.5: Entropia function. The horizontal axis represents the share of players with identity 1 (θ_1) in the population. The vertical axis represents the level of conflict (Φ).

Based on this consideration of conflicting preferences, we want to know what are the conditions, in terms of Φ, for players to choose a specialized or a hybrid equilibrium. In order to achieve this we characterize efficiency as a measure of utilitarian welfare and denote a network as efficient if it maximizes the aggregate utility of all players, given the distribution of identities (i.e., the level of conflict in preferences). Lemma 3 presents this arguments.

Lemma 3. *The strongly efficient configuration of the game Γ is the complete network specialized in the prescribed behavior for the majority, for any level of conflict. So that:*

(i) if $\Phi = 0$, such that $\theta_1 = 1(0)$, $x_i^* = 1(0)$ and $k_i = (n-1)\ \forall\ i \in N$

(ii) if $0 < \Phi < 1$*, such that* $\theta_1 > \theta_0 > 0$*,* $x_i^* = 1$ *and* $k_i = (n-1)\ \forall\ i \in N$

(iii) if $\Phi = 1$*, such that* $\theta_1 = \theta_0$*, either* $x_i^* = 1$ *or* $x_i^* = 0$*, and* $k_i = (n-1)\ \forall\ i \in N$

Proof: The first element follows because a network in which all players who adopt the same behavior are affiliated dominates in payoffs any less connected network. Moreover, such a network will rank the highest if all players are choosing the behavior they like. For the case of $\Phi = 0$ this is the Satisfactory Specialized (S_S) configuration.

To prove the second element we compare two networks. A satisfactory hybrid configuration (S_H), in which all players choose the action they like, and a frustrated specialized configuration (F_S), in which all players choose the action of the majority. It follows from the statement above that a F_S in the action preferred by the minority will be dominated in payoffs, given $\alpha > \beta$. Also, it follows from Lemma 2 that such networks are pairwise stable, so that $k_i = n - 1$ for all players in the F_S, and $k_i = \theta_i n$ for players in the S_H.

Consider a distribution of identities such that there are $\theta_1 n$ players with identity 1 and $\theta_0 n = n(1 - \theta_1)$ players with identity 0. The aggregate payoffs of the F_S network are given by:

$$\begin{aligned} v(F_S) &= \sum_{i=1}^{\theta_1 n} \alpha n + c(n-1) + \sum_{i=1}^{\theta_0 n} \beta n + c(n-1) \\ &= n[\theta_1(\alpha n - c(n-1)) + (1-\theta_1)(\beta n - c(n-1))] \end{aligned} \tag{1.6}$$

The aggregate payoffs of the S_H network are given by:

$$\begin{aligned} v(S_H) &= \sum_{i=1}^{\theta_1 n} \alpha(\theta_1 n) + c(\theta_1(n-1)) + \sum_{i=1}^{\theta_0 n} \alpha\theta_0 n + c(\theta_0(n-1)) \\ &= n[\theta_1(\alpha\theta_1 n - c(\theta_1(n-1))) + (1-\theta_1)(\alpha(n-\theta_1 n) - c(n-\theta_1(n-1)))] \end{aligned} \tag{1.7}$$

where it is straightforward to check that

$$v(F_S) > v(S_H) \text{ for } \theta_1 \geq \frac{1}{2} \tag{1.8}$$

The third point is easy to prove under the conditions exposed so far, because if $\Phi = 1$ then $\theta_1 = \frac{1}{2}$, and Equation 1.8 states that under that level of θ_1 the aggregate generated profit in a Frustrated Specialized configuration is higher than in a Satisfactory Hybrid one. Obviously the profit is the same if the action chosen by all players is 0 or 1, since $\theta_0 = \theta_1 = \frac{1}{2}$.

The intuition of this Lemma is that if all players behave as prescribed for the majority, so that the entire population is specialized, this is strictly better from a social welfare perspective, than if each player adopts her preferred behavior and the population segregates. That is, when social influence in the population is exerted by a majority, it is socially better for the members of the minority to choose the behavior that goes against their identity but increases their benefits from the complementarities of their neighbors. Specialization is socially better even if the share of each social category in the population is exactly divided into equal parts. Particularly, when this is the case, socially there is no difference on which behavior players specialize in. Both specializing in 1 or 0 gives the same aggregate value. Nonetheless, this is the aggregate welfare and for cases with a strict majority it is not always the case that the minority maximizes individual payoffs by following this strategy. In fact, a player i from the minority gets higher payoffs in the satisfactory specialized network in which each component is completely connected as long as $\theta_i > \frac{(\beta-c)}{(\alpha-c)} > \frac{1}{2}\frac{(\alpha-\beta)}{(\alpha-c)}$.[7]

Going back to our example on the adoption of technologies we have that a network is pairwise stable if all players purchasing MacOS are connected and none of them relates to anyone purchasing Windows, and viceverza. Furthermore, if those who like MacOS more than Windows are a majority, it is better for everyone in the society to buy this operative system, even for those who like Windows. By doing so they can all relate

[7]This comes from the comparison of choosing the behavior of the majority in the complete network or the preferred choice in the network segregated into two components: $\alpha\theta_i n - c(\theta_i n - 1) \geq \beta n - c(n-1)$.

between each other and obtain greater benefits from the compatibility of their choices than if they had segregated into clusters of Mac users and Windows users.

Finally, note that there is an important consideration when relating efficiency and pairwise stability. If the satisfactory hybrid equilibrium emerges, so that players are segregated by identities in two components each choosing the preferred action of the players in the component, there is no smooth transition to the specialized frustrated (efficient) configuration. Once a player has entered a pairwise stable but non efficient network, there are no individual incentives to move to the efficient one. A player who is part of the majority has only incentives to link to a player from the minority if she knows the other will choose her frustrated action. A player who is part of the minority has no incentives to unilaterally or bilaterally deviate to the component where the majority is segregated, because she would need multiple changes to be connected to all of them. Such a transition requires a stronger restriction than *dyadic* coalitions as modeled in pairwise stability.

In conclusion, our identity-based model and equilibrium characterization points to the following considerations. First of all, players are always better off coordinating with all their neighbors in the same behavior, because social influence from others results in greater benefits from the complementarities of the interaction. If this is not the case, a player will rather eliminate a relationship with an *uncoordinated* neighbor. However, given the interaction between identity-based payoffs and the effect of others' behavior on a player's choices, it is not always the case that players adopt the behavior prescribed for their identity. Our model of conflicting preferences shows that depending on the distribution of identities (i.e., the level of conflict in the population) some equilibria dominate others. In particular, equilibria in which all players are integrated into one same component and the behavior prescribed for the majority is the specialized action are the socially efficient networks. Moreover, in many cases, as long as the size of the minority is not big enough, these *frustrated* equilibria are also dominant on individual payoffs. Notice that in this order of ideas, the share of the population that a given identity occupies can determine whether players belonging to this group will be governed by social pressure and sacrifice their identity-based preferences for their social interaction benefit. In the next section we describe the experimental study we have used to test our game theoretic model.

1.5 The experiment

Our theoretical model gives account of the way equilibrium takes place in network games in which identities and social influence are at play. To test the results of our theory we designed an experimental game which replicates our identity-based model in the laboratory. Our interest is to evaluate the interplay between individual preferences and social influence by assessing the effect that different levels of conflict in preferences have on individual and aggregate behavior. As our game-theoretic analysis shows, there are multiple configurations in equilibrium that are likely to emerge and their likelihood depends on the strength of individual identities and on the influence exerted by others.

1.5.1 The experimental game

There are 15 subjects in a one-shot network game interaction. Each subject at the beginning of the interaction is informed about a symbol she is assigned to, either a *square* or a *circle*. The two symbols represent the artificially generated social categories to which subjects can belong to. Participants were also informed of how many of the remaining 14 subjects in the population had been assigned to each category (how many were circles and how many were squares).

The experimental game replicates the two-stage structure of our game theoretic model. In the first stage, *affiliation*, subjects simultaneously decided to whom in the group of 15 subjects they wanted to propose a link to (see Figure 1.6). Subjects were also assigned an identification number from 1 to 15 to facilitate the linking process. The identification numbers were randomly associated to the social categories but kept the same for all groups (i.e., subject with identification number 12 always belonged to the social category square). The cost of proposing a link is $c = 2$ and, only if two subjects proposed to each other a connection between them was created.

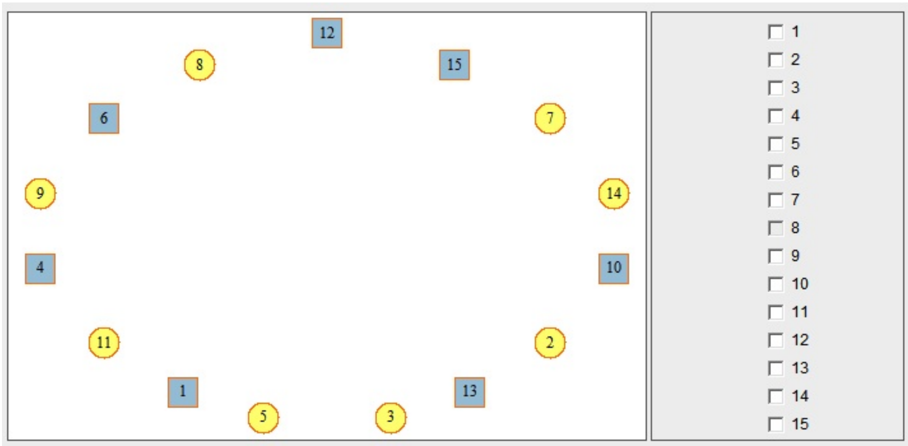

Figure 1.6: Screen of connection proposals.

In the second stage, subjects were informed about the proposals made and connections formed in their group (see Figure 1.7). That is, subjects were informed of the social network that resulted from the affiliation stage. Then, they had to choose an action *up* or *down*. Up (down) gives $\alpha = 6$ points to a subject with identity *square* (*circle*) for every neighbor she coordinates with in the same choice. Down (up) gives her $\beta = 4$ points. These choices represent the identity-prescribed behavior from our model.

The total number of points earned is calculated with the payoff function in Equation 1.1. This linear payoff function makes it straight forward

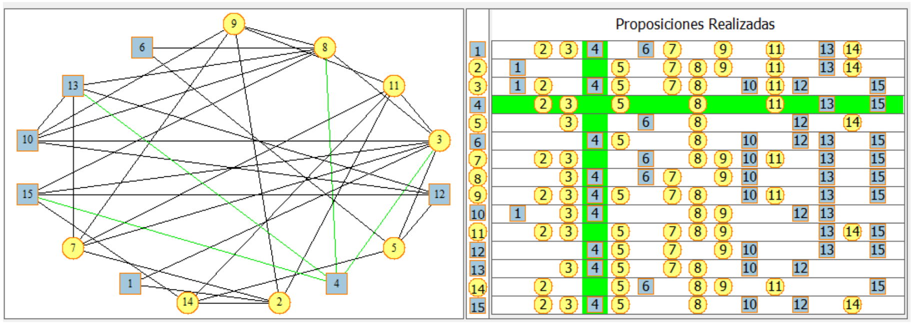

Figure 1.7: Screen with resulting network and proposals made.

for participants to calculate their expected payoffs in any situation given their individual preferences and the behavior of others (i.e., social influence). In addition, all subjects received a printed table illustrating the points they can get for any level of connections (from 1 to 14) and any choice in which they coordinated on (up or down).

Therefore, in the experiment subjects are assigned to one of two social categories. Each of the categories has a behavior that gives more benefits if chosen (in isolation) by a subject belonging to that category. Subjects, knowing their own identity and that of all other participants, propose to establish relationships between them. Only if proposals are mutual a network connection is formed. Once they observe the resulting network, subjects simultaneously choose an action (i.e., adopt a behavior). For every connected neighbor with whom a subject coordinates in her behavior, she benefits as a consequence of the strategic complementarities of the interaction. If two subjects are connected but choose differently, they do not benefit from the relationship but have to pay the cost of affiliation.

If two subjects are not connected, they cannot influence each other, because their choices have no effect on the other's payoffs. In this way our experimental game addresses the question of how individual preferences and social influence affect the decision-making process of whom to relate with and what behavior to adopt.

1.5.2 Experimental design and treatments

The experiment consists of a two-stage network game played for 25 periods by groups of 15 subjects. It was conducted in the Laboratory of Experimental Economics (LINEEX) at the University of Valencia in November 2012. Subjects interacted through computer terminals and the experiment was programmed using z-Tree (Fischbacher, 2007).

Upon arrival subjects drew a ballot to be randomly assigned to a seat in the laboratory. At the beginning of the experiment instructions were read out loud to all subjects to guarantee that they all received the same information (you can see a full copy of the instructions in appendix 1.A). Instructions also appeared on their screens. At the end of the experiment each subject answered a debriefing questionnaire. The standard conditions of anonymity and non-deception were implemented in the experiment. In every period subjects were randomly matched using a strangers protocol, so that each round represented an independent one-shot interaction with no reputation effects. Identities were randomly assigned in the first round and kept constant along the 25 interactions and only group composition varied (also the assigned identification number varied). That is, the social category a subject belonged to was always the same for all rounds. The first five periods were trial rounds.

To evaluate the effect that the level of conflict in preferences has upon outcomes, the interplay between individual preferences and social influence, we used the distribution of identities in the groups as our experimental variable. We implemented three treatments that systematically vary this feature: No conflict, Low conflict and High conflict (see Table 1.2). In all our treatments we kept the social category *square* to be the majority. Therefore, for all treatments the socially efficient and individual payoff dominant outcome is the complete network specialized in choosing *up*, the prescribed behavior for the majority.

Treatment	Majority	Minority	Session
No conflict	15	0	30
Low conflict	12	3	45
High conflict	8	7	45

Table 1.2: Number of subjects in the majority and the minority per treatment, and size of the subject sample per treatment/session.

Our experimental design captures an important mixed-motive social situation derived from our theory, which results from the incorporation of identities into the analysis. Subjects earn more by coordinating their choices with others, maximizing payoffs when the entire group integrates and coordinates in one same choice (social motive). This motive results from the effect that social influence has on behavior. But subjects disagree in their preferences about which action to coordinate on (individual motive). This motive results from the effect that individual preferences have on behavior. From this, we derive contrasting hypotheses for the equilibrium selection strategies. On one hand, we use the category motivation in the identity literature stating that if there are artificially induced identities, subjects are more likely to favor their in-group. On the other, we use the payoff dominant motivation from the literature on social influence in network interactions, which is independent of the identities of the players, and states that if subjects can decide with whom to connect they are more likely to coordinate in the equilibrium that gives them the highest payoffs. Thus, hypotheses 1a and 2a, the identity-dominant hypotheses, are a result of how identity predicts equilibrium selection in our game. Hypotheses 1b and 2b, the payoff-dominant hypotheses, are a result of how social influence predicts equilibrium selection for rational payoff maximizers in our game. The hypotheses for the affiliation stage of the experimental game are:

Hypothesis 1a. *(Identity-dominant affiliation) The higher level of conflict the higher the tendency to propose connections only to the in-group.*

Hypothesis 1b. *(Payoff-dominant affiliation) The level of conflict does not affect the tendency to propose connections to the in-group (the same amount of links to in-group and out-group, adjusted for group size).*

The affiliation hypothesis argue that the probability of linking with one's in-group or out-group is the same if subjects aim to maximize payoffs. However, if subjects rather strengthen their social identity, it is more likely to be connected to one's in-group. Therefore, integration between identities is predicted for all treatments by Hypothesis 1b, and segregation is predicted for treatments with conflicting preferences by Hypothesis 1a.

Hypothesis 2a. *(Identity-dominant behavior) The higher the level of conflict the more likely subjects will adopt the behavior they prefer as prescribed by their social category.*

Hypothesis 2b. *(Payoff-dominant behavior) The level of conflict will have no effect and subjects will adopt the behavior preferred by the majority.*

The behavior adoption hypotheses state that if identity is more salient than social influence (i.e., payoffs), the treatments with positive level of conflict are more likely to result in satisfactory hybrid action profiles (each subject chooses the behavior she *prefers*). Otherwise, the frustrated specialized action profile will be the outcome (subjects in the majority choose what they prefer and subjects in the minority choose what they do not prefer). Particularly, for the No Conflict treatment the satisfactory specialized outcome is predicted. Consequently, we use this treatment as our baseline condition.

Finally, we derive point predictions from our theory in relation to our characterization of equilibrium and the selection criteria modeled. Notice, nevertheless, that as argued by Camerer (2003), it is unlikely that equilibrium is reached instantaneously in one-shot games. It has been pointed along the extensive experimental research on rational behavior that the idea of instant equilibration is so unnatural that perhaps an equilibrium should not be thought of as a prediction which is vulnerable to falsification at all. A more useful perspective should be to perceive equilibrium predictions as the limiting outcome of an unspecified learning process that unfolds over time. This means that we could expect to observe learning from the repetition of the interactions in the experiment. In this view, equilibrium is the end of the story of how strategic thinking, optimization, and equilibration (or learning) work, not the beginning (one-shot) or the middle (equilibration). The following are the hypotheses on equilibrium derived from our game theoretic model:

Hypothesis 3. *(Subgame Perfection) The higher the number of one-shot interactions subjects are part of, the more likely the difference between links proposed and links formed will be reduced.*

This prediction is derived for the affiliation stage of our network game from the backward induction process. Finally, the hypothesis on pairwise stability is derived from our modeling of equilibrium selection:

Hypothesis 4. *(Pairwise stability) The higher the number of one-shot* *interactions subjects are part of, the more likely subjects choosing the same action will be neighbors.*

From these previous hypotheses we can state that if learning is manifested along the repeated interactions, subjects choosing the same behavior are more likely to be connected, regardless of whether identities or social influence motivate their behavior. Specifically for the payoff dominant strategies networks will be completely connected into a single component, so that the efficient configuration will emerge. Otherwise, the segregated configuration where players are separated into social categories should be observed. Nonetheless, whether it is one or two components, these hypotheses predict that networks will tend to be more dense along time, leading towards the pairwise stable predicted configurations.

1.5.3 Experimental procedures, data and methods

All subjects in our experiment were students from the campus of social sciences of the University of Valencia (Spain). Subjects were recruited through online recruitment systems. In total 120 subjects participated in three sessions, one for each treatment (No, Low and High Conflict). There were 30, 45 and 45 participants in each session, respectively. Each session lasted between 90 and 120 minutes and no one participated in more than one session. On average everyone earned 16.5 euros, including a show-up fee of 5 euros.

To conclude this section, we describe the measures we use to test the hypotheses presented above, and the way we developed our analytical strategy. Recall that in reference to a subject, others either belong to her in-group, when they share her identity, or to her out-group, when identities are different.

In-group favoritism. To assess a subject's favoritism to propose connections to the in-group rather than the out-group, the number of proposals sent by a subject to the in-group was divided by the subject's total number of proposals sent. In-group favoritism could range from a maximum of 1 where all proposals were sent to the in-group to a minimum of 0 where all proposals were sent to the out-group. A value of 0.5 denoted equal preferences for sending proposals to both the in-group and the out-group.

Reciprocation. Reciprocation was operationalized as a subject's number of reciprocated proposals (i.e., realized connections) divided by a subject's total number of proposals, regardless of group membership. Reciprocation had a maximum of 1 (0) when all proposals were reciprocated (rejected), and hence no coordination problem occurred in the affliation stage.

Pairwise stability. Pairwise stability was measured with a subject's number of realized connections with in-group members as compared to the total possible connections with this group. That is, the number of in-group members minus the subject. Subjects in the No Conflict condition could realize up to 14 in-group connections, subjects in the Low Conflict condition could realize up to 11 (majority) or 2 (minority) connections, and subjects in the High Conflict condition could realize up to 7 (majority) or 6 (minority) connections. Again, a value of 1 expressed maximum pairwise stability. That is, a subject sent proposals to all of her in-group members of which all proposals were reciprocated, resulting in the subject's connection with every in-group member. Note that this measure disregarded activities with the out-group.

Analytical strategy. The data structure at hand did not permit standard ordinary least square regression modeling. Standard regression models base on the assumption that observations are measured independently from one another. This independence assumption was violated in our data: The experiment included 120 subjects who each played 20 one-shot interactions, so that a total of 2,400 interactions (Level 1) were nested within clusters of 120 subjects (Level 2). Interactions belonging to the same subject could not be assumed to occur independently from one another, as different subjects likely followed varying behavioral tendencies. For example, throughout all interactions, some subjects may have systematically favored in-group members more than may have other subjects.

Multilevel regression modeling is a methodology for the analysis of complex data patterns with a focus on nesting (Snijders and Bosker, 2012). Such models allow variability at multiple levels of observations, namely variability between interactions (Level 1) and variability between subjects (Level 2). While the interpretation of these models is comparable to standard regression models, they additionally assume the intercept (and sometimes the slope) to be randomly varied for each of the 120 subjects. These models, in the following referred to as mixed-models, allowed subjects to differ in their general behavior with regard to in-group favoritism, reciprocation and pairwise stability. Three separate models were run for in-group favoritism, reciprocation and pairwise stability.

1.6 Results

Our experimental study assesses the interplay between individual preferences and social influence by varying the level of conflict in preferences in a network game with strategic complementarities. In this section we describe our main findings beginning with a descriptive discussion of the behavior of the participants. The data show that nearly all choices corresponded with the subjects' preference. We observed that 99.3 percent of the decisions on behavior adoption where such that the prescribed behavior for the social categories was selected. For the affiliation criteria it was found that 99.4 percent of the connections were formed between subjects choosing the same behavior. Table 1.3 presents an overview of the proposals sent and reciprocated (i.e., the realized connections) for the different experiment conditions and groups. In-group favoritism and reciprocation were most prominent in the No Conflict condition. Stronger in-group favoritism related to increased reciprocation (Pearson's correlation coefficient: $r = .69, p < .001$), which in turn was associated with greater pairwise stability ($r = .69, p < .001$).

Hypothesis 1a (Identity-dominant affiliation) expected that higher level of conflict would lead to greater favoritism for in-group proposals. The alternative *Hypothesis 1b (Payoff-dominant affiliation)* stated that no such effect would occur. Table 1.4 presents the results from the mixed-effects regression models. The constant of 0.90 indicates that in-group favoritism was generally high: put aside all other variables (experimental conditions, group membership and development over periods), it could be predicted that subjects send proposals to members from their own group in 90 percent of the cases. According to the negative and significant parameter estimate in Model A, subjects in the Low Conflict condition showed less in-group favoritism than subjects in the No Conflict condition (which served as the reference category). Subjects in the High Conflict condition did not differ significantly in their favoritism from the

		Experimental condition					
		No Conflict		Low Conflict		High Conflict	
	Group	M	SD	M	SD	M	SD
Proposals [a]							
to in-group	Majority	12.87	2.47	9.82	1.95	6.50	1.27
	Minority	n/a	n/a	1.96	0.24	5.97	0.17
to out-group	Majority	n/a	n/a	0.21	0.59	0.28	0.92
	Minority	n/a	n/a	1.22	2.67	0.11	0.48
Connections [a]							
to in-group	Majority	11.95	2.90	8.79	2.17	6.06	1.41
	Minority	n/a	n/a	1.92	0.31	5.95	0.25
to out-group	Majority	n/a	n/a	0.04	0.21	0.00	0.00
	Minority	n/a	n/a	0.14	0.46	0.00	0.00
In-group favoritism [b]							
to in-group	Majority	1.00	0.00	0.98	0.07	0.95	0.15
	Minority	n/a	n/a	0.83	0.30	0.99	0.06
Reciprocation [b]							
to in-group	Majority	0.92	0.12	0.87	0.12	0.89	0.18
	Minority	n/a	n/a	0.84	0.27	0.98	0.07
Pairwise stability [b]							
to in-group	Majority	0.85	0.21	0.80	0.20	0.87	0.20
	Minority	n/a	n/a	0.96	0.15	0.99	0.04

Note:[a] Means and standard deviations for proposals and connections represent absolute numbers.

[b] Means and standard deviations for in-group favoritism, reciprocation and pairwise stability represent relative shares (percentages).

Table 1.3: Subjects' proposals and connections within and between groups (across all periods).

No Conflict group. This suggests that in-group favoritism was greater in the High Conflict group than in the Low Conflict group, supporting *Hypothesis 1a* over *Hypothesis 1b*.

	Model A In-group favoritism		Model B Reciprocation		Model C Pairwise stability	
	B	SE	B	SE	B	SE
No Conflict (ref.)						
Low Conflict	-0.04*	(0.02)	−0.06*	(0.02)	-0.05	(0.03)
High Conflict	-0.01	(0.02)	−0.01	(0.02)	-0.01	(0.03)
Period	0.01***	(0.00)	0.03***	(0.00)	0.04***	(0.00)
Period squared	-0.000***	(0.00)	−0.001***	(0.00)	−0.001***	(0.00)
Minority (ref.)						
Majority	0.04	(0.02)	−0.04*	(0.02)	−0.14***	(0.02)
Constant	0.90***	(0.03)	0.79***	(0.02)	0.76***	(0.03)
$N_{observations}$	2,399		2,399		2,399	
$N_{individuals}$	120		120		120	
$Var_{observations}$	0.01	0.00	0.01	0.00	0.01	0.00
$Var_{individuals}$	0.01	0.00	0.01	0.00	0.01	0.00
Log likelihood	2,361.30		1,783.91		1,489.50	

Note:Unstandardized coefficients. Standard errors in parentheses. $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$.

Table 1.4: Mixed-effects regression models on favoritism, reciprocation and connectivity

Hypotheses on behavior adoption stated that if subjects were more influenced by their identities the higher the level of conflict *Hypothesis 2a (Identity-dominant behavior)* they were more likely to behave as prescribed for their social category (i.e., according to their individual preference). Alternatively *Hypothesis 2b (Payoff-dominant behavior)* expected subjects to be more influenced by their social context choosing the behavior prescribed for the majority. As mentioned above, 99.3 percent of the choices corresponded to the behavior prescribed for each subject's individual preference, so that there is essentially no variation between the choices across time, subjects identity or experimental condition. Thus, the evidence suggests that regardless of the level of conflict, subjects' behavior is influenced by identities above social pressure. To summarize the findings regarding affiliation criteria and behavior adoption we present the following result:

Result 1. *In the presence of conflicting preferences, individual identities are more salient than social influence. Therefore, segregation arises between social categories.*

Hypothesis 3 (Subgame perfection) expected learning and thus increases of reciprocation with higher number of one-shot interactions, in the following referred to as period. In support of this, the positive and significant parameter estimate for period in Model B shows that reciprocation increased by 0.03 percent points with every additional interaction. This effect summed up to a total gain in 60 percent points over the whole experiment of 20 rounds. That is, identity is more salient than social influence as a behavioral criterion. By pursuing the prescribed behavior for their social category, subjects segregate. In consequence, the conflicting aspect of the interaction is put aside. The two components in the network appear as if they were two isolated populations. Once this takes place and subjects end up in a network such that those around them share their same identity (in-group), then social influence takes a relevant role again. Subjects start behaving more and more in accordance with the predictions of social influence aiming to connect with all those around them. The effect that experience and learning brings is that subjects end up decreasing the gap between the connections they propose and the connections they form, maximizing the complementarities of coordinating with their neighbors.

Similarly to the latter hypothesis, *Hypothesis 4 (Pairwise stability)* stated an increase in connections within groups with an increasing number of one-shot interactions. That is, not only subjects will coordinate more along time so that the links proposed are formed. But also, subjects will tend to form more links along time. Also supporting this assumption, the positive and significant parameter estimate for period in Model C shows that pairwise stability increased by 0.04 percent points with every additional interaction. It was reasonable to assume that the learning curve for reciprocation and pairwise stability increased steeply at the beginning and flattened out toward very high numbers of interactions, e.g. because a near-maximum had been reached in earlier interactions. The small but significant squared effects for period show indeed that both reciprocation and pairwise stability did not increase significantly anymore in later experiment periods, namely after period 10, suggesting a curvilinear learning effect. These findings are illustrated in Figure 1.8 and summarized in the next result:

Result 2. *In the presence of conflicting preferences, when segregation* *arises between social categories, subjects aim to maximize the benefits of social influence from those around them through denser networks.*

Additional tests showed that there was a learning effect in all experiment conditions, further supporting our assumptions. The learning curve was steepest in the No Conflict condition, but flattest in the High Conflict condition. The predictive margins for the different conditions are plotted in Figure 1.9.

Besides differences between experimental conditions and learning over periods, the regression models yielded interesting findings with regard to group membership. As presented by the negative and significant parameter estimate in Model C, subjects in the majority group reached less pairwise stability than those in the minority group. This effect occurred net of the different experimental conditions. Figure 1.10 shows that the difference between majority and minority group persisted throughout the entire period of the experiment. However, differences became smaller toward high numbers of one-shot interactions, which was mainly due to the learning effect in the majority group. While on average subjects in the minority reached maximum pairwise stability of 1 in period 4, subjects in the majority reached their maximum of 0.95 only in period 20. This last finding is presented in the next result:

Result 3. *In the presence of conflicting preferences, when segregation arises between social categories, being the minority facilitates coordina-*

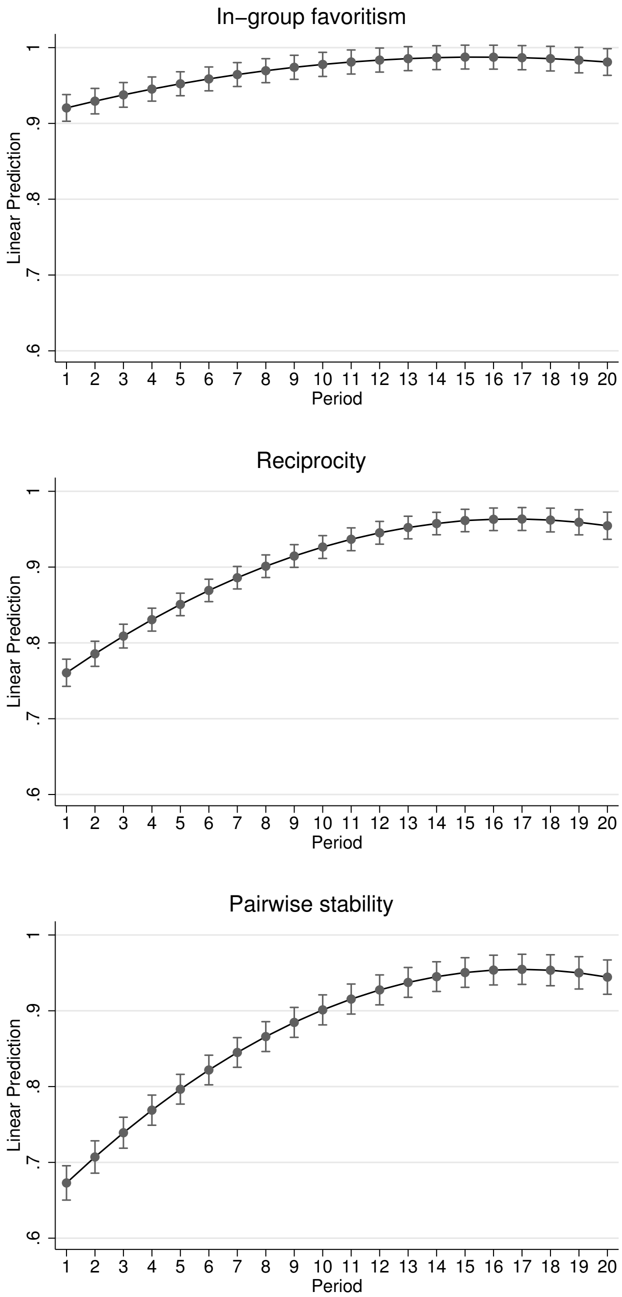

Figure 1.8: Predictive margins by period.

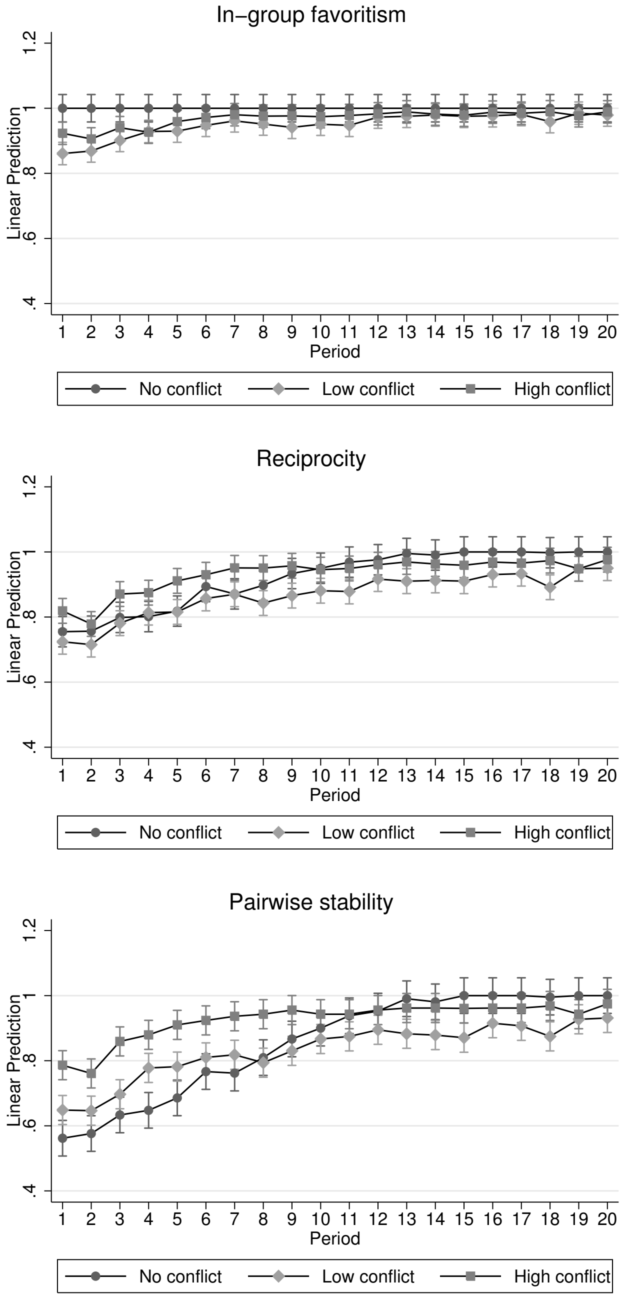

Figure 1.9: Predictive margins by experiment condition and period.

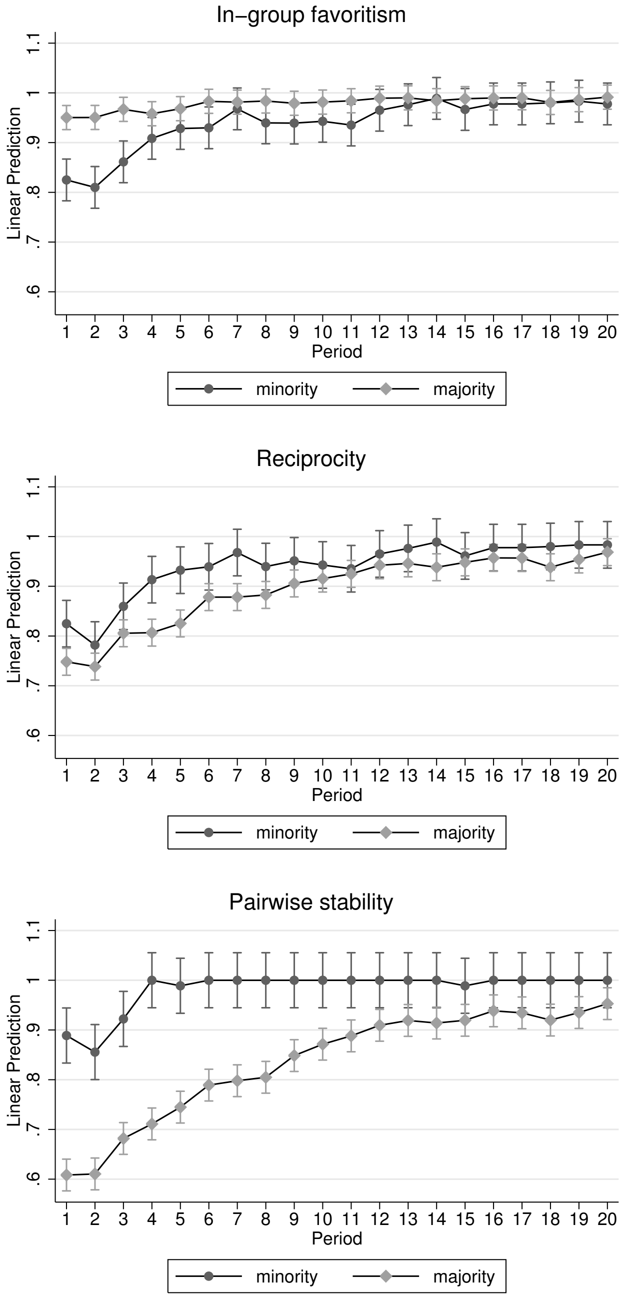

Figure 1.10: Predictive margins by group and period.

tion and stability. Majority groups find it harder to reach affiliation consensus, which is not the case when all subjects have aligned preferences in the population.

1.7 Discussion

In this article we have argued that in social interactions with strategic complementarities the interplay between individual preferences and social influence can decisively affect outcomes of how people relate to each other and what choices they make in terms of their behavior. To elaborate this argument, we proposed a model in which actors have conflicting preferences about the behavior they want to choose but are interested in coordinating in the same behavior with more than less of those around them. Following research on identity theory (Akerlof and Kranton, 2010) we characterized identities as the result of belonging to social categories. Each social category has a prescribed behavior which represent an actor's individual preference. Following research on social influence in network interactions (Jackson, 2009; Hernández et al., 2013) we characterized in which way the behavior of others influences our decisions. To analyze the interaction of these forces that affect our decision-making process we developed a game theoretic model of network interactions with heterogeneous populations (i.e., people belonging to different social categories) and empirically tested the predictions of our theory by means of an experimental design that allowed us to control the social context, and thus the interplay between individual preferences and social influence. We first comment on our theoretical results from the model and then on the empirical findings from our experiment.

Our model, which is an extension of the work by (Hernández et al., 2013), indicated that the choices an actor makes about what behavior to adopt depend on her identity and the influence of others around her. An actor wants to coordinate with the highest number of neighbors making the same choice and prefers coordination on the action prescribed for her identity. As a consequence, the level of social influence needed to choose what we like is necessarily lower than the pressure we need from those around us to behave in a different way. However, this result allows for multiple outcomes depending on where in the network the influence in exerted. It is possible that all actors behave in the same way, so that the network is specialized, or that actors behave in different ways, so that the network is hybrid. Moreover, it is also possible that actors in specialized networks are all from the same social category and are all choosing what they like, so that the network is satisfactory, or there are actors from both social categories, so that some of them are not following the prescribed behavior for their identity and are frustrated.

Given the multiplicity of equilibrium outcomes that arise from games with strategic complementarities we characterized different equilibrium selection criteria. On one hand we used an adapted version of pairwise stability (Jackson and Wolinsky, 1996). If actors can coordinate bilaterally the creation of a new link, in case they are not connected, or if individually they can eliminate any relationship that is not beneficial, then only very particular network structures can result. Pairwise stable configurations would be those in which every actor is connected to all other actors who are choosing the same as her, and every actor is connected only to those choosing the same as her. This means that at the network level, the only pairwise stable configurations are either a completely connected network where every actor is behaving in the same way, or a network separated into two completely intra-connected networks, where actors in each component behave the same but not between components. Finally, we ranked the social efficiency of the resulting networks and found that the network where all actors are connected and their behavior is the same is the one that gives the highest social benefit, as long as the behavior chosen is that of the majority. This, regardless of the social composition of the population.

Using our example of the acquisition of technologies, what our model shows is that people who prefer one technology over the other will only choose what they dislike if the social pressure from those around them is stronger than the support they get to choose what is prescribed for their identity. People who like MacOS need much more pressure to buy Windows than to buy MacOS. This result is very important for threshold models because it points to the way identities and social influence interact in the adoption of different behaviors. Our model then follows to show that if people can also select the relationships they want to maintain or eliminate, the resulting network configurations are those in which MacOS users have no relation with Windows users and vice versa. Moreover, it will be likely that if actors can coordinate by pairs on what relationships to maintain, all MacOS users are linked together and all Windows users are linked together. This because individually each actor can draw out from her interactions the strategic complementarities of relating with others whose choices are compatible to theirs. As a consequence of this, the most beneficial outcome in social terms is when all actors are achieving such complementarities from all others in the population (i.e., the network is completely connected), they are all coordinating in the same behavior (i.e., the network is specialized), and the behavior is the one preferred by the majority. Thus, if MacOS users are a majority, society is at its best when all users are MacOS users even if some of them prefer Windows.

To test our theory we designed an experimental study in which we varied the composition of the population for three conditions: No Conflict, Low Conflict and High Conflict. In this way, we could assess what role individual identities and social influence play when they are interacting together but their intensity is varied. Our main empirical findings suggest that when there are different social categories, so that there are conflicting preferences about what behavior to adopt, individual identities are more salient than social influence. Therefore, networks segregate into two components. Each component has the characteristics of a satisfactory specialized network with a homogeneous population. That is, all subjects in a component belong to the same identity, they choose the behavior they prefer given their identity, and only connect with others who belong to their social category. This first result reinforces the categorization argument of identity theory showing how identities can be so strong that are used to help focalize equilibrium selection. However, the strength of individual preferences leads to two undesirable situations. In terms of relational structures, segregation between social categories is dominant. In terms of social outcomes, inefficiency is pervasive. Thus, the same force that helps individuals reduce risk and relate to others hurts society in an important way.

As a consequence, the outcome dominant in payoffs, the one that is most efficient from the societal perspective is not achievable. However, this is not because individuals are not aware of the complementarities that they could exert from relating to more than to less neighbors but because of the presence of conflict in preferences. Moreover, our second empirical result states that when there is conflict in individual preferences and segregation arises between social categories, subjects aim to maximize the complementarities from the social interactions with those around them and try to increase their number of relationships to a maximum. Thus, when a conflict between identities and social influence in relation to payoffs is latent, an actor's identity is more salient. However, once segregation emerges, so that identities are not in conflict anymore, actors social influence to each other becomes more salient so that they aim to connect completely within their component. The conflict in preferences makes the payoff dominant structure unreachable but within the segregated configuration leads to the payoff dominant case for such types of networks. This points to the tension between stability and efficiency that has been so relevant and pervasive in network studies (Jackson and Wolinsky, 1996; Jackson, 2009), but introduces the effect of identities in it, bowing that the stable networks emerge because of the interplay between identities and "selective" social influence. That is, only influence from those around me who are like me (in-group).

Our third empirical result is a surprising observation. When there are conflicting preferences and individuals segregate favoring only their in-groups, being in the minority facilitates coordination and stability. So, the minority groups tended to completely connect between them from early stages but the majority failed to do so until the very end of the interactions. Although this could be considered as a consequence of group size, because majorities are larger than minorities the coordination problem is greater, the failing of coordination was not present when the majority was absolute. That is, in the case of No Conflict, when all subjects belong to the majority, and grow size was the largest, they did not show the same limitations in maximizing the complementarities of their social connections by reaching pairwise stable networks. This result complements the existing work on in-group bias in identity theory.

As mentioned before, when identification is experimentally induced (i.e., minimal group paradigm), in-group bias has been significantly observed. (Leonardelli and Brewer, 2001) even observed this for cases where there was a majority and a minority. Our results complement these findings on the literature by showing that in network interactions with conflicting preferences in-group bias is observed but groups in numerical minorities express more bias than those in numerical majorities. Our results go in accordance with what has been empirically found by (Mullen et al., 1992; Otten et al., 1996) who have observed, outside of the lab, bias in group size both when groups are real or artificial.

Some potential limitations of our work warrant further discussion. Compared to other works on identities (Akerlof and Kranton, 2000), we model social categories as fixed while their works have assumed that individuals can choose their individual identity and not only their behavior. Our main aim was to understand the adoption of behavior when given identities and social influence are at play in context of conflicting preferences. Accordingly, we decided to maintain the identity assumptions central to our approach. Fixed social categories are common in research on identities (i.e., race, gender, nationality) and our model can be extended to include variable identities in further research.

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Appendix

1.A Appendix: Subjects' instructions

INSTRUCCIONES

Bienvenido/a. Vas a participar en un experimento económico. Por favor lee atentamente las siguientes instrucciones. Si tienes alguna pregunta, por favor levanta la mano y uno de los experimentalistas se acercará a ti para responder a tus preguntas de manera individualizada. Durante el experimento no está permitido comunicarte con otros participantes, ni utilizar tu teléfono móvil, ni realizar ninguna otra tarea con el ordenador que no sea la propia realización del experimento.

En este experimento vas a acumular puntos, que serán convertidos a euros. El número de puntos que obtengas depende de tus propias decisiones y de las decisiones de los demás participantes.

Este experimento consta de 25 rondas, de las cuales las primeras 5 serán de prueba. Al comienzo de cada ronda todos los participantes son aleatoriamente divididos en grupos de 15 personas, identificadas del 1 al 15 (es decir, 1, 2, 3, … , 15). El ordenador asigna de manera aleatoria un símbolo a cada participante: *círculo* o *cuadrado.* El símbolo de cada participante se mantendrá constante a lo largo de todo el experimento, pero la composición de los grupos y la numeración cambiarán de manera aleatoria en cada ronda. Todos los participantes conocerán la numeración y símbolos de todos los miembros de su grupo, pero no su identidad.

En cada ronda tomarás dos decisiones, y constará de 4 fases:

1. Elegirás de los 14 miembros de tu grupo a cuales quieres proponer una conexión.
2. Serás informado de las proposiciones de todos los miembros del grupo y de la red que se ha formado. Una conexión se forma si ambos participantes han propuesto dicha conexión.
3. Elegirás una acción: *arriba* o *abajo*.
4. Serás informado de las decisiones que se hayan tomado en esa ronda y de los puntos que hayas obtenido.

Tus elecciones, conexión y acción, así como la de los demás participantes de tu grupo, determinan el total de puntos que puedes obtener en cada ronda. **Cada proposición de una conexión, incluso si no se llega a formar, tiene un coste de 2 puntos**. Recibes puntos por cada participante al que estés conectado que elija la misma acción que tú. Dicho número de puntos depende de la acción elegida y de tu símbolo:

Eres ***círculo:***

- Si eliges ***arriba*** recibes **6 puntos por cada** coordinación con tus conexiones**.**
- Si eliges ***abajo*** recibes **4 puntos por cada** coordinación con tus conexiones.

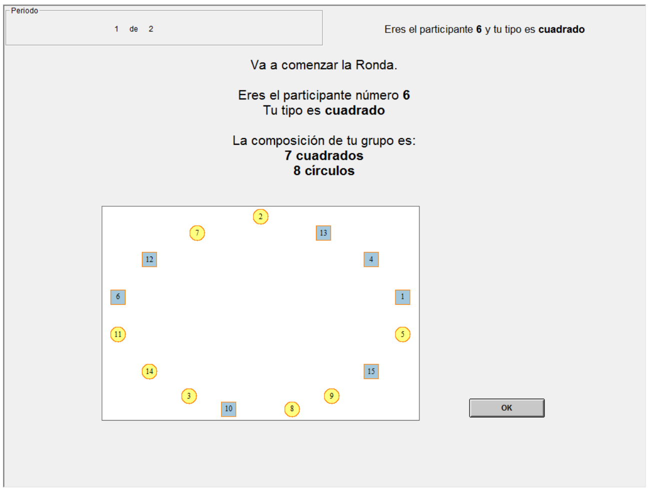
Eres ***cuadrado:***

- Si eliges ***abajo*** recibes **6 puntos por cada** coordinación con tus conexiones**.**
- Si eliges ***arriba*** recibes **4 puntos por cada** coordinación con tus conexiones.

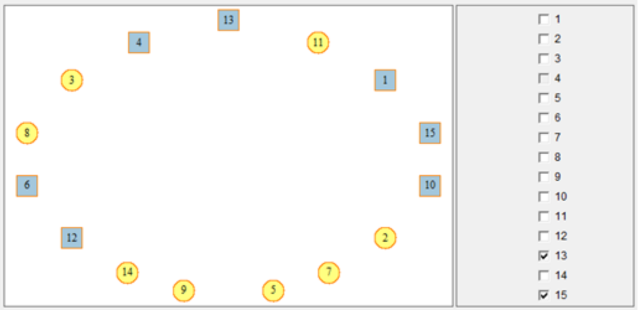
A continuación tienes las instrucciones detalladas de cada fase junto con ejemplos (ten en cuenta que los números, tipos, redes y acciones son a modo de ejemplo y no tienen por qué producirse en el transcurso del experimento):

Inicio de Periodo:

Al inicio de cada periodo conocerás tu número y tipo, así como los de todos los miembros de tu grupo. Tu tipo será el mismo a lo largo de todo el experimento, no así tu número ni la composición de tu grupo.

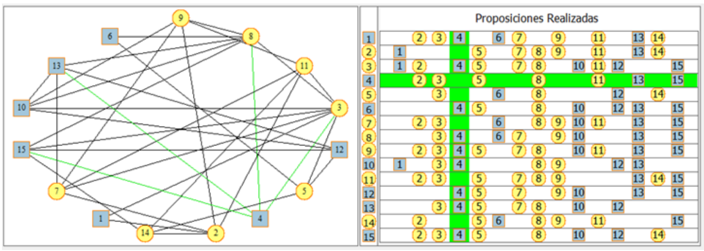


Fase 1 - Propuestas:



La primera decisión que debes tomar es a qué miembros de tu grupo quieres proponer una conexión. Para ello debes marcar la casilla al lado de su número en la lista de la derecha. En el ejemplo de arriba, les propones conexión a los participantes 13 y 15.

Fase 2 - Conexiones:



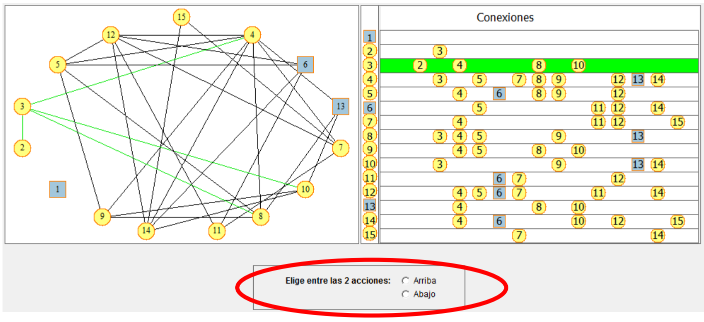
Una vez todos los participantes han realizado sus propuestas verás la red formada. Una conexión se forma cuando los dos participantes la proponen mutuamente. En el dibujo de la red verás tus conexiones resaltadas en color verde, y en la tabla verás también en color verde la fila correspondiente a las propuestas que tú has realizado y la columna correspondiente a las propuestas que has recibido.

En el ejemplo de arriba eres el participante **4**. Has propuesto conexión a los siguientes participantes:

- De tipo círculo: 2, **3**, 5, **8** y 11.
- De tipo cuadrado: **13** y **15**.

A ti te han propuesto conexión los participantes 1, **3**, 6, **8**, 9, 10, 12, **13** y **15**. Por tanto tienes una conexión con 3, 8, 13 y 15, que son los participantes a los que propusiste conexión y a su vez te la propusieron a ti. Es decir, las conexiones finales son la intersección entre las proposiciones realizadas y recibidas.

Fase 3 - Acción:



Una vez formada la red, en la siguiente fase deberás decidir tu acción, *arriba* o *abajo*. Ahora seguirás viendo la red formada, pero en la tabla de la derecha verás únicamente las conexiones de cada participante. Recuerda que lo que ganarás será:

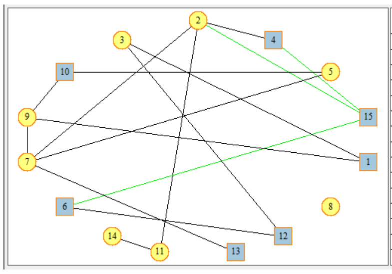
Si eres **círculo** ○:

- 6 puntos por cada coordinación en ↑
- 4 puntos por cada coordinación en ↓

Si eres **cuadrado** □:

- 6 puntos por cada coordinación en ↓
- 4 puntos por cada coordinación en ↑

Fase 4 - Resumen:



Participante	Proposición Realizada	Proposición Recibida	Acción	Coordinación	Puntos
1			▲		0
2	■	■	▼	■	4
3		■	▲		0
4	■	■	▼	■	4
5			▼		0
6	■	■	▲		-2
7			▲		0
8		■	▲		0
9		■	▼		0
10			▲		0
11			▲		0
12	■		▲		-2
13			▼		0
14		■	▲		0
15			▼		6

En la última fase verás un resumen de lo ocurrido en la ronda: las proposiciones que realizaste, las que te hicieron a ti, la acción elegida por cada participante, si os coordinasteis o no y los puntos obtenidos por la interacción con cada uno.

En el ejemplo de arriba eres el participante 15 de tipo cuadrado, y has elegido la acción abajo. Veamos detalladamente tu interacción con algunos participantes:

- Con 2: Le propusiste y te propuso, y os habéis coordinado. Ganas: 6 – (coste de la propuesta) = 6 – 2 = 4
- Con 6: Hay conexión pero no coordinación. Pagas el coste de la propuesta (2).
- Con 12: Le propusiste y él no. Pagas el coste de la propuesta.
- Con 14: Él te propuso y tú no. Él paga el coste de la propuesta, tú no.
- Con 15: "Contigo mismo" siempre te coordinas.

A continuación tienes una tabla para ayudarte a calcular los puntos totales que puedes obtener por coordinarte con tus vecinos (los costes de las propuestas de conexión **NO** están descontados, recuerda que cada proposición cuesta **2** puntos):

Si eres ○ y elijes △
Si eres □ y elijes ▽

Coordinaciones	Puntos
0	6
1	12
2	18
3	24
4	30
5	36
6	42
7	48
8	54
9	60
10	66
11	72
12	78
13	84
14	90

Si eres ○ y elijes ▽
Si eres □ y elijes △

Coordinaciones	Puntos
0	4
1	8
2	12
3	16
4	20
5	24
6	28
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Chapter 2

Analysis of Strategies to Promote Cooperation in Distributed Service Discovery

Abstract:

New systems can be designed, developed, and managed as societies of agents that interact with each other by offering and providing services. These systems can be viewed as complex networks where nodes are bounded rational agents. In order to deal with complex goals, they require the cooperation of the other agents to be able to locate the required services. In this pa- per, we present a theoretical model that formalizes

the interactions among agents in a search process. We present a repeated game model where the actions that are involved in the search process have an associated cost. Also, if the task arrives to an agent that can perform it, there is a reward for agents that collaborated by forwarding queries. We propose a strategy that is based on random-walks, and we study under what conditions the strategy is a Nash Equilibrium. We performed several experiments in order to validate the model and the strategy and to analyze which network structures are more appropriate to promote cooperation.

2.1 Introduction

Social computing has emerged as a discipline in different fields such as Economics, Psychology, and Computer Science. Computing can be seen as a social activity rather than as an individual one. New systems are designed, developed, and managed as societies of independent entities or agents that offer services and interact with each other by providing and consuming these services (Luck et al., 2005). These systems and applications can be formally represented through formal models from the field of Complex Networks (Newman, 2011). This area provides a sound theoretical basis for the development of models that help us to reason about how distributed systems are organized (Kleinberg, 2006).

Complex Network models have been used in different contexts such as social networks (collaboration, music, religious networks), economic networks (trade, tourism, employment networks), Internet (structure and traffic networks), bio-molecular networks, and computer science networks among others (Costa et al., 2011; Newman, 2003).

In systems of this kind, one of the challenges is the design of efficient search strategies to be able to locate the resources or services required by entities in order to deal with complex goals (Ban et al., 2010; Newman, 2003; Del Val et al., 2014). Taking into account the autonomy of the entities that participate in the search process, three levels of search decentralization can be considered. We consider that at the first level the search process is centralized when there is a common protocol that is adopted by all the entities of the system and this protocol dictates the actions that must be followed (i.e., the protocol specifies the entity that starts the process, the sequence of participation of entities, and the target). At the second level this protocol can be relaxed. The entities adopt that protocol, and, therefore, they carry out the same set of actions, but the search path (i.e., the sequence of entities that participate in the search process) is not specified. At the third level, a decentralized search can be considered when there is a protocol adopted by all the entities that specifies the set of available actions. However, these entities can decide whether or not they are going to follow the protocol. It would not be desirable that to impose the same behavior on all the nodes if it takes away their individual choice, (i.e., it would be desirable that all the nodes would follow the protocol willingly). Therefore, we have looked for a concept of stability within the strategies of the entities of the system. This concept, which comes from Game Theory, is known as Nash Equilibrium.

As an application scenario, we consider a P2P system that is modeled as a multi-agent system. Agents act on behalf of users playing the role of a service provider or service consumer. Agents that play the role of service consumers should be able to locate services, make contracts agreements, and receive and present results (Sierra et al., 2011). Agents that play the role of service providers should be able to manage the access to services and ensure that contracts are fulfilled. By considering the system as a network, it is assumed that all the information is distributed among the agents. Since agents only have a local view of the network, the collaboration of other agents is required in order to reach the target. During a search process, agents can carry out a set of actions: create a task that must be performed by a qualified agent, forward the task to one or several neighbors if they do not know how to solve the task, or perform the task if they can provide the required service. The cooperation of agents forwarding queries plays a critical role in the success of the search process (Del Val et al., 2013). This action facilitates the location of a resource based on local knowledge. However, in our scenario, this action has an associated cost and agents are free to decide whether or not the forwarding action is profitable to them based on its cost and the expected reward.

In this paper, we propose a model to formally describe the distributed search for services in a network as a game. Specifically, we use the repeated games framework to model both the process that a task follows through the network and the global task-solving process. In the former, each period is a decision stage for the agent who is in possession of the task. In the latter, a project is generated in each period and randomly assigned to an agent in the network.

Our intention is to analyze the relationship between the cost of forwarding the task and the reward that agents obtain later when the task is solved in order to guarantee that cooperation is a stable behavior in the game. We called this reward α. We establish a bound for the total length of the search process total length using Mean First Passage Time (MFPT), which is the average number of steps necessary to go from an agent i to another agent j in the same network. Therefore, the structure of the network also characterizes α through the MFPT, and, consequently, the network structure influences the agents' behavior. In order to verify this, we ran simulations to contrast the possible differences among network structures. The results show that the structure of the network has a significant influence on the emergence of cooperation. The structure that offers the best results is the Scale-Free structure since its diameter is closer to the limit of steps in the search process than the other network structures.

The paper is organized as follows. Section 2.3 presents a repeated game model to formalize the search process of services in agent networks. In Section 2.4, some strategies that agents can follow in the repeated game are analyzed in order to determine whether or not they are at a Nash Equilibrium. Section 2.5 describes several experiments we performed to empirically validate the theoretical results in different network structures as well as to analyze the influence of the network structure and to determine which structure facilitates the emergence of cooperation in the proposed repeated game. Section 2.2 presents other works related to cooperation emergence in distributed environments. Finally, Section 2.6 presents the conclusions.

2.2 Related Work

Random-walk strategies have been presented as an alternative search strategy to *flooding* strategies (Chawathe et al., 2003; Yang and Garcia-Molina, 2002; Lopes and Botelho, 2008) since they reduce the traffic in the system and provide better results (Lv et al., 2002; Zhong, 2006).

A random-walk search algorithm selects a neighbor randomly each time to forward the message to (Gkantsidis et al., 2006). There are many search proposals that navigate networks using random-walk since they do no require specific knowledge and can be applied in several domains. Some of these works have introduced modifications such as using random-walk from multiples sources (Zhou, 2008; Pu and Pei, 2010) or adding information about routes (Lee et al., 2009; Backstrom and Leskovec, 2011; Cajueiro, 2009) in attempt to improve the search efficiency. The influence of network structural properties on random-walk has also been studied. For instance, some of the properties that have been evaluated are: the mean first-passage time (MFPT) from one node to another (Zhang et al., 2011; Roberts and Haynes, 2011; Tejedor et al., 2011), how the structural heterogeneity affects the nature of the diffusive and relaxation dynamics of the random walk (Noh and Rieger, 2004), and the biased random-walk process based on preferential transition probability (Fronczak and Fronczak, 2009).

One of the common assumptions in network search is that all the agents have homogeneous behavior and that all of them are going to cooperate by forwarding messages. However, this does not correspond with real scenarios. In real large-scale networks, decisions are often made by each agent independently, based on that agent's preferences or objectives. Game Theoretic models are well suited to explain these scenarios

(MacKenzie and DaSilva, 2006). Game theory studies the interaction of autonomous agents that make their own decisions while trying to optimize their goals. Game Theory provides a suite of tools that may be effectively used in modeling interactions among agents with different behaviors (Srivastava et al., 2005).

There are works in the area of Game Theory that focus on the routing problem in networks where there are selfish agents. Specifically, this problem has been studied in wireless and ad-hoc networks (Srivastava et al., 2005; MacKenzie and DaSilva, 2006). Numerous approaches use reputation (Jaramillo and Srikant, 2010) (i.e., techniques based on monitoring the nodes' behavior from a cooperation perspective) or price-based techniques (Janzadeh et al., 2009) (i.e., a node receives a payment for its cooperation in forwarding network messages and also pays other nodes which participate in forwarding its messages) to deal with selfish agents. One of the drawbacks of reputation systems is that nodes whose reputation values are higher than a threshold are treated equally. Therefore, a node can maintain its reputation value just above the threshold to obtain the same benefit as nodes with higher reputation levels. One of the problems of the Price-based techniques is that they are not fair with nodes located in region with low traffic that have few opportunities to earn credit. Li et al. (Li and Shen, 2012) integrate both techniques and propose a game theory model for analyzing the integrated system.

However, this approach does not consider the influence of the underlying structure in the cooperation emergence.

To understand the social behavior of the systems it is important to consider the network structure. There are several works that analyze the influence of the network structure when the agents of the networks do not follow homogeneous behavior. These works study how structural parameters such as clustering or degree distribution affect the emergence and maintenance of cooperative behavior among agents (Pujol et al., 2005; Ohtsuki et al., 2006). Hofmann et al. (Hofmann et al., 2011) present a critical study about the evolution of cooperation in agent societies. The authors conclude that there is a dependence of cooperation on parameters such as network topology, interaction game, state update rules and initial fraction of cooperators.

The proposal presented in this paper analyzes through a game theory model the problem of cooperation emergence in the context of decentralized search. It differs from previous approaches in several ways. First, we considered a game that fits better with the characteristics of decentralized search than other games proposed in the literature that are based on the often studied Prisioner's Dilemma. Second, agents decision about cooperation is based on an utility function that takes into account the network topology properties. Moreover, the utility function also considers a limit in the number of possible steps to reach the target agent.

This feature is important in distributed systems in order to avoid traffic overhead. Third, the strategy that agents follow is based on a search mechanism that is often used in network navigation and does not require specific domain knowledge. Therefore, the model can be easily applied in different search contexts. Finally, in order to promote cooperation, instead of using a reputation or price-based mechanisms, we use a mechanisms based on incentives provided by the system. We formally and experimentally analyze which is the minimum required reward in order to consider the strategy a Nash Equilibrium.

2.3 The Model

Consider a finite set of agents $N = \{1, 2, \ldots, n\}$ that are connected by undirected links in a fixed network represented by the adjacency matrix g. A link between two agents i and j, such that $i, j \in N$, is represented by $g_{ij} = g_{ji} = 1$, where $g_{ij} = 0$ means that i and j are not connected. The set of neighbors of agent i is

$$N_i = \{j | g_{ij} = 1\}$$

For simplicity we assume that $g_{ii} = 0$ so all neighbors in $N_i(g)$ are different from i. The number of neighbors that agent i has (its degree of connection) is denoted by $k_i = |N_i(g)|$, which is the cardinality of the set $N_i(g)$. Alternatively, we use the adjacency matrix to represent the network, which is denoted by A. A link between agents i and j is represented by $A_{ij} = 1$, and by $A_{ij} = 0$ if there is no link.

We consider that each agent has a type (service) $\theta_i \in [0, 1]$ that represents the degree of ability of agent i. Let $\rho \in [0, 1]$ be a task that must be carried out by one of the agents in the network. We assume that there is at least one agent $i \in N$ such that i is suitable to perform the task, which means that, for a fixed ε, its type θ_i is 'similar' to the task ρ, *i.e.*, $|\theta_i - \rho| \leq \varepsilon$.

We define an N-person network game Γ_ρ^∞ that takes place in g. Each agent has a set of actions $A_i = \{\emptyset, 1, 2, \ldots, N_i, \infty\}$, where:

- ∞ means the agent itself does the task
- $\{1, 2, \ldots, N_i\}$ means forwarding the task to one of the agent's N_i neighbors
- $\emptyset$ means doing nothing

In the first period of the game, a task ρ is uniformly assigned to a randomly selected agent. Beginning at stage 1, the task passes through the network stage by stage. At stage $t > 0$ each agent chooses one of the above actions depending on whether or not the task is in the agent's node.

The action perform by an agent where the task is not in the agent's node is considered to be $\emptyset$ or "doing nothing". We associate a null payoff to this action. At stage 1, agent $i(1)$ chooses one action from its action set.

At the initial stage, if the first agent, $j \in \{1, \ldots, N\}$ chooses to do the task itself because its type is ε close to the task ρ, then the game ends. Agent j gets a payoff of $1 - |\rho - \theta_j|$, which depends on its type θ_j and the task ρ. The more similar the type and the task are, the greater the payoff is. The rest of the agents can do any action in their action sets. More specifically, let $c > 0$ be the cost of forwarding the task. If an agent forwards the task, at some point it, the agent may earn a payoff $\alpha > c$ if the task ends successfully. If an agent chooses the action $\emptyset$, the payoff is 0 if the agent did not forward any task in a previous period or the agent did forward the task but the task ended unsuccessfully (i.e.,nobody chose ∞).

Formally:

$$u_i^t(a_i, a_{-i}; j) = \begin{cases} 1 - |\rho - \theta_i| & \text{if } a_i^t = \infty \\ -c & \text{if } a_i^t \in \{1, \ldots, N_i\} \\ 0 & \text{if } a_i^t = \emptyset \wedge \nexists t' < t : a_i^{t'} \in \{1, \ldots, N_i\} \\ \alpha & \text{if } a_i^t = \emptyset \wedge \exists t' < t : a_i^{t'} \in \{1, \ldots, N_i\} \wedge \exists j \in N : a_j^t = \infty \end{cases}$$

By choosing actions at stage t, agents are informed of actions that are chosen in previous stages of the game. Therefore, let us consider a complete information set-up. Formally, let $H_t, t = 1, \ldots$, be the cartesian product $A \times A$ $t-1$ times, i.e., $H_t = A^{t-1}$, with the common set-theoretic identification $A^0 = \oslash$, and let $H = \cup_{t \geq 0} H_t^{\infty}$. A pure strategy σ^i for agent i is a mapping from H to $A^i, \sigma^i : H \rightarrow A^i$. Obviously, H is a disjoint union of H_t, $t = 1, \ldots, T$ and σ_t^i:$H_t \rightarrow A^i$ as the restriction of σ^i to H_t.

The payoff function of each agent when the game is repeated a certain number of times and when the task starts at any agent is formalized as:

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{t>0} u_i^t(a_i, a_{-i}; j) = \frac{1}{N} \sum_{j=1}^{N} \sum_{t=1}^{\infty} u_i^t(a_i, a_{-i}; j)$$

This induces an order in the payoffs for each strategy σ_i that each agent i chooses given the action profiles σ_{-i}, which allows us to rank them and calculate the Nash Equilibriums of the game.

In order to characterize the set of feasible and individual rational level to define the set of equilibria payoffs of the repeated game (*i.e.*, the equilibrium payoff attained as a consequence of the well-known Folk Theorem) (Fudenberg and Maskin, 1986), we have to establish the min-max level in pure actions. The min-max strategy for agent i is the one that guarantees the highest possible payoff in the action profile that is the worst case scenario for agent i. This is sometimes called the reservation payoff. Formally,

$$\bar{u_i} = \min_{a_{-i}} \max_{a_i} u_i(a_i, a_{-i}), a_i \in A_i, a_{-i} \in A_{-i} \tag{2.1}$$

In our set-up for the one-shot payoff function, the min-max strategy is $\emptyset$ and, therefore, the min-max level is 0. An action profile $(\sigma_1^*, \ldots, \sigma_n^*)$ is a Nash equilibrium in the network game Γ, if and only if

$$u_i(\theta_i, \sigma_1^*, \ldots, \sigma_n^*) \geq u_i(\theta_i, \sigma_1^*, \ldots, \hat{\sigma}_i, \ldots, \sigma_n^*), \quad \forall\, \sigma_i^* \neq \hat{\sigma}_i,\ i \in N,\ \text{and}\ \theta_i \in \Theta.$$

We define the set of feasible payoff vectors as

$$F := \text{conv}\{u(a), a \in \mathcal{A}\}.$$

The set of *strictly individually rational payoff vectors* (relative to the min-max value in pure strategies) is

$$V := \{x = (x_1, \ldots, x_n) \in F \colon x_i > \bar{u_i} \ \ \forall i \in N\}\,.$$

Folk theorems in the context of game theory establish feasible payoffs for repeated games. Each Folk Theorem considers a subclass of games and identifies a set of payoffs that are feasible under an equilibrium strategy profile. Since there are many possible subclasses of games and several concepts of equilibrium, there are many Folk Theorems.

The Folk Theorem states any payoff profile in V can be implemented as a Nash equilibrium payoff if δ is large enough. The intuition behind the Folk Theorem is that any combination of payoffs such that each agent gets at least its min-max payoff is sustainable in a repeated game, provided each agent believes the game will be repeated with high probability. For instance, the punishment imposed on an agent who deviates is that the agent will be held to its min-max payoff for all subsequent rounds of the game. Therefore, the short-term gain obtained by deviating is offset by the loss of payoff in future rounds. Of course, there may be other, less radical (less grim) strategies that also lead to the feasibility of some of those payoffs. The good news from the Folk Theorem is that a wide range of payoffs may be sustainable in equilibrium. The bad news is that, there may exist a multiple number of equilibria.

2.4 Equilibrium strategies

In this section, we study which strategy profiles are a Nash Equilibrium in the game Γ_ρ^∞. Namely, we start defining the *Nobody works* strategy, which basically consists of doing nothing, even in the case that an agent can perform the task. We prove that the *Nobody works* strategy is not a Nash Equilibrium in the game. Then we consider the so-called random-walk strategy. In this strategy, an agent is not able to solve the project, it uniformly and randomly chooses one of its neighbors to forward the task to. We establish the conditions under which the strategy profile every agent plays the random-walk strategy is a Nash Equilibrium. We enrich the model by adding a threshold for the number of times that a task can be forwarded and we also study under which conditions is a Nash Equilibrium.

2.4.1 Nobody works

One possible strategy is the strategy we call *Nobody works*, in which every agent always chooses the action $\emptyset$ and consequently gets a payoff of 0. One of our model's assumptions is that for all possible task ρ there exists an agent that is able to perform it. Let that agent be i, and let its type be θ_i. From our payoff criterion, we can state that in some period t the project will start at agent i and agent i will be able to solve it. In that case, if agent i chooses the ∞ action (doing the project), agent i gets a payoff of $1 - |\rho - \theta_i| > 0$; therefore, the *Nobody works* strategy is not an equilibrium strategy.

2.4.2 Random Walk

In this subsection, we study the case where all agents play a behavioral strategy $\sigma_i^\infty : H^{t-1} \rightarrow \Delta(A_i)$, which leads to the well-known dynamics of “random-walk". We call this behavioral strategy the random-walk strategy.

Let us formally define the random-walk strategy. At each stage t, agent i performs one of the three actions that are possible:

- the $\emptyset$ action if no task arrives.
- the ∞ action if agent i's type θ_i is close to the task ρ_t.
- the forwarding action when the task arrives and agent i cannot solve it, agent i uniformly and randomly chooses one of its neighbors to forward the task to.

This strategy is a “myopic" strategy since agents do not update the expected payoff. Each agent i will uniformly and randomly choose one of its neighbors to continue searching for the agent that can solve the task ρ. Recall that in our game for all task ρ, there exists an agent k^* such that agent i can do the task ρ (*i.e.*, $|\theta_{k*} - \rho| > \varepsilon$). As a consequence of the random-walk strategy we can assert the existence of a finite time $0 \leq \tilde{t} < \infty$ and $k^* \in \{1, \ldots, N\}$ such that $a^{\tilde{t}}_{K*} = \infty$. Therefore, given a task ρ, the achieved payoff for each agent first depends on whether or not agent i was part of the path of searching for the agent that did the task. If agent i did not in the procedure, then agent i gets 0, which is the $\min - \max$ value.

Now, suppose that i is part of the path. Let us define some parameters that take part in the utility function. We refer to the probability of an agent being capable of performing the task as γ_i, and since it is the same for all agents , we simply call it simply γ. P_x^∞ is the probability that the task reaches a specific agent x in the long run, and the previously defined parameters α and c are the reward and the cost of forwarding the task, respectively.

Hence, the utility function of the game Γ_ρ^∞ for agent i is:

$$u_i(\theta_i, \sigma_i, \sigma_{-i}) = P_i^\infty \left(\gamma(1 - |\rho - \theta_i|) + (1-\gamma)(P_{k^*}^\infty(\alpha - c) + (1 - P_{k^*}^\infty)(-c)\right)) \tag{2.2}$$

The following proposition states that the strategy profile in which every agent plays a random-walk strategy is a Nash equilibrium in the game Γ_ρ^∞.

Proposition 2. *The strategy profile* $(\sigma_1^\infty, \ldots, \sigma_n^\infty)$ *is a Nash Equilibrium in the game* Γ_ρ^∞.

Proof. Let i be an agent such that agent i selects a strategy $\sigma_i^\emptyset \neq \sigma_i^\infty$; let t be a time period such that the task ρ arrives to i; let t' be another time period such that $t' \neq t$ and let $|\theta_i - \rho| < \varepsilon$. The strategy $\sigma_i^{\emptyset}$ is formally defined as

$$(\sigma_i^{\emptyset})^t : H^t \rightarrow A_i \begin{cases} (\sigma_i^{\emptyset})^t & = \infty \\ \forall t \neq t', (\sigma_i^{\emptyset})^t & = \emptyset \end{cases} \tag{2.3}$$

When selecting that strategy, if i is able to afford the task agent i does it, and that is the only profit that agent i eventually gets because it never forwards the task. Consequently, agent i's utility function is

$$u_i(\theta_i, \sigma_i^{\emptyset}, \sigma_{-i}^{\infty}) = P_i^{\infty}\left(\gamma(1 - (\rho - \theta_i))\right) \tag{2.4}$$

In order to prove that the strategy profile $(\sigma_1^{\infty}, \ldots, \sigma_n^{\infty})$ is a Nash Equilibrium the utility function described in 2.2 must be greater or equal to the utility function specified in 2.4

$$\begin{aligned} P_i^{\infty}\left(\gamma(1 - (\rho - \theta_i)) + (1-\gamma)(P_{k*}^{\infty}(\alpha - c) + (1 - P_{k*}^{\infty})(-c)\right) &\geq P_i^{\infty}\left(\gamma(1 - (\rho - \theta_i))\right) \\ (1-\gamma)(P_{k*}^{\infty}(\alpha - c) + (1 - P_{k*}^{\infty})(-c)) &\geq 0 \end{aligned}$$

Since $0 < \gamma < 1$, $(1 - \gamma)$ is always positive. Then

$$\begin{aligned} P_{k*}^{\infty}(\alpha - c) + (1 - P_{k*}^{\infty})(-c) &\geq 0 \\ \alpha &\geq \frac{c}{P_{k*}^{\infty}} \end{aligned}$$

By the definition of random-walk dynamics, in the long term, a task ρ will always find the agent k^* that is capable of solving it, so $P_{k*}^{\infty} = 1$. Since $\alpha > c$ by assumption, the strategy profile $(\sigma_1^{\infty}, \ldots, \sigma_n^{\infty})$ is a Nash Equilibrium in the game Γ_{ρ}^{∞}.

□

Now we enrich the model by introducing a "time" condition to solve the task. It makes sense to limit the rewards for efforts to solve or forward the task to a time limit within which the task must be solved (i.e., efforts are only rewarded if the task is solved in a certain number of time periods).

2.4.3 Random-walk strategy with a finite number of steps

An interesting measure for establishing the limit of steps that a task ρ can take to be solved is the Mean First Passage Time (hereafter MFPT). The MFPT between two nodes i and j of a network is defined as the average number of steps to go from i to j in that particular network (Zhang et al., 2011). Therefore, we define the strategy σ_i^{τ} for an agent i, which consists of forwarding the task to a randomly selected neighbor only if it has advanced a number $t_i < \tau$ times, where τ is the average MFPT of the network (which we formally define below).

From equation 34 of (Zhang et al., 2011), we know that the MFPT from any agent to a particular agent j in a network is defined as:

$$\langle T_j \rangle = \frac{1}{1-\pi_j}\sum_{i=1}^{N}\pi_i T_{ij} =$$
$$= \frac{1}{1-\pi_j}\sum_{k=2}^{N}\left(\frac{1}{1-\lambda_k}\psi_{kj}^2\sum_{i=1}^{N}\frac{k_i}{k_j}\right) - \frac{1}{1-\pi_j}\sum_{k=2}^{N}\left(\frac{1}{1-\lambda_k}\psi_{kj}\sqrt{\frac{K}{k_j}}\sum_{i=1}^{N}\psi_{ki}\sqrt{\frac{k_i}{K}}\right)$$

where k_i and k_j are the degree of agents i and j, respectively, ψ_k is the kth eigenvector of S corresponding to the kth eigenvalue λ_k (with $S = D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$, A being the adjacency matrix of the network, D being the diagonal degree matrix of the network, and the eigenvalues being rearranged as $1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_N \geq -1$) and $\pi_j = d_j/K$ (with $K = \sum_{j=1}^{N} d_j$).

It follows from Eq. (6) from the same article that $\sum_{i=1}^{N}\psi_{ki}\sqrt{\frac{k_i}{K}} = \sum_{i=1}^{N}\psi_{ki}\psi_{1i} = 0$. Thus, the second term is equal to 0. So

$$\langle T_j \rangle = \frac{1}{1-\pi_j}\sum_{k=2}^{N}\left(\frac{1}{1-\lambda_k}\psi_{kj}^2\sum_{i=1}^{N}\frac{k_i}{k_j}\right) = \frac{1}{1-\pi_j}\frac{K}{k_j}\sum_{k=2}^{N}\frac{1}{1-\lambda_k}\psi_{kj}^2 \tag{2.5}$$

We define the maximum number of steps that must be taken for every task ρ to be solved as the average $\langle T_j \rangle$ for all $j \in N$, which we denote as

τ. Formally:

$$\tau = \sum_{j=1}^{N} \frac{\langle T_j \rangle}{N} \tag{2.6}$$

We now define a new game Γ_{ρ}^{τ}. In this game, if it takes more than τ steps to solve the task, the game ends and the collaborating agents get no reward. In the following, we explain the equilibrium strategies for the game Γ_{ρ}^{τ}.

Let us define some new parameters that play a role in the new game: the number of steps a task has advanced until it reaches agent i is t_i; $Q_{i,k*}^{\tau-t_i}$ is the probability that the task reaches an agent k^* starting from agent i in $\tau - t_i$ or less steps and $P_{s,i}^{\tau}$ is the probability that the task reaches an agent i starting from agent s in τ or less steps.

In order to formally define $P_{s,i}^{\tau}$, we use the adjacency matrix of the network (denoted as A) and one of its properties which states that the values (i, j) of A^n indicate the number of paths of length n between i and j in that network. $P_{s,i}^{\tau}$ can be defined as the number of paths of length τ or less between s and i divided by the total number of paths with the same length starting at s but ending at any possible agent j of the network. Formally:

$$P^{\tau}_{s,i} = \frac{\sum\limits_{t=1}^{\tau}(A^t)_{si}}{\sum\limits_{j=1}^{N}\left(\sum\limits_{t=1}^{\tau}(A^t)_{sj}\right)} \tag{2.7}$$

Let us define $r_i^{\tau-t_i}$, or simply r_i, as the number of agents that the task can reach starting from i in $\tau - t_i$ or less steps. For this purpose, we use the Reachability matrix, (denoted as R), which is defined as

$$\forall i,j \in N, (R^{\tau-t_i})_{ij} = \begin{cases} 1 & \text{if there exists at least one path between } i \text{ and } j \\ & \text{ of length } \tau - t_i \text{ or less} \\ 0 & \text{otherwise} \end{cases} \tag{2.8}$$

The process for obtaining R from the adjacency matrix is straightforward.

Then, we formally define r_i as

$$r_i = \sum_{j=1}^{N}(R^{\tau-t_i})_{ij} \tag{2.9}$$

To define $Q^{\tau-t_i}_{i,k*}$, we compute the probability that none of the reachable agents for agent i is able to solve the task, which is $(1-\gamma)^{r_i}$. Then $Q^{\tau-t_i}_{i,k*}$ is defined as

$$Q_{i,k*}^{\tau-t_i} = 1 - (1-\gamma)^{r_i} \tag{2.10}$$

Hence, the utility function of the game Γ_ρ^τ for an agent i when all agents play the strategy σ^τ is

$$u_i(\theta_i, \sigma_i^\tau, \sigma_{-i}^\tau) = P_{s,i}^\tau \left(\gamma(1-(\rho-\theta_i)) + (1-\gamma)(Q_{i,k*}^{\tau-t_i}(\alpha-c) + (1-Q_{i,k*}^{\tau-t_i})(-c)\right) \tag{2.11}$$

Now we study a bound for α for which the strategy profile $(\sigma_1^\tau, \ldots, \sigma_n^\tau)$ is a Nash Equilibrium in the game Γ_ρ^τ.

Proposition 3. *If* $\alpha_i \geq \dfrac{c}{1-(1-\gamma)^{r_i}}$, *the strategy profile* $(\sigma_1^\tau, \ldots, \sigma_n^\tau)$ *is a Nash Equilibrium in the game* Γ_ρ^τ.

Proof. The proof proceeds exactly like the proof for Proposition 2 but substituting the proper probabilities $P_{s,i}^\tau$ and $Q_{i,k*}^{\tau-t_i}$. Finally, we have

$$\alpha \geq \frac{c}{Q_{i,k*}^{\tau-t_i}} \tag{2.12}$$

By substituting 2.10 in 2.12, we have

$$\alpha \geq \frac{c}{1-(1-\gamma)^{r_i}} \tag{2.13}$$

The fact that α depends on r_i implies that each agent has its own bound for alpha which depends on the agent's connectivity (α becomes α_i).

□

This means that the network structure has a deep impact on α bounds. In high clustered networks, r_i is high for each agent i, and, consequently, α_i is low. The opposite occurs in low clustered networks (e.g., Erdös-Renyi networks) where α_i is uniform among all agents. In networks with a non-uniform degree distribution (e.g., scale-free networks), average α may be similar to the α for Erdös-Renyi networks, but it varies a lot between hub and terminal agents.

2.5 Experiments

In this section, we validate the proposed mathematical model for service search in different network structures. Specifically, we focus on how the structural parameters of the networks influence the required reward α in order to promote cooperation (i.e., forwarding tasks) and improve the success of the search process. The structural parameters are represented by the parameter r_i (see Equation 2.9). For the evaluation, we compare the success rate of the searches and the average agent utility in different network structures. The network structures considered in the experiments are: Random, Scale-free, and Small-World networks.

2.5.1 Experimental Design

Each network in the experiments is undirected and has 100 agents. We also tested different sizes of networks, but the conclusions were similar to those obtained with 100 agents and we do not include them here. The structural properties of the networks are shown in Table 2.1. Each agent has a type (service) $\theta_i \in [0, 1]$ that represents the degree of ability of agent. The degree of ability is uniformly distributed among the agents. A task ρ is generated and assigned to an agent following a uniform probability distribution. Each agent has a set of actions to choose from when it receives a task: doing the task if the similarity between its ability and the task ρ is under a threshold $|\theta_i - \rho| < \varepsilon$, forwarding the task based on the expected reward (Formula 2.13), or doing nothing. The forwarding action has an associated cost $c = 5$. A task ρ is successfully solved when an agent that has an ability that is similar enough to the task ($|\theta_i - \rho| < \varepsilon$) in less than τ steps. For the experiments, the value of the τ is the Log(MFPT). We use this concave transformation to obtain clear results and illustrate the impact on the parameter with the structure of

topology	N	edges	avDg	std	clust	dens	τ = Log(mfpt)	d	diameter/τ
Random	100.0	200.00	4.00	1.46	0.02	0.04	5.00	7.09	1.418
	100.0	300.00	6.00	1.82	0.03	0.06	4.00	5.00	1.25
ScaleFree	100.0	197.00	3.94	3.94	0.02	0.04	5.00	5.09	1.09
	100.0	293.00	5.86	5.08	0.04	0.06	5.00	4.27	0.854
SmallWorld	100.0	200.00	4.00	1.02	0.08	0.04	5.00	7.55	1.55
	100.0	300.00	6.00	1.27	0.10	0.06	4.00	5.45	1.36

Table 2.1: Network structural properties: topology, number of agents, number of edges, average degree of connection of agents, standard deviation of the degree distribution, clustering, density, τ = Log(Mean First Passage Time),diameter, ratio diameter/τ.

the network. The value for the ε parameter is 0.1. We executed each experiment over 10 networks of each type and we generated 1,000 tasks ρ in each network.

2.5.2 The Influence of Structural Properties and α

In this section, we analyze the influence of network structural properties and reward α in the search process. We consider values for α in the range [4.99975, 5.0005] in order to see the effects on the search process (see Figure 2.1). In this interval, we observe the effects of considering values for α that are lower than the cost of the forwarding action (c=5), values that are equal to the cost of the forwarding action, and values that are greater than the cost of the forwarding action. With values of α lower than or equal to c, the success rate was around 20%. This percentage represents the number of tasks that can be solved directly by the first agent that receives the task. Values of α that are strictly superior

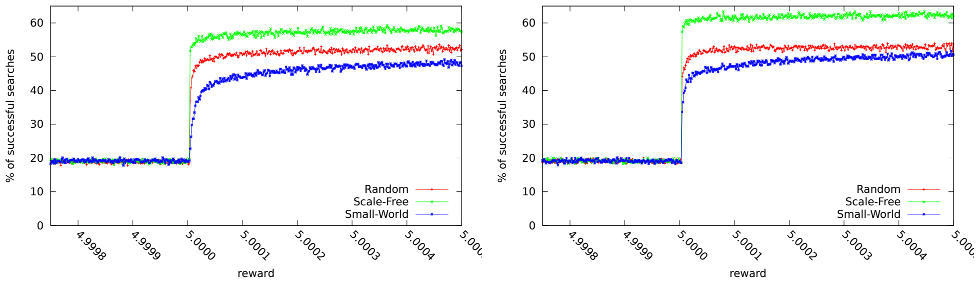

Figure 2.1: Influence of α values in the percentage of successful searches in different network structures of 100 agents. (Left) Network structures with an average degree of connection of 4. (Right) Network structures with an average degree of connection of 6.

to the cost of the forwarding action ($\alpha > c$) provide an increase in the success rate of the search process (see Figure 2.1 Left). The structural properties of the network considered in the search process have an important influence on the success rate. We observe that there are significant differences between the results in Scale-Free, Random, and Small-World networks. Scale-Free provided better results than the other networks since its structural properties increased the number of agents that could be reachable. The diameter of the network is closer to τ than the diameters of other network models (see Table 2.1). Another example of the influence of structural properties is the average degree of connection of the agents. As the average degree of connection increases, the number of reachable agents increases and so does the probability of finding the required agent. Therefore, agents estimate that it is profitable to forward the task to their neighbors (see Figure 2.1 Right).

The structural properties and the reward value α also influence the average utility obtained by an agent. In this experiment, we analyzed values of α in the range [0, 60]. We considered a wider range in order to see the values that made the average agent utility positive and how this utility evolves (see Figure 2.2). Values of α lower than or equal to c provided a utility equal to 0 since agents estimate that the expected reward was not enough to compensate the cost of the forwarding action. Values of α that were in the interval (5, 10] made some agents estimate that the forwarding action was going to be profitable. Although the value for α was enough for agents to consider forwarding tasks, their utility was not always positive for all the agents. Therefore, the average utility had a negative value. The interval (5, 10] for α values could be considered risky. The average utility became positive with α values greater than 10 (see Figure 2.2 Left). In this experiment, the network structure also had a significant influence. The Scale-Free network provided higher values of utility than the Random or Small-Word networks. This difference was also observed when we increased the average degree of connection of agents (see Figure 2.2 Right).

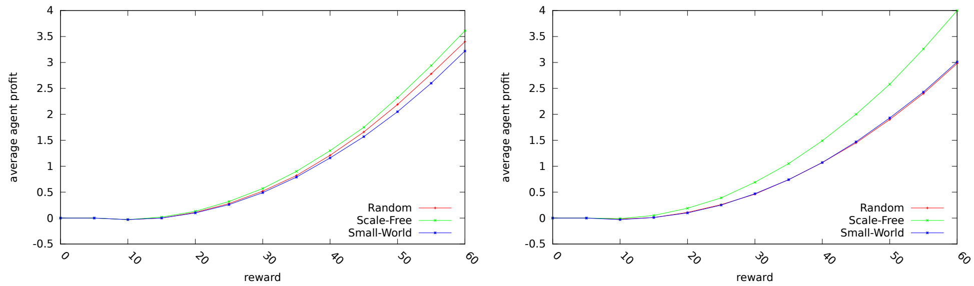

Figure 2.2: Influence of α values on the utility in different network structures of 100 agents. (Left) Network structures with an average degree of connection of 4. (Right) Network structures with an average degree of connection of 6.

2.6 Conclusions

In this paper, we have analyzed the distributed search of resources in networks that model societies of agents. These agents offer services and interact with each other by providing and consuming these services. The actions of these agents have an associated cost and not all of the agents have homogeneous behavior. We have proposed the use of Game Theory to formally model the interactions between the agents as a repeated game, and we have described a strategy that is based on the simple well-known random-walk strategy. We have also established the conditions under which the random-walk strategy is a Nash Equilibrium. The strategy proposed has been extended by adding a constraint for contexts where the number of times a task can be forwarded is restricted. The conditions under which this extended strategy is a Nash Equilibrium have also been analyzed. Finally, we validated the proposed model and the latest strategy in different types of networks. The results show that in order to promote cooperation among the agents of the network, the expected reward should be greater than the cost of the forwarding action. Moreover, the network structure has an important influence on the success of the search process and in the average utility of the system. Scale-Free structural parameters facilitate the success of the search process because their structural properties increase the number of agents that can be reached. The experiments also show that even though there are certain values of the reward that are enough to promote cooperation, these values are not enough to obtain a positive average utility value.

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Chapter 3

Bayesian model of peer review problem

Abstract:

Peer review is a system that subjects scientific work to scrutiny of experts in the field. A fundamental problem of the peer review process is that it introduces conflicting interests or moral hazard problems in a variety of situations. We model the process from a game-theoretical approach. We assume rational agents, where authors have a type which models their “quality" or “ability" as producers of scientific articles, and the referees can have different types regarding their behavior emitting evaluations of them: *unreliable*, *cheater* or *fair*. We characterize the Bayesian Nash Equilibrium of the game of different scenarios achieving both pooling and

separating equilibria.

3.1 Introduction

Peer review is a system that subjects scientific work to scrutiny of experts in the field. If the standards of scientific rigor, technical correctness, novelty and the criterion of sufficient interest are approved. Usually two or three peers evaluate the standards of scientific rigor, technical correctness, novelty and the criterion of sufficient interest of a scientific work (e.g., a submission to a journal or a conference). A scientific authority, such as the editor of a journal, take the experts opinion into account and decides if the work should be published (Meadows and Meadow, 1998; Thurner and Hanel, 2011; Carvalho and Larson, 2013a).

A fundamental problem of the peer review process is that it introduces conflicting interests or moral hazard problems in a variety of situations. It is clear that in the presence of referees with conflicting interests the quality selection aspect of the peer review system will work sub-optimally. The issue between individual versus aggregate optimization is well studied in Economics where agents act as maximizers of their individual utility instead of maximizers of a common goal. Moreover from Computer Science there are an enormous interest of this phenomena as a social and complex interaction among racional individuals.

The high level of competitiveness for reaching scientific positions and funding (Fanelli, 2010), and possible conflicts of interests, might have lead to cases of misconduct (Squazzoni, 2010). Nationality, language, speciality, reputation and gender biases have been evidenced in (Godlee et al., 1998) and (Wenneras and Wold, 2001). This situation has motivated several studies about the process.

The strategic behavior of scientists is being studied recently from different approaches.

Agent Based Models such as (Thurner and Hanel, 2011), (Squazzoni and Gandelli, 2012), (Paolucci and Grimaldo, 2014) and (Cabotà and Squazzoni, 2013) care about this and its consequences in the system. (Baier, 2012) links the motivation and behavior of scientists to knowledge growth and scientific innovations, taking a look at how they coordinate and add to scientific progress as utility-driven agents.

Other studies model the process from a game-theoretical approach. (Leek et al., 2011) develop a theoretical model for peer-review described in terms of payoffs for author and referee behavior, and they analyze it to determine the properties of optimal strategies under both open and closed peer review. In the closed approach, players spent more time solving problems than reviewing, while in the open games there was greater balance between reviewing and submitting. Also in the open review there are more cooperativeness. They observed that when a submitter and reviewer acted cooperatively, the accuracy of the reviews increased by 11 percent. (Squazzoni et al., 2013) describe an experiment that models Peer Review under different incentive conditions, assuming that the quality of peer review depends on a cooperative problem between editors, authors and referees, and propose a modified version of the “investment game". Results of the experiment demonstrate that offering incentives to referees decreases the quality and efficiency of the review process.

Other works have defined a Bayesian model of the Peer Review Process (Park et al., 2013), (Carvalho and Larson, 2013a) and (Carvalho and Larson, 2013b). In (Carvalho and Larson, 2013b) authors propose a scoring method built on scoring rules to induce honest reporting and illustrate its application to the Peer-Review process using a Bayesian Model.

There are several contributions aimed at counteracting the effects of misbehavior in science. (Grimaldo and Paolucci, 2013) propose an agent-based model implementing a program committee update mechanism based on disagreement control that removes rational cheating. (Cabotà and Squazzoni, 2014) studied different editorial policies, concluding that selecting referees of good quality, might counteract evaluation bias.

Concerning the dilemma of taking into account a manuscript quality or the author's reputation when reviewing, (Thorngate and Chowdhury, 2014) propose a weighted rank of a manuscript taking into account the manuscript quality (as perceived by two reviewers) and the track record of the author (number of previous publications). They present it as a possible approach for resolving disagreements among reviewers' assessments. Results show that increases in the weight given to track record decrease the proportion of best articles published, decline the percentage of best authors who accumulate the best track records and favor authors who develop a track record of publications in the first cycles of journal publication.

Summarizing, some works have explored different kinds of behavior, other works have tried to adjust them empirically and experimentally. In this article we study the bayesian equilibrium of different behaviors of authors and referees, which lead us to stable and plausible situations from a game-theoretical perspective.

We apply the framework of Game Theory to model the Peer Review problem. We assume rational agents, which play the role of authors or referees. Authors have a type which models their "quality" or "ability" as producers of scientific articles, and the referees can have different types regarding their behavior emitting evaluations of them: *unreliable*, *cheater* or *fair*. We model such behavior assigning a different utility function for each type. The Game Theory formalization studies agents' strategic behaviors, which allows us to obtain a stability result, the well known Bayesian Nash Equilibrium. This concept establishes stable behaviors with respect to agent's unilateral deviations, what implies that agents have no individual incentives to change their strategy if everybody "sticks to the plan". In this framework, authors choose their best response investing resources to produce articles, what determines how close the quality of the paper is to their ability. The level of resources they invest depends critically on the probability distribution that determines which type of referee is evaluating their paper. The knowledge of this distribution allow us to characterize the Bayesian Nash Equilibrium in different scenarios. Our approach is based on the diversity of cost functions, understanding the cost as the investment agents must make to do their actions: authors to produce their papers and referees evaluating them.

We characterize the Bayesian Nash Equilibrium strategy profile in different scenarios. In the first scenario the type of the author is common knowledge and a constant cost function is present in every agent utility function. We find a pooling equilibrium where all players choose the exact amount of effort equal to author's type, i.e authors produce articles of the quality they are capable of and the referees evaluate them fairly. Then we modify the cost functions for the different types of referees, making them linear and quadratic. In each case we characterize the corresponding separating Bayesian Nash Equilibrium and doing some comparative statistics show that, generally, higher percentages of *unre-*

liable and *cheater* referees may result in lower quality papers and evaluations. Finally we characterize the Bayesian Nash Equilibrium when author's type is not exactly known by the referees, who are only aware of its probability distribution.

The paper is organized as follows. Section 3.2 presents a bayesian game model to formalize the peer review process between an author and a referee. In Section 3.3 we characterize the Bayesian Nash Equilibrium for a variety of cost functions for the referees, and we do some comparative statistics over the different parameters of the model. Finally, Section 3.4 presents the final conclusions of the article and some future research.

3.2 The Model

Consider Γ a two-player game under incomplete information where player 1 is called the author (A) and player 2 the referee (R). The type set of the author is the compact set $[0,1]$ and the type set of referee is the discrete set $\{u, ch, f\}$. We assume independence between author's type and referee's type. The nature chooses a realization of each type for both players with the following law:

- $\theta_A \in [0,1]$ with probability distribution p.

- $\theta_R \in \{u, ch, f\}$, with prior probability $P_j = Prob(\theta_R = j)$ such that $0 \leq P_j \leq 1$ for $j \in \{u, ch, f\}$ and $P_u + P_{ch} + P_f = 1$.

The author has as private information the realization $\theta_A \in [0, 1]$. The value θ_A can be understood as her capability, assuming that $\theta_A = 0$ means the lower strength and $\theta_A = 1$ corresponds with the higher capability, in other words, a pope in the field. Neverthless, the author does not the type of the referee who later will revise her paper. On the other hand, the referee knows whether his type is unreliable, cheater or fair but not the realized type of the author.

The set of actions of player is for both the compact set $[0, 1]$.

The timing of game is as follows:

- Once the author and the referee know their type, the author A sends an article A_s to the referee R to be evaluated. The action played by the author is $A_s \in [0, 1]$.
- In the second stage the referee R sees the action of the author A and then sends and evaluation R_s of the article to A knowing the realization of his type. The action played by the referee is $R_s \in [0, 1]$.

Consider a pair of types (θ_A, θ_R). The payoff functions for each player depends on (θ_A, θ_R) and the action pairs chosen by both players. More specifically for the referee we can write the following three payoff functions:

$$\begin{aligned} U_R^u(R_s, A_s, \theta_A) &= (1-(R_s-\theta_A)^2) - c(u) & (3.1) \\ U_R^{ch}(R_s, A_s, \theta_A) &= \alpha(1-(R_s-A_s)^2) \\ &\quad +(1-\alpha)(1-(R_s-\theta_A)^2) - c(ch) & (3.2) \\ U_R^f(R_s, A_s, \theta_A) &= (1-(R_s-A_s)^2) - c(f) & (3.3) \end{aligned}$$

Where $c(u)$, $c(ch)$ and $c(f)$ are the costs the referee pays for the effort he invests evaluating A_s when he is *unreliable*, *cheater* or *fair* respectively. Note that an unreliable referee takes only into account the author's type, a fair one only the article and a cheater does a convex combination of both terms, weighted by a parameter α. This models the behavior of referees who only focus on the author's name, the quality the article produced or both, respectively.

As the type set of the author is the $[0, 1]$, we can write the author payoff function as:

$$U_A(R_s, A_s, \theta_A) = \alpha(1-(R_s-A_s)^2)+(1-\alpha)(1-(A_s-\theta_A)^2)-c(A) \quad (3.4)$$

where $c(A)$ is the cost the author pays for the effort he invests generating A_s. The author always produces a paper taking into account the evaluation he thinks will get and how close it is to his own capabilities (θ_A).

A strategy σ_A for the author is a map from her type set to her action set: $\sigma_A : [0,1] \Rightarrow [0,1]$. Respectively, a strategy σ_R for the type is a map: $\sigma_R : \{u, ch, f\} \Rightarrow [0,1]$.

As a usual average criterium, the average payoff of each player given his type allows us to compute the average payoff for each player:

$$E[U_A(\sigma_R, A_s, \theta_A)] = P_u U_A(R_s^u, A_s, \theta_A) + P_{ch} U_A(R_s^{ch}, A_s, \theta_A) + P_f U_A(R_s^f, A_s, \theta_A) \tag{3.5}$$

where $\sigma(\theta_A) = A_s$ and

$$E[U_R(R_s, \sigma_A, \theta_R)] = \int_0^1 U_R^{\theta_R}(R_s, A_s, \theta_A) dp(\theta_A) \tag{3.6}$$

where $\sigma(\theta_R) = R_s$ for $\theta_R \in \{u, ch, f\}$.

Definition 5. *A pair* (σ_A^*, σ_R^*) *is a perfect bayesian equilibrium if:*

- $E[U_A(\sigma_R^*, A_s^*, \theta_A)] \geq E[U_A(\sigma_R^*, A_s, \theta_A)]$ *and*
- $E[U_R(R_s^*, \sigma_A^*, \theta_R)] \geq E[U_R(R_s, \sigma_A^*, \theta_R)]$

In this study we assume independence between the author and referee types. As well, we depict several instances of this game taking into account different cost function for the referee and the distribution probability of $p(\theta_A)$.

3.3 Results

In this section, we drive for different cases of cost functions for the referee players by studying existence and characterization of some bayesian equilibrium. We differentiate the cases where the equilibrium is a pooling equilibria and those that each type of referee may act differently.

3.3.1 Bayesian Equilibrium when $c(u) = c$, $c(ch) = c$, $c(f) = c$, and $c(A) = c$ and $p(\theta_A)$ is a Dirac measure at θ_A

In this subsection we characterize a bayesian equilibria with a constant cost function equal for all referee types. First, we formulate the proposition that states the bayesian equilibrium of the game and later we propose a static comparative of the equilibrium behavior attending different cases.

Proposition 4. *For* Γ *with* $c(u) = c$*,* $c(ch) = c$*,* $c(f) = c$*,* $c(autor) = c$ *and* $p(\theta_A)$ *a Dirac measure at* θ_A *the pair* $(\sigma_R, \sigma_A) = ((\theta_A, \theta_A, \theta_A), \theta_A)$ *is a pooling bayesian equilibium.*

Proof. We obtain the best responses for each type of referee maximizing the utility functions from equations (3.1), (3.2) and (3.3):

$$\begin{aligned} \frac{\partial u_R^u}{\partial R_s} &= -2(R_s - \theta_A) = 0 \\ R_s^*(u) &= \theta_A \end{aligned} \tag{3.7}$$

$$\begin{aligned} \frac{\partial U_R^{ch}}{\partial R_s} &= -2(1-\alpha)\left(R_s - \theta_A\right) - 2\alpha\left(R_s - A_s\right) - c = 0 \\ R_s^*(ch) &= \alpha A_s + (1-\alpha)\theta_A \end{aligned} \tag{3.8}$$

$$\begin{aligned} \frac{\partial U_R^{f}}{\partial R_s} &= 2A_s - 2R_s \\ R_s^*(f) &= A_s \end{aligned} \tag{3.9}$$

Now we substitute each referee's best response in author's expected utility function from equation (3.5):

$$\begin{aligned} E[U_A(R_s^*, A_s, \theta_A)] &= \alpha P_u \left(1 - (A_s - \theta_A)^2\right) + \alpha P_{\text{ch}} \left(1 - (\alpha - 1)^2 (A_s - \theta_A)^2\right) \\ &\quad + \alpha P_f + (1 - \alpha) \left(1 - (A_s - \theta_A)^2\right) - c \end{aligned} \tag{3.10}$$

To obtain the best response for the author, we maximize the previous equation:

$$\begin{aligned} \frac{\partial E[U_A(R_s^*, A_s, \theta_A)]}{\partial A_s} &= -2(\alpha - 1)\alpha P_{\text{ch}} \left(-(\alpha - 1)\theta_A + \alpha A_s - A_s\right) \\ &\quad + 2\alpha P_u (\theta_A - A_s) - 2(1 - \alpha)(A_s - \theta_A) = 0 \\ A_s^* &= \theta_A \end{aligned} \tag{3.11}$$

Now we substitute (3.11) in equations (3.7), (3.8) and (3.9):

$$R_s^*(u) = \theta_A \tag{3.12}$$

$$R_s^*(ch) = \theta_A \tag{3.13}$$

$$R_s^*(f) = \theta_A \tag{3.14}$$

We show now that the best responses we obtained satisfy definition (5) for the author

$$
\begin{aligned}
E[U_A(R_s^*, A_s^* = \theta_A, \theta_A)] &= \alpha P_u + \alpha P_{\text{ch}} + \alpha P_f + (1-\alpha) - c \\
&> \alpha P_u \left(1 - (A_s - \theta_A)^2\right) + \alpha P_{\text{ch}} \left(1 - (\alpha - 1)^2 (A_s - \theta_A)^2\right) \\
&\quad + \alpha P_f + (1-\alpha) \left(1 - (A_s - \theta_A)^2\right) - c \\
&= E[U_A(R_s^*, A_s \neq \theta_A, \theta_A)]
\end{aligned}
$$

and for each type of referee

$$
\begin{aligned}
U_R^u(R_s^* = \theta_A, A_s = \theta_A, \theta_A) &= 1 - c \\
&> (1 - (R_s - \theta_A)^2) - c = U_R^u(R_s \neq \theta_A, A_s = \theta_A, \theta_A)
\end{aligned}
$$

$$
\begin{aligned}
U_R^{ch}(R_s^* = \theta_A, A_s = \theta_A, \theta_A) &= \alpha + (1-\alpha) - c \\
&> \alpha(1 - (R_s - \theta_A)^2) + (1-\alpha)(1 - (R_s - \theta_A)^2) - c \\
&= U_R^{ch}(R_s^* \neq \theta_A, A_s = \theta_A, \theta_A)
\end{aligned}
$$

$$
\begin{aligned}
U_R^f(R_s^* = \theta_A, A_s = \theta_A, \theta_A) &= 1 - c \\
&> (1 - (R_s - \theta_A)^2) - c = U_R^f(R_s \neq \theta_A, A_s = \theta_A, \theta_A)
\end{aligned}
$$

□

3.3.2 Bayesian Equilibrium when $c(u) = c$, $c(ch) = c$, $c(f) = c$, and $c(A) = c$ and $p(\theta_A)$ is not a Dirac measure at θ_A

In this subsection we characterize a bayesian equilibria considering a constant cost function equal for all referee types and $p(\theta_A)$ is not a Dirac measure at θ_A, i.e. θ_A is not exactly known by the referee but how it is distributed.

Proposition 5. *For Γ with $c(u) = c$, $c(ch) = c$, $c(f) = c$, and $c(autor) = c$ and $p(\theta_A)$ is the distribution of θ_A, there exists a separating bayesian equilibrium in pure strategies.*

Proof. First, we obtain from equations (3.1), (3.2) and (3.3) the utility functions when $p(\theta_A)$ is not a Dirac measure at θ_A:

$$
\begin{aligned}
&U_R^u(R_s, A_s, \theta_A) = \\
&\int_0^1 \big((1-(R_s-\theta_A)^2)-c\big)d(p(\theta_A)) = \\
&-c[p(\theta_A)]_0^1 + \int_0^1 (1-(R_s-\theta_A)^2)p'(\theta_A)d(\theta_A) = \\
&(-c-R_s^2+1)[p(\theta_A)]_0^1 + 2R_s\int_0^1 \theta_A p'(\theta_A)d(\theta_A) - \int_0^1 \theta_A^2 p'(\theta_A)d(\theta_A) = \\
&(-c-R_s^2+1)[p(\theta_A)]_0^1 + 2R_s(p(\theta_A=1)-1) - (p(\theta_A=1)-2E[\theta_A]) = \\
&p(\theta_A=0)\left(R_s^2+c-1\right) + p(\theta_A=1)\left(-R_s^2+2R_s-c\right) - 2R_s + 2E[\theta_A]
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
&U_R^f(R_s, A_s, \theta_A) = \\
&\int_0^1 \big((1-(R_s-A_s)^2)-c\big)d(p(\theta_A)) = \\
&-c[p(\theta_A)]_0^1 + \int_0^1 (1-(R_s-A_s)^2)p'(\theta_A)d(\theta_A) = \\
&p(\theta_A=1)\left((1-(R_s-A_s)^2)-c\right) - p(\theta_A=0)\left((1-(R_s-A_s)^2)-c\right)
\end{aligned}
\tag{3.16}
$$

$$
\begin{aligned}
&U_R^{ch}(R_s, A_s, \theta_A) = \qquad (3.17)\\
&\int_0^1 \big(\alpha(1-(R_s-A_s)^2) + (1-\alpha)(1-(R_s-\theta_A)^2) - c\big) d(p(\theta_A)) =\\
&\int_0^1 \Big(\big(\alpha(1-(R_s-A_s)^2) + (1-\alpha)(1-(R_s-\theta_A)^2) - c\big)p'(\theta_A)\Big) d(\theta_A) =\\
&\alpha(1-(R_s-A_s)^2)\int_0^1 p'(\theta_A)d(\theta_A) + (1-\alpha)\int_0^1 (1-(R_s-\theta_A)^2)p'(\theta_A)d(\theta_A)\\
&-c\int_0^1 p'(\theta_A)d(\theta_A) =\\
&\alpha(1-(R_s-A_s)^2)[p(\theta_A)]_0^1 - c[p(\theta_A)]_0^1\\
&+p(\theta_A=1)(1-\alpha)\left(1+2R_s+\frac{2E[\theta_A]}{p(\theta_A=1)}\right) - 2R_s =\\
&p(\theta_A=1)\Bigg(\alpha((R_s-A_s)^2 + R_s(A_s-1)) - R_s - c + 1)\\
&-(1-\alpha)\Big(1+2R_s+\frac{2E[\theta_A]}{p(\theta_A=1)}\Big)\Bigg)\\
&-p(\theta_A=0)(\alpha((R_s-A_s)^2 + R_s(A_s-1)) - R_s - c + 1) - 2R_s =\\
&p(\theta_A=1)(\alpha(R_s^2 - A_s^2 - R_sA_s + R_s) + (\alpha-1)\frac{2E[\theta_A]}{p(\theta_A=1)} - 3R_s - c)\\
&-p(\theta_A=0)(\alpha(R_s^2 + A_s^2 - R_sA_s - R_s) - R_s - c + 1) - 2R_s =\\
&\frac{1}{p(\theta_A=1)}\big(2E[\theta_A](\alpha-1) + p(\theta_A=1)(\alpha - 1 + 2(\alpha-2)R_s\\
&+p(\theta_A=0)(R_s(1+\alpha+\alpha A_s - \alpha R_s) - \alpha A_s^2 + c - 1)\\
&-p(\theta_A=1)(R_s(1+\alpha+\alpha A_s - \alpha R_s) - \alpha A_s^2 + c - 1))\big)
\end{aligned}
$$

We obtain the best responses for each type of referee maximizing the utility functions from equations (3.15), (3.17) and (3.16):

$$\frac{\partial u_R^u}{\partial R_s} = -2p(\theta_A = 0)R_s + p(\theta_A = 1)\left(-2R_s - 2\right) - 2 = 0$$

$$R_s^*(u) = \frac{1 - p(\theta_A = 1)}{p(\theta_A = 0) + p(\theta_A = 1)} \tag{3.18}$$

$$\frac{\partial U_R^{ch}}{\partial R_s} = 2(\alpha - 2) + p(\theta_A = 0)\left(\alpha + \alpha A_s - 2\alpha R_s + 1\right)$$

$$- p(\theta_A = 1)\left(\alpha + \alpha A_s - 2\alpha R_s + 1\right) = 0$$

$$R_s^*(ch) = \frac{\alpha + \alpha A_s + \frac{2(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)} + 1}{2\alpha} \tag{3.19}$$

$$\frac{\partial U_R^f}{\partial R_s} = 2\left(p(\theta_A = 0) - p(\theta_A = 1)\right)\left(R_s - A_s\right) = 0$$

$$R_s^*(f) = A_s \tag{3.20}$$

Now we substitute each referee's best response in author's expected utility function from equation (3.5):

$$
\begin{aligned}
E[U_A(R_s^*, A_s, \theta_A)] &= \alpha P_u \left(1 - \left(A_s + \frac{p(\theta_A = 1) - 1}{p(\theta_A = 0) + p(\theta_A = 1)}\right)^2\right) \\
&+ \alpha P_{\text{ch}} \left(1 - \frac{1}{4}\left(A_s - \frac{\alpha + \frac{2(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)} + 1}{\alpha}\right)^2\right) \\
&+ \alpha P_f + (1-\alpha)\left(1 - (A_s - \theta_A)^2\right) - c \qquad (3.21)
\end{aligned}
$$

To obtain the best response for the author, we maximize the previous equation:

$$
\begin{aligned}
\frac{\partial E[U_A(A_s, R_s^*, \theta_A)]}{\partial A_s} &= \frac{1}{2}\Big(-4(\alpha-1)\theta_A - A_s\left(-4\alpha + \alpha P_{\text{ch}} + 4\alpha P_u + 4\right) \\
&+ P_{\text{ch}}\left(\alpha + \frac{2(\alpha-2)}{p(\theta_A = 0) - p(\theta_A = 1)} + 1\right) \\
&- \frac{4\alpha\left(p(\theta_A = 1) - 1\right) P_u}{p(\theta_A = 0) + p(\theta_A = 1)}\Big) = 0
\end{aligned}
$$

$$
A_s^* = \frac{(\alpha-1)\theta_A - \frac{1}{4}P_{\text{ch}}\left(\alpha + \frac{(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)} + 1\right) + \frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{\alpha - \frac{\alpha - P_{ch}}{4} - \alpha P_u - 1} \qquad (3.22)
$$

Now we substitute (3.22) in equations (3.18), (3.19) and (3.20):

$$R_s^*(u) = \frac{1 - p(\theta_A = 1)}{p(\theta_A = 0) + p(\theta_A = 1)} \tag{3.23}$$

$$R_s^*(ch) = \frac{2(\alpha-1)\theta_A - \frac{1}{2}P_{\text{ch}}\left(\alpha + \frac{2(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)} + 1\right) + \frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{4(\alpha - \frac{\alpha P_{\text{ch}}}{4} - \alpha P_u - 1)} + \frac{(\alpha-2)}{\alpha(p(\theta_A=0) - p(\theta_A=1))} + \frac{\alpha+1}{2\alpha} \tag{3.24}$$

$$R_s^*(f) = \frac{(\alpha-1)\theta_A - \frac{1}{4}P_{\text{ch}}\left(\alpha + \frac{(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)} + 1\right) + \frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{\alpha - \frac{\alpha - P_{ch}}{4} - \alpha P_u - 1} \tag{3.25}$$

The proof that the conditions of definition (5) are satisfied follows the same procedure as in subsection (3.3.1). Therefore, the expression of the pure bayesian equilibrium pair is $(\sigma_R, \sigma_A) = \Bigg(\bigg(\frac{1-p(\theta_A=1)}{p(\theta_A=0)+p(\theta_A=1)},$

$\frac{2(\alpha-1)\theta_A-\frac{1}{2}P_{\text{ch}}\left(\alpha+\frac{2(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)}+1\right)+\frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{4(\alpha-\frac{\alpha P_{\text{ch}}}{4}-\alpha P_u-1)} + \frac{(\alpha-2)}{\alpha(p(\theta_A=0)-p(\theta_A=1))} + \frac{\alpha+1}{2\alpha},$

$\frac{(\alpha-1)\theta_A-\frac{1}{4}P_{\text{ch}}\left(\alpha+\frac{(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)}+1\right)+\frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{\alpha-\frac{\alpha-P_{ch}}{4}-\alpha P_u-1}\bigg),$

$\frac{(\alpha-1)\theta_A-\frac{1}{4}P_{\text{ch}}\left(\alpha+\frac{(\alpha-2)}{p(\theta_A=0)-p(\theta_A=1)}+1\right)+\frac{2\alpha(p(\theta_A=1)-1)P_u}{p(\theta_A=0)+p(\theta_A=1)}}{\alpha-\frac{\alpha-P_{ch}}{4}-\alpha P_u-1}\Bigg).$

$\square$

3.3.3 Bayesian Equilibrium when $c(u) = c$, $c(ch) = cR_s$, $c(f) = c$, and $c(A) = c$ and $p(\theta_A)$ is a Dirac measure at θ_A

In this subsection we characterize a bayesian equilibria when the cost function associated to the cheater Referee is an increasing linear function on R_s. In this case, when the author's type is known by the referee, it matters the shape of the cost function generating a separating equilibria. First, we formulate the proposition that states the bayesian equilibrium of the game and later we propose a static comparative of the equilibrium behavior attending different cases.

Proposition 6. *For Γ with $c(u) = c$, $c(ch) = cR_s$, $c(f) = c$, and $c(A) = c$ the pair $(\sigma_R, \sigma_A) = ((\theta_A, \alpha A_s + (1-\alpha)\theta_A - \frac{c}{2}, A_s), \theta_A + \frac{(\alpha-1)\alpha c P_{ch}}{-2\alpha+2\alpha((\alpha-1)^2 P_{ch}+P_u)+2})$ is a bayesian equilibium.*

Proof. We obtain the best responses for each type of referee maximizing the utility functions from equations (3.1), (3.2) and (3.3):

$$\frac{\partial u_R^u}{\partial R_s} = -2(R_s - \theta_A) = 0$$
$$R_s^*(u) = \theta_A \tag{3.26}$$

$$\frac{\partial U_R^{ch}}{\partial R_s} = -2(1-\alpha)\,(R_s - \theta_A) - 2\alpha\,(R_s - A_s) - c = 0$$
$$R_s^*(ch) = \alpha A_s + (1-\alpha)\theta_A - \frac{c}{2} \tag{3.27}$$

$$\frac{\partial U_R^f}{\partial R_s} = 2A_s - 2R_s$$
$$R_s^*(f) = A_s \tag{3.28}$$

Now we substitute each referee's best response in author's expected utility function from equation (3.5):

$$\begin{aligned} E[U_A(R_s^*, A_s, \theta_A)] &= \alpha P_u \left(1 - (A_s - \theta_A)^{\,2}\right) \\ &\quad + \alpha P_{\text{ch}} \left(1 - \frac{1}{4}\,(2(\alpha-1)\theta_A - 2(\alpha-1)A_s + c)^{\,2}\right) \\ &\quad + \alpha P_f + (1-\alpha)\left(1 - (A_s - \theta_A)^{\,2}\right) - c \end{aligned} \tag{3.29}$$

To obtain the best response for the author, we maximize the previous equation:

$$\begin{aligned}\frac{\partial E[U_A(A_s, R_s^*, \theta_A)]}{\partial A_s} &= -2(\alpha-1)\alpha P_{\text{ch}}\left(-(\alpha-1)\theta_A+\alpha A_s-A_s-\frac{c}{2}\right)\\ &\quad +2\alpha P_u\left(\theta_A-A_s\right)-2(1-\alpha)\left(A_s-\theta_A\right)\\ A_s^* &= \theta_A+\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha\left((\alpha-1)^2 P_{\text{ch}}+P_u\right)+2} \qquad (3.30)\end{aligned}$$

Now we substitute (3.30) in equations (3.26), (3.27) and (3.28):

$$R_s^*(u) = \theta_A \qquad (3.31)$$

$$R_s^*(ch) = \theta_A+\frac{(\alpha-1)c\left(\alpha P_{\text{ch}}+1\right)-\alpha c P_u}{-2\alpha+2\alpha\left((\alpha-1)^2 P_{\text{ch}}+P_u\right)+2} \qquad (3.32)$$

$$R_s^*(f) = \theta_A+\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha\left((\alpha-1)^2 P_{\text{ch}}+P_u\right)+2} \qquad (3.33)$$

The proof that the conditions of definition (5) are satisfied follows the same procedure as in subsection (3.3.1).

□

When the payoffs of authors and referees apply a constant cost function equal for all of them, the best response is to follow the author's type (i.e. her capability or track record) faithfully.

When the payoff of fair referees considers a linear cost function, while the payoff of the rest of referees and the author apply a constant cost function, fair referees underrate contributions, that continue to be sent by the authors and reviewed by unreliable and cheater referees strictly following the author's type.

Best responses' analysis

Authors and fair referees provide their best response by applying a variation to the author's type (see equations 3.30 and 3.33) that can be reformulated as follows:

$$A_s^* = R_s^*(f) = \theta_A + \Delta_1\theta_A \tag{3.34}$$

where:

$$\Delta_1\theta_A = \frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha + 2\alpha\left((\alpha-1)^2 P_{\text{ch}} + P_u\right) + 2} \tag{3.35}$$

Figure 3.1 shows the values taken by $\Delta_1\theta_A$ when $c = 1$ and α varies from 0 to 1. Respectively, the X-axis and the Y-axis represent the prior probability of having ch and u as the referee's type. Thus, only the area under the diagonal renders valid values, since it corresponds to probability combinations in which $P_u + P_{ch} + P_f = 1$.

Note how extreme values of α (i.e. $\alpha = 0$ and $\alpha = 1$) lead authors and fair referees not to apply any variation but to respond with the exact author's type ($\Delta_1\theta_A = 0$). On the other hand, intermediate values of α always yield negative values of $\Delta_1\theta_A$ that result in a reduction of the author's type. This reduction is smaller for low values of α (i.e. when the author's payoff function puts more weight on the difference between his track record and the evaluation received) and it increases when either α (i.e. the importance given to the difference between the evaluation and the quality of the contribution) or the probability of finding a cheater referee gets higher. In other words, authors would better reduce slightly their capability when they face constant maximum cost functions and act in competitive scenarios, a behaviour that will be imitated by fair referees. On the contrary, under the same conditions, unreliable referees choose as their best response not to modify the author's type (see equation 3.31), which implies overrating contributions.

In turn, the best response of cheater referees is obtained by applying a different variation to the author's type (see equation 3.32) that can now be formulated as follows:

$$R_s^*(ch) = \theta_A + \Delta_2\theta_A \tag{3.36}$$

where:

$$\Delta_2\theta_A = \frac{(\alpha - 1)c\,(\alpha P_{\text{ch}} + 1) - \alpha c P_u}{-2\alpha + 2\alpha\left((\alpha - 1)^2 P_{\text{ch}} + P_u\right) + 2} \tag{3.37}$$

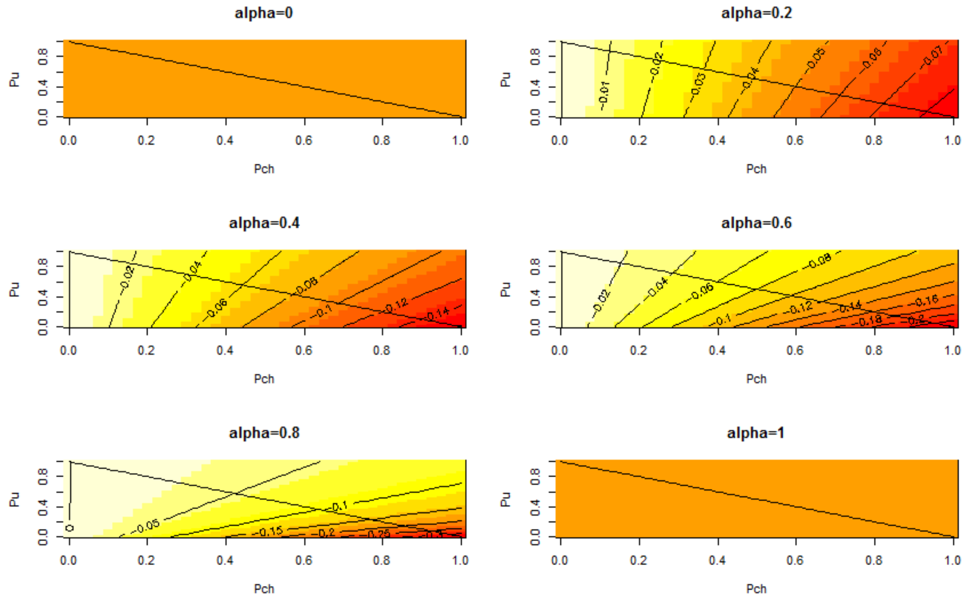

Figure 3.1: $\Delta_1\theta_A$ applied by: (a) authors and fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = c$ and $c(A) = c$; (b) just authors when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s$ and $c(A) = c$; (c) authors and fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$

Figure 3.2 shows the values taken by $\Delta_2\theta_A$ when $c = 1$ and α varies from 0 to 1. The plot formatting is similar to that followed in Figure 3.1 and so is the interpretation of the figures. Though, we appreciate how cheater referees reduce the author's type to a greater extent (also when $\alpha = 1$), then providing evaluations that are considerably lower than those coming from unreliable and fair referees. Therefore, the linear cost used on the payoff function of cheater referees makes them undervalue the contributions.

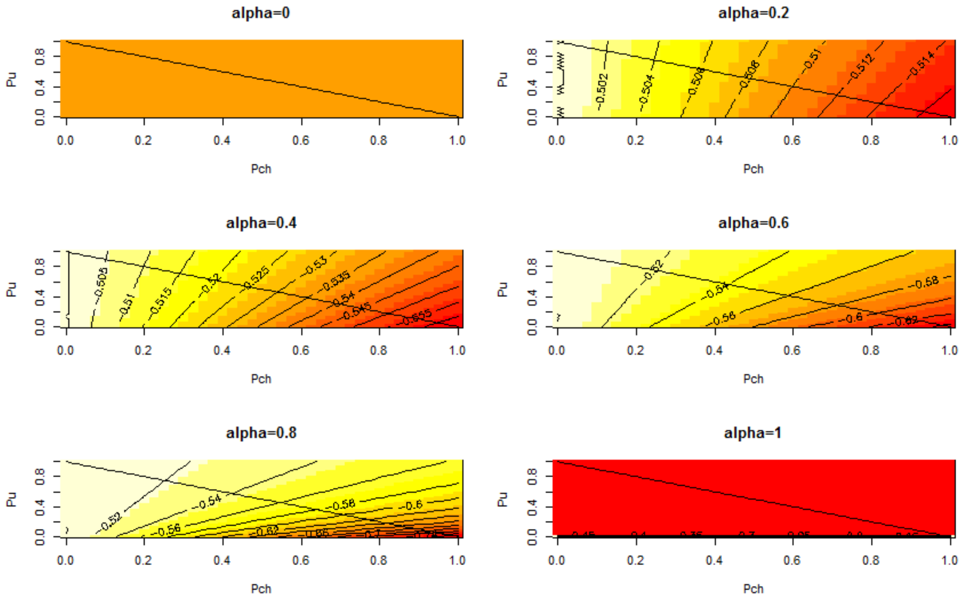

Figure 3.2: $\Delta_2\theta_A$ applied by cheater referees when: (a) $c(u) = c$, $c(ch) = cR_s$, $c(f) = c$, and $c(A) = c$; (b) $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s$, and $c(A) = c$

3.3.4 Bayesian Equilibrium when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s$, and $c(A) = c$ and $p(\theta_A)$ is a Dirac measure at θ_A

In this subsection we characterize a bayesian equilibria when the cost function associated to both cheater and fair Referee is an increasing linear function on R_s.

Proposition 7. *For* Γ *with* $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s$, *and* $c(A) = c$ *the pair* $(\sigma_R, \sigma_A) = \Big((\theta_A, \theta_A + \frac{(\alpha-1)c(\alpha P_{ch}+1)-\alpha c P_u}{-2\alpha+2\alpha((\alpha-1)^2 P_{ch}+P_u)+2},$

$\theta_A + \frac{(\alpha-1)\alpha c P_{ch}}{-2\alpha+2\alpha((\alpha-1)^2 P_{ch}+P_u)+2} - \frac{c}{2}), \theta_A + \frac{(\alpha-1)\alpha c P_{ch}}{-2\alpha+2\alpha((\alpha-1)^2 P_{ch}+P_u)+2}\Big)$ *is a bayesian equilibium.*

Proof. Following the same process as in the previous subsections, we obtain the best responses for both players, which characterize the Bayesian Nash Equilibrium:

$$R_s^*(u) = \theta_A \tag{3.38}$$

$$R_s^*(ch) = \alpha A_s + (1-\alpha)\theta_A - \frac{c}{2} \tag{3.39}$$

$$R_s^*(f) = A_s - \frac{c}{2} \tag{3.40}$$

Now we obtain author's best response

$$A_s^* = \theta_A + \frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha + 2\alpha\left((\alpha-1)^2 P_{\text{ch}} + P_u\right) + 2} \tag{3.41}$$

and referees' best responses

$$\begin{aligned} R_s^*(u) &= \theta_A && (3.42) \\ R_s^*(ch) &= \theta_A + \frac{(\alpha-1)c\,(\alpha P_{\mathrm{ch}}+1) - \alpha c P_u}{-2\alpha + 2\alpha\,((\alpha-1)^2 P_{\mathrm{ch}} + P_u) + 2} && (3.43) \\ R_s^*(f) &= \theta_A + \frac{(\alpha-1)\alpha c P_{\mathrm{ch}}}{-2\alpha + 2\alpha((\alpha-1)^2 P_{\mathrm{ch}} + P_u) + 2} - \frac{c}{2} && (3.44) \end{aligned}$$

The proof that the conditions of definition (5) are satisfied follows the same procedure as in subsection (3.3.1).

□

Best responses' analysis

When the cost function of authors and unreliable referees is constant and that of cheater and fair referees grows linearly, authors provide their best response by applying a small reduction to their type (see equation 3.41) that can be expressed as follows:

$$A_s^* = \theta_A + \Delta_1\theta_A \tag{3.45}$$

where:

$$\Delta_1\theta_A = \frac{(\alpha-1)\alpha c P_{\mathrm{ch}}}{-2\alpha + 2\alpha\,((\alpha-1)^2 P_{\mathrm{ch}} + P_u) + 2} \tag{3.46}$$

Figure 3.1 shows the values taken by $\Delta_1\theta_A$ when $c = 1$ and α varies from 0 to 1. Respectively, the X-axis and the Y-axis represent the prior probability of having ch and u as the referee's type. Thus, only the area under the diagonal renders valid values, since it corresponds to probability combinations in which $P_u + P_{ch} + P_f = 1$. A detailed interpretation of these results can be found in section 3.3.3. Essentially, authors submit contributions that are barely under their type or capability value.

Secondly, cheater referees apply a greater reduction (see equation 3.43) that can be represented as follows:

$$R_s^*(ch) = \theta_A + \Delta_2\theta_A \tag{3.47}$$

where:

$$\Delta_2\theta_A = \frac{(\alpha - 1)c\,(\alpha P_{\text{ch}} + 1) - \alpha c P_u}{-2\alpha + 2\alpha\,((\alpha - 1)^2 P_{\text{ch}} + P_u) + 2} \tag{3.48}$$

Figure 3.2 shows the values taken by $\Delta_2\theta_A$ when $c = 1$ and α varies from 0 to 1. The linear cost used on the payoff function of cheater referees makes them undervalue the contributions whereas unreliable referees trust the author's track record (see equation 3.42).

Lastly, fair reviewers give their best response by also reducing the author's type (see equation 3.44) in the following form:

$$R_s^*(f) = \theta_A + \Delta_3\theta_A \tag{3.49}$$

where:

$$\Delta_3\theta_A = \frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha + 2\alpha((\alpha-1)^2 P_{\text{ch}} + P_u) + 2} - \frac{c}{2} \tag{3.50}$$

Figure 3.3 shows the values taken by $\Delta_3\theta_A$ when $c = 1$ and α varies from 0 to 1. It is worth mentioning that, by comparing Figure 3.2 and Figure 3.3, the reduction applied by fair referees is shown to be equal or even higher than that applied by cheater referees.

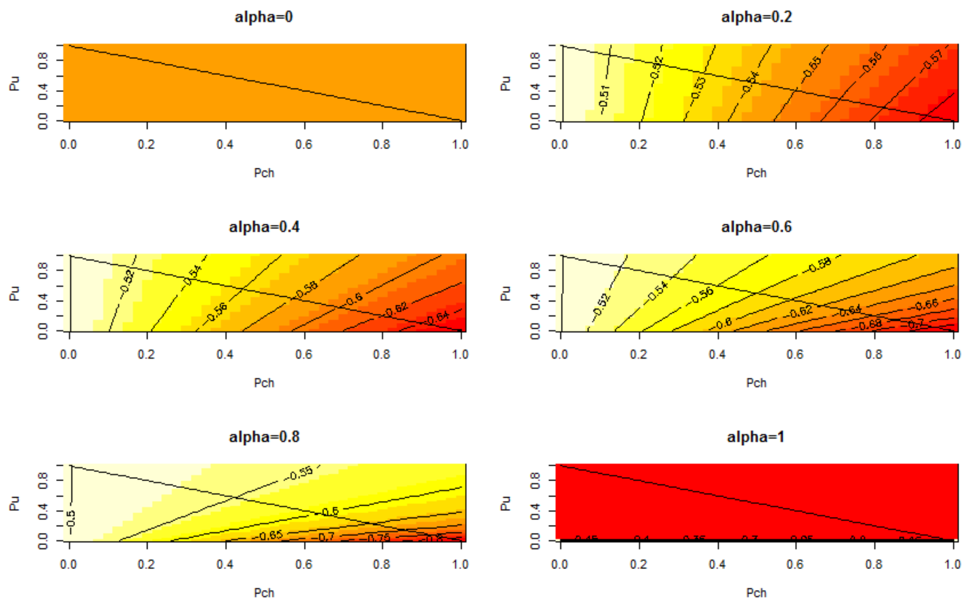

Figure 3.3: $\Delta_3\theta_A$ applied by fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s$, and $c(A) = c$

3.3.5 Bayesian Equilibrium when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$ and $p(\theta_A)$ is a Dirac measure at θ_A

In this subsection we characterize a bayesian equilibria when the cost function associated to the cheater referee is an increasing linear function and for the fair a quadratic one, both on R_s.

Proposition 8. *For* Γ *with* $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, $c(A) = c$ *and* $p(\theta_A)$ *a Dirac measure at* θ_A *there exists a separating bayesian equilibrium in pure strategies.*

Proof. Following the same process as in the previous subsections, we obtain the best responses for both players, which characterize the Bayesian Nash Equilibrium:

$$R_s^*(u) = \theta_A \tag{3.51}$$

$$R_s^*(ch) = \alpha A_s + (1-\alpha)\theta_A - \frac{c}{2} \tag{3.52}$$

$$R_s^*(f) = \frac{A_s}{c+1} \tag{3.53}$$

Now we obtain author's best response

$$A_s^* = \frac{1}{2\left(\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)\right)} * \left(\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha\left((\alpha-1)^2 P_{\text{ch}}+P_u\right)+2} + \theta_A\right) \tag{3.54}$$

and referees' best responses

$$R_s^*(u) = \theta_A \tag{3.55}$$

$$R_s^*(ch) = \frac{c}{2}\left(\frac{(\alpha-1)\alpha^2(c+1)^2 P_{\text{ch}}}{\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)} - 1\right) + \left(1 - \frac{\alpha^2 c^2 P_f}{\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)}\right)\theta_A \tag{3.56}$$

$$R_s^*(f) = \frac{1}{2\left(\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)\right)} * \left(\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha\left((\alpha-1)^2 P_{\text{ch}}+P_u\right)+2} + \theta_A\right) \tag{3.57}$$

The proof that the conditions of definition (5) are satisfied follows the same procedure as in subsection (3.3.1). Therefore, the expression of the pure bayesian equilibrium pair is $(\sigma_R, \sigma_A) = \Bigg(\Big(\theta_A,$

$\frac{c}{2}\left(\frac{(\alpha-1)\alpha^2(c+1)^2 P_{\text{ch}}}{\alpha c^2 P_f+(\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}}+(c+1)^2(-\alpha+\alpha P_u+1)} - 1\right)$
$+ \left(1 - \frac{\alpha^2 c^2 P_f}{\alpha c^2 P_f+(\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}}+(c+1)^2(-\alpha+\alpha P_u+1)}\right)\theta_A,$

$$\frac{1}{2\left(\alpha c^2 P_f+(\alpha-1)^2\alpha(c+1)^2P_{\text{ch}}+(c+1)^2(-\alpha+\alpha P_u+1)\right)}\left(\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha((\alpha-1)^2P_{\text{ch}}+P_u)+2}+\theta_A\right)\Bigg),$$
$$\frac{1}{2\left(\alpha c^2 P_f+(\alpha-1)^2\alpha(c+1)^2P_{\text{ch}}+(c+1)^2(-\alpha+\alpha P_u+1)\right)}\left(\frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha+2\alpha((\alpha-1)^2P_{\text{ch}}+P_u)+2}+\theta_A\right)\Bigg).$$

□

Best responses' analysis

When the cost function of authors and unreliable referees is constant, that of cheater referees grows linearly and the one used for fair referees grows quadratically, authors and fair referees provide their best response by modifying their type (see equations 3.54 and 3.57) using the following transformation:

$$A_s^* = R_s^*(f) = m_1 * (\theta_A + \Delta_1\theta_A) \tag{3.58}$$

where:

$$m_1 = \frac{1}{2\left(\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha + \alpha P_u + 1\right)\right)} \tag{3.59}$$

$$\Delta_1\theta_A = \frac{(\alpha-1)\alpha c P_{\text{ch}}}{-2\alpha + 2\alpha\left((\alpha-1)^2 P_{\text{ch}} + P_u\right) + 2} \tag{3.60}$$

On the one hand, $\Delta_1\theta_A$ has proven to be a small reduction of the author's type (for a throughout analysis, see Figure 3.1 along with its interpretation in subsection 3.3.3). On the other hand, Figure 3.4 shows the values taken by the multiplication factor m_1 when $c = 1$ and α varies from 0 to 1. Again, the X-axis and the Y-axis represent the prior probability of having respectively ch and u as the referee's type. Thus, only the area under the diagonal renders valid values, since it corresponds to probability combinations in which $P_u + P_{ch} + P_f = 1$. Note that this factor actually entails a reduction of the author's type, since all values are positive and lower than the unity.

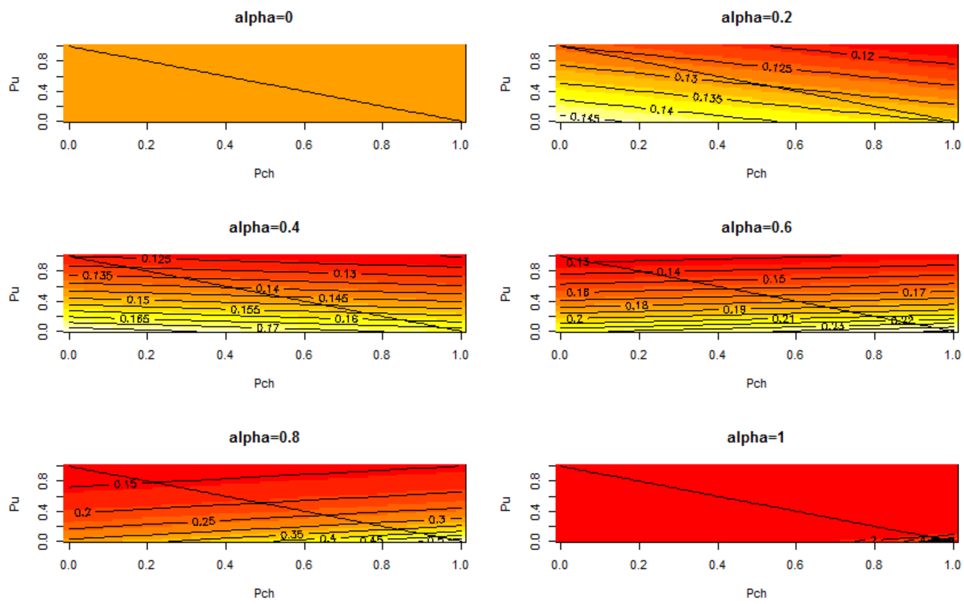

Figure 3.4: Multiplication factor m_1 applied by authors and fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$.

To evaluate how m_1 behaves with respect to c and α, we use the partial differential equations (3.61) and (3.62). The multiplication factor m_1 grows with cost (c) as shown in Figure 3.5, where all values are positive. However, m_1 decreases when more importance is given to the difference between evaluations and the quality of contributions (α). This is shown in Figure 3.6, where all differential values are negative. Anyhow, these opposing forces do not prevent m_1 from decrementing the author's type, which is the action taken by author an fair referees.

$$\frac{\partial m_1}{\partial c} = -\frac{\alpha(\alpha-1)^2(c+1)P_{\text{ch}} + \alpha c P_f + (c+1)\left(-\alpha+\alpha P_u+1\right)}{\left(\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)\right)^2} \tag{3.61}$$

$$\frac{\partial m_1}{\partial \alpha} = -\frac{c^2 P_f + (\alpha-1)(3\alpha-1)(c+1)^2 P_{\text{ch}} + (c+1)^2\left(P_u-1\right)}{2\left(\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha+\alpha P_u+1\right)\right)^2} \tag{3.62}$$

The best response of cheater referees modifies the author's type (see equation 3.56) by now using the following transformation:

$$R_s^*(ch) = m_2 * \theta_A + \Delta_4\theta_A \tag{3.63}$$

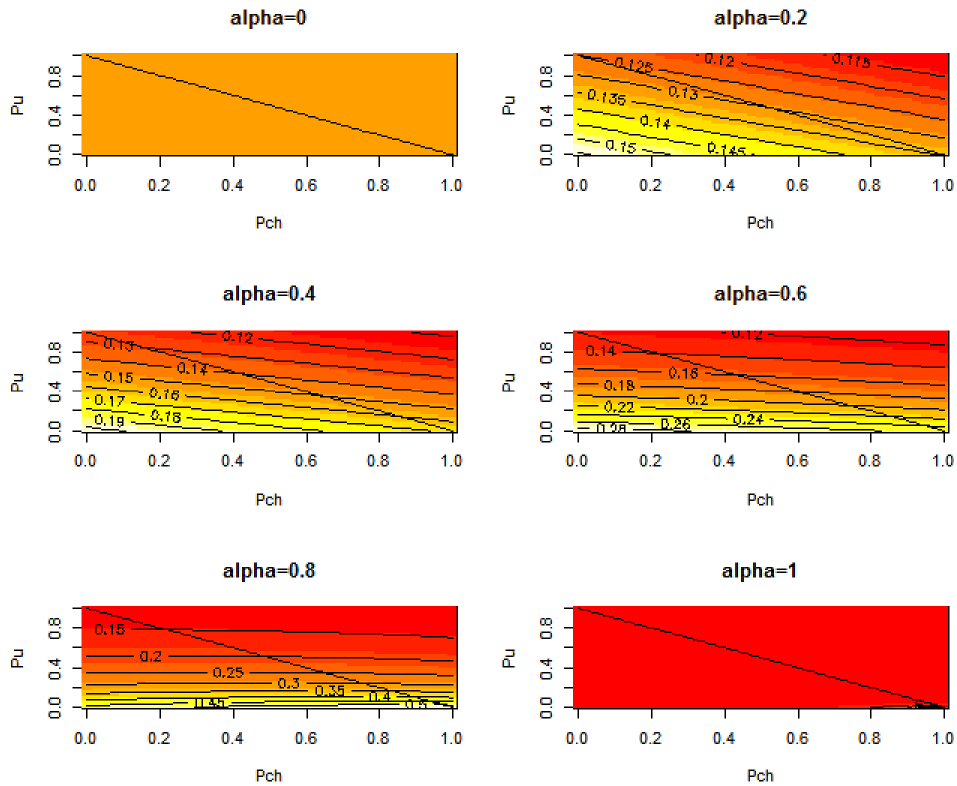

Figure 3.5: $\dfrac{\partial m_1}{\partial c}$ applied by authors and fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$.

where:

$$m_2 = 1 - \frac{\alpha^2 c^2 P_f}{\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha + \alpha P_u + 1\right)} \tag{3.64}$$

$$\Delta_4\theta_A = \frac{1}{2}c\left(\frac{(\alpha-1)\alpha^2(c+1)^2 P_{\text{ch}}}{\alpha c^2 P_f + (\alpha-1)^2\alpha(c+1)^2 P_{\text{ch}} + (c+1)^2\left(-\alpha + \alpha P_u + 1\right)} - 1\right) \tag{3.65}$$

Figure 3.8 shows the values taken by the multiplication factor m_2 when

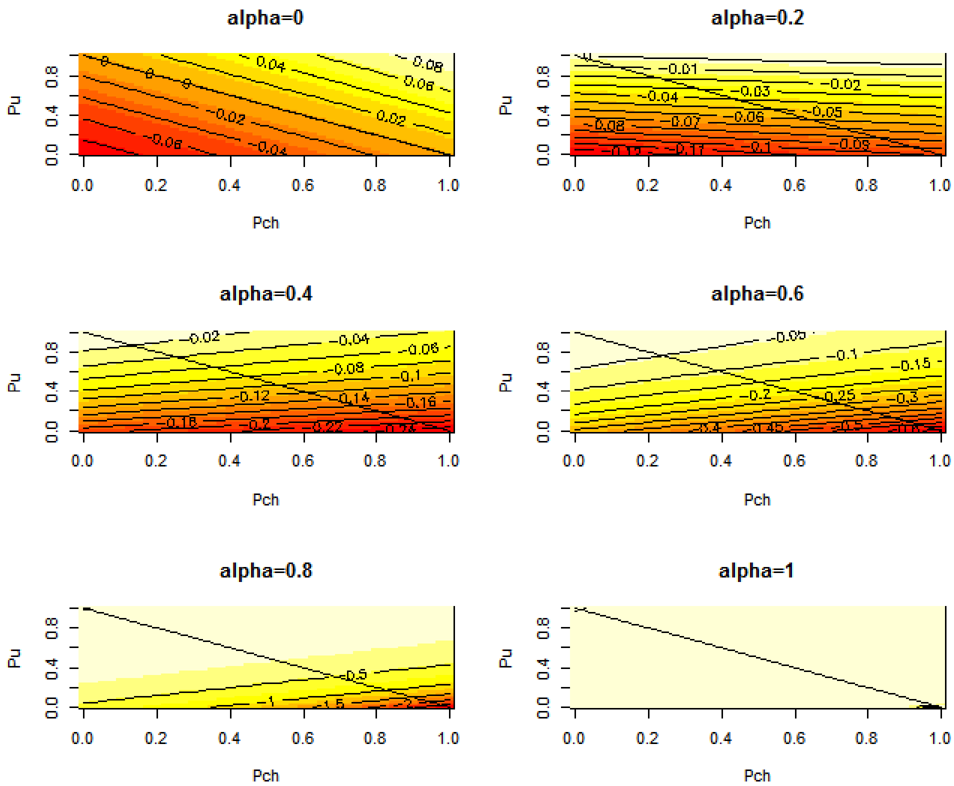

Figure 3.6: $\dfrac{\partial m_1}{\partial \alpha}$ applied by authors and fair referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$.

$c = 1$ and α varies from 0 to 1. As it also occurred with fair referees, this factor reduces the quality of contributions, since all valid values (see the area below the diagonal) are lower than 1. Similarly, $\Delta_4\theta_A$ is again a reduction of the author's type as it can be seen in Figure 3.7, where all values are negative and quantitatively similar to the reduction performed by cheater referees in the rest of scenarios.

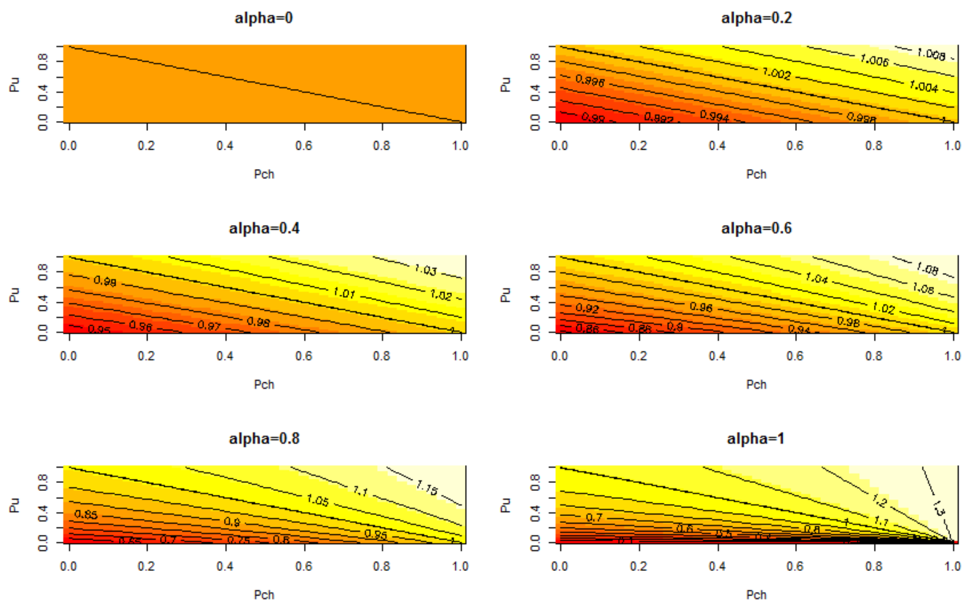
alpha=0
alpha=0.2
alpha=0.4
alpha=0.6
alpha=0.8
alpha=1
Pu
Pch

Figure 3.7: $\Delta_4\theta_A$ applied by cheater referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$.

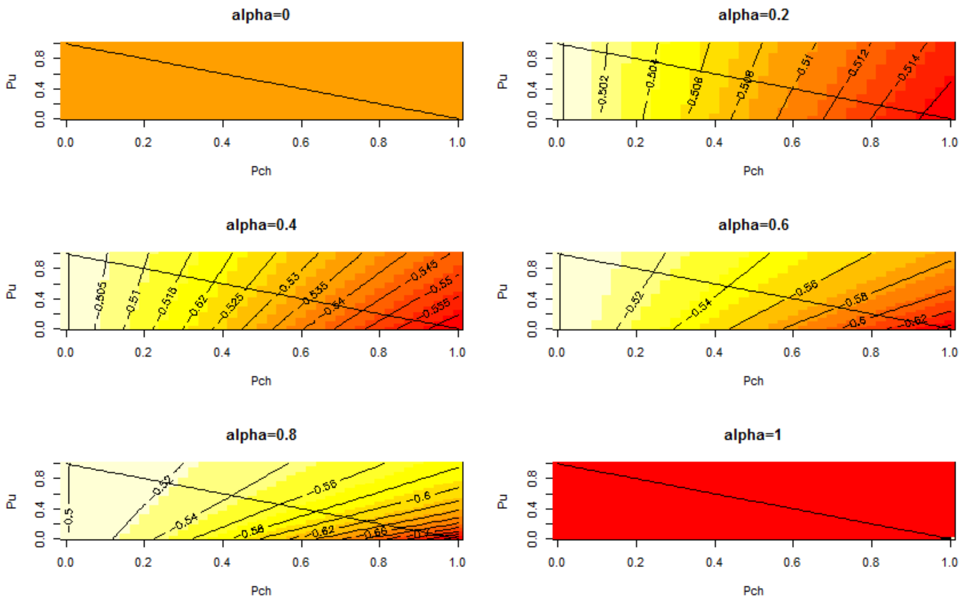

Figure 3.8: Multiplication factor m_2 applied by cheater referees when $c(u) = c$, $c(ch) = cR_s$, $c(f) = cR_s^2$, and $c(A) = c$.

3.4 Concluding remarks

In this paper we propose a Game Theory model for the peer review problem. We model the charisma of possible referees using a combination of both the effect of identity of the author versus the quality of the article and the shape of the cost function; in other words, how non-blind procedure and how much referees engage on the evaluation procedure may affect the quality and technical correctness of the submitted articles and the evaluations they get.

This analysis has been done under the assumption of only one iteration. This implies that the roles of referee and author are not exchangeable and no reputation based behavior emerges. An important issue we can tackle is the same question that we address in this paper but under a dynamic environment. It is well-known that an author in one stage may be a referee and in the future an author, and viceversa. We would like to answer as much as possible the previos question in future research.

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Conclusions

L'objectiu principal de la tesi ha sigut utilitzar la teoria de jocs per analitzar diversos problemes d'interacció entre agents. Aquesta potent eina ens permet trobar estratègies d'estabilitat assumint que les parts enfrontades es comporten com agents econòmics que pretenen maximitzar la seua utilitat individual. Establim plans d'actuació on cap jugador té incentius unilaterals a canviar la seua estratègia, fixant les accions de la resta. A més, hem enriquit l'anàlisi dels models presentats incloent el supost d'informació incompleta, situació en la que hem fet servir el concepte d'equilibri bayesià. Açò ens ha permés treballar amb la premisa de que part de la informació del joc és desconeguda per a un o més dels jugadors que prenen part en ell, fet que es produeix en multitud d'interaccions econòmiques en la realitat. Per als tres treballs que composen la tesi hem plantejat un joc que modela el problema a tractar i hem caracteritzat els prefils d'estratègies d'equilibri, especificant les condicions que els diversos paràmetres del model han de complir per a que així siga. La teoria de jocs, doncs, ha resultat ser una eina adequada i potent per a satisfer els objectius de la tesi. Adicionalment, hem fet servir eines al nostre abast per a obtenir dades empíriques i fer estàtica comparativa que ens ha ajudat a contrastar les conclusions presentades analíticament.

El primer capítol fa servir l'Economia experimental per obtenir evidència empírica del comportament dels subjectes als quals se'ls presenta el joc, analitzat prèviament des del punt de vista teòric. Aquesta branca de l'economia ha agafat un gran impuls els darrers anys, i es presenta com una eina útil per a obtenir dades sota condicions de control i replicabilitat. Nosaltres la hem utilitzat per contrastar els resultats d'equlibri calculats, observant que els subjectes no s'han comportat guiats purament per l'incentiu econòmic individual i que, per tant, han influït altres factors a l'hora de decidir com la identitat pròpia i l'afiliació al grup.

El segon capítol presenta el nivell de recompenses a la cooperació que ha d'implementar un sistema de cerca de serveis en xarxa per a que tots els agents col.laboren amb el procés seguint una estratègia *Random Walk*. A més, presentem un anàlisi mitjançant simulacions multi-agent de diverses estructures de xarxa, factor clau en l'èxit del procés i la utilitat acumulada pels agents. Concluïm que la estructura *Scale-free* és la millor en ambdós aspectes.

En el tercer i últim capítol analitzem els problemes de conflicte d'interesos i risc moral que apareixen en el sistema d'avaluació d'articles d'invesitgació conegut com a *peer review.* Modelem el problema com un joc amb informació incompleta, on un autor envia un article que serà revisat per un científic desconegut per a ell. El comportament del revisors el modelem amb distintes funcions d'utilitat per als distints tipus possibles, variant en cada cas la funció de cost. Per cada escenari exposem les condicions necessàries i les accions que composen els perfils d'estratègies d'equilibri, *pooling* en uns casos i separador en altres. Realitzem estàtica comparativa per a concloure que , a menys proporció d'avaluadors 'honrats', menor resulta la qualitat dels articles produïts.