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Tri-bimaximal neutrino mixing and neutrinoless double beta decay

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We present a tri-bimaximal lepton mixing scheme where the neutrinoless double beta decay rate has a lower bound which correlates with the ratio $\alpha \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ well determined by current data, as well as with the unknown Majorana CP phase ϕ_{12} characterizing the solar neutrino sub-system. For the special value $\phi_{12} = \frac{\pi}{2}$ (opposite CP-sign neutrinos) the $\beta\beta_{0\nu}$ rate vanishes at tree level when $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} = 3/80$, only allowed at 3σ. For all other cases the rate is nonzero, and lies within current and projected experimental sensitivities close to $\phi_{12} = 0$. We suggest two model realizations of this scheme in terms of an $A_4 \times Z_2$ and $A_4 \times Z_4$ flavour symmetries.

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Current neutrino oscillation data [1, 2, 3, 4, 5, 6, 7] indicate a peculiar pattern [8] of neutrino masses and mixings quite at variance with the structure of the Cabibbo-Kobayashi-Maskawa quark mixing matrix [9]. However they do not yet fully determine the absolute scale of neutrino masses nor shed any light on the issue of leptonic CP violation, two demanding challenges left for future experiments.

Lacking a basic theory for the origin of mass one needs theoretical models restricting the pattern of fermion masses and mixings and providing guidance for future experimental searches. An attractive phenomenological ansatz for leptons is the Harrison-Perkins-Scott (HPS) mixing [10]

$$U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \qquad (1)$$

which predicts the following values for the lepton mixing angles: $\tan^2\theta_{\rm atm} = 1$, $\sin^2\theta_{\rm Chooz} = 0$ and $\tan^2\theta_{\rm sol} = 0.5$, providing a good first approximation to the values [8] indicated by neutrino oscillation experiments [1, 2, 3, 4, 5].

As noted earlier [11], when the charged lepton mass matrix M_l obeys

$$M^l M^{l\dagger} = U_\omega\, M^{l2}_{diag}\, U^\dagger_\omega;$$

where U_ω is the "magic" unitary matrix

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

and the neutrino mass matrix has the form

$$M_\nu \sim \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix},$$

the resulting lepton mixing matrix has exactly the tri-bimaximal structure given in Eq. (1).

Here we consider schemes where neutrinos get mass *a la seesaw*, defined by the following mass matrices,

$$M^l \sim \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix} = U_\omega\, M^l_{diag}\, U^\dagger_\omega;$$

$$m_D \sim \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}; M_R \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This "texture" constitutes a new ansatz for the lepton sector that can be realized (see below) in the framework of A_4-based flavour symmetry models. The assumed symmetry of the Dirac mass term holds in $SO(10)$ models where it comes from a 16 16 10 Yukawa coupling. In contrast with other existing tri-bimaximal A_4 based schemes, the gauge singlet seesaw mass term characterizing the heavy right-handed neutrinos is also a flavour singlet, instead of the neutrino Dirac mass term. This makes the scheme extremely predictice, as it involves as free parameters only the two modulii and the relative phase between a and b.

After the seesaw mechanism, one obtains the effective light neutrino mass matrix M_ν given as

$$M_\nu = m_D \frac{1}{M_R} m_D^T \sim \begin{pmatrix} a^2 & 0 & 0 \\ 0 & a^2+b^2 & 2ab \\ 0 & 2ab & a^2+b^2 \end{pmatrix}. \qquad (2)$$

Rewriting the effective light neutrino mass matrix in the basis where charged leptons are diagonal one finds

$$\mathcal{M}_\nu \equiv \begin{pmatrix} a^2+\frac{4ab}{3}+\frac{2b^2}{3} & -\frac{1}{3}b(2a+b) & -\frac{1}{3}b(2a+b) \\ -\frac{1}{3}b(2a+b) & \frac{1}{3}b(4a-b) & a^2-\frac{2ab}{3}+\frac{2b^2}{3} \\ -\frac{1}{3}b(2a+b) & a^2-\frac{2ab}{3}+\frac{2b^2}{3} & \frac{1}{3}b(4a-b) \end{pmatrix}.$$

This matrix is fully determined by two complex parameters a and b, which imply three physical real parameters, namely two moduli and a relative phase, which is the only source of leptonic CP violation in the scheme.

We note that $\mathcal{M}_\nu$ is $\mu \leftrightarrow \tau$ invariant, so it gives $\theta_{13}=0$ and $\sin^2\theta_{23}=1/2$ as predictions. The state $(1,1,1)^t$ is an eigenstate of $\mathcal{M}_\nu$ with eigenvalue a^2, so the neutrino mass matrix $\mathcal{M}_\nu$ is diagonalized by the tri-bimaximal mixing matrix, leading then to $\tan^2\theta_{12}=0.5$. The three neutrino mass eigenvalues are

$$\{m_1, m_2, m_3\} = \{(a+b)^2, a^2, -(a-b)^2\}.$$

Data from neutrino oscillation experiments [1, 2, 3, 4, 5] determine pretty well two of the three parameters on the left-hand side [8], namely the solar and atmospheric mass-square splittings. The remaining observable is precisely the neutrino-exchange amplitude for neutrinoles double beta decay, given by

$$\langle m_\nu \rangle \equiv |m_{ee}| = |a^2 + \frac{4ab}{3} + \frac{2b^2}{3}|.$$

This parameter can be given as a function of the three independent model parameters, which we choose to express in terms of the observables $\Delta m^2_{\rm atm}$, α and the relative phase between a and b. The latter is directly related to the Majorana CP phase [12, 13, 14, 15] characterizing the solar neutrino sub-system, ϕ_{12}, in a symmetric parametrization of the lepton mixing matrix where all phases appear attached to the corresponding mixing angle [12, 13].

First we note that our scheme is compatible with negligible neutrinoless double beta decay, vanishing at the tree level i. e. $m_{ee}=0$. This happens only when CP is conserved with opposite CP parities [16, 17] between ν_1 and ν_2 and for

$$\alpha = \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = \frac{3}{80} = 0.0375, \qquad (3)$$

as seen in Fig. 1, which is currently allowed at 3 σ. For all other values of the CP phase the model gives a lower bound on the neutrinoless double beta decay which we display in Fig. 2, which we call the "Niemeyer" plot [32]. This plot exhibits two dips characterized by very small $\beta\beta_{0\nu}$ amplitudes, which correspond to almost full destructive interference between opposite CP sign neutrinos ν_1 and ν_2.

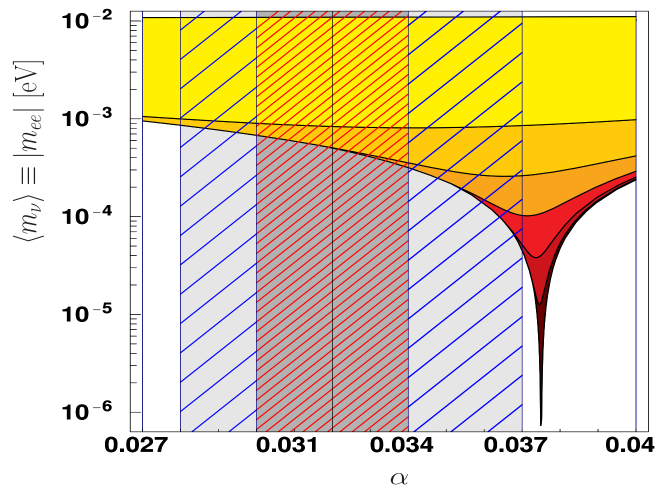

FIG. 1: Lower bound on the $\beta\beta_{0\nu}$ amplitude parameter m_{ee} as function of $\alpha \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ for different values of the Majorana phase $\phi_{12} = -\pi/2 + t$ where $t = 0$ (dark brown), 0.001 (brown), 0.004 (red), 0.011 (dark orange), 0.029 (orange), 0.089 (yellow). The 1, 2 and 3σ ranges for α are also shown.

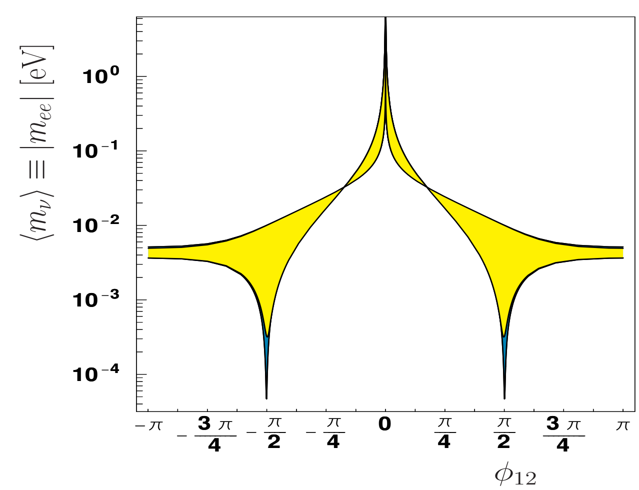

FIG. 2: Lower bound on the neutrinoless double beta decay amplitude parameter m_{ee} as function of the Majorana CP phase ϕ_{12} for α within the 1σ (yellow) and 2σ (blue) ranges.

Notice that in the central region around the other CP conserving point $\phi_{12}=0$ the $\beta\beta_{0\nu}$ amplitude is sizeable, and depends very sensitively on the Majorana phase ϕ_{12}, as displayed in Fig. 3.

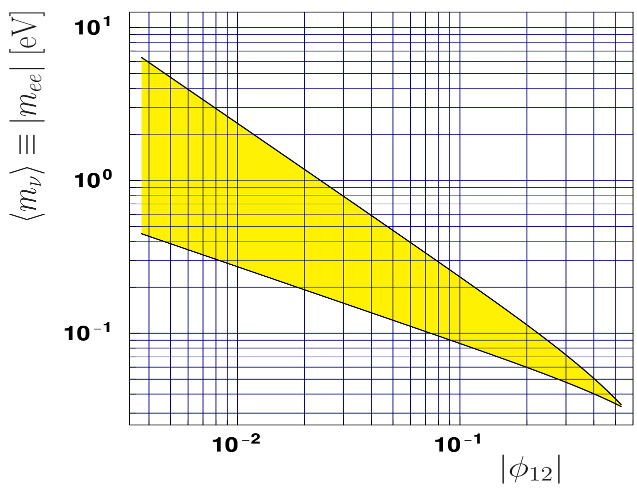

FIG. 3: Zoom of the region giving the maximum value for the lower bound on m_{ee} in Fig. 2.

It is a non-trivial task to produce a consistent flavour symmetry leading to a structure of the effective neutrino mass matrix $\mathcal{M}_\nu$ that has, at least as a first approximation, the desired predictive pattern.

Here we suggest two possible realizations based on an A_4 flavour symmetry for the neutrino mass matrix. The discrete group A_4 is a relatively small and simple flavour group consisting of the 12 even permutations among four objects. It has a three-dimensional irreducible representation appropriate to describe the three generations observed. Originally, A_4 was proposed [18, 19] for understanding degenerate neutrino spectrum with nearly maximal atmospheric neutrino mixing angle. More recently, predictions for the solar neutrino mixing angle have also been incorporated within the so-called tri-bimaximal neutrino mixing schemes [20, 21, 22, 23, 24, 25, 26].

In our phenomenological A_4-based flavor symmetry schemes the neutrino mass comes from type-I seesaw mechanism with right-handed Majorana mass matrix proportional to the identity matrix. In both models leptons transform as A_4-triplets, while the standard Higgs is a flavour singlet [33]. The lepton and scalar content of the models are specified in Tables I and II.

The $A_4 \times Z_2$ invariant Lagrangian characterizing the first model is renormalizable, and given by

$$\mathcal{L} = \lambda_0 (L l^c) h + \lambda (L l^c H) \\ + \lambda'_0 (L \nu^c) \varphi + \lambda' (L \nu^c \Phi) + \lambda_R (\nu^c \nu^c) \xi.$$

where the first term involves an A_4-invariant coupling λ_0 that provides α in M^l, while the second involves a tensor

fields	L_i	l_i^c	ν_i^c	h	H_i	φ	Φ	ξ
$SU(2)_L$	2	1	1	2	2	2	2	1
A_4	3	3	3	1	3	1	3	1
Z_2	+	+	−	+	+	−	−	+

TABLE I: Lepton multiplet structure of model I

λ_{ijk} with components β and γ, and similarly for the next two terms. Note that in this case there are additional SU(2) $\otimes$ U(1) doublet Higgs scalar bosons H_i, Φ_i, φ transforming non-trivially under the flavour symmetry. We assume that these develop non-zero vacuum expectation values (vevs), with the structure

$$\langle H_i \rangle \sim (1,1,1); \quad \langle \Phi_i \rangle \sim (0,0,1)$$

Similar vev alignment condition has been used in Ref. [27]. Note that the two zeros in m_D follow from the alignment condition $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = 0$.

In contrast the second model contains only one SU(2) $\otimes$ U(1) doublet Higgs boson and its $A_4 \times Z_4$ symmetric leading-order Lagrangian is written as

$$\mathcal{L} = \lambda_0 (L l^c) h \xi_1 + \lambda (L l^c \phi) h \\ + \lambda'_0 (L \nu^c \phi') \tilde{h} + \lambda' (L \nu^c) h \, \xi_2 + \lambda_R (\nu^c \nu^c) \xi_3$$

where λ_R is dimensionless while the others scale as inverse mass. Note the appearance of gauge singlet scalars ϕ, ϕ' and ξ_i, transforming non-trivially under the flavour symmetry and coupling non-renormalizably to the lepton doublets. Their only renormalizable is the one giving rise to the large Majorana mass term. We assume that these "flavon" fields develop non-zero vacuum expectation values (vevs), with the structure

$$\langle \phi \rangle \sim (1,1,1); \quad \langle \phi' \rangle \sim (0,0,1)$$

Note that either way we obtain the desired predictive charged lepton and neutrino mass matrices discussed above.

fields	L_i	l_i^c	ν_i^c	h	ϕ	ϕ'	ξ_1	ξ_2	ξ_3
$SU(2)_L$	2	1	1	2	1	1	1	1	1
A_4	3	3	3	1	3	3	1	1	1
Z_4	1	ω^3	ω	1	ω	ω^3	ω	ω^3	ω^2

TABLE II: Lepton multiplet structure of model II

In summary, here we have proposed two A_4-based flavour symmetries leading to tri-bimaximal lepton mixing, namely $\tan^2\theta_{\rm atm} = 1$, $\sin^2\theta_{\rm Chooz} = 0$ and $\tan^2\theta_{\rm sol} =$

0.5. Although this implies a boring scenario for upcoming long baseline oscillation experiments [28, 29] aiming to probe θ_{13} and leptonic CP violation in oscillations, we have analysed its implications for neutrinoless double beta decay. We have seen how the $\beta\beta_{0\nu}$ amplitude parameter m_{ee} has a lower bound which correlates with the ratio $\alpha \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ well determined by current neutrino oscillation data, as well as with the Majorana CP violating phase ϕ_{12}. Accelerator neutrino oscillation experiments like MINOS, T2K and NOvA are expected to improve the determination of α in the not-too-distant future.

For the special value $\phi_{12} = \frac{\pi}{2}$ (opposite CP-sign neutrinos) one finds that $\beta\beta_{0\nu}$ vanishes at tree level when $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} = 3/80$. However this is only allowed at 3σ, as seen from Fig. 1, at 1 σ we currently have a lower bound $|m_{ee}| \gtrsim$ few $\times 10^{-4}$ eV. For all other cases one has a nonzero $\beta\beta_{0\nu}$ decay rate, with CP conservation with same CP-sign neutrinos already excluded. We have also presented in Fig. 3 the lower bound in the region close to $\phi_{12} = 0$, corresponding to the case of same CP-parity neutrinos, where neutrinoless double beta decay could soon be observed.

All our considerations refer to an effective low-energy model which assumes the vev alignment conditions, and the symmetry of the neutrino Dirac mass matrix, a relation which holds in the framework of $SO(10)$ unification [30, 31]. A more complete picture formulated at the unified level is outside the scope of this letter. In principle the structure presented here can be lifted to the $SO(10)$ level, though fitting the flavour structure of quarks will require additional fields and/or symmetries. In such more complete scenario exact tri-bimaximality would be just a first approximation, corrections leading to calculable deviations from the predictions reported here. These issues will be taken up elsewhere.

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[32] Due to its similarity to the columns of "Palacio da Alvorada" designed by this brilliant Brazilian arquitect.
[33] Quarks are also flavor singlets, hence the quark sector is completely standard and its flavor structure unpredicted.