Supersymmetric seesaw type II: CERN LHC and lepton flavour violating phenomenology

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Abstract

We study the supersymmetric version of the type-II seesaw mechanism assuming minimal supergravity boundary conditions. We calculate branching ratios for lepton flavour violating (LFV) scalar tau decays, potentially observable at the LHC, as well as LFV decays at low energy, such as $l_i \to l_j + \gamma$ and compare their sensitivity to the unknown seesaw parameters. In the minimal case of only one triplet coupling to the standard model lepton doublets, ratios of LFV branching ratios can be related unambigously to neutrino oscillation parameters. We also discuss how measurements of soft SUSY breaking parameters at the LHC can be used to indirectly extract information of the seesaw scale.

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I. INTRODUCTION

Neutrinos have mass and non-trivial mixing angles, as neutrino oscillation experiments have shown [1, 2, 3, 4, 5]. If neutrinos are *Majorana* particles, their mass at low energy is described by a unique dimension-5 operator [6]

$$m_\nu = \frac{f}{\Lambda}(HL)(HL). \tag{1}$$

Using only renormalizable interactions, there are only three tree-level realizations of this operator [7]. The first one is the exchange of a heavy fermionic singlet. This is the celebrated seesaw mechanism [8, 9, 10], which we will call seesaw type-I. The second possibility is the exchange of a scalar triplet [11, 12]. This is commonly known as seesaw type-II. And lastly, one could also add one (or more) fermionic triplets to the field content of the SM [13]. This is called seesaw type-III in [7], although this nomenclature is not universally accepted [1].

The dimension-5 operator of eq. (1) could also be generated at loop level. As the classical examples for loop generated neutrino masses we only mention the Zee model [17] (1-loop) and the Babu-Zee model [18] (2-loop), although many more models exist in the literature. A list of generic 1-loop realizations of eq. (1) can also be found in [7].

At "low" energies one can neither decide whether tree-level or loop physics generates eq. (1), nor can any measurements of neutrino angles, phases or masses distinguish between the different tree-level realizations of the seesaw discussed above. Observables outside the neutrino sector are needed to ultimately learn about the origin of eq. (1). For loop generated neutrino masses, f in eq. (1) can be a very small number and the scale Λ at which new physics appears can be quite low, probably accessible at future accelerators such as the LHC or an ILC. The "classical" tree-level realizations of the seesaw, unfortunately, can not be put to the test in such a direct way. This can be straightforwardly understood by inverting eq. (1), which results in $\Lambda \sim f\left(\frac{0.05 \text{ eV}}{m_\nu}\right)10^{15}$ GeV.

Indirect inside into the high-energy world might be possible in supersymmetric versions of the seesaw. In the renormalization group equations for the soft SUSY breaking slepton mass parameters terms proportional to the neutrino Yukawa couplings appear. If the scale where

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[1] Barr and Dorsner [14], for example, add additional singlets to the seesaw type-I. This version of the seesaw - which the authors call type-III - might be named "double seesaw, variant-II" to distinguish it from the original double seesaw [15], see also the related work in [16].

the right-handed neutrinos and/or the triplet decouple is below the scale at which SUSY breaks, lepton flavour violating (LFV) entries in the Yukawa matrices then induce LFV off-diagonals in the slepton mass matrices. This effect potentially leads to large values for lepton flavour violating lepton decays, such as $\mu \to e + \gamma$, even if the soft masses are completely flavour blind at high scale, as was first pointed out for the case of seesaw type-I in [19]. It is maybe not surprising then that with the increasingly convincing experimental evidence for non-zero neutrino masses a number of articles have studied the prospects for observing LFV processes, both at low energies and at future colliders, within the supersymmetric seesaw [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

Despite the fact that a minimal seesaw type-II has fewer free parameters than the seesaw type-I, type-I seesaw has received considerably more attention in the literature. Probably one of the reasons for this preference is gauge coupling unification. As is well known [34, 35], the SM gauge couplings unify within the minimal supersymmetric standard model (MSSM) at a scale around $M_G \simeq 2 \times 10^{16}$ GeV, if the SUSY particles have masses around the electroweak scale. Adding gauge singlets does not destroy this nice feature of the MSSM. However, a scalar triplet with mass below the GUT scale changes the running of g_1 and g_2 in an unwanted way and gauge coupling unification is lost [36]. A simple way to cure this defect of the seesaw-II consists in adding only complete $SU(5)$ multiplets (or GUT multiplets which can be decomposed into complete $SU(5)$ multiplets) to the standard model particle content. In this way the scale where couplings unify remains the same (at one loop level), only the value of the GUT coupling changes [37].

In this paper we calculate lepton flavour violating branching ratios of the scalar tau as well as LFV lepton decays at low energies, such as $l_i \to l_j + \gamma$ and $l_i \to 3l_j$. For definiteness, we assume minimal Supergravity (mSugra) boundary conditions and fit the observed neutrino masses by a seesaw mechanism of type-II. We will discuss two different realizations. The first one is based on adding one pair of triplets to the MSSM, from which only one couples to the standard model leptons. This is the simplest supersymmetric version of the type-II seesaw. The second model we consider consists in adding a pair of **15** and **$\overline{15}$** multiplets to the MSSM particle content [36]. This second option allows to maintain gauge coupling unification also for $M_{15} \ll M_G$.

We compare the sensitivities of low-energy and accelerator measurements and study their dependence on the unknown seesaw and SUSY parameters. Absolute values of LFV stau

decays and LFV lepton decays depend very differently on the unknown SUSY parameters. For a light SUSY spectrum, say slepton masses below 200 GeV, the current upper bound on $\mathrm{Br}(\mu \to e + \gamma)$ limits seriously the possibility to observe LFV scalar tau decays. However, for heavier sparticles low energy data very rapidly looses its constraining power and large LFV at the LHC is allowed by current data.

While absolute values of LFV observables depend very strongly on the soft SUSY breaking parameters, we discuss how ratios of LFV branching ratios can be used to eliminate most of the dependence on the unknown SUSY spectrum. I.e. ratios such as, for example, $\mathrm{Br}(\tilde{\tau}_2 \to e + \chi_1^0)/\mathrm{Br}(\tilde{\tau}_2 \to \mu + \chi_1^0)$ are constants for fixed neutrino parameters over large parts of the supersymmetric parameter space. Measurements of such ratios would allow to extract valuable information about the seesaw parameters: In the minimal type-II seesaw case these ratios can be calculated as function of measurable low-energy neutrino data. For the more involved case of the $\mathbf{15} + \overline{\mathbf{15}}$ model this simple connection is lost in general, but relations to neutrino data can be (re-) established in some simple, extreme cases for the Yukawa matrix $\mathbf{Y}_{15}$. We therefore study such ratios in some detail, first analytically then numerically.

The presence of new non-singlet states below the GUT scale does not only affect the running of gauge couplings but also the evolution of the soft SUSY breaking parameters. Measurements of soft SUSY masses at the LHC and at a possible ILC therefore contain indirect information about the physics at higher energy scales [28, 38]. ¿From the different soft scalar and gaugino masses one can define certain "invariants", i.e. parameter combinations which are nearly constant over large ranges of the mSugra parameter space [39], at least in leading order approximation. If the measured values of all the invariants depart from the mSugra expectation in a consistent way, one could gain some indirect estimate of the mass scale of the new particles, the scale of the seesaw type-II. We discuss first some leading order analytical approximation, before showing by numerical calculation the limitations of the simplified analytical approach. While the different invariants indeed contain useful information about the high energy physics, reliable quantitative conclusions about the mass scale of the $\mathbf{15}$ require highly precise measurements of soft masses as well as a full numerical 2-loop analysis.

The rest of this paper is organized as follows. In the next section we will recall the basic features of the supersymmetric seesaw type-II and discuss a $SU(5)$ motivated variant, which adds a pair of $\mathbf{15}$ and $\overline{\mathbf{15}}$. Section III then discusses analytical solutions for the RGEs

and presents estimates for slepton mixing angles and the corresponding LFV observables. In Section IV we present our numerical results for LFV decays at low energies and accelerators. This numerical study demonstrates the reliability of our analytical approximations for the LFV observables. We then discuss soft masses and the seesaw type-II scale, demonstrating by a numerically exact calculation that for soft masses the leading order approximations are not accurate enough to draw quantitative conclusions. We then summarize in section V.

II.   SETUP: MSUGRA WITH SEESAW TYPE II

In this section, to set up the notation, we briefly recall the main features of the seesaw type-II and mSugra. We then outline a simple $SU(5)$ motivated model based on the work of [36].

A.   Supersymmetric seesaw with triplet(s)

In supersymmetry at least two $SU(2)$ triplet states $T_{1,2}$ with opposite hypercharge are needed to cancel anomalies. Thus, the minimal SUSY potential including triplets can be written as

$$W = W_{\text{MSSM}} + \frac{1}{\sqrt{2}}\left(Y_T^{ij}L_iT_1L_j + \lambda_1 H_1 T_1 H_1 + \lambda_2 H_2 T_2 H_2\right) + M_T T_1 T_2. \tag{2}$$

Here T_1 (T_2) are supermultiplets with hypercharge $Y = 1$ ($Y = -1$) and $H_{1,2}$ are the standard Higgs doublets with $Y = \mp 1/2$. The matrix Y_T is complex symmetric, $\lambda_{1,2}$ are arbitrary constants and M_T gives mass to the triplets, supposedly at a very high scale. Note that only T_1 couples to the SM leptons, thus in the minimal (supersymmetric) model with two triplets the only source of lepton flavour violation resides in the matrix Y_T.

Integrating out the heavy triplets at their mass scale the dimension-5 operator of eq. (1) is generated and after electro-weak symmetry breaking the resulting neutrino mass matrix can be written as

$$m_\nu = \frac{v_2^2}{2}\frac{\lambda_2}{M_T}Y_T. \tag{3}$$

where v_2 is the vacuum expectation value of Higgs doublet H_2 and we use the convention $\langle H_i \rangle = \frac{v_i}{\sqrt{2}}$. Note that eq. (3) depends on the energy scale. m_ν is measured at low energies,

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whereas for the calculation of m_ν we need to know λ_2, Y_T and M_T as input paramters at the high scale. One can use an iterative procedure to find the high scale parameters from the low energy measured quantities, as explained in section IV. In the basis where the charged lepton masses are diagonal, eq. (3) is diagonalized by

$$\hat{m}_\nu = U^T \cdot m_\nu \cdot U, \tag{4}$$

where the neutrino mixing matrix U is, in standard notation [40], given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Here $s_{ij} \equiv \sin\theta_{ij}$ ($c_{ij} = \cos\theta_{ij}$). For Majorana neutrinos, U contains three phases: δ is the (Dirac-) CP violating phase, which appears in neutrino oscillations, and $\alpha_{1,2}$ are Majorana phases, which can only be observed in lepton number violating processes. Neutrino oscillation experiments can be fitted with either a normal hierarchical spectrum (NH), or with inverted hierarchy (IH). If one does not insist in ordering the neutrino mass eigenstates m_{ν_i}, $i = 1, 2, 3$ with respect to increasing mass, the matrix U can describe both possibilities without re-ordering of angles. In this convention, which we will use in the following, $m_{\nu_1} \simeq 0$ ($m_{\nu_3} \simeq 0$) corresponds to normal (inverse) hierarchy and s_{12}, s_{13} and s_{23} are the solar ($s_\odot$), reactor (s_R) and atmosperic angle (s_{Atm}) for both type of spectra.

Note that

$$\hat{Y}_T = U^T \cdot Y_T \cdot U \tag{6}$$

i.e. Y_T is diagonalized by *the same matrix as* m_ν. If all neutrino eigenvalues, angles and phases were known, Y_T would be fixed up to an overall constant which can be easily estimated to be

$$\frac{M_T}{\lambda_2} \simeq 10^{15}\text{GeV} \left(\frac{0.05 \text{ eV}}{m_\nu}\right). \tag{7}$$

At this points it might be worth recalling the main differences between seesaw type-II and seesaw type-I. In seesaw type-I there is one non-zero mass eigenstate for the light neutrinos for each right-handed neutrino added to the model. In contrast, seesaw-II can produce three non-zero neutrino masses with only one triplet. Thus the minimal model for seesaw type-II with only one triplet coupling to L has less parameters than seesaw type-I. We can count the new parameters in eq. (2): Y_T being complex symmetric has 9 parameters. Additionally

we have $\lambda_{1,2}$ and M_T. All three could in principle be complex. However, field redefinitions on T_1 and T_2 can be applied to remove two of the three phases, thus there is a a total of 13 parameters. Note, however, that only 11 of them are related to neutrino physics. Since we have the freedom to write down eq. (2) in the basis, where the charged lepton mass matrix is diagonal, we only have to add three charged lepton masses to the counting of free parameters.[2] This number should be compared to the 21 free parameters in seesaw type-I for three right-handed neutrinos [41]. At low energies a maximum of 12 parameters can be fixed by measuring lepton properties: 3 neutrino and 3 charged lepton masses, 3 angles and 3 phases. Thus from neutrino data neither seesaw type-II nor seesaw type-I can be completely reconstructed. However, especially important in the following is the fact, see eq. (6), that low-energy neutrino angles are directly related to the high-energy Yukawa matrix in seesaw-II, whereas no such simple connection exists in the seesaw type-I, see also the discussion in [22].

B. $SU(5)$ inspired model with $\mathbf{15} + \overline{\mathbf{15}}$

In this section we outline the basics of an $SU(5)$ inspired model, which adds a pair of $\mathbf{15}$ and $\overline{\mathbf{15}}$ to the MSSM particle spectrum [36]. Our numerical calculations will all be based on this variant, since it allows to maintain gauge coupling unification for $M_T \ll M_G$, as discussed in the introduction.

Under $SU(3) \times SU_L(2) \times U(1)_Y$ the $\mathbf{15}$ decomposes as

$$\mathbf{15} = S + T + Z \tag{8}$$
$$S \sim (6, 1, -\frac{2}{3}), \qquad T \sim (1, 3, 1), \qquad Z \sim (3, 2, \frac{1}{6}).$$

T has the same quantum numbers as the triplet T_1 discussed above. The $SU(5)$ invariant superpotential reads as

$$W = \frac{1}{\sqrt{2}}\mathbf{Y}_{15}\bar{5} \cdot 15 \cdot \bar{5} + \frac{1}{\sqrt{2}}\lambda_1\bar{5}_H \cdot 15 \cdot \bar{5}_H + \frac{1}{\sqrt{2}}\lambda_2 5_H \cdot \overline{15} \cdot 5_H + \mathbf{Y}_5 10 \cdot \bar{5} \cdot \bar{5}_H \tag{9}$$
$$+ \mathbf{Y}_{10} 10 \cdot 10 \cdot 5_H + M_{15} 15 \cdot \overline{15} + M_5 \bar{5}_H \cdot 5_H$$

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[2] In the non-supersymmetric version of seesaw-II $\frac{\lambda_2}{M_T} \to \frac{\mu}{M_T^2}$, with μ having dimension of mass, but the number of parameters related with neutrino physics does not change.

Here, $\bar{5} = (d^c, L)$, $10 = (u^c, e^c, Q)$, $5_H = (t, H_2)$ and $\bar{5}_H = (\bar{t}, H_1)$. Below the GUT scale in the $SU(5)$-broken phase the potential contains the terms

$$\frac{1}{\sqrt{2}}(Y_T L T_1 L + Y_S d^c S d^c) + Y_Z d^c Z L + Y_d d^c Q H_1 + Y_u u^c Q H_2 + Y_e e^c L H_1 \qquad (10)$$

$$+ \frac{1}{\sqrt{2}}(\lambda_1 H_1 T_1 H_1 + \lambda_2 H_2 T_2 H_2) + M_T T_1 T_2 + M_Z Z_1 Z_2 + M_S S_1 S_2 + \mu H_1 H_2$$

The first term in eq. (10) is responsible for the generation of the neutrino masses in the same way as discussed for the triplet-only case in the previous subsection. Y_d, Y_u and Y_e generate quark and charged lepton masses in the usual manner. However, in adddition there are the matrices Y_S and Y_Z, which, in principle, are not determined by any low-energy data. In the calculation of LFV observables in supersymmetry both matrices, Y_T and Y_Z, contribute. For the case of a complete **15**, apart from threshold corrections, $Y_T = Y_S = Y_Z$. One can recover the results for the simplest triplet-only model, as far as lepton flavour violation is concerned, by putting $Y_S = Y_Z = 0$.

As long as $M_Z \sim M_S \sim M_T \sim M_{15}$ gauge coupling unification will be mantained. The equality need not be exact for successful unification. In our numerical studies we have taken into account the different running of these mass parameters but we decouple them all at the scale $M_T(M_T)$ because the differences are small.

III. ANALYTICAL RESULTS

A. Approximate solutions for the RGEs

In mSugra one has in total five parameters at the GUT scale [42]. These are usually chosen to be M_0, the common scalar mass, $M_{1/2}$, the gaugino mass parameter, A_0, the common trilinear parameter, $\tan\beta = \frac{v_2}{v_1}$ and the sign of μ. For the full set of RGEs for the **15** + $\overline{\mathbf{15}}$ see [36]. In the numerical calculation, presented in the next section, we solve the exact RGEs. However, the following approximative solutions are very helpful in gaining a qualitative understanding.

The gauge couplings are given as

$$\alpha_1(m_Z) = \frac{5\alpha_{em}(m_Z)}{3\cos^2\theta_W}, \qquad \alpha_2(m_Z) = \frac{\alpha_{em}(m_Z)}{\sin^2\theta_W}, \tag{11}$$

$$\alpha_i(m_{SUSY}) = \frac{\alpha_i(m_Z)}{1 - \frac{\alpha_i(m_Z)}{4\pi}b_i^{SM}\log\frac{m_{SUSY}^2}{m_Z^2}},$$

$$\alpha_i(M_T) = \frac{\alpha_i(m_{SUSY})}{1 - \frac{\alpha_i(m_{SUSY})}{4\pi}b_i\log\frac{M_T^2}{m_{SUSY}^2}},$$

$$\alpha_i(M_G) = \frac{\alpha_i(M_T)}{1 - \frac{\alpha_i(M_T)}{4\pi}(b_i + \Delta b_i)\log\frac{M_G^2}{M_T^2}}.$$

$b^{SM} = (b_1, b_2, b_3)^{SM} = (\frac{41}{10}, -\frac{19}{6}, -7)$ for SM and $b = (b_1, b_2, b_3)^{MSSM} = (\frac{33}{5}, 1, -3)$ for MSSM. M_T denotes the mass of the triplet (15-plet). For the case of the complete 15-plet one finds $\Delta b_i = 7$ whereas for the case with triplets-only one finds $\Delta b_1 = 18/5$, $\Delta b_2 = 4$ and $\Delta b_3 = 0$. Using the equality $\alpha_1(M_G) = \alpha_2(M_G)$ determines the GUT-scale M_G via

$$\log\frac{M_G^2}{M_T^2} = \frac{1}{\alpha_1(m_{SUSY})\alpha_2(m_{SUSY})(b_1 + \Delta b_1 - b_2 - \Delta b_2)} \tag{12}$$
$$\cdot \left(4\pi(\alpha_2(m_{SUSY}) - \alpha_1(m_{SUSY})) + \alpha_1(m_{SUSY})\alpha_2(m_{SUSY})(b_2 - b_1)\log\frac{M_T^2}{m_{SUSY}^2}\right)$$

Note, that in the case of the complete 15-plet M_G is independent of M_T. For the gaugino masses one finds

$$M_i(m_{SUSY}) = \frac{\alpha_i(m_{SUSY})}{\alpha(M_G)}M_{1/2}. \tag{13}$$

Eq. (13) implies that the ratio M_2/M_1, which is measured at low-energies, has the usual mSugra value, but the relationship to $M_{1/2}$ is changed. Neglecting the Yukawa couplings $\mathbf{Y}_{15}$, see below, for the soft mass parameters of the first two generations one obtains

$$m_{\tilde{f}}^2 = M_0^2 + \sum_{i=1}^{3} c_i^{\tilde{f}}\left(\left(\frac{\alpha_i(M_T)}{\alpha(M_G)}\right)^2 f_i + f_i'\right)M_{1/2}^2, \tag{14}$$

$$f_i = \frac{1}{b_i}\left(1 - \left[1 + \frac{\alpha_i(M_T)}{4\pi}b_i\log\frac{M_T^2}{m_Z^2}\right]^{-2}\right),$$

$$f_i' = \frac{1}{b_i + \Delta b_i}\left(1 - \left[1 + \frac{\alpha(M_G)}{4\pi}(b_i + \Delta b_i)\log\frac{M_G^2}{M_T^2}\right]^{-2}\right). \tag{15}$$

The various coefficients $c_i^{\tilde{f}}$ are given in table I.

$\tilde{f}$	$\tilde{E}$	$\tilde{L}$	$\tilde{D}$	$\tilde{U}$	$\tilde{Q}$
$c_1^{\tilde{f}}$	$\frac{6}{5}$	$\frac{3}{10}$	$\frac{2}{15}$	$\frac{8}{15}$	$\frac{1}{30}$
$c_2^{\tilde{f}}$	0	$\frac{3}{2}$	0	0	$\frac{3}{2}$
$c_3^{\tilde{f}}$	0	0	$\frac{8}{3}$	$\frac{8}{3}$	$\frac{8}{3}$

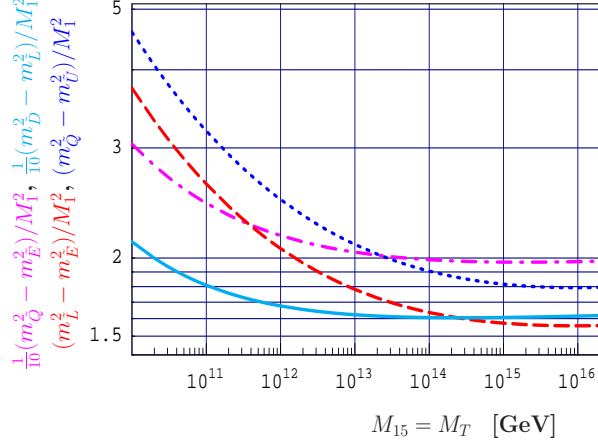TABLE I: Coefficients $c_i^{\tilde{f}}$ for eq. (14).

FIG. 1: Four different "invariant" combinations of soft masses (left) versus the mass of the **15**-plet, $M_{15} = M_T$. The plot assumes that the Yukawa couplings $\mathbf{Y}_{15}$ are negligibly small. The calculation is at 1-loop order in the leading-log approximation.

Individual SUSY masses depend strongly on the initial values for M_0 and $M_{1/2}$. However, one can form different combinations, such as

$$(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2 = \left(\frac{\alpha(M_G)}{\alpha_1(m_{SUSY})}\right)^2 \left(\frac{3}{2}\left[\left(\frac{\alpha_2(m_T)}{\alpha(m_G)}\right)^2 f_2 + f_2'\right] - \frac{9}{10}\left[\left(\frac{\alpha_1(m_T)}{\alpha(m_G)}\right)^2 f_1 + f_1'\right]\right),$$

which, to first approximation, are constants over large regions of mSugra space. We will call such combinations "invariants".

Figure (1) shows four different invariants as a function of $M_{15} = M_T$, calculated using eqs (13) - (14). For $M_T = M_G$ one reaches the mSugra limit. For lower values of M_T one obtains a logarithmic dependence on the value of M_T. If all the different invariants depart from their mSugra values in a consistent way, measurements of these parameter combinations can be used to obtain indirect information about the seesaw scale. In practice the "invariants" do depend on the SUSY spectrum and thus, indirectly still depend to some degree on the initial values of M_0 and $M_{1/2}$. We will discuss this point in more details in the numerical section.

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For the off-diagonal elements of the slepton mass matrix, we will discuss only the left sector, since right slepton mass parameters do not run to first order approximation [36]. In our numerical calculation we do solve the RGEs exactly and confirm this expectation. Off-diagonal elements are induced in $m_{\tilde{L}}^2$ due to the non-trivial flavour structure of the matrices Y_T and Y_Z. Y_T and Y_Z appear symmetrically in the RGEs [36]. Since only Y_T can be fixed from low-energy data, for a general Y_Z the off-diagonal entries of $m_{\tilde{L}}^2$ do not follow any correlation with low-energy physics. For this reason in the following we will consider two extreme cases: (a) $Y_Z = Y_T$, we will call this the 15-plet case; and (b) $Y_Z = 0$, we will refer to this as the triplet case.

For $m_{\tilde{L}}^2$ one finds the following approximation in the case of the 15-plet:

$$
\begin{aligned}
\Delta m_{\tilde{L},ij}^2 = & -\frac{1}{16\pi^2} \left(Y_T^\dagger Y_T \right)_{ij} \int_0^{\log \frac{M_G^2}{M_T^2}} \left(18 M_0^2 + \left(\frac{34}{5} f_1'(t) + 30 f_2'(t) + 16 f_3'(t) \right) M_{1/2}^2 \right. \\
& + 3(A_0 - \frac{9}{68} M_1'(t) - \frac{7}{8} M_2'(t))^2 \\
& \left. + 3(A_0 - \frac{7}{204} M_1'(t) - \frac{3}{8} M_2'(t) - \frac{4}{3} M_3'(t))^2 \right) dt \quad (16)
\end{aligned}
$$

$$
M_i'(t) = M_{1/2} \left(1 - \frac{1}{1 + \frac{1}{4\pi}(b_i + \Delta b_i)\alpha(M_G)t} \right) \quad (17)
$$

In case of the triplet one finds

$$
\begin{aligned}
\Delta m_{\tilde{L},ij}^2 = & -\frac{1}{16\pi^2} \left(Y_T^\dagger Y_T \right)_{ij} \int_0^{\log \frac{M_G^2}{M_T^2}} \left(9 M_0^2 + \left(\frac{27}{5} f_1'(t) + 21 f_2'(t) \right) M_{1/2}^2 \right. \\
& \left. + 3(A_0 - \frac{9}{68} M_1'(t) - \frac{7}{8} M_2'(t))^2 \right) dt \quad (18)
\end{aligned}
$$

The integration over t can be done analytically leading to corrections to the formulas for $(\Delta m_{\tilde{L}}^2)_{ij}$.

We have found that the approximation formulas shown above work less well than the corresponding formulas for the seesaw type-I case and only give a rough order of magnitude estimate. The reason for this difference is, that in seesaw type-I the Y_ν hardly run, unless left neutrinos are very degenerate, as either the Yukawas themselfes are small or in the case of large Yukwas the contribution from gauge and (top) Yukawa couplings nearly cancel each other. Such a cancellation does not take place in case of Y_T, thus leading to significantly stronger dependence of Y_T on the renormalization scale and consequently larger differences between the numerical solutions and eqs (16) and (18).

However, we have found that it is possible to improve the accuracy of the approximation formulas using the results of the next subsection, see eq. (25). The idea here is to replace the running Yukawa coupling Y_T by the measured low-energy neutrino masses and angles times the unkonw coupling λ_2. In case λ_2 is sufficiently small, this parameter does run very little and eqs (16) and (18) agree already very well with the numerical results. For large values of λ_2, we can find approximate solutions for the RGE for this parameter, following the procedure outlined in [28].

We define

$$X \equiv \frac{\lambda_2^2}{4\pi}, \quad Y_t \equiv \frac{y_t^2}{4\pi}. \tag{19}$$

The solution for the RGE for λ_2 is then given in terms of X(t) by $(t = \log \frac{M_G^2}{Q^2})$

$$X(t) = \frac{X(M_G)u_X(t)}{1 + \frac{7}{2\pi}X(M_G)\int_0^t u_X(t')dt'}, \tag{20}$$

$$u_X(t') = \frac{(1 + \frac{6}{2\pi}Y_t(M_G)t')^{1/\frac{6}{2\pi}}}{E_X(t')},$$

$$E_X(t') = \left(1 + \frac{b_1 + \Delta b_1}{2\pi}\alpha_1(M_G)t'\right)^{\frac{1}{2\pi}\frac{9}{5}/(b_1+\Delta b_1)} \left(1 + \frac{b_2 + \Delta b_2}{2\pi}\alpha_2(M_G)t'\right)^{\frac{1}{2\pi}7/(b_2+\Delta b_2)}.$$

we have found that, assuming an approximately constant Y_t, the above equations become easy to solve and describe the running of λ_2 to a rather good approximation. Eq. (20) and eq. (25), together with eqs (16) and (18) then allow to estimate LFV entries in $m_{\tilde{L}}^2$ up to an accuracy of typically a few percent.

B. Analytical results for flavour violating processes

Here we concentrate exclusively on the left-slepton sector. Taking into account the discussion given above this is expected to be a reasonable first approximation. The left-slepton mass matrix is diagonalized by a matrix $R^{\tilde{l}}$, which in general can be written as a product of three Euler rotations. However, if the mixing between the different flavour eigenstates is sufficiently small, the three differenct angles can be estimated by the following simple formula

$$\theta_{ij} \simeq \frac{(\Delta m_{\tilde{L}}^2)_{ij}}{(\Delta m_{\tilde{L}}^2)_{ii} - (\Delta m_{\tilde{L}}^2)_{jj}}. \tag{21}$$

LFV decays are directly proportional to the squares of these mixing angles as long as all angles are small. Taking the ratio of two decays, for example,

$$\frac{Br(\tilde{\tau}_2 \to e + \chi_1^0)}{Br(\tilde{\tau}_2 \to \mu + \chi_1^0)} \simeq \left(\frac{\theta_{\tilde{e}\tilde{\tau}}}{\theta_{\tilde{\mu}\tilde{\tau}}}\right)^2 \simeq \left(\frac{(\Delta m_{\tilde{L}}^2)_{13}}{(\Delta m_{\tilde{L}}^2)_{23}}\right)^2, \tag{22}$$

one expects that (a) all the unknown SUSY mass parameters and (b) the denominators of eq. (21) cancel approximately. To calculate estimates for different ratios of branching ratios we define

$$r_{kl}^{ij} \equiv \frac{|(\Delta m_{\tilde{L}}^2)_{ij}|}{|(\Delta m_{\tilde{L}}^2)_{kl}|} \tag{23}$$

where the observable quantities are $(r_{kl}^{ij})^2$. Of course, only two of the three possible combinations that can be formed are independent.

We next derive some analytical formulas for $(r_{kl}^{ij})^2$ in terms of observable neutrino parameters. The neutrino Yukawa coupling Y_T can be written in terms of observable parameters

$$Y_T = \frac{2M_T}{v_2^2 \lambda_2} m_\nu = \frac{2M_T}{v_2^2 \lambda_2} U^* \cdot \mathrm{diag}(m_1, m_2, m_3) \cdot U^\dagger. \tag{24}$$

The running of the soft-SUSY breaking slepton mass matrix $(m_{\tilde{L}}^2)_{ij}$ is proportional to the parameter combination $(Y_T^\dagger Y_T)_{ij}$. [3] This combination can again be expressed in terms of low-energy neutrino observables times an unknown scale:

$$\begin{aligned} (Y_T^\dagger Y_T)_{ij} &= \left[\frac{2M_T}{v_2^2 \lambda_2}\right]^2 \left(U \cdot \mathrm{diag}(m_1^2, m_2^2, m_3^2) \cdot U^\dagger\right)_{ij} \\ &\equiv \widetilde{m}^{-2} \sum_k U_{ik} U_{jk}^* m_k^2, \end{aligned} \tag{25}$$

The different off-diagonal entries are explicitly given as

$$(Y_T^\dagger Y_T)_{12} = \widetilde{m}^{-2} \left[U_{11} U_{21}^* m_1^2 + U_{12} U_{22}^* m_2^2 + U_{13} U_{23}^* m_3^2\right], \tag{26}$$

$$(Y_T^\dagger Y_T)_{13} = \widetilde{m}^{-2} \left[U_{11} U_{31}^* m_1^2 + U_{12} U_{32}^* m_2^2 + U_{13} U_{33}^* m_3^2\right],$$

$$(Y_T^\dagger Y_T)_{23} = \widetilde{m}^{-2} \left[U_{21} U_{31}^* m_1^2 + U_{22} U_{32}^* m_2^2 + U_{23} U_{33}^* m_3^2\right].$$

Inserting the convention for the matrix U from eq. (5) results in

$$(Y_T^\dagger Y_T)_{12} \propto c_{12} s_{12} c_{13} c_{23} (m_2^2 - m_1^2) - c_{13} s_{13} s_{23} e^{-i\delta} \{(m_3^2 - m_2^2) + c_{12}^2 (m_2^2 - m_1^2)\}, \tag{27}$$

$$(Y_T^\dagger Y_T)_{13} \propto c_{12} s_{12} c_{13} s_{23} (m_1^2 - m_2^2) - c_{13} s_{13} c_{23} e^{-i\delta} \{(m_3^2 - m_2^2) + c_{12}^2 (m_2^2 - m_1^2)\},$$

$$(Y_T^\dagger Y_T)_{23} \propto s_{23} c_{23} \left((s_{12}^2 - c_{12}^2)(m_2^2 - m_1^2) + c_{13}^2 \{(m_3^2 - m_2^2) + c_{12}^2 (m_2^2 - m_1^2)\}\right)$$
$$- s_{12} c_{12} s_{13} (c_{23}^2 e^{-i\delta} - s_{23}^2 e^{i\delta})(m_2^2 - m_1^2).$$

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[3] For the triplet-only case. For the **15** case we assume $Y_Z = Y_T$ at M_G, see the previous subsection.

Note, that the off-diagonals can be expressed as a function of mass squared differences only, i.e. there is no dependence on the overall neutrino mass scale. However, again note that eq. (27) depends on the energy scale, see the discussion below eq. (3) and in section (IV). Also it is worth mentioning that with the convention of U from eq. (5) the Majorana phases cancel in eq. (27).

As a starting approximation for the following discussion, let us assume that the lepton mixing matrix has exact tri-bimaximal (TBM) form [43]

$$U = U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{28}$$

As is well-known, eq. (28) is an excellent first-order approximation to the measured neutrino mixing angles [44]. For these values eq. (27) simplifies to

$$|(Y_T^\dagger Y_T)_{12}| \propto \frac{1}{3}\Delta m_\odot^2 \tag{29}$$

$$|(Y_T^\dagger Y_T)_{13}| \propto \frac{1}{3}\Delta m_\odot^2,$$

$$|(Y_T^\dagger Y_T)_{23}| \propto \frac{1}{2}\Delta m_{\text{Atm}}^2.$$

The ratios $(r_{13}^{12}) = 1$ and $(r_{23}^{12}) = (r_{23}^{13}) = \frac{2}{3}\frac{\Delta m_\odot^2}{\Delta m_{\text{Atm}}^2}$ result.

For the general mixing matrix one can derive $(r_{kl}^{ij})^2$ using eq. (27). For the currently allowed ranges of the neutrino parameters, the most important unknown turns out to be s_{13}, as fig. (2) demonstrates. In this figure $(r_{kl}^{ij})^2$ are shown as function of s_{13}^2 for $\tan^2\theta_A = 1$ and $\tan^2\theta_\odot = 1/2$, as well as for the Δm^2 fixed at their best fit point values [44]. Currently $s_{13}^2 \leq 0.05$ at $3\,\sigma$ c.l. r_{kl}^{ij} strongly depend on the value of s_{13} and there exists a special value of s_{13}, for which either (r_{23}^{12}) or (r_{23}^{13}) even vanish, due to a cancellation between the different terms in eq. (27). Note, however, that (r_{23}^{12}) and (r_{23}^{13}) can not vanish simultaneously. Note also, that for $\tan^2\theta_A = 1$, $(r_{23}^{12})^2$ and $(r_{23}^{13})^2$ are symmetric under the exchange of $\delta = 0 \leftrightarrow \delta = \pi$. Also, for non-zero values of s_{13} the results depend on the assumed hierarchy of the left neutrinos and the simultaneous exchange of the cases (normal hierarchy) NH $\leftrightarrow$ IH (inverse hierarchy) and $(\delta = 0) \leftrightarrow (\delta = \pi)$ leads to the same values for the different r_{kl}^{ij} in case of $\tan^2\theta_A = 1$.

Table (II) shows the currently allowed ranges for the $(r_{kl}^{ij})^2$ for $s_{13} = 0$ and s_{13}^{max} for different assumptions about the remaining neutrino parameters for the different cases of NH and
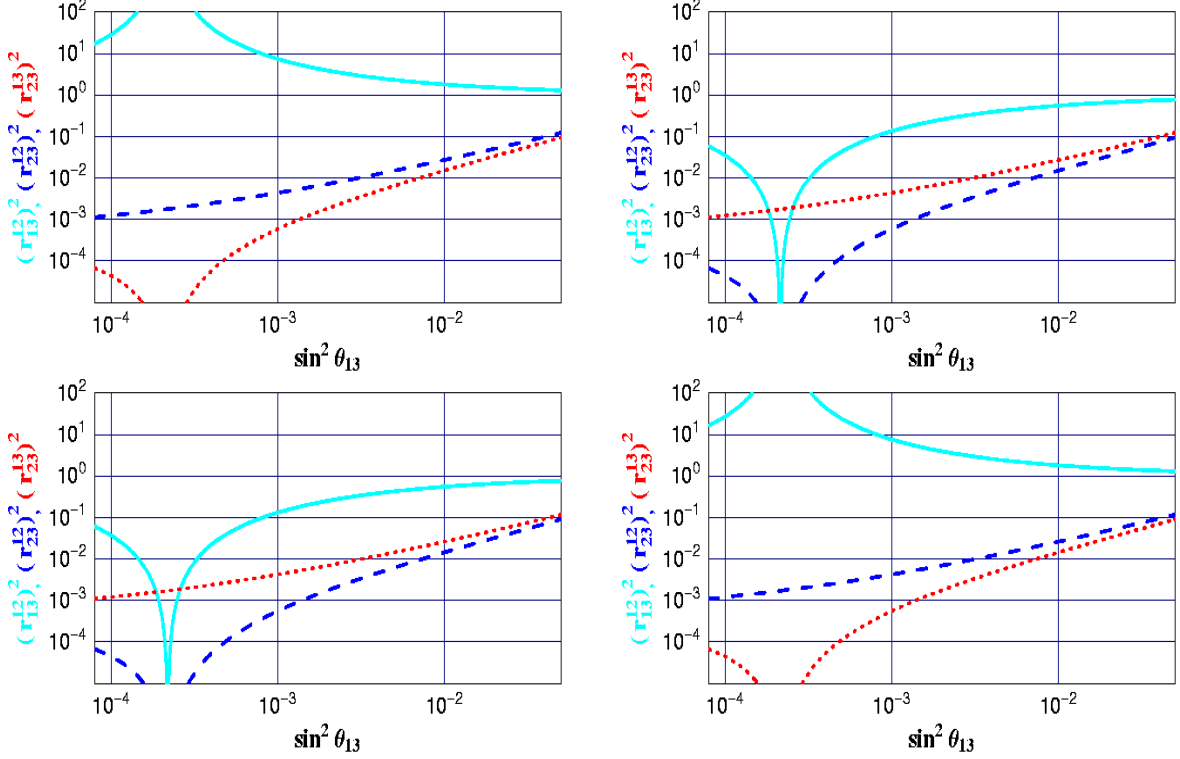
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FIG. 2: Square ratios $(r_{13}^{12})^2$ (light blue line, full line), $(r_{23}^{12})^2$ (blue line, dashed line) and $(r_{23}^{13})^2$ (red line, dotted line) versus s_{13}^2 for NH (upper panels), IH (lower panels) for $\delta = 0$ (left panels) and $\delta = \pi$ (right panels). The other light neutrino parameters have been fixed to their b.f.p. values. Note, that for $\tan^2 \theta_A = 1$, $(r_{23}^{12})^2$ and $(r_{23}^{13})^2$ are symmetric under the exchange of $\delta = 0 \leftrightarrow \delta = \pi$. Also the simultaneous exchange of NH $\leftrightarrow$ IH and $(\delta = 0) \leftrightarrow (\delta = \pi)$ leads to the same values for the different r_{kl}^{ij}, in case of $\tan^2 \theta_A = 1$.

IH. These values serve to indicate the allowed variations for r_{kl}^{ij} due to other parameters than s_{13}. As stated above, the allowed variation on s_{13} is most important for the "uncertainties" in $(r_{kl}^{ij})^2$. However, also the current error bar on $\tan^2 \theta_A$ leads to a sizeable variation on r_{kl}^{ij}. Since Δm_{Atm}^2 and $\Delta m_{\odot}^2$ are now known to much better precision than the neutrino angles, their variation is much less important for the $(r_{kl}^{ij})^2$, as table (II) demonstrates.

Finally, recall that all results presented in this subsection are based on the assumption that one of the extreme cases, $Y_Z = Y_T$ or $Y_Z = 0$, is realized. The former corresponds to the SU(5) inspired model with a complete **15** of section (II B), whereas the latter corresponds to the simplest triplet-only model discussed in section (II A). However, we stress that departures in the ratios of LFV branching ratios from the values calculated in this subsection should not be interpreted as "ruling out" the seesaw type-II. Rather they should

		NH		IH	
		$\delta = 0$	$\delta = \pi$	$\delta = 0$	$\delta = \pi$
$s_{12} = 1/\sqrt{3}$	$(r_{13}^{12})^2$	1		1	
$s_{23} = 1/\sqrt{2}$	$(r_{23}^{12})^2$	$[2.8, 7.4] \times 10^{-4}$		$[2.8, 7.2] \times 10^{-4}$	
$s_{13} = 0$	$(r_{23}^{13})^2$	$[2.8, 7.4] \times 10^{-4}$		$[2.8, 7.2] \times 10^{-4}$	
$s_{12} = 1/\sqrt{3}$	$(r_{13}^{12})^2$	$[1.2, 1.4]$	$[0.71, 0.81]$	$[0.71, 0.81]$	$[1.2, 1.4]$
$s_{23} = 1/\sqrt{2}$	$(r_{23}^{12})^2$	$[0.12, 0.13]$	$[0.091, 0.096]$	$[0.085, 0.093]$	$[0.11, 0.12]$
$s_{13} = s_{13}^{\max}$	$(r_{23}^{13})^2$	$[0.091, 0.096]$	$[0.12, 0.13]$	$[0.11, 0.12]$	$[0.085, 0.093]$
$s_{12} \neq 1/\sqrt{3}$	$(r_{13}^{12})^2$	$[0.49, 1.94]$		$[0.49, 1.94]$	
$s_{23} \neq 1/\sqrt{2}$	$(r_{23}^{12})^2$	$[1.8, 12] \times 10^{-4}$		$[1.8, 12] \times 10^{-4}$	
$s_{13} = 0$	$(r_{23}^{13})^2$	$[1.8, 12] \times 10^{-4}$		$[1.8, 12] \times 10^{-4}$	
$s_{12} \neq 1/\sqrt{3}$	$(r_{13}^{12})^2$	$[0.63, 3.0]$	$[0.35, 1.7]$	$[0.35, 1.7]$	$[0.63, 3.0]$
$s_{23} \neq 1/\sqrt{2}$	$(r_{23}^{12})^2$	$[0.094, 0.18]$	$[0.062, 0.15]$	$[0.060, 0.15]$	$[0.089, 0.18]$
$s_{13} = s_{13}^{\max}$	$(r_{23}^{13})^2$	$[0.061, 0.15]$	$[0.093, 0.18]$	$[0.088, 0.17]$	$[0.058, 0.14]$

TABLE II: The parameters r_{kl}^{ij} are given for several values of the neutrino mixing angles. $s_{13}^{\max}$ is the experimentally allowed maximum value: $(s_{13}^{\max})^2 = 0.050$ at (3σ) c.l. NH and IH are normal and inverted hierarchy of neutrino masses, respectively. The intervals correspond to (3σ) experimental allowed range of neutrino oscillation parameters: $s_{12}^2 = 0.26 - 0.40$, $s_{23}^2 = 0.34 - 0.67$, $\Delta m_{\odot}^2 = (7.1 - 8.3) \times 10^{-5} \text{eV}^2$ and $\Delta m_{\text{Atm}}^2 = (2.0 - 2.8) \times 10^{-3} \text{eV}^2$. In the top two rows only the mass squared splittings are varied, while for the lower set also angles are allowed to vary.

be interpreted in the sense that one has to go beyond minimal scenarios.

IV. NUMERICAL RESULTS

In this section we present our numerical calculations. All results presented below have been obtained with the lepton flavour violating version of the program package SPheno [45]. Calculations are done for the 15-plet case, using the assumption $Y_Z = Y_T$ at M_G, as discussed above. Unless mentioned otherwise, we fit neutrino mass squared differences to their best fit values [44] and the angle to TBM values. Our numerical procedure is as follows. Inverting

the seesaw equation, see eq. (3), one can get a first guess of the Yukawa couplings for any fixed values of the light neutrino masses (and angles) as a function of the corresponding triplet mass for any fixed value of λ_2. This first guess will not give the correct Yukawa couplings, since the neutrino masses and mixing angles are measured at low energy, whereas for the calculation of m_ν we need to insert the parameters at the high energy scale. However, we can use this first guess to run numerically the RGEs to obtain the exact neutrino masses and angles (at low energies) for these input parameters. The difference between the results obtained numerically and the input numbers can then be minimized in a simple iterative procedure until convergence is achieved. As long as neutrino Yukawas are not too close to one we reach convergence in a few steps. However, in seesaw type-II the Yukawas run stronger than in seesaw type-I, thus our initial guess can deviate sizeably from the exact Yukawas. Since neutrino data requires at least one neutrino mass to be larger than about 0.05 eV, we do not find any solutions for $M_T \gtrsim \lambda_2 \cdot 10^{15}$ GeV.

We have implemented the effects of the additional triplets (15-plets) including the two-loop contributions to the RGEs for gauge couplings and gaugino masses, one-loop contributions to the remaining MSSM parameters and one-loop RGES for the new parameters in SPheno. For consistency we have also included 1-loop threshold corrections for gauge couplings and gaugino mass parameters at the scale corresponding to the mass of the triplet. The MSSM part is implemented at the 2-loop level and, thus, in principle one should also include the effect of the 15-plets consistently for all parameters at this level. However, the correct fit of the neutrino data require that either the triplet (15-plet) Yukawa couplings are small and/or that M_T is close to M_G implying that the ratio M_T/M_G is significantly smaller than M_G/m_Z and thus one expects only small effects.

A. Numerical results for LFV

The analytical results presented in the previous section allow to estimate ratios of branching ratios for LFV decays. For absolute values of the branching ratios, as well as for cross-checking the reliability of the analytical estimates, one has to resort to a numerical calculation. Below we show results only for a few "standard" mSugra points, namely for SPS3 [46] and SPS1a' [47]. However, we have checked with a number of other points that our results for ratios of branching ratios are generally valid.
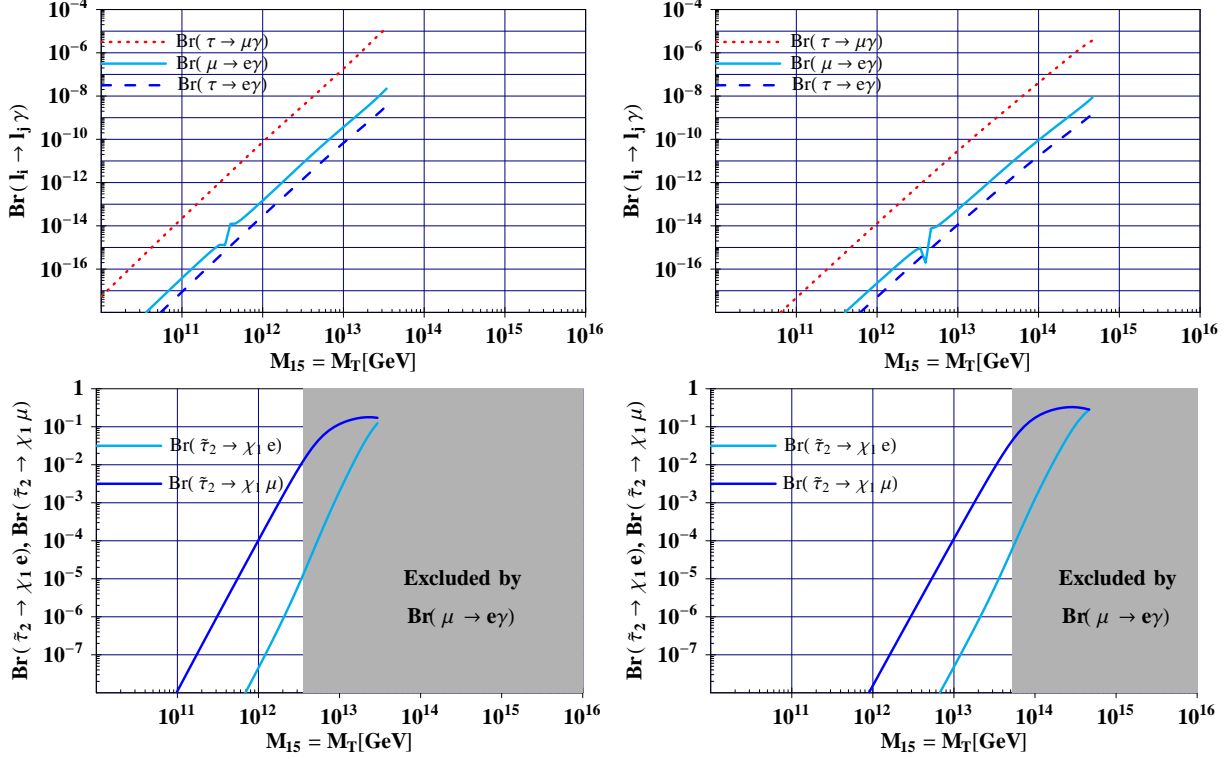
FIG. 3: Lepton flavour violating branching ratios versus $M_T = M_{15}$ for the standard mSugra point SPS3 for two values of λ_2. To the left $\lambda_2 = 0.05$, to the right $\lambda_2 = 0.5$. The plots show $Br(l_i \to l_j + \gamma)$ (top) and $Br(\tilde{\tau}_2 \to e, \mu + \chi_1^0)$ (bottom). Ratios of the different branching ratios follow closely the analytical expectations. The regios excluded by the current upper limit on $Br(\mu \to e + \gamma)$ is shown also in the lower plot.

Fig. (3) shows examples of LFV decays for the mSugra point SPS3 as a function of $M_T = M_{15}$ for two different values of λ_2. The upper plots show $Br(l_i \to l_j + \gamma)$, while the lower ones show $Br(\tilde{\tau}_2 \to e, \mu + \chi_1^0)$. We have also calculated $Br(l_i \to 3l_j)$, but these are not shown in the plots, because they follow very well the approximate relation [24, 48]

$$\frac{\text{Br}(l_i \to 3l_j)}{\text{Br}(l_i \to l_j + \gamma)} \simeq \frac{\alpha}{3\pi}\Big(\log(\frac{m_{l_i}^2}{m_{l_j}^2}) - \frac{11}{4}\Big). \tag{30}$$

All LFV branching ratios show a very strong dependence on the value of M_T and due to the stronger running of parameters in the seesaw type-II case, compared to the seesaw type-I, the dependence on the seesaw scale is stronger than in seesaw-I [49]. See also the discussion in section (III).

For the calculation shown in fig. (3), we have fitted the neutrino angles to exact tribimaximal values. One sees that, as long as the different LFV branching ratios are small,

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ratios of branching ratios are constants, which follow very well the analytical expectations. Currently the most important phenomenological constraints comes from the upper limit on $Br(\mu \to e + \gamma)$, $Br(\mu \to e + \gamma) \leq 1.2 \cdot 10^{-11}$ [40]. Note that the "dip" in $Br(\mu \to e + \gamma)$ is due to a level-crossing of selectron and smuon mass eigenstates. [4] For SPS3 one finds that this limit rules out $Br(\tilde{\tau}_2 \to \mu + \chi_1^0)$ larger than a few percent, the exact number depending on the unknown parameter λ_2. Fig. (3) to the left (right) shows results for $\lambda_2 = 0.05$ ($\lambda_2 = 0.5$). Recall that neutrino physics fixes only M_T/λ_2. However, note also that the upper limit on $Br(\tilde{\tau}_2 \to \mu + \chi_1^0)$ depends only weakly on λ_2.
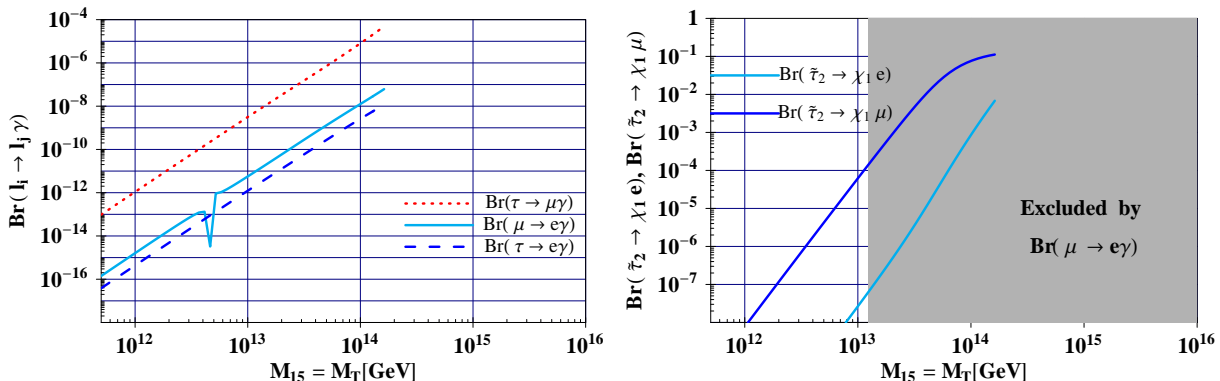

FIG. 4: As fig. (3), but for the mSugra standard point SPS1a' and for $\lambda_2 = 0.5$. For a slepton spectrum as light as expected for SPS1a' $Br(\mu \to e + \gamma)$ rules out the possibility to observe large lepton flavour violating slepton decays at the LHC.

It is well-known that absolute values of LFV branching ratios depend very strongly on the SUSY spectrum, for example $Br(\mu \to e + \gamma) \propto 1/m_{SUSY}^8$ [21]. Since both left-sleptons as well as (lightest) neutralino and chargino are approximately a factor of two heavier for SPS3 than for SPS1a', one expects that $Br(\mu \to e + \gamma)$ gives a strong constraint on the observability of LFV at the LHC for SPS1a'. This is confirmed numerically, as shown in fig. (4), which shows $Br(l_i \to l_j + \gamma)$ and $Br(\tilde{\tau}_2 \to e, \mu + \chi_1^0)$ as function of $M_T = M_{15}$ for the example of $\lambda_2 = 0.5$. Given the current limit on $Br(\mu \to e + \gamma)$ one expects $Br(\tilde{\tau}_2 \to \mu + \chi_1^0) \lesssim$ (few) 10^{-4}. Note that again we have fitted neutrino angles to tri-bimaximal values in this

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[4] In mSugra the left-selectron is usually slightly lighter than the left-smuon. In the mSugra plus seesaw case, for both type-I and type-II seesaw, the additional Yukawas change the running of the slepton masses. For large Yukawas (i.e. large M_T) fitting current neutrino data requires couplings such that the smuon mass runs faster to smaller values than the selectron mass. However, the splitting between seletron and smuon mass eigenstates is expected to be too small to be measurable in most parts of the parameter space, see the discussion in the next subsection.

calculation and that ratios of LVF branching ratios follow closely the analytical expressions.

B. Sparticles Masses and seesaw scale

As disdussed in the analytic section, the running of soft parameters allows, in principle, an indirect determination of the seesaw scale. In this section we discuss numerical results for the running of the "invariants" defined above. Although below we show plots only for the combination $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ we have checked numerically that all invariants shown in fig. (1) behave qualitatively in the same way.
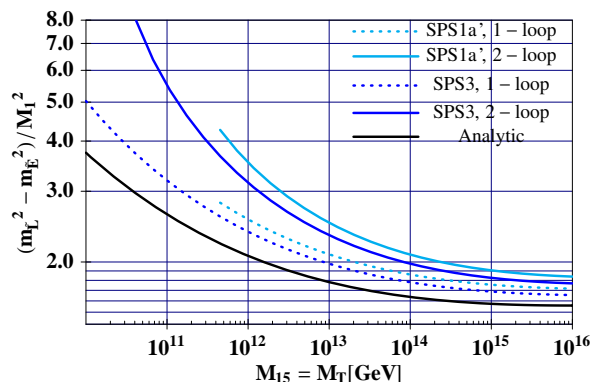

FIG. 5: "Invariant" $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$, calculated with negligibly small Yukawa couplings for two mSugra standard points. The figure shows a comparsion of different calculation. The curve labeled "Analytic" uses the formulas presented in the previous section. 1-loop and 2-loop stand for exactly solved numerical calculations using 1-loop and 2-loop RGEs. Note the significant shift when going from 1-loop order to 2-loop order.

Fig. (5) shows $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ as a function of $M_T = M_{15}$ for SPS1a' and SPS3 comparing different calculations. This plot assumes that the Yukawas of the 15-plet are negligibly small, i.e. neutrino mass are *not correctly fitted in this calculation*. The black line is the analytical calculation based on 1-loop RGEs and the leading-log approximation with an assumed $m_{SUSY} = 1$ TeV. The dotted lines are the numerically exact results for this invariant using 1-loop RGEs, while the full lines are the exact results using 2-loop RGEs. Obviously the "invariant" does depend to a certain degree on the mSugra point, as already pointed out in section (III). However, we also find a considerable upward shift of $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$, when going from the 1-loop to the 2-loop calculation. Since the dependence of $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ on the value of M_T is only logarithmic, even such a moderate change in the invariant is

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important, if one wants to extract an indirect estimate on M_T from such a measurement. Note that for the point SPS1a' the calculation stops at $M_{15} \sim 10^{11.6}$ GeV, the lowest value of M_{15} for which correct electro-weak symmetry breaking occurs.

We have checked by an exact numerical calculation that the other invariants shown in section (III) suffer from similar changes when going from 1-loop order to 2-loop. In other words, if one wants to learn about the seesaw scale from measurements of the soft masses, a careful analysis at 2-loop order will be necessary. Also note that, due to the logarithmic dependence on M_T, highly precise measurements will be necessary, especially if M_T is large, say $M_T \geq 10^{12-13}$ GeV.
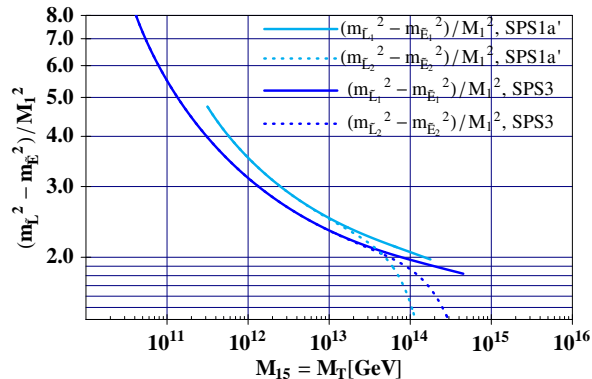

FIG. 6: "Invariant" $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ calculated with Yukawa couplings fitted to neutrino data, for an arbitrary choice of $\lambda_2 = 0.5$. The calculation uses 2-loop RGEs. Results are shown for SPS1a' and SPS3. Neutrino angles are assumed to have exact TBM values.

Fig. (6) shows $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ calculated with Yukawa couplings fitted to neutrino data, for an arbitrary choice of $\lambda_2 = 0.5$. The calculation uses 2-loop RGEs and results are shown again for the mSugra standard points SPS1a' and SPS3. For M_T low, say $M_T \leq 10^{13}$ GeV or so in this example, Yukawa couplings which explain current neutrino data are too small to induce any significant effect in the determination of $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$.

However, for larger values of M_T sizeable differences between fig. (5) and fig. (6) show up. First of all, for negligibly small Yukawas the calculation can vary M_T freely up to the GUT scale. If instead we insist to fit neutrino masses, such large values for M_T are not allowed. The downward turn in $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ is due to Yukawas, which if larger than $\mathcal{O}(0.1)$ contribute sizeable in the running of the soft parameters. In the example shown in this figure $\lambda_2 = 0.5$ has been chosen. For smaller values of λ_2 again for fixed values of the Yukawa couplings to fit neutrino masses a lower M_T is required. Correspondingly, for

smaller λ_2 the effect of the Yukawas is seen for smaller values of M_T.

It is also found that slepton mass parameters of the first and second generation run differently for large values of M_T, see fig. (6). This difference can be traced to the fact that we have fitted neutrino angles to take exact TBM values. In this limit, $m_{\tilde{L}_1}^2 \propto \Delta m_\odot^2$, while $m_{\tilde{L}_2}^2 \propto \Delta m_{\text{Atm}}^2$. Thus, at the largest values of M_T sizeable mass differences between 1st and 2nd generation sleptons show up. This difference is expected to be smaller for non-zero values of s_{13}. Note, however, that for the example points shown in fig. (6), there is the upper limit on M_T from $Br(\mu \to e + \gamma)$, discussed in the last subsection. For SPS1a' $M_T \lesssim 1.5 \cdot 10^{13}$ GeV, for SPS3 $M_T \lesssim 6 \cdot 10^{13}$ GeV. This limits the range of M_T where differences between 1st and 2nd generation slepton masses might be observable. We mention that a recent paper [50] claims that mass differences between smuons and selectrons can be measured very accurately, even at the LHC. Depending on the mSugra point $(m_{\tilde{\mu}}^2 - m_{\tilde{e}}^2)/(m_{\tilde{\mu}}^2 + m_{\tilde{e}}^2)$ as small as $O(10^{-4})$ might be measurable [50] provided the leptons have sufficient energy to pass the experimental cuts.
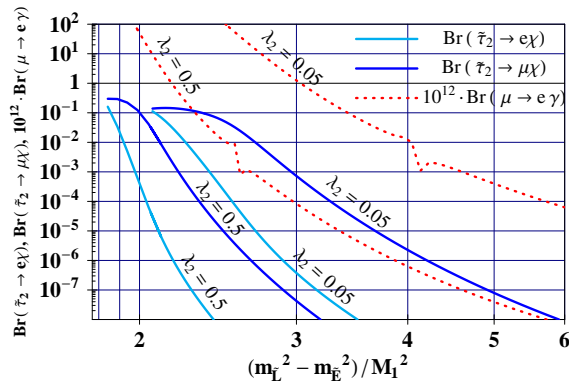

FIG. 7: Branching ratios for LFV decays versus $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ for SPS3 for two different values of λ_2. Measuring both types of observables allow in principle to disentangle λ_2 and M_T.

All observables discussed so far are sensitive only to a combination of M_T and λ_2. If, however, both LFV decays as well as $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$ could be measured in the future, one could disentangle the two parameters, in principle, by combining both measurements. This is demonstrated in fig. (7), which shows LFV decays, $Br(\mu \to e + \gamma)$ and $Br(\tilde{\tau}_2 \to e, \mu + \chi_1^0)$ versus $(m_{\tilde{L}}^2 - m_{\tilde{E}}^2)/M_1^2$, for two different values of λ_2. Note again that the "dip" in $Br(\mu \to e + \gamma)$ is due to a level-crossing of selectron and smuon mass eigenstates. However, again we warn that a full 2-loop calculation is needed, before any quantitative conclusions could be drawn from such a measurement.

V. CONCLUSIONS

We have studied phenomenological implications of the supersymmetric version of the type-II seesaw within mSugra. We have calculated lepton flavour violating observables, such as $Br(l_i \rightarrow l_j + \gamma)$ and LFV scalar tau decays. We have found branching ratios for LFV violating stau decays are large enough to be detectable at the LHC in principle. We have pointed out that in the simplest case of only one triplet coupling to the SM leptons, ratios of LFV branching ratios can be calculated from low energy neutrino data only. However, for the case of a complete **15** multiplet the situation is not as straightforward. In the SU(5) inspired model the Yukawa couplings Y_T and Y_Z are related to the $\mathbf{Y_{15}}$ and the conclusions remain unchanged. However, allowing Y_T and Y_Z to be free parameters, the relation with neutrino physics is lost. Thus, seesaw type-II can not be ruled out by any LFV measurements in general. Instead measuring ratios of LFV branching ratios can be understood as a consistency check for the minimal seesaw type-II models.

We have also calculated the soft masses as a function of the seesaw parameters. As discussed in some detail, there are certain combinations of soft masses, which are approximately constants over large regions of mSugra space. These "invariants" contain indirect information about the seesaw scale. Measuring SUSY masses as precisely as possible will therefore allow to constrain the scale of seesaw type-II indirectly. However, theoretically there are many possibilities, why any single of the "invariants" we discussed could depart from the simplest mSugra expectations. Only a consistent departure of several "invariants", together with measurements of LFV processes, could therefore be taken as a hint for seesaw type-II.

APPENDIX A: CONTRIBUTIONS TO THE β FUNCTIONS

Using general formulas by [51] we obtain for the RGEs of the gauge couplings:

$$\frac{dg_a}{dt} = \frac{1}{16\pi^2}B_a^{(1)}g_a^3 + \left(\frac{1}{16\pi^2}\right)^2 g_a^3\left(B_{ab}^{(2)}g_b^2 + C_a^b Tr\left(Y_b Y_b^\dagger\right) + D_a^b|\lambda_b|^2\right) \tag{A1}$$

with

$$B_1 = b_1 + \frac{3}{5}\left(\frac{8}{3}n_S + 3n_T + \frac{1}{6}n_Z\right)$$

$$B_2 = b_2 + \frac{3}{5}\left(\frac{8}{3}n_S + 3n_T + \frac{1}{6}n_Z\right)$$

$$B_3 = b_3 + \frac{3}{5}\left(\frac{8}{3}n_S + 3n_T + \frac{1}{6}n_Z\right) \tag{A2}$$

$$B_{ab}^{(2)} = b_{ab}^{(2)} + b_{ab}^{(2,S)}n_S + b_{ab}^{(2,T)}n_T + b_{ab}^{(2,Z)}n_Z \tag{A3}$$

$$C_a^{u,d,l,S,T,Z} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} & \frac{18}{5} & \frac{27}{5} & \frac{14}{5} \\ 6 & 6 & 2 & 0 & 7 & 6 \\ 4 & 4 & 0 & 6 & 0 & 4 \end{pmatrix} \quad , \quad D_a^b = \begin{pmatrix} \frac{27}{5} & \frac{27}{5} \\ 7 & 7 \\ 0 & 0 \end{pmatrix} \tag{A4}$$

and $(b_1, b_2, b_3) = (33/5, 1, -3)$,

$$b_{ab}^{(2)} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} \ , \ b_{ab}^{(2,S)} = \begin{pmatrix} \frac{128}{75} & 0 & \frac{64}{3} \\ 0 & 0 & 0 \\ \frac{8}{3} & 0 & \frac{145}{3} \end{pmatrix} \ , \ b_{ab}^{(2,T)} = \begin{pmatrix} \frac{108}{25} & \frac{72}{5} & 0 \\ \frac{24}{5} & 24 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \ b_{ab}^{(2,Z)} = \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{21}{2} & 8 \\ \frac{1}{15} & 3 & \frac{34}{3} \end{pmatrix} \tag{A5}$$

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