IFIC/10-19

Discrete dark matter

M. Hirsch,[1,*] S. Morisi,[1,†] E. Peinado,[1,‡] and J. W. F. Valle[1,§]

[1] *AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València Edificio Institutos de Paterna, Apt 22085, E–46071 Valencia, Spain*

(Dated: November 11, 2010)

We propose a new motivation for the stability of dark matter (DM). We suggest that the same non-abelian discrete flavor symmetry which accounts for the observed pattern of neutrino oscillations, spontaneously breaks to a Z_2 subgroup which renders DM stable. The simplest scheme leads to a scalar doublet DM potentially detectable in nuclear recoil experiments, inverse neutrino mass hierarchy, hence a neutrinoless double beta decay rate accessible to upcoming searches, while $\theta_{13} = 0$ gives no CP violation in neutrino oscillations.

PACS numbers: 95.35.+d 11.30.Hv 14.60.-z 14.60.Pq 12.60.Fr 14.60.St 23.40.Bw

Introduction The existence of dark matter (DM) plays a central role in the modeling of structure formation and galaxy evolution, affecting also the cosmic microwave background. Despite the strong evidence in favor of DM, its detailed nature remains rather elusive. Viable particle physics candidates for dark matter must be electrically neutral, and provide the correct relic abundance. Therefore they must be stable over cosmological time scales. A simple way to justify the stability of the DM is by *assuming* some parity symmetry Z_2, which might arise from the spontaneous breaking of an abelian U(1) gauge symmetry [1–3] [1], or from a non-abelian discrete symmetry, as might be the case in some string models [4].

Non abelian discrete symmetries are motivated by neutrino oscillation data [5, 6]. Here we propose that the same symmetry explaining neutrino mixing angles is also responsible for the dark matter stability. In our simplest type-I seesaw [7] realization the flavor symmetry A_4 spontaneously breaks to Z_2 providing a stable DM candidate. We extend the scalar sector of the standard model by adding three Higgs doublets transforming as a triplet of A_4 we show that there is a consistent pattern of vacuum expectation values (vevs) for which only one of the three extra Higgs doublets takes a vev, while the other two give rise to the dark matter candidate. The model accounts for the observed pattern of mixing angles [8] indicated by current neutrino oscillation data, predicting $\theta_{13} = 0$ and inverted spectrum of neutrino masses. It will therefore be tested in upcoming double beta and neutrino oscillation searches [9], while the dark matter has potentially detectable rates within reach of nuclear recoil experiments.

Model We assign matter fields to irreducible representations of A_4, the group of even permutations of four objects, isomorphic to the symmetry group of the tetrahedron. All elements are generated from two elements S and T with $S^2 = T^3 = (ST)^3 = \mathcal{I}$. A_4 has four irreducible representations, three singlets 1, $1'$ and $1''$ and one triplet. In the basis where S is real diagonal,

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ ; \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\ ; \tag{1}$$

one has the following triplet multiplication rules,

$$\begin{aligned} (ab)_1 &= a_1b_1 + a_2b_2 + a_3b_3\,; \\ (ab)_{1'} &= a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3\,; \\ (ab)_{1''} &= a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3\,; \\ (ab)_{3_1} &= (a_2b_3, a_3b_1, a_1b_2)\,; \\ (ab)_{3_2} &= (a_3b_2, a_1b_3, a_2b_1)\,, \end{aligned} \tag{2}$$

where $\omega^3 = 1$, $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$. We assign the standard model Higgs doublet H, to the singlet 1, and we assume three additional Higgs doublets transforming as an A_4 triplet, namely $\eta = (\eta_1, \eta_2, \eta_3) \sim 3$. We have four right-handed neutrinos, three transforming as an A_4 triplet $N_T = (N_1, N_2, N_3)$, and one singlet N_4. The lepton and Higgs assignments of our model is in table I. The resulting Yukawa Lagrangian is

	L_e	L_μ	L_τ	l_e^c	l_μ^c	l_τ^c	N_T	N_4	H	η
$SU(2)$	2	2	2	1	1	1	1	1	2	2
A_4	1	$1'$	$1''$	1	$1''$	$1'$	3	1	1	3

TABLE I: Summary of relevant model quantum numbers

$$\begin{aligned} \mathcal{L} = {} & y_e L_e l_e^c H + y_\mu L_\mu l_\mu^c H + y_\tau L_\tau l_\tau^c H + \\ & + y_1^\nu L_e (N_T\eta)_1 + y_2^\nu L_\mu (N_T\eta)_{1''} + y_3^\nu L_\tau (N_T\eta)_{1'} + \\ & + y_4^\nu L_e N_4 H + M_1 N_T N_T + M_2 N_4 N_4 + \text{h.c.} \end{aligned}$$

This way H is responsible for quark and charged lepton masses, the latter automatically diagonal. Note that we do not discuss the quark sector, assumed to be blind to A_4, namely all left and right-handed up and down-type quarks transform trivially under A_4, their mass and

[1] In supersymmetry a viable DM particle is the neutralino, whose stability stems from the imposition of the so-called R-parity.

mixing hierarchies might arise from an extra family symmetry, for example, Frogatt-Nielsen-like [10]. Neutrino masses arise from H and η, see below. The relevant terms of the scalar potential are of the form

$$\begin{aligned} V &= \mu_\eta^2 \eta^\dagger \eta + \mu_H^2 H^\dagger H + \lambda_1 [H^\dagger H]^2 + \lambda_2 [\eta^\dagger \eta]_1^2 + \\ &+ \lambda_3 [\eta^\dagger \eta]_{1'} [\eta^\dagger \eta]_{1''} + \lambda_4 [\eta^\dagger \eta^\dagger]_{1'} [\eta \eta]_{1''} + \lambda'_4 [\eta^\dagger \eta^\dagger]_1 [\eta \eta]_1 \\ &+ \lambda_5 \textstyle\sum_i [\eta^\dagger \eta]_{3_i} [\eta^\dagger \eta]_{3_i} + \lambda'_5 ([\eta^\dagger \eta]_{3_1} [\eta^\dagger \eta]_{3_2} + h.c.) + \\ &+ \lambda_6 (\textstyle\sum_{i,j} [\eta^\dagger \eta^\dagger]_{3_i} [\eta \eta]_{3_j} + h.c) + \lambda_7 [\eta^\dagger \eta]_1 H^\dagger H + \\ &+ \lambda'_7 [\eta^\dagger H] H^\dagger \eta + \lambda_8 \left([\eta^\dagger \eta^\dagger]_1 H H + h.c \right) + \\ &+ \lambda_9 \left([\eta^\dagger \eta]_{3_1} \eta^\dagger H + h.c \right) + \lambda'_9 \left([\eta^\dagger \eta]_{3_2} \eta^\dagger H + h.c \right) + \\ &+ \lambda_{10} \left([\eta^\dagger \eta^\dagger]_{3_1} \eta H + h.c \right) + \lambda'_{10} \left([\eta^\dagger \eta^\dagger]_{3_2} \eta H + h.c \right) + \\ &+ \lambda_{11} [\eta^\dagger \eta^\dagger]_1 [\eta \eta]_1 \end{aligned} \tag{3}$$

where $i, j = 1, 2$, and $[...]_{3_i}$ means the product of two triplets contracted into a triplet of A_4, see eq. (2), $[...]_1$ means the product of two triplets contracted into a singlet of A_4 and so on. Note that $[\eta^\dagger \eta]_{1,1',1''} \equiv [\eta \eta^\dagger]_{1,1',1''}$, $[\eta\eta]_{3_1} \equiv [\eta\eta]_{3_2}$ and so on.

We have studied the minimization of the potential V solving the equations $\partial V / \partial v_i = 0$ where v_i are the vevs of the fields H, η_1, η_2 and η_3. For simplicity we assume real vevs. We have checked that for suitable parameter choices of the potential V, an allowed local minimum is

$$\langle H^0 \rangle = v_h \neq 0, \quad \langle \eta_1^0 \rangle = v_\eta \neq 0 \quad \langle \eta_{2,3}^0 \rangle = 0 \,, \tag{4}$$

with the eigenvalues of the Hessian $\partial^2 V / \partial v_i \partial v_j$ all positive.

Note that the alignment $\langle \eta \rangle \sim (1,0,0)$ breaks spontaneously A_4 to Z_2 since $(1,0,0)$ is invariant under the S generator in eq. (1). The Z_2 is defined as

$$\begin{array}{lll} N_2 \to -N_2 \,, & h_2 \to -h_2 \,, & A_2 \to -A_2 \,, \\ N_3 \to -N_3 \,, & h_3 \to -h_3 \,, & A_3 \to -A_3 \,. \end{array} \tag{5}$$

This residual symmetry is responsible for the stability of our DM candidate and the stability of the minimum. Note that the potential cannot break spontaneously A_4 into Z_3 because in this case the alignment $\langle \eta \rangle \sim (1,1,1)$ is not a minimum unless a fine tuning in the parameters $\lambda_9 + \lambda_{10} = 0$ is assumed. This atractive feature reminds of the inert doublet model [11], with the difference that here it follows naturally from the underlying flavor symmetry which accounts for neutrino oscillations.

We have four Higgs doublets[2] giving three physical charged scalar bosons, plus four neutral scalars, and three pseudoscalars. After electroweak symmetry breaking we can write

$$\begin{aligned} H &= \begin{pmatrix} 0 \\ v_h + h \end{pmatrix}, \quad \eta_1 = \begin{pmatrix} \eta_1^+ \\ v_\eta + h_1 + iA_1 \end{pmatrix}, \\ \eta_2 &= \begin{pmatrix} \eta_2^+ \\ h_2 + iA_2 \end{pmatrix}, \quad \eta_3 = \begin{pmatrix} \eta_3^+ \\ h_3 + iA_3 \end{pmatrix}. \end{aligned} \tag{6}$$

There are 3 physical charged scalar bosons, 4 CP even and 3 CP odd neutral scalars. The mass of the neutral scalar fields is block diagonal with the standard model Higgs h mixed with h_1, but not with the scalar fields with zero vev's $h_{2,3}$.

Dark matter The lightest combination of the stable scalar fields h_2, h_3 plays the role of our dark matter particle, which we will denote generically by η_{DM}. We list below all interactions of η_{DM}:

1. Yukawa interactions

$$\eta_{_{DM}}\, \overline{\nu}_i N_{2,3} \,, \tag{7}$$

where $i = e,\, \mu,\, \tau$.

2. Higgs-Vector boson couplings

$$\begin{array}{ll} \eta_{_{DM}} \eta_{_{DM}} ZZ \,, & \eta_{_{DM}} \eta_{_{DM}} WW \,, \\ \eta_{_{DM}} \eta_{2,3}^{\pm} W^{\pm} Z \,, & \eta_{_{DM}} \eta_{2,3}^{\pm} W^{\pm} \,, \\ \eta_{_{DM}} A_{2,3} Z \,. & \end{array} \tag{8}$$

3. Scalar interactions from the Higgs potential:

$$\begin{array}{ll} \eta_{_{DM}}\, A_1 A_2 h \,, & \eta_{_{DM}}\, A_1 A_3 h_1 \,, \\ \eta_{_{DM}}\, A_1 A_2 h_1 \,, & \eta_{_{DM}}\, A_1 A_3 h \,, \\ \eta_{_{DM}}\, A_2 A_3 h_3 \,, & \eta_{_{DM}}\, h_1 h_3 h \\ \eta_{_{DM}} \eta_{_{DM}} hh \,, & \eta_{_{DM}} \eta_{_{DM}} h_1 h_1 \,. \end{array} \tag{9}$$

After electroweak symmetry breaking, the vevs v_h and v_η are generated, so that additional terms are obtained from those in Eq. (9) by replacing $h \to v_h$ and $h_1 \to v_\eta$. The flavor symmetry A_4 is broken down to the residual Z_2 symmetry in Eq. (5), implying the stability of our dark matter candidate. As we will see, despite the many mass and coupling parameters appearing in the potential, eq. (3), for $M_\eta \gg M_z$, only two determine the relic dark matter abundance and its direct detection rates.

Relic Density Assuming that our DM candidate arises as thermal relic in the early universe, one of the most important requirements one must check is its relic abundance. For definiteness we require that η_{DM} makes up all the observed DM. For $M_\eta \gg M_z$ the most important annihilation and coanihilation processes are those with vector bosons, though for large $\lambda \gtrsim g_2$, where $16\lambda^2 = (\lambda_7 + \lambda'_7 + 2\lambda_8)^2 + (2\lambda_2 - \lambda_3 - 2\lambda_4 + \lambda'_4 + 2\lambda_5 +$

[2] Lepton flavor violating processes are suppressed by the large right-handed neutrino scale.

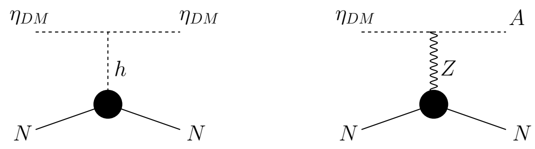

FIG. 1: Feynman diagrams relevant for direct DM detection. Elastic scattering (left) is generically more important than inelastic (right).

$\lambda_5' + \lambda_6)^2$, annihilation into Higgs bosons plays an important role, see Eq. (11). The DM abundance can be approximated as [12]

$$\frac{n_{\rm DM}(T)}{s(T)} \approx \sqrt{\frac{180}{\pi\ g_*}} \frac{1}{M_{\rm Pl}\ T_f \langle\sigma_A v\rangle}, \qquad (10)$$

where $\frac{M_\eta}{T_f} \approx 26$ and $g_* = 106.75 + n$ is the number of SM degrees of freedom plus $1 \leq n \leq 12$ degrees of freedom arising from the extra scalars, and $M_{Pl} = 1.22 \times 10^{19} GeV$ is the Planck scale. The cross section for $\eta_{DM}\eta_{DM} \rightarrow VV$ where V are vector bosons in the limit of massless final states, is given by [12]

$$\langle\sigma_A v\rangle \simeq \frac{3g_2^4 + g_Y^4 + 6g_2^2 g_Y^2 Y^2 + 4\lambda^2}{256\pi\ M_\eta^2}, \qquad (11)$$

where $Y = 1/2$ is the weak hypercharge, $g_2 = \sqrt{4\pi\alpha/(1 - M_W^2/M_Z^2)}$ and $g_Y = \sqrt{4\pi\alpha} M_Z/M_W$. From these equations it follows that, in order to provide the correct relic abundance $\Omega_{\rm DM}h^2 = 0.110 \pm 0.006$ i.e. $n_{\rm DM}/s = (0.40 \pm 0.02) eV/M_\eta$ [13], a correlation between the mass of the dark matter M_η and the quartic coupling constant λ is required. For simplicity if we take the limit of small λ we obtain a mass for the DM candidate of $M_\eta \approx 0.51$ TeV. For large λ values we have that the DM mass M_η scales as λ.

Direct detection The quartic couplings $\eta^\dagger\eta H^\dagger H$ and $\eta^\dagger\eta^\dagger HH$ give an interaction of the DM candidate with the nucleon through the interchange of the SM Higgs boson. Hence our DM candidate can be detected through the elastic scattering with a nucleus $\eta_{DM}N \rightarrow \eta_{DM}N$ via the exchange of a Higgs, or through inelastic scattering with a nucleus $\eta_{DM}N \rightarrow AN$ with the exchange of a Z boson, see Fig. 1, where A is the lightest pseudoscalar, in general a mixture of A_2 and A_3.

Barring fine-tuned choices of parameters for which the threshold for inelastic scattering opens up, the detection will be dominated by the elastic process, whose cross section is given by [14]

$$\sigma_{\rm el}({\rm nucleon}) \approx \lambda^2 \frac{1}{1 + (\tan\beta)^2} \left(\frac{100\ {\rm GeV}}{M_h}\right)^4 \times \times \left(\frac{50\ {\rm GeV}}{M_\eta}\right)^2 \left(5 \times 10^{-42}\ {\rm cm}^2\right), \qquad (12)$$

where $\tan\beta = v_h/v_\eta$. Note that all uncertainties associated with the nuclear form factor in Eq. (12), have been neglected. From the requirement of correctly reproducing the relic density, Eqs. (10) and (11), one can find an expression for λ as function of the DM mass, M_η. Using this relation and eq. (12) one can plot the estimated cross section for the direct detection for each value of $\tan\beta$ and mass of the Higgs, M_h, as illustrated in Fig. 2 [3]. The figure has been generated using [15] and compares the experimental sensitivities with our model expectations, fixing $m_H = 120$ GeV and three $\tan\beta$ values. This choice of Higgs mass is motivated by the LEP bounds $m_H > 114$ GeV, which however is not strictly valid in our model due to the additional Higgs doublets.

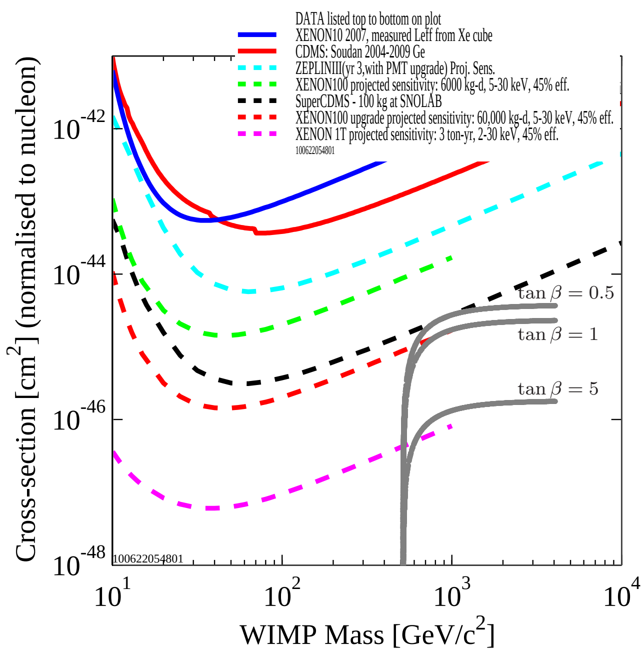

FIG. 2: Elastic DM scattering cross section with a nucleon versus DM mass. We compare present [16, 17] and future [18, 19] sensitivities with our model expectations, for $m_H = 120$ GeV and $\tan\beta = 0.5,\ 1,\ 5$ (grey solid lines).

Neutrino phenomenology Our model has four heavy right-handed neutrinos, and is a special case, called (3,4),

[3] Here we focus on the region $M_\eta \gg M_z$. The interesting case of light DM will be treated elsewhere.

of the general type-I seesaw mechanism [20]. After electroweak symmetry breaking, it is characterized by Dirac and Majorana mass terms given as

$$m_D = \begin{pmatrix} x_1 & 0 & 0 & y_1 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \end{pmatrix}, M_R = \mathrm{diag}(M_1, M_1, M_1, M_2)\ , \tag{13}$$

so that the light neutrinos get Majorana mass by means of the type-I seesaw relation $m_\nu = -m_{D_{3\times4}} M^{-1}_{R_{4\times4}} m^T_{D_{3\times4}}$ the light-neutrinos mass matrix M_ν being given as

$$\begin{pmatrix} \frac{x_1^2}{M_1} + \frac{y_1^2}{M_2} & \frac{x_1 x_2}{M_1} & \frac{x_1 x_3}{M_1} \\ \frac{x_1 x_2}{M_1} & \frac{x_2^2}{M_1} & \frac{x_2 x_3}{M_1} \\ \frac{x_1 x_3}{M_1} & \frac{x_2 x_3}{M_1} & \frac{x_3^2}{M_1} \end{pmatrix} = \begin{pmatrix} y^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}. \tag{14}$$

It falls within the class of scaling matrices introduced in Ref. [21]. This form of the light neutrino mass matrix has an inverse hierarchical neutrino mass spectrum and a zero eigenvalue with $m_3 = 0$ and corresponding eigenvector $(0, -c/b, 1)^T$ implying a vanishing reactor mixing angle $\theta_{13} = 0$. One can see explicitly that the solar and atmospheric square mass differences and mixing angles indicated by neutrino oscillation data [8] can indeed be fitted by taking, as an example, the tri-bimaximal (TBM) *ansatz* [22]. When $b = c$ and $y^2 = 2c^2 - ac$ the neutrino mass matrix Eq. (14) is $\mu - \tau$ invariant yielding maximal atmospheric mixing, $\sin^2\theta_{23} = 1/2$ and $M_{\nu,11} + M_{\nu,13} = M_{\nu,22} + M_{\nu,23}$, which gives the TBM value of the solar angle, $\sin\theta_{12}^2 = 1/3$, in good agreement with experimental data within one σ. The eigenvalues are $\{m_1, m_2, m_3\} = \{2ac + 2c^2, 2c^2 - ac, 0\}$, which can fit the two mass-squared differences required to account for the observed pattern of neutrino oscillations. By relaxing the condition $b = c$ and $y^2 = 2c^2 - ac$ one generates deviations from the TBM limit, while keeping $\theta_{13} = 0$. Note the above imples a neutrinoless double beta decay effective mass parameter in the range 0.03 to 0.05 eV at 3 σ, within reach of upcoming experiments [23].

Conclusions In summary we have suggested that DM stability follows from the same non-abelian discrete flavor symmetry which accounts for the observed pattern of neutrino oscillations. In the realization we have given we have an A_4 symmetry which spontaneously breaks to a Z_2 parity that stabilizes a scalar doublet dark matter, potentially detectable in nuclear recoil experiments, as well as accelerators. Despite the complexity of the scalar potential, in the heavy dark matter limit both the relic dark matter abundance and its direct detection cross section depend just on the DM mass and a single coupling strength parameter. The model is also manifestly unifiable and agrees with electroweak searches as well as precision tests, as will be shown elsewhere. Our simple example gives $0\nu\beta\beta$ rates accessible to upcoming experiments and no CP violation in neutrino oscillations.

Acknowledgments We thank Marco Taoso for useful discussions. This work was supported by the Spanish MICINN under grants FPA2008-00319/FPA and MULTIDARK Consolider CAD2009-00064, by Prometeo/2009/091, by the EU grant UNILHC PITN-GA-2009-237920. S. M. is supported by a Juan de la Cierva contract.

* Electronic address: mahirsch@ific.uv.es
† Electronic address: morisi@ific.uv.es
‡ Electronic address: epeinado@ific.uv.es
§ Electronic address: valle@ific.uv.es

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