Proposal for generalised Supersymmetry Les Houches Accord for see-saw models and PDG numbering scheme

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Abstract

The SUSY Les Houches Accord (SLHA) 2 extended the first SLHA to include various generalisations of the Minimal Supersymmetric Standard Model (MSSM) as well as its simplest next-to-minimal version. Here, we propose further extensions to it, to include the most general and well-established see-saw descriptions (types I/II/III, inverse, and linear) in both an effective and a simple gauged extension of the MSSM framework. In addition, we generalise the PDG numbering scheme to reflect the properties of the particles.

1. INTRODUCTION

If neutrinos are *Majorana* particles, their mass at low energy is described by a unique dimension-5 operator [1]

$$m_\nu = \frac{f}{\Lambda}(HL)(HL). \tag{1}$$

Using only renormalisable interactions, there are exactly three tree-level models leading to this operator [2]. The first one is the exchange of a heavy fermionic singlet, called the right-handed neutrino. This is the celebrated see-saw mechanism [3, 4, 5, 6, 7], nowadays called see-saw type I. The second possibility is the exchange of a scalar $SU(2)_L$ triplet [8, 9]. This is commonly known as see-saw type II. And lastly, one could also add one (or more) fermionic triplets to the field content of the SM [10]. This is known as see-saw type III. The see-saw mechanism provides a rationale for the observed smallness of neutrino masses, by the introduction of the inverse of some large scale Λ. In see-saw type I, for example, Λ is equal to the mass(es) of the right-handed neutrinos. Since these are $SU(2)_L$ singlets, their masses can take any value, and with neutrino masses as indicated by the results from oscillation experiments $m_\nu \sim \sqrt{\Delta m_A^2} \sim 0.05$ eV, where Δm_A^2 is the atmospheric neutrino mass splitting, and couplings of order $\mathcal{O}(1)$, the scale of the see-saw is estimated to be very roughly $m_{SS} \sim 10^{15}$ GeV. This value is close to, but slightly lower than, the scale of

grand unification. In addition there exist see-saw models with large couplings at the electroweak scale, such as the linear [11] and inverse [12] see-saw models.

In the MSSM the gauge couplings unify nearly perfectly at an energy scale close to $m_G \simeq 2 \times 10^{16}$ GeV. Adding new particles which are charged under the SM group at a scale below m_G tends to destroy this attractive feature of the MSSM, unless the new superfields come in complete $SU(5)$ multiplets. For this reason, within supersymmetric see-saw models, one usually realizes the type II see-saw by adding **15**-plets [2, 13, 14, 15] and the type III see-saw by the addition of **24**-plets [16, 15, 17].

Just like with any other extensions of the SM based on supersymmetry (SUSY), those implementing see-saw realisations have seen several different conventions used over the years, many of which have become widespread. Such a proliferation of conventions has some drawbacks though from a calculational point of view: results obtained by different authors or computing tools are not always directly comparable. Indeed, to enable this comparison, a consistency check of all the relevant conventions and the implementation of any necessary translations thereof must first be made. Needless to say, this is a time-consuming and rather error-prone task.

To remedy this problem, the original SUSY Les Houches Accord (SLHA1) was proposed [18]. The SLHA1 uniquely defined a set of conventions for SUSY models together with a common interface between codes. The latter can be broadly categorised in terms of four different kinds of tools: (i) spectrum calculators (which calculate the SUSY mass and coupling spectrum, assuming some SUSY-breaking terms and a matching of SM parameters to known data); (ii) observables calculators (packages which calculate one or more of the following: inclusive cross sections, decay partial widths, relic dark matter densities and indirect/precision observables); (iii) Monte Carlo (MC) event generators (which calculate exclusive cross sections through explicit simulation of high-energy particle collisions, by including resonance decays, parton showering, hadronisation and underlying-event effects); (iv) SUSY fitting programs (which fit SUSY model parameters to data). (See http://www.ippp.dur.ac.uk/montecarlo/BSM/ for an up-to-date collection and description of such tools.) Further, SLHA1 provided users with input and output in a common format, which is more readily comparable and transferable. In short, the basic philosophy was to specify a unique set of conventions for SUSY extensions of the SM together with generic file structures to be communicated across the four types of codes above, (i) − (iv), based on the transfer of three different ASCII files: one for model input, one for spectrum calculator output and one for decay calculator output.

The original protocol, SLHA1, was strictly limited to the MSSM with real parameters and R-parity conservation neglecting generation mixing. An expanded version was proposed in [19] (see also [20] in Ref. [21]), known as SLHA2, whereby various MSSM generalisations were included: $i.e.$, those involving CP, R-parity and flavour violation as well as the simplest extension of the MSSM, the so-called next-to-MSSM (NMSSM). Herein, we follow and extend the development of this protocol started at Les Houches 2011 [22], by including the most general and well-established see-saw descriptions in both an effective and a simple gauged extension of the MSSM. The new conventions and control switches described here comply with those of SLHA2 (and, retrospectively, also SLHA1) unless explicitly mentioned in the text. Our effort here is parallelled by other generalisations of the previous accords documented in [22], altogether eventually contributing to the definition of a future release of the original protocol.

2. THE SEE-SAW MECHANISM

In this section, we discuss different implementations of the see-saw mechanism. As already stated in the introduction, the aim of the see-saw mechanism is to explain the neutrino masses and mixing angles. This is done by linking the tiny masses to other parameters which are of the naturally expected order. The general idea can be summarised by writing down the most general mass matrix combining left-handed neutrino (L), right-handed neutrino (R) and additional singlet fields carrying lepton number (S):

$$\begin{pmatrix} m_{LL} & m_{LR} & m_{LS} \\ m_{LR}^T & m_{RR} & m_{RS} \\ m_{LS}^T & m_{RS}^T & m_{SS} \end{pmatrix} \qquad (2)$$

Looking at specific limits of this matrix, we can recover the different see-saw realizations: $m_{LL} = m_{LS} = m_{RS} = 0$ leads to type I, type III is obtained in the same limit but with m_{RR} stemming from $SU(2)_L$ triplets. $m_{LL} = m_{RR} = m_{LS} = 0$ is the characteristic matrix for inverse see-saw , while $m_{LL} = m_{RR} = m_{SS} = 0$ is the standard parametrisation of the linear see-saw . What most of these different see-saw models have in common is the way the tiny neutrino masses are recovered, just by suppressing them with very high scales for the new fields. This is strictly true for the type I/II/III models. The linear and inverse see-saw versions work slightly differently: the heaviness of the new fields is reduced at the price of introducing a relatively small dimensionful parameter, usually connected to an explicit violation of the lepton number.

In the following subsection we discuss models which can explain the origin of the distinct neutrino mass matrices.

2.1. Type I/II/III

The simplest see-saw models describe neutrino masses with an effective operator arising after integrating out heavy superfields. While one generation of 15-plets is sufficient to explain the entire neutrino data, this is not the case with just one 24-plet if $SU(5)$-invariant boundary conditions are assumed on the new parameters and more generations have to be included. We will therefore treat the number of generations of singlets, 15- and 24-plets as free parameter. Bearing this in mind, the models will be described with minimal addition of superfields, in the basis after $SU(5)$ symmetry breaking (see table 1 [1]). All these fields are integrated out during the RGE evaluation, such that, at the SUSY scale, only the particle content of the MSSM remains.

We give in the following the unified equations which would lead to a mixed scenario of see-saw types I, II, and III. The Eqs (3), (7), and (10) are specific to type I, Eqs (4), (8), and (11) refer to type II and Eqs (5), (9), and (12) refer to type III.

Type I					
SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1) \otimes SU(2) \otimes SU(3)$	R-parity of fermion
$\hat{\nu}^c$	$\tilde{\nu}^c$	ν^c	n_1	$(0, \mathbf{1}, \mathbf{1})$	$+$
Type II					
SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1) \otimes SU(2) \otimes SU(3)$	R-parity of fermion
$\hat{T}$	$\tilde{T}$	T	n_{15}	$(1, \mathbf{3}, \mathbf{1})$	$-$
$\hat{\bar{T}}$	$\tilde{\bar{T}}$	$\bar{T}$	n_{15}	$(-1, \mathbf{3}, \mathbf{1})$	$-$
$\hat{S}$	$\tilde{S}$	S	n_{15}	$(-\frac{2}{3}, \mathbf{1}, \mathbf{6})$	$-$
$\hat{\bar{S}}$	$\tilde{\bar{S}}^*$	$\bar{S}^*$	n_{15}	$(\frac{2}{3}, \mathbf{1}, \overline{\mathbf{6}})$	$-$
$\hat{Z}$	$\tilde{Z}$	Z	n_{15}	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$	$-$
$\hat{\bar{Z}}$	$\tilde{\bar{Z}}$	$\bar{Z}$	n_{15}	$(-\frac{1}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$-$
Type III					
SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1) \otimes SU(2) \otimes SU(3)$	R-parity of fermion
$\hat{W}_M$	$\tilde{W}_M$	W_M	n_{24}	$(0, \mathbf{3}, \mathbf{1})$	$+$
$\hat{G}_M$	$\tilde{G}_M$	G_M	n_{24}	$(0, \mathbf{1}, \mathbf{8})$	$+$
$\hat{B}_M$	$\tilde{B}_M$	B_M	n_{24}	$(0, \mathbf{1}, \mathbf{1})$	$+$
$\hat{X}_M$	$\tilde{X}_M$	X_M	n_{24}	$(\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$+$
$\hat{\bar{X}}_M$	$\tilde{\bar{X}}_M$	$\bar{X}_M$	n_{24}	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3})$	$+$

Table 1: New chiral superfields appearing in the effective type I/II/III see-saw models. While $n_{15} = 1$ is sufficient to explain neutrino data, n_1 and n_{24} must be at least 2.

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[1]We use always the convention that all given $U(1)$ charges are those appearing in the covariant derivative, i.e. $\partial_\mu - igQA_\mu$

The combined superpotential of all three types can be written as

$$W - W_{\mathrm{MSSM}} = W_I + W_{II} + W_{III}$$

where

$$W_I = Y_\nu \, \hat{\nu}^c \, \hat{L} \, \hat{H}_u + \frac{1}{2} M_{\nu^c} \, \hat{\nu}^c \, \hat{\nu}^c \tag{3}$$

$$\begin{aligned}
W_{II} = {} & \frac{1}{\sqrt{2}} Y_T \, \hat{L} \, \hat{T} \, \hat{L} + \frac{1}{\sqrt{2}} Y_S \, \hat{d}^c \, \hat{S} \, \hat{d}^c + Y_Z \, \hat{d}^c \, \hat{Z} \, \hat{L} \\
& + \frac{1}{\sqrt{2}} \lambda_1 \, \hat{H}_d \, \hat{T} \, \hat{H}_d + \frac{1}{\sqrt{2}} \lambda_2 \, \hat{H}_u \, \hat{\bar{T}} \, \hat{H}_u + M_T \, \hat{T} \, \hat{\bar{T}} + M_Z \, \hat{Z} \, \hat{\bar{Z}} + M_S \, \hat{S} \, \hat{\bar{S}}
\end{aligned} \tag{4}$$

$$\begin{aligned}
W_{III} = {} & \sqrt{\frac{3}{10}} Y_B \, \hat{H}_u \, \hat{B}_M \, \hat{L} + Y_W \, \hat{H}_u \, \hat{W}_M \, \hat{L} + Y_X \, \hat{H}_u \, \hat{\bar{X}}_M \, \hat{d}^c \\
& + M_X \, \hat{X}_M \, \hat{\bar{X}}_M + \frac{1}{2} M_W \, \hat{W}_M \, \hat{W}_M + \frac{1}{2} M_G \, \hat{G}_M \, \hat{G}_M + \frac{1}{2} M_B \, \hat{B}_M \, \hat{B}_M
\end{aligned} \tag{5}$$

For the MSSM part we use the conventions

$$W_{MSSM} = Y_u \, \hat{u}^c \, \hat{Q} \, \hat{H}_u - Y_d \, \hat{d}^c \, \hat{Q} \, \hat{H}_d - Y_e \, \hat{e}^c \, \hat{L} \, \hat{H}_d + \mu \, \hat{H}_u \, \hat{H}_d \tag{6}$$

The soft-breaking terms can be split into three categories: terms stemming from the superpotential couplings when replacing the fermions with their scalar superpartners ($L_{SB,W}$), the scalar soft-breaking masses for each chiral superfield ($L_{SB,\phi}$) and the soft-breaking masses for the gauginos ($L_{SB,\lambda}$). Since the gauge sector is not modified, $L_{SB,\lambda}$ reads as in the MSSM. The soft-breaking terms stemming from the superpotential are

$$L_{\mathrm{SB},W} - L_{\mathrm{SB},W,\mathrm{MSSM}} = L_{\mathrm{SB},W}^I + L_{\mathrm{SB},W}^{II} + L_{\mathrm{SB},W}^{III}$$

where

$$L_{\mathrm{SB},W}^I = T_\nu \, \tilde{\nu}^c \, \tilde{L} \, H_u + \frac{1}{2} B_{\nu^c} \, \tilde{\nu}^c \, \tilde{\nu}^c + \mathrm{H.c.} \tag{7}$$

$$\begin{aligned}
L_{\mathrm{SB},W}^{II} = {} & \frac{1}{\sqrt{2}} T_T \, \tilde{L} \, \tilde{T} \, \tilde{L} + \frac{1}{\sqrt{2}} T_S \, \tilde{d}^c \, \tilde{S} \, \tilde{d}^c + T_Z \, \tilde{d}^c \, \tilde{Z} \, \tilde{L} + \frac{1}{\sqrt{2}} T_1 \, H_d \, \tilde{T} \, H_d \\
& + \frac{1}{\sqrt{2}} T_2 \, H_u \, \tilde{\bar{T}} \, H_u + B_T \, \tilde{T} \, \tilde{\bar{T}} + B_Z \, \tilde{Z} \, \tilde{\bar{Z}} + B_S \, \tilde{S} \, \tilde{\bar{S}} + \mathrm{h.c.}
\end{aligned} \tag{8}$$

$$\begin{aligned}
L_{\mathrm{SB},W}^{III} = {} & \sqrt{\frac{3}{10}} T_B \, H_u \, \tilde{B}_M \, \tilde{L} + T_W \, H_u \, \tilde{W}_M \, \tilde{L} + T_X \, H_u \, \tilde{\bar{X}}_M \, \tilde{d}^c + B_X \, \tilde{X}_M \, \tilde{\bar{X}}_M \\
& + \frac{1}{2} B_W \, \tilde{W}_M \, \tilde{W}_M + \frac{1}{2} B_G \, \tilde{G}_M \, \tilde{G}_M + \frac{1}{2} B_B \, \tilde{B}_M \, \tilde{B}_M + \mathrm{H.c.}
\end{aligned} \tag{9}$$

while the soft-breaking scalar masses read

$$L_{\mathrm{SB},\phi} - L_{\mathrm{SB},\phi,MSSM} = L_{\mathrm{SB},\phi}^I + L_{\mathrm{SB},\phi}^{II} + L_{\mathrm{SB},\phi}^{III}$$

where

$$L_{\mathrm{SB},\phi}^I = -(\tilde{\nu}^c)^\dagger m_{\nu^c}^2 \tilde{\nu}^c \tag{10}$$

$$L_{\mathrm{SB},\phi}^{II} = -m_S^2 \tilde{S}^* \tilde{S} - m_{\bar{S}}^2 \tilde{\bar{S}}^* \tilde{\bar{S}} - m_T^2 \tilde{T}^* \tilde{T} - m_{\bar{T}}^2 \tilde{\bar{T}}^* \tilde{\bar{T}} - m_Z^2 \tilde{Z}^* \tilde{Z} - m_{\bar{Z}}^2 \tilde{\bar{Z}}^* \tilde{\bar{Z}} \tag{11}$$

$$\begin{aligned}
L_{\mathrm{SB},\phi}^{III} = {} & -\tilde{B}_M^\dagger m_B^2 \tilde{B}_M - \tilde{W}_M^\dagger m_W^2 \tilde{W}_M - \tilde{G}_M^\dagger m_G^2 \tilde{G}_M \\
& - \tilde{X}_M^\dagger m_X^2 \tilde{X}_M - \tilde{\bar{X}}_M^\dagger m_{\bar{X}}^2 \tilde{\bar{X}}_M
\end{aligned} \tag{12}$$

4

GUT conditions and free parameters. Since the new interactions in Eq.s (4) to (5) are the result of $SU(5)$-invariant terms, it is natural to assume a unification of the different couplings at the GUT scale.

$$M_T = M_Z = M_S \equiv M_{15}, \qquad Y_S = Y_T = Y_Z \equiv Y_{15} \tag{13}$$

$$Y_B = Y_W = Y_X \equiv Y_{24}, \qquad M_X = M_W = M_G = M_B \equiv M_{24} \tag{14}$$

In the same way the bi- and trilinear soft-breaking terms unify and they are connected to the superpotential parameters by

$$B_{\nu^c} \equiv B_0 M_{\nu^c}, \qquad T_\nu \equiv A_0 Y_\nu \tag{15}$$

$$B_{15} \equiv B_0 M_{15}, \qquad T_{15} \equiv A_0 Y_{15} \tag{16}$$

$$B_{24} \equiv B_0 M_{24}, \qquad T_{24} \equiv A_0 Y_{24} \tag{17}$$

In case of CMSSM-like boundary conditions, this leads to the following free parameters

$$B_0, \ M_{\nu^c}, \ Y_\nu \ M_{15}, \lambda_1, \lambda_2, \ Y_{15}, \ M_{24}, \ Y_{24} \tag{18}$$

in addition to the well-known MSSM parameters

$$m_0, \ M_{1/2}, \ A_0, \ \tan\beta, \ \text{sign}(\mu) \tag{19}$$

In principle, this B_0 is not the same as the B for the Higgs, though in a minimal case they may be defined to be equal at the GUT scale. Furthermore, $T_i = A_0 Y_i$ holds at the GUT scale.

Effective neutrino masses.. The effective neutrino mass matrices appearing in type I/II/III at SUSY scale are

$$m_\nu^I = -\frac{v_u^2}{2} Y_\nu^T M_R^{-1} Y_\nu \tag{20}$$

$$m_\nu^{II} = \frac{v_u^2}{2} \frac{\lambda_2}{M_T} Y_T. \tag{21}$$

$$m_\nu^{III} = -\frac{v_u^2}{2} \left(\frac{3}{10} Y_B^T M_B^{-1} Y_B + \frac{1}{2} Y_W^T M_W^{-1} Y_W \right). \tag{22}$$

2.2. Inverse and linear see-saw

The inverse and linear see-saw realisations are obtained in models that provide three generations of a further gauge singlet carrying lepton number in addition to three generations of the well-known right-handed neutrino superfields, here $\hat{\nu}^c$ (see table 2).

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3)$	lepton number	R-parity of fermion
$\hat{\nu}^c$	$\tilde{\nu}^c$	ν^c	n_{ν^c}	$(0, \mathbf{1}, \mathbf{1})$	$+1$	$+$
$\hat{N}_S$	$\tilde{N}_S$	N_S	n_{N_S}	$(0, \mathbf{1}, \mathbf{1})$	-1	$+$

Table 2: New chiral superfields appearing in models with inverse and linear see-saw.

The only additional terms in the superpotential which are allowed by conservation of gauge quantum numbers are

$$W - W_{\mathrm{MSSM}} = Y_\nu \, \hat{\nu}^c \, \hat{L} \, \hat{H}_u + M_R \, \hat{\nu}^c \, \hat{N}_S + \begin{cases} \frac{1}{2} \mu_N \, \hat{N}_S \, \hat{N}_S & \text{inverse see-saw} \\ Y_{LN} \, \hat{N}_S \, \hat{L} \, \hat{H}_u & \text{linear see-saw} \end{cases}. \tag{23}$$

It is important to note that the last term in each model breaks lepton number explicitly, but are expected for different reasons.

The soft-breaking terms read

$$L_{\text{SB},W} = L_{\text{SB},W,\text{MSSM}} + T_\nu \, \tilde{\nu}^c \, \tilde{L} \, H_u + B_R \, \tilde{\nu}^c \, \tilde{N}_S + \left\{ \begin{array}{ll} \frac{1}{2} B_N \, \tilde{N}_S \, \tilde{N}_S & \text{inverse see-saw} \\ T_{LN} \, \tilde{N}_S \, \tilde{L} H_u & \text{linear see-saw} \end{array} \right\} + \text{H.c.} \qquad (24)$$

$$L_{\text{SB},\phi} = L_{\text{SB},\phi,\text{MSSM}} - (\tilde{\nu}^c)^\dagger m_{\nu^c}^2 \tilde{\nu}^c - \tilde{N}_S m_N^2 \tilde{N}_S^* . \qquad (25)$$

while $L_{\text{SB},\lambda}$ is again the same as for the MSSM. It is necessary to split the sneutrinos and the singlets into their scalar and pseudoscalar components:

$$\tilde{\nu}_L = \frac{1}{\sqrt{2}} \left(\sigma_L + i\phi_L \right), \ \tilde{\nu}^c = \frac{1}{\sqrt{2}} \left(\sigma_R + i\phi_R \right), \ \tilde{N}_S = \frac{1}{\sqrt{2}} \left(\sigma_S + i\phi_S \right) . \qquad (26)$$

In comparison to the MSSM, additional mixings between fields take place: the left- and right-handed scalar components mix with the scalar component of the singlet fields. The same holds for the pseudoscalar components. Furthermore, the neutrinos mix with the fermionic singlet fields to build up 9 Majorana fermions. All three appearing 9×9 mass matrices can be diagonalised by unitary matrices. We define the basis for the mass matrices as

- Scalar sneutrinos: $(\sigma_L, \sigma_R, \sigma_S)^T$

- Pseudoscalar sneutrinos: $(\phi_L, \phi_R, \phi_S)^T$

- Neutrinos: $(\nu_L, \nu^c, N_S)^T$

The neutrino mass matrix then reads

$$\begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu & 0 \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & 0 & M_R \\ 0 & M_R^T & \mu_N \end{pmatrix} \text{ (inverse) or } \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu & \frac{v_u}{\sqrt{2}} Y_{LN} \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & 0 & M_R \\ \frac{v_u}{\sqrt{2}} Y_{LN}^T & M_R^T & 0 \end{pmatrix} \text{ (linear)}, \qquad (27)$$

Note, the presence of v_u in all terms of the first column and row is just coincidence caused by the given, minimal particle content. For more general models different VEVs can appear.

Free parameters. If CMSSM-like boundary conditions are assumed, the following new free parameters arise:

$$M_R, \ Y_{LN}, \ \mu_N, \ B_0 \qquad (28)$$

in addition of those given in Eq. (19).

Calculating the eigenvalues of the above mass matrices, it can be seen that the light neutrino masses are linear functions of Y_{LN} in the linear see-saw models, while the neutrino masses are linearly proportional to μ_N, as in the inverse see-saw models. The neutrino masses in the two models read

$$m_\nu^{LS} \simeq \frac{v_u^2}{2} \left(Y_\nu (Y_{LN} M_R^{-1})^T + (Y_{LN} M_R^{-1}) Y_\nu^T \right) , \qquad (29)$$

$$m_\nu^{IS} \simeq \frac{v_u^2}{2} Y_\nu (M_R^T)^{-1} \mu_N M^{-1} Y_\nu^T . \qquad (30)$$

Hence we propose that both models, and any combination of the two, be specified by extending Y_ν to a $3 \times (n_{\nu^c} + n_{N_S})$ Yukawa matrix, incorporating Y_{LN} as Y_ν^{ij} with i running from $n_{\nu^c} + 1$ to $n_{\nu^c} + n_{N_S}$. Implementation of each model consists of zeros being specified in the appropriate entries in the relevant matrices (*e.g.* specifying that the elements of Y_ν corresponding to Y_{LN} are zero recovers the inverse see-saw model, while specifying that μ_N is zero recovers the linear see-saw model).

2.3. See-saw in models with $U(1)_R \times U(1)_{B-L}$ gauge sector

Models based on a $SO(10)$ GUT theory can lead to a gauge sector containing the product group $U(1)_R \times U(1)_{B-L}$, through the breaking pattern

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad \to SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}. \quad (31)$$

The $U(1)_R \times U(1)_{B-L}$ factors will be subsequently broken to the hypercharge $U(1)_Y$ of the SM. However, it is possible that this final breaking scale is just around the TeV scale without spoiling gauge unification [23]. This can therefore lead to interesting phenomenology and can have important impact on the Higgs sector [24]. The first version of these models included a linear see-saw mechanism, but it has been shown that also the inverse see-saw can be included [25]. Further, the minimal (type-I) see-saw can also be included if the Higgs fields responsible for the $U(1)_R \times U(1)_{B-L}$ breaking carry twice the traditional $U(1)$ charges.

Notice that, in general, these models contain not only gauge couplings per each Abelian gauge group, but also so-called 'off-diagonal couplings', as discussed in Appendix A. The minimal particle content for such model extending the MSSM, leading to the spontaneous breaking of $U(1)_R \times U(1)_{B-L}$ and to neutrino masses, is given in table 3. This particle content consists of 3 generations of 16-plets of $SO(10)$, 2 additional Higgs fields and 3 generations of a singlet field. The vector superfields are given in table 4.

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1)_{B-L} \otimes SU(2) \otimes U(1)_R \otimes SU(3)$
Matter fields (fermionic components have positive R-parity)
$\hat{Q}$	$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	$(\frac{1}{6}, \mathbf{2}, 0, \mathbf{3})$
$\hat{L}$	$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	3	$(-\frac{1}{2}, \mathbf{2}, 0, \mathbf{1})$
$\hat{u}^c$	$\tilde{u}^c$	u^c	3	$(-\frac{1}{6}, \mathbf{1}, -\frac{1}{2}, \overline{\mathbf{3}})$
$\hat{d}^c$	$\tilde{d}^c$	d^c	3	$(-\frac{1}{6}, \mathbf{1}, \frac{1}{2}, \overline{\mathbf{3}})$
$\hat{\nu}^c$	$\tilde{\nu}^c$	ν^c	3	$(\frac{1}{2}, \mathbf{1}, -\frac{1}{2}, \mathbf{1})$
$\hat{e}^c$	$\tilde{e}^c$	e^c	3	$(\frac{1}{2}, \mathbf{1}, \frac{1}{2}, \mathbf{1})$
$\hat{N}_S$	$\tilde{N}_S$	N_S	n_{N_S}	$(0, \mathbf{1}, 0, \mathbf{1})$
Higgs fields (scalar components have positive R-parity)
$\hat{H}_d$	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\tilde{H}_d = \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	1	$(0, \mathbf{2}, -\frac{1}{2}, \mathbf{1})$
$\hat{H}_u$	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\tilde{H}_u = \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	1	$(0, \mathbf{2}, \frac{1}{2}, \mathbf{1})$
For minimal see-saw
$\hat{\delta}_R$	δ_R^0	$\tilde{\delta}_R^0$	1	$(-1, \mathbf{1}, 1, \mathbf{1})$
$\hat{\bar{\delta}}_R$	$\bar{\delta}_R^0$	$\tilde{\bar{\delta}}_R^0$	1	$(1, \mathbf{1}, -1, \mathbf{1})$
For linear and inverse see-saw
$\hat{\xi}_R$	ξ_R^0	$\tilde{\xi}_R^0$	1	$(-\frac{1}{2}, \mathbf{1}, \frac{1}{2}, \mathbf{1})$
$\hat{\bar{\xi}}_R$	$\bar{\xi}_R^0$	$\tilde{\bar{\xi}}_R^0$	1	$(\frac{1}{2}, \mathbf{1}, -\frac{1}{2}, \mathbf{1})$
Fields integrated out (scalar components have positive R-parity)
$\hat{\xi}_L$	ξ_L^0	$\tilde{\xi}_L^0$	1	$(\frac{1}{2}, \mathbf{2}, 0, \mathbf{1})$
$\hat{\bar{\xi}}_L$	$\bar{\xi}_L^0$	$\tilde{\bar{\xi}}_L^0$	1	$(-\frac{1}{2}, \mathbf{2}, 0, \mathbf{1})$

Table 3: Chiral Superfields appearing in models with $U(1)_R \times U(1)_{B-L}$ gauge sector which incorporate minimal, linear and inverse see-saw mechanisms.

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}'$	$\lambda_{\tilde{B}'}$	B'	$U(1)$	g_{BL}	B-L
$\hat{W}_L$	λ_L	W_L	SU(2)	g_L	left
$\hat{B}_R$	$\lambda_{\tilde{B}_R}$	B_R	$U(1)$	g_R	right
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_s	color

Table 4: Vector superfields appearing in models with $U(1)_R \times U(1)_{B-L}$ gauge sector.

Superpotential. We assume for the following discussion that the superpotential can contain the following terms:

$$W - W_{MSSM} = Y_\nu \, \hat{\nu}^c \, \hat{L} \, \hat{H}_u \, - \mu_\delta \, \hat{\bar{\delta}}_R \hat{\delta}_R \, + Y_M \hat{\nu}^c \hat{\delta}_R \hat{\nu}^c \tag{32}$$

for the realization of the minimal see-saw and

$$W - W_{MSSM} = Y_\nu \, \hat{\nu}^c \, \hat{L} \, \hat{H}_u \, - \mu_\xi \, \hat{\bar{\xi}}_R \hat{\xi}_R \, + Y_{N\nu^c} \hat{N}_S \hat{\nu}^c \hat{\xi}_R$$

$$+ \begin{cases} \frac{1}{2}\mu_N \hat{N}_S \hat{N}_S & \text{inverse see-saw} \\ Y_{LS} \, \hat{L} \, \hat{\xi}_L \, \hat{N}_S \, + Y_{LR} \hat{\xi}_L \hat{\xi}_R \hat{H}_d \, + \mu_L \hat{\xi}_L \hat{\bar{\xi}}_L & \text{linear see-saw} \end{cases} \tag{33}$$

for linear and inverse see-saw, respectively. Notice that, again, the $\hat{N}_S$ superfield carries lepton number and that therefore the term μ_N provides its explicit violation. Since $\mu_L \gg m_{SUSY}$, the fields $\hat{\xi}_L$ and $\hat{\bar{\xi}}_L$ are integrated out and create an effective operator $\frac{Y_{LR}Y_{LS}}{\mu_L}\hat{L}\,\hat{N}_S\hat{H}_d\hat{\xi}_R$.

Soft-breaking terms. The soft-breaking terms in the matter sector are

$$L_{\text{SB},W} - L_{\text{SB},W,\text{MSSM}} = T_\nu \, \tilde{\nu}^c \, \tilde{L} \, H_u \, - B_\delta \, \bar{\delta}_R \, \delta_R, + T_M \tilde{\nu}^c \delta_R^0 \tilde{\nu}^c + \text{H.c.}$$

$$L_{\text{SB},\phi} - L_{\text{SB},\phi,\text{MSSM}} = - m_\delta^2 |\delta_R|^2 - m_{\bar{\delta}}^2 |\bar{\delta}_R|^2 - (\tilde{\nu}^c)^\dagger m_{\nu^c}^2 \tilde{\nu}^c \tag{34}$$

respectively

$$L_{\text{SB},W} - L_{\text{SB},W,\text{MSSM}} = T_\nu \, \tilde{\nu}^c \, \tilde{L} \, H_u \, - B_\xi \, \bar{\xi}_R \, \xi_R, + T_{N\nu^c} \tilde{N}_S \tilde{\nu}^c \xi_R$$

$$+ \begin{cases} \frac{1}{2} B_N \tilde{N}_S \tilde{N}_S & \text{inverse see-saw} \\ T_{LS} \, \tilde{L} \, \xi_L \, \tilde{N}_S \, + T_{LR} \xi_L \xi_R H_d \, + B_L \xi_L \bar{\xi}_L & \text{linear see-saw} \end{cases} + \text{H.c.} \tag{35}$$

$$L_{\text{SB},\phi} - L_{\text{SB},\phi,\text{MSSM}} = - \tilde{N}_S^\dagger m_N^2 \tilde{S} - m_\xi^2 |\xi_R|^2 - m_{\bar{\xi}}^2 |\bar{\xi}_R|^2 - (\tilde{\nu}^c)^\dagger m_{\nu^c}^2 \tilde{\nu}^c - m_{\xi_L}^2 |\xi_L|^2 - m_{\bar{\xi}_L}^2 |\bar{\xi}_L|^2 \tag{36}$$

While the soft-breaking gaugino sector reads

$$L_{\text{SB},\lambda} = \frac{1}{2} \left(-\lambda_{\tilde{B}'}^2 M_{B-L} - 2\lambda_{\tilde{B}'} \lambda_{\tilde{B}_R} M_{RB} - M_2 \lambda_{\tilde{W},i}^2 - M_3 \lambda_{\tilde{g},\alpha}^2 - \lambda_{\tilde{B}_R}^2 M_R + \text{H.c.} \right) \tag{37}$$

The term $\lambda_{\tilde{B}'} \lambda_{\tilde{B}_R} M_{RB}$ is a consequence of the presence of two Abelian gauge groups, see Appendix A.

Symmetry breaking. Since it is assumed that in these models the scale of spontaneous symmetry breaking to the SM gauge group is near the TeV scale, it is possible to restrict ourselves to a direct one-step breaking pattern (*i.e.*, $SU(2)_L \times U(1)_R \times U(1)_{B-L} \to U(1)_{\text{EM}}$). This breaking pattern takes place when the Higgs fields in the left and right sectors receive VEVs. We can parametrise the scalar fields as follows:

$$H_d^0 = \frac{1}{\sqrt{2}} \left(v_d + \sigma_d + i\phi_d \right), \ H_u^0 = \frac{1}{\sqrt{2}} \left(v_u + \sigma_u + i\phi_u \right) \tag{38}$$

$$X_R = \frac{1}{\sqrt{2}} \left(v_{X_R} + \sigma_{X_R} + i\phi_{X_R} \right), \ \bar{X}_R = \frac{1}{\sqrt{2}} \left(v_{\bar{X}_R} + \sigma_{\bar{X}_R} + i\phi_{\bar{X}_R} \right) . \tag{39}$$

with $X = \xi, \delta$ It is useful to define the quantities $v_R = v_{X_R}^2 + v_{\bar{X}_R}^2$ and $\tan\beta_R = \frac{v_{\bar{X}_R}}{v_{X_R}}$, in analogy to $v^2 = v_d^2 + v_u^2$ and to $\tan\beta = \frac{v_u}{v_d}$.

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Particle mixing. Additional mixing effects take place in the gauge and Higgs sectors due to the additional gauge fields considered, besides the neutrino and sneutrino cases. The three neutral gauge bosons B', B_R and W^3 mix to form three mass eigenstates: the massless photon, the well-known Z boson, and a Z' boson. This mixing can be parameterised by a unitary 3×3 matrix which diagonalises the mass matrix of the gauge bosons, such as

$$(\gamma, Z, Z')^T = U^{\gamma Z Z'} (B', B_R, W^3)^T \,. \tag{40}$$

Similarly, this model contains 7 neutralinos which are an admixture of the three neutral gauginos, of the two neutral components of the Higgsino doublets and of the two additional fermions coming from the right sector. The mass matrix, written in the basis $\left(\lambda_{\tilde{B}'}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_{\tilde{B}_R}, \tilde{X}_R, \tilde{\tilde{X}}_R \right)$ (with $X = \xi, \delta$), can be diagonalised by a unitary matrix, here denoted with Z^N. In the Higgs sector we choose the mixing basis and rotation matrices to be, respectively,

- Scalar Higgs fields: $\left(\sigma_d, \sigma_u, \sigma_{X_R}, \sigma_{\bar{X}_R} \right)^T$ and Z^H

- Pseudoscalar Higgs fields: $\left(\phi_d, \phi_u, \phi_X, \phi_{\bar{X}_R} \right)^T$ and Z^A .

The neutrino and sneutrino sectors are similar to the case discussed in section 2.2: the scalar fields are decomposed into their *CP*-even and odd components according to eq. (26). The mass matrices are defined in the same basis. If all terms of Eq. (33) are present, the resulting masses of the light neutrinos are a result of a mixed linear and an inverse see-saw. The mass matrix for inverse seesaw is analog to left matrix given in Eq. (27) with M_R replaced by $\frac{1}{\sqrt{2}} Y_{N\nu^c} v_{\xi_R} \equiv \tilde{Y}$, while the mass matrix for linear seesaw is given by the right matrix in Eq. (27) and the replacement $Y_{LS} \to \frac{Y_{LS} Y_{LR}}{\sqrt{2} \mu_L} v_\eta$. For the minimal realization with the superpotential given in Eq. (32), the neutrino mass matrix is given by

$$\begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & \frac{2 v_{\delta_R}}{\sqrt{2}} Y_M \end{pmatrix} \tag{41}$$

Free parameters. If CMSSM-like boundary conditions are assumed, the following new free parameters arise in addition of those given in Eq. (19) in inverse seesaw

$$Y_{N\nu^c}, \ \mu_N, \ B_0, \ \tan\beta_R, \ \mathrm{sign}(\mu_\xi) \ M_{Z'} \tag{42}$$

and in addition for linear seesaw

$$Y_{LS}, \ Y_{LR}, \ \mu_L \tag{43}$$

Here we have assumed that the parameters μ_ξ and B_ξ are fixed by the tadpole equations. The relationships of the soft trilinear terms to the Yukawa couplings are as before, and $B_N = B_0 \mu_N$.

2.4. See-saw in models with $U(1)_Y \times U(1)_{B-L}$ gauge sector

The final category of models considered here includes an additional $B - L$ gauge group tensored to the SM gauge groups, *i.e.* $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The corresponding vector superfields are given in table 5. The minimal version of these models [26, 27, 28] extends the MSSM particle content with three generations of right-handed superfields. Two additional scalars, singlets with respect to SM gauge interactions but carrying $B - L$ charge, are added to break $U(1)_{B-L}$, as well as allowing for a Majorana mass term for the right-handed neutrino superfields. Furthermore, two new lepton fields per generation can be included to specifically implement the inverse see-saw mechanism [29], as well as the linear see-saw realisation if further two doublet fields ($\hat{\rho}$ and $\hat{\bar{\rho}}$) are considered, to be integrated out. All particles and their quantum numbers are given in table 6. This table contains also the charge assignment under a Z_2 symmetry which is just present in the case of the inverse and linear see-saw models[2].

---

[2]Notice that in comparison to Ref. [29], the charge assignments of the new particles in the inverse see-saw model, as well as the Z_2 symmetry, have been redefined for consistency with the similar minimal model of Ref.[26].

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}$	$\lambda_{\tilde{B}}$	B	$U(1)$	g_1	hypercharge
$\hat{W}$	$\lambda_{\tilde{W}}$	W^-	SU(2)	g_2	left
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_3	color
$\hat{B}'$	$\lambda_{\tilde{B}'}$	B'	$U(1)$	g_B	B-L

Table 5: Vector superfields appearing in models with $U(1)_Y \times U(1)_{B-L}$ gauge sector.

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$U(1)_Y \otimes$ SU(2) $\otimes$ SU(3) $\otimes U(1)_{B-L}$	Z_2 inverse SS
Matter fields (fermionic components have positive R-parity)					
$\hat{Q}$	$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3}, \frac{1}{6})$	$+$
$\hat{L}$	$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, -\frac{1}{2})$	$+$
$\hat{d}^c$	$\tilde{d}^c$	d^c	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{6})$	$+$
$\hat{u}^c$	$\tilde{u}^c$	u^c	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{6})$	$+$
$\hat{e}^c$	$\tilde{e}^c$	e^c	3	$(1, \mathbf{1}, \mathbf{1}, \frac{1}{2})$	$+$
$\hat{\nu}^c$	$\tilde{\nu}^c$	ν^c	3	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$	$+$
Higgs fields (scalar components have positive R-parity)					
$\hat{\eta}$	η	$\tilde{\eta}$	1	$(0, \mathbf{1}, \mathbf{1}, -1)$	$-$
$\hat{\bar{\eta}}$	$\bar{\eta}$	$\tilde{\bar{\eta}}$	1	$(0, \mathbf{1}, \mathbf{1}, +1)$	$-$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$	$+$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$	$+$
Additional field for inverse see-saw (fermionic components have positive R-parity)					
$\hat{N}_S$	$\tilde{N}_S$	N_S	n_{N_S}	$(0, \mathbf{1}, \mathbf{1}, -\frac{1}{2})$	$-$
$\hat{N}'_S$	$\tilde{N}'_S$	N'_S	n_{N_S}	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$	$+$
Fields integrated out (for linear see-saw)					
$\hat{\rho}$	ρ	$\tilde{\rho}$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1}, 1)$	$-$
$\hat{\bar{\rho}}$	$\bar{\rho}$	$\tilde{\bar{\rho}}$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, -1)$	$-$

Table 6: Chiral Superfields appearing in models with $U(1)_Y \times U(1)_{B-L}$ gauge sector. The minimal particle content is needed for see-saw type I, the additional fields can be used to incorporate inverse or linear see-saw. The $SU(2)_L$ doublets are named as in table 3. The Z_2 in the last column is not present in the minimal model but just in the model with inverse and linear see-saw. The number of generations of $\hat{N}'_S$ must match those of $\hat{N}_S$ for anomaly cancellation.

The additional terms for the superpotential in comparison to the MSSM read

$$W - W_{\text{MSSM}} = Y_\nu \hat{\nu}^c \hat{l} \hat{H}_u + \begin{cases} Y_{\eta\nu^c} \hat{\nu}^c \hat{\eta} \hat{\nu}^c - \mu_\eta \hat{\eta} \hat{\bar{\eta}} & \text{minimal see-saw} \\ Y_{IS} \hat{\nu}^c \hat{\eta} \hat{N}_S + \mu_N \hat{N}_S \hat{N}_S & \text{inverse see-saw} \\ Y_{IS} \hat{\nu}^c \hat{\eta} \hat{N}_S + Y_{LS} \hat{L} \hat{\rho} \hat{N}_S + Y_{LR} \hat{\rho} \hat{\eta} \hat{H}_d + \overline{Y}_{LR} \hat{\bar{\rho}} \hat{\bar{\eta}} \hat{H}_u + \mu_\rho \hat{\rho} \hat{\bar{\rho}} & \text{linear see-saw} \end{cases} .$$

The extra parity is not required in the minimal case, thus the see-saw term $Y_{\eta\nu^c} \hat{\nu}^c \hat{\eta} \hat{\nu}^c$ is allowed, whereas the latter is forbidden in the other see-saw realisations precisely due to the Z_2 symmetry. In the inverse see-saw model the $\mu_N \hat{N}_S \hat{N}_S$ term plays an important role, even though since $\hat{N}_S$ carries $B - L$ charge, this term violates $B - L$ charge conservation similarly to the terms in Eq. (23) and Eq. (33) which break lepton number. This term is assumed to come from higher-order effects. (A similar term for the $\hat{N}'_S$ field

is possible but not relevant, since the N'_S does not take part in the mixing with the neutrinos.) A possible bilinear term $\hat{N}_S\hat{N}'_S$ is forbidden by the Z_2 symmetry given in the last column of Table 6. This discrete symmetry also forbids terms like $\hat{N}_S\hat{N}_S\eta$ or $\hat{N}'_S\hat{N}'_S\bar{\eta}$ as well as the μ_η-term that is necessary to obtain a pure minimal see-saw scenario. In case of linear see-saw heavy fields $\hat{\rho}$ and $\hat{\bar{\rho}}$ are present which get integrated at $\mu_\rho \gg M_{SUSY}$ similar to the linear see-saw in $U(1)_R \times U(1)_{B-L}$. This creates an effective operator of the form $\frac{Y_{LR}Y_{LS}}{\mu_\rho}\hat{L}\,\hat{N}_S\hat{H}_d\hat{\eta}$. Notice that it is assumed that in linear see-saw the term μ_N is not generated at higher loop-level.

The additional soft-breaking terms are written as follows:

$$L_{\text{SB},W} - L_{\text{SB},W,\text{MSSM}} = T_\nu \tilde{\nu}^c \tilde{l} H_u$$

$$+ \left\{ \begin{array}{ll} T_{\eta\nu^c}\tilde{\nu}^c\eta\tilde{\nu}^c - B_\eta\eta\bar{\eta} & \text{minimal see-saw} \\ T_{IS}\tilde{\nu}^c\eta\tilde{N}_S + B_N\tilde{N}_S\tilde{N}_S & \text{inverse see-saw} \\ T_{IS}\tilde{\nu}^c\eta\tilde{N}_S + T_{LS}\tilde{L}\rho\tilde{N}_S + T_{LR}\rho\eta H_d + \overline{T}_{LR}\bar{\rho}\bar{\eta}H_u + B_\rho\rho\bar{\rho} & \text{linear see-saw} \end{array} \right\} + \text{H.c.} \quad (44)$$

$$L_{\text{SB},\phi} - L_{\text{SB},\phi,\text{MSSM}} = -m_\eta^2|\eta|^2 - m_{\bar{\eta}}^2|\bar{\eta}|^2 - (\tilde{N}_S^* m_N^2 \tilde{N}_S + \tilde{N}_{S'}^* m_{N'}^2 \tilde{N}'_S + m_\rho^2|\rho|^2 + m_{\bar{\rho}}^2|\bar{\rho}|^2) \quad (45)$$

$$L_{\text{SB},\lambda} = \frac{1}{2}\left(-\lambda_{\tilde{B}}^2 M_1 - \lambda_{\tilde{B}}\lambda_{\tilde{B}'}M_{BB'} - M_2\lambda_{\tilde{W},i}^2 - M_3\lambda_{\tilde{g},\alpha}^2 - \lambda_{\tilde{B}'}^2 M_{BL} + \text{H.c.}\right) \quad (46)$$

The soft-breaking terms m_ρ^2 and $m_{\bar{\rho}}^2$ are just present in linear see-saw, while m_N^2 and $m_{N'}^2$ only exists for linear and inverse see-saw. To break $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ to $U(1)_{em}$, the neutral MSSM Higgs fields and the new SM scalar singlets acquire VEVs:

$$H_d^0 = \frac{1}{\sqrt{2}}\left(v_d + \sigma_d + i\phi_d\right), \ H_u^0 = \frac{1}{\sqrt{2}}\left(v_u + \sigma_u + i\phi_u\right) \quad (47)$$

$$\eta = \frac{1}{\sqrt{2}}\left(\sigma_\eta + v_\eta + i\phi_\eta\right), \ \bar{\eta} = \frac{1}{\sqrt{2}}\left(\sigma_{\bar{\eta}} + v_{\bar{\eta}} + i\phi_{\bar{\eta}}\right) \quad (48)$$

We also define here

$$v^2 = v_d^2 + v_u^2, \qquad \tan\beta = \frac{v_u}{v_d} \quad (49)$$

as well as

$$v_x = v_{\eta_R}^2 + v_{\bar{\eta}_R}^2, \qquad \tan\beta' = \frac{v_{\bar{\eta}}}{v_\eta} \quad (50)$$

The left- and right-handed sneutrinos are decomposed into their scalar and pseudoscalar components according to Eq. (26). Similarly, in the inverse see-saw model, the scalar component of $\hat{N}_S$ reads

$$\tilde{N}_S = \frac{1}{\sqrt{2}}\left(\sigma_S + i\phi_S\right) \quad (51)$$

The additional mixing effects which take place in this model are similar to the case of $U(1)_R \times U(1)_{B-L}$ discussed in section 2.3. In the gauge sector three neutral gauge bosons appear which mix to give rise to the massless photon, the Z boson, and a Z' boson:

$$(\gamma, Z, Z')^T = U^{\gamma Z}(B, W^3, B')^T . \quad (52)$$

Notice that when the kinetic mixing is neglected, the B' field decouples and the mass matrix of the gauge bosons becomes block diagonal, where the upper 2×2 block reads as in the SM. In the matter sector we choose the basis and the mixing matrices which diagonalise the mass matrices respectively as follows:

- Neutralinos: $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_{\tilde{B}'}, \tilde{\eta}, \tilde{\bar{\eta}}\right)^T$ and Z^N

- Scalar Higgs fields: $(\sigma_d, \sigma_u, \sigma_\eta, \sigma_{\bar{\eta}})^T$ and Z^H

- Pseudoscalar Higgs fields: $(\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}})^T$ and Z^A

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- Scalar sneutrinos: $(\sigma_L, \sigma_R, \sigma_S)^T$ and $Z^{\sigma v}$

- Pseudoscalar sneutrinos: $(\phi_L, \phi_R, \phi_S)^T$ and $Z^{\phi v}$

- Neutrinos: $(\nu_L, \nu^c, N_S)^T$ and U^V

where the σ_S, ϕ_S, and N_S are only present in the inverse see-saw case. The neutrino mass matrices for the minimal, inverse and linear see-saw cases can be written as

$$
\begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & \frac{2 v_\eta}{\sqrt{2}} Y_{\eta \nu^c} \end{pmatrix} \text{ or } \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu & 0 \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & 0 & \frac{v_\eta}{\sqrt{2}} Y_{IS} \\ 0 & \frac{v_\eta}{\sqrt{2}} Y_{IS}^T & \mu_N \end{pmatrix} \text{ or } \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu & \frac{\tilde{Y}}{2} v_d v_\eta \\ \frac{v_u}{\sqrt{2}} Y_\nu^T & 0 & \frac{v_\eta}{\sqrt{2}} Y_{IS} \\ \frac{\tilde{Y}^T}{2} v_d v_\eta & \frac{v_\eta}{\sqrt{2}} Y_{IS}^T & 0 \end{pmatrix} \tag{53}
$$

respectively. $\tilde{Y}$ is the running value of the effective $\frac{Y_{LR} Y_{LS}}{\mu_\rho}$ caused by the heavy superfields ρ and $\bar{\rho}$. In the minimal case, the see-saw of type I is recovered. The light neutrino mass matrix can be approximated in this case by

$$
m_\nu \simeq -\frac{v_u^2}{2\sqrt{2} v_\eta} Y_\nu Y_{\eta \nu^c}^{-1} Y_\nu \,. \tag{54}
$$

In the inverse see-saw case, the light neutrino masses can be written as

$$
m_\nu \simeq -\frac{v_u^2}{2} Y_\nu^T (Y_{IS}^T)^{-1} \mu_N Y_{IS} Y_\nu \tag{55}
$$

Free parameters. If the minimum conditions for the vacuum are solved with respect to μ, B_μ, μ_η and B_η, the following parameters can be treated as free in addition of those given in Eq. (19):

$$
Y_{\eta \nu^c}, \ Y_{IS}, \ \mu_N, \ B_0, \ \tan\beta', \ \text{sign}(\mu_\eta), \ M_{Z'} \tag{56}
$$

where we have used again $B_N = B_0 \mu_N$.

3. TOWARDS SLHAv3

The purpose of this paper is to define the extensions of the SUSY Les Houches Accords required to incorporate the models described above. In this context and with respect to the implementation of further models in future, it is helpful to propose some general rules for the naming of blocks and the allocation of PDG particle codes, which we present below.

We would like to point out that the SLHA conventions allow for redundant information to be contained in additional blocks. Hence, in the interests of backward compatibility, we propose that if the following names for the blocks are used, but in cases where the old SLHA1/2 blocks would also suffice, they should also be written in addition, *e.g.* if there are only four neutralinos, `NMIX` should be written as well as `NEUTRALINORM`, both containing the same information.

Block names for input and output. Blocks used to give parameters as input end with "`IN`", while the corresponding values as output are written in blocks without the ending "`IN`". For example, the gauge couplings at the output scale are given in the `GAUGE` block, while one could define them as input at the input scale with the `GAUGEIN` block.

Block names for mixing matrices. In SLHA 2 the neutralino and Higgs mixing matrices of the MSSM and NMSSM were named differently. However, the information about the current model is already given in the block `MODSEL` and the renaming is thus redundant. Therefore, in order to prevent a confusing amount of names for mixing matrices in different models we propose to use always the same names for the following mixing matrices regardless of their dimension:

- Scalar Higgs mixing matrix: `SCALARRM`

- Pseudoscalar Higgs mixing matrix: `PSEUDOSCALARRM`

- Charged Higgs mixing matrix: `CHARGEDSCALARRM`

- Neutralino mixing matrix: `NEUTRALINORM` (corresponds to `NMIX` in SLHA1/2)

- Chargino mixing matrices: `CHARGINOPLUSRM` and `CHARGINOMINUSRM` (corresponds to `VMIX` and `UMIX` respectively in SLHA1/2)

- Up-squark mixing matrix: `UPSQUARKRM` (corresponds to `USQMIX` in SLHA1/2)

- Down-squark mixing matrix: `DOWNSQUARKRM` (corresponds to `DSQMIX` in SLHA1/2)

- Charged slepton mixing matrix: `CHARGEDSLEPTONRM` (corresponds to `SELMIX` in SLHA1/2)

- Sneutrino mixing matrix: `SNEUTRINORM` (corresponds to `SNUMIX` in the MSSM). In the case of splitting the sneutrinos into real and imaginary parts, `SNEUTRINOEVENRM` for the CP-even and `SNEUTRINOODDRM` for the CP-odd states should be used (corresponding to `SNSMIX` and `SNAMIX` in SLHA1/2)

To distinguish these names from the SLHA1/2-specific names we have always used the suffix `RM` for Rotation Matrix.

Block names for couplings. For the naming of blocks which correspond to couplings and soft-breaking terms we propose to use names which already give information about the meaning of the parameter: the new names contain abbreviations for the involved fields and start with a prefix to assign the meaning of the parameters. The different prefixes should be

- `Y`: trilinear superpotential coupling

- `T`: trilinear softbreaking coupling

- `M`: bilinear superpotential coupling

- `B`: bilinear softbreaking coupling

- `L`: linear superpotential coupling

- `S`: linear softbreaking coupling

- `M2`: soft breaking scalar mass-squared

Because of the prefixes which already give information about the spin of the involved particles, it is not necessary to distinguish between fermions and scalars when naming the blocks. However, this requires the convention that all Majorana mass terms for fermions from chiral superfields appear in the superpotential, leaving only bilinear mass terms for the scalars in the soft SUSY-breaking Lagrangian. We propose the following names:

- MSSM: H_u: `HU`, H_d: `HD`, d^c: `D`, u^c: `U`, q: `Q`, e^c: `E`, l: `L`

- See-saw types I-III: ν^c: `NUR`, S: `15S`, $\bar{S}$: `15SB`, Z: `15Z`, $\bar{Z}$: `15ZB`, T: `15T`, $\bar{T}$: `15TB`, G: `24G`, W: `24W`, X: `24X`, $\bar{X}$: `24XB`, B: `24B`

- inverse and linear see-saw: S: `SL`

- $U(1)_R \times U(1)_{B-L}$ models: ξ_R: `CR`, $\bar{\xi}_R$: `CRB`, ξ_L: `CL`, $\bar{\xi}_L$: `CLB`

- $U(1)_Y \times U(1)_{B-L}$ models: η: `BIL`, $\bar{\eta}$: `BILB`, N_S: `NS`, N_S': `NSP`, ρ: `RHO`, $\bar{\rho}$: `RHOB`,

Using these conventions, the block `YU` for the up-type Yukawa coupling in the MSSM would be replaced by `YHUQU` and the block name for the μ-term would be `MHUHD`.

EXTPAR and MSOFT. In the MSSM the input parameters for the gaugino masses and the Higgs soft-breaking masses are given in `EXTPAR`. However, their output is given in `MSOFT`. Similarly, the singlet couplings in the NMSSM are defined by `EXTPAR` but their output is in `NMSSMRUN`. This is in some conflict with the general rule to use always the the input name plus `IN` as output. Therefore, we propose that all new one-dimensional soft-breaking parameters and couplings involved in the see-saw models presented here are not given in `EXTPAR` as input, but rather in the given output with the prefix `IN`.

PDG particle numbering scheme. We would like to introduce a consistent numbering scheme for the additional elementary particles introduced by such models, such that further particles may be added without worrying about accidentally using a pre-existing code, which also allows one to gather some information about the particle. While the proposed scheme would give new numbers to particles which already have codes, we restrict ourselves just to giving our new particles unique codes, while hoping that codes compliant with the new standard would be able to read either the old code or the new code for particles. The proposal is a signed nine digit integer for each mass eigenstate, which should easily fit in a 32-bit integer.

We note that the existing scheme is already mildly inconsistent, insofar as adding a fourth generation already has a PDG numbering scheme (17 for the extra neutrino and 18 for the τ'), yet SLHA2 uses 100017 and 100018 for CP-odd sneutrinos which are degrees of freedom from the first three generations. We note also that the Flavour Les Houches Accord [30] uses 17 and 18 for summing over the three Standard Model generations.

Since the particle code is a signed integer, pairs of conjugate particles are assigned the same code with different signs. We propose a convention for which particle gets the positive sign:

- If the particle is self-conjugate, there is only the positive sign. If the fermion is Majorana, it only has positive sign. If the scalar part of the superfield can be written in terms of self-conjugate CP eigenstates, it should be. If it cannot, the scalar field should be assigned a sign according to the rules below.

- If the particle has non-zero electric charge, the state with positive electric charge is taken as the particle (hence the positron are taken as the particle).

- If the particle is electrically neutral, but has baryon number B or lepton number L, the state with positive $B - L$ is taken as the particle (hence antineutrinos are taken as the particle).

- If the particle has $B - L = 0$ according to traditional assignment, but is still baryon- or lepton-like, a "temporary" $B - L$ is assigned. If the particle has colour charge a temporary B is first assigned by finding the combination of triplets and antitriplets which could combine to form the particle's representation: adding $+1/3$ for each triplet and $-1/3$ for each antitriplet, the combination which has the *lowest magnitude* of temporary B is assigned; *e.g.* an octet may be formed by a triplet with an antitriplet, giving $B = 0$, or by three triplets, giving $B = 1$, or three antitriplets, giving $B = -1$: in this case, $B = 0$ as the lowest $|B|$ is assigned. This temporary $B - L$ is only for the purposes of determining which state is taken to be the particle with positive code.

- If the neutral, colourless particle has no natural assignment by (temporary) $B - L$, there are still a few cases:

  - the fermion is massless: the left-handed fermion is given the positive code, thus in supersymmetric models, the scalars left-chiral superfields also are given positive codes if their fermions are massless.

  - the fermion has a Dirac mass: this does not occur in the MSSM, NMSSM, or any of the extended models described here. However it is conceivable. We propose that the model builder is responsible for deciding to assign a temporary lepton number to one of the fields and thus fix the convention.

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Under this scheme, quarks and squarks would have the same signs as they already have in the PDG conventions, leptons would have the opposite sign (*e.g.* the muon would have a negative code), and charginos and W bosons would have the same signs.

Once the sign is fixed, the digits are as follows:

1st digit: 1 if it is a mass eigenstate which has an admixture of a Standard Model gauge eigenstate (including the Higgs doublet), 2 if it does not mix with the Standard Model particles.

2nd digit: twice the spin of the particle.

3rd digit: the CP nature: complex bosons (spin 0, 1, or 2) and Dirac fermions have 0, while (massless or massive) scalar bosons, massless vector bosons, massless tensor bosons, and Majorana fermions have 1. Pseudoscalar bosons (massless or massive) and massive vector and tensor bosons have 2.

4th and 5th digits: a 2-digit number for the $SU(3)$ representation; $SU(3)$ singlets have 00; representations up to dimension 64 have either the Dynkin labels for an unbarred representation, or 99 minus the Dynkin labels for the barred representation. These representations are enumerated in [31]. Any particles with $SU(3)$ representation that does not fit into this scheme are assigned 99 (for instance those of dimension 65 or greater).

6th and 7th digits: a 2-digit number for the electric charge; six times the absolute value of the electric charge, relative to the electron, up to a maximum of 98/6. Any particle with a charge that does not fit into this scheme is assigned 99. For example, an electron would have 06, while a down quark would have 02, and a doubly-charged scalar would have 12.

8th and 9th digits: a generation number; 01 should be given to the lightest particle of any group which share the same first seven digits, 02 to the 2nd-lightest, and so on.

Consequently what might be considered to be the same model with or without R-parity will have different codes for some of its mass eigenstates. In the MSSM, with R-parity there would be three charged antileptons 110000601, 110000602, and 110000603, and two charginos 210000601 and 210000602, while without R-parity there would be five charged antileptons 110000601, 110000602, 110000603, 110000604 and 110000605, and no 210000601 or 210000602.

Additionally, the codes for the neutrinos will depend on whether they are Dirac or Majorana in the considered model.

4. EXTENSIONS TO SLHA

In this section we describe the implementation of the models presented in section 2 using the conventions defined in the previous section. Note that all parameters can be implemented in complex forms and the corresponding information can be passed by using the corresponding blocks starting with "IM" [19].

4.1. Block format

In the following, all blocks with a single index are to be written in the FORTRAN format (1x,I5,3x,1P,E16.8,0P,3x,#,1x,A) (the same format as the SLHA1 blocks HMIX, GAUGE, *etc.*). These blocks will be denoted as "rank one". All blocks with two indices are to be written in the FORTRAN format (1x,I2,1x,I2,3x,1P,E16.8,0P,3x,#,1x,A) (the same format as the SLHA1 blocks NMIX, UMIX, *etc.*). These blocks will be denoted as "rank two". All blocks with two indices are to be written in the FORTRAN format (1x,I2,1x,I2,1x,I2,3x,1P,E16.8,0P,3x,#,1x,A) (the same format as the SLHA2 blocks RVLAMLLE, RVLAMLQD, *etc.*). These blocks will be denoted as "rank three".

4.2. Blocks required for each model

Each model requires the presence of certain blocks. Some blocks are common to several models. We summarise the blocks needed for each of the models described in section 2 under the entry for selecting this model in the MODSEL block 4.3.1.

4.3. Extra Flags In Existing Blocks

4.3.1. Block `MODSEL`

Flag 3 (particle content) has further switches, arranged in groups: 11X is for effective realizations, 12X non-effective models, 13X for $U(1)_R \times U(1)_{B-L}$ models, and 14X is for $U(1)_Y \times U(1)_{B-L}$ models, the initial digit 1 indicating see-saw. 2XY might be used for fourth-generation models, 3XY for another set of models, and so on. In addition to the sets of required blocks listed for each model below, the `SEESAWGENERATIONS` block must also be given, and the appropriate flags set.

- Effective models

  110: Most general $SU(5)$-invariant effective see-saw (combination of types I-III); all the blocks required for types I-III below are required.

  111: type I see-saw; the following blocks are required: MNURNURIN, BNURNURIN, M2NURNURIN, YNURLHUIN, TNURLHUIN.

  112: type II see-saw ($SU(5)$ version); the following blocks are required: M15S15SBIN, B15S15SBIN, M215SIN, M215SBIN, M15T15TBIN, B15T15TBIN, M215TIN, M215TBIN, M15Z15ZBIN, B15Z15ZBIN, M215ZIN, M215ZBIN, YD15SDIN, TD15SDIN, YL15TLIN, TL15TLIN, YD15ZLIN, TD15ZLIN, YHD15THDIN, THD15THDIN, YHU15TBHUIN, THU15TBHUIN. Any or all of the blocks M15S15SBIN, M15T15TBIN, M15Z15ZBIN may be left absent if the block M15IN is present, and the missing terms are taken as copies of M15IN. Similarly for YD15SDIN, YL15TLIN, YD15ZLIN and Y15IN, and TD15SDIN, TL15TLIN, TD15ZLIN and Y15IN times A_0.

  113: type III see-saw ($SU(5)$ version); the following blocks are required: M24W24WIN, B24W24WIN, M224WIN, M24G24GIN, B24G24GIN, M224GIN, M24B24BIN, B24B24BIN, M224BIN, M24X24XBIN, B24X24XBIN, M224XIN, M224XBIN, YHU24BLIN, THU24BLN, YHU24WLIN, THU24WLN, YHU24BLIN, THU24BLN, YHU24XBDIN, THU24XBDIN. Any or all of the blocks M24W24WIN, M24G24GIN, M24B24BIN, M24X24XBIN may be left absent if the block M24IN is present, and the missing terms are taken as copies of M24IN. Similarly for B24W24WIN, B24G24GIN, B24B24BIN, B24X24XBIN and B24IN; YHU24BLIN, YHU24WLIN, YHU24XBDIN and Y24IN; and THU24BLIN, THU24WLIN, THU24XBDIN and Y24IN times A_0.

  114: type II see-saw (minimal version, only triplets); the following blocks are required: M15T15TBIN, B15T15TBIN, M215TIN, M215TBIN, YL15TLIN, TL15TLIN, YHD15THDIN, THD15THDIN, YHU15TBHUIN, THU15TBHUIN. If TL15TLIN is absent, YL15TLIN times A_0 is used in its place.

  115: type III see-saw (minimal version, only triplets); the following blocks are required: M24W24WIN, B24W24WIN, M224WIN, YHU24WLIN, THU24WLN. If THU24WLN is absent, YHU24WLIN times A_0 is used in its place.

- Linear and inverse see-saw:

  120 : combined inverse and linear see-saw; the following blocks are required: MNURSIN, BNURSIN, MNSNSIN, BNSNSIN, M2NSIN, M2NURIN, YNURLHUIN, TNURLHUIN. Purely inverse or linear see-saw is specified by appropriate zeroes in the block entries.

- $U(1)_R \times U(1)_{B-L}$:

  131 : YNURLHUIN, MDRDRBIN, YNURDRNURIN, TNURLHUIN, BDRDRBIN, TNURDRNURIN, M2NURNURIN, M2DRIN, M2DRBIN

  132 : inverse see-saw; the following blocks are required in addition to those required by the $U(1)_R \times U(1)_{B-L}$ minimal see-saw: MCRCRBIN, BCRCRBIN, YNSNURCRIN, TNSNURCRIN, M2CRCRIN, M2CRBCRBIN, MNSNSIN, BNSNSIN, M2NSIN, YNSNURCRIN, TNSNURCRIN.

  133 : linear see-saw; the following blocks are required in addition to those required by the $U(1)_R \times U(1)_{B-L}$ linear see-saw: YLCLNSIN, YCLCRHDIN, TLCLNSIN, TCLCRHDIN, MCLCLBIN, BCLCLBIN, M2CLIN, M2LCBIN.

- $U(1)_Y \times U(1)_{B-L}$:

    141 : minimal see-saw; the following blocks are required: `MBILBILBIN`, `BBILBILBIN`, `M2NURIN`, `M2BILIN`, `M2BILBIN`, `YNURLHUIN`, `TNURLHUIN`, `YNURBILNURIN`, `TNURBILNURIN`.

    142 : inverse see-saw; the following blocks are required in addition to those required by the $U(1)_Y \times U(1)_{B-L}$ minimal see-saw: `MNSNSIN`, `BNSNSIN`, `M2NSIN`, `YNSNURBILIN`, `TNSNURBILIN`.

    143 : linear see-saw; the following blocks are required in addition to those required by the $U(1)_Y \times U(1)_{B-L}$ minimal see-saw: `YNSNURBILIN`, `TNSNURBILIN`, `YLRHONSIN`, `TLRHINSIN`, `YRHOETAHDIN`, `TRHOETAHDIN`, `MRHORHOB`, `BRHORHOB`, `M2RHOIN`, `M2RHOBIN`, `M2NSIN`.

In addition, the $U(1)_R \times U(1)_{B-L}$ and $U(1)_Y \times U(1)_{B-L}$ models require that the appropriate entries in `GAUGEIN`, `MSOFTIN`, `MINPAR`, and `EXTPAR` are set correctly.

4.3.2. Block `MINPAR`

We extend the `MINPAR` block to include the input parameters necessary for the addition of an extra Abelian gauge group. We do not reproduce the existing entries here since there are quite a few. We only show those that are new and those that have new interpretations.

2: The common soft mass term for all the the gauginos.

6: The cosine of the phase of the μ' parameter.

7: The ratio of the two vacuum expectation values that either break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$ or break $U(1)_{B-L}$ leaving $U(1)_Y$ intact.

8: The mass of the Z' boson.

9: The common bilinear mass parameter B_0.

4.3.3. Block `GAUGE`

We extend the `GAUGE` block to include the couplings necessary for the addition of an extra Abelian gauge group. We reproduce the existing entries here for completeness.

1: The coupling $g_1(Q)$ (also known as $g'(Q)$ in some conventions) of the $U(1)_Y$ gauge in models which have it, or the coupling $g_R(Q)$ of the $U(1)_R$ gauge in such models where the $U(1)_R$ in combination with another $U(1)$ gauge breaks down to $U(1)_Y$.

2: The coupling $g_2(Q)$ (also known as $g(Q)$ in some conventions) of the $SU(2)_L$ gauge.

3: The coupling $g_3(Q)$ of the $SU(3)_c$ gauge.

4: The coupling $g_{BL}(Q)$ of the $U(1)_{B-L}$ gauge.

14: The off-diagonal coupling $g_T(Q)$ of the two $U(1)$ gauges in the triangle basis.

4.3.4. Block `MSOFT`

We extend the `MSOFT` block to include the soft mass terms necessary for the addition of an extra Abelian gauge group. We do not reproduce the existing entries here since there are quite a few. We only show those that are new and those that have new interpretations.

1: The soft mass term for the gaugino of the $U(1)_Y$ gauge in models which have it, or the gaugino of the $U(1)_R$ gauge in such models where the $U(1)_R$ in combination with another $U(1)$ gauge breaks down to $U(1)_Y$.

4: The soft mass term for the gaugino of the $U(1)_{B-L}$ gauge.

5: The soft mass term mixing the gauginos of the two $U(1)$ gauges.

4.3.5. Block `EXTPAR`

We extend the `EXTPAR` block to include the parameters necessary for the addition of an extra Abelian gauge group. We do not reproduce the existing entries here since there are quite a few. We only show those that are new and those that have new interpretations. Bear in mind that just as in the MSSM, contradictory inputs should not be given.

124: The tree-level mass-squared $m_{\phi_R}^2$ of the pseudoscalar formed by the doublets breaking $SU(2)_R$.

126: The pole mass m_{ϕ_R} of the pseudoscalar formed by the doublets breaking $SU(2)_R$.

4.4. Extended blocks superseding existing blocks

4.4.1. Block `XPMNSRM` (rank two), superseding `UPMNS` (rank one)

In the super-PMNS basis the mass matrices of the charged leptons and the three light neutrinos are diagonal and the relevant generation mixing information is given in the PMNS matrix. In the SLHA2 the input block `UPMNSIN` was defined in terms of three mixing angles and three phases [19] whereas for the output the complete 3×3 mixing has to be given in the block `UPMNS`. In general more than 3 neutrinos contribute and thus we propose that similarly the generalised PMNS matrix, which is rectangular, shall be given for both input and output. In the basis where, as above, the charged lepton and the extended neutrino mass matrices are diagonal, this matrix corresponds to the coupling between the left-handed charged leptons with the W boson and the neutrinos divided by $g/\sqrt{2}$.

4.5. New blocks

Blocks used to give parameters as input end with "`IN`", while the corresponding values as output are written in blocks without the ending "`IN`".

4.5.1. Block `SEESAWGENERATIONS` (rank one)

This block specifies the number of generations of the extra fields of the see-saw models.

1: The number n_{ν^c} of right-handed neutrino generations in the type I, inverse, linear, and $U(1)_Y \times U(1)_{B-L}$ see-saw models.

2: The number n_{N_S} of neutrino-like singlet generations in the inverse, linear, and $U(1)_Y \times U(1)_{B-L}$ see-saw models, and also the number of parity-odd neutrino-like singlet generations in $U(1)_Y \times U(1)_{B-L}$ inverse see-saw models.

15: The number n_{15} of 15-plet generations in the type II model.

24: The number n_{24} of 24-plet generations in the type III model.

4.5.2. Block `SCALARRM` (rank two)

This block specifies the mixing matrix of the neutral scalar Higgs bosons. The gauge eigenstates that are rotated into the mass-ordered mass eigenstates are ordered as H_d^0, H_u^0, then a model-dependent ordering. For see-saw types II and III, there are no further entries, because these fields are assumed to be integrated out. For $U(1)_R \times U(1)_{B-L}$, they are $\xi_R^0, \bar{\xi}_R^0$. For $U(1)_Y \times U(1)_{B-L}$, they are $\eta, \bar{\eta}$.

4.5.3. Block `PSEUDOSCALARRM` (rank two)

This block specifies the mixing matrix of the neutral pseudoscalar Higgs bosons. They are ordered analogously to how `SCALARRM` is ordered.

4.5.4. Block `GAUGEIN` (rank one)

This block specifies the gauge couplings at the input scale. The entries are analogous to those of the `GAUGE` block.

4.5.5. Block `MSOFTIN` (rank one)

This block specifies the soft mass terms at the input scale. The entries are analogous to those of the `MSOFT` block.

4.5.6. Superpotential mass matrix blocks (rank two)

The superpotential mass matrix blocks are `MNURNUR(IN)` for M_{ν^c}, `M15S15SB(IN)` for M_S, `M15T15TB(IN)` for M_T, `M15Z15ZB(IN)` for M_Z, `M24W24W(IN)` for M_W, `M24G24G(IN)` for M_G, `M24B24B(IN)` for M_B, `M24X24XB(IN)` for M_X, `MNURS(IN)` for M_R, `MNSNS(IN)` for μ_N, `MCRCRB(IN)` for μ_ξ, and `MBILBILB(IN)` for μ_η. `M15(IN)` for M_{15} is used when M_S, M_T, and M_Z are set to a common value. `M24(IN)` for M_{24} is used when M_W, M_B, M_G, and M_X are set to a common value. Though some of the models are described with only a single generation of some types of field, we allow for extra generations, and thus define all the mass matrix blocks as being rank two, even though the minimal case would use only the $(1,1)$ entry of some of them.

4.5.7. Soft SUSY-breaking mass matrix blocks (rank two)

The soft SUSY-breaking mass matrix blocks are `M2NUR(IN)` for $m_{\tilde{\nu}^c}^2$, `M215S(IN)` for m_S^2, `M215SB(IN)` for $m_{\bar S}^2$, `M215T(IN)` for m_T^2, `M215TB(IN)` for $m_{\bar T}^2$, `M215Z(IN)` for m_Z^2, `M215ZB(IN)` for $m_{\bar Z}^2$, `M224W(IN)` for m_W^2, `M224G(IN)` for m_G^2, `M224B(IN)` for m_B^2, `M224X(IN)` for m_X^2, `M224XB(IN)` for $m_{\bar X}^2$, `M2NS(IN)` for m_N^2, `M2CR(IN)` for m_ξ^2, `M2CRB(IN)` for $m_{\bar\xi}^2$, `M2BIL(IN)` for m_η^2, `M2BILB(IN)` for $m_{\bar\eta}^2$, and `M2NSP(IN)` for $m_{N'}^2$. Though some of the models are described with only a single generation of some types of field, we allow for extra generations, and thus define all the mass matrix blocks as being rank two, even though the minimal case would use only the $(1,1)$ entry of some of them.

4.5.8. Soft SUSY-breaking bilinear matrix blocks (rank two)

The soft SUSY-breaking bilinear matrix blocks are `BNURNUR(IN)` for $B_{\tilde{\nu}^c}$, `B15S15SB(IN)` for B_S, `B15T15TB(IN)` for B_T, `B15Z15ZB(IN)` for B_Z, `B24W24W(IN)` for B_W, *`B24G24G(IN)`* for B_G, `B24B24B(IN)` for B_B, `B24X24XB(IN)` for B_X, `BNURS(IN)` for B_R, `BNSNS(IN)` for B_N, `BCRCRB(IN)` for B_ξ, and `BBILBILB(IN)` for B_η. `B15(IN)` for B_{15} is used when B_S, B_T, and B_Z are set to a common value. `B24(IN)` for B_{24} is used when B_W, B_B, B_G, and B_X are set to a common value.

4.5.9. Yukawa coupling matrix blocks (rank three)

The first digit is the generation index of the 15-plet or 24-plet field in type II and III models respectively, and is 1 for the other models. The 2nd pair of indices correspond to the usual indices of the minimal cases. The Yukawa coupling matrix blocks are `YNURLHU(IN)` for Y_ν, `YL15TL(IN)` for Y_T, `YD15SD(IN)` for Y_S, `YD15ZL(IN)` for Y_Z, `YHD15THD(IN)` for λ_1, `YHU15TBHU(IN)` for λ_2, `YHU24BL(IN)` for Y_B, `YHU24WL(IN)` for Y_W, `YHU24XBD(IN)` for Y_X, `YNSLHU(IN)` for Y_{LN}, `YNSNURCR(IN)` for $Y_{N\nu^c}$, `YNURBILNUR(IN)` for $Y_{\eta\nu^c}$, and `YNSNURBIL(IN)` for Y_{IS}. `Y15(IN)` for Y_{15} is used when Y_S, Y_T, and Y_Z are set to a common value. `Y24(IN)` for Y_{24} is used when Y_B, Y_W, and Y_X are set to a common value.

4.5.10. Soft SUSY-breaking trilinear matrix blocks (rank three)

There is a soft SUSY-breaking trilinear matrix block for each Yukawa coupling matrix block, with the same name but with the initial "Y" replaced by "T", corresponding to the "T_{blah}" associated with the "Y_{blah}" matrices. However, there is no `T15(IN)` for T_{15} or `T24(IN)` for T_{24}; instead, in such constrained parameter sets, $A_0 \times Y_{15/24}$ is used.

4.5.11. Block `GAMZZPRM` (rank two)

This is the gauge boson rotation matrix $U^{\gamma ZZ'}$ given at the SUSY scale.

5. SOFTWARE TO STUDY SEE-SAW MODELS

We give in the following a short overview of software which support at least some of the models presented here.

- **SPheno** [32, 33]:
  SPheno is a spectrum calculator written in Fortran. It supports the MSSM with and without bilinear R-parity violation as well as high-scale extensions with the same particle content as the MSSM at the SUSY scale. So far, seesaw type I-III of the models discussed here are implemented. The For type-II it can be chosen between the variant with two $SU(2)_L$ 15-plets as well as the version with only one $SU(2)_L$ triplet. For all seesaw models the entire two-loop RGEs are included and the threshold corrections to gauge couplings and gaugino masses at the threshold scales are calculated. SPheno performs also a calculation of flavour observables like $l \to l'\gamma$ and $l \to 3l'$.

- **SuSeFLAV** [34]:
  SuSeFLAV is a spectrum calculator written in Fortran. It calculates SUSY spectrum for MSSM with conserved R-parity and its extension for inputs at high scale. It is specifically geared up studying type-I seesaw in great detail. Various input options for neutrino Yukawa couplings, CKM-like, PMNS-like and R-parameterisation are provided for the user. For MSSM and type I seesaw case it uses full two loop RGEs including flavour mixing and also calculates the threshold corrections at one loop level. SuSeFLAV also calculates flavour observables like $l \to l'\gamma$, $l \to 3l'$ and $(g_\mu - 2)$ at low energy.

- **SARAH** [35, 36, 37]:
  SARAH is a Mathematica package to derive analytical expressions for the masses, vertices, RGEs and one-loop corrections for a given SUSY model. This information can be used by SARAH to write model files for CalcHep/CompHep [38, 39], WHIZARD [40], FeynArts/FormCalc [41, 42] and Madgraph [43]. The model files for CalcHep can also be used with MicrOmegas [44] for relic density calculation. In addition, the output of Fortran source-code is possible which can be compiled with SPheno. This gives the possibility to create a full-fledged spectrum calculator based on 2-loop RGEs and 1-loop mass corrections for any model implemented in SARAH. Also routines for the calculation of flavour observables as well as for decay widths and branching ratios are written. Also input files for HiggsBounds [45, 46] are created by the SPheno modules of SARAH. So far, seesaw type I-III, inverse and linear seesaw as well as the models with $U(1)_Y \times U(1)_{B-L}$ gauge sector a part of the public version of SARAH.

- **SUSY Toolbox** [47]:
  The SUSY toolbox is a collection of scripts to create an environment consisting of SPheno, CalcHep, SARAH, SSP, HiggsBounds, MicrOmegas and WHIZARD to study extensions of the MSSM. These scripts give the possibility for an automatised implementation of a new model in all tools based on the implementation in SARAH.

6. CONCLUSIONS AND OUTLOOK

In this contribution we propose an extension of the existing SLHA accords to include several see-saw models. Firstly, this requires new blocks to be defined for the additional couplings and masses needed within the various models. In this connection, we do not restrict ourselves to individual versions of the latter but also allow for combinations of such models in order to be as general as possible. Secondly, several new particles have to be postulated. For these, we propose a 9-digit scheme for the corresponding PDG-codes, which could in fact be of more general use than for see-saw models only, yet it needs to be tested extensively against the properties of existing and possibly new SUSY models before widespread adoption. One issue that has to be addressed as next step is the proper definition of the additional parameters in the so-called super-PMNS basis. Moreover, also $SU(2)_R$ models are not yet covered in this proposal.

ACKNOWLEDGEMENTS

Appendix A. Kinetic mixing

It is well known that in models with several $U(1)$ gauge groups, kinetic mixing terms

$$- \chi_{ab} \hat{F}^{a,\mu\nu} \hat{F}^b_{\mu\nu}, \quad a \neq b \tag{A.1}$$

between the field-strength tensors are allowed by gauge and Lorentz invariance [48], as $\hat{F}^{a,\mu\nu}$ and $\hat{F}^{b,\mu\nu}$ are gauge invariant quantities by themselves, see *e.g.* [49]. Even if these terms are absent at tree level at a particular scale, they might be generated by RGE effects [50, 51]. This happens usually if the two Abelian gauge groups cannot be embedded in a larger gauge group simultaneously or if incomplete gauge multiplets of the fundamental theory are integrated out. It is easier to work with non-canonical covariant derivates instead of off-diagonal field-strength tensors such as in eq. (A.1). The equivalence of both approaches has been shown in [50, 52]. We show here the special case of two Abelian gauge groups $U(1)_A \times U(1)_B$. The covariant derivatives has the form

$$D_\mu = \partial_\mu - i Q_\phi^T G A \tag{A.2}$$

where Q_ϕ is a vector containing the charges of the field ϕ with respect to the two Abelian gauge groups, G is the gauge coupling matrix

$$G = \begin{pmatrix} g_{AA} & g_{AB} \\ g_{BA} & g_{BB} \end{pmatrix} \tag{A.3}$$

and A contains the gauge bosons $A = (A_\mu^A, A_\mu^B)^T$. As long as the two Abelian gauge groups are unbroken there is freedom to rotate the gauge bosons. It is convenient to choose a basis in which G gets a triangle form

$$G' = \begin{pmatrix} g & \tilde{g} \\ 0 & g' \end{pmatrix} \tag{A.4}$$

Mixing effects of Abelian gauge groups appear not only in the gauge sector but also for the gauginos because also terms of the form

$$M_{AB} \lambda_A \lambda_B \tag{A.5}$$

are allowed by gauge and Lorentz invariance [52, 53].

Appendix B. PDG CODES AND EXTENSIONS

We summarise here our codes for existing and for the extra particle content of the see-saw models. In the first column of each table, the conventional PDG number is shown.

Coloured Fermions		
PDG	Particle	PDG.IX
2	u	110100401
4	c	110100402
6	t	110100403
-1	$\bar{d}$	110890201
-3	$\bar{s}$	110890202
-5	$\bar{b}$	110890203
1000021	$\tilde{g}$	211110001

(a)

Colourless, Charged Fermions		
PDG	Particle	PDG.IX
-11	e^+	110000601
-13	μ^+	110000602
-15	τ^+	110000603
1000024	$\tilde{\chi}_1^+$	210000601
1000037	$\tilde{\chi}_2^+$	210000602

(b)

Neutral Fermions		
PDG	Particle	PDG.IX
-12/12	$\bar{\nu}_1^D$ / ν_1^M	110000001 /111000001
-14/14	$\bar{\nu}_2^D$ / ν_2^M	110000002 /111000002
-16/16	$\bar{\nu}_3^D$ / ν_3^M	110000003 /111000003
	$\bar{\nu}_4^D$ / ν_4^M	110000004 /111000004
	$\bar{\nu}_5^D$ / ν_5^M	110000005 /111000005
	$\bar{\nu}_6^D$ / ν_6^M	110000006 /111000006
	$\bar{\nu}_7^D$ / ν_7^M	110000007 /111000007
	$\bar{\nu}_8^D$ / ν_8^M	110000008 /111000008
	$\bar{\nu}_9^D$ / ν_9^M	110000009 /111000009
1000022	$\tilde{\chi}_1^0$	211000001
1000023	$\tilde{\chi}_2^0$	211000002
1000025	$\tilde{\chi}_3^0$	211000003
1000035	$\tilde{\chi}_4^0$	211000004
1000045	$\tilde{\chi}_5^0$	211000005
	$\tilde{\chi}_6^0$	211000006
	$\tilde{\chi}_7^0$	211000007

(c)

Table B.7: (a) The down-type quarks are considered antiparticles due to having negative electric charge, hence the anti-downs are the defining states, and since they are colour antitriplets, their colour digits are 89 $(99 - 10)$.
(b) In the case of R-parity violation, these fields mix to eigenstates with codes `1100006XY`.
(c) ν^D are Dirac neutrinos while ν^M are Majorana neutrinos. In the case of R-parity violation, the fields mix forming (Majorana) mass eigenstates with PDG.IX codes `1110000XY`.

Coloured Scalars

PDG	Particle	PDG.IX
1000002	$\tilde{u}_1$	200100401
1000004	$\tilde{u}_2$	200100402
1000006	$\tilde{u}_3$	200100403
2000002	$\tilde{u}_4$	200100404
2000004	$\tilde{u}_5$	200100405
2000006	$\tilde{u}_6$	200100406
-1000001	$\tilde{d}_1^*$	200890201
-1000003	$\tilde{d}_2^*$	200890202
-1000005	$\tilde{d}_3^*$	200890203
-2000001	$\tilde{d}_4^*$	200890204
-2000003	$\tilde{d}_5^*$	200890205
-2000005	$\tilde{d}_6^*$	200890206

(a)

Colourless, Charged Scalars

PDG	Particle	PDG.IX
-1000011	$\tilde{e}_1^+$	200000601
-1000013	$\tilde{e}_2^+$	200000602
-1000015	$\tilde{e}_3^+$	200000603
-2000011	$\tilde{e}_4^+$	200000604
-2000013	$\tilde{e}_5^+$	200000605
-2000015	$\tilde{e}_6^+$	200000606
37	H^+	100000601

(b)

Vector Bosons

PDG	Particle	PDG.IX
21	g	121110001
22	γ	121000001
23	Z	122000001
32	Z'	122000002
33	Z''	122000003
24	W^+	120000601
34	W'^+	120000602

(c)

Table B.8: (a) New PDG code for coloured scalars.
(b) In the case of R-parity violation, the fields mix to eigenstates with PDG.IX codes `1000006XY`.
(c) The W^+ is considered to be the particle (and hence the W^- the antiparticle) since it has positive electric charge.

Neutral Scalars		
PDG	Particle	PDG.IX
25	h_1	101000001
35	h_2	101000002
45	h_3	101000003
	h_4	101000004
36	A_1^0	102000001
46	A_2^0	102000002
-1000012/1000012	$\tilde{\nu}_1^*$ / $\mathrm{Re}(\tilde{\nu}_1)$	200000001 / 201000001
-1000014/1000014	$\tilde{\nu}_2^*$ / $\mathrm{Re}(\tilde{\nu}_2)$	200000002 / 201000002
-1000016/1000016	$\tilde{\nu}_3^*$ / $\mathrm{Re}(\tilde{\nu}_3)$	200000003 / 201000003
1000017	$\mathrm{Im}(\tilde{\nu}_1)$	202000001
1000018	$\mathrm{Im}(\tilde{\nu}_2)$	202000002
1000019	$\mathrm{Im}(\tilde{\nu}_3)$	202000003
	$\tilde{\nu}_4^*$ / $\mathrm{Re}(\tilde{\nu}_4)$ / $\mathrm{Im}(\tilde{\nu}_4)$	200000004 / 201000004 / 202000004
	$\tilde{\nu}_5^*$ / $\mathrm{Re}(\tilde{\nu}_5)$ / $\mathrm{Im}(\tilde{\nu}_5)$	200000005 / 201000005 / 202000005
	$\tilde{\nu}_6^*$ / $\mathrm{Re}(\tilde{\nu}_6)$ / $\mathrm{Im}(\tilde{\nu}_6)$	200000006 / 201000006 / 202000006
	$\tilde{\nu}_7^*$ / $\mathrm{Re}(\tilde{\nu}_7)$ / $\mathrm{Im}(\tilde{\nu}_7)$	200000007 / 201000007 / 202000007
	$\tilde{\nu}_8^*$ / $\mathrm{Re}(\tilde{\nu}_8)$ / $\mathrm{Im}(\tilde{\nu}_8)$	200000008 / 201000008 / 202000008
	$\tilde{\nu}_9^*$ / $\mathrm{Re}(\tilde{\nu}_9)$ / $\mathrm{Im}(\tilde{\nu}_9)$	200000009 / 201000009 / 202000009

Table B.9: In the case of R-parity violation but CP conservation, the CP-even and CP-odd components mix separately to form eigenstates with PDG.IX codes `1010000XY` and `1020000XY`, respectively. If CP violation is present, the numbers are `1000000XY` and `2000000XY` respectively (no R-parity violation) or just `1000000XY` (with R-parity violation).

Appendix C. Choice of basis

We propose in this section a choice of basis to fix ambiguities in the different models. This is to be understood as a generalization of the SCKM and SPMNS basis of the MSSM [20]. We focus here only on scenarios with three generations of right-handed superfields $\hat{\nu}^c$. In case of more or less generations some rules might have to be adjusted individually, see for instance [54]. Before we start with the new models presented here, we recapitulate first the SCKM and SPMNS basis of the MSSM.

Appendix C.1. SCKM and SPMNS basis

In the SCKM basis the quark Yukawa matrices in W_{MSSM} are diagonal.

$$(\hat{Y}_d)_{ii} = (U_d^\dagger Y_d^T V_d)_{ii} \qquad (\hat{Y}_u)_{ii} = (U_u^\dagger Y_u^T V_u)_{ii} \tag{C.1}$$

This is reached by a rotation of the quarks

$$d_L^o = V_d d_L \,, \quad u_L^o = V_u u_L \,, \quad d_R^o = U_d d_R \,, \quad u_R^o = U_u u_R \,, \tag{C.2}$$

The entire flavor structure can be absorbed in the CKM matrix defined as

$$V_{\text{CKM}} = V_u^\dagger V_d \,, \tag{C.3}$$

In addition, the following re-parametrization of the soft-breaking squark masses takes place

$$\hat{m}_{\tilde{q}}^2 \equiv V_d^\dagger m_{\tilde{q}}^2 V_d \,, \quad \hat{m}_{\tilde{u}}^2 \equiv U_u^\dagger m_{\tilde{u}}^{2\,T} U_u \,, \quad \hat{m}_{\tilde{d}}^2 \equiv U_d^\dagger m_{\tilde{d}}^{2\,T} U_d \,, \tag{C.4}$$

and the trilinear soft-breaking masses are defined as

$$\hat{T}_U \equiv U_u^\dagger T_U^T V_u \,, \quad \hat{T}_D \equiv U_d^\dagger T_D^T V_d \,, \tag{C.5}$$

In the SPMNS basis the effective neutrino mass term in Eq. (1) is diagonalized by a rotation of the neutrino fields

$$\nu^o = V_\nu \nu \,, \tag{C.6}$$

i.e.

$$(\hat{m}_\nu)_{ii} = (V_\nu^T m_\nu V_\nu)_{ii} \tag{C.7}$$

In addition, also the lepton Yukawa coupling is diagonalized by a rotation of the charged lepton fields

$$e_L^o = V_e e_L \quad \text{and} \quad e_R^o = U_e e_R \,. \tag{C.8}$$

The equivalent diagonalised charged lepton Yukawa matrix is

$$(\hat{Y}_e)_{ii} = (U_e^\dagger Y_e^T V_e)_{ii} \tag{C.9}$$

The PMNS basis can be defined by using the mixing matrices V_e and V_ν as

$$U_{PMNS} = V_e^\dagger V_\nu \,, \tag{C.10}$$

and contains the information about the neutrino mxing. It's standard parametrization is given by

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{C.11}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The angles θ_{12}, θ_{13} and θ_{23} are the solar neutrino angle, the reactor (or CHOOZ) angle and the atmospheric neutrino mixing angle, respectively. δ is the Dirac phase and α_i are Majorana phases. The recent experimental values of the angles are given in [55].

After that change of basis, the leptonic soft-breaking terms are written in the new basis as

$$\hat{m}_{\tilde{l}}^2 = V_e^\dagger m_{\tilde{l}}^2 V_e \,, \quad \hat{m}_{\tilde{e}}^2 = U_e^\dagger m_{\tilde{e}}^{2\,T} U_e \,, \tag{C.12}$$

$$\hat{T}_e = U_e^\dagger T_e^T V_e \tag{C.13}$$

Appendix C.2. Choice of basis in seesaw type I – III

Type I. In seesaw type-I we fix the additional superfield by diagonalizing M_{ν^c} due to a rotation of ν^c

$$U_{\nu^c}^\dagger M_{\nu^c} U_{\nu^c} = \hat{M}_{\nu^c}^{ii} \tag{C.14}$$

with

$$\nu^{c,0} = U_{\nu^c} \nu^c. \tag{C.15}$$

The neutrino Yukawa coupling can be re-expressed by

$$\hat{Y}_\nu = V_\nu^\dagger Y_\nu^T U_{\nu^c} \tag{C.16}$$

with the running $\overline{DR}$ value of the rotation matrix V_ν defined in Eq. (C.6). To fulfill neutrino data, $\hat{Y}_\nu$ can be calculated in this basis using the approach by Casas-Ibarra [56]

$$\hat{Y}_\nu = \sqrt{2}\frac{i}{v_u}\sqrt{\hat{M}_{\nu^c}} \cdot R \cdot \sqrt{\hat{m}_\nu} \cdot U_{PMNS}^\dagger, \tag{C.17}$$

where the $\hat{m}_\nu$ is a diagonal matrices containing the neutrino masses. R is in general a complex orthogonal matrix.

Type II. In type II with one generation of 15-plets no new freedom in the field definition arises. The neutrino data is mostly given by the matrix Y_T after $SU(5)$ breaking. This matrix is diagonalized by the same matrix as m_ν. If all neutrino eigenvalues, angles and phases were known, Y_T would be fixed up to an overall constant which can be easily estimated to be

$$\frac{M_T}{\lambda_2} \simeq 10^{15}\text{GeV} \left(\frac{0.05 \text{ eV}}{m_\nu}\right). \tag{C.18}$$

If a superposition of type-I and type-II is present, it is possible to find valid value for the superpotential parameters to get correct neutrino data as discussed in Ref. [57].

If several generations of 15-plets are added we propose that the corresponding bilinear term has to be diagonalized to fix the basis

$$U_{\overline{15}}^\dagger M_{15} V_{15} = \hat{M}_{15}^{ii} \tag{C.19}$$

with

$$\overline{15}^0 = U_{\overline{15}}\overline{15} \qquad 15^0 = V_{15}15. \tag{C.20}$$

If no GUT unifcation for the components of the 15-plets is assumed this diagonalization procedure has to be performed for the different components separately.

Type-III. In case of several generations of 24-plets their phases are fixed by diagonalizing the bilinear term M_{24}

$$U_{24}^\dagger M_{24} U_{24} = \hat{M}_{24}^{ii} \tag{C.21}$$

with

$$24^0 = U_{24}24. \tag{C.22}$$

Appendix C.3. Choice of basis in linear and inverse seesaw

We define here the basis for three generations of right-handed sneutrino superfields $\hat{\nu}^c$ and three generations of additional singlet superfields $\hat{N}_S$ by demanding M_R to be diagonal

$$U_N^\dagger M_R^T U_{\nu^c} = \hat{M}_R^{ii} \tag{C.23}$$

with

$$\nu^{c,0} = U_{\nu^c}\nu^c \qquad N_S^0 = U_N N_S \tag{C.24}$$

The other superpotential parameters are rotated as

$$\hat{Y}_\nu = V_\nu^\dagger Y_\nu^T U_{\nu^c} \qquad \hat{\mu}_N = U_N^\dagger \mu_N U_N \qquad \hat{Y}_{LS} = V_\nu^\dagger Y_{LS}^T U_N \tag{C.25}$$

while the soft-breaking terms read in the new basis

$$\hat{B}_R = U_N^\dagger B_R^T U_{\nu^c} \qquad \hat{B}_N = U_N^\dagger B_N U_N \tag{C.26}$$

$$\hat{T}_\nu = V_\nu^\dagger T_\nu^T U_{\nu^c} \qquad \hat{T}_{LS} = V_\nu^\dagger T_{LS}^T U_N \tag{C.27}$$

$$\hat{m}_{\nu^c}^2 = U_{\nu^c}^\dagger m_{\nu^c}^{2,T} U_{\nu^c} \qquad \hat{m}_N^2 = U_N^\dagger m_N^{2,T} U_N \tag{C.28}$$

To get correct neutrino data Y_ν has to be choosen apropiatly. This can be done by using the parametrization given in Ref. [58]. The formula in case of inverse seesaw reads

$$Y_\nu = \frac{\sqrt{2}}{v_u} U_{PMNS} \sqrt{\hat{m}_\nu} R^T \sqrt{\hat{\mu}}^{-1} \hat{M}_R \tag{C.29}$$

Here, $\hat{m}_\nu$ and R are the matrices already introduced in Eq. (C.17). In case of linear seesaw Y_ν can be calculated by

$$Y_\nu = \frac{2}{v_u^2} U_{PMNS} \sqrt{\hat{m}_\nu} A^T \sqrt{\hat{m}_\nu} U_{PMNS}^T \hat{Y}_{LN}^{-1} \hat{M}_R \tag{C.30}$$

with

$$A = \begin{pmatrix} \frac{1}{2} & a & b \\ -a & \frac{1}{2} & c \\ -b & -c & \frac{1}{2} \end{pmatrix} \tag{C.31}$$

and real numbers a, b, c.

Appendix C.4. Choice of basis in seesaw scenearios in $U(1)_R \times U(1)_{B-L}$ gauge sector

In case of minimal seesaw we fix the basis by demanding

$$U_{\nu^c}^\dagger Y_M U_{\nu^c} = \hat{Y}_M^{ii} \tag{C.32}$$

while for the linear and inverse realization of the seesaw, $Y_{N\nu^c}$ has to be diagonal

$$U_N^\dagger Y_{N\nu^c}^T U_{\nu^c} = \hat{Y}_{N\nu^c}^{ii}. \tag{C.33}$$

Both conditions can be fullfilled by a rotation of ν^c respectively ν^c and N_S

$$\nu^{c,0} = U_{\nu^c} \nu^c \qquad N_S^0 = U_N N_S. \tag{C.34}$$

The other superpotential parameters are rotated as

$$\hat{Y}_\nu = V_\nu^\dagger Y_\nu^T U_{\nu^c} \qquad \hat{\mu}_N = U_N^\dagger \mu_N U_N \qquad \hat{Y}_{LS} = V_\nu^\dagger Y_{LS}^T U_N \tag{C.35}$$

while the soft-breaking terms read in the new basis

$$\hat{T}_M = U_{\nu^c}^\dagger T_M U_{\nu^c} \qquad\qquad \hat{T}_{N\nu^c} = U_N^\dagger T_{N\nu^c}^T U_{\nu^c} \tag{C.36}$$

$$\hat{T}_\nu = V_\nu^\dagger T_\nu^T U_{\nu^c} \qquad \hat{T}_{LS} = V_\nu^\dagger T_{LS}^T V_N \tag{C.37}$$

$$\hat{B}_N = U_N^\dagger B_N U_N \tag{C.38}$$

$$\hat{m}_{\nu^c}^2 = U_{\nu^c}^\dagger m_{\nu^c}^{2,T} U_{\nu^c} \qquad \hat{m}_N^2 = U_N^\dagger m_N^{2,T} U_N \tag{C.39}$$

The neutrino Yukawa couplings can be choose in analogy to Eq. (C.17),Eq. (C.29) and Eq. (C.30).

Appendix C.5. Choice of basis in seenearios in $U(1)_Y \times U(1)_{B-L}$ gauge sector

In case of minimal seesaw we fix the basis by demanding

$$U_{\nu^c}^\dagger Y_{\eta\nu^c} U_{\nu^c} = \hat{Y}_{\eta\nu^c}^{ii} \tag{C.40}$$

while for the linear and inverse realization of the seesaw, $Y_{N\nu^c}$ has to be diagonal

$$U_N^\dagger Y_{IS}^T U_{\nu^c} = \hat{Y}_{IS}^{ii} \,. \tag{C.41}$$

Both conditions can be fullfilled by a rotation of ν^c respectively ν^c and N_S

$$\nu^{c,0} = U_{\nu^c}\nu^c \qquad N_S^0 = U_N N_S \,. \tag{C.42}$$

The other superpotential parameters are rotated as

$$\hat{Y}_\nu = V_\nu^\dagger Y_\nu^T U_{\nu^c} \qquad \hat{\mu}_N = U_N^\dagger \mu_N U_N \qquad \hat{Y}_{LS} = V_\nu^\dagger Y_{LS}^T U_N \tag{C.43}$$

while the soft-breaking terms read in the new basis

$$\hat{T}_{\eta\nu^c} = U_{\nu^c}^\dagger T_{\eta\nu^c} U_{\nu^c} \qquad\qquad \hat{T}_{IS} = U_N^\dagger T_{IS}^T U_{\nu^c} \tag{C.44}$$

$$\hat{T}_\nu = V_\nu^\dagger T_\nu^T U_{\nu^c} \qquad \hat{T}_{LS} = V_\nu^\dagger T_{LS}^T U_N \tag{C.45}$$

$$\hat{B}_N = U_N^\dagger B_N U_N \tag{C.46}$$

$$\hat{m}_{\nu^c}^2 = U_{\nu^c}^\dagger m_{\nu^c}^{2,T} U_{\nu^c} \qquad \hat{m}_N^2 = U_N^\dagger m_N^{2,T} U_N \tag{C.47}$$

Since the phase of the field $N_{S'}$ appearing in linear and inverse seesaw has no physical impact it is not necessary to fix it.

As for $U(1)_R \times U(1)_{B-L}$, the neutrino Yukawa couplings can be choose in analogy to Eq. (C.17),Eq. (C.29) and Eq. (C.30).

Appendix D. Tree-level mass matrices

We give here the mass matrices of the different models which change in comparison to the MSSM. The equations are based on the LaTeX output for the Mathematica package SARAH [35, 36, 37].
For a given basis Φ, the scalar mass matrix m^2 is defined as

$$L = -\Phi^\dagger m^2 \Phi \tag{D.1}$$

while for fermions the conventions are for Majorana mass matrices m_M and Dirac mass matrices m_D

$$L = -\Psi^T m_M \Psi \tag{D.2}$$

$$L = -\Psi_1^T m_D \Psi_2 \tag{D.3}$$

with basis vectors Ψ, Ψ_1 and Ψ_2 given in Weyl spinors.

Appendix D.1. Linear and inverse seesaw

We show here the mass matrices for an imaginary model which has all terms of the inverse and linear at once. The mass matrices for the physical relevant models, separated linear and inverse seesaw, can easily obtained by setting the unnecessary terms to zero.

Appendix D.1.1. Mass matrix for pseudo scalar sneutrinos
Basis: $(\sigma_L, \sigma_R, \sigma_S)$

$$m_{\nu^i}^2 = \begin{pmatrix} m_{11} & m_{21}^T & m_{31}^T \\ m_{21} & m_{22} & m_{32}^T \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \tag{D.4}$$

$$m_{11} = \frac{1}{8}\Big(2v_u^2\big(2\Re\big(Y_{LN}^T Y_{LN}^*\big) + 2\Re\big(Y_\nu^T Y_\nu^*\big)\big) + 8\Re\big(m_l^2\big) + \big(g_1^2 + g_2^2\big)\mathbf{1}\big(-v_u^2 + v_d^2\big)\Big) \tag{D.5}$$

$$m_{21} = \frac{1}{2}\frac{1}{\sqrt{2}}\Big(2v_d\Re\big(\mu Y_\nu^*\big) + v_u\big(2\Re\big(M_\nu Y_{LN}^*\big) - 2\Re\big(T_\nu\big)\big)\Big) \tag{D.6}$$

$$m_{22} = \frac{1}{4}\Big(2v_u^2\Re\big(Y_\nu Y_\nu^\dagger\big) + 4\Re\big(m_{\nu^c}^2\big) + 4\Re\big(M_\nu M_\nu^\dagger\big)\Big) \tag{D.7}$$

$$m_{31} = \frac{1}{4}\frac{1}{\sqrt{2}}\Big(4v_d\Re\big(\mu Y_{LN}^*\big) + v_u\big(4\Re\big(M_\nu^T Y_\nu^*\big) + 4\Re\big(\mu_N Y_{LN}^*\big) - 4\Re\big(T_{LN}\big)\big)\Big) \tag{D.8}$$

$$m_{32} = \frac{1}{4}\Big(2m_{\nu^c s}^{2,T} + 2v_u^2\Re\big(Y_{LN} Y_\nu^\dagger\big) - 4\Re\big(B_\nu^T\big) + 4\Re\big(\mu_N M_\nu^\dagger\big)\Big) \tag{D.9}$$

$$m_{33} = \frac{1}{8}\Big(2\big(2v_u^2\Re\big(Y_{LN} Y_{LN}^\dagger\big) + 4\Re\big(M_\nu^T M_\nu^*\big) - 2\Re(B_N)\big) - 4\Re\big(B_N\big) + 8\Re\big(m_N^2\big) + 8\Re\big(\mu_N \mu_N^*\big)\Big) \tag{D.10}$$

This matrix is diagonalized by Z^i:

$$Z^i m_{\nu^i}^2 Z^{i,\dagger} = m_{2,\nu^i}^{dia} \tag{D.11}$$

Appendix D.1.2. Mass matrix for scalar sneutrinos
Basis: (ϕ_L, ϕ_R, ϕ_S)

$$m_{\nu^R}^2 = \begin{pmatrix} m_{11} & m_{21}^T & m_{31}^T \\ m_{21} & m_{22} & m_{32}^T \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \tag{D.12}$$

$$m_{11} = \frac{1}{8}\Big(2v_u^2\Big(2\Re\big(Y_{LN}^T Y_{LN}^*\big) + 2\Re\big(Y_\nu^T Y_\nu^*\big)\Big) + 8\Re\big(m_l^2\big) + \big(g_1^2 + g_2^2\big)\mathbf{1}\big(-v_u^2 + v_d^2\big)\Big) \tag{D.13}$$

$$m_{21} = \frac{1}{2}\frac{1}{\sqrt{2}}\Big(-2v_d\Re\big(\mu Y_\nu^*\big) + v_u\big(2\Re\big(M_\nu Y_{LN}^*\big) + 2\Re\big(T_\nu\big)\big)\Big) \tag{D.14}$$

$$m_{22} = \frac{1}{4}\Big(2v_u^2\Re\big(Y_\nu Y_\nu^\dagger\big) + 4\Re\big(m_{\nu^c}^2\big) + 4\Re\big(M_\nu M_\nu^\dagger\big)\Big) \tag{D.15}$$

$$m_{31} = \frac{1}{4}\frac{1}{\sqrt{2}}\Big(-4v_d\Re\big(\mu Y_{LN}^*\big) + v_u\big(4\Re\big(M_\nu^T Y_\nu^*\big) + 4\Re\big(\mu_N Y_{LN}^*\big) + 4\Re\big(T_{LN}\big)\big)\Big) \tag{D.16}$$

$$m_{32} = \frac{1}{4}\Big(2m_{\nu^c s}^{2,T} + 2v_u^2\Re\big(Y_{LN} Y_\nu^\dagger\big) + 4\Re\big(B_\nu^T\big) + 4\Re\big(\mu_N M_\nu^\dagger\big)\Big) \tag{D.17}$$

$$m_{33} = \frac{1}{8}\Big(2\big(2v_u^2\Re\big(Y_{LN} Y_{LN}^\dagger\big) + 4\Re\big(M_\nu^T M_\nu^*\big) + 2\Re(B_N)\big) + +4\Re\big(B_N\big) + 8\Re\big(m_N^2\big) + 8\Re\big(\mu_N \mu_N^*\big)\Big) \tag{D.18}$$

This matrix is diagonalized by Z^R:

$$Z^R m_{\nu^R}^2 Z^{R,\dagger} = m_{2,\nu^R}^{dia} \tag{D.19}$$

Appendix D.1.3. Mass matrix for neutrinos

Basis: $\big(\nu_L, \nu_R, \tilde{S}\big)$

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}v_u Y_\nu^T & \frac{1}{\sqrt{2}}v_u Y_{LN}^T \\ \frac{1}{\sqrt{2}}v_u Y_\nu & 0 & M_\nu \\ \frac{1}{\sqrt{2}}v_u Y_{LN} & M_\nu^T & \mu_N \end{pmatrix} \tag{D.20}$$

This matrix is diagonalized by U^V:

$$U^{V,*} m_\nu U^{V,\dagger} = m_\nu^{dia} \tag{D.21}$$

Appendix D.2. See-saw in models with $U(1)_R \times U(1)_{B-L}$ gauge sector

We use in the following

$$v_{X_R}^2 = \begin{cases} 2v_{\delta_R}^2 \text{ minimal see-saw} \\ v_{\xi_R}^2 \text{ linear and inverse see-saw} \end{cases} , \qquad v_{\bar{X}_R}^2 = \begin{cases} 2v_{\bar{\delta}_R}^2 \text{ minimal see-saw} \\ v_{\bar{\xi}_R}^2 \text{ linear and inverse see-saw} \end{cases}$$

Appendix D.2.1. Mass matrix for down squarks

Basis: $\big(\tilde{d}_{L,\alpha}, \tilde{d}_{R,\alpha}\big)$

$$m_{\tilde{d}}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}}\big(v_d T_d^\dagger - v_u Y_d^\dagger \mu\big) \\ \frac{1}{\sqrt{2}}\big(v_d T_d - v_u \mu^* Y_d\big) & m_{22} \end{pmatrix} \tag{D.22}$$

$$m_{11} = \frac{1}{24}\Big(\mathbf{1}\big(3g_L^2\big(v_u^2 - v_d^2\big) + g_{BL}^2\big(v_{\bar{X}_R}^2 - v_{X_R}^2\big) + g_{BL}\tilde{g}\big(-v_{\bar{X}_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\big)\big) + 24m_q^2$$
$$+ 12v_d^2 Y_d^\dagger Y_d\Big) \tag{D.23}$$

$$m_{22} = \frac{1}{24}\Big(\mathbf{1}\big(3\big(g_R^2 + \tilde{g}^2\big)\big(-v_{\bar{X}_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\big) + g_{BL}^2\big(v_{X_R}^2 - v_{\bar{X}_R}^2\big) + g_{BL}\tilde{g}\big(4v_{\bar{X}_R}^2 - 4v_{X_R}^2 - v_u^2 + v_d^2\big)\big)$$
$$+ 24m_d^2 + 12v_d^2 Y_d Y_d^\dagger\Big) \tag{D.24}$$

This matrix is diagonalized by Z^D:

$$Z^D m_{\tilde{d}}^2 Z^{D,\dagger} = m_{2,\tilde{d}}^{dia} \tag{D.25}$$

Appendix D.2.2. Mass matrix for pseudo scalar sneutrinos
minimal seesaw. Basis: (σ_L, σ_R)

$$m_{\nu^i}^2 = \begin{pmatrix} m_{11} & m_{21}^T \\ m_{21} & m_{22} \end{pmatrix} \tag{D.26}$$

$$m_{11} = \frac{1}{8}\Big(\mathbf{1}\Big(2g_{BL}^2\big(-v_{\delta_R}^2 + v_{\bar\delta_R}^2\big) + g_{BL}\tilde{g}\big(2v_{\delta_R}^2 - 2v_{\bar\delta_R}^2 - v_u^2 + v_d^2\big) + g_L^2\big(-v_u^2 + v_d^2\big)\Big) + 8\Re\big(m_l^2\big)$$
$$+ 4v_u^2\Re\big(Y_v^T Y_v^*\big)\Big) \tag{D.27}$$

$$m_{21} = \frac{1}{4}\Big(-2\sqrt{2}v_d\Re\big(\mu Y_v^*\big) + v_u\big(2\sqrt{2}\Re\big(T_\nu\big) - v_{\delta_R}\big(2\Re\big(Y_M^T Y_v^*\big) + 2\Re\big(Y_M Y_v^*\big)\big)\big)\Big) \tag{D.28}$$

$$m_{22} = \frac{1}{8}\Big(\mathbf{1}\Big(2g_{BL}^2 v_{\bar\delta_R}^2 + g_{BL}\tilde{g}\big(-4v_{\bar\delta_R}^2 - v_d^2 + v_u^2\big) + \big(g_R^2 + \tilde{g}^2\big)\big(2v_{\bar\delta_R}^2 - v_u^2 + v_d^2\big)\Big) + 2\sqrt{2}\mu_\delta v_{\bar\delta_R}Y_M^\dagger$$
$$+ 2\sqrt{2}\mu_\delta v_{\bar\delta_R}Y_M^* + 8\Re\big(m_\nu^2\big) + 4v_u^2\Re\big(Y_v Y_v^\dagger\big)$$
$$+ 2v_{\delta_R}^2\big(2\Re\big(Y_M^T Y_M^*\big) + 2\Re\big(Y_M^T Y_M^\dagger\big) + 2\Re\big(Y_M Y_M^*\big) + 2\Re\big(Y_M Y_M^\dagger\big) - \big(g_R^2 + \big(-\tilde{g} + g_{BL}\big)^2\big)\mathbf{1}\big)$$
$$- 2\sqrt{2}v_{\delta_R}\big(2\Re\big(T_M\big) + 2\Re\big(T_M^T\big)\big) + 2\sqrt{2}v_{\bar\delta_R}\mu_\delta^*\big(Y_M + Y_M^T\big)\Big) \tag{D.29}$$

linear and inverse seesaw. Basis: $(\sigma_L, \sigma_R, \sigma_S)$

$$m_{\nu^i}^2 = \begin{pmatrix} m_{11} & m_{21}^T & \frac{1}{2}v_{\xi_R}v_u\Re\big(Y_v^T Y_{N\nu^c}^\dagger\big) \\ m_{21} & m_{22} & m_{32}^T \\ \frac{1}{2}v_{\xi_R}v_u\Re\big(Y_{N\nu^c}Y_v^*\big) & m_{32} & m_{33} \end{pmatrix} \tag{D.30}$$

$$m_{11} = \frac{1}{8}\Big(4v_u^2\Re\big(Y_v^T Y_v^*\big) + 8\Re\big(m_l^2\big) + \mathbf{1}\big(g_{BL}^2\big(-v_{\xi_R}^2 + v_{\bar\xi_R}^2\big) + g_{BL}\tilde{g}\big(-v_u^2 - v_{\xi_R}^2 + v_{\bar\xi_R}^2 + v_d^2\big) + g_L^2\big(-v_u^2 + v_d^2\big)\big)\Big) \tag{D.31}$$

$$m_{21} = -\frac{1}{2}\frac{1}{\sqrt{2}}\Big(2v_d\Re\big(\mu Y_v^*\big) - 2v_u\Re\big(T_\nu\big)\Big) \tag{D.32}$$

$$m_{22} = \frac{1}{8}\Big(\mathbf{1}\big(g_{BL}^2\big(-v_{\xi_R}^2 + v_{\bar\xi_R}^2\big) + g_{BL}\tilde{g}\big(-2v_{\bar\xi_R}^2 + 2v_{\xi_R}^2 - v_d^2 + v_u^2\big) - \big(g_R^2 + \tilde{g}^2\big)\big(-v_{\xi_R}^2 - v_d^2 + v_{\bar\xi_R}^2 + v_u^2\big)\big)$$
$$+ 8\Re\big(m_\nu^2\big) + 2\big(2v_u^2\Re\big(Y_v Y_v^\dagger\big) + 2v_{\xi_R}^2\Re\big(Y_{N\nu^c}^T Y_{N\nu^{c*}}\big)\big)\Big) \tag{D.33}$$

$$m_{32} = -\frac{1}{2}\frac{1}{\sqrt{2}}\Big(2v_{\bar\xi_R}\Re\big(\mu_\xi Y_{N\nu^c,*}\big) + v_{\xi_R}\big(-2\Re\big(T_{N\nu^c}\big) + 4\Re\big(\mu_N Y_{N\nu^{c*}}\big)\big)\Big) \tag{D.34}$$

$$m_{33} = \frac{1}{8}\Big(4\big(8\Re\big(\mu_N\mu_N^*\big) - B_N\big) - 4B_N^* - 8\Re\big(B_N\big) + 8\Re\big(m_N^2\big) + v_{\xi_R}^2\big(2v_d^2\Re\big(\tilde{W}_{LS}^T\tilde{W}_{LS}^*\big) + 4\Re\big(Y_{N\nu^c}Y_{N\nu^c}^\dagger\big)\big)\Big) \tag{D.35}$$

This matrix is diagonalized by Z^i:

$$Z^i m_{\nu^i}^2 Z^{i,\dagger} = m_{2,\nu^i}^{dia} \tag{D.36}$$

Appendix D.2.3. Mass matrix for scalar sneutrinos
minimal seesaw. Basis: (ϕ_L, ϕ_R)

$$m_{\nu^R}^2 = \begin{pmatrix} m_{11} & m_{21}^T \\ m_{21} & m_{22} \end{pmatrix} \tag{D.37}$$

$$m_{11} = \frac{1}{8}\Big(\mathbf{1}\Big(2g_{BL}^2\big(-v_{\bar\delta_R}^2 + v_{\delta_R}^2\big) + g_{BL}\tilde g\big(2v_{\bar\delta_R}^2 - 2v_{\delta_R}^2 - v_u^2 + v_d^2\big) + g_L^2\big(-v_u^2 + v_d^2\big)\Big) + 8\Re\big(m_l^2\big)$$
$$+ 4v_u^2\Re\big(Y_v^T Y_v^*\big)\Big) \tag{D.38}$$

$$m_{21} = \frac{1}{4}\Big(-2\sqrt{2}v_d\Re\big(\mu Y_v^*\big) + v_u\big(2\sqrt{2}\Re\big(T_\nu\big) + v_{\delta_R}\big(2\Re\big(Y_M^T Y_v^*\big) + 2\Re\big(Y_M Y_v^*\big)\big)\big)\Big) \tag{D.39}$$

$$m_{22} = \frac{1}{8}\Big(\mathbf{1}\Big(2g_{BL}^2 v_{\bar\delta_R}^2 + g_{BL}\tilde g\big(-4v_{\bar\delta_R}^2 - v_d^2 + v_u^2\big) + \big(g_R^2 + \tilde g^2\big)\big(2v_{\bar\delta_R}^2 - v_u^2 + v_d^2\big)\Big) - 2\sqrt{2}\mu_\delta v_{\bar\delta_R} Y_M^\dagger$$
$$- 2\sqrt{2}\mu_\delta v_{\bar\delta_R} Y_M^* + 8\Re\big(m_\nu^2\big) + 4v_u^2\Re\big(Y_v Y_v^\dagger\big)$$
$$+ 2v_{\delta_R}^2\big(2\Re\big(Y_M^T Y_M^*\big) + 2\Re\big(Y_M^T Y_M^\dagger\big) + 2\Re\big(Y_M Y_M^*\big) + 2\Re\big(Y_M Y_M^\dagger\big) - \big(g_R^2 + \big(-\tilde g + g_{BL}\big)^2\big)\mathbf{1}\big)$$
$$+ 2\sqrt{2}v_{\delta_R}\big(2\Re\big(T_M\big) + 2\Re\big(T_M^T\big)\big) - 2\sqrt{2}v_{\bar\delta_R}\mu_\delta^*\big(Y_M + Y_M^T\big)\Big) \tag{D.40}$$

linear and inverse seesaw. Basis: (ϕ_L, ϕ_R, ϕ_S)

$$m_{\nu R}^2 = \begin{pmatrix} m_{11} & m_{21}^T & \frac{1}{2}v_{\xi_R}v_u\Re\big(Y_v^T Y_{N\nu^c}^\dagger\big) \\ m_{21} & m_{22} & m_{32}^T \\ \frac{1}{2}v_{\xi_R}v_u\Re\big(Y_{N\nu^c}Y_v^*\big) & m_{32} & m_{33} \end{pmatrix} \tag{D.41}$$

$$m_{11} = \frac{1}{8}\Big(4v_u^2\Re\big(Y_v^T Y_v^*\big) + 8\Re\big(m_l^2\big) + \mathbf{1}\big(g_{BL}^2\big(-v_{\bar\xi_R}^2 + v_{\xi_R}^2\big) + g_{BL}\tilde g\big(-v_u^2 - v_{\xi_R}^2 + v_{\bar\xi_R}^2 + v_d^2\big) + g_L^2\big(-v_u^2 + v_d^2\big)\big)\Big) \tag{D.42}$$

$$m_{21} = -\frac{1}{2}\frac{1}{\sqrt{2}}\Big(2v_d\Re\big(\mu Y_v^*\big) - 2v_u\Re\big(T_\nu\big)\Big) \tag{D.43}$$

$$m_{22} = \frac{1}{8}\Big(\mathbf{1}\Big(g_{BL}^2\big(-v_{\xi_R}^2 + v_{\bar\xi_R}^2\big) + g_{BL}\tilde g\big(-2v_{\xi_R}^2 + 2v_{\bar\xi_R}^2 - v_d^2 + v_u^2\big) - \big(g_R^2 + \tilde g^2\big)\big(-v_{\bar\xi_R}^2 - v_d^2 + v_{\xi_R}^2 + v_u^2\big)\Big)$$
$$+ 8\Re\big(m_\nu^2\big) + 2\big(2v_u^2\Re\big(Y_v Y_v^\dagger\big) + 2v_{\xi_R}^2\Re\big(Y_{N\nu^c}^T Y_{N\nu^c *}\big)\big)\Big) \tag{D.44}$$

$$m_{32} = \frac{1}{2}\frac{1}{\sqrt{2}}\Big(-2v_{\bar\xi_R}\Re\big(\mu_\xi Y_{N\nu^c,*}\big) + v_{\xi_R}\big(2\Re\big(T_{N\nu^c}\big) + 4\Re\big(\mu_N Y_{N\nu^c *}\big)\big)\Big) \tag{D.45}$$

$$m_{33} = \frac{1}{8}\Big(4\big(8\Re\big(\mu_N\mu_N^*\big) + B_N\big) + 4B_N^* + 8\Re\big(B_N\big) + 8\Re\big(m_N^2\big) + v_{\xi_R}^2\big(2v_d^2\Re\big(\tilde W_{LS}^T \tilde W_{LS}^*\big) + 4\Re\big(Y_{N\nu^c}Y_{N\nu^c}^\dagger\big)\big)\Big) \tag{D.46}$$

This matrix is diagonalized by Z^R:

$$Z^R m_{\nu R}^2 Z^{R,\dagger} = m_{2,\nu R}^{dia} \tag{D.47}$$

Appendix D.2.4. Mass matrix for up squarks

Basis: $(\tilde u_{L,\alpha}, \tilde u_{R,\alpha})$

$$m_{\tilde u}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}}\big(-v_d Y_u^\dagger \mu + v_u T_u^\dagger\big) \\ \frac{1}{\sqrt{2}}\big(-v_d\mu^* Y_u + v_u T_u\big) & m_{22} \end{pmatrix} \tag{D.48}$$

$$m_{11} = \frac{1}{24}\Big(\mathbf{1}\big(3g_L^2\big(v_d^2 - v_u^2\big) + g_{BL}^2\big(v_{\bar X_R}^2 - v_{X_R}^2\big) + g_{BL}\tilde g\big(-v_{\bar X_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\big)\big) + 24m_q^2$$
$$+ 12v_u^2 Y_u^\dagger Y_u\Big) \tag{D.49}$$

$$m_{22} = \frac{1}{24}\Big(\mathbf{1}\big(3\big(g_R^2 + \tilde g^2\big)\big(-v_{X_R}^2 - v_u^2 + v_{\bar X_R}^2 + v_d^2\big) + g_{BL}^2\big(v_{X_R}^2 - v_{\bar X_R}^2\big) + g_{BL}\tilde g\big(-2v_{\bar X_R}^2 + 2v_{X_R}^2 - v_u^2 + v_d^2\big)\big)$$
$$+ 24m_u^2 + 12v_u^2 Y_u Y_u^\dagger\Big) \tag{D.50}$$

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This matrix is diagonalized by Z^U:

$$Z^U m_{\tilde{u}}^2 Z^{U,\dagger} = m_{2,\tilde{u}}^{dia} \tag{D.51}$$

Appendix D.2.5. Mass matrix charged sleptons

Basis: $(\tilde{e}_L, \tilde{e}_R)$

$$m_{\tilde{e}}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}}\left(v_d T_e^\dagger - v_u Y_e^\dagger \mu\right) \\ \frac{1}{\sqrt{2}}\left(v_d T_e - v_u \mu^* Y_e\right) & m_{22} \end{pmatrix} \tag{D.52}$$

$$m_{11} = \frac{1}{8}\left(4v_d^2 Y_e^\dagger Y_e + 8m_l^2 + \mathbf{1}\left(g_{BL}^2\left(v_{X_R}^2 - v_{\bar{X}_R}^2\right) + g_{BL}\tilde{g}\left(-v_{X_R}^2 - v_u^2 + v_{\bar{X}_R}^2 + v_d^2\right) + g_L^2\left(v_u^2 - v_d^2\right)\right)\right) \tag{D.53}$$

$$m_{22} = \frac{1}{8}\left(\mathbf{1}\left(g_{BL}^2\left(v_{\bar{X}_R}^2 - v_{X_R}^2\right) + g_{BL}\tilde{g}\left(v_u^2 - v_d^2\right) + \left(g_R^2 + \tilde{g}^2\right)\left(-v_{\bar{X}_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\right)\right) + 8m_e^2 \right.$$
$$\left. + 4v_d^2 Y_e Y_e^\dagger\right) \tag{D.54}$$

This matrix is diagonalized by Z^E:

$$Z^E m_{\tilde{e}}^2 Z^{E,\dagger} = m_{2,\tilde{e}}^{dia} \tag{D.55}$$

Appendix D.2.6. Mass matrix for scalar Higgs
minimal seesaw. Basis: $(\sigma_d, \sigma_u, \sigma_\delta, \sigma_{\bar{\delta}})$

$$m_h^2 = \begin{pmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{21} & m_{22} & m_{32} & m_{42} \\ m_{31} & m_{32} & m_{33} & m_{43} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \tag{D.56}$$

$$m_{11} = \frac{1}{8}\Big(8m_{H_d}^2 + g_L^2\big(3v_d^2 - v_u^2\big) - g_R^2\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 - 3v_d^2 + v_u^2\big)$$
$$+ \tilde{g}\big(2g_{BL}\big(-v_{\bar{\delta}_R}^2 + v_{\delta_R}^2\big) - \tilde{g}\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 - 3v_d^2 + v_u^2\big)\big) + 8|\mu|^2\Big) \tag{D.57}$$

$$m_{21} = -\frac{1}{4}\Big(g_L^2 + g_R^2 + \tilde{g}^2\Big)v_d v_u - \Re\Big(B_\mu\Big) \tag{D.58}$$

$$m_{22} = \frac{1}{8}\Big(8m_{H_u}^2 - g_L^2\big(-3v_u^2 + v_d^2\big) + g_R^2\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 + 3v_u^2 - v_d^2\big)$$
$$+ \tilde{g}\big(-2g_{BL}\big(-v_{\bar{\delta}_R}^2 + v_{\delta_R}^2\big) + \tilde{g}\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 + 3v_u^2 - v_d^2\big)\big) + 8|\mu|^2\Big) \tag{D.59}$$

$$m_{31} = -\frac{1}{2}\Big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\Big)v_{\delta_R}v_d \tag{D.60}$$

$$m_{32} = \frac{1}{2}\Big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\Big)v_{\delta_R}v_u \tag{D.61}$$

$$m_{33} = \frac{1}{4}\Big(4m_\delta^2 + g_{BL}^2\big(-2v_{\bar{\delta}_R}^2 + 6v_{\delta_R}^2\big) + g_{BL}\tilde{g}\big(-12v_{\delta_R}^2 + 4v_{\bar{\delta}_R}^2 - v_u^2 + v_d^2\big)$$
$$+ \big(g_R^2 + \tilde{g}^2\big)\big(-2v_{\bar{\delta}_R}^2 + 6v_{\delta_R}^2 - v_d^2 + v_u^2\big) + 4|\mu_\delta|^2\Big) \tag{D.62}$$

$$m_{41} = \frac{1}{2}\Big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\Big)v_{\bar{\delta}_R}v_d \tag{D.63}$$

$$m_{42} = -\frac{1}{2}\Big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\Big)v_{\bar{\delta}_R}v_u \tag{D.64}$$

$$m_{43} = -\Big(g_R^2 + \big(-\tilde{g} + g_{BL}\big)^2\Big)v_{\delta_R}v_{\bar{\delta}_R} - \Re\Big(B_\delta\Big) \tag{D.65}$$

$$m_{44} = \frac{1}{4}\Big(4m_{\bar{\delta}}^2 - 2g_{BL}^2\big(-3v_{\bar{\delta}_R}^2 + v_{\delta_R}^2\big) + g_{BL}\tilde{g}\big(-12v_{\bar{\delta}_R}^2 + 4v_{\delta_R}^2 - v_d^2 + v_u^2\big)$$
$$- \big(g_R^2 + \tilde{g}^2\big)\big(2v_{\delta_R}^2 - 6v_{\bar{\delta}_R}^2 - v_d^2 + v_u^2\big) + 4|\mu_\delta|^2\Big) \tag{D.66}$$

linear and inverse seesaw. Basis: $\big(\sigma_d, \sigma_u, \sigma_{\bar{\xi}}, \sigma_\xi\big)$

$$m_h^2 = \begin{pmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{21} & m_{22} & m_{32} & m_{42} \\ m_{31} & m_{32} & m_{33} & m_{43} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \tag{D.67}$$

$$m_{11} = \frac{1}{8}\Big(8m_{H_d}^2 + g_L^2\big(3v_d^2 - v_u^2\big) - g_R^2\big(-3v_d^2 - v_{\xi_R}^2 + v_{\bar\xi_R}^2 + v_u^2\big)$$
$$+ \tilde{g}\big(g_{BL}\big(-v_{\xi_R}^2 + v_{\bar\xi_R}^2\big) - \tilde{g}\big(-3v_d^2 - v_{\xi_R}^2 + v_{\bar\xi_R}^2 + v_u^2\big)\big) + 8|\mu|^2\Big) \tag{D.68}$$

$$m_{21} = -\frac{1}{4}\big(g_L^2 + g_R^2 + \tilde{g}^2\big)v_d v_u - \Re\big(B_\mu\big) \tag{D.69}$$

$$m_{22} = \frac{1}{8}\Big(8m_{H_u}^2 - g_L^2\big(-3v_u^2 + v_d^2\big) + g_R^2\big(3v_u^2 - v_{\xi_R}^2 - v_d^2 + v_{\bar\xi_R}^2\big)$$
$$+ \tilde{g}\big(g_{BL}\big(-v_{\xi_R}^2 + v_{\bar\xi_R}^2\big) + \tilde{g}\big(3v_u^2 - v_{\xi_R}^2 - v_d^2 + v_{\bar\xi_R}^2\big)\big) + 8|\mu|^2\Big) \tag{D.70}$$

$$m_{31} = -\frac{1}{4}\big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\big)v_{\xi_R}v_d \tag{D.71}$$

$$m_{32} = \frac{1}{4}\big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\big)v_{\xi_R}v_u \tag{D.72}$$

$$m_{33} = \frac{1}{8}\Big(8m_\xi^2 + g_{BL}^2\big(3v_{\xi_R}^2 - v_{\bar\xi_R}^2\big) + g_{BL}\tilde{g}\big(2v_{\xi_R}^2 - 6v_{\bar\xi_R}^2 - v_u^2 + v_d^2\big)$$
$$+ \big(g_R^2 + \tilde{g}^2\big)\big(3v_{\xi_R}^2 - v_{\bar\xi_R}^2 - v_d^2 + v_u^2\big) + 8|\mu_\xi|^2\Big) \tag{D.73}$$

$$m_{41} = \frac{1}{4}\big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\big)v_{\bar\xi_R}v_d \tag{D.74}$$

$$m_{42} = -\frac{1}{4}\big(\tilde{g}\big(-g_{BL} + \tilde{g}\big) + g_R^2\big)v_{\bar\xi_R}v_u \tag{D.75}$$

$$m_{43} = \frac{1}{4}\Big(-4\Re\big(B_\xi\big) - \big(g_R^2 + \big(-\tilde{g} + g_{BL}\big)^2\big)v_{\xi_R}v_{\bar\xi_R}\Big) \tag{D.76}$$

$$m_{44} = \frac{1}{8}\Big(8m_{\bar\xi}^2 - g_{BL}^2\big(-3v_{\bar\xi_R}^2 + v_{\xi_R}^2\big) + g_{BL}\tilde{g}\big(2v_{\xi_R}^2 - 6v_{\bar\xi_R}^2 - v_d^2 + v_u^2\big)$$
$$- \big(g_R^2 + \tilde{g}^2\big)\big(-3v_{\bar\xi_R}^2 - v_d^2 + v_{\xi_R}^2 + v_u^2\big) + 8|\mu_\xi|^2\Big) \tag{D.77}$$

This matrix is diagonalized by Z^H:

$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \tag{D.78}$$

Appendix D.2.7. Mass matrix for pseudo scalar Higgs minimal seesaw. Basis: $(\phi_d, \phi_u, \phi_\delta, \phi_{\bar\delta})$

$$m_{A_0}^2 = \begin{pmatrix} m_{11} & \Re\big(B_\mu\big) & 0 & 0 \\ \Re\big(B_\mu\big) & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & \Re\big(B_\delta\big) \\ 0 & 0 & \Re\big(B_\delta\big) & m_{44} \end{pmatrix} \tag{D.79}$$

$$m_{11} = \frac{1}{8}\Big(8m_{H_d}^2 + g_L^2\big(-v_u^2 + v_d^2\big) + g_R^2\big(2v_{\bar{\delta}_R}^2 - 2v_{\delta_R}^2 - v_u^2 + v_d^2\big)$$
$$+ \tilde{g}\big(2g_{BL}\big(-v_{\delta_R}^2 + v_{\bar{\delta}_R}^2\big) + \tilde{g}\big(2v_{\bar{\delta}_R}^2 - 2v_{\delta_R}^2 - v_u^2 + v_d^2\big)\big) + 8|\mu|^2\Big) \tag{D.80}$$

$$m_{22} = \frac{1}{8}\Big(8m_{H_u}^2 + g_L^2\big(-v_d^2 + v_u^2\big) + g_R^2\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 - v_d^2 + v_u^2\big)$$
$$+ \tilde{g}\big(-2g_{BL}\big(-v_{\delta_R}^2 + v_{\bar{\delta}_R}^2\big) + \tilde{g}\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 - v_d^2 + v_u^2\big)\big) + 8|\mu|^2\Big) \tag{D.81}$$

$$m_{33} = \frac{1}{4}\Big(4m_\delta^2 + 2g_{BL}^2\big(-v_{\delta_R}^2 + v_{\bar{\delta}_R}^2\big) + g_{BL}\tilde{g}\big(4v_{\bar{\delta}_R}^2 - 4v_{\delta_R}^2 - v_u^2 + v_d^2\big) + \big(g_R^2 + \tilde{g}^2\big)\big(-2v_{\delta_R}^2 + 2v_{\bar{\delta}_R}^2 - v_d^2 + v_u^2\big)$$
$$+ 4|\mu_\delta|^2\Big) \tag{D.82}$$

$$m_{44} = \frac{1}{4}\Big(4m_\delta^2 - 2g_{BL}^2\big(-v_{\bar{\delta}_R}^2 + v_{\delta_R}^2\big) + g_{BL}\tilde{g}\big(-4v_{\bar{\delta}_R}^2 + 4v_{\delta_R}^2 - v_d^2 + v_u^2\big)$$
$$- \big(g_R^2 + \tilde{g}^2\big)\big(-2v_{\bar{\delta}_R}^2 + 2v_{\delta_R}^2 - v_d^2 + v_u^2\big) + 4|\mu_\delta|^2\Big) \tag{D.83}$$

linear and inverse seesaw. Basis: $\big(\phi_d, \phi_u, \phi_{\bar{\xi}}, \phi_\xi\big)$

$$m_{A_0}^2 = \begin{pmatrix} m_{11} & \Re\big(B_\mu\big) & 0 & 0 \\ \Re\big(B_\mu\big) & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & \Re\big(B_\xi\big) \\ 0 & 0 & \Re\big(B_\xi\big) & m_{44} \end{pmatrix} \tag{D.84}$$

$$m_{11} = \frac{1}{8}\Big(8m_{H_d}^2 + g_L^2\big(-v_u^2 + v_d^2\big) + g_R^2\big(-v_u^2 - v_{\xi_R}^2 + v_{\bar{\xi}_R}^2 + v_d^2\big)$$
$$+ \tilde{g}\big(g_{BL}\big(-v_{\bar{\xi}_R}^2 + v_{\xi_R}^2\big) + \tilde{g}\big(-v_u^2 - v_{\xi_R}^2 + v_{\bar{\xi}_R}^2 + v_d^2\big)\big) + 8|\mu|^2\Big) \tag{D.85}$$

$$m_{22} = \frac{1}{8}\Big(8m_{H_u}^2 + g_L^2\big(-v_d^2 + v_u^2\big) + g_R^2\big(-v_{\bar{\xi}_R}^2 - v_d^2 + v_{\xi_R}^2 + v_u^2\big)$$
$$+ \tilde{g}\big(g_{BL}\big(-v_{\xi_R}^2 + v_{\bar{\xi}_R}^2\big) + \tilde{g}\big(-v_{\bar{\xi}_R}^2 - v_d^2 + v_{\xi_R}^2 + v_u^2\big)\big) + 8|\mu|^2\Big) \tag{D.86}$$

$$m_{33} = \frac{1}{8}\Big(8m_\xi^2 + g_{BL}^2\big(-v_{\bar{\xi}_R}^2 + v_{\xi_R}^2\big) + g_{BL}\tilde{g}\big(2v_{\xi_R}^2 - 2v_{\bar{\xi}_R}^2 - v_u^2 + v_d^2\big) + \big(g_R^2 + \tilde{g}^2\big)\big(-v_{\xi_R}^2 - v_d^2 + v_{\bar{\xi}_R}^2 + v_u^2\big)$$
$$+ 8|\mu_\xi|^2\Big) \tag{D.87}$$

$$m_{44} = \frac{1}{8}\Big(8m_\xi^2 + g_{BL}^2\big(-v_{\xi_R}^2 + v_{\bar{\xi}_R}^2\big) + g_{BL}\tilde{g}\big(-2v_{\xi_R}^2 + 2v_{\bar{\xi}_R}^2 - v_d^2 + v_u^2\big) - \big(g_R^2 + \tilde{g}^2\big)\big(-v_{\bar{\xi}_R}^2 - v_d^2 + v_{\xi_R}^2 + v_u^2\big)$$
$$+ 8|\mu_\xi|^2\Big) \tag{D.88}$$

This matrix is diagonalized by Z^A:

$$Z^A m_{A_0}^2 Z^{A,\dagger} = m_{2,A_0}^{dia} \tag{D.89}$$

Using the solution of the tadpole equations the mass matrix can also be written as

$$m_{A^0}^2 = \begin{pmatrix} B_\mu \tan\beta & B_\mu & 0 & 0 \\ B_\mu & B_\mu \cot\beta & 0 & 0 \\ 0 & 0 & B_X \tan\beta_R & B_X \\ 0 & 0 & B_X & B_X \cot\beta_R \end{pmatrix}. \tag{D.90}$$

with $X = \xi, \delta$. This leads to following the masses of the physical states:

$$m_{A^0}^2 = \frac{2B_\mu}{\sin 2\beta}, \qquad m_{A_X^0}^2 = \frac{2B_X}{\sin 2\beta_R}. \tag{D.91}$$

Appendix D.2.8. Mass matrix for charged Higgs

Basis: $\left(H_d^-, H_u^{+,*}\right)$

$$m_{H^-}^2 = \begin{pmatrix} m_{11} & \frac{1}{4}g_L^2 v_d v_u + B_\mu^* \\ \frac{1}{4}g_L^2 v_d v_u + B_\mu & m_{22} \end{pmatrix} \tag{D.92}$$

$$m_{11} = \frac{1}{8}\Big(8m_{H_d}^2 + g_R^2\Big(-v_{X_R}^2 - v_u^2 + v_{\bar{X}_R}^2 + v_d^2\Big) + g_L^2\Big(v_d^2 + v_u^2\Big)$$
$$+ \tilde{g}\Big(g_{BL}\Big(v_{X_R}^2 - v_{\bar{X}_R}^2\Big) + \tilde{g}\Big(-v_{X_R}^2 - v_u^2 + v_{\bar{X}_R}^2 + v_d^2\Big)\Big) + 8|\mu|^2\Big) \tag{D.93}$$

$$m_{22} = \frac{1}{8}\Big(8m_{H_u}^2 + g_R^2\Big(-v_{\bar{X}_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\Big) + g_L^2\Big(v_d^2 + v_u^2\Big)$$
$$+ \tilde{g}\Big(g_{BL}\Big(v_{\bar{X}_R}^2 - v_{X_R}^2\Big) + \tilde{g}\Big(-v_{\bar{X}_R}^2 - v_d^2 + v_{X_R}^2 + v_u^2\Big)\Big) + 8|\mu|^2\Big) \tag{D.94}$$

This matrix is diagonalized by Z^+:

$$Z^+ m_{H^-}^2 Z^{+,\dagger} = m_{2,H^-}^{dia} \tag{D.95}$$

Appendix D.2.9. Mass matrix for neutralinos

minimal seesaw. Basis: $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_R, \tilde{\delta}, \tilde{\bar{\delta}}\right)$

$$m_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -\frac{1}{2}\tilde{g}v_d & \frac{1}{2}\tilde{g}v_u & \frac{1}{2}M_{BR} & m_{61} & m_{71} \\
0 & M_2 & \frac{1}{2}g_L v_d & -\frac{1}{2}g_L v_u & 0 & 0 & 0 \\
-\frac{1}{2}\tilde{g}v_d & \frac{1}{2}g_L v_d & 0 & -\mu & -\frac{1}{2}g_R v_d & 0 & 0 \\
\frac{1}{2}\tilde{g}v_u & -\frac{1}{2}g_L v_u & -\mu & 0 & \frac{1}{2}g_R v_u & 0 & 0 \\
\frac{1}{2}M_{BR} & 0 & -\frac{1}{2}g_R v_d & \frac{1}{2}g_R v_u & M_4 & g_R v_{\delta_R} & -g_R v_{\bar{\delta}_R} \\
m_{61} & 0 & 0 & 0 & g_R v_{\delta_R} & 0 & -\mu_\delta \\
m_{71} & 0 & 0 & 0 & -g_R v_{\bar{\delta}_R} & -\mu_\delta & 0
\end{pmatrix} \tag{D.96}$$

$$m_{61} = \Big(-g_{BL} + \tilde{g}\Big)v_{\delta_R} \tag{D.97}$$

$$m_{71} = \Big(-\tilde{g} + g_{BL}\Big)v_{\bar{\delta}_R} \tag{D.98}$$

linear and inverse seesaw. Basis: $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_R, \tilde{\xi}, \tilde{\bar{\xi}}\right)$

$$m_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -\frac{1}{2}\tilde{g}v_d & \frac{1}{2}\tilde{g}v_u & \frac{1}{2}M_{BR} & m_{61} & m_{71} \\
0 & M_2 & \frac{1}{2}g_L v_d & -\frac{1}{2}g_L v_u & 0 & 0 & 0 \\
-\frac{1}{2}\tilde{g}v_d & \frac{1}{2}g_L v_d & 0 & -\mu & -\frac{1}{2}g_R v_d & 0 & 0 \\
\frac{1}{2}\tilde{g}v_u & -\frac{1}{2}g_L v_u & -\mu & 0 & \frac{1}{2}g_R v_u & 0 & 0 \\
\frac{1}{2}M_{BR} & 0 & -\frac{1}{2}g_R v_d & \frac{1}{2}g_R v_u & M_4 & \frac{1}{2}g_R v_{\xi_R} & -\frac{1}{2}g_R v_{\bar{\xi}_R} \\
m_{61} & 0 & 0 & 0 & \frac{1}{2}g_R v_{\xi_R} & 0 & -\mu_\xi \\
m_{71} & 0 & 0 & 0 & -\frac{1}{2}g_R v_{\bar{\xi}_R} & -\mu_\xi & 0
\end{pmatrix} \tag{D.99}$$

with

$$m_{61} = \frac{1}{2}\Big(-g_{BL} + \tilde{g}\Big)v_{\xi_R} \tag{D.100}$$

$$m_{71} = \frac{1}{2}\Big(-\tilde{g} + g_{BL}\Big)v_{\bar{\xi}_R} \tag{D.101}$$

This matrix is diagonalized by N:

$$N^* m_{\tilde{\chi}^0} N^\dagger = m_{\tilde{\chi}^0}^{dia} \tag{D.102}$$

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Appendix D.2.10. Mass matrix for neutrinos
minimal seesaw. Basis: (ν_L, ν^c)

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} v_u Y_v^T \\ \frac{1}{\sqrt{2}} v_u Y_v & \frac{2}{\sqrt{2}} v_{\delta_R} Y_M \end{pmatrix} \tag{D.103}$$

linear and inverse seesaw. Basis: (ν_L, ν^c, N)

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} v_u Y_v^T & \tilde{Y} v_d v_{\xi_R} \\ \frac{1}{\sqrt{2}} v_u Y_v & 0 & \frac{1}{\sqrt{2}} v_{\xi_R} Y_{N\nu^c}^T \\ \tilde{Y}^T v_d v_{\xi_R} & \frac{1}{\sqrt{2}} Y_{N\nu^c} v_{\xi_R} & 2\mu_N \end{pmatrix} \tag{D.104}$$

$\tilde{Y}$ is the running, effective operator $\sim \hat{L} \hat{N}_S \hat{H}_d \hat{\xi}_R$ obtained by integrating out ξ_L and $\bar{\xi}_L$. The mass matrix is diagonalized by Z_ν^V:

$$U^{V,*} m_\nu U^{V,\dagger} = m_\nu^{dia} \tag{D.105}$$

Appendix D.3. See-saw in models with $U(1)_Y \times U(1)_{B-L}$ gauge sector

We give here the mass matrices for an imaginary model which would contain all terms in the Lagrangian of the minimal $U(1)_Y \times U(1)_{B-L}$ model as well of the model with inverse seesaw. The mass matrices for the physical relevant models are given in the limit that the unwanted interactions vanish.

Appendix D.3.1. Mass matrix for down squarks

Basis: $\left(\tilde{d}_{L,\alpha}, \tilde{d}_{R,\alpha} \right)$

$$m_{\tilde{d}}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}} \left(v_d T_d^\dagger - v_u \mu Y_d^\dagger \right) \\ \frac{1}{\sqrt{2}} \left(v_d T_d - v_u \mu^* Y_d \right) & m_{22} \end{pmatrix} \tag{D.106}$$

$$m_{11} = -\frac{1}{24} \Big(\mathbf{1} \Big(2 \big(g_B^2 + \tilde{g}^2 \big) \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) + 3g_2^2 \big(v_d^2 - v_u^2 \big) + g_1^2 \big(v_d^2 - v_u^2 \big) + g_1 \tilde{g} \big(2v_\eta^2 - 2v_{\bar{\eta}}^2 - v_u^2 + v_d^2 \big) \Big)$$
$$- 12 \big(2m_q^2 + v_d^2 Y_d^\dagger Y_d \big) \Big) \tag{D.107}$$

$$m_{22} = \frac{1}{24} \Big(12 v_d^2 Y_d Y_d^\dagger + 24 m_d^{2,*} + \mathbf{1} \Big(- \big(2g_1 - \tilde{g} \big) \big(2\tilde{g} \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) + g_1 \big(v_d^2 - v_u^2 \big) \big) + 2g_B^2 \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) \Big) \Big) \tag{D.108}$$

This matrix is diagonalized by Z^D:

$$Z^D m_{\tilde{d}}^2 Z^{D,\dagger} = m_{2,\tilde{d}}^{dia} \tag{D.109}$$

Appendix D.3.2. Mass matrix for up-squarks

Basis: $(\tilde{u}_{L,\alpha}, \tilde{u}_{R,\alpha})$

$$m_{\tilde{u}}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}} \left(- v_d \mu Y_u^\dagger + v_u T_u^\dagger \right) \\ \frac{1}{\sqrt{2}} \left(- v_d \mu^* Y_u + v_u T_u \right) & m_{22} \end{pmatrix} \tag{D.110}$$

$$m_{11} = \frac{1}{24} \Big(\mathbf{1} \Big(- 2 \big(g_B^2 + \tilde{g}^2 \big) \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) + 3g_2^2 \big(v_d^2 - v_u^2 \big) + g_1^2 \big(v_u^2 - v_d^2 \big) + g_1 \tilde{g} \big(- 2v_\eta^2 + 2v_{\bar{\eta}}^2 - v_d^2 + v_u^2 \big) \Big)$$
$$+ 24 m_q^2 + 12 v_u^2 Y_u^\dagger Y_u \Big) \tag{D.111}$$

$$m_{22} = \frac{1}{24} \Big(12 v_u^2 Y_u Y_u^\dagger + 24 m_u^2 + \mathbf{1} \Big(2g_B^2 \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) + \big(4g_1 + \tilde{g} \big) \big(2\tilde{g} \big(v_\eta^2 - v_{\bar{\eta}}^2 \big) + g_1 \big(v_d^2 - v_u^2 \big) \big) \Big) \Big) \tag{D.112}$$

This matrix is diagonalized by Z^U:

$$Z^U m_{\tilde{u}}^2 Z^{U,\dagger} = m_{2,\tilde{u}}^{dia} \tag{D.113}$$

with

Appendix D.3.3. Mass matrix for charged sleptons

Basis: $(\tilde{e}_L, \tilde{e}_R)$

$$m_{\tilde{e}}^2 = \begin{pmatrix} m_{11} & \frac{1}{\sqrt{2}}\left(v_d T_e^\dagger - v_u \mu Y_e^\dagger\right) \\ \frac{1}{\sqrt{2}}\left(v_d T_e - v_u \mu^* Y_e\right) & m_{22} \end{pmatrix} \tag{D.114}$$

$$m_{11} = \frac{1}{8}\Big(\mathbf{1}\Big(2\big(g_B^2 + \tilde{g}^2\big)\big(v_\eta^2 - v_{\bar\eta}^2\big) + g_1^2\big(v_d^2 - v_u^2\big) + g_1\tilde{g}\big(2v_\eta^2 - 2v_{\bar\eta}^2 - v_u^2 + v_d^2\big) + g_2^2\big(v_u^2 - v_d^2\big)\Big)$$
$$+ 8m_l^2 + 4v_d^2 Y_e^\dagger Y_e\Big) \tag{D.115}$$

$$m_{22} = \frac{1}{8}\Big(4v_d^2 Y_e Y_e^\dagger + 8m_e^2 - \mathbf{1}\Big(\big(2g_1 + \tilde{g}\big)\big(2\tilde{g}\big(v_\eta^2 - v_{\bar\eta}^2\big) + g_1\big(v_d^2 - v_u^2\big)\big) + 2g_B^2\big(v_\eta^2 - v_{\bar\eta}^2\big)\Big)\Big) \tag{D.116}$$

This matrix is diagonalized by Z^E:

$$Z^E m_{\tilde{e}}^2 Z^{E,\dagger} = m_{2,\tilde{e}}^{dia} \tag{D.117}$$

Appendix D.3.4. Mass matrix for pseudo scalar sneutrinos

(ϕ_L, ϕ_R, ϕ_S)

$$m_{\nu^i}^2 = \begin{pmatrix} m_{11} & m_{21}^T & \frac{1}{2}v_u v_\eta \Re\big(Y_\nu^\dagger Y_{IS}\big) \\ m_{21} & m_{22} & m_{32}^T \\ \frac{1}{2}v_u v_\eta \Re\big(Y_{IS} Y_\nu^*\big) & m_{32} & m_{33} \end{pmatrix} \tag{D.118}$$

$$m_{11} = \frac{1}{8}\Big(\mathbf{1}\Big(2\big(g_B^2 + \tilde{g}^2\big)\big(-v_{\bar\eta}^2 + v_\eta^2\big) + g_1^2\big(-v_u^2 + v_d^2\big) + g_1\tilde{g}\big(-2v_{\bar\eta}^2 + 2v_\eta^2 - v_u^2 + v_d^2\big) + g_2^2\big(-v_u^2 + v_d^2\big)\Big)$$
$$+ 8\Re\big(m_l^2\big) + 4v_u^2\Re\big(Y_\nu^\dagger Y_\nu\big)\Big) \tag{D.119}$$

$$m_{21} = \frac{1}{4}\Big(-2\sqrt{2}v_d \Re\big(\mu^* Y_\nu^\dagger\big) + v_u\big(2\sqrt{2}\Re\big(T_\nu\big) - 4v_\eta\Re\big(Y_{\eta\nu^c} Y_\nu^*\big)\big)\Big) \tag{D.120}$$

$$m_{22} = \frac{1}{8}\Big(g_1\tilde{g}\mathbf{1}\big(-v_d^2 + v_u^2\big) + 8\Re\big(m_\nu^2\big) + 2v_{\bar\eta}\big(4\sqrt{2}\Re\big(Y_{\eta\nu^c}\mu_\eta^*\big) + \big(g_B^2 + \tilde{g}^2\big)\mathbf{1}v_{\bar\eta}\big) + 4v_u^2\Re\big(Y_\nu Y_\nu^\dagger\big)$$
$$+ 2v_\eta\big(-4\sqrt{2}\Re\big(T_{\eta\nu^c}\big) + v_\eta\big(2\Re\big(Y_{IS} Y_{IS}^*\big) + 8\Re\big(Y_{\eta\nu^c} Y_{\eta\nu^c}^*\big) - \big(g_B^2 + \tilde{g}^2\big)\mathbf{1}\big)\big)\Big) \tag{D.121}$$

$$m_{32} = \frac{1}{4}\Big(-2\sqrt{2}v_{\bar\eta}\Re\big(Y_{IS}\mu_\eta^*\big) + v_\eta\big(2\sqrt{2}\Re\big(T_{IS}\big) - 4\sqrt{2}\Re\big(\mu_N Y_{IS}^*\big) - 4v_\eta\Re\big(Y_{IS} Y_{\eta\nu^c}^*\big)\big)\Big) \tag{D.122}$$

$$m_{33} = \frac{1}{8}\Big(\mathbf{1}\Big(-2\big(g_B^2 + \tilde{g}^2\big)\big(-v_{\bar\eta}^2 + v_\eta^2\big) + g_1\tilde{g}\big(-v_d^2 + v_u^2\big)\Big)$$
$$+ 4\Big(-2\Re\big(B_N\big) + 2\Re\big(m_N^2\big) + 8\Re\big(\mu_N\mu_N^*\big) - 2\Re(B_N)\Big) + 4v_\eta^2\Re\big(Y_{IS} Y_{IS}^*\big)\Big) \tag{D.123}$$

This matrix is diagonalized by Z^i:

$$Z^i m_{\nu^i}^2 Z^{i,\dagger} = m_{2,\nu^i}^{dia} \tag{D.124}$$

Appendix D.3.5. Mass matrix for scalar sneutrinos

Basis: $(\sigma_L, \sigma_R, \sigma_S)$

$$m_{\nu^R}^2 = \begin{pmatrix} m_{11} & m_{21}^T & \frac{1}{2}v_u v_\eta \Re\big(Y_\nu^\dagger Y_{IS}\big) \\ m_{21} & m_{22} & m_{32}^T \\ \frac{1}{2}v_u v_\eta \Re\big(Y_{IS} Y_\nu^*\big) & m_{32} & m_{33} \end{pmatrix} \tag{D.125}$$

$$m_{11} = \frac{1}{8}\Big(\mathbf{1}\Big(2\Big(g_B^2 + \tilde{g}^2\Big)\Big(-v_{\bar{\eta}}^2 + v_{\eta}^2\Big) + g_1^2\Big(-v_u^2 + v_d^2\Big) + g_1\tilde{g}\Big(-2v_{\bar{\eta}}^2 + 2v_{\eta}^2 - v_u^2 + v_d^2\Big) + g_2^2\Big(-v_u^2 + v_d^2\Big)\Big)$$
$$+ 8\Re\Big(m_l^2\Big) + 4v_u^2\Re\Big(Y_\nu^\dagger Y_\nu\Big)\Big) \tag{D.126}$$

$$m_{21} = \frac{1}{4}\Big(-2\sqrt{2}v_d\Re\Big(\mu^* Y_\nu\Big) + v_u\Big(2\sqrt{2}\Re\Big(T_\nu\Big) + 4v_\eta\Re\Big(Y_{\eta\nu^c}Y_\nu^*\Big)\Big)\Big) \tag{D.127}$$

$$m_{22} = \frac{1}{8}\Big(g_1\tilde{g}\mathbf{1}\Big(-v_d^2 + v_u^2\Big) + 8\Re\Big(m_\nu^2\Big) + 2v_{\bar{\eta}}\Big(-4\sqrt{2}\Re\Big(Y_{\eta\nu^c}\mu_\eta^*\Big) + \Big(g_B^2 + \tilde{g}^2\Big)\mathbf{1}v_{\bar{\eta}}\Big) + 4v_u^2\Re\Big(Y_\nu Y_\nu^\dagger\Big)$$
$$+ 2v_\eta\Big(4\sqrt{2}\Re\Big(T_{\eta\nu^c}\Big) + v_\eta\Big(2\Re\Big(Y_{IS}Y_{IS}^*\Big) + 8\Re\Big(Y_{\eta\nu^c}Y_{\eta\nu^c}^*\Big) - \Big(g_B^2 + \tilde{g}^2\Big)\mathbf{1}\Big)\Big)\Big) \tag{D.128}$$

$$m_{32} = \frac{1}{4}\Big(-2\sqrt{2}v_{\bar{\eta}}\Re\Big(Y_{IS}\mu_\eta^*\Big) + v_\eta\Big(4v_\eta\Re\Big(Y_{IS}Y_{\eta\nu^c}^*\Big) + \sqrt{2}\Big(2\Re\Big(T_{IS}\Big) + 4\Re\Big(\mu_N Y_{IS}^*\Big)\Big)\Big)\Big) \tag{D.129}$$

$$m_{33} = \frac{1}{8}\Big(4\Big(2\Re\Big(B_N\Big) + 2\Re\Big(m_N^2\Big) + 8\Re\Big(\mu_N\mu_N^*\Big) + 2\Re(B_N)\Big) + 4v_\eta^2\Re\Big(Y_{IS}Y_{IS}^*\Big)$$
$$+ \mathbf{1}\Big(-2\Big(g_B^2 + \tilde{g}^2\Big)\Big(-v_{\bar{\eta}}^2 + v_\eta^2\Big) + g_1\tilde{g}\Big(-v_d^2 + v_u^2\Big)\Big)\Big) \tag{D.130}$$

This matrix is diagonalized by Z^R:
$$Z^R m_{\nu^R}^2 Z^{R,\dagger} = m_{2,\nu^R}^{dia} \tag{D.131}$$

Appendix D.3.6. Mass matrix for scalar Higgs
Basis: $(\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}})$

$$m_h^2 = \begin{pmatrix} m_{11} & m_{21} & \frac{1}{2}g_1\tilde{g}v_d v_\eta & -\frac{1}{2}g_1\tilde{g}v_d v_{\bar{\eta}} \\ m_{21} & m_{22} & -\frac{1}{2}g_1\tilde{g}v_u v_\eta & \frac{1}{2}g_1\tilde{g}v_u v_{\bar{\eta}} \\ \frac{1}{2}g_1\tilde{g}v_d v_\eta & -\frac{1}{2}g_1\tilde{g}v_u v_\eta & m_{33} & m_{43} \\ -\frac{1}{2}g_1\tilde{g}v_d v_{\bar{\eta}} & \frac{1}{2}g_1\tilde{g}v_u v_{\bar{\eta}} & m_{43} & m_{44} \end{pmatrix} \tag{D.132}$$

$$m_{11} = \frac{1}{8}\Big(2g_1\tilde{g}\Big(v_\eta^2 - v_{\bar{\eta}}^2\Big) + 8m_{H_d}^2 + 8|\mu|^2 + g_1^2\Big(3v_d^2 - v_u^2\Big) + g_2^2\Big(3v_d^2 - v_u^2\Big)\Big) \tag{D.133}$$

$$m_{21} = -\frac{1}{4}\Big(g_1^2 + g_2^2\Big)v_d v_u - \Re\Big(B_\mu\Big) \tag{D.134}$$

$$m_{22} = \frac{1}{8}\Big(2g_1\tilde{g}\Big(-v_\eta^2 + v_{\bar{\eta}}^2\Big) + 8m_{H_u}^2 + 8|\mu|^2 - g_1^2\Big(-3v_u^2 + v_d^2\Big) - g_2^2\Big(-3v_u^2 + v_d^2\Big)\Big) \tag{D.135}$$

$$m_{33} = \frac{1}{4}\Big(2\Big(g_B^2 + \tilde{g}^2\Big)\Big(3v_\eta^2 - v_{\bar{\eta}}^2\Big) + 4m_\eta^2 + 4|\mu_\eta|^2 + g_1\tilde{g}\Big(v_d^2 - v_u^2\Big)\Big) \tag{D.136}$$

$$m_{43} = -\Big(g_B^2 + \tilde{g}^2\Big)v_\eta v_{\bar{\eta}} - \Re\Big(B_\eta\Big) \tag{D.137}$$

$$m_{44} = \frac{1}{4}\Big(-2\Big(g_B^2 + \tilde{g}^2\Big)\Big(-3v_{\bar{\eta}}^2 + v_\eta^2\Big) + 4m_{\bar{\eta}}^2 + 4|\mu_\eta|^2 + g_1\tilde{g}\Big(v_u^2 - v_d^2\Big)\Big) \tag{D.138}$$

This matrix is diagonalized by Z^H:
$$Z^H m_h^2 Z^{H,\dagger} = m_{2,h}^{dia} \tag{D.139}$$

Appendix D.3.7. Mass matrix for pseudo scalar Higgs
Basis: $(\sigma_d, \sigma_u, \sigma_\eta, \sigma_{\bar{\eta}})$

$$m_{A^0}^2 = \begin{pmatrix} m_{11} & \Re\Big(B_\mu\Big) & 0 & 0 \\ \Re\Big(B_\mu\Big) & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & \Re\Big(B_\eta\Big) \\ 0 & 0 & \Re\Big(B_\eta\Big) & m_{44} \end{pmatrix} \tag{D.140}$$

$$m_{11} = \frac{1}{8}\left(2g_1\tilde{g}\left(v_\eta^2 - v_{\bar\eta}^2\right) + 8m_{H_d}^2 + 8|\mu|^2 + g_1^2\left(v_d^2 - v_u^2\right) + g_2^2\left(v_d^2 - v_u^2\right)\right) \tag{D.141}$$

$$m_{22} = \frac{1}{8}\left(2g_1\tilde{g}\left(-v_\eta^2 + v_{\bar\eta}^2\right) + 8m_{H_u}^2 + 8|\mu|^2 + g_1^2\left(v_u^2 - v_d^2\right) + g_2^2\left(v_u^2 - v_d^2\right)\right) \tag{D.142}$$

$$m_{33} = \frac{1}{4}\left(2\left(g_B^2 + \tilde{g}^2\right)\left(v_\eta^2 - v_{\bar\eta}^2\right) + 4m_\eta^2 + 4|\mu_\eta|^2 + g_1\tilde{g}\left(v_d^2 - v_u^2\right)\right) \tag{D.143}$$

$$m_{44} = \frac{1}{4}\left(-2\left(g_B^2 + \tilde{g}^2\right)\left(v_\eta^2 - v_{\bar\eta}^2\right) + 4m_{\bar\eta}^2 + 4|\mu_\eta|^2 + g_1\tilde{g}\left(v_u^2 - v_d^2\right)\right) \tag{D.144}$$

This matrix is diagonalized by ZA:

$$Z^A m_{A^0}^2 Z^{A,\dagger} = m_{2,A^0}^{dia} \tag{D.145}$$

Using the solution of the tadpole equations the mass matrix can also be written as

$$m_{A^0}^2 = \begin{pmatrix} B_\mu \tan\beta & B_\mu & 0 & 0 \\ B_\mu & B_\mu \cot\beta & 0 & 0 \\ 0 & 0 & B_{\mu'} \tan\beta' & B_{\mu'} \\ 0 & 0 & B_{\mu'} & B_{\mu'} \cot\beta' \end{pmatrix}. \tag{D.146}$$

what gives the masses of the physical states:

$$m_{A^0}^2 = \frac{2B_\mu}{\sin 2\beta} \ , \qquad m_{A_\eta^0}^2 = \frac{2B_{\mu'}}{\sin 2\beta'} \ . \tag{D.147}$$

Appendix D.3.8. Mass matrix for charged Higgs
  Basis: $\left(H_d^-, H_u^{+,*}\right)$

$$m_{H^-}^2 = \begin{pmatrix} m_{11} & \frac{1}{4}g_2^2 v_d v_u + B_\mu^* \\ \frac{1}{4}g_2^2 v_d v_u + B_\mu & m_{22} \end{pmatrix} \tag{D.148}$$

$$m_{11} = \frac{1}{8}\left(2g_1\tilde{g}\left(v_\eta^2 - v_{\bar\eta}^2\right) + 8m_{H_d}^2 + 8|\mu|^2 + g_1^2\left(v_d^2 - v_u^2\right) + g_2^2\left(v_d^2 + v_u^2\right)\right) \tag{D.149}$$

$$m_{22} = \frac{1}{8}\left(2g_1\tilde{g}\left(-v_\eta^2 + v_{\bar\eta}^2\right) + 8m_{H_u}^2 + 8|\mu|^2 + g_1^2\left(v_u^2 - v_d^2\right) + g_2^2\left(v_d^2 + v_u^2\right)\right) \tag{D.150}$$

This matrix is diagonalized by Z^+:

$$Z^+ m_{H^-}^2 Z^{+,\dagger} = m_{2,H^-}^{dia} \tag{D.151}$$

Appendix D.3.9. Mass matrix for neutralinos
  Basis: $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}', \tilde{\eta}, \tilde{\bar\eta}\right)$

$$m_{\tilde\chi^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & \frac{1}{2}M_{BB'} & -\tilde{g}v_\eta & \tilde{g}v_{\bar\eta} \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 & 0 & 0 \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu & 0 & 0 & 0 \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu & 0 & 0 & 0 & 0 \\ \frac{1}{2}M_{BB'} & 0 & 0 & 0 & M_{B-L} & -g_B v_\eta & g_B v_{\bar\eta} \\ -\tilde{g}v_\eta & 0 & 0 & 0 & -g_B v_\eta & 0 & -\mu_\eta \\ \tilde{g}v_{\bar\eta} & 0 & 0 & 0 & g_B v_{\bar\eta} & -\mu_\eta & 0 \end{pmatrix} \tag{D.152}$$

This matrix is diagonalized by N:

$$N^* m_{\tilde\chi^0} N^\dagger = m_{\tilde\chi^0}^{dia} \tag{D.153}$$

Appendix D.3.10. Mass matrix for neutrinos
  Basis: (ν_L, ν^c, N)

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} v_u Y_\nu & \frac{\tilde{Y}}{2} v_d v_\eta \\ \frac{1}{\sqrt{2}} v_u Y_\nu^T & \frac{1}{\sqrt{2}} v_\eta \left(Y_{\eta\nu^c} + Y_{\eta\nu^c}^T \right) & \frac{1}{\sqrt{2}} v_\eta Y_i S \\ \frac{\tilde{Y}^T}{2} v_d v_\eta & \frac{1}{\sqrt{2}} v_\eta Y_{IS}^T & 2\mu_N \end{pmatrix} \tag{D.154}$$

$\tilde{Y}$ is the running, effective operator $\sim \hat{L} \hat{N}_S \hat{H}_d \hat{\eta}$ obtained by integrating out ρ and $\bar{\rho}_L$. This matrix is diagonalized by U^V:

$$U^{V,*} m_\nu U^{V,\dagger} = m_\nu^{dia} \tag{D.155}$$

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