B-L-violating Masses in Softly Broken Supersymmetry.

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Abstract

We prove a general low-energy theorem establishing a generic relation between the neutrino Majorana mass and the superpartner sneutrino B-L-violating "Majorana"-like mass term. The theorem states that, if one of these two quantities is non-zero the other one is also non-zero and, vice versa, if one of them vanishes the other vanishes, too. The theorem is a consequence of the underlying supersymmetry (SUSY) and valid for any realistic gauge model with weak scale softly broken SUSY.

Neutrinos are believed to be massive particles. Despite the lack of unambiguous experimental confirmation of this belief there are insisting indications for non-zero neutrino masses from cosmology, the solar and atmospheric neutrino puzzles (for recent review see [1]) as well as from recent LSND results on possible $\bar{\nu}_e - \bar{\nu}_\mu$ neutrino oscillations [2].

Among the known explanations for the extreme smallness of the neutrino mass compared to masses of the other fermions the most natural one is based on the see-saw mechanism [3]. It leads to a B-L-violating Majorana mass term for the neutrino. Various 1-loop contributions to the neutrino self-energy, widely discussed in the literature [4]-[9], also induce a small Majorana mass for neutrinos. Furthermore, the Grand Unification paradigm definitely prefers a Majorana mass for neutrinos. Due to these arguments, it has become a common trend to think of neutrinos as Majorana particles.

In supersymmetric (SUSY) models the neutrino ν has its scalar superpartner the sneutrino $\tilde{\nu}$. Given that they are components of the same superfield one may suspect a certain interplay between the neutrino and sneutrino properties at low energies as a relic of the underlying supersymmetry.

In the present note we prove a low-energy theorem establishing an intimate relation between the neutrino Majorana mass term and the B-L-violating as well as B-L-conserving sneutrino mass terms. Our consideration refers to the general structure of the low-energy effective Lagrangian assuming weak scale softly broken supersymmetry and stability of the ground state after electro-weak symmetry breaking. The proof of the low-energy theorem, consisting of three statements, is based on symmetry arguments and is of a general significance.

The effective Lagrangian of a generic model of weak scale supersymmetry contains after electro-weak symmetry breaking the following terms

$$\begin{aligned} \mathcal{L} &= -\sqrt{2} g \epsilon_i \cdot \overline{\nu}_L \chi_i \tilde{\nu}_L - g \epsilon_i^- \cdot \overline{e}_L \chi_i^- \tilde{\nu}_L - g \epsilon_i^+ \cdot \overline{\nu}_L \chi_i^+ \tilde{e}_L + \\ &+ \frac{g}{\sqrt{2}} \cdot \overline{\nu}_L \gamma^\mu e_L W_\mu^+ + g \cdot \bar{\chi}_i \gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \chi_j^+ W_\mu^- + ... + h.c. \end{aligned} \quad (1)$$

Dots denote other terms which are not essential for further consideration. Here, $\tilde{\nu}_L$ and $\tilde{e}_L$ represent scalar superpartners of the left-handed neutrino ν_L and electron e_L fields. The chargino $\chi_i^\pm$ and neutralino χ_i are superpositions of the gaugino and the higgsino fields. The contents of these superpositions depends on the model. Note that the neutralino is a Majorana field $\chi_i^c = \chi_i$. The explicit form of the coefficients ϵ, $\epsilon_i^\pm$ and $O_{ij}^{L,R}$ is also unessential. For the case of the MSSM one can find them, for instance in [10]. Eq. (1) is a general consequence of the underlying weak scale supersymmetry (softly broken) and the spontaneously broken electro-weak gauge symmetry.

Now let us assume the neutrino acquires a Majorana mass

$$\mathcal{L}_M^\nu = -\frac{1}{2}(m_M^\nu \overline{\nu^c} \nu + h.c.), \quad (2)$$

where $\nu = \nu^c$ is a Majorana field. The further proof does not depend on the mechanism generating this mass term in the low-energy Lagrangian. For the sake of simplicity and without any loss of generality we ignore possible neutrino mixing.

Prove the following statements.

<u>Statement 1</u>: If $m_M^\nu \neq 0$ then in the low-energy Lagrangian also the sneutrino "Majorana"-like B-L-violating mass term is present

$$\mathcal{L}_M^{\tilde{\nu}} = -\frac{1}{2}(\tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L + h.c.) \quad (3)$$

with $\tilde{m}_M^2 \neq 0$. Note that $\tilde{m}_M^2$ is not a positively defined parameter.

<u>Statement 2</u> is inverse to the statement 1: If $\tilde{m}_M^2 \neq 0$ in Eq. (3), then in Eq. (2) $m_M^\nu \neq 0$.

First notice that in the presence of a non-zero Majorana neutrino mass term in Eq. (2) the "Majorana"-like sneutrino mass term in Eq. (3) is generated at the 1-loop level as shown in Fig.1(a) with neutrino and neutralino internal lines. An opposite statement based on the 1-loop diagram in Fig.1(b) is also true. The B-L-violating sneutrino propagator in Fig.1(b) is proportional to $\tilde{m}_M^2$ and explicitly derived below. Here we do not need detailed calculations and write down schematically

$$\tilde{m}_M^2 = \frac{1}{4\pi}(g^2 + g'^2) m_M^{\nu} M_{\chi_i} \gamma_i + A_{\tilde{\nu}}, \tag{4}$$

$$m_M^{\nu} = \frac{1}{4\pi}(g^2 + g'^2) \frac{\tilde{m}_M^2}{\tilde{m}_D^2} M_{\chi_i} \beta_i + A_{\nu}. \tag{5}$$

Here m_D is a mass parameter explained below, g and g' are the $SU(2)_L \times U_{1Y}$ gauge coupling constants, M_{χ_i} are the neutralino masses and β_i, γ_i are functions depending on neutralino mixing coefficients and on the masses of particles in the loop. $A_{\tilde{\nu}}$ and A_{ν} represent any other possible contributions. The explicit form of A_i, β_i, γ_i is not essential for us. Important is just the presence of a correlation between $\tilde{m}_M^2$ and m_M^{ν} which we write down in general as $\tilde{m}_M^2 = f(m_M^{\nu}),\ m_M^{\nu} = \phi(\tilde{m}_M^2)$. Now we are going to prove that

$$\tilde{m}_M^2 = f(m_M^{\nu} = 0) = 0, \quad m_M^{\nu} = \phi(\tilde{m}_M^2 = 0) = 0. \tag{6}$$

One can expect such properties of the functions f and ϕ from Eqs. (4)-(5). Indeed, $\tilde{m}_M^2 = 0$ in the left-hand side of Eq. (4) strongly disfavors $m_M^{\nu} \neq 0$. Similarly, $m_M^{\nu} = 0$ in the left-hand side of Eq. (5) strongly disfavors $\tilde{m}_M^2 \neq 0$. This is because vanishing left-hand sides of Eqs. (4)-(5) requires either vanishing of both terms in the right-hand sides or their net cancelation. The latter is unlikely since it implies unnatural fine-tuning of certain parameters. More serious, it is unstable under radiative corrections. Even if the fine-tuning was done by hand it would be spoiled in higher orders of perturbation theory. To guarantee the cancelation of both terms in the right-hand sides of Eqs. (4)-(5) in all orders of perturbation theory one needs a special unbroken symmetry. The Lagrangian (1) does not posses any continuous symmetry having non-trivial B-L transformation properties. However, there might be an appropriate discrete symmetry.

Let us specify this discrete symmetry group by the following field transformations

$$\begin{aligned} \nu &\rightarrow \eta_{\nu}\nu, \quad \tilde{\nu} \rightarrow \eta_{\tilde{\nu}}\tilde{\nu}, \quad e_L \rightarrow \eta_e e_L, \quad \tilde{e}_L \rightarrow \eta_{\tilde{e}}\tilde{e}_L, \\ W^+ &\rightarrow \eta_W W^+, \quad \chi_i \rightarrow \eta_{\chi_i}\chi_i, \quad \chi^+ \rightarrow \eta_{\chi^+}\chi^+. \end{aligned} \tag{7}$$

Here η_i are phase factors. Since the Lagrangian (1) is assumed to be invariant under these transformations one obtains the following relations

$$\begin{aligned} \eta_\nu^* \eta_{\tilde{\nu}} \eta_{\chi_i} &= 1, \quad \eta_e \eta_{\chi^+} \eta_{\tilde{\nu}}^* = 1, \\ \eta_e \eta_W \eta_\nu^* &= 1, \quad \eta_W^* \eta_{\chi^+} \eta_{\chi_i}^* = 1, \quad ... \end{aligned} \tag{8}$$

Dots denote other relations which are not essential here. The complete set of these equations defines the admissible discrete symmetry group of the Lagrangian in Eq. (1).

Solving these equations, one finds

$$\eta_\nu^2 = \eta_{\tilde{\nu}}^2. \tag{9}$$

This relation proves the statements 1,2 and the corresponding properties expressed by Eqs. (6). To see this we note that the B-L-violating mass terms in Eq. (2) or in Eq. (3) are forbidden by this symmetry if $\eta_\nu^2 \neq 1$ or $\eta_{\tilde{\nu}}^2 \neq 1$. Contrary, if $\eta_\nu^2 = 1$, this mass term is not protected by the symmetry and appears in higher orders of perturbation theory, even if it does not exist at the tree-level. Relation (9) claims that if the neutrino Majorana mass term in Eq. (2) is forbidden, *i.e.* $m_M^\nu = 0$, then the sneutrino Majorana-like mass term in Eq. (3) is also forbidden, $\tilde{m}_M = 0$, and vice versa. If one of them is not forbidden then both are not forbidden. Thus, statements 1 and 2 as well as the explicit relations in Eqs. (6) are proven.

One can derive the following corollary from statements 1,2.

Corollary: If one of the two B-L-violating masses, either m_M^ν or $\tilde{m}_M^2$, vanishes, then the other one vanishes too.

Let us turn to the last statement.

Statement 3: In the presence of $\tilde{m}_M^2 \neq 0$ in Eq. (3) there must exist a "Dirac"-like B-L-conserving sneutrino mass term

$$\mathcal{L}_D^{\tilde{\nu}} = -\tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L \tag{10}$$

with $\tilde{m}_D^2 \geq |\tilde{m}_M^2|$.

To prove this statement consider the combined sneutrino mass term $\mathcal{L}_{mass}^{\tilde{\nu}} = \mathcal{L}_M^{\tilde{\nu}} + \mathcal{L}_D^{\tilde{\nu}}$ and use the real field representation for the complex scalar sneutrino field $\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}$, where $\tilde{\nu}_{1,2}$ are real fields. Then

$$\mathcal{L}_{mass}^{\tilde{\nu}} = -\frac{1}{2}(\tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L + h.c.) - \tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L = -\frac{1}{2}\tilde{m}_1^2 \tilde{\nu}_1^2 - \frac{1}{2}\tilde{m}_2^2 \tilde{\nu}_2^2 \tag{11}$$

where $\tilde{m}_{1,2}^2 = \tilde{m}_D^2 \pm |\tilde{m}_M^2|$. Assume the vacuum state is stable. Then $\tilde{m}_{1,2}^2 \geq 0$ or $\tilde{m}_D^2 \geq |\tilde{m}_M^2|$, otherwise vacuum is unstable and the subsequent spontaneous symmetry breaking occurs via non-zero vacuum expectation values of the sneutrino fiels $< \tilde{\nu}_i > \neq 0$. The broken symmetry in this case is the R-parity. It is a discrete symmetry defined as $R_p = (-1)^{3B+L+2S}$, where S, B and L are the spin, the baryon and the lepton quantum number.

This completes the proof of the theorem consisting of the above three statements.

From the above consideration it follows that a self-consistent structure of mass terms of the neutrino-sneutrino sector is

$$\mathcal{L}_{mass}^{\nu\tilde{\nu}} = -\frac{1}{2}(m_M^\nu \overline{\nu^c}\nu + h.c) - \frac{1}{2}(\tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L + h.c.) - \tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L. \tag{12}$$

The Dirac neutrino mass term $m_D^\nu(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$ can also be introduced but it is not required by the self-consistency arguments.

It is instructive to derive an explicit form of the above mentioned B-L-violating sneutrino propagator. It can be done by the use of the real field representation as in Eq. (11). Let us consider for comparison both the B-L-conserving $\Delta_{\tilde{\nu}}^D$ and the B-L-violating $\Delta_{\tilde{\nu}}^M$ sneutrino propagators

$$\begin{aligned}
&\Delta_{\tilde{\nu}}^D(x-y) =< 0|T(\tilde{\nu}(x)\tilde{\nu}^*(y))|0 >= && (13)\\
&= \frac{1}{2} < 0|T(\tilde{\nu}_1(x)\tilde{\nu}_1(y))|0 > \; + \frac{1}{2} < 0|T(\tilde{\nu}_2(x)\tilde{\nu}_2(y))|0 >= \\
&= -\frac{i}{2}(\Delta_{\tilde{m}_1}(x-y) + \Delta_{\tilde{m}_2}(x-y)), \\
&\Delta_{\tilde{\nu}}^M(x-y) =< 0|T(\tilde{\nu}(x)\tilde{\nu}(y))|0 >= && (14)\\
&= \frac{1}{2} < 0|T(\tilde{\nu}_1(x)\tilde{\nu}_1(y))|0 > \; - \frac{1}{2} < 0|T(\tilde{\nu}_2(x)\tilde{\nu}_2(y))|0 >= \\
&= -\frac{i}{2}(\Delta_{\tilde{m}_1}(x-y) - \Delta_{\tilde{m}_2}(x-y)),
\end{aligned}$$

where

$$\Delta_{\tilde{m}_i}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{\tilde{m}_i^2 - k^2 - i\epsilon} \tag{15}$$

is the ordinary propagator for a scalar particle with mass $\tilde{m}_i$. Using the definition of $\tilde{m}_{1,2}$ as in Eq. (11) one finds

$$\Delta_{\tilde{\nu}}^D(x) \;\; = \;\; \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{m}_D^2 - k^2}{(\tilde{m}_1^2 - k^2 - i\epsilon)(\tilde{m}_2^2 - k^2 - i\epsilon)} e^{-ikx}, \tag{16}$$

$$\Delta^M_{\tilde{\nu}}(x) \;=\; -\tilde{m}^2_M \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{(\tilde{m}^2_1 - k^2 - i\epsilon)(\tilde{m}^2_2 - k^2 - i\epsilon)}. \qquad (17)$$

It is seen that in absence of the B-L-violating sneutrino Majorana-like mass term $\tilde{m}^2_M = 0$ the B-L-violating propagator vanishes while the B-L-conserving one becomes the ordinary propagator of a scalar particle with mass $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_D$.

In presence of the B-L-violating sneutrino Majorana-like mass term the complex scalar sneutrino field splits into two real mass eigenstate fields $\tilde{\nu}_{1,2}$ with different masses $\tilde{m}_{1,2}$. The square mass splitting is $2\tilde{m}^2_M$.

This sneutrino mass splitting parameter can be probed by searching for B-L-violating exotic processes such as neutrinoless double beta decay. It is obvious from Eq. (4)-(5) that certain constraints on $\tilde{m}^2_M$ can be also obtained from the experimental upper bound on the neutrino mass. We are going to analyze these constraints in a separate paper.

In summary, we have proven a low-energy theorem for weak scale softly broken supersymmetry relating the B-L-violating mass terms of the neutrino and the sneutrino.

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Figure Captions

Fig.1. 1-loop contributions to (a) the neutrino Majorana mass m_M^ν and (b) the sneutrino B-L-violating mass $\tilde{m}_M^2$. Crossed (s)neutrino lines correspond to the B-L-violating propagators.

Figure 1.a

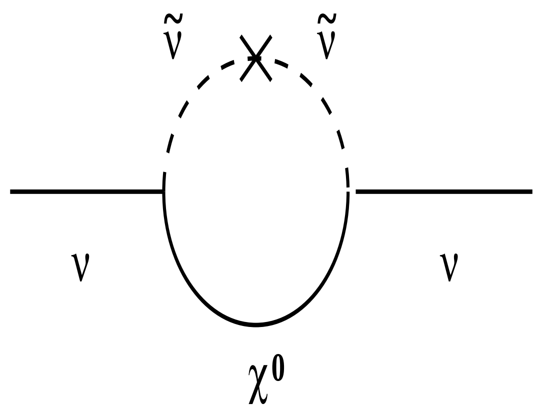

Figure 1.b