R-parity Conserving Supersymmetry, Neutrino Mass and Neutrinoless Double Beta Decay.

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Abstract

We consider contributions of R-parity conserving softly broken supersymmetry (SUSY) to neutrinoless double beta ($0\nu\beta\beta$) decay via the (B-L)-violating sneutrino mass term. The latter is a generic ingredient of any weak-scale SUSY model with a Majorana neutrino mass. The new R-parity conserving SUSY contributions to $0\nu\beta\beta$ are realized at the level of box diagrams. We derive the effective Lagrangian describing the SUSY-box mechanism of $0\nu\beta\beta$-decay and the corresponding nuclear matrix elements. The 1-loop sneutrino contribution to the Majorana neutrino mass is also derived.

Given the data on the $0\nu\beta\beta$-decay half-life of ^{76}Ge and the neutrino mass we obtain constraints on the (B-L)-violating sneutrino mass. These constraints leave room for accelerator searches for certain manifestations of the 2nd and 3rd generation (B-L)-violating sneutrino mass term, but are most probably too tight for first generation (B-L)-violating sneutrino masses to be searched for directly.

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1 Introduction

Neutrinoless double beta ($0\nu\beta\beta$) decay [1, 2] is a unique example of a nuclear process which allows one to probe for lepton number violation. Given the fact that the standard model conserves L at the classical level, $0\nu\beta\beta$ decay can proceed only via non-SM interactions. Therefore - taking into account the stringent experimental limits [3] - $0\nu\beta\beta$ decay is extremely sensitive to physics beyond the standard model.

The simplest source of lepton number violation directly leading to $0\nu\beta\beta$ decay is a finite Majorana neutrino rest mass. It violates lepton number by two units $\Delta L = 2$ precisely what is necessary for $0\nu\beta\beta$-decay. The corresponding contribution is described by the tree-level diagram of the 4th order in the weak coupling constant with one Majorana neutrino propagator as shown in Fig.1(a).

Supersymmetric extensions of the SM bring in new sources of lepton number violation and, as a result, new mechanisms of $0\nu\beta\beta$-decay. Even with the minimal matter and Higgs field content there are renormalizable L and B violating terms in the superpotential and in the sector of soft-supersymmetry breaking interactions which are not forbidden by gauge symmetry. These terms violate not only L and B but also R-parity defined as $R_p = (-1)^{3B+L+2S}$ with S being a particle spin. Models containing such interaction terms are usually referred to as SUSY models with explicit R_P-breaking [4]. (R_P can also be broken spontaneously via a non-zero vacuum expectation value of the sneutrino field $< \tilde{\nu} > \neq 0$ [5].)

Formerly it was widely believed that supersymmetry can contribute to $0\nu\beta\beta$ decay only if R_P is broken [6]-[10]. The corresponding mechanisms have been comprehensively studied in the last few years [7]-[11]. In particular, it was shown that the current experimental limits from non-observation of $0\nu\beta\beta$-decay sets upper bounds on certain $R\!\!\!/_p$ Yukawa coupling constants which are more stringent [7, 8] than previously known from various accelerator and non-accelerator experiments. Moreover, they turned out to be more stringent than those from some forthcoming accelerator experiments. This conclusion has put $0\nu\beta\beta$-decay forward as an interesting probe of supersymmetry.

Although there are no compelling theoretical arguments for R-parity conservation, there exist a number of well-known phenomenological drawbacks for supersymmetric models in which R_P is violated. Maybe the most serious one is the instability of the lightest SUSY particle. As a result, the supersymmetric solution for the dark matter problem is lost unless $R\!\!\!/_p$Yukawa coupling constants λ become unnaturally small, typically $\lambda \leq 10^{-16}$.

In view of this and other problems for $\not{R}_p$SUSY there arises a natural question whether R_P-violation is an inevitable condition for SUSY to contribute to $0\nu\beta\beta$-decay. The present paper addresses this question. We will demonstrate that there is a non-trivial R-parity conserving SUSY contribution to $0\nu\beta\beta$-decay. In Fig.1(b) we present an example of a diagram associated with the lowest order contribution to $0\nu\beta\beta$-decay within the R-parity conserving minimal supersymmetric standard model (MSSM) with a Majorana neutrino mass m_M^ν. This particular example gives an explicit answer to the above question: R-parity violation is not a necessary condition for a SUSY contribution to $0\nu\beta\beta$-decay.

It is also obvious that this SUSY contribution is strongly suppressed compared to the non-SUSY diagram in Fig.1(a). This is because it is of higher order in perturbation theory, contains heavy sparticles in intermediate states and receives a typical suppression due to the loop integration. Moreover, this diagram is proportional to the very small factor m_M^ν/p_F where $p_F \approx 80$MeV is the nucleon Fermi momentum. The latter is also true for the simplest non-SUSY diagram in Fig. 1(a). The reason is common for both diagrams. In fact, both the SM and the MSSM interactions conserve lepton number L. Therefore, the only source for $\Delta L = 2$ violation, necessary for $0\nu\beta\beta$-decay to proceed, is the Majorana neutrino mass term. If $m_M^\nu = 0$ lepton number would be a conserved quantity and the $0\nu\beta\beta$-decay amplitude should vanish, $R_{0\nu\beta\beta} = 0$. Such a behavior corresponds to $R_{0\nu\beta\beta} \sim m_M^\nu/p_F$ in the limit of small m_M^ν.

Thus, the diagram in Fig.1(b), given its very small contribution to $0\nu\beta\beta$-decay, provides just a principal demonstration of the fact that $0\nu\beta\beta$-decay can be triggered by R-parity conserving supersymmetry. No practical consequences can be obtained from this new diagram in the sense of establishing new constraints either on SUSY parameters or on m_M^ν from non-observation of $0\nu\beta\beta$-decay.

However, we will show that there are other R-parity conserving SUSY contributions via the lepton number violating sneutrino mass term. As was shown in [12] the Majorana neutrino mass, the (B-L)-violating sneutrino mass and the $0\nu\beta\beta$-decay amplitude are generically connected to each other. Namely, non-vanishing of one of these three quantities implies non-zero values of the remaining two. Thus, the $0\nu\beta\beta$-amplitude should always contain a contribution corresponding to the (B-L)-violating sneutrino mass term.

We will study this contribution and extract constraints on the (B-L)-violating sneutrino mass term from the current experimental data on $0\nu\beta\beta$-

decay of ^{76}Ge.

This paper is organized as follows. In section 2 we give a short account on the structure of neutrino-sneutrino mass terms and the theorem which establishes the above mentioned relations between the neutrino and sneutrino masses and $0\nu\beta\beta$-decay. Sect.3 is devoted to some general properties of possible SUSY contributions to $0\nu\beta\beta$-decay. In this section we specify the box diagrams describing the R_P-conserving SUSY contribution. Our approach to the derivation of the corresponding $0\nu\beta\beta$-transition operators and nuclear matrix elements is outlined in sect. 4. Sect. 5 deals with $0\nu\beta\beta$-decay constraints on the (B-L)-violating sneutrino mass. In section 6 we calculate the sneutrino contribution to the Majorana neutrino mass and derive then limits from experimental data on neutrino masses. We then close with a short summary.

2 Structure of the Neutrino-Sneutrino Mass Terms

As shown in [12], the self-consistent form of the neutrino and sneutrino mass terms is

$$\mathcal{L}^{\nu\tilde{\nu}}_{mass} = -\frac{1}{2}(m^{\nu}_{M}\overline{\nu^{c}_{L}}\nu_L + h.c.) - \frac{1}{2}(\tilde{m}^2_M\tilde{\nu}_L\tilde{\nu}_L + h.c.) - \tilde{m}^2_D\tilde{\nu}^{*}_L\tilde{\nu}_L. \tag{1}$$

where $\nu = \nu^c$ is a Majorana field. The first two terms violate the global (B-L) symmetry while the last one respects it. The first term is a Majorana mass term of the neutrino. We call the second term a "Majorana"-like mass, while the third one is referred to as a "Dirac"-like sneutrino mass term. This reflects an analogy with Majorana and Dirac mass terms for neutrinos. In the presence of the right handed neutrino field ν_R the Dirac neutrino mass term $m^{\nu}_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$ could also be included in Eq. (1) but it is not required by the self-consistency arguments. Note that $\tilde{m}^2_M$ is not a positively defined parameter.

Eq. (1) is a generic consequence of weak-scale softly broken supersymmetry and does not depend on the specific mechanism of mass generation in the low-energy theory. For the sake of simplicity and without any loss of generality we ignore possible neutrino mixing.

The low-energy theorem proven in ref. [12] relates the following three (B-L)-violating quantities: the neutrino Majorana mass m^{ν}_M, the "Majorana"-like sneutrino mass $\tilde{m}_M$ and the amplitude of $0\nu\beta\beta$-decay $R_{0\nu\beta\beta}$. Here we shortly describe the proof of this theorem.

It is relatively easy to see that if at least one of the quantities is non-zero the two others are generated in higher orders of perturbation theory as demonstrated in Fig. 2, where only dominant diagrams are shown. Internal lines in these diagrams are neutralinos χ_i, gluinos $\tilde{g}$, charginos $\chi^{\pm}$, selectron $\tilde{e}$, u-squark $\tilde{u}$ and sneutrino $\tilde{\nu}$. The latter is to be identified with the (B-L)-violating "Majorana" propagator proportional to $\tilde{m}_M^2$. The sneutrino "Majorana" propagator was explicitly derived in ref. [12] and will be given below.

The various diagrams lead to relations among the three (B-L)-violating observables, which we write down schematically

$$z_i = \sum_{i \neq j} a_{ij} \cdot z_j + \mathcal{A}_i. \tag{2}$$

Here, z_i can stand for $z_i = m_M^\nu$, $\tilde{m}_M^2$, $R_{0\nu\beta\beta}$. The coefficients a_{ij} correspond to contributions of the diagrams in Fig.2(a)-(f) so that $i, j = a, b, c, d, e, f$. Terms $\mathcal{A}_i$ represent any other possible contributions. The explicit form of a_{ij} and $\mathcal{A}_i$ is not essential in the following. Important is only the presence of a correlation between m_M^ν, $\tilde{m}_M^2$, $R_{0\nu\beta\beta}$, expressed by Eq. (2).

Now we are going to prove that if $z_{i_1} = 0$ then $z_{i_2} = z_{i_3} = 0$ (the same will be true for any permutation). On the basis of Fig. 2 and Eqs. (2) one can expect such properties of the set of observables z_i. Indeed $z_{i_1} = 0$ in the left-hand side of Eq. (2) strongly disfavors $z_{j_2} \neq 0$ and $z_{j_3} \neq 0$, because it requires either all the three terms in the right-hand sides to vanish or their net cancelation. The latter is "unnatural". Even if such a cancelation would be done by hand, using (unnatural) fine-tuning of certain parameters, in some specific order of perturbation theory, it would be spoiled again in higher orders of perturbation theory. The cancelation of all terms in the right-hand side of Eqs. (2) in all orders of perturbation theory could only be guaranteed by a special unbroken symmetry. Let us envisage this possibility in details.

The effective Lagrangian of a generic model of weak scale softly broken supersymmetry contains after electro-weak symmetry breaking the following terms [13]

$$\begin{aligned}\mathcal{L} &= -\sqrt{2} g \epsilon_i \cdot \overline{\nu}_L \chi_i \tilde{\nu}_L - g \epsilon_i^- \cdot \overline{e}_L \chi_i^- \tilde{\nu}_L - g \epsilon_i^+ \cdot \overline{\nu}_L \chi_i^+ \tilde{e}_L + \\ &+ \frac{g}{\sqrt{2}} (\overline{\nu}_L \gamma^\mu e_L + \overline{u}_L \gamma^\mu d_L) W_\mu^+ + g \cdot \bar{\chi}_i \gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \chi_j^+ W_\mu^- \\ &+ \ldots + h.c.\end{aligned} \tag{3}$$

Dots denote other terms which are not essential for our further considerations. Here, $\tilde{\nu}_L$ and $\tilde{e}_L$ represent scalar superpartners of the left-handed neutrino ν_L

and electron e_L fields. The chargino $\chi_i^{\pm}$ and neutralino χ_i are superpositions of the gaugino and the higgsino fields. The contents of these superpositions depend on the model. Note that the neutralino is a Majorana field $\chi_i^c = \chi_i$. The explicit form of the coefficients ϵ_i, $\epsilon_i^{\pm}$ and $O_{ij}^{L,R}$ is also unessential. For the case of the MSSM one can find them, for instance in [13]. Eq. (3) is a general consequence of the underlying weak scale softly broken supersymmetry and the spontaneously broken electro-weak gauge symmetry.

The Lagrangian (3) does not posses any continuous symmetry having non-trivial (B-L) transformation properties. Recall, that $U(1)_{B-L}$ is assumed to be broken since we admit (B-L)-violating mass terms in Eq. (1). However, there might be an appropriate unbroken discrete symmetry. Let us specify this discrete symmetry group by the following field transformations

$$\begin{aligned} \nu &\rightarrow \eta_\nu \nu, \quad \tilde{\nu} \rightarrow \eta_{\tilde{\nu}} \tilde{\nu}, \quad e_L \rightarrow \eta_e e_L, \quad \tilde{e}_L \rightarrow \eta_{\tilde{e}} \tilde{e}_L, \\ W^+ &\rightarrow \eta_{W} W^+, \quad \chi_i \rightarrow \eta_{\chi_i} \chi_i, \quad \chi^+ \rightarrow \eta_{\chi^+} \chi^+, \quad q_L \rightarrow \eta_q q_L. \end{aligned} \tag{4}$$

Here η_i are phase factors. Since the Lagrangian (3) is assumed to be invariant under these transformations one obtains the following relations

$$\begin{aligned} \eta_\nu^* \eta_{\tilde{\nu}} \eta_{\chi_i} &= 1, \quad \eta_e \eta_{\chi^+} \eta_{\tilde{\nu}}^* = 1, \quad ... \\ \eta_e \eta_{W} \eta_\nu^* &= 1, \quad \eta_W^* \eta_{\chi^+} \eta_{\chi_i}^* = 1, \quad \eta_d \eta_W \eta_u^* = 1 \quad .... \end{aligned} \tag{5}$$

Dots denote other relations which are not essential here. The complete set of these equations defines the admissible discrete symmetry group of the Lagrangian in Eq. (3).

Let us find the transformation property of the operator structure responsible for $0\nu\beta\beta$-decay under this group. At the quark level $0\nu\beta\beta$-decay implies the transition $dd \rightarrow uuee$, described by the effective operator

$$\mathcal{O}_{0\nu\beta\beta} = \alpha_i \cdot \bar{u} \Gamma_i^{(1)} d \cdot \bar{u} \Gamma_i^{(1)} d \cdot \bar{e} \Gamma_i^{(2)} e^c, \tag{6}$$

where α_i are numerical constants, $\Gamma_i^{(k)}$ are certain combinations of Dirac γ matrices. The $0\nu\beta\beta$-decay amplitude $R_{0\nu\beta\beta}$ is related to the matrix element of this operator

$$R_{0\nu\beta\beta} \sim < 2e^-(A, Z+2)|\mathcal{O}_{0\nu\beta\beta}|(A, Z) > \tag{7}$$

where (A, Z) is a nucleus with the atomic weight A and the total charge Z. The operator in Eq. (6) transforms under the group (4) as follows

$$\mathcal{O}_{0\nu\beta\beta} \rightarrow \eta_{0\nu\beta\beta}^2 \mathcal{O}_{0\nu\beta\beta} \tag{8}$$

with

$$\eta_{0\nu\beta\beta} = \eta_d \eta_u^* \eta_e^* \tag{9}$$

Solving Eqs. (5), (9), one finds

$$\eta_\nu^2 = \eta_{\tilde{\nu}}^2 = (\eta_{0\nu\beta\beta}^*)^2. \tag{10}$$

This relation proves the statements 1,2. To see this we note that the observable quantity $z_i = (m_M^\nu,\ \tilde{m}_M^2,\ R_{0\nu\beta\beta})$ is forbidden by this symmetry if the corresponding discrete group factor is non-trivial, *i.e.* $\eta_i^2 \neq 1$. Contrary, if $\eta_i^2 = 1$, this quantity is not protected by the symmetry and appears in higher orders of perturbation theory, even if it is not included at the tree-level. Relation (10) claims that if one of the z_i is forbidden then the two others are also forbidden and, vice versa, if one of them is not forbidden they are all not forbidden.

This completes the proof of the theorem relating the neutrino Majorana mass m_M^ν, the "Majorana"-like sneutrino mass $\tilde{m}_M$ and the amplitude of $0\nu\beta\beta$-decay $R_{0\nu\beta\beta}$. The proven theorem can be considered as a supersymmetric generalization of the well-known theorem [14] relating only neutrino Majorana mass and the neutrinoless double beta decay amplitude.

Let us show that the "Dirac"-like (B-L) conserving sneutrino mass $\tilde{m}_D$ should also be present in the theory to ensure the stability of the vacuum state. Towards this end consider the last two terms of Eq. (1) which we denote as $\mathcal{L}_{mass}^{\tilde{\nu}}$ and use the real field representation for the complex scalar sneutrino field

$$\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}, \tag{11}$$

where $\tilde{\nu}_{1,2}$ are real fields. Then

$$\mathcal{L}_{mass}^{\tilde{\nu}} = -\frac{1}{2}(\tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L + h.c.) - \tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L = -\frac{1}{2}\tilde{m}_1^2 \tilde{\nu}_1^2 - \frac{1}{2}\tilde{m}_2^2 \tilde{\nu}_2^2 \tag{12}$$

where

$$\tilde{m}_{1,2}^2 = \tilde{m}_D^2 \pm |\tilde{m}_M^2| \tag{13}$$

Assume the vacuum state is stable. Then $\tilde{m}_{1,2}^2 \geq 0$, i.e. $\tilde{m}_D^2 \geq |\tilde{m}_M^2|$, otherwise the vacuum is unstable and subsequent spontaneous symmetry breaking occurs via non-zero vacuum expectation values of the sneutrino fields $< \tilde{\nu}_i > \neq 0$. The broken symmetry in this case is the R-parity. It is a discrete symmetry defined as $R_p = (-1)^{3B+L+2S}$, where S, B and L are the spin, the baryon and the lepton quantum number.

Therefore, as indicated at the beginning of this section the self-consistent structure of the mass terms of the neutrino-sneutrino sector is given by Eq. (1). The mass parameter $\tilde{m}_M$ gives a measure of sneutrino-antisneutrino mixing $(\tilde{\nu} - \tilde{\nu}^*)$. This (B-L)-violating effect is an evident manifestation of the 2nd term in Eq. (1). On the other hand, a finite $\tilde{m}_M$ gives rise to splitting the complex scalar field $\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}$ into two real mass eigenfields $\tilde{\nu}_{1,2}$ with the masses $\tilde{m}^2_{1,2} = \tilde{m}^2_D \pm |\tilde{m}^2_M|$. According to the above definition, $\tilde{\nu}_1$ is the CP-even state while $\tilde{\nu}_2$ is the CP-odd one.

Let us write down the explicit form of the above mentioned (B-L)-violating "Majorana" propagator $\Delta^M_{\tilde{\nu}}$ for the sneutrino [12] which is necessary for our subsequent considerations. It can be derived by the use of the real field representation as in Eq. (12). For comparison we also give the (B-L)-conserving "Dirac" $\Delta^D_{\tilde{\nu}}$ sneutrino propagator,

$$\Delta^D_{\tilde{\nu}}(x-y) = i < 0|T(\tilde{\nu}_L(x)\tilde{\nu}^\dagger_L(y)|0 >= \frac{1}{2}(\Delta_{\tilde{m}_1}(x-y) + \Delta_{\tilde{m}_2}(x-y)), \quad (14)$$

$$\Delta^M_{\tilde{\nu}}(x-y) = i < 0|T(\tilde{\nu}_L(x)\tilde{\nu}_L(y)|0 >== \frac{1}{2}(\Delta_{\tilde{m}_1}(x-y) - \Delta_{\tilde{m}_2}(x-y)), \quad (15)$$

where

$$\Delta_{\tilde{m}_i}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{\tilde{m}^2_i - k^2 - i\epsilon} \quad (16)$$

is the ordinary propagator for a scalar particle with mass $\tilde{m}_i$. Using the definition of $\tilde{m}_{1,2}$ as in Eq. (12) one finds

$$\Delta^D_{\tilde{\nu}}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{m}^2_D - k^2}{(\tilde{m}^2_1 - k^2 - i\epsilon)(\tilde{m}^2_2 - k^2 - i\epsilon)} e^{-ikx}, \quad (17)$$

$$\Delta^M_{\tilde{\nu}}(x) = -\tilde{m}^2_M \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{(\tilde{m}^2_1 - k^2 - i\epsilon)(\tilde{m}^2_2 - k^2 - i\epsilon)}. \quad (18)$$

It is seen that in absence of the (B-L)-violating sneutrino "Majorana"-like mass term $\tilde{m}^2_M = 0$ the (B-L)-violating propagator $\Delta^M_{\tilde{\nu}}$ vanishes while the (B-L)-conserving one $\Delta^D_{\tilde{\nu}}$ becomes the ordinary propagator of a scalar particle with mass $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_D$.

Majorana neutrino fields can propagate in a virtual state conserving the (B-L) quantum number as well as violating it. In the Majorana representation of the neutrino field

$$\nu = P_L \nu_L + P_R (\nu_L)^c \tag{19}$$

where $\nu^c = \nu$, the corresponding (B-L)-conserving (S^D) and (B-L)-violating (S^M) propagators can be written as

$$\begin{aligned} S^D(x-y) &= i < 0|T(\nu_L(x)\bar{\nu}_L(y)|0 >= iP_L < 0|T(\nu(x)\bar{\nu}(y)|0 > P_R = \quad (20)\\ &= \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu k^\mu}{m_\nu^2 - k^2 - i\epsilon} e^{-ik(x-y)}, \\ S^M(x-y) &= i < 0|T(\nu_L(x)\overline{\nu_L^c}(y)|0 >= P_L < 0|T(\nu(x)\bar{\nu}(y)|0 > P_L = \quad (21)\\ &= m_\nu \int \frac{d^4k}{(2\pi)^4} \frac{1}{m_\nu^2 - k^2 - i\epsilon} e^{-ik(x-y)}, \end{aligned}$$

where $m_\nu \equiv m_M^\nu$ for simplicity. Therefore, the effect of (B-L)-violation originating from the neutrino propagator is proportional to the Majorana neutrino mass while a (B-L)-conserving contribution to $0\nu\beta\beta$ decay via neutrino propagation in the Dirac mode does essentially not depend on the neutrino mass and leads to a contribution proportional to the mean neutrino momentum in a nucleus. The latter is typically of the order of the Fermi momentum $\sim p_F \approx 80$MeV. As a result such a contribution, if it exists, is greatly enhanced compared to the Majorana mass contribution.

3 MSSM contribution to $0\nu\beta\beta$-decay. General properties and Effective Lagrangian

In this section we are considering the general properties of the R_P-conserving MSSM contribution to $0\nu\beta\beta$-decay and derive the corresponding effective Lagrangian in terms of color-singlet quark charged currents.

Within our supersymmetric framework there are two sources of lepton number violation, the Majorana neutrino mass, m_M^ν, and the (square of the) "Majorana"-like sneutrino mass, $\tilde{m}_M^2$. Both of them violate - by construction - lepton number by two units. $0\nu\beta\beta$ decay also violates L by two units. The $0\nu\beta\beta$ amplitude $R_{0\nu\beta\beta}$ will therefore be proportional

$$R_{0\nu\beta\beta} \sim m_M^\nu \mathcal{O}^{(1)} + \tilde{m}_M^2 \mathcal{O}^{(2)}, \tag{22}$$

with $\mathcal{O}^{(i)}$ representing some matrix elements. Contributions proportional to m_M^ν can be classified by $\mathcal{O}^{(1)} = \mathcal{O}^{(1)}_{(SM)} + \mathcal{O}^{(1)}_{(SUSY)}$, where the first part stands symbolically for the usual mass mechanism diagram, see Fig. 1(a), while the second part summarizes all kinds of diagrams involving virtual SUSY particles. An example of this type of contribution was given in the introduction, see Fig. 1(b). Clearly, all diagrams of $0\nu\beta\beta$ decay involving SUSY particles must have at least 6 basic vertices. They are thus of higher order compared to Fig. 1(a) and can be safely neglected: $\mathcal{O}^{(1)} = \mathcal{O}^{(1)}_{(SM)} + \mathcal{O}^{(1)}_{(SUSY)} \cong \mathcal{O}^{(1)}_{(SM)}$.

Let us consider $R_{0\nu\beta\beta}$ in more details. At the quark level neutrinoless double beta decay is induced by the transition of two d quarks into two u quarks and two electrons. This process is schematically represented by the diagram in Fig. 3(a) encoding all possible contributions to $0\nu\beta\beta$-decay. In the R_P conserving supersymmetric model it is useful to decompose the basic diagram Fig. 3(a) as shown in Fig. 3(b-g).

Given that the low-energy theory contains a light neutrino there are always contributions involving a long-distance interaction component associated with the light neutrino exchange. Let us use this fact for a decomposition of the quark-lepton $0\nu\beta\beta$-effective vertex in Fig. 3(a) into different parts explicitly showing the presence of the neutrino line, Fig. 3(b-d). It is also instructive to isolate the SUSY contributions in the form of effective vertices induced by heavy SUSY particles exchange. This decomposition is indicated by the small black spots corresponding to the short-distance, approximately point-like, effective SUSY induced interactions, Fig. 3(c-g). The first diagram, Fig. 3(b), corresponds to the conventional standard model Majorana neutrino exchange contribution mentioned above. The crossed neutrino line indicates the lepton number violating Majorana neutrino propagator, S^M. Diagrams Fig. 3(c,d) correspond to the neutrino accompanied SUSY contributions. Here, neutrino lines are uncrossed and correspond to lepton number conserving neutrino propagators, S^D. (As noted above we neglect SUSY diagrams with lepton number violating propagators proportional to the small neutrino mass m^{ν}_M.) The last three diagrams represent the purely supersymmetric contribution and are of short-ranged nature.

This decomposition allows one to apply, if necessary, a Fierz rearrangement to the approximately point-like black-spot SUSY vertices in Fig. 3 and represent them in the form of a product of color-singlet quark charged currents and a leptonic part. Such a representation is crucial for the derivation of the $0\nu\beta\beta$-transition operators in the non-relativistic impulse approximation and for the subsequent nuclear structure calculations discussed in the sect. 4.

Assuming the Fierz rearrangement applied to the point-like SUSY vertices one can write down the general form of the effective Lagrangian reproducing the decomposition in Fig. 3 within 4th order of perturbation theory. It can be written in the form:

$$\begin{aligned}\mathcal{L}_{0\nu\beta\beta} &= \mathcal{L}_{W\bar{f}f} + \frac{\lambda_i^{(1)}}{m_{SUSY}^{n_1}} j_i \cdot \bar{e}\Gamma_{i(1)}\nu_L^c + + \frac{\lambda_i^{(2)}}{m_{SUSY}^{n_2}} W_\mu^- \cdot \bar{e}\Gamma^\mu_{i(2)}\nu_L^c \\ &+ \frac{\lambda_i^{(3)}}{m_{SUSY}^{n_3}} W_\mu^- W_\nu^- \cdot \bar{e}\Gamma^{\mu\nu}_{i(3)} e^c + \frac{\lambda_i^{(4)}}{m_{SUSY}^{n_4}} j_i^\mu W_\mu^- \cdot \bar{e}\Gamma_{i(4)} e^c \\ &+ \frac{\lambda_{ij}^{(5)}}{m_{SUSY}^{n_5}} j_i j_j \cdot \bar{e}\Gamma_{ij(5)} e^c,\end{aligned} \tag{23}$$

where the SM term $\mathcal{L}_{W\bar{f}f}$ is also introduced (see Eq. (68) in Appendix A). Color-singlet local diquark operators are defined as

$$j_i = \bar{u}^\alpha \mathcal{O}_i d_\alpha \tag{24}$$

with α being a color index. The objects $\Gamma_{i(k)}$ and $\mathcal{O}_i$ are constructed of Dirac gamma matrices as well as derivatives.

Effective couplings $\lambda^{(k)}$ are dimensionless constants. Different terms are scaled out by the characteristic SUSY breaking mass scale m_{SUSY} with an appropriate degree n_i to accommodate correct physical dimension of the corresponding term. As seen from the leptonic part of the effective Lagrangian (23), the first term conserves lepton number ($\Delta L = 0$) while the remaining five terms violate it by two units ($\Delta L = 2$).

In the effective Lagrangian (23) we neglected possible L-conserving terms with the lepton operator structure $\sim \bar{e}\Gamma_i\nu_L$. Their contributions to the $0\nu\beta\beta$-amplitude are strongly suppressed compared to the contributions of the similar L-violating terms $\sim \bar{e}\Gamma_i\nu_L^c$. To see this fact let us have a closer look at the corresponding leading order diagrams in Fig. 3(c,d). The bottom parts of these diagrams are the SM charged current (SMCC) interactions of the form $(\bar{u}\gamma_\mu P_L d)(\bar{e}\gamma^\mu P_L \nu_n)\cdot U_{en}$, while the top parts correspond to the effective SUSY vertices. If they are given by the 2nd and 3rd terms of the effective Lagrangian Eq. (23), the resulting leptonic tensor $\mathcal{L}_{SUSY}$ can then be written schematically as

$$\mathcal{L}_{\Delta L=2} \sim \bar{e}\gamma_\mu P_L < 0|T(\nu_k\bar{\nu}_n)|0 > P_R\Gamma e^c \cdot U_{ek}U_{en}^* \sim \bar{e}\gamma_\mu\gamma_\nu P_R\Gamma e^c \cdot q^\nu/q^2, \tag{25}$$

where q is the neutrino momentum. In the right hand side neutrino masses $m_M^{\nu_n}$ are neglected since $m_M^{\nu_n} << \langle q\rangle$, where $\langle q\rangle \sim p_F \approx 100$MeV is the average momentum of a neutrino propagating in a nucleus (p_F is the nucleon Fermi momentum). The mixing matrix elements disappear on the right hand side due to the unitarity relation $U_{ek}U^*_{en}\delta_{kn} = 1$. This should be compared with the contribution of the possible L-conserving terms $\sim \bar{e}\Gamma_i\nu_L$ which we neglected in the effective Lagrangian in (23). The leptonic tensor in this case takes the form

$$\mathcal{L}_{\Delta L=0} \sim \bar{e}\Gamma_i P_L < 0|T(\nu_k\bar{\nu}_n)|0 > P_L\gamma_\rho e^c \cdot U_{ek}U_{en} \sim \bar{e}\Gamma_i\gamma_\rho P_R e^c \cdot \langle m_\nu\rangle/q^2, \quad (26)$$

where $\langle m_\nu\rangle = m_M^{\nu_n}U^2_{en}$. The same structure appears in the standard neutrino mass mechanism with the SMCC at both ends of the virtual neutrino line.

Comparing Eqs. (25) and (26) one can see that the SUSY contribution corresponding to the $\Delta L = 2$ operators receives from the leptonic sector a huge enhancement compared to the contribution of the $\Delta L = 0$ operators. In fact

$$\mathcal{L}_{\Delta L=2}/\mathcal{L}_{\Delta L=0} \sim p_F/\langle m_\nu\rangle \sim 10^8 \cdot (1\text{eV}/\langle m_\nu\rangle) \quad (27)$$

For this reason we neglected the SUSY induced $\Delta L = 0$ operators in the effective Lagrangian (23).

Now let us turn from the general consideration to the concrete case of the SUSY contribution to $0\nu\beta\beta$-decay within the MSSM. The following Lagrangian terms are relevant to $0\nu\beta\beta$-quark transitions in Fig. 3(a)

$$\mathcal{L}_{int} = \mathcal{L}_{W\bar{f}f} + \mathcal{L}_{W\tilde{f}\tilde{f}} + \mathcal{L}_{\chi^+ f\tilde{f}} + \mathcal{L}_{\chi f\tilde{f}} + \mathcal{L}_{\tilde{g}\tilde{q}q} + \mathcal{L}_{W\chi^+\chi}. \quad (28)$$

The Lagrangian terms in the r.h.s. are explicitly given in Appendix A.

Starting from this Lagrangian one can find 14 dominant diagrams proportional to $\tilde{m}^2_M$ which contribute to the $0\nu\beta\beta$-quark transition in Fig.3(a). They are listed in Fig.4. (Note, that in addition to the graphs shown, there exist several graphs corresponding simply to an exchange of two of the external momenta and are not shown for brevity.) As seen, all diagrams in Fig.4 fall into 5 classes represented by the last 5 diagrams of the decomposition in Fig.3. The supersymmetric part of these diagrams, as discussed above, can be parameterized by the effective Lagrangian $\mathcal{L}_{0\nu\beta\beta}$ given in the general form of Eq. (23). One can reconstruct a specific form of this Lagrangian in the MSSM comparing diagrams in Fig.3 and Fig.4 and separating the basic SUSY vertices denoted in Fig.3 by the black spots representing five different terms of the effective Lagrangian $\mathcal{L}_{0\nu\beta\beta}$ in Eq. (23). The next step of the derivation is based on the standard approximate procedure relying on the fact that all intermediate particles involved in evaluation of the effective SUSY vertices are heavy SUSY particles with typical masses of order m_{SUSY}. As a result these vertices can approximately be represented in the form of local operators. The local form of the SUSY operators allows one to apply the Fierz rearrangement and to collect the quark fields in the color-singlet quark charged currents. Straightforward realization of this strategy leads to an effective Lagrangian with the following leading order operators violating the lepton number L by 2 units

$$\begin{aligned}\mathcal{L}_{\Delta L=2} &= -(\eta^{(1)}_{WW}+\eta^{(2)}_{WW}+\eta^{(3)}_{WW})\ \frac{W^-_\mu W^{-\ \mu}}{m_{SUSY}}\cdot\bar{e}(1+\gamma_5)e^c + \\ &+ \ (\eta_{\tilde{g}\tilde{u}}+\eta_{\tilde{g}\tilde{d}})\ \frac{j^\mu_{AV} j_{\mu AV}}{m^5_{SUSY}}\cdot\bar{e}(1+\gamma_5)e^c.\end{aligned} \tag{29}$$

The color-singlet quark charged currents are defined as usual

$$j^\mu_{\rm AV} = \cos\theta_c\ \bar{u}\gamma^\mu(1-\gamma_5)d. \tag{30}$$

Note that since we take only the leading order contributions into account in Eq. (29) not all possible terms of the decomposition Eq. (23) are retained in Eq. (29). Naively one might have expected that the diagrams in Fig. 4 (and the corresponding terms in Eq. (23)) are ordered with respect to decreasing importance, since a larger number of heavy sparticles in the loops results in larger (loop) suppression factors. However, the explicit calculation shows that this is not the case. Terms corresponding in structure to the 2nd, 3rd and the 5th terms in Eq. (23) are suppressed by the helicity structure of the basic MSSM interactions and/or typical suppression factors of order m_e/m_{SUSY} from left-right sfermion mixing or higgsino-like interactions.

Retaining only leading contributions to the operator structures in Eq. (29), the dimensionless lepton number violating parameters take the form

$$\begin{aligned}\eta_{\tilde{g}\tilde{d}} &= \frac{g_s^2 g^4}{72}\Big(\frac{\tilde{m}_M}{m_{SUSY}}\Big)^2 \sum_{i,j} U_{i1}V_{i1}U_{j1}V_{j1}\Big(\frac{m_{\chi^\pm_j}}{m_{SUSY}}\Big)\Big(\frac{m_{\chi^\pm_i}}{m_{SUSY}}\Big)\times \\ &\times\ \Big(\frac{m_{\tilde{g}}}{m_{SUSY}}\Big)\mathcal{G}(m_{\tilde{g}}, m_{\chi^\pm_j}, m_{\chi^\pm_i}),\end{aligned} \tag{31}$$

$$\eta_{\tilde{g}\tilde{u}} = \frac{g_s^2 g^4}{72}\Big(\frac{\tilde{m}_M}{m_{SUSY}}\Big)^2 \sum_{i,j} V^2_{i1}V^2_{j1}\Big(\frac{m_{\tilde{g}}}{m_{SUSY}}\Big)\mathcal{F}(m_{\tilde{g}}, m_{\chi^\pm_j}, m_{\chi^\pm_i}), \tag{32}$$

$$\eta^{(1)}_{WW} = \frac{g^4}{4}\Big(\frac{\tilde{m}_M}{m_{SUSY}}\Big)^2 \sum_{i,j,k} V_{k1}V_{j1}\times \tag{33}$$

$$
\begin{aligned}
&\times \quad \Big[\mathcal{O}^L_{ik}\mathcal{O}^L_{ij}\Big(\frac{m_{\chi_i}}{m_{SUSY}}\Big)\mathcal{J}(m_{\chi_i}, m_{\chi^\pm_j}, m_{\chi^\pm_k}) \\
&+ \quad \mathcal{O}^R_{ik}\mathcal{O}^L_{ij}\Big(\frac{m_{\chi^\pm_k}}{m_{SUSY}}\Big)\mathcal{J}(m_{\chi_i}, m_{\chi^\pm_j}, m_{\chi^\pm_k}) \\
&+ \quad \mathcal{O}^R_{ik}\mathcal{O}^R_{ij}\Big(\frac{m_{\chi^\pm_j}}{m_{SUSY}}\Big)\Big(\frac{m_{\chi^\pm_k}}{m_{SUSY}}\Big)\Big(\frac{m_{\chi_i}}{m_{SUSY}}\Big)\mathcal{I}(m_{\chi_i}, m_{\chi^\pm_j}, m_{\chi^\pm_k})\Big],
\end{aligned}
$$

$$
\begin{aligned}
\eta^{(2)}_{WW} &= \frac{g^4}{4}\Big(\frac{\tilde{m}_M}{m_{SUSY}}\Big)^2\sum_{i,j}\mathcal{J}(m_{\tilde{e}}, m_{\chi_i}, m_{\chi^\pm_j})\epsilon_{L_i}(e)V_{j1} \times \qquad (34)\\
&\times \quad \Big[\mathcal{O}^R_{ij}\Big(\frac{m_{\chi^\pm_j}}{m_{SUSY}}\Big) + \mathcal{O}^L_{ij}\Big(\frac{m_{\chi_i}}{m_{SUSY}}\Big)\Big],
\end{aligned}
$$

$$
\eta^{(3)}_{WW} = \frac{g^4}{4}\Big(\frac{\tilde{m}_M}{m_{SUSY}}\Big)^2\sum_{i}\mathcal{J}(m_{\chi_i}, m_{\tilde{e}}, m_{\tilde{e}})\epsilon^2_{L_i}(e)\Big(\frac{m_{\chi_i}}{m_{SUSY}}\Big) \qquad (35)
$$

The dimensionless loop factors $\mathcal{F}(m_i), \mathcal{G}(m_i), \mathcal{J}(m_i)$ and $\mathcal{I}(m_i)$ are given in Appendix B. They depend on the sparticle masses m_i in the corresponding loop. Recall that g and g_s are the $SU(2)_L$ and $SU(3)_c$ coupling constants. Further definitions on couplings and mixing parameters can be found in Appendix A.

The next step of the calculation deals with reformulating the problem in terms of nucleon degrees of freedom instead of quark ones. This is relevant for the nuclear structure part of the calculations.

4 From quark to nuclear level

So far the discussion has focussed on particle physics aspects, deriving the low-energy effective Lagrangian in Eq. (29) formulated in terms of quark fields. However our goal is the calculation of the amplitude $R_{0\nu\beta\beta}$ of $0\nu\beta\beta$-decay which is a nuclear process proceeding not at the level of quark degrees of freedom but at the level of nucleon ones. Formally one can write down

$$
\begin{aligned}
\mathcal{R}_{0\nu\beta\beta} &= \, < (A, Z+2), 2e^-|S-1|(A,Z) >= \qquad (36)\\
&= \, < (A, Z+2), 2e^-|Texp[i\int d^4x\mathcal{L}_{0\nu\beta\beta}(x)]|(A,Z) >
\end{aligned}
$$

where the effective Lagrangian $\mathcal{L}_{0\nu\beta\beta} = \mathcal{L}_{W\bar{f}f} + \mathcal{L}_{\Delta L=2}$ is given by Eqs. (68), (29). The nuclear structure is involved via the initial (A,Z) and the final (A, Z+2) nuclear states having the same atomic weight A, but different electric charges Z and Z+2. The standard framework for the calculation of this nuclear matrix element is the non-relativistic impulse approximation(NRIA). It implies the substitution of the quark current ${j^\mu}_{AV}$ in the effective Lagrangian $\mathcal{L}_{0\nu\beta\beta}$ in Eq. (36) by the non-relativistic nucleon current

$$ {j^\mu}_{\mathrm{AV}} \stackrel{\mathrm{NRIA}}{\longrightarrow} J^\mu. $$

The latter is an incoherent sum over individual nucleon currents of a nucleus and is given by the formula [15]

$$ J^\mu(\mathbf{x}) \;=\; \sum_i \tau_+^{(i)} \left[(f_V - f_A C_i) g^{\mu 0} - (f_A \sigma_i^k + f_V D_i^k) g^{\mu k}\right] \frac{m_A^3}{8\pi} e^{-m_A|\mathbf{x}-\mathbf{r_i}|} \tag{37} $$

Here $f_V \approx 1,\ f_A \approx 1.261$, $g^{\mu\nu}$ is the metric tensor, $\tau_+^{(i)}$ is the isospin raising operator, $\mathbf{r}_i$ is the position of the ith nucleon, the superscript k stands for the spatial component. C_i and $\mathbf{D}_i$ are the well-known scalar and the vector nuclear recoil terms given by [2]

$$ C_i \;=\; \frac{1}{2m_P}\left[(\mathbf{p}_i + \mathbf{p}_i') \cdot \boldsymbol{\sigma}_i \;-\; \frac{f_P}{f_A}(E_i - E_i')\ \mathbf{q}_i \cdot \boldsymbol{\sigma}_i\right] \tag{38} $$

$$ \mathbf{D}_i \;=\; \frac{1}{2m_P}\left[(\mathbf{p}_i + \mathbf{p}_i') \;+\; i(1 - 2m_P \frac{f_W}{f_V}) \mathbf{q}_i \times \boldsymbol{\sigma}_i\right], \tag{39} $$

Here $(\mathbf{p}_i, E_i)$ and $(\mathbf{p}_i', E_i')$ are initial and final 3-momentum and energy of the ith nucleon and the 3-momentum transfer is $\mathbf{q}_i = \mathbf{p}_i - \mathbf{p}_i'$. The nucleon couplings obey the relations

$$ f_W/f_V = -(\mu_p - \mu_n)/(2m_P) \approx 3.7/(2m_P), \quad f_P/f_A = 2m_P/m_\pi^2, \tag{40} $$

where m_π is the pion mass and $\mu_{p(n)}$ is the proton (neutron) magnetic moment.

Since we are interested in the dominant contributions only, recoil terms will be neglected in the rest of this paper and have been given above for completeness.

The exponential factor in Eq. (37) is introduced instead of the local delta function in order to take into account the finite nucleon size. It is the Fourier transform of the nucleon form factor $F(\mathbf{q}^{\,2})$ in the conventional parameterization a dipole form

$$ F(\mathbf{q}^2) = \left(1 + \frac{\mathbf{q}^2}{m_A^2}\right)^{-2} \tag{41} $$

with $m_A = 0.85$GeV. The finite nucleon size effects are known [16] to be important for the short-distance contributions to the $0\nu\beta\beta$-amplitude such as those corresponding to the dominant terms in the effective Lagrangian (29).

Now, starting from Eq. (36), it is straightforward to calculate the $0\nu\beta\beta$-amplitude $R_{0\nu\beta\beta}$ within the non-relativistic impulse approximation. The final result for the $0^+ \to 0^+$ transition amplitude can be written as follows

$$R_{0\nu\beta\beta}(0^+ \to 0^+) \;\; = \;\; \frac{\eta^{SUSY}}{m^5_{SUSY}}\sqrt{2}\cdot C^{-1}_{0\nu}\Big[\bar{e}(1+\gamma_5)e^c\Big] < F|\Omega^{SUSY}|I > . \quad (42)$$

The normalization factor is

$$C_{0\nu} = \frac{4\pi}{m_P m_e}\frac{R_0}{f^2_A}. \quad (43)$$

Here, R_0 is the nuclear radius, m_P and m_e are the proton and the electron masses.

The effective lepton number violating parameter is defined as

$$\eta^{SUSY} \;\; = \;\; (\eta_{\tilde{g}\tilde{d}} + \eta_{\tilde{g}\tilde{u}}) + g^2\Big(\frac{m_{SUSY}}{M_W}\Big)^4(\eta^{(1)}_{WW} + \eta^{(2)}_{WW} + \eta^{(3)}_{WW}). \quad (44)$$

In Eq. (42) we have introduced the transition operator Ω^{SUSY} in order to separate the nuclear physics part of the calculation from the particle physics one. Having the transition operator one can calculate the corresponding nuclear matrix element for any $0\nu\beta\beta$-decaying candidate isotope within any specific model of nuclear structure. From now on we define

$$\mathcal{M}^{SUSY} =< F|\Omega^{SUSY}|I > \quad (45)$$

This nuclear matrix element is found to be equal to those for heavy neutrino exchange [17]

$$\mathcal{M}^{SUSY} = \Big\{\mathcal{M}_{F,N} - \mathcal{M}_{GT,N}\Big\}, \quad (46)$$

where

$$\mathcal{M}_{GT,N} = \Big(\frac{m^2_A}{m_e m_p}\Big)\langle F|\Omega_{GT,N}|I\rangle \quad (47)$$

where

$$\langle F|\Omega_{GT,N}|I\rangle = \langle F|\sum_{i\neq j}\tau^{(i)}_+\tau^{(j)}_+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\left(\frac{R_0}{r_{ij}}\right)F_N(x_A)|I\rangle \quad (48)$$

and

$$\mathcal{M}_{F,N} = \Big(\frac{m_A^2}{m_e m_p}\Big)(\frac{f_V}{f_A})^2\langle F|\Omega_{F,N}|I\rangle \tag{49}$$

where

$$\langle F|\Omega_{F,N}|I\rangle = \langle F|\sum_{i\neq j}\tau_+^{(i)}\tau_+^{(j)}\left(\frac{R_0}{r_{ij}}\right)F_N(x_A)|I\rangle. \tag{50}$$

Here, $F_N(x_A)$ is the short-ranged potential

$$F_N(x_A) \quad = \quad 4\pi m_A^6 r_{ij}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{(m_A^2+\mathbf{q}^2)^4}e^{i\mathbf{q}\mathbf{r}_{ij}} \tag{51}$$

with $x_A = m_A r_{ij}$. This potential takes into account the finite nucleon size, see Eq. (41), its analytic solution is given by the formula

$$F_N(x) = \frac{x}{48}(3+3x+x^2)e^{-x}. \tag{52}$$

The above definitions are general in the sense that one can apply nuclear wave functions of any nuclear structure model for their calculation. For the following analysis, on the other hand, numerical values for the matrix elements are needed.

In our numerical analysis we will use the following value for the $0\nu\beta\beta$ decay of ^{76}Ge [17]

$$\mathcal{M}^{SUSY} = 289. \tag{53}$$

This value is based on a pn-QRPA model [18], which has been discussed already several times in the literature [18] and has been applied previously to calculations of the R-parity violating contributions to $0\nu\beta\beta$ decay [8, 10], as well as to $0\nu\beta\beta$ decay in left-right symmetric models [17]. We will therefore not repeat the details of the calculation here, and refer for brevity to [18]. Uncertainties associated with the pn-QRPA have been discussed in [8] for $\mathcal{M}_{F,N}$ and $\mathcal{M}_{GT,N}$.

5 $0\nu\beta\beta$ constraints on (B-L)-violating Sneutrino Mass

The $0\nu\beta\beta$-decay amplitude given in Eq. (42) leads to the following half-life formula

$$[T_{1/2}^{0\nu\beta\beta}(0^+\to 0^+)]^{-1} = G_{01}\frac{4m_P^2}{G_F^4}\left|\frac{\eta^{SUSY}}{m_{SUSY}^5}\mathcal{M}^{SUSY}\right|^2. \tag{54}$$

Here G_{01} is the standard phase space factor tabulated for various nuclei in [2].

Eq. (54) takes into account only the contributions from sneutrino exchange. There might be other contributions which we assume not to cancel the SUSY contribution. If there is no unnatural fine tuning between different contributions we may retain only the SUSY one in deriving upper bounds for the lepton number violating parameters.

The most stringent experimental lower limit on $0\nu\beta\beta$-decay has been obtained for ^{76}Ge [3]

$$T_{1/2}^{0\nu\beta\beta-exp}(0^+ \to 0^+) \ \geq \ 1.0 \times 10^{25} years \quad 90\% \text{ c.l.} \tag{55}$$

Combining this bound with Eq. (54) and the numerical value of the nuclear matrix element $\mathcal{M}^{SUSY}$ given in Eq. (53) we get the following constraint on the effective MSSM parameter

$$\eta^{SUSY} \leq 1.0 \times 10^{-8} \Big(\frac{m_{SUSY}}{100\text{GeV}}\Big)^5 \tag{56}$$

Since we are interested in deriving constraints on the (B-L)-violating sneutrino mass $\tilde{m}_M$ from η^{SUSY}, we will adopt the following simplifying assumptions. Assume all SUSY particle masses to be equal to the effective SUSY breaking scale m_{SUSY} introduced in Eq. (23) and consider two limiting cases for the lightest neutralino χ composition. In the first case it is assumed to be a pure B-ino as suggested by the SUSY solution of the dark matter problem [19], in the second a pure Higgsino. These two cases can be understood as extreme cases, and actual values for $\tilde{m}_M$ for other choices of the neutralino composition should therefore lie in between the two extreme values given below. In the Higgsino case essentially only the last three graphs in Fig.4 with gluino lines contribute to $0\nu\beta\beta$-decay. As seen from Eq. (31) the corresponding lepton number violating parameter $\eta_{\tilde{q}}$ does not depend on the neutralino composition and survives in this limiting case. With the currently accepted [20] values of the gauge coupling constants and W-boson mass we derive

$$\tilde{m}_M \ \leq \ 2\Big(\frac{m_{SUSY}}{100\text{GeV}}\Big)^{3/2}\text{GeV}, \quad \chi \ \sim \ \tilde{B}, \tag{57}$$

$$\tilde{m}_M \ \leq \ 11\Big(\frac{m_{SUSY}}{100\text{GeV}}\Big)^{7/2}\text{GeV}, \quad \chi \ \sim \ \tilde{H}. \tag{58}$$

6 Neutrino mass constraints

As already mentioned, the sneutrino contributes to the Majorana neutrino mass m_M^ν at the 1-loop level via the (B-L)-violating propagator Eq. (18)

proportional to the sneutrino (B-L)-violating mass parameter $\tilde{m}_M^2$. The corresponding diagram given in Fig. 2(f) gives rise to an induced Majorana neutrino mass δm^ν. Thus, in the presence of a non-zero tree-level contribution $m_M^{\nu(tree)}$ the total neutrino mass is

$$m_M^\nu = m_M^{\nu(tree)} + \delta m^\nu. \tag{59}$$

As seen from Eq. (18) the (B-L)-violating sneutrino propagator in momentum space is a rapidly decreasing function of momentum q. In the ultraviolet limit it behaves as $\Delta_{\tilde{\nu}}^M(q) \sim 1/q^4$ unlike an ordinary scalar propagator decreasing only as $\sim 1/q^2$. As a result, the 1-loop diagram in Fig. 2(f) leads to a finite loop integral. Hence, the sneutrino induced Majorana neutrino mass δm^ν is a calculable object. It is given by the formula

$$\delta m^{\nu(i)} = g^2 \sum_{k=1}^{4} \epsilon_{L_k}^{\nu^2} m_{\chi k} \tilde{m}_{M(i)}^2 I(\tilde{m}_M^2, \tilde{m}_D^2, m_{\chi k}) \tag{60}$$

where the subscript i stands for generation. The neutralino-neutrino-sneutrino coupling is $\epsilon_{L_k}^\nu = \tan\theta_W \mathcal{N}_{k1} - \mathcal{N}_{k2}$. The loop integral is defined as

$$I(\tilde{m}_M^2, \tilde{m}_D^2, m_{\chi_k}) = -i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(\tilde{m}_1^2 - q^2)} \frac{1}{(\tilde{m}_2^2 - q^2)} \frac{1}{(m_{\chi_k}^2 - q^2)}. \tag{61}$$

For an approximate numerical estimation we take all superpartner masses equal to the common mass scale of supersymmetry breaking $m_{\chi_k} \approx \tilde{m}_1 \approx \tilde{m}_2 \approx m_{SUSY}$. Then in this approximation one gets for the lightest neutralino χ contribution the following constraint on the sneutrino mass splitting parameter

$$\tilde{m}_{M(i)} \leq \frac{1.7 \cdot 10^{-2}}{|\tan\theta_W \mathcal{N}_{11} - \mathcal{N}_{12}|} \left(\frac{m_{SUSY}}{100\text{GeV}}\right)^{1/2} \left(\frac{m_{\nu_{(i)}}^{exp}}{1\text{eV}}\right)^{1/2} \text{GeV}. \tag{62}$$

Here $m_{\nu_{(i)}} \leq m_{\nu_{(i)}}^{exp}$ are the best laboratory limits on the neutrino masses which can be summarized as [20] $m_{\nu_{(e)}}^{exp} = 15\text{eV}$, $m_{\nu_{(\mu)}}^{exp} = 170\text{KeV}$, $m_{\nu_{(\tau)}}^{exp} = 23\text{MeV}$. Eq. (62) assumes that there is no significant cancellation in Eq. (59) between the tree and 1-loop level contributions.

If the neutralino is B-ino dominant, than we derive the following limits on the sneutrino mass splitting parameter

$$\tilde{m}_{M(e)} \leq 120\text{MeV}, \quad \tilde{m}_{M(\mu)} \leq 13\text{GeV}, \quad \tilde{m}_{M(\tau)} \leq 149\text{GeV}. \tag{63}$$

Thus, for the second and third generation sneutrinos large splittings are not excluded by experimental data.

An interesting question to ask is whether there are certain loopholes in the constraints on the sneutrino mass splitting derived from the experimental upper limits on neutrino masses. It could happen, for instance that the lightest neutralino is higgsino dominated, in which case one would expect that the bound (62) might have to be relaxed considerably. However, one should remember that all neutralino states contribute to (60), so that even if there is no constraint from the lightest neutralino the other mass eigenstates will provide a finite contribution to the neutrino mass.

In order to investigate this question a little bit more quantitatively we did a numerical scan of the SUSY parameter space, calculating upper bounds on $\tilde{m}_M$ taking into account all four neutralino states. In this case instead of Eq. (62) we have

$$\tilde{m}_{M(i)} \leq \frac{1.7 \cdot 10^{-2}\mathrm{GeV}}{(\sum_{i=1}^{4}(\tan\theta_W N_{i1} - N_{i2})^2 y_i C_3(x,y_i))^{1/2}} \left(\frac{m_{SUSY}}{100\mathrm{GeV}}\right) \left(\frac{m^{exp}_{\nu_{(i)}}}{1\mathrm{eV}}\right)^{1/2}. \quad (64)$$

Here $x = \tilde{m}_D/m_{SUSY}$ and $y_i = m_{\chi_i}/m_{SUSY}$. $C_3(x,y_i)$ takes into account the fact that the masses of the particles in the loop are no longer taken to be equal. In the limit where $\tilde{m}_M \ll \tilde{m}_D$ the 1-loop integral $C_3(x,y_i)$ is given by

$$C_3(x,y) = 2\frac{x^2 + y^2(\mathrm{Log}(y^2) - \mathrm{Log}(x^2) - 1)}{(y^2-x^2)^2} \quad (65)$$

$C_3(x,y)$ is normalized such that it approaches 1 in the limit where x and y approach 1. We let the parameters of the neutralino mass matrix vary from $(0-1000)$ GeV for μ and M_2, $\tan\beta$ from $(1-50)$, for both positive and negative μ. The unification condition $M_1 = (5/3)\tan^2\theta_W M_2$ has been assumed in this calculation. We required the lightest mass eigenstate to be heavier than 20 GeV, motivated by the LEP measurements. About 10^8 solutions were calculated. From these we calculated the "average constraint" and an "absolute" upper bound. These are:

$$\tilde{m}_{M(i)} \leq 60(125)\left(\frac{m^{exp}_{\nu_{(i)}}}{1\mathrm{eV}}\right)^{1/2} \mathrm{MeV}. \quad (66)$$

on average ("absolute"), if $\tilde{m}_D \approx m_{SUSY} = 100$ GeV is assumed. These numbers are about a factor of 2 (4) less stringent than taking only the lightest neutralino (being bino) fixed at 100 GeV. This simply reflects the fact that within the above-mentioned parameter ranges many solutions exist where even the lightest neutralino mass state can be considerably heavier than 100 GeV. On the other hand, it seems that within the typical range of SUSY parameters, the constraint on $\tilde{m}_M$ is essentially "stable" and has to be taken seriously. (This conclusion remains unchanged even if we drop the unification assumption on M_1, although the bounds might have to be slightly relaxed in some cases.)

Note that, in principle, more stringent limits on $\tilde{m}_M$ than in Eq. (63) could be derived from the upper bounds on the neutrino mass given by non-observation of $0\nu\beta\beta$ decay. However, in this case the situation is more complex, since $0\nu\beta\beta$ decay measures an effective neutrino mass $\langle m_M^\nu \rangle = \sum' U_{ej}^2 m_j$, where U_{ej} are mixing coefficients connecting the weak and the mass eigenstate basis for neutrinos. Thus limits on $\tilde{m}_M$ derived from the neutrino mass limit of $0\nu\beta\beta$ decay will also depend on assumptions on neutrino mixing. If one assumes for simplicity $U_{ej} \approx \delta_{e1}$ one could derive $\tilde{m}_{M(e)} \leq 22$ MeV from the data on ^{76}Ge [3].

7 Conclusion

In summary, we have proven a low-energy theorem for weak scale softly broken supersymmetry relating the (B-L)-violating mass terms of the neutrino and the sneutrino as well as the amplitude of neutrinoless double beta decay. This theorem can be regarded as a supersymmetric generalization of the well-known theorem [14] relating only the neutrino Majorana mass and the neutrinoless double beta decay amplitude.

According to Eq. (12) the parameter $\tilde{m}_M$ describes a splitting in the sneutrino mass spectrum. This splitting leads to mixing in the sneutrino-antisneutrino $(\tilde{\nu} - \tilde{\nu}^c)$ system and to the effect of lepton number violating $\tilde{\nu} - \tilde{\nu}^c$ oscillations [21], [22].

The mass splitting parameter $\tilde{m}_M$ is constrained by the experimental data on neutrinoless double beta decay $0\nu\beta\beta$ and the neutrino mass discussed in the present paper. The neutrino mass constraint on $\tilde{m}_M$ is found to be more stringent then the direct $0\nu\beta\beta$-decay constraint. This is opposite to the conclusion reached for R-parity violating SUSY, for which the direct double beta decay constraints have been found to be more stringent than those derivable from the neutrino mass [7, 8]. However, in contrast to the neutrino mass limits, the corresponding constraint on $\tilde{m}_M$ from neutrinoless double beta decay is completely independent of assumptions about neutralino masses and mixings.

The constraints derived here, nevertheless, leave quite some room for accelerator searches for sneutrino mediated (B-L)-violating effects for the second and third generation. The sneutrino mass splitting parameter $\tilde{m}_M$ might be searched for at future colliders such as the NLC or a first muon collider [21], [22]. Probably, dedicated searches for Majorana sneutrinos have a chance of detecting positive signal, within the above discussed low-energy limits. For $\tilde{\nu}_e$, on the other hand, these limits seem to be too stringent and accelerator experiments should not be expected to give positive signals.

Acknowledgments

We thank V.A. Bednyakov, for helpful discussions. The research described in this publication was made possible in part (S.G.K.) by Grant GNTP 315 NUCLON from the Russian ministry of science. M.H. would like to thank the Deutsche Forschungsgemeinschaft for financial support by grants kl 253/8-2 and 446 JAP-113/101/0.

8 Appendix A. Supersymmetric Lagrangian terms contributing to the $0\nu\beta\beta$-decay amplitude

In the presence of the (B-L)-violating (s)neutrino masses, given in Eq. (1), $0\nu\beta\beta$-decay is triggered by the following terms of the MSSM Lagrangian

$$\mathcal{L}_{MSSM} = \mathcal{L}_{W\bar{f}f} + \mathcal{L}_{W\tilde{f}\tilde{f}} + \mathcal{L}_{\chi^+ f\tilde{f}} + \mathcal{L}_{\chi f\tilde{f}} + \mathcal{L}_{\tilde{g}\tilde{q}q} + \mathcal{L}_{W\chi^+\chi} \qquad (67)$$

The individual terms can be found in the standard sources like ref. [13], [23]. Let us list them explicitly.

a) *Gauge boson-fermion-fermion*

This is nothing but the usual standard model charged-current Lagrangian,

$$\begin{aligned} \mathcal{L}_{W\bar{f}f} &= -\frac{g}{\sqrt{2}}\left[W^+_\mu(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L) + \text{h.c.}\right] && (68) \\ &\equiv \mathcal{L}_{W\bar{q}q} + \mathcal{L}_{W\bar{l}l}. && (69) \end{aligned}$$

b) *Gauge boson-sfermion-sfermion*

Only the charged-current part of this type of the MSSM interactions is of interest in $0\nu\beta\beta$ decay:

$$
\begin{aligned}
\mathcal{L}_{W\tilde{f}\tilde{f}} &\equiv \mathcal{L}_{W\tilde{q}\tilde{q}} + \mathcal{L}_{W\tilde{l}\tilde{l}} && (70)\\
&= -\frac{ig}{\sqrt{2}} W^{+}_{\mu}(\tilde{u}^{*}_{L} \overleftrightarrow{\boldsymbol{\partial}}^{\mu} \tilde{d}_{L}) - \frac{ig}{\sqrt{2}} W^{+}_{\mu}(\tilde{\nu}^{*}_{L} \overleftrightarrow{\boldsymbol{\partial}}^{\mu} \tilde{e}_{L}) + \text{h.c.} && (71)
\end{aligned}
$$

Note that $\overleftrightarrow{\boldsymbol{\partial}}^{\mu}$ is defined by $\overleftrightarrow{\boldsymbol{\partial}}^{\mu} = \overleftarrow{\partial}^{\mu} - \overrightarrow{\partial}^{\mu}$.

c) *Chargino-fermion-sfermion*

$$
\begin{aligned}
\mathcal{L}_{\chi^{+}f\tilde{f}} &= C^{u}_{LL} \cdot \bar{u}_{L}\chi^{+}_{i}\tilde{d}_{L} + C^{d}_{LL} \cdot \bar{d}_{L}\chi^{c+}_{i}\tilde{u}_{L} + C^{u}_{LR} \cdot \bar{u}_{L}\chi^{+}_{i}\tilde{d}_{R} + && (72)\\
&+ C^{u}_{RL} \cdot \bar{u}_{R}\chi^{+}_{i}\tilde{d}_{L} + C^{d}_{RL} \cdot \bar{d}_{R}\chi^{-}_{i}\tilde{u}_{L} + C^{d}_{LR} \cdot \bar{d}_{L}\chi^{-}_{i}\tilde{u}_{R} + \\
&+ C^{\nu}_{LL} \cdot \bar{\nu}_{L}\chi^{+}_{i}\tilde{e}_{L} + C^{e}_{LL} \cdot \bar{e}_{L}\chi^{-}_{i}\tilde{\nu}_{L} + \text{h.c.},
\end{aligned}
$$

where the following shorthands have been defined:

$$
\begin{aligned}
C^{u,\nu}_{LL} &= -gU_{i1}, \quad C^{d,e}_{LL} = -gV_{i1},\\
C^{u}_{LR} &= \frac{gm_{d}}{\sqrt{2}m_{W}cos\beta}U_{i2}, \quad C^{u}_{RL} = \frac{gm_{u}}{\sqrt{2}m_{W}sin\beta}V^{*}_{i2},\\
C^{d}_{RL} &= \frac{gm_{d}}{\sqrt{2}m_{W}cos\beta}U^{*}_{i2}, \quad C^{d}_{LR} = \frac{gm_{u}}{\sqrt{2}m_{W}sin\beta}V_{i2}.
\end{aligned}
$$

Coefficients C_{LL}, C_{RR} are fermion-sfermion couplings to the gaugino component of the chargino while C_{LR}, C_{RL} describe couplings to the Higgsino component. The latter are proportional to the fermion mass and, therefore, can be neglected in the fermion-sfermion sector as is done in the present paper.

The chargino mixing matrices U_{ij} and V_{ij} are defined from the diagonalization of the chargino mass matrix $M_{\chi^{\pm}}$

$$
U^{*} \cdot M_{\chi^{\pm}} V^{\dagger} = Diag\{m_{\chi^{\pm}}\}, \tag{73}
$$

For details see Ref. [23].

d) *Neutralino-fermion-sfermion*

The neutralino interaction can be written as

$$
\begin{aligned}
\mathcal{L}_{\chi f\tilde{f}} &= -\sqrt{2}g\ [\epsilon^{\psi}_{L(i)}\bar{\psi}_{L}\chi_{i}\tilde{\psi}_{L} + \epsilon^{\psi}_{R(i)}\bar{\psi}_{R}\chi_{i}\tilde{\psi}_{R} + && (74)\\
&+ \epsilon^{\psi}_{LR(i)}\bar{\psi}_{L}\chi_{i}\tilde{\psi}_{R} + \epsilon^{\psi}_{RL(i)}\bar{\psi}_{R}\chi_{i}\tilde{\psi}_{L}] - \text{h.c.},
\end{aligned}
$$

where $\psi_L = u_L, d_L, e_L, \nu_L$ and their scalar superpartners $\tilde{\psi}_L = \tilde{u}_L, \tilde{d}_L, \tilde{e}_L, \tilde{\nu}_L$. The corresponding chiral coefficients are

$$\epsilon^{\psi}_{L(i)} = -T_3(\psi)\mathcal{N}_{i2} + \tan\theta_W[T_3(\psi) - Q(\psi)]\mathcal{N}_{i1},$$

$$\epsilon^{\psi}_{R(i)} = Q(\psi)\tan\theta_W\mathcal{N}_{i1}, \tag{75}$$

$$\epsilon^{u}_{LR(i)} = \frac{m_u}{m_W sin\beta}\mathcal{N}^*_{j4}, \tag{76}$$

$$\epsilon^{u}_{RL(i)} = \frac{m_u}{m_W sin\beta}\mathcal{N}_{j4}, \tag{77}$$

$$\epsilon^{d}_{LR(i)} = \frac{m_d}{m_W cos\beta}\mathcal{N}^*_{j3}, \tag{78}$$

$$\epsilon^{d}_{RL(i)} = \frac{m_d}{m_W cos\beta}\mathcal{N}_{j3}. \tag{79}$$

Coefficients N_{ij} are elements of the orthogonal neutralino mixing matrix diagonalizing the neutralino mass matrix M_χ. In the $\not{R}_p$ MSSM the neutralino mass matrix is identical to the MSSM one [13] and in the basis of fields $(\tilde{B}, \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)$ has the form:

$$M_\chi = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}, \tag{80}$$

where $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $t_W = \tan\theta_W$, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$. The angle β is defined as $\tan\beta = < H^0_2 > / < H^0_1 >$. Here $< H^0_2 >$ and $< H^0_1 >$ are vacuum expectation values of the neutral components H^0_2 and H^0_1 of the Higgs doublet fields with weak hypercharges $Y(H^0_2) = +1$ and $Y(H^0_1) = -1$, respectively. The mass parameters M_1, M_2 are the soft $SU(2)_L$ and $U(1)_Y$ gaugino masses. In grand unification scenarios they are related to each other as follows

$$M_1 = \frac{5}{3}\tan^2\theta_W \cdot M_2 \tag{81}$$

By diagonalizing the mass matrix (80) one can obtain four neutralinos χ_i with masses m_{χ_i} and the field content

$$\chi_i = \mathcal{N}_{i1}\tilde{B} + \mathcal{N}_{i2}\tilde{W}^3 + \mathcal{N}_{i3}\tilde{H}^0_1 + \mathcal{N}_{i4}\tilde{H}^0_2. \tag{82}$$

Recall again that we use notations $\tilde{W}^3$, $\tilde{B}$ for neutral $SU(2)_L \times U(1)$ gauginos and $\tilde{H}^0_1$, $\tilde{H}^0_2$ for higgsinos which are the superpartners of the two neutral Higgs boson fields H^0_1 and H^0_2.

We apply a diagonalization by means of a real orthogonal matrix $\mathcal{N}$. Therefore the coefficients $\mathcal{N}_{ij}$ are real and masses m_{χ_i} are either positive or negative. The sign of the mass coincides with the CP-parity of the corresponding neutralino mass eigenstate χ_i. If necessary, a negative mass can be always made positive by a redefinition [23] of the neutralino field χ_i. It leads to a redefinition of the relevant mixing coefficients $\mathcal{N}_{ij} \to i \cdot \mathcal{N}_{ij}$.

e) *Gluino-squark-quark*

The gluino interaction is given by

$$\mathcal{L}_{\tilde{g}} = -\sqrt{2} g_3 \frac{\lambda^{(a)}_{\alpha\beta}}{2} \left(\bar{q}^{\alpha}_L \tilde{g}^{(a)} \tilde{q}^{\beta}_L - \bar{q}^{\alpha}_R \tilde{g}^{(a)} \tilde{q}^{\beta}_R \right) + h.c., \tag{83}$$

Here $\lambda^{(a)}$ are 3×3 Gell-Mann matrices ($a = 1, ..., 8$). Superscripts α, β in eq. (83) are color indices.

f) *Gauge boson-chargino-neutralino*

In the notation of ref. [13]

$$\mathcal{L}_{W\chi^+\chi} = g W^-_{\mu} \bar{\chi}_i \gamma^{\mu} \Big(\mathcal{O}^L_{ij} P_L + \mathcal{O}^R_{ij} P_R \Big) \chi^+_j + \text{h.c.}, \tag{84}$$

where

$$\mathcal{O}^L_{ij} = -\frac{1}{\sqrt{2}} \mathcal{N}_{i4} V^*_{j2} + \mathcal{N}_{i2} V^*_{j1} \tag{85}$$

$$\mathcal{O}^R_{ij} = +\frac{1}{\sqrt{2}} \mathcal{N}^*_{i3} U_{j2} + \mathcal{N}^*_{i2} U_{j1}. \tag{86}$$

Further details and conventions on the definitions used can be found in the paper by Haber and Kane [13].

9 Appendix B. Box Integrals

In this appendix some relevant formulas for the calculation of the loop integrals are summarized. There are four kinds of integrals encountered in the Eqs. (31)-(35):

$$\begin{aligned} \mathcal{G}(m_1, m_2, m_3) &= -i \int \frac{d^4 k}{(2\pi)^4} \qquad (87) \\ \times & \frac{m^{10}_{SUSY}}{(\tilde{m}^2_1 - k^2)(\tilde{m}^2_2 - k^2)(m^2_{\tilde{d}} - k^2)^2 (m^2_1 - k^2)(m^2_2 - k^2)(m^2_3 - k^2)}, \end{aligned}$$

$$\mathcal{F}(m_1,m_2,m_3) = -i\int \frac{d^4k}{(2\pi)^4} \times \frac{m_{SUSY}^8 k^2}{(\tilde{m}_1^2-k^2)(\tilde{m}_2^2-k^2)(m_{\tilde{u}}^2-k^2)^2(m_1^2-k^2)(m_2^2-k^2)(m_3^2-k^2)} \tag{88}$$

$$\mathcal{I}(m_1,m_2,m_3) = -i\int \frac{d^4k}{(2\pi)^4} \times \frac{m_{SUSY}^6}{(\tilde{m}_1^2-k^2)(\tilde{m}_2^2-k^2)(m_1^2-k^2)(m_2^2-k^2)(m_3^2-k^2)} \tag{89}$$

$$\mathcal{J}(m_1,m_2,m_3) = -i\int \frac{d^4k}{(2\pi)^4} \times \frac{m_{SUSY}^4 k^2}{(\tilde{m}_1^2-k^2)(\tilde{m}_2^2-k^2)(m_1^2-k^2)(m_2^2-k^2)(m_3^2-k^2)} \tag{90}$$

All four integrals are finite and can be calculated analytically using standard methods. Simple solutions can be found for the case when the masses of all particles in the loops are assumed to be equal to m_{SUSY}. In this limit one finds

$$\begin{aligned} \mathcal{G}(m_{SUSY}) &= (480\pi^2)^{-1}, \quad \mathcal{F}(m_{SUSY}) = (960\pi^2)^{-1}, \\ \mathcal{I}(m_{SUSY}) &= \mathcal{J}(m_{SUSY}) = (192\pi^2)^{-1}. \end{aligned} \tag{91}$$

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Figure Captions

Fig.1.: (a) the conventional Majorana neutrino mass mechanism of $0\nu\beta\beta$-decay; (b) an example of a R_P conserving SUSY contribution to the $0\nu\beta\beta$-decay.

Fig.2.: Lowest order perturbation theory diagrams representing the relation between the neutrino Majorana mass m_M^ν, the "Majorana"-like (B-L)-violating sneutrino mass $\tilde{m}_M$ and the amplitude of neutrinoless double beta decay $R_{0\nu\beta\beta}$. (a) the neutrino and (b) an example of sneutrino contribution to the $0\nu\beta\beta$-decay amplitude $R_{0\nu\beta\beta}$. $0\nu\beta\beta$-vertex contribution to (c) the neutrino Majorana mass and (d) to the "Majorana"-like sneutrino mass; (e) neutrino contribution to the sneutrino "Majorana"-like mass and (f) sneutrino contribution to the neutrino Majorana mass. Crossed (s)neutrino lines correspond to the (B-L)-violating propagators.

Fig.3.: The decomposition of the $0\nu\beta\beta$-effective vertex. (a) to the left: The $0\nu\beta\beta$-effective vertex. To the right the decomposition: (b-d) first line: neutrino-accompanied contributions to $0\nu\beta\beta$ decay, (e-g) second line: purely supersymmetric contributions without neutrinos.

Fig.4.: R_P conserving MSSM contributions to the $0\nu\beta\beta$-decay amplitude. Leading order diagrams, see also the text.

Figure 1.(a)

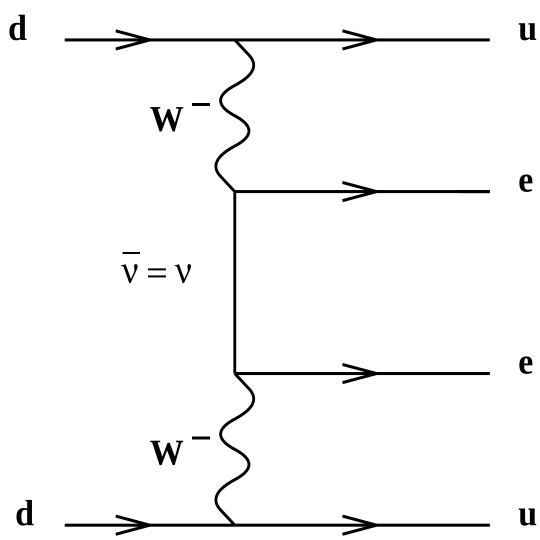


Figure 1.(b)

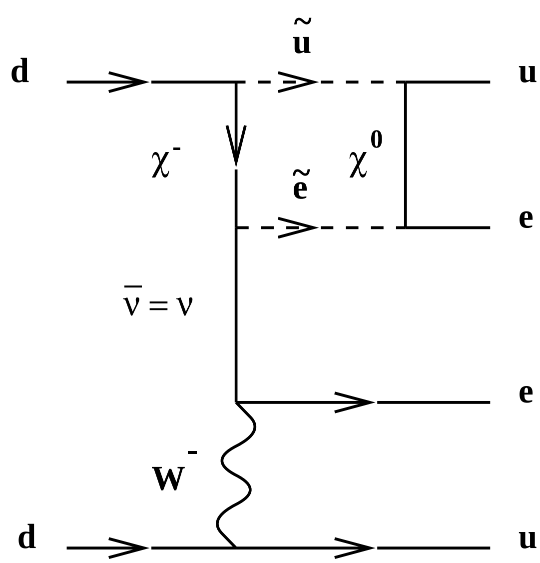


Figure 2.a

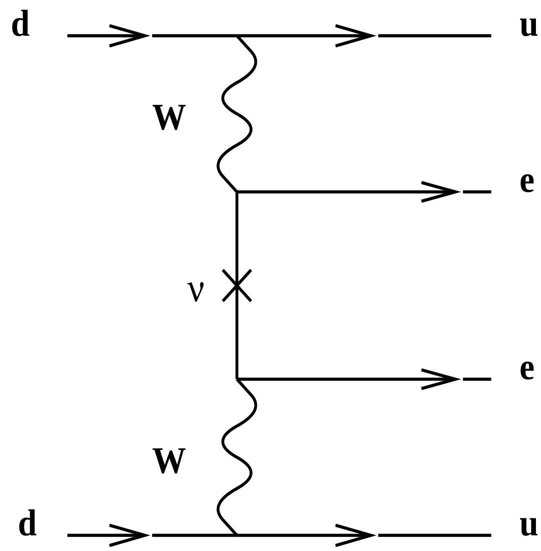
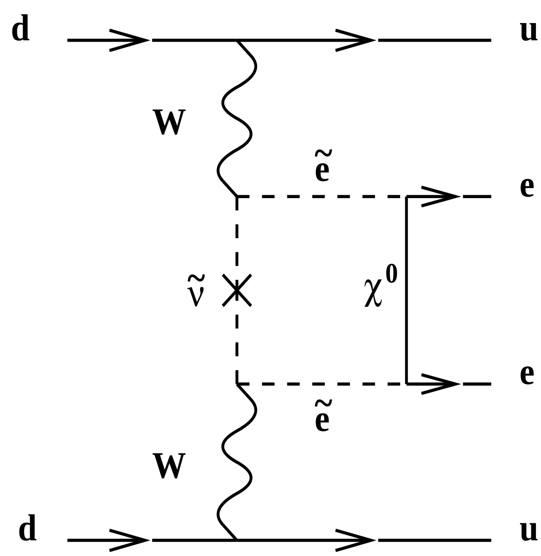


Figure 2.b



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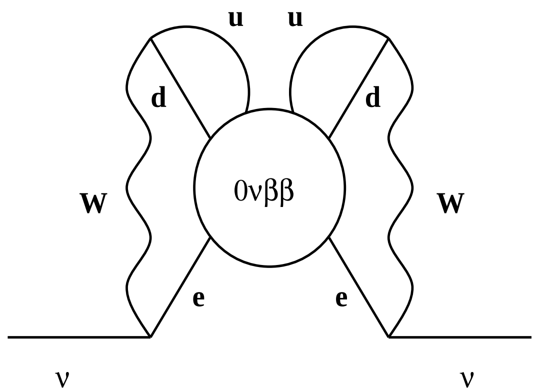

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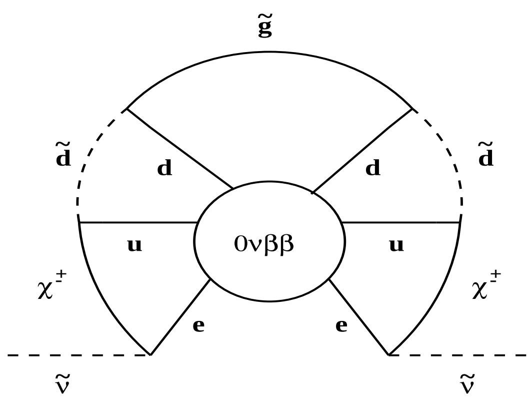

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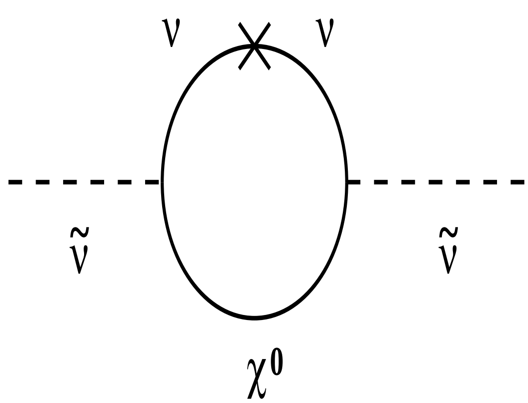
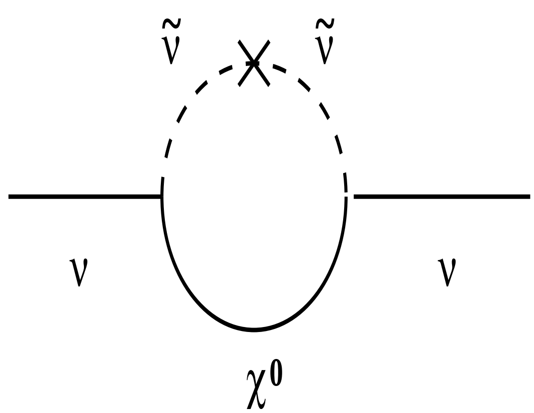


Figure 2.f



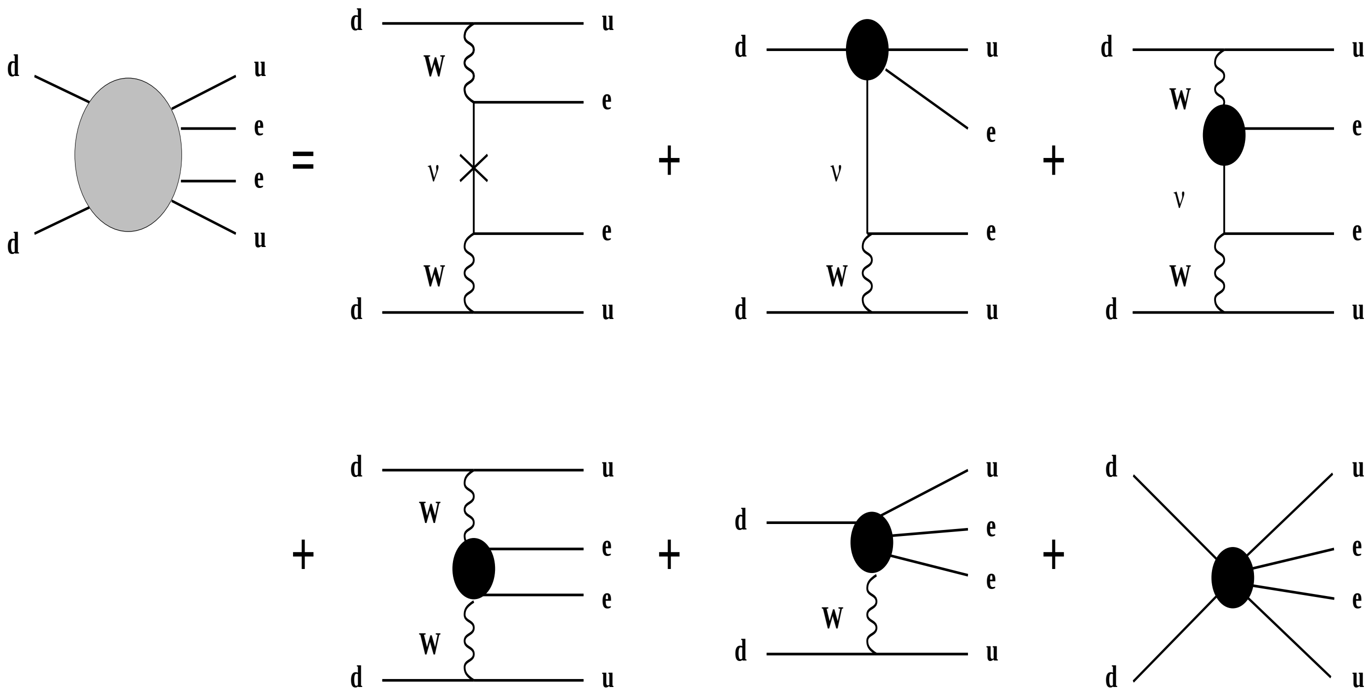
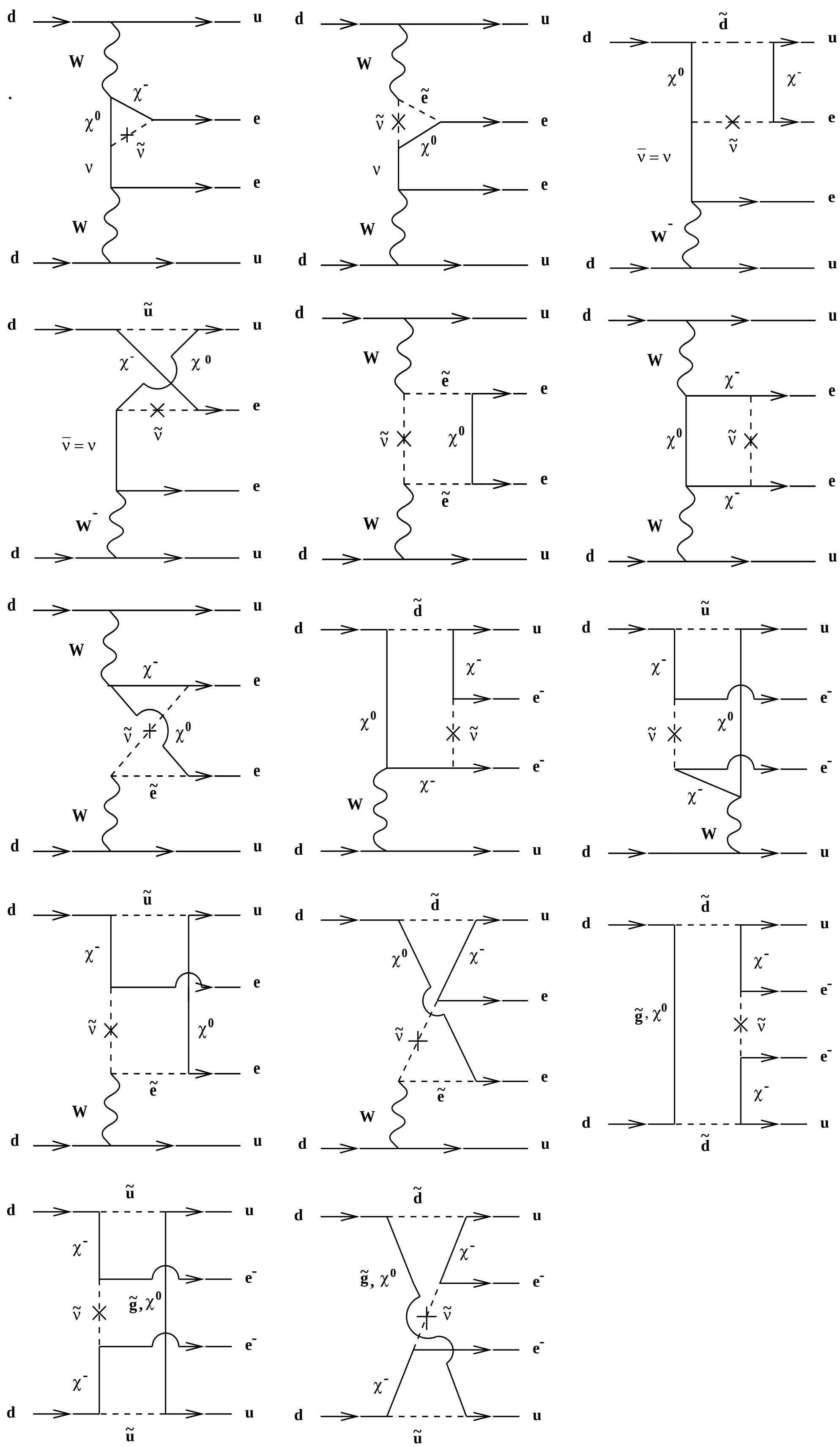


Figure 3:

Figure 4: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