Phenomenological Tests of Supersymmetric A_4 Family Symmetry Model of Neutrino Mass

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Abstract

Recently Babu, Ma and Valle proposed a model of quark and lepton mixing based on A_4 symmetry[1]. Within this model the lepton and slepton mixings are intimately related. We perform a numerical study in order to derive the slepton masses and mixings in agreement with present data from neutrino physics. We show that, starting from three-fold degeneracy of the neutrino masses at a high energy scale, a viable low energy neutrino mass matrix can indeed be obtained in agreement with constraints on lepton flavour violating μ and τ decays. The resulting slepton spectrum must necessarily include at least one mass below 200 GeV which can be produced at the LHC. The predictions for the absolute Majorana neutrino mass scale $m_0 \geq 0.3$ eV ensure that the model will be tested by future cosmological tests and $\beta\beta_{0\nu}$ searches. Rates for lepton flavour violating processes $\ell_j \to \ell_i + \gamma$ in the range of sensitivity of current experiments are typical in the model, with $\mathrm{BR}(\mu \to e\gamma) \gtrsim 10^{-15}$ and the lower bound $\mathrm{BR}(\tau \to \mu\gamma) > 10^{-9}$. To first approximation, the model leads to maximal leptonic CP violation in neutrino oscillations.

1 Introduction

The remarkable experimental achievements in neutrino physics [2, 3, 4, 5, 6, 7, 8, 9] have provided great insight in the neutrino masses and mixings. In particular it is now well established that the leptonic mixing matrix is rather different from the quark mixing matrix [10]. The structure of the mixings suggested by experiment involves a large mixing angle describing solar neutrino oscillations, a maximal one describing atmospheric neutrino oscillations, and a small one to account for reactor neutrino data. This is in sharp contrast to the three small mixings that characterize the quark sector and poses a challenge to models of the origin of the flavour structure.

In this paper we perform a detailed study of the model put forward by Babu, Ma and Valle in Ref.[1]. The model offers a simple and coherent picture of the quark and lepton mixings. Both mixing matrices are generated by radiative corrections, but with different tree-level structures fixed at some high energy scale, which we will denote by M_N. Small off-diagonal corrections to a hierarchical mass matrix will give small mixing angles. In contrast large mixing is a natural consequence of small corrections to degenerate energy levels. Therefore, the quark mixings are pushed to be small due to the hierarchical structure of their masses. Whereas, the large solar mixing angle is achieved due to degeneracy of the neutrino masses at tree-level. The two other leptonic mixing angles are fixed by the family symmetry.

The model uses A_4 family symmetry, where A_4 is the symmetry group of the tetrahedron or equivalently the group of even permutations of four elements. The family symmetry is broken at the high energy scale M_N, which is imagined to be around the scale of grand unification (of order 10^{16} GeV). However, the model is not explicitly embedded into any grand unification group. In order to have a natural stabilization of the different energy scales involved, low energy supersymmetry (SUSY) is used. Besides, as will be discussed below, the soft SUSY breaking terms constitute a necessary ingredient of the model, implying sizeable flavour changing interactions.

In addition to the usual fields in the Minimal Supersymmetric Standard Model (MSSM) a number of heavy fermion and Higgs fields are introduced. Within the A_4 family sym-

metry scheme, this implies:

(i) the quark mass matrices are hierarchical and aligned, hence giving $V_{\mathrm{CKM}}(M_N) = I$. As a result the low energy CKM angles are naturally small;

(ii) all three neutrino masses are exactly degenerate at M_N, with an off-diagonal $\nu_\mu - \nu_\tau$ texture. The atmospheric mixing angle is thereby predicted to be maximal and this feature is kept even after the leading radiative corrections;

(iii) the electron neutrino has no mixing with the state separated with the atmospheric mass scale, or in the usual terminology the U_{e3} element vanishes at tree-level;

(iv) if non-vanishing, U_{e3} is purely imaginary [1], to leading order. This in turn means that the Dirac CP-phase is maximal, a feature we refer to as maximal CP-violation;

(v) to leading order the Majorana phases [11, 12] are constrained to be 1 or i [13] and, although physical, do not give rise to genuine CP-violating effects [14, 15].

Within SUSY theories new contributions to flavour changing processes arise from the exchange of squarks and slepton. In particular the contributions are non-zero if the scalar mass matrices are off-diagonal in the basis where the corresponding fermionic mass matrices are diagonal (the super-CKM basis). The experimental bounds on flavour violating (FV) interactions in the quark sector are very strong, whereas the bounds in the lepton sector are somewhat less severe. It is a general problem to achieve sufficient suppression of the SUSY FV contributions. This is the well-known SUSY flavour problem. A popular way to suppress the magnitude of SUSY FV is to assume that slepton masses are universal at the Planck scale in the super-CKM basis. In such so-called Minimal Supergravity (mSUGRA) scenario [16, 17, 18] RGE running down to the electroweak scale gives naturally small calculable off-diagonal flavour violating terms.

A necessary ingredient of the present model is that the soft SUSY breaking terms are flavour dependent. We should therefore be especially worried about the strong constraints on flavour violation. In fact, in order to get sufficient splitting of the degenerate neutrinos, large mixings and large mass splittings in the slepton sector are required. In particular for smaller values of the overall neutrino mass scale, larger off-diagonal elements of the slepton mass matrix are necessary, in potential conflict with observation.

Our approach will therefore be to derive the possible low-energy slepton masses and

mixings by using the present knowledge of the neutrino mass matrix. Although severely constrained by bounds on lepton flavour violation as well as the overall neutrino mass scale, we show that the model is indeed viable. We give the predictions for lepton flavour violations processes, such as $\tau \rightarrow \mu\gamma$. These are within experimental reach in the very near future. We also note that the bounds derived here can be applied to any model having the same tree-level form of the neutrino mass matrix as in the A_4 model and using SUSY FV corrections to split the degeneracy. Rates for lepton flavour violating in other models such as the CP violating version of the neutrino unification model considered in Ref. [19], and the inverse-hierarchy model in Ref. [20] may be treated in a similar way.

The plan of the paper is as follows. In Sec. 2 we describe the model, and the structure of the radiative corrections, in Sec. 3 we give the numerical results for the phenomenological FV observables and the absolute scale of neutrino mass, and conclude in Sec. 4.

2 The supersymmetric A_4 model

In this section we will give an outline of the model. For further details we refer to the original paper in Ref.[1] and related work [21, 22, 23]. As already mentioned in the introduction, the A_4 group is the symmetry group of even permutations of four elements. It has four irreducible representations; three independent singlets, which we denote as $\underline{1}, \underline{1}'$ and $\underline{1}''$ and one A_4 triplet $\underline{3}$.

The usual MSSM fields are assigned the following transformation properties under A_4

$$\hat{Q}_i = (\hat{u}_i, \hat{d}_i) \text{ and } \hat{L}_i = (\hat{\nu}_i, \hat{e}_i) \sim \underline{3}, \quad \hat{\phi}_{1,2} \sim \underline{1} \tag{1}$$

$$\hat{u}_1^c, \hat{d}_1^c, \hat{e}_1^c \sim \underline{1}, \quad \hat{u}_2^c, \hat{d}_2^c, \hat{e}_2^c \sim \underline{1}', \quad \hat{u}_3^c, \hat{d}_3^c, \hat{e}_3^c \sim \underline{1}'' \tag{2}$$

Extra SU(2) singlet heavy quark, lepton and Higgs superfields transforming as A_4 triplets are added, as follows,

$$\hat{U}_i, \ \hat{U}_i^c, \ \hat{D}_i, \ \hat{D}_i^c, \ \hat{E}_i, \ \hat{E}_i^c, \ \hat{N}_i^c, \ \hat{\chi}_i, \sim \underline{3} \tag{3}$$

We also assume an extra Z_3 symmetry under which all superfields are singlets, except the SU(2) singlet Higgs superfield $\hat{\chi} \sim \omega$ (A_4 triplet) and the SU(2) singlet superfields $\hat{u}_i^c, \hat{d}_i^c, \hat{e}_i^c \sim \omega^2$, where $\omega = e^{2\pi i/3}$ is the cube root of unity.

The superpotential is then given by

$$
\begin{aligned}
\hat{W} \;=\; & M_U \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h_{ijk}^u \hat{U}_i \hat{u}_j^c \hat{\chi}_k \\
+ \; & M_D \hat{D}_i \hat{D}_i^c + f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_1 + h_{ijk}^d \hat{D}_i \hat{d}_j^c \hat{\chi}_k \\
+ \; & M_E \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{E}_i^c \hat{\phi}_1 + h_{ijk}^e \hat{E}_i \hat{e}_j^c \hat{\chi}_k \\
+ \; & \frac{1}{2} M_N \hat{N}_i^c \hat{N}_i^c + f_N \hat{L}_i \hat{N}_i^c \hat{\phi}_2 + \mu \hat{\phi}_1 \hat{\phi}_2 \\
+ \; & \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3
\end{aligned}
\tag{4}
$$

Note that the Z_3 symmetry is explicitly broken by the soft supersymmetric mass term M_χ. On the other hand the A_4 symmetry gets spontaneously broken at the high scale by the $\langle \chi_i \rangle$ vev's lying along the F-flat direction given as $\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u = -M_\chi/h_\chi$. This solution is therefore invariant under supersymmetry, which is a necessary requirement as we want to have low energy SUSY. In fact, the low energy effective theory of the model is nothing but the MSSM. Note that supersymmetry is thus only broken by TeV scale soft breaking terms. These will also break the A_4 symmetry and constitute the very source of the threshold corrections to the neutrino masses. The electroweak symmetry is broken by the vev's of the two Higgs doublets. As usual we define $\tan(\beta) = v_2/v_1$, where $\langle \phi_i^0 \rangle = v_i$.

The charged lepton mass matrix linking (e_i, E_i) to (e_j^c, E_j^c) is restricted by the family symmetry to the simple form

$$
M_{eE} =
\begin{pmatrix}
0 & 0 & 0 & f_e v_1 & 0 & 0 \\
0 & 0 & 0 & 0 & f_e v_1 & 0 \\
0 & 0 & 0 & 0 & 0 & f_e v_1 \\
h_1^e u & h_2^e u & h_3^e u & M_E & 0 & 0 \\
h_1^e u & h_2^e \omega u & h_3^e \omega^2 u & 0 & M_E & 0 \\
h_1^e u & h_2^e \omega^2 u & h_3^e \omega u & 0 & 0 & M_E
\end{pmatrix}.
\tag{5}
$$

The ω factors arise due to the way triplets and singlets form A_4-invariant combinations [22]. This mass matrix is sufficiently simple to allow for an analytic diagonalization. It is of see-saw type and the effective 3×3 low energy mass matrix, M_ℓ^{eff}, can be diagonalized by $M_\ell^{\text{diag.}} = U_L M_\ell^{\text{eff}} I$, where the left diagonalization matrix reads

$$
U_L = \frac{1}{\sqrt{3}}
\begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{pmatrix}.
\tag{6}
$$

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The Yukawa couplings h_i^e, $i = 1, 2, 3$, are chosen such that the three eigenvalues of M_ℓ^{eff}, given by $m_i = \sqrt{3} f_e v_1 u / M_E \sqrt{1 + (h_i^e u)^2 / M_E^2} \, h_i^e$, agree with the measured masses of the electron, muon and tau leptons. Similarly, the up-type and down-type quark mass matrices have the same structure as in Eq(5) [22]. Therefore, they will be simultaneously diagonalized, implying $V_{\text{CKM}} = I$ at the high scale. An elegant feature of the model is that, since the quark masses are hierarchical, the low-energy CKM mixing angles are naturally small, as they arise due to small calculable radiative corrections. A realistic V_{CKM} matrix can indeed be ascribed to radiative corrections coming from the soft SUSY breaking scalar quark (squark) mixing terms, starting from the tree-level identity matrix. This can be done obeying all experimental constraints (in particular bounds on flavour changing processes), as already shown in Ref. [24].

Here we focus on the neutrino masses. Rotating to the flavour basis, where the charged leptons mass matrix is diagonal, the 6×6 Majorana mass-matrix for $(\nu_e, \nu_\mu, \nu_\tau, N_1^c, N_2^c, N_3^c)$ takes the simplest (type-I) see-saw form

$$\begin{pmatrix} 0 & U_L f_N v_2 \\ U_L^T f_N v_2 & M_N \end{pmatrix}.$$ (7)

so that the effective low-energy neutrino mass matrix, is given by

$$M_\nu^0 = \frac{f_N^2 v_2^2}{M_N} U_L^T U_L = \frac{f_N^2 v_2^2}{M_N} \lambda_0, \qquad \lambda_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$ (8)

Thus the tree-level neutrino mass matrix at the M_N scale has exactly degenerate neutrinos, $m_1 = m_2 = m_3$, and exact maximal atmospheric mixing.

Let us turn to look at the sources of the radiative correction to the mass matrices. There are in general two kinds of radiative corrections; the standard renormalization effects arising when running from M_N to the electroweak scale, using supersymmetric renormalization group equations (RGE's), and the low-energy threshold corrections.

Starting with degenerate neutrinos at some high energy scale, and using the standard minimal supergravity RGE's, it is not possible to obtain a suitable neutrino spectrum [25, 26]. Moreover, renormalization group evolution can not produce corrections to the textures zeros in M_ν^0, since the RGE corrections, in the flavour basis, are proportional
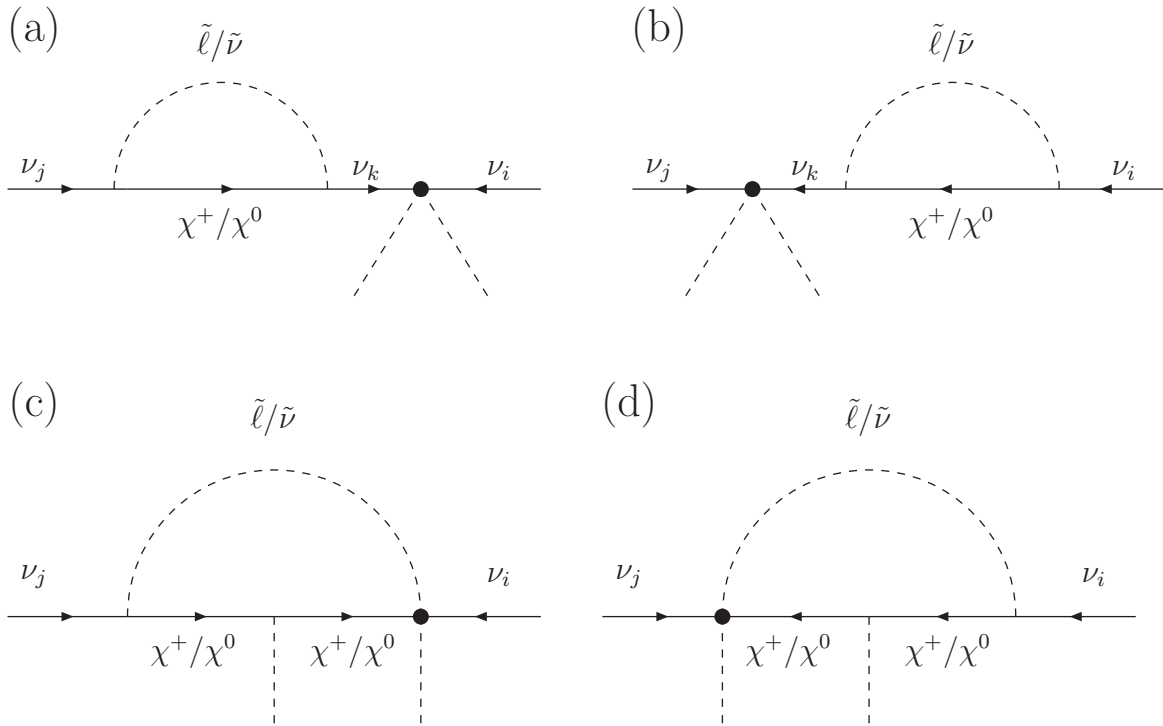
Figure 1: Feynman diagram responsible for "wave-function" (top) and "vertex" (bottom) radiative corrections to neutrino mass. The fat vertex indicates an effective dimension-5 operator obtained by integrating out the heavy right-handed neutrinos

to the original mass element [27, 28, 29, 30]. However, it is clear that small corrections to the tree-level texture zeros are necessary in order to obtain a realistic mass matrix.

Invoking threshold corrections from flavour violating (FV) soft SUSY breaking terms allows us to obtain both adequate neutrino mass splittings [19, 31] as well as mixing angles. We now show explicitly how a fully realistic low energy neutrino mass matrix can be obtained when one includes the radiative corrections coming from FV scalar lepton (slepton) interactions. In our numerical programs we include these corrections.

The RGE effect can be approximated by

$$M_{\alpha\beta}(M_S) \simeq \left[1 - \frac{m_\alpha^2 + m_\beta^2}{16\pi^2 v^2 \cos^2(\beta)} \log(M_N/M_S)\right] M_{\alpha\beta}(M_N) . \tag{9}$$

Let us for the moment just consider the τ Yukawa coupling. Then defining

$$\delta_\tau \equiv \frac{m_\tau^2}{8\pi^2 v^2 \cos^2(\beta)} \log(M_N/M_S) \tag{10}$$

we get the values: $\delta_\tau \simeq \mathcal{O}(10^{-5})$ for $\tan(\beta) = 1$ and $\delta_\tau \simeq \mathcal{O}(10^{-3})$ for $\tan(\beta) = 15$. Here, we have put $M_N = 10^{12}$ GeV and $M_S = 1000$ GeV.

In the following we calculate the radiative threshold corrections coming from the soft SUSY breaking terms. At one-loop these contributions to the neutrino masses arise from the diagrams shown in Fig.1. For the evaluation we make some approximations. First of all, we will not consider the full 6×6 slepton mass matrix but restrict to the 3×3 left-left part of it. The charged slepton mass matrix, in the super-CKM basis, may be written as

$$M_{\tilde{\ell}}^2 = \begin{pmatrix} M_{LL,\tilde{\ell}}^2 & (A - \mu \tan(\beta))m_\ell \\ (A - \mu \tan(\beta))m_\ell & M_{RR}^2 \end{pmatrix} \tag{11}$$

where each entry is a 3×3 matrix and

$$(M_{LL,\tilde{\ell}}^2)_{ij} = M_{L,ij}^2 - \frac{1}{2}(2m_W^2 - m_Z^2)\cos(2\beta)\delta_{ij} + m_\ell^2 \delta_{ij} \tag{12}$$

The 3×3 sneutrino mass matrix is given by

$$(M_{LL,\tilde{\nu}}^2)_{ij} = M_{L,ij}^2 + \frac{1}{2}m_Z^2 \cos(2\beta)\delta_{ij} . \tag{13}$$

Although there are small differences in the sneutrino and slepton left-left mass matrices we will assume that they are identical, i.e. $M_{LL,\tilde{\ell}}^2 = M_{LL,\tilde{\nu}}^2 = M_{LL}^2$. Indeed the soft

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breaking terms are expected to give the largest contribution. Consequently the sleptons and sneutrinos have identical mixing matrices and eigenvalues. Let us define the mixing matrix such that

$$\tilde{\ell}'_\alpha = R_{i\alpha}\tilde{\ell}_i , \qquad i = 1, 2, 3 \tag{14}$$

where $\tilde{\ell}'_\alpha$ is the flavour eigenstate and $\tilde{\ell}_i$ is a mass eigenstate. Then the mass eigenvalues can be written as $R^\dagger M^2_{\text{diag.}} R = M^2_{LL}$. The contribution from the right-sleptons in the diagrams in Fig.1 are suppressed with the Yukawa couplings squared. Hence, at least for small $\tan(\beta)$ the approximation of only using the 3×3 slepton mass matrix is reasonable. As we will discuss below, the solution in agreement with data on lepton flavour violations have indeed relatively small values of $\tan(\beta)$.

Now, as M_{LL} is hermitian, it is easily realized that the structure of the one-loop corrections to the Majorana neutrino mass matrix can be written as

$$\lambda^{1-\text{loop}} = \lambda^0 \hat{\delta} + (\hat{\delta})^T \lambda^0 , \qquad \hat{\delta} = \begin{pmatrix} \delta_{ee} & \delta^*_{e\mu} & \delta^*_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & \delta^*_{\mu\tau} \\ \delta_{e\tau} & \delta_{\mu\tau} & \delta_{\tau\tau} \end{pmatrix} \tag{15}$$

Therefore, the form of the neutrino mass matrix may be approximated by

$$M^{1-\text{loop}}_\nu = m_0 \begin{pmatrix} 1 + 2\delta + 2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1 + \delta - 2\delta_\tau \\ \delta''^* & 1 + \delta - 2\delta_\tau & \delta \end{pmatrix} , \tag{16}$$

Since the value of $\delta_0 \equiv \delta_{\mu\mu} + \delta_{\tau\tau} - 2\delta_{\mu\tau}$ does not affect the mixing angles, it has been absorbed in m_0, the overall neutrino mass scale. Moreover, in Eq. (16) we have defined

$$\delta = 2\delta_{\mu\tau} , \qquad \delta'' = \delta^*_{e\mu} + \delta_{e\tau} \tag{17}$$

$$\delta' = \delta_{ee} - \delta_{\mu\mu}/2 - \delta_{\tau\tau}/2 - \delta_{\mu\tau} \tag{18}$$

Note that, without loss of generality, by redefining ν_μ and ν_τ, one can always make the parameter δ real. Thus, due to the special form of $M^{1-\text{loop}}_\nu$ implied by the flavour symmetry, the phase of δ can be rotated away, even though the neutrinos are Majorana particles.

The general form of the light neutrino mixing matrix in any gauge theory of the weak interaction containing $SU(2) \otimes U(1)$ singlet leptons was given in Ref. [11]. For

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the case where the isosinglets get super-heavy masses, as in the present case, it can be approximated as a unitary matrix, U, which may be written as the product of three complex rotation matrices involving three angles and three phases, two of which are the Majorana CP-violating phases [11, 12].

In the present case, due to the flavour symmetry which restricts the form of M_ν as given in Eq.(16), we have that the atmospheric mixing angle is maximal, and not affected by the radiative corrections. Moreover, with this parametrization the tree-level value of the "reactor" angle, s_{13}, is zero [1]. The model also implies that $s_{13}\cos(\delta_{\rm CP}) = 0$, where $\delta_{\rm CP}$ is the Dirac CP-phase [13]. Therefore, in this model a non-zero value of s_{13} implies maximal CP-violation in the leptonic sector [1]. On the other hand, $s_{13} = 0$ is equivalent to δ''^2 being real [13]. The property of maximal CP violation gives interesting perspectives for discovery of leptonic CP-violation in future long-baseline neutrino oscillation experiments. Finally, Majorana CP phases affecting $\Delta L = 2$ processes such as $\beta\beta_{0\nu}$ take on CP conserving values, 0 or $\pi/2$.

In the following numerical analysis we will assume that the mass matrix is real, in which case it can be diagonalized analytically. The eigenvalues are given by

$$
\begin{aligned}
m_1 &= m_0\left(1 + 2\delta + \delta' - \delta_\tau - \sqrt{\delta'^2 + 2\delta''^2 + 2\delta'\delta_\tau + \delta_\tau^2}\right) \\
m_2 &= m_0\left(1 + 2\delta + \delta' - \delta_\tau + \sqrt{\delta'^2 + 2\delta''^2 + 2\delta'\delta_\tau + \delta_\tau^2}\right) \\
m_3 &= m_0(-1 + 2\delta_\tau)
\end{aligned}
\tag{19}
$$

Hence, assuming $\delta_\tau, \delta', \delta'' \ll \delta$ the mass squared differences can be approximated by [2]

$$
\Delta m^2_{\rm atm} \simeq 4m_0^2\delta
\tag{20}
$$

$$
\Delta m^2_{\rm sol} \simeq 4m_0^2\sqrt{\delta'^2 + 2\delta''^2 + 2\delta'\delta_\tau + \delta_\tau^2}
\tag{21}
$$

The solar angle is given by

$$
\tan^2(\theta_{\rm sol}) = \frac{2\delta''^2}{(\delta' + \delta_\tau - \sqrt{\delta'^2 + 2\delta''^2 + 2\delta'\delta_\tau + \delta_\tau^2})^2}
\tag{22}
$$

---

[1]We define the most split neutrino mass as m_3 and require $m_2 > m_1$, therefore the reactor angle is always given by s_{13}.

[2]Note that since the maximal angle has to go along with the atmospheric mass scale, these mass squared differences can not be swapped around.

Although generated by threshold effects, the solar angle is naturally expected to be large, thanks to the quasi-degenerate neutrino spectrum. Note that if $\delta' = -\delta_\tau$ the effect of the corrections from δ' and δ_τ is equal to having $\delta_0 = 2\delta'$ and thus amounts to an overall shift of the mass scale. Nevertheless, in this case the solar angle would become maximal, which is now excluded by experiments [10].

Furthermore, as δ' and δ_τ arise from different physics, there is no reason for this fine-tuning to take place. Therefore, the numerical value of δ_τ can not be much bigger than the solar mass scale, more precisely $\delta_\tau \stackrel{<}{\sim} 5 \times 10^{-4}$, implying that $\tan(\beta) > 10$ is disfavored, as it will destroy the agreement with the solar data. This can be seen explicitly in Fig. 7.

The analytic expression for the radiative corrections to the neutrino masses are

$$
\begin{aligned}
\delta_{\alpha\beta}^{(a)\chi^+} &= \sum_{i=1}^{3}\sum_{A=1}^{2} |U_{A1}|^2 \frac{g^2}{16\pi^2} B_1(m_{\chi_A^+}^2, m_{\tilde{\ell}_i}^2) R_{i\alpha} R_{i\beta}^* , \\
\delta_{\alpha\beta}^{(a)\chi^0} &= \sum_{i=1}^{3}\sum_{A=1}^{4} |gN_{A2} - g'N_{A1}|^2 \frac{1}{32\pi^2} B_1(m_{\chi_A^0}^2, m_{\tilde{\nu}_i}^2) R_{i\alpha} R_{i\beta}^* , \\
\delta_{\alpha\beta}^{(c)\chi^+} &= \sum_{i=1}^{3}\sum_{A=1}^{2}\sum_{B=1}^{2} |U_{A1}V_{B2}|^2 \frac{g^2}{4\pi^2} C_{00}(m_{\chi_A^+}^2, m_{\chi_B^+}^2, m_{\tilde{\ell}_i}^2) R_{i\alpha} R_{i\beta}^* , \\
\delta_{\alpha\beta}^{(c)\chi^0} &= \sum_{i=1}^{3}\sum_{A=1}^{4}\sum_{B=1}^{4} |gN_{A2} - g'N_{A1}|^2 |N_{B4}|^2 \frac{1}{8\pi^2} C_{00}(m_{\chi_A^0}^2, m_{\chi_B^0}^2, m_{\tilde{\nu}_i}^2) R_{i\alpha} R_{i\beta}^* ,
\end{aligned}
\tag{23}
$$

where we have evaluated the Feynman diagrams at zero external momentum, which is an excellent approximation as the neutrino masses are tiny. Here $\delta_{\alpha\beta}^{(a)\chi^+}, \alpha, \beta = e, \mu, \tau$, is the contributions from the chargino/slepton diagram in Fig.1(a), with analogous notation for the other contributions. The value of the $\delta_{\alpha\beta}, \alpha, \beta = e, \mu, \tau$ in Eq. (23) is the sum of the four contributions given above. In the above formula, U, V are the chargino mixing matrices and $m_{\chi_A^+}, A = 1, 2$ are chargino masses. N is the neutralino mixing matrix and $m_{\chi_A^0}, A = 1, .., 4$ are the neutralino masses. The coupling constant of the $SU(2)$ gauge group is denoted g and of $U(1)$ is g'. Furthermore B_1 and C_{00} are Passarino-Veltman functions given by

$$
B_1(m_0^2, m_1^2) = -\frac{1}{2}\Delta_\epsilon + \frac{1}{2}\ln\left(\frac{m_0^2}{\mu^2}\right) + \frac{-3 + 4t - t^2 - 4t\ln(t) + 2t^2\ln(t)}{4(t^2 - 1)}
\tag{24}
$$

where $t = m_1^2/m_0^2$ and

$$
C_{00}(m_0^2, m_1^2, m_2^2) = \frac{1}{8}(3 + 2\Delta_\epsilon) - \frac{1}{4}\ln\frac{m_0^2}{\mu^2} + \frac{-2r_1^2(r_2 - 1)\ln(r_1) + 2r_2^2(r_1 - 1)\ln(r_2)}{8(r_1 - 1)(r_2 - 1)(r_1 - r_2)}
\tag{25}
$$

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where $r_1 = m_1^2/m_0^2$ and $r_2 = m_2^2/m_0^2$. We have used dimensional reduction, with $\epsilon = 4 - n$ and n is the number of space-time dimensions. The term $\Delta_\epsilon = \frac{2}{\epsilon} - \gamma + 4\ln(4\pi)$, where γ is Eulers constant, is divergent as $\epsilon \to 0$. However, the unitarity conditions

$$\sum_i R_{i\alpha} R_{i\beta}^* = \delta_{\alpha\beta} \tag{26}$$

ensure that the infinities and the μ^2 dependent terms cancel in the corrections to the neutrino mass matrix given in Eqs. (17)-(18). Therefore, the final result does not depend on the renormalization scheme.

Let us shortly comment on the effect coming from the diagonalization of the low energy charged lepton mass matrix. The neutrino mass matrix in Eq.(23) is written in the tree-level flavour basis. Therefore, the radiative corrections to the charged lepton masses will result in small corrections to the mixing angles in U. Indeed, for the case of quarks, we attribute the value of V_{CKM} to this type of radiative corrections. Nonetheless, as the dominant quark corrections originate from gluino exchange, whereas the charged leptons only receive contributions from bino exchange, we expect that the charged lepton mixings will be much smaller. Let us denote the matrix which diagonalize the low energy charged lepton mass matrix by $U_\ell = I + \epsilon_{\text{rad}}$. In the following we will only consider to first order in the radiative corrections. In this case the matrix ϵ_{rad} will be anti-hermitian. The neutrino mass matrix M_ν^{FB} in the flavour basis will be given by

$$M_\nu^{\text{FB}} = U_\ell M_\nu U_\ell^T = U_\ell(\lambda^0 + \hat{\delta}\lambda^0 + \lambda^0\hat{\delta})U_\ell^T \simeq \lambda^0 + \hat{\delta}\lambda^0 + \lambda^0\hat{\delta}^T + \epsilon_{\text{rad}}\lambda^0 + \lambda^0\epsilon_{\text{rad}}^T . \tag{27}$$

Clearly this amounts to performing the substitution $\hat{\delta} \to \hat{\delta} + \epsilon_{\text{rad}}$, but with the important difference that ϵ_{rad} is anti-hermitian whereas $\hat{\delta}$ is hermitian. Consequently, the atmospheric mixing angle will get small deviations from maximal, while the CP phase will depart from its leading order value $\pi/4$.

3   Numerical Analysis and Results

Here we test the validity of the model by performing a numerical analysis. We determine the allowed parameter space by searching randomly, within certain ranges, for points in
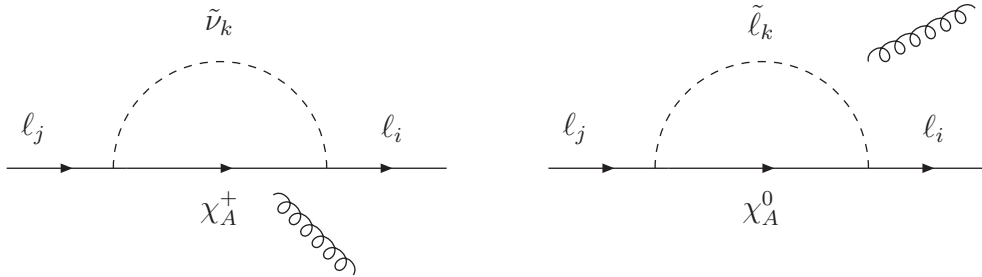
Figure 2: Supersymmetric contributions to the flavour violating charged lepton decay.

agreement with all experimental data, considering only the radiative corrections from the diagrams shown in Fig. 1 and the effect of the RGE running in Eq. (10). The main phenomenological restrictions come from neutrino data and LFV constraints.

As discussed above, a non vanishing value of the reactor angle, s_{13}, can only be accomplished if there is CP-violation in the model. From the formulæ's for the radiative corrections to the neutrino mass matrix given in Eq.(23), it is clear that all CP-violation originate from the slepton mixing matrix $R_{i\alpha}$. This is true even considering the RGE effects, as the tree-level mass matrix is real, and CP-phases cannot be generated by RGE running. Therefore, complex phases in the off-diagonal left-left sleptons masses are necessary. In Ref.[32] is has been shown that these off-diagonal phases are allowed to be large. Although, the phases in the slepton mass matrix will give contributions to the electric dipole moment (EDM) of the electron, they can be cancelled with contributions of other SUSY phases, such as the phase of the μ term or phases of the gaugino masses. Therefore, the experimental bounds on the electron EDM will not necessarily restrict the maximum achievable magnitude of U_{e3}.

To get sufficient suppression of LFV is a general problem in SUSY models. This is even more so in the A_4 model, since it requires flavour violation in the slepton sector. The strongest bounds on lepton flavour violating processes come from $\ell_j \to \ell_i\gamma$ and the contributions, due to exchange of SUSY particles are shown in Fig. 2. The present bounds[3] on these processes [34] are

$$BR(\mu \to e\gamma) \ < \ 1.2 \times 10^{-11} \ ,$$

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[3]Recently a new bound of 3.1×10^{-7} at 90% C.L. on $BR(\tau \to \mu\gamma)$ is given [33].

$$BR(\tau \to \mu\gamma) \quad < \quad 1.1 \times 10^{-6} \, , \tag{28}$$

$$BR(\tau \to e\gamma) \quad < \quad 2.7 \times 10^{-6} \, . $$

Explicit formulas for the SUSY contributions can be found in Ref.[35]. We have used the full 6×6 slepton mass matrix in order to get the slepton mixings. In so-doing we assumed, for simplicity, that the LR and RR sectors of the slepton mass matrix are flavour diagonal. In this approximation the only source of flavour violation comes therefore from the LL sector. We have compared our numerical results for the branching ratios against the ones in Ref. [36, 37] and found agreement.

In total there are 10 parameters: the slepton masses and mixing angles. The two gaugino masses M_1 for the $U(1)$ gauge group and M_2 for the $SU(2)$ gauge group. The value of the μ term and $\tan(\beta)$. For the numerical calculations we take all SUSY masses in the range 100 GeV to 1000 GeV. If SUSY masses are much larger supersymmetry will no longer solve the hierarchy problem, which is indeed one of the strongest arguments for SUSY. The results that we present in the following are quite naturally dependent on the upper cut on the SUSY masses. If larger masses are admitted, the allowed parameter space will be larger. Furthermore, all parameters are taken to be real and therefore $s_{13} = 0$ is obtained. Hence, the reactor bound is automatically satisfied. The neutrino parameters are taken within 3σ ranges allowed by the most recent solar, atmospheric, reactor and accelerator data, taken from Ref. [10]. The most relevant parameters in our analysis are the solar angle and mass squared difference, as well as the atmospheric mass squared difference. These two mass splittings may potentially conflict with the overall mass scale m_0 for the degenerate neutrinos, fixed at the large scale where the flavour symmetry holds. The absolute neutrino mass scale is constrained by cosmology and by $(\beta\beta)_{0\nu}$ experiments. Using the recent data from WMAP [38] and 2df galaxy survey [39] a bound on the sum of neutrino masses in the range $0.7 - 1.0$ eV (95% CL.) has been claimed [40, 41]. However, a more recent re-analysis dropping prior assumptions gives a less stringent bound of 1.8 eV which leads to [42]

$$m_0 < 0.6 \, \text{eV} \, . \tag{29}$$

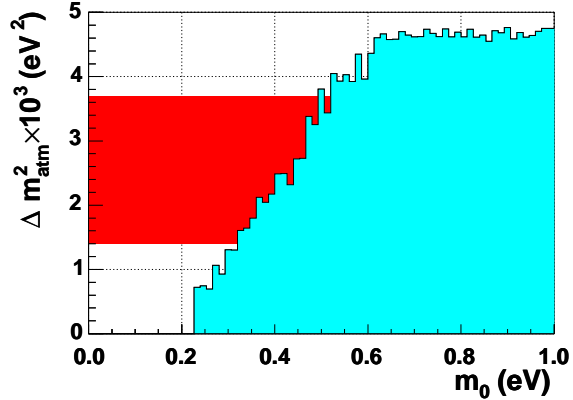Similarly, from neutrinoless double beta decay an upper bound is obtained, less strict,

Figure 3: The light shaded histogram shows the maximum possible value of the atmospheric mass squared difference as a function of m_0. The dark shaded region is the current 3σ allowed region for $\Delta m^2_{\rm ATM}$ from [10].

considering the nuclear matrix element uncertainties (there also exists claims in favor of degenerate neutrino [43, 44]. See, however, Ref. [45]).

Here we take the conservative upper limit of 0.6 eV on the magnitude of the Majorana neutrino mass m_0. In order to obtain the measured atmospheric mass squared difference, approximately given by Eq.(20), a minimum value for the δ parameter, defined in Eq.(17), is needed. Similarly to arrive at the right solar mass squared difference, roughly given by Eq.(21), the values of δ' and δ'', defined in Eqs.(17)-(18), should be around $10^{-5} - 10^{-4}$. Furthermore, it is clear from Eq.(22), that $\delta'' \sim \delta'$ is necessary in order to obtain a large solar mixing. The numerical study gives the bound $|\delta'/\delta''| > 0.1$.

It is non-trivial to obtain large enough values for the δ, δ' and δ'' parameters. To produce a large value of δ, large mass splittings as well as large mixing in the $\tilde{\mu} - \tilde{\tau}$ sector are needed. The upper bound on the mass gaps will give a maximum value for δ which, through the relation to $\Delta m^2_{\rm ATM}$, imposes a minimum value for m_0. In Fig.3 the maximum achievable value of $\Delta m^2_{\rm ATM}$ is plotted against the value of m_0. A conservative lower bound

$$m_0 \geq 0.3 \, {\rm eV} \tag{30}$$

is derived. This is very close to the present limit from experiments, which we take as Eq. 29.
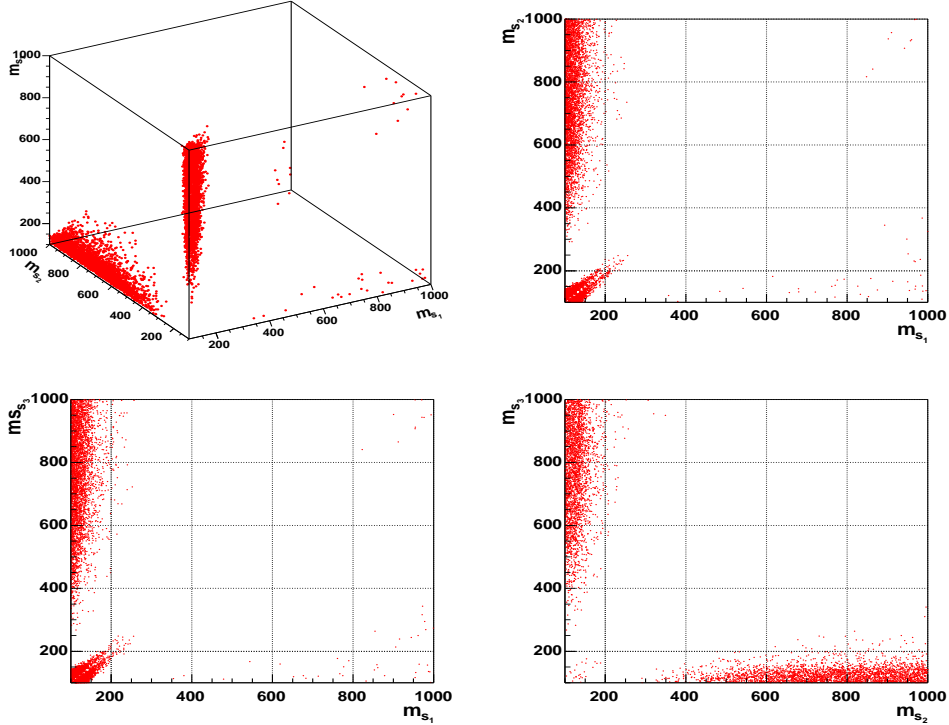
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Figure 4: The slepton and sneutrino masses for the normal hierarchy case, $\delta < 0$

The spectrum for the charged sleptons, which is taken to be the same as that of the sneutrinos, fall in two different classes. One group, which we will call the normal hierarchy, have two low mass sleptons and the third mass rather large (around 800 GeV). The second group, denoted the inverted hierarchy case, has two rather large masses in the neighborhood of 700 GeV and the lightest mass typically at 150 GeV. In either case, at least one slepton mass lies below ~ 200 GeV which is detectable, for example, at the LHC. Most points fall into the case of normal hierarchy, which as a matter of fact often corresponds to a normal hierarchy for the neutrinos as well ($\delta < 0$).

In Fig.4 we display the slepton masses, and in Fig.5 the mixing angles are shown for the normal hierarchy. It is clearly seen that the spectrum contains two large mixing angles and one small mixing angle, needed to suppress the decay $\mu \to e\gamma$. Also the degeneracy of two of the sleptons helps to minimize the LFV. As a rule of thumb there is at least one pair of sleptons with a mass splitting of less than 40 GeV. The rough spectrum needed is schematized in Fig.6. Although there is room for substantial deviations from
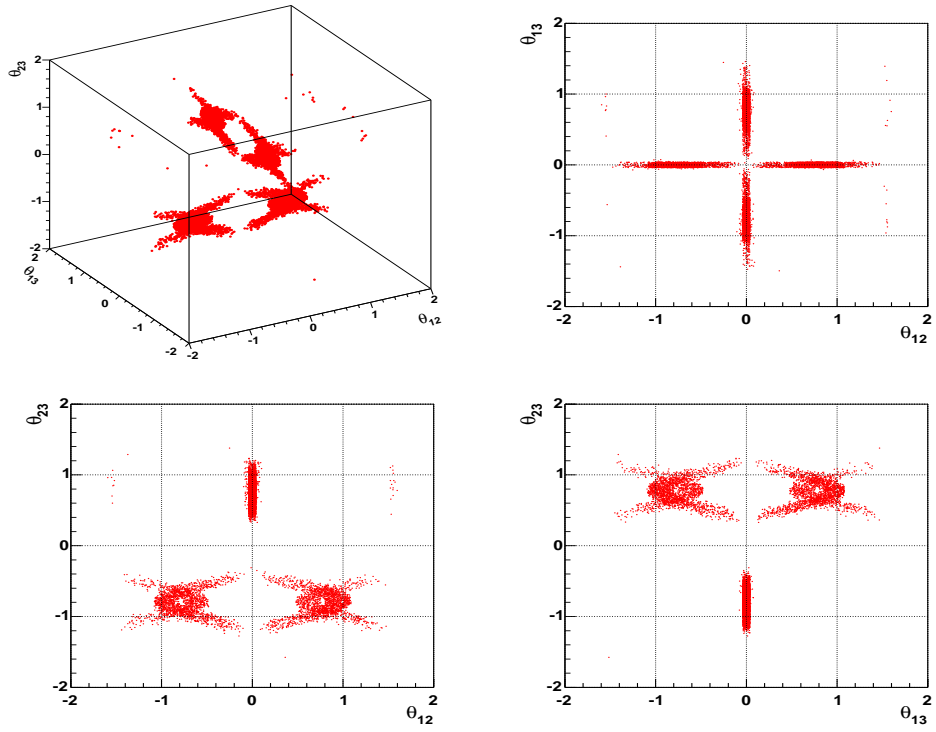
Figure 5: The slepton and sneutrino mixing angles for the normal hierarchy case, $\delta < 0$
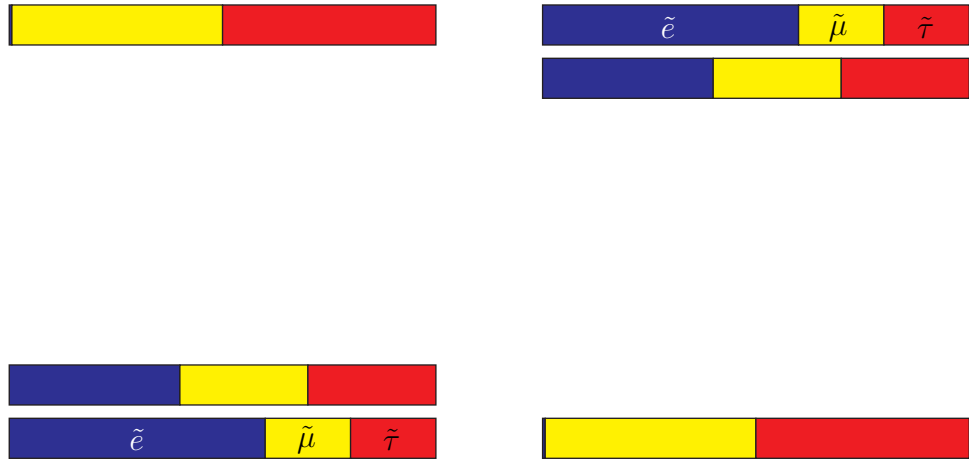

Figure 6: The rough form of the slepton and sneutrino spectrum in the case of normal hierarchy (left) or inverted hierarchy (right).

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the spectrum shown, the similarity with the neutrino spectrum is quite striking: the large mixings in the slepton sector are rather correlated to the large mixings in the neutrino sector. We also obtain a lower bound on the value of the μ parameter around 500 GeV, although in the case of inverted hierarchical slepton spetrum there are a few points with $\mu \sim 200$ GeV. Thus in most cases the second chargino, which is almost pure Higgsino is rather heavy.

As there is at least one low–mass slepton present in the model, one could suspect that a large contribution to the anomalous magnetic moment of the muon will result. We have explicitely calculated the magnitude, as in Ref. [46, 47]. The rough order of magnitude is 10×10^{-10}, which is too small to explain the BNL result [48]. As is well-known the contribution to $g - 2$ has the same sign as the μ-term, thereby disfavoring negative values for the μ parameter.

An important outcome of our study is the prediction for the LFV radiative charged lepton decays [4] $\ell_i \rightarrow \ell_j \gamma$. As seen in Fig. 7 a lower bound of 10^{-9} for $\mathrm{BR}(\tau \rightarrow \mu\gamma)$ is found. This is within reach of the future BaBar and Belle search [49]. The model also leads to sizeable rates for muon-electron conversion. We find that $\mathrm{BR}(\mu \rightarrow e\gamma)$ is constrained to be larger than about 10^{-15} and therefore stands good changes of being observed at future LFV searches, such as those taking place at PSI. As already noted, Fig. 7 indicates the existence of an upper bound on the value of $\tan(\beta)$ in this model. Taking the new constraint given in Ref. [33], this upper limit would be $\tan(\beta) \lesssim 7$.

In conclusion our analysis shows that, although we can obtain a realistic model in a fully consistent range of the SUSY parameter space, the model is rather strongly restricted and will be tested in the near future in a crucial way.

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[4]Note that, due to the presence of isosinglet charged leptons, this model implies the existence of tree-level LFV decays such as $\mu \rightarrow 3e$. However, due to the large value of M_E, close to the A_4 scale, their expected magnitude would be too small to be phenomenologically relevant.
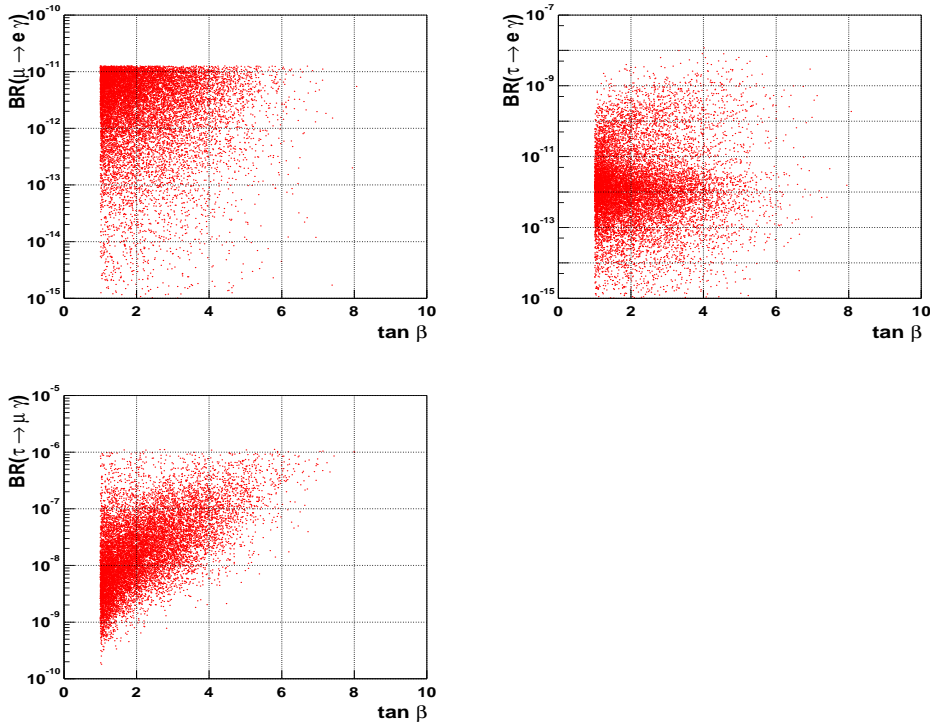
Figure 7: The branching ratios for the processes $\ell_i \to \ell_j \gamma$ as a function of $\tan(\beta)$.

4 Conclusion

The A_4 model that we have studied provides a complete picture of the flavour structure, especially it offers a common mechanism to obtain viable quark and lepton mixing matrices. The flavour dependence of the soft supersymmetry breaking terms acts as the source of the radiative corrections to the fermion masses. It splits the degeneracy of the neutrino masses as well as the alignment of the quark masses. We have shown that starting from a three-fold degeneracy at a high energy scale, it is possible to obtain a mass matrix in complete agreement with all current neutrino data. Within the model the lepton and slepton mixings are intimately related, with one slepton mass lying below 200 GeV. The flavour composition of this state ensures that it will be detectable at future collider experiments, such as the LHC.

The radiative corrections restrict the form of the neutrino mass matrix imply,

- Maximal atmospheric mixing.

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- Maximal leptonic CP violation (unless $U_{e3} = 0$).

Note that the maximality of leptonic CP violation is a feature of the leading order approximation and may acquire sizeable corrections.

The absolute Majorana neutrino mass scale for the quasi-degenerate neutrinos is shown to be larger than 0.3 eV and is bounded from above by cosmology as in Eq. (29), and therefore lies in the range of sensitivity of upcoming searches for neutrinoless double beta decay [50], and tritium beta decay [51].

We have also shown how the model is fully consistent with current data on lepton flavour violation. The predictions of lepton flavour violating charged lepton decays lie in a range accessible to future tests. We find, for example, that the $BR(\mu \to e\gamma)$ lies close to the current experimental limits, although parameters can easily be chosen so that the bound is obeyed. On the other hand we find a lower bound for the $\tau \to \mu\gamma$ decay branching ratio, $BR(\tau \to \mu\gamma) > 10^{-9}$.

Let us also mention the fact that the study we have performed is not only restricted to the specific A_4 model presented in section (2). Any model with the neutrino mass matrix given by λ_0 at some high energy scale and with supersymmetric flavour changing radiative corrections will have the same constraints as presented in this work.

5 Acknowledgements

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