

# Predictive flavour symmetries of the neutrino mass matrix

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Here we propose an  $A_4$  flavour symmetry model which implies a lower bound on the neutrinoless double beta decay rate, corresponding to an effective mass parameter  $M_{ee} \gtrsim 0.03$  eV, and a direct correlation between the expected magnitude of CP violation in neutrino oscillations and the value of  $\sin^2 \theta_{13}$ , as well as a nearly maximal CP phase  $\delta$ .

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Unless flavour symmetries are assumed, particle masses and mixings are generally undetermined in gauge theories. Understanding mass and mixing constitutes one of the biggest challenges in elementary particle physics. Current observations do not determine all elements of the effective neutrino mass matrix  $\mathcal{M}_\nu$  completely and this will be a great challenge even for future experiments. Therefore theoretical ideas restricting the structure of  $\mathcal{M}_\nu$  are needed in order to guide future searches. One such input studied extensively is the assumption that some entries in the neutrino mass matrix vanish [1]. While the phenomenological implications of the assumed zeros in the texture of  $\mathcal{M}_\nu$  are straightforward to derive [2], it is a non-trivial task to produce a good symmetry leading to such zeros and a diagonal charged lepton mass matrix simultaneously. Although for any desired texture structure of the mass matrices such a symmetry is in principle always present, this symmetry and the associated Higgs content are sometimes discouragingly complex [3].

Here we propose a predictive flavour symmetry for leptons based on a relatively small and simple flavour group, namely  $A_4$  or its  $Z_3$  subgroup, and briefly analyse its phenomenological implications. We show how this provides a simple means of understanding some of the two-zero textures of  $\mathcal{M}_\nu$  studied earlier [2].

The discrete group  $A_4$  is a 12 element group consisting of even permutations among four objects. The group is small enough to lead to a simple model but large enough to give interesting predictions. The distinguishing feature of  $A_4$  compared to other smaller discrete groups is

	$L_1$	$L_2$	$L_3$	$l_{Ri}$	$\nu_{Ri}$	$\Phi_i$	$\Delta$
$SU(2)$	2	2	2	1	1	2	3
$U(1)$	-1	-1	-1	-2	0	1	2
$A_4$	1	1'	1''	3	3	3	1' or 1''

TABLE I: Lepton multiplet structure of the model

the presence of a 3 dimensional irreducible representation appropriate to describe the three generations. This has been exploited in a number of variants. Originally, the  $A_4$  was proposed [4, 5] for understanding degenerate neutrino spectrum with nearly maximal atmospheric neutrino mixing angle. More recently, predictions for the solar neutrino mixing angle have also been incorporated in so-called tri-bi-maximal [6] neutrino mixing schemes [7, 8, 9, 10, 11, 12]. There also exist attempts at unified  $A_4$  models [13]. The resulting models however are not always simple and usually require many Higgs fields. Here we show that a very simple model based on  $A_4$  leads to two-zero textures for  $\mathcal{M}_\nu$ .

The  $L_i$  are assigned to the triplet representation in all the  $A_4$  models proposed so far. Here we propose the opposite assignment indicated in Table I, where the  $L_i$  are assigned to the 1, 1', 1'' representations. The  $l_{Ri}$  as well as the Higgs doublets responsible for lepton masses transform as  $A_4$  triplets, while the (undisplayed) quarks and the  $SU(2)$  Higgs doublet that gives their masses are all singlets under  $A_4$ . This leads to the following terms responsible for the lepton masses:

$$\begin{aligned}
 -\mathcal{L} = & h_1 \bar{L}_1 (l_R \Phi)_1 + h_2 \bar{L}_2 (l_R \Phi)'_1 + h_3 \bar{L}_3 (l_R \Phi)''_1 \\
 & + h_{1D} \bar{L}_1 (\nu_R \Phi)_1 + h_{2D} \bar{L}_2 (\nu_R \Phi)'_1 + h_{3D} \bar{L}_3 (\nu_R \Phi)''_1 \\
 & + \frac{M}{2} \nu_{Ri}^T C \nu_{Ri} + \text{H.c.} , \quad (1)
 \end{aligned}$$

where the quantities in parenthesis denote products of two  $A_4$ -triplets  $l_R$  (or  $\nu_R$ ) and  $\Phi$  forming the representations 1, 1', 1'' respectively. Note that Eq. (1) includes the

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most general terms allowed by the symmetry and field content in Table I. Hence, in contrast to many other  $A_4$  models, here one does not need to impose any additional symmetry to forbid unwanted terms.

Earlier studies on  $A_4$  have shown that it is possible to obtain a minimum of the Higgs potential with equal vacuum expectation values (vevs) [4]

$$\langle \Phi_1^0 \rangle = \langle \Phi_2^0 \rangle = \langle \Phi_3^0 \rangle \equiv \frac{v}{\sqrt{3}}. \quad (2)$$

This minimum leads to charged lepton and Dirac neutrino mass matrices  $M_l$  and  $m_D$  given by, respectively

$$M_l = v \text{diag}(h_1, h_2, h_3)U \\ m_D = v \text{diag}(h_{1D}, h_{2D}, h_{3D})U,$$

with

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \omega \equiv e^{\frac{2\pi i}{3}}. \quad (3)$$

The above  $M_l, m_D$  imply that the symmetry basis  $L_i$  also corresponds to the mass basis and only the right handed fields need to be redefined. As a result, the neutrino mass matrix following from eqs.(1, 2) after the seesaw diagonalization [14] is already in the flavour basis and is given by

$$\mathcal{M}_{\nu f}^I = m_D M_R^{-1} m_D^T = \frac{v^2}{M} \begin{pmatrix} h_{1D}^2 & 0 & 0 \\ 0 & 0 & h_{2D}h_{3D} \\ 0 & h_{2D}h_{3D} & 0 \end{pmatrix}. \quad (4)$$

This has the same zero textures as obtained in [5] except that only two (instead of three) neutrinos are degenerate. As noted in [5], this texture by itself is not complete and one needs to modify it. For example, one can supersymmetrize the above scenario and use radiative corrections to split the degeneracy and obtain predictions for the mixing angles and masses as in [5].

Here we choose a different approach, introducing a triplet field  $\Delta$  [15] transforming either as a  $1''$  or as a  $1'$  under  $A_4$ , as in Table I. In the first case a small induced vev  $\langle \Delta^0 \rangle \equiv u$  for its neutral component leads to a type-II neutrino mass matrix contribution given as

$$\mathcal{M}_{\nu}^{II} = \begin{pmatrix} 0 & \lambda u & 0 \\ \lambda u & 0 & 0 \\ 0 & 0 & \lambda' u \end{pmatrix}, \quad (5)$$

where  $\lambda, \lambda'$  are two Yukawa couplings (another hybrid model based on  $A_4$  and using both type-I and type-II contributions to neutrino masses has been considered in

[16]). The total neutrino mass matrix is given by the sum of eq.(4) and (5) and has the form

$$\mathcal{M}_{\nu} = \begin{pmatrix} a & x & 0 \\ x & 0 & b \\ 0 & b & y \end{pmatrix}, \quad (6)$$

where  $a, b$  and  $x, y$  refer to the type-I and type-II contributions, respectively. The above arguments provide a simple derivation of the two-zero texture classified as  $B_1$  in Ref. [1]).

Alternatively, had the triplet been assigned to the  $1'$  representation of  $A_4$  then we would have obtained

$$\mathcal{M}_{\nu} = \begin{pmatrix} a & 0 & x \\ 0 & y & b \\ x & b & 0 \end{pmatrix}, \quad (7)$$

a texture classified as  $B_2$  in [1]. It is possible to modify the assignment of various  $L_i$  fields among different singlet representations of  $A_4$ . This either results in one of the two above textures, or in a texture which is not viable phenomenologically. Thus the realization of the the  $A_4$  flavour symmetry proposed here leads to just two viable two-zero textures, which are quite predictive as we will show.

The vacuum structure given in eq.(2) breaks the  $A_4$  preserving a  $Z_3$  subgroup [9]. This  $Z_3$  in our case gets broken spontaneously by the triplet vacuum expectation value. Thus the type-I contribution in eq.(4) is  $Z_3$  invariant. Interestingly, the converse and more powerful statement is also true. One can argue the above two-zero texture to be a consequence of the (spontaneously broken)  $Z_3$  symmetry instead of the full  $A_4$  as we now show.

The  $Z_3$  group under consideration is generated by  $(1, z, z^2)$ ,  $z^3 = 1$  with the leptons transforming as

$$L_i \rightarrow Z_{ij}^L L_j, \\ (l_{Ri}, \nu_{Ri}) \rightarrow Z_{ij}^R (l_{Rj}, \nu_{Rj}), \quad (8)$$

where  $Z^L = \text{diag}(1, \omega, \omega^2)$  and

$$Z^R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (9)$$

Note that the fields which earlier transformed as triplets under  $A_4$  are now put into a reducible representation of the  $Z_3$  group. Let us now demand that  $M_l, m_D$  and  $M_R$  are invariant under the above defined  $Z_3$ . This implies

$$Z^{L\dagger} M_l Z^R = M_l; \quad Z^{L\dagger} m_D Z^R = m_D; \quad Z_R^T M_R Z^R = M_R. \quad (10)$$

It is straightforward to show that the above invariance implies that both  $M_I, m_D$  must have the form

$$\begin{pmatrix} X & X & X \\ A & \omega A & \omega^2 A \\ B & \omega^2 B & \omega B \end{pmatrix} \quad (11)$$

The above form coincides with that obtained in eq. (3) with proper identification of parameters. The right handed neutrino mass matrix now has the following general form [21]

$$\begin{pmatrix} M_1 & M_2 & M_2 \\ M_2 & M_1 & M_2 \\ M_2 & M_2 & M_1 \end{pmatrix} \quad (12)$$

In spite of this more complicated form, it is easy to see that the type-I contribution has exactly the same zero texture as in eq.(4) which is therefore more general than its derivation through the seesaw mechanism used here. It simply follows from the  $Z_3$  invariance of the effective neutrino mass matrix:

$$Z^{LT} \mathcal{M}_\nu Z^L = \mathcal{M}_\nu \quad (13)$$

irrespective of the underlying dynamics. For example, the same form would arise in a model without the right handed neutrinos but containing a  $Z_3$ -singlet Higgs triplet with a non-zero vev. As in the  $A_4$  case, one can introduce a triplet  $H$  transforming as  $\omega^2$  under  $z$ , and whose vev will now break  $Z_3$  to give the required two-zero texture as in eq. (5).

We now turn to the phenomenological implications. The main feature of two-zero texture models, such as the ones derived here, is their power in predicting the as yet undetermined neutrino parameters. Current neutrino oscillation experiments determine two mass splittings  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\text{sol}}^2$  and the corresponding mixing angles  $\theta_{12}$  and  $\theta_{23}$ , with some sensitivity on  $\theta_{13}$  which is bounded [17]. The Dirac CP phase will be probed in future oscillation experiments. Similarly, the absolute neutrino mass scale will be probed by future cosmological observations [18], tritium beta decays [19] and neutrinoless double beta decay experiments [20] with improved sensitivity. The latter will also shed light on the two Majorana CP phases which are hard to test otherwise, as they do not affect lepton number conserving processes. The general  $3 \times 3$  light neutrino mass matrix  $\mathcal{M}_\nu$  in the flavour basis contains *a priori* nine independent real parameters, once the three unphysical phases associated with the charged lepton fields are removed. In contrast, in the proposed model all the above nine parameters are given in terms of only five unknowns. Hence the number

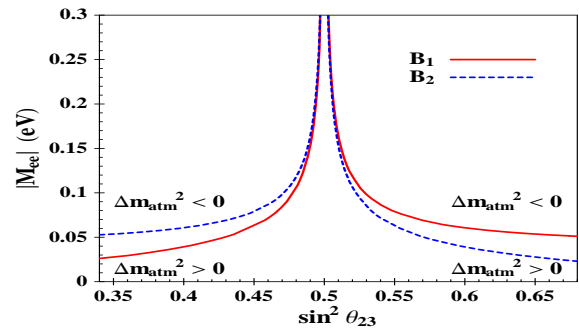


FIG. 1: Lower bound on neutrinoless double beta decay.

of physical parameters characterizing the charged current weak interaction is reduced with respect to what is expected in the general case [15].

We now illustrate these predictions. We first consider the mass parameter characterizing neutrinoless double beta decay  $|M_{ee}|$  which depends mainly on  $\theta_{23}$ , as illustrated in Fig. 1. A remarkable feature of our  $A_4$  flavour symmetry model is that it implies the lower bound  $|M_{ee}| \gtrsim 0.03$  eV, as seen in Fig. 1. This prediction correlates with the maximality of the atmospheric mixing angle and lies within the range of planned experiments. The bound hardly depends on other parameters. For example, in contrast to Ref. [10], it shows no strong dependence with the value of the relevant Majorana phase. This follows from the more stringent lower bound on the lightest neutrino mass obtained in the present model. We note that  $|M_{ee}|$  has, however, some dependence on the value of  $\Delta m_{\text{atm}}^2$  and the bound corresponds to  $\Delta m_{\text{atm}}^2 = 2 \times 10^{-3}$  eV<sup>2</sup>.

We now turn to the predictions for CP violation and the parameter  $\delta$ . As seen in Fig. 2, both for the  $B_1$  (left

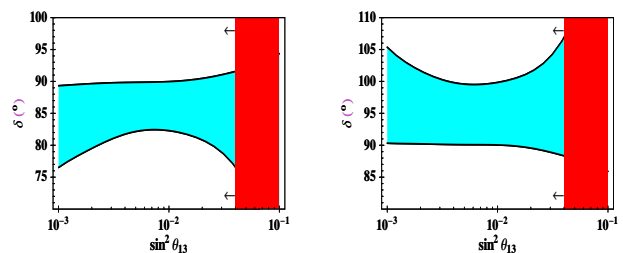


FIG. 2: Near-maximal CP violation in neutrino oscillations.

panel) and  $B_2$  cases (right panel) our model predicts the near maximality of the CP violation in neutrino oscillations. The predicted CP violating parameter  $\delta$  depends mainly on  $\theta_{13}$  which is currently only bounded by oscillation data [17].

The rephasing invariant magnitude  $|J|$  of CP violation in neutrino oscillations is defined as

$$J = \text{Im}[K_{11}K_{22}K_{12}^*K_{21}^*] = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta, \quad (14)$$

where  $K_{ij}$  are the elements of the leptonic mixing matrix. As seen in Fig. 3, which holds for both B1 and B2 models, one finds that  $|J|$  is directly correlated with the value of  $\sin^2 \theta_{13}$ , to be probed in the next generation of high sensitivity neutrino oscillation experiments such as Double Chooz. The width of the band reflects the current uncertainties in the neutrino oscillation parameters [17].

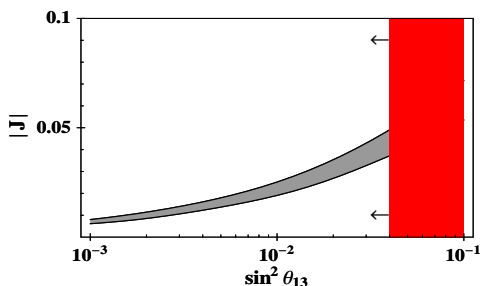


FIG. 3: CP violation in neutrino oscillations versus  $\sin^2 \theta_{13}$ .

In summary, here we have proposed an  $A_4$  flavour symmetry for leptons which leads to a near-maximal CP phase  $\delta$  and correlates the invariant measure of CP violation in neutrino oscillations with the magnitude of  $\sin^2 \theta_{13}$  to be probed in future neutrino oscillation experiments. Moreover, it implies a lower bound  $|M_{ee}| \gtrsim 0.03$  eV for the mass parameter characterizing neutrinoless double beta decay, also accessible to planned experiments. All these features already emerge from an effective  $Z_3$  invariance of the larger  $A_4$  symmetry. However, the structure of  $M_R$  is different in the  $A_4$  model and the effective  $Z_3$  model. Hence, for example, some phenomenological details related to leptogenesis could be different. These issues will be taken up elsewhere.

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