

# Minimal supergravity radiative effects on the tri-bimaximal neutrino mixing pattern

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## Abstract

We study the stability of the Harrison-Perkins-Scott (HPS) mixing pattern, assumed to hold at some high energy scale, against supersymmetric radiative corrections. We work in the framework of a reference minimal supergravity model (mSUGRA) where supersymmetry breaking is universal and flavor-blind at unification. The radiative corrections considered include both RGE running as well as threshold effects. We find that in this case the solar mixing angle can only increase with respect to the HPS reference value, while the atmospheric and reactor mixing angles remain essentially stable. Deviations from the solar angle HPS prediction towards lower values would signal novel contributions from physics beyond the simplest mSUGRA model.

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## I. INTRODUCTION

The discovery of neutrino oscillations [1, 2, 3, 4, 5] has indicated a very peculiar structure of lepton mixing [6], quite distinct from that of quarks. These data have triggered a rush of papers attempting to understand the values of the leptonic mixing angles from underlying symmetries at a fundamental level. An attractive possibility is that the observed pattern of lepton mixing results from some kind of flavour symmetry, such as  $A_4$ , valid at a some superhigh energy scale where the dimension-five neutrino mass operator arises [7].

Here we reconsider the Harrison-Perkins-Scott (HPS) mixing pattern [9] within a simple reference model approach. Our only assumption is that at the high energy scale the tree-level neutrino mass matrix  $m_\nu^{\text{tree}}$  is diagonalized by the so-called HPS matrix, taken as,

$$U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (1)$$

which corresponds to the following mixing angle values:

$$\begin{aligned} \tan^2 \theta_{\text{ATM}} &= \tan^2 \theta_{23}^0 = 1, \\ \sin^2 \theta_{\text{Chooz}} &= \sin^2 \theta_{13}^0 = 0, \\ \tan^2 \theta_{\text{SOL}} &= \tan^2 \theta_{12}^0 = 0.5. \end{aligned} \quad (2)$$

These predictions which hold at high energies may be regarded as a good first approximation to the observed values [6] indicated by oscillation experiments [1, 2, 3, 4, 5]. The diagonal neutrino mass matrix can be written as  $\hat{m}_\nu^{\text{tree}} = U_{\text{HPS}}^T \cdot m_\nu^{\text{tree}} \cdot U_{\text{HPS}} = \text{diag}(m_1, m_2, m_3)$ , so that the tree-level neutrino mass matrix becomes

$$m_\nu^{\text{tree}} = \begin{pmatrix} \frac{2}{3}m_1 + \frac{1}{3}m_2 & -\frac{1}{3}m_1 + \frac{1}{3}m_2 & -\frac{1}{3}m_1 + \frac{1}{3}m_2 \\ -\frac{1}{3}m_1 + \frac{1}{3}m_2 & \frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3 & \frac{1}{6}m_1 + \frac{1}{3}m_2 - \frac{1}{2}m_3 \\ -\frac{1}{3}m_1 + \frac{1}{3}m_2 & \frac{1}{6}m_1 + \frac{1}{3}m_2 - \frac{1}{2}m_3 & \frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3 \end{pmatrix}. \quad (3)$$

This form corresponds to a specific structure for the dimension-five lepton number violating operator. For example, it constitutes the most general ansatz that follows from a basic  $A_4$  symmetry for the neutrino mass matrix and the quark mixing matrix [7]. One of the central open questions in neutrino physics is to identify the exact mechanism of producing Fig. 1. As a first step, here we will adopt a model-independent approach of considering the implications of Eq. (3) assuming only the evolution expected in flavor-blind softly broken minimal supergravity at unification. This will provide us with a reference value that can be useful in the future for treating different models of neutrino mass [8].

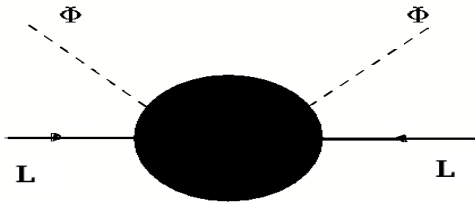


Figure 1: Dimension five operator responsible for neutrino mass.

## II. RADIATIVE CORRECTIONS

It has already been noted that radiative corrections present in the Standard Model renormalization group equations (RGEs), leave the HPS “reference” predictions essentially stable [10]. In addition to Minimal Supersymmetric Standard Model RGE evolution, here we consider also the effect of one-loop threshold effects [11]. We will first consider the evolution of the neutrino oscillation parameters that follow from Eq. (3), which covers both the cases of degenerate as well as hierarchical neutrino masses. The radiatively corrected neutrino mass matrix in this case becomes

$$m_\nu^{\text{1-loop}} = m_\nu^{\text{tree}} + \hat{\delta}^T \cdot m_\nu^{\text{tree}} + m_\nu^{\text{tree}} \cdot \hat{\delta}, \quad (4)$$

where

$$\hat{\delta} = \begin{pmatrix} \delta'_{ee} & \delta_{\mu e} & \delta_{\tau e} \\ \delta_{e\mu} & \delta'_{\mu\mu} & \delta_{\tau\mu} \\ \delta_{e\tau} & \delta_{\mu\tau} & \delta'_{\tau\tau} \end{pmatrix}. \quad (5)$$

The diagonal elements include the threshold correction and the RGE running

$$\delta'_{\alpha\alpha} = \delta_{\alpha\alpha} + \delta_\alpha, \quad (6)$$

where the RGE running effect is [12]

$$\delta_\alpha = \frac{-h_\alpha^2}{16\pi^2} \ln \left( \frac{M_{\text{GUT}}}{M_{\text{EWSB}}} \right). \quad (7)$$

In order to get the analytic expressions for the threshold corrections, we proceed as in Ref. [13]. However, now we do not neglect Yukawa couplings, taking into account the fact that right- and left-handed charged sleptons mix. Therefore, the analytic expressions for

the deltas are

$$\begin{aligned}
\delta_{\alpha\beta}^{(a)\chi^+} &= \sum_{i=1}^6 \sum_{A=1}^2 \frac{1}{16\pi^2} (gU_{A1}^* R_{i\alpha}^{\tilde{\ell}} - h_\alpha U_{A2}^* R_{i\alpha+3}^{\tilde{\ell}}) (gU_{A1} R_{i\beta}^{\tilde{\ell}*} - h_\beta U_{A2} R_{i\beta+3}^{\tilde{\ell}*}) \\
&\quad \times B_1(m_{\chi_A^+}^2, m_{\tilde{\ell}_i}^2), \\
\delta_{\alpha\beta}^{(a)\chi^0} &= \sum_{i=1}^3 \sum_{A=1}^4 \frac{1}{32\pi^2} |gN_{A2} - g'N_{A1}|^2 R_{i\alpha}^{\tilde{\nu}} R_{i\beta}^{\tilde{\nu}*} B_1(m_{\chi_A^0}^2, m_{\tilde{\nu}_i}^2), \\
\delta_{\alpha\beta}^{(c)\chi^+} &= \sum_{i=1}^6 \sum_{A=1}^2 \sum_{B=1}^2 \frac{1}{4\pi^2} (gU_{A1}^* R_{i\alpha}^{\tilde{\ell}} - h_\alpha U_{A2}^* R_{i\alpha+3}^{\tilde{\ell}}) gU_{A1} |V_{B2}|^2 R_{i\beta}^{\tilde{\ell}*} C_{00}(m_{\chi_A^+}^2, m_{\chi_B^+}^2, m_{\tilde{\ell}_i}^2), \\
\delta_{\alpha\beta}^{(c)\chi^0} &= \sum_{i=1}^3 \sum_{A=1}^4 \sum_{B=1}^4 \frac{1}{8\pi^2} |gN_{A2} - g'N_{A1}|^2 |N_{B4}|^2 R_{i\alpha}^{\tilde{\nu}} R_{i\beta}^{\tilde{\nu}*} C_{00}(m_{\chi_A^0}^2, m_{\chi_B^0}^2, m_{\tilde{\nu}_i}^2),
\end{aligned} \tag{8}$$

where we have evaluated the Feynman diagrams at zero external momentum, which is an excellent approximation as the neutrino masses are tiny. Here  $\delta_{\alpha\beta}^{(a,c)\chi^+}$ , ( $\alpha, \beta = e, \mu, \tau$ ), are the contributions from the chargino/charged slepton diagrams in Fig. 2 (a,c), respectively, while  $\delta_{\alpha\beta}^{(a,c)\chi^0}$  are the contributions from the neutralino/sneutrino diagrams. The values of the  $\delta_{\alpha\beta}$ 's, in Eqs. (5) and (6) are the sum of the four contributions given above. Analogous contributions exist corresponding to the symmetrized terms in Eq. (4), required by the Pauli principle, as displayed in Fig. 2 (b,d). In the above formulas,  $U$  and  $V$  are the chargino mixing matrices and  $m_{\chi_A^+}$ , ( $A = 1, 2$ ), are chargino masses, while  $N$  is the neutralino mixing matrix with  $m_{\chi_A^0}$ , ( $A = 1, \dots, 4$ ), denoting the neutralino masses. Finally, the matrices  $R^{\tilde{\ell}/\tilde{\nu}}$  denote the slepton/sneutrino mixing matrices, respectively. The coupling constant of the  $SU(2)$  gauge group is denoted  $g$  and that of  $U(1)$  is  $g'$ . Here  $h_\alpha$  is the charged lepton Yukawa coupling in the basis where the charged lepton masses are diagonal. Furthermore  $B_1$  and  $C_{00}$  are Passarino-Veltman functions given by

$$B_1(m_0^2, m_1^2) = -\frac{1}{2}\Delta_\epsilon + \frac{1}{2} \ln \left( \frac{m_0^2}{M_{\text{EWSB}}^2} \right) + \frac{-3 + 4t - t^2 - 4t \ln(t) + 2t^2 \ln(t)}{4(t-1)^2}, \tag{9}$$

where  $t = m_1^2/m_0^2$  and

$$C_{00}(m_0^2, m_1^2, m_2^2) = \frac{1}{8}(3 + 2\Delta_\epsilon) - \frac{1}{4} \ln \left( \frac{m_0^2}{M_{\text{EWSB}}^2} \right) + \frac{-2r_1^2(r_2-1) \ln(r_1) + 2r_2^2(r_1-1) \ln(r_2)}{8(r_1-1)(r_2-1)(r_1-r_2)}, \tag{10}$$

where  $r_1 = m_1^2/m_0^2$  and  $r_2 = m_2^2/m_0^2$ . We have used dimensional regularization, with  $\epsilon = 4-n$  and  $n$  is the number of space-time dimensions. The term  $\Delta_\epsilon = (2/\epsilon) - \gamma + 4 \ln(4\pi)$ , where  $\gamma$  is Euler's constant, is divergent as  $\epsilon \rightarrow 0$ .

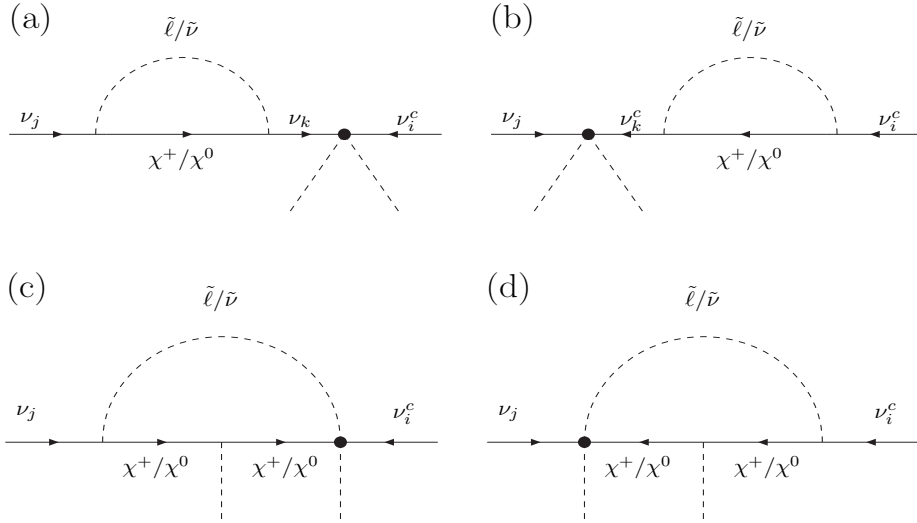


Figure 2: Feynman diagrams responsible for neutrino mass radiative corrections. The blob indicates an effective Lagrangian term obtained from integrating out the heavy right-handed neutrinos

### III. CORRECTIONS TO MIXING ANGLES: NUMERICAL RESULTS

We now describe our numerical procedure. In order to compute the magnitude of the radiative corrections expected in the HPS ansatz we work in the framework of a reference minimal supergravity model approach, with universal flavor-blind soft supersymmetry breaking terms at unification. Therefore the off-diagonal elements in the matrix in Eq. (5) are all zero <sup>1</sup>

$$\delta_{e\mu} = \delta_{e\tau} = \delta_{\mu\tau} = \delta_{\mu e} = \delta_{\tau e} = \delta_{\tau\mu} = 0 . \quad (11)$$

We first have used the SPheno package [14] to calculate spectra and mixing matrices within mSUGRA within the ranges:  $M_{1/2}, M_0, A_0 \in [100, 1000]$  GeV,  $A_0$  with both signs,  $\tan\beta \in [2.5, 50]$  and  $\mu$  with both signs. Then we have calculated the RGE running, Eq. (7), and the threshold corrections, Eqs. (8). We have explicitly checked that the dominant contribution to  $\delta'_{\alpha\alpha}$ , defined in Eq. (6), always comes from the threshold corrections for  $\alpha = e, \mu$ . Also for  $\alpha = \tau$ , threshold corrections are usually more important than RGE running contributions, typically

$$\delta_{\alpha\alpha} \sim \mathcal{O}(10^{(-4, -3)}), \quad \forall\alpha \quad (12)$$

<sup>1</sup> Nonzero off-diagonal elements may arise due to running, see discussion.

while

$$\delta_e \sim \mathcal{O}(10^{(-11,-9)}) \quad \delta_\mu \sim \mathcal{O}(10^{(-7,-4)}) \quad \delta_\tau \sim \mathcal{O}(10^{(-4,-2)}). \quad (13)$$

Note that only for very large values of  $\tan\beta$ , the RGE effect  $\delta_\tau$  is slightly larger than the threshold corrections  $\delta_{\tau\tau}$ . Using these radiative corrections we have computed the delta matrix in Eq. (5) and inserted it in the neutrino mass matrix at 1-loop given in Eq. (4). We have then numerically diagonalized the 1-loop neutrino mass matrix in Eq. (4) in order to obtain the neutrino masses and mixing angles.

Notice that the HPS scheme only fixes neutrino mixing angles. Thus, the neutrino masses are free parameters. Making use of this freedom, we have used an iterative procedure in order to choose the parameters  $m_1$ ,  $m_2$  and  $m_3$ , so that the numerically calculated 1-loop neutrino masses are such that the solar and atmospheric squared-mass splittings  $\Delta m_{\text{SOL}}^2$  and  $\Delta m_{\text{ATM}}^2$  reproduce the current best fit point value. In our numerical calculation we concentrate on normal hierarchy. We will comment on the case of inverse hierarchy at the end of the next section.

The numerically calculated atmospheric and reactor neutrino mixing angles at low energies do not deviate significantly from its HPS reference value at high energies. Indeed, the numerical results are:

$$\begin{aligned} \tan^2 \theta_{\text{ATM}} &\lesssim \tan^2 \theta_{23}^0 + \mathcal{O}(10^{-1}), \\ \sin^2 \theta_{\text{Chooz}} &\lesssim \sin^2 \theta_{13}^0 + \mathcal{O}(10^{-7}). \end{aligned} \quad (14)$$

However, the solar neutrino mixing angle can be significantly affected. In Fig. 3, we have plotted the maximum deviation of the solar angle from the HPS reference value for  $\tan\beta \in [2.5, 50]$ , as a function of  $m_{\nu_1}$ , for both extreme  $CP$  parity combinations for  $m_{\nu_1}$  and  $m_{\nu_2}$ : same sign (left panel) and opposite sign (right panel). All the other  $CP$  possibilities lie in between these two extreme cases. As can be seen, the solar mixing angle remains essentially stable in the case of opposite  $CP$  signs, while deviations are maximal in the case of same  $CP$  signs. In this case, the solar mixing angle always increases with respect to the HPS value, irrespective of mSUGRA parameters. Moreover we can get a rough upper bound on  $m_{\nu_1}$  of order

$$m_{\nu_1} \lesssim 0.2 \text{ eV} \quad (15)$$

for the mSUGRA parameter values:  $M_{1/2} = 100 \text{ GeV}$ ,  $M_0 = -A_0 = 10^3 \text{ GeV}$ ,  $\mu > 0$  and  $\tan\beta = 2.5$ . Note that the upper bound is sensitive to the values of  $\tan\beta$ . For higher values of  $\tan\beta$  the radiative corrections are larger, implying a more stringent bound on  $m_{\nu_1}$ , as indicated by the upper boundary of the red (dark) band of the left panel in Fig. 3. Here we have fixed solar and atmospheric mass squared splittings at their best-fit values from

Ref. [6]. However, we have explicitly checked that the effect of letting  $\Delta m_{\text{ATM}}^2$  and  $\Delta m_{\text{SOL}}^2$  vary within their current  $3\sigma$  allowed range is negligible, i. e. the bands obtained at the extreme values almost coincide with the ones in Fig. 3.

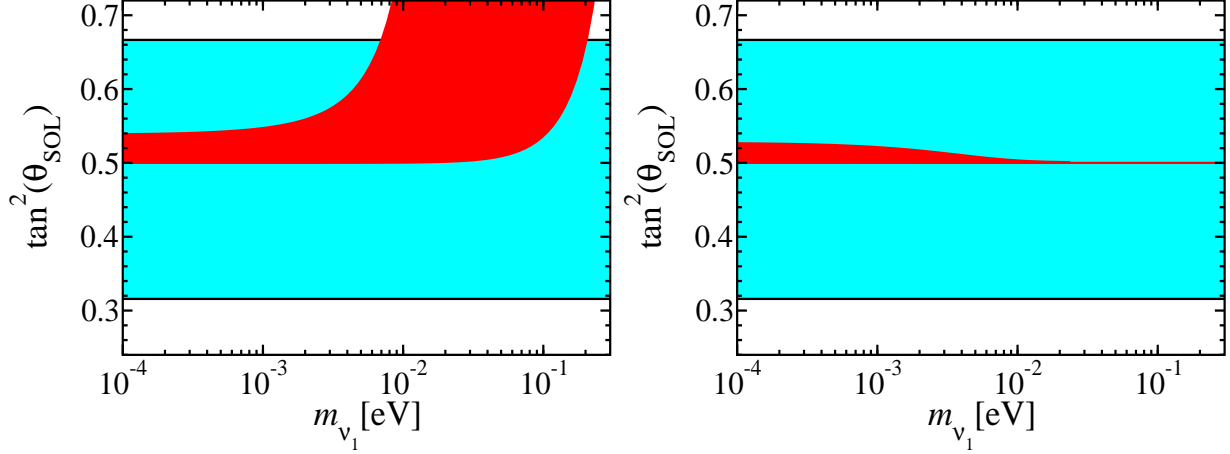


Figure 3: Upper bound for the solar mixing parameter  $\tan^2 \theta_{\text{SOL}}$ , as a function of  $m_{\nu_1}$  (in eV), for  $\tan \beta = 2.5$  (lower border of the red band) and  $\tan \beta = 50$  (upper border of the red band). On the left panel,  $m_{\nu_1}$  and  $m_{\nu_2}$  have the same  $CP$  sign. On the right panel,  $m_{\nu_1}$  and  $m_{\nu_2}$  have opposite  $CP$  sign. The neutrino mass splittings are assumed to have their best fit value from [6]. The horizontal band corresponds to the  $3\sigma$  allowed range for  $\tan^2 \theta_{\text{SOL}}$  [6].

#### IV. ANALYTICAL UNDERSTANDING

The numerical results presented above can be understood analytically as follows. If we perform the original HPS rotation to the 1-loop neutrino mass matrix in Eq. 4, we get:

$$\hat{m}_{\nu}^{1\text{-loop}} = U_{\text{HPS}}^{\text{T}} \cdot m_{\nu}^{1\text{-loop}} \cdot U_{\text{HPS}} \quad (16)$$

$$= \begin{pmatrix} (1 + \delta_{11})m_1 & \delta_{12}^{m_1}m_1 + \delta_{12}^{m_2}m_2 & \delta_{13}^{m_1}m_1 + \delta_{13}^{m_3}m_3 \\ \delta_{12}^{m_1}m_1 + \delta_{12}^{m_2}m_2 & (1 + \delta_{22})m_2 & \delta_{23}^{m_2}m_2 + \delta_{23}^{m_3}m_3 \\ \delta_{13}^{m_1}m_1 + \delta_{13}^{m_3}m_3 & \delta_{23}^{m_2}m_2 + \delta_{23}^{m_3}m_3 & (1 + \delta_{33})m_3 \end{pmatrix}, \quad (17)$$

where

$$\begin{aligned} \delta_{11} &= \frac{1}{3}(4\delta'_{ee} + \delta'_{\mu\mu} + \delta'_{\tau\tau} - 2\delta_{e\mu} - 2\delta_{\mu e} - 2\delta_{e\tau} - 2\delta_{\tau e} + \delta_{\mu\tau} + \delta_{\tau\mu}), \\ \delta_{22} &= \frac{2}{3}(\delta'_{ee} + \delta'_{\mu\mu} + \delta'_{\tau\tau} + \delta_{e\mu} + \delta_{\mu e} + \delta_{e\tau} + \delta_{\tau e} + \delta_{\mu\tau} + \delta_{\tau\mu}), \\ \delta_{33} &= \delta'_{\mu\mu} + \delta'_{\tau\tau} - \delta_{\mu\tau} - \delta_{\tau\mu}, \end{aligned}$$

$$\begin{aligned}
\delta_{12}^{m_1} &= \frac{1}{3\sqrt{2}}(2\delta'_{ee} - \delta'_{\mu\mu} - \delta'_{\tau\tau} - \delta_{e\mu} + 2\delta_{\mu e} - \delta_{e\tau} + 2\delta_{\tau e} - \delta_{\mu\tau} - \delta_{\tau\mu}) , \\
\delta_{12}^{m_2} &= \frac{1}{3\sqrt{2}}(2\delta'_{ee} - \delta'_{\mu\mu} - \delta'_{\tau\tau} + 2\delta_{e\mu} - \delta_{\mu e} + 2\delta_{e\tau} - \delta_{\tau e} - \delta_{\mu\tau} - \delta_{\tau\mu}) , \\
\delta_{13}^{m_1} &= \frac{1}{2\sqrt{3}}(\delta'_{\mu\mu} - \delta'_{\tau\tau} - 2\delta_{\mu e} + 2\delta_{\tau e} + \delta_{\mu\tau} - \delta_{\tau\mu}) , \\
\delta_{13}^{m_3} &= \frac{1}{2\sqrt{3}}(\delta'_{\mu\mu} - \delta'_{\tau\tau} - 2\delta_{e\mu} + 2\delta_{e\tau} - \delta_{\mu\tau} + \delta_{\tau\mu}) , \\
\delta_{23}^{m_2} &= \frac{1}{\sqrt{6}}(-\delta'_{\mu\mu} + \delta'_{\tau\tau} - \delta_{\mu e} + \delta_{\tau e} - \delta_{\mu\tau} + \delta_{\tau\mu}) , \\
\delta_{23}^{m_3} &= \frac{1}{\sqrt{6}}(-\delta'_{\mu\mu} + \delta'_{\tau\tau} - \delta_{e\mu} + \delta_{e\tau} + \delta_{\mu\tau} - \delta_{\tau\mu}) .
\end{aligned} \tag{18}$$

The matrix in Eq. (17) should be nearly diagonal and its off-diagonal elements determine the deviations from tri-bimaximality. We define the following parameters characterizing the deviations from tri-bimaximality:

$$\epsilon_{ij} \simeq \frac{1}{2} \tan(2\epsilon_{ij}) = \frac{(\hat{m}_\nu^{1\text{-loop}})_{ij}}{(\hat{m}_\nu^{1\text{-loop}})_{jj} - (\hat{m}_\nu^{1\text{-loop}})_{ii}} , \tag{19}$$

so that

$$\begin{aligned}
\theta_{\text{ATM}} &\equiv \theta_{23} \simeq \theta_{23}^0 + \epsilon_{23} , \\
\theta_{\text{Chooz}} &\equiv \theta_{13} \simeq \theta_{13}^0 + \epsilon_{13} , \\
\theta_{\text{SOL}} &\equiv \theta_{12} \simeq \theta_{12}^0 + \epsilon_{12} .
\end{aligned} \tag{20}$$

Substituting the matrix elements in Eq. (17) into Eq. (19), we get:

$$\epsilon_{23} = \frac{\delta_{23}^{m_2} m_2 + \delta_{23}^{m_3} m_3}{(-1 - \delta_{22})m_2 + (1 + \delta_{33})m_3} , \tag{21}$$

$$\epsilon_{13} = \frac{\delta_{13}^{m_1} m_1 + \delta_{13}^{m_3} m_3}{(-1 - \delta_{11})m_1 + (1 + \delta_{33})m_3} , \tag{22}$$

$$\epsilon_{12} = \frac{\delta_{12}^{m_1} m_1 + \delta_{12}^{m_2} m_2}{(-1 - \delta_{11})m_1 + (1 + \delta_{22})m_2} . \tag{23}$$



Taking into account that for mSUGRA the off-diagonal elements in the matrix in Eq. (5) are all zero, see Eq.(11), the  $\delta$ 's in Eq. (18) become

$$\begin{aligned}
\delta_{11} &= \delta_{11}^0 = \frac{1}{3}(4\delta'_{ee} + \delta'_{\mu\mu} + \delta'_{\tau\tau}) , \\
\delta_{22} &= \delta_{22}^0 = \frac{2}{3}(\delta'_{ee} + \delta'_{\mu\mu} + \delta'_{\tau\tau}) , \\
\delta_{33} &= \delta_{33}^0 = \delta'_{\mu\mu} + \delta'_{\tau\tau} , \\
\delta_{12}^{m1} &= \delta_{12}^{m2} = \delta_{12}^0 = \frac{1}{3\sqrt{2}}(2\delta'_{ee} - \delta'_{\mu\mu} - \delta'_{\tau\tau}) , \\
\delta_{13}^{m1} &= \delta_{13}^{m3} = \delta_{13}^0 = \frac{1}{2\sqrt{3}}(\delta'_{\mu\mu} - \delta'_{\tau\tau}) , \\
\delta_{23}^{m2} &= \delta_{23}^{m3} = \delta_{23}^0 = \frac{-1}{\sqrt{6}}(\delta'_{\mu\mu} - \delta'_{\tau\tau}) .
\end{aligned} \tag{24}$$

The deviations of the neutrino mixing angles from the HPS value given in Eqs. (21-23) then become

$$\epsilon_{23} = \frac{\delta_{23}^0(m_2 + m_3)}{(-1 - \delta_{22}^0)m_2 + (1 + \delta_{33}^0)m_3} , \tag{25}$$

$$\epsilon_{13} = \frac{\delta_{13}^0(m_1 + m_3)}{(-1 - \delta_{11}^0)m_1 + (1 + \delta_{33}^0)m_3} , \tag{26}$$

$$\epsilon_{12} = \frac{\delta_{12}^0(m_1 + m_2)}{(-1 - \delta_{11}^0)m_1 + (1 + \delta_{22}^0)m_2} . \tag{27}$$

If  $\epsilon_{12}$ , given in Eq. (27), is always positive,  $\theta_{\text{sol}}$  can only increase, see Eq. (20). The denominator in Eq. (27) can be approximated to

$$(-1 - \delta_{11}^0)m_1 + (1 + \delta_{22}^0)m_2 \simeq -m_1 + m_2 > 0 \tag{28}$$

and hence, by assumption, is always positive. The sign of  $\epsilon_{12}$  will be the sign of  $\delta_{12}^0$  given by Eq. (24). Considering the expressions for the deltas given in Eq. (8) and bearing in mind that the Passarino-Veltmann functions depend rather smoothly on their arguments, we can take them out of the sum. The following very rough estimations of the threshold corrections result

$$\begin{aligned}
\delta_{\alpha\alpha} &\simeq \frac{1}{32\pi^2}(3g^2(B_1 + 4C_{00}) + g'^2(B_1 + 4C_{00})) , \quad (\alpha = e, \mu) , \\
\delta_{\tau\tau} &\simeq \frac{1}{32\pi^2}(3g^2(B_1 + 4C_{00}) + g'^2(B_1 + 4C_{00}) + 2h_\tau^2 B_1) ,
\end{aligned} \tag{29}$$

where we have neglected the charged lepton Yukawa couplings for  $\alpha = e, \mu$ . Using

$$\lim_{m_{L_i}^2 \rightarrow \infty} \frac{B_1(m_{\chi_A}^2, m_{L_i}^2)}{C_{00}(m_{\chi_A}^2, m_{\chi_B}^2, m_{L_i}^2)} = -2 , \tag{30}$$

Eq. (29) becomes

$$\begin{aligned}\delta_{\alpha\alpha} &\simeq \frac{-B_1}{32\pi^2}(3g^2 + g'^2), \quad (\alpha = e, \mu), \\ \delta_{\tau\tau} &\simeq \frac{-B_1}{32\pi^2}(3g^2 + g'^2 - 2h_\tau^2).\end{aligned}\tag{31}$$

Therefore, the contribution of the threshold corrections to  $\delta_{12}^0$  is roughly

$$2\delta_{ee} - \delta_{\mu\mu} - \delta_{\tau\tau} \simeq \frac{-B_1}{16\pi^2}h_\tau^2.\tag{32}$$

Besides the threshold correction contributions, one has also to consider the RGE running contribution. Here the dominant part obviously is  $\delta_\tau$ , given in Eq. (7). The approximated expression for  $\delta_{12}^0$ , defined in Eq. (27), is then

$$\delta_{12}^0 \simeq \frac{1}{3\sqrt{2}}(2\delta_{ee} - \delta_{\mu\mu} - \delta_{\tau\tau} - \delta_\tau) \simeq \frac{1}{3\sqrt{2}}\frac{h_\tau^2}{16\pi^2}\left[-B_1 + \ln\left(\frac{M_{\text{GUT}}}{M_{\text{EWSB}}}\right)\right].\tag{33}$$

Considering that in the limit where the slepton mass goes to infinity, the Passarino-Veltman function  $B_1$  behaves as

$$\lim_{m_{\tilde{L}_i}^2 \rightarrow \infty} B_1(m_{\chi_A}^2, m_{\tilde{L}_i}^2) \simeq \frac{1}{2}\ln\left(\frac{m_{\tilde{L}_i}^2}{m_{\chi_A}^2}\right),\tag{34}$$

one obtains, from Eq. (33),

$$\delta_{12}^0 \simeq \frac{1}{3\sqrt{2}}(2\delta_{ee} - \delta_{\mu\mu} - \delta_{\tau\tau} - \delta_\tau) \simeq \frac{1}{3\sqrt{2}}\frac{h_\tau^2}{16\pi^2}\left[\ln\left(\frac{M_{\text{GUT}}}{M_{\text{EWSB}}}\right) - \ln\left(\frac{m_{\tilde{L}_i}}{m_{\chi_A}}\right)\right],\tag{35}$$

which is always positive, thus explaining why  $\epsilon_{12} > 0$ . Note that although the threshold corrections are in general larger than the RGE contributions, in  $\delta_{12}^0$  there is a cancellation among the threshold corrections so that the  $\delta_\tau$  RGE contribution becomes the relevant term. We have numerically checked that

$$2\delta_{ee} - \delta_{\mu\mu} - \delta_{\tau\tau} \sim \mathcal{O}(10^{(-6, -3)}).\tag{36}$$

This cancellation among the threshold corrections is the reason why the solar neutrino mixing angle can only increase with respect its HPS reference value.

We now turn to the other two neutrino mixing angles. In the mSUGRA framework the deviations from the HPS predictions are much smaller than found for the solar mixing parameter, and fit within their current experimental  $3\sigma$  allowed range given in Ref. [6] for acceptable  $m_{\nu_1}$  values. The reason for this can be understood from Eqs. (25-27). On the one hand, the deltas on the numerators, given by Eq. (24), are all of the same order. For small values of  $m_{\nu_1}$  the deviations are all negligible, since they are all proportional to the

previous deltas. For large  $m_{\nu_1}$  values the neutrino masses are nearly degenerate so that the numerators in Eqs. (25-27) are all of the same order. The denominators in Eqs. (25-27) can be approximated as

$$(-1 - \delta_{22}^0)m_2 + (1 + \delta_{33}^0)m_3 \simeq m_3 - m_2 , \quad (37)$$

$$(-1 - \delta_{11}^0)m_1 + (1 + \delta_{33}^0)m_3 \simeq m_3 - m_1 , \quad (38)$$

$$(-1 - \delta_{11}^0)m_1 + (1 + \delta_{22}^0)m_2 \simeq m_2 - m_1 . \quad (39)$$

Although these mass differences are very small,  $m_3 - m_2$  and  $m_3 - m_1$  are larger than  $m_2 - m_1$ , thus making  $\epsilon_{23}$  and  $\epsilon_{13}$  smaller than  $\epsilon_{12}$ .

We now comment briefly on inverse hierarchy. As can be seen from Eqs. (37-39), for inverse hierarchy,  $m_2 - m_1$  is still much smaller than  $m_3 - m_2$  or  $m_3 - m_1$ , while the latter two just change sign but not the magnitude. We therefore expect that the above discussion remains essentially correct also for inverse hierarchy.

We should stress that we have considered so far the  $CP$  conserving case HPS ansatz, with same- $CP$ -sign neutrino mass eigenvalues,

$$m_1, m_2, m_3 > 0 . \quad (40)$$

However, for all other CP combinations the denominators in Eqs. (25-27) are larger such that the deviations from HPS mixing pattern become smaller and correspondingly relax the bound in Eq. (15). In particular for the case of opposite CP signs there is no bound, as seen in right panel in Fig. 3.

## V. SUMMARY AND DISCUSSION

We have studied the stability of the HPS mixing ansatz that could arise from a flavor symmetry valid at some high energy scale, against supersymmetric radiative corrections (RGE running and threshold effects). We have adopted a model-independent minimal supergravity framework where supersymmetry breaking is universal and flavor-blind at unification. In this case we have found that the solar mixing angle can only be increased with respect to the HPS reference value. Under the assumption that all neutrino masses have the same  $CP$ -sign, this sets a rough upper bound on the mass of the lightest neutrino which, in turn, implies that the neutrinoless double beta decay rate is also bounded as a function of the mSUGRA parameters. In contrast, in the case of opposite CP signs there is no bound on the lightest neutrino mass. We have also shown that the atmospheric and reactor mixing angles remain essentially stable in all cases. It should not be surprising that the effect of radiative corrections is more important for the solar angle than for the others. It simply reflects the fact that the solar is the smallest of the two neutrino mass splittings.

We stress that in our approach we have assumed only that the matrix  $m_\nu^{\text{tree}}$  is diagonalized by the HPS matrix at the unification scale and this gets modified only by minimal supergravity radiative corrections, universal and flavor-blind at unification. This concerns the structure of the dimension-five operator, Fig. 1. Additional radiative corrections [12] to the solar angle HPS prediction are expected, if the neutrino mass arises *a la* seesaw [15, 16, 17, 18]. Their magnitude will be determined by the strength of the Yukawa coupling characterizing the Dirac neutrino mass entry in the seesaw mass matrix [19]. This will depend strongly on the details of the model, in particular, on whether Higgs triplets are present in the seesaw [17] or on whether the seesaw is extended [20]. Scrutinizing the schemes for which it is possible to decrease the solar mixing angle value predicted by the HPS mixing pattern towards its currently preferred best fit point value will be considered elsewhere [21], together with the related issue of the lepton flavor violating processes that would be expected in these schemes.

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