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On the full Boltzmann equations for Leptogenesis

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Abstract: We consider the full Boltzmann equations for standard and soft leptogenesis, instead of the usual integrated Boltzmann equations which assume kinetic equilibrium for all species. Decays and inverse decays may be inefficient for thermalising the heavy-(s)neutrino distribution function, leading to significant deviations from kinetic equilibrium. We analyse the impact of using the full kinetic equations in the case of a previously generated lepton asymmetry, and find that the washout of this initial asymmetry due to the interactions of the right-handed neutrino is larger than when calculated via the integrated equations. We also solve the full Boltzmann equations for soft leptogenesis, where the lepton asymmetry induced by the soft SUSY-breaking terms in sneutrino decays is a purely thermal effect, since at $T = 0$ the asymmetry in leptons cancels the one in sleptons. In this case, we obtain that in the weak washout regime ($K \lesssim 1$) the final lepton asymmetry can change up to a factor four with respect to previous estimates.

Keywords: Neutrino Physics, Beyond Standard Model, Leptogenesis.

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1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1]. It is usually assumed that the tiny neutrino masses are generated via the (type I) seesaw mechanism [2] and thus the new singlet neutral leptons with heavy (lepton number violating) Majorana masses can produce dynamically a lepton asymmetry through out of equilibrium decays. Eventually, this lepton asymmetry is partially converted into a baryon asymmetry due to fast $B-L$ violating sphaleron processes.

Most studies of leptogenesis use the integrated Boltzmann equations to follow the evolution of the heavy particle number density and the lepton asymmetry. This approach assumes Maxwell–Boltzmann statistics, as well as kinetic equilibrium for all particles, including the heavy species. This assumption is normally justified in freeze-out calculations, where elastic scattering is assumed to be much faster than inelastic reactions. However, in the present context, kinetic equilibrium for the heavy species would have to be maintained basically by the decays and inverse decays alone, and it is not obvious that the integrated Boltzmann equation is always a good approximation. In general, $1 \leftrightarrow 2$ processes are more inefficient for thermalization compared to $2 \leftrightarrow 2$ processes, and in some parameter ranges there can be large deviations from kinetic equilibrium.

In [3], the impact of this difference on the lepton asymmetry produced during leptogenesis was studied, and it was found that in the strong washout regime the final asymmetry is changed by $15-30\%$ when the full Boltzmann equations are used. In this work we extend the study to two different scenarios not considered previously, in which the effects can be sizeable:

(i) Preexisting lepton asymmetry. A lepton asymmetry that has been previously generated, for instance by the next-to-lightest right handed neutrino, N_2, tends to be washed out by the interactions of the lightest one, N_1. If $M_{N_1} \ll 10^9$ GeV, the lepton asymmetry generated in its decay can be neglected and given that the N_1 are thermally produced at temperatures close to M_1, this washout is exponential and therefore a change of order 20% may be important. We have used the full Boltzmann equations to calculate the evolution of the lepton asymmetry, created during N_2 decay, at lower temperatures, $T \sim M_1 \ll M_2$. We have also generalize them to the flavoured leptogenesis case.

(ii) Soft leptogenesis. Since right-handed neutrino masses and therefore leptogenesis are usually associated to a very high energy scale, a supersymmetric scenario is desirable in order to stabilize the hierarchy between the leptogenesis scale and the electroweak one. It has been shown [4–6] that supersymmetry-breaking terms can play an important role in the generation of a lepton asymmetry in sneutrino decays: they remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and also provide new sources of lepton number and CP violation. As a consequence, the mixing between the two sneutrino states generates a CP asymmetry in the decay, which can be sizable because of the resonant effect [7] of the two nearly-degenerate states. This scenario has been termed "soft leptogenesis", since the soft terms and not flavour physics provide the necessary mass splitting and CP-violating phase. It has also been studied in the minimal supersymmetric triplet seesaw model [8, 9] and in the inverse seesaw scenario [10]. An important difference with respect to the standard leptogenesis mechanism, is that the lepton asymmetry produced through soft leptogenesis is a pure thermal effect, because at $T = 0$ the asymmetry in leptons cancels the one in sleptons. Only at finite temperature the difference between the fermion and boson statistics leads to a non-vanishing lepton and CP asymmetry, so in this case the use of the full Boltzmann equations is mandatory. Several approximations have been used in the literature [6, 9], and we will compare our exact results with them.

This paper is organized as follows. In section 2, we derive the full Boltzmann equations for standard leptogenesis and we study the washout of a previously generated lepton asymmetry. Section 3 is devoted to soft leptogenesis. In section 4, we present our conclusions, and more technical details concerning the full Boltzmann equations are described in the appendices.

2. Standard leptogenesis

In this section, we review the full Boltzmann equations relevant for leptogenesis, in a simplified model which includes only the heavy right-handed neutrino (N_i) decays, inverse decays, and resonant scattering. The off-shell $2 \leftrightarrow 2$ scattering processes mediated by N_i have only small effects for $T < 10^{12}$ GeV and can be neglected in first approximation [11,12]. We then investigate how the use of the full evolution equations affects the final lepton number asymmetry. We do not include thermal corrections [13] and we consider only initial zero abundance of the heavy neutrinos.

The CP asymmetry in the decay of the right-handed neutrino N_i is:

$$\epsilon_i = \frac{|A(N_i \to LH)|^2 - \left|A(N_i \to \bar{L}\bar{H})\right|^2}{|A(N_i \to LH)|^2 + \left|A(N_i \to \bar{L}\bar{H})\right|^2} = \frac{|A(N_i \to LH)|^2 - \left|A(N_i \to \bar{L}\bar{H})\right|^2}{\left|A_D^i\right|^2} , \quad (2.1)$$

where we implicitly sum over all flavours, since, at this point, we work in the single flavour approximation. Its decay width is

$$\Gamma_i = \frac{\left|A_D^i\right|^2}{16\pi M_i} = \frac{M_i}{8\pi} \sum_\alpha |Y_{\alpha i}|^2 , \quad (2.2)$$

where $Y_{\alpha i}$ are the Yukawa couplings of the heavy neutrinos.

We will denote f_a the phase-space density of a particle species a, so its number density is given by

$$n_a = g_a \int \frac{d^3p}{(2\pi)^3} f_a(\bar{p}) , \quad (2.3)$$

where g_a is the number of internal degrees of freedom. In order to eliminate the dependence in the expansion of the Universe, as usual, we write the equations in terms of the abundances, $Y_a = n_a/s$, being s the entropy density,

$$s = \frac{2\pi^2}{45} g_* T^3 \quad (2.4)$$

with g_* the number of relativistic degrees of freedom at temperature T, so that $g_* = 106.75$ in the Standard Model (SM) and $g_* = 228.75$ in the Minimal Supersymmetric Standard Model (MSSM), which we consider in Sec. 3.

We will study the time evolution of the right-handed neutrino distribution f_{N_i}, and the lepton asymmetry distribution $f_{\mathcal{L}} = f_L - f_{\bar{L}}$. Due to the fast gauge interactions, to a good approximation the Higgs field and the leptons are in kinetic equilibrium. Moreover, the Higgs number asymmetry is not conserved due to the large top Yukawa coupling, so we neglect the Higgs chemical potential*. Then, we consider the following distributions:

$$f_H^{eq} = (e^{E_H/T} - 1)^{-1} , \quad (2.5)$$

$$f_L = (e^{(E_L-\mu)/T} + 1)^{-1} , \qquad f_{\bar{L}} = (e^{(E_{\bar{L}}+\mu)/T} + 1)^{-1} , \quad (2.6)$$

where we have introduced a chemical potential for the leptons, μ. Following [3], we use that $\mu/T \ll 1$ and we approximate:

$$f_{\mathcal{L}} = \frac{2\, e^{E_L/T}}{(e^{E_L/T} + 1)^2} \frac{\mu}{T} + \mathcal{O}((\frac{\mu}{T})^3) , \quad (2.7)$$

$$f_L + f_{\bar{L}} \simeq 2 f_L^{eq} + \mathcal{O}((\mu/T)^2) . \quad (2.8)$$

*The effect of keeping the Higgs number asymmetry has been studied in [14], and could lead to a reduction of the final baryon asymmetry of $\mathcal{O}(1)$.

where ,

$$f_L^{eq} = (e^{E_L/T} + 1)^{-1} \,. \tag{2.9}$$

Then, the lepton asymmetry $Y_{\mathcal{L}} = n_{\mathcal{L}}/s$ is given by

$$Y_{\mathcal{L}} = \mu \frac{T^2}{3s} + \mathcal{O}((\frac{\mu}{T})^3) \,. \tag{2.10}$$

Note that in this section we have neglected the thermal masses $m_H(T)$, $m_L(T)$, therefore $E_{H,L} = |\bar{p}_{H,L}| \equiv p_{H,L}$. A thorough study of thermal leptogenesis can be found in [13], where it has been shown that when thermal masses are taken into account, at sufficiently high temperature, the Higgs becomes heavier than N_i and the decay $N_i \to HL$ is kinematically forbidden. At higher temperatures, the Higgs becomes heavy enough to allow the decay $H \to N_i L$, where also a CP asymmetry ϵ_H is produced. However this asymmetry turns out to have a negligible effect on the final results for leptogenesis, so we do not consider it here.

Using the formalism of appendix A, we can write the full Boltzmann equations for the heavy neutrino and the lepton asymmetry. We assume that the later is small, so we work to first order in ϵ and $Y_{\mathcal{L}}$. It is convenient to use the dimensionless variables $z_i = M_i/T$, $\bar{E}_i = E_{N_i}/T$ and $y_a = p_a/T$ $(a = N_i, H, L)$ to eliminate the dependence on the expansion rate of the Universe, so we obtain †:

$$\frac{\partial f_{N_i}}{\partial z_i} = \frac{K_i z_i^2}{y_i \bar{E}_i} \int_{\frac{\bar{E}_i - y_i}{2}}^{\frac{\bar{E}_i + y_i}{2}} dy_H \left[f_H^{eq} f_L^{eq} (1 - f_{N_i}) - f_{N_i} (1 - f_L^{eq})(1 + f_H^{eq}) \right] \,, \tag{2.11}$$

$$\begin{aligned} \frac{\partial f_{\mathcal{L}}}{\partial z_i} = \frac{K_i z_i^2}{y_L^2} \int_{y_L + \frac{z_i^2}{4y_L}}^{\infty} d\bar{E}_i \Big\{ &\epsilon_i \, (f_{N_i} - f_{N_i}^{eq}) \left[(1 - f_L^{eq})(1 + f_H^{eq}) - f_H^{eq} f_L^{eq} \right] - \\ &- \frac{1}{2} f_{\mathcal{L}} (f_H^{eq} + f_{N_i}) \Big\} \,, \end{aligned} \tag{2.12}$$

where we have defined the decay parameter $K_i \equiv \Gamma_i / H(T = M_i)$, which controls whether or not N_i decays are in equilibrium.

Equations (2.11) and (2.12) can be integrated numerically for given values of K_i and ϵ_i. Note that within our first order calculation, in the evolution equation for the heavy neutrino distribution Eq. (2.11) only the *equilibrium* Higgs and lepton distributions appear, so it can be solved independently of the lepton asymmetry. Using the approximate expression Eq. (2.7) for $f_{\mathcal{L}}$, the integration of Eq. (2.12) over the dimensionless lepton momentum, y_L, leads to the following evolution equation for the chemical potential,

$$\begin{aligned} \frac{1}{T}\frac{d\mu}{dz_i} = \frac{3K_i z_i^2}{\pi^2} \int_{z_i}^{\infty} d\bar{E}_i \int_{\frac{\bar{E}_i - y_i}{2}}^{\frac{\bar{E}_i + y_i}{2}} dy_L \Big\{ &- \frac{\mu}{T} \frac{e^{y_L}}{(1 + e^{y_L})^2} (f_H^{eq} + f_{N_i}) + \\ &+ \epsilon_i \, (f_{N_i} - f_{N_i}^{eq}) \left[(1 - f_L^{eq})(1 + f_H^{eq}) - f_H^{eq} f_L^{eq} \right] \Big\} \,, \end{aligned} \tag{2.13}$$

†Our equation for the evolution of $f_{\mathcal{L}}$ is slightly different from the one in [3], due to the fact that they add the term $2\epsilon_i (1 - f_{N_i}) f_H^{eq} 2 f_L^{eq}$ (in our notation) because of the resonant part of the $LH \leftrightarrow \bar{L}\bar{H}$ scattering. However, this resonant contribution involves $f_{N_i}^{eq}$ instead of f_{N_i}, as we show in the appendix, so we have an extra term $4\epsilon_i f_H^{eq} f_L^{eq} (f_{N_i} - f_{N_i}^{eq})$. We have checked that this difference is numerically very small and does not change the main results.

Recall that μ is related to the lepton asymmetry $Y_{\mathcal{L}}$ by Eq. (2.10).

It is straightforward to verify that if Maxwell–Boltzmann statistics and kinetic equilibrium for all species are assumed, so that the phase space distributions are

$$f_{N_i} = \frac{Y_{N_i}}{Y_{N_i}^{eq}} e^{-\bar{E}_i} , \qquad f_H^{eq} = e^{-\bar{E}_H} , \qquad f_{L,\bar{L}} = e^{-\bar{E}_L \pm \mu/T} , \tag{2.14}$$

the above equations can be easily integrated and one recovers the usual integrated Boltzmann equations for leptogenesis:

$$\frac{dY_{N_i}}{dz_i} = -K_i z_i (Y_{N_i} - Y_{N_i}^{eq}) \frac{\mathcal{K}_1(z_i)}{\mathcal{K}_2(z_i)} , \tag{2.15}$$

$$\frac{dY_{\mathcal{L}}}{dz_i} = \epsilon_i \, K_i z_i (Y_{N_i} - Y_{N_i}^{eq}) \frac{\mathcal{K}_1(z_i)}{\mathcal{K}_2(z_i)} - \frac{z_i^3}{4} K_i \mathcal{K}_1(z_i) Y_{\mathcal{L}} , \tag{2.16}$$

where $\mathcal{K}_1(z_i)$ and $\mathcal{K}_2(z_i)$ are the modified Bessel functions of the second kind of order 1 and 2.

We have solved Eqs. (2.11) and (2.13) in the case that the lepton asymmetry is generated by N_1, the lightest right-handed neutrino. The results for the N_1 abundance normalized to the equilibrium one Y_N/Y_N^{eq} are shown in Fig. II, left panel (blue solid line). As was already noticed in [3], at high temperatures the equilibration rate of the heavy neutrino is faster when the full Boltzmann equation is used, so the abundance is larger than the one obtained assuming kinetic equilibrium (pink dashed line).

After conversion by sphaleron transitions, the resulting baryon asymmetry is related to the generated lepton asymmetry $Y_{\mathcal{L}}$ by [15]

$$Y_B = \frac{12}{37} Y_{\mathcal{L}} \tag{2.17}$$

In Fig. I we plot our results for the Baryon Asymmetry of the Universe (BAU) Y_B (blue solid line) and the ones obtained using the integrated equations (pink dashed line) for $K_1 = 1, 10$ and $\epsilon_1 = 10^{-6}$. We find, in agreement with [3], that the difference between the two approaches is at most 20% for $K_1 \gtrsim 1$. For smaller values of K_1 the difference can be larger, but in this limit our results are no longer valid because we have not included scattering which can enhance the neutrino production and change our results.

Next, we focus on a different scenario: we assume that a sizeable lepton asymmetry has been produced initially (for instance, during N_2 decay), and we study the washout of this asymmetry by N_1 interactions, relevant at $T \sim M_1 \ll M_2$, when the lepton asymmetry generated by N_1 decays is negligible, which is always the case if $M_{N_1} \ll 10^9$ GeV [16, 17]. Here, as discussed in [18], such lepton asymmetry may survive in two ways: (i) because N_1 interactions are weak, so it only washes out a small part of the lepton asymmetry; (ii) if the N_1 decays when only the τ Yukawa interactions are in equilibrium, at 10^9 GeV $< T < 10^{12}$ GeV, generically a part of the preexisting lepton asymmetry is always protected from N_1 washout due to flavour effects. We first study the effect of using the full Boltzmann equations to follow the evolution of the preexisting lepton asymmetry, due to N_1 interactions, in the unflavoured case.

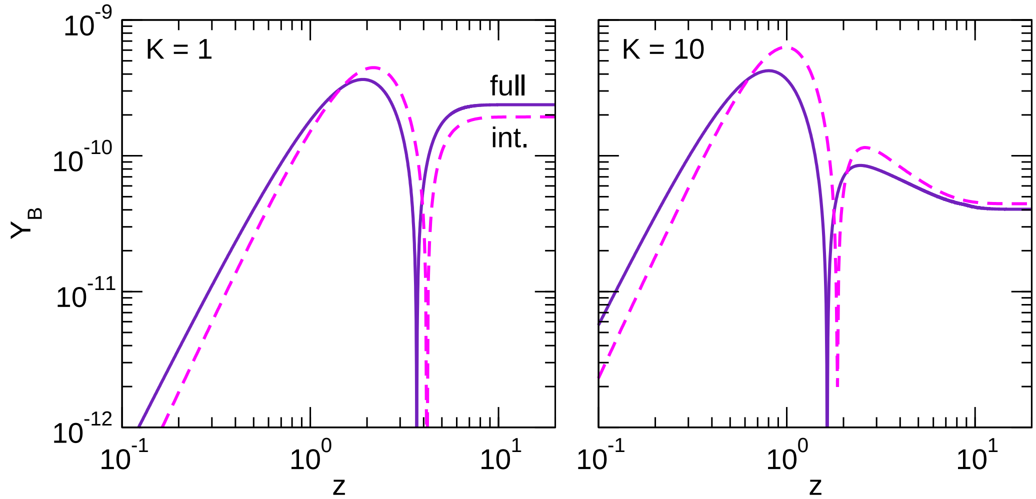

Figure I: BAU obtained from the integration of the full Boltzmann equations, Eqs. (2.11) and (2.13), (blue solid line) and the approximate integrated equations, Eq. (2.15),(2.16) (pink dashed line) for $K_1 = 1$ (left) and 10 (right), and $\epsilon_1 = 10^{-6}$.

We therefore solve eqs. (2.11) and (2.13) with the initial conditions at $z_1 = M_1/T << 1$ of zero N_1 initial abundance and $Y_{\mathcal{L}} = 10^{-9}$, for different values of the decay parameter K_1 and $\epsilon_1 = 0$, i.e., we neglect the lepton asymmetry generated by N_1 decays. In this approximation, the Boltzmann equation for the lepton asymmetry can be solved analytically and we obtain:

$$Y_{\mathcal{L}}(z) = Y_{\mathcal{L}}(0) \exp\left\{ -\frac{3K_1}{\pi^2} \int_0^z dz_1 z_1^2 \int_{z_1}^{\infty} d\bar{E}_1 \int_{\frac{\bar{E}_1 - y_1}{2}}^{\frac{\bar{E}_1 + y_1}{2}} dy_L \frac{e^{y_L}}{(1+e^{y_L})^2} (f_H^{eq} + f_{N_1}) \right\} \quad (2.18)$$

We show our results in Fig. II. As we explained before, the N_1 abundance (left panel) is independent of the lepton asymmetry (to the order we are considering), so it is the same as in the previous case. In the right panel we show the evolution of the baryon asymmetry, as a function of $z = M_1/T$. We obtain that the use of the full Boltzmann equations always decreases the final lepton asymmetry, as compared with the standard approximation. This can be understood from Eq. (2.18): in the usual Maxwell–Bolzmann approximation, only the f_H^{eq} distribution appears in the exponential washout factor. However in the full equation there is an extra term, f_{N_i}, and thus the washout is stronger. We can see in this figure that for $K_1 \leq 1$ the final asymmetry is only slightly reduced with respect to the integrated equations and this suppression increases with K_1.

In Fig. III, we show the amount of initial lepton asymmetry that needs to be produced by N_2 in order to obtain the observed BAU after N_1 washout, as a function of the parameter K_1. The coloured region indicates the values of Y_L^{in} and K_1 for which we obtain Y_B within 50% of the experimental value $Y_B = (8.75 \pm 0.23) \times 10^{-11}$ [19]. In the case considered here of hierarchical right-handed neutrinos, the asymmetry generated by N_2 is typically $Y_L^{N_2} \lesssim 10^{-8}$. Therefore, in typical scenarios, only for $K_1 \leq 3$ we can expect a sufficient asymmetry to be generated. In fact, as shown in [17], we usually need $K_1 \leq 3$ in $SO(10)$-

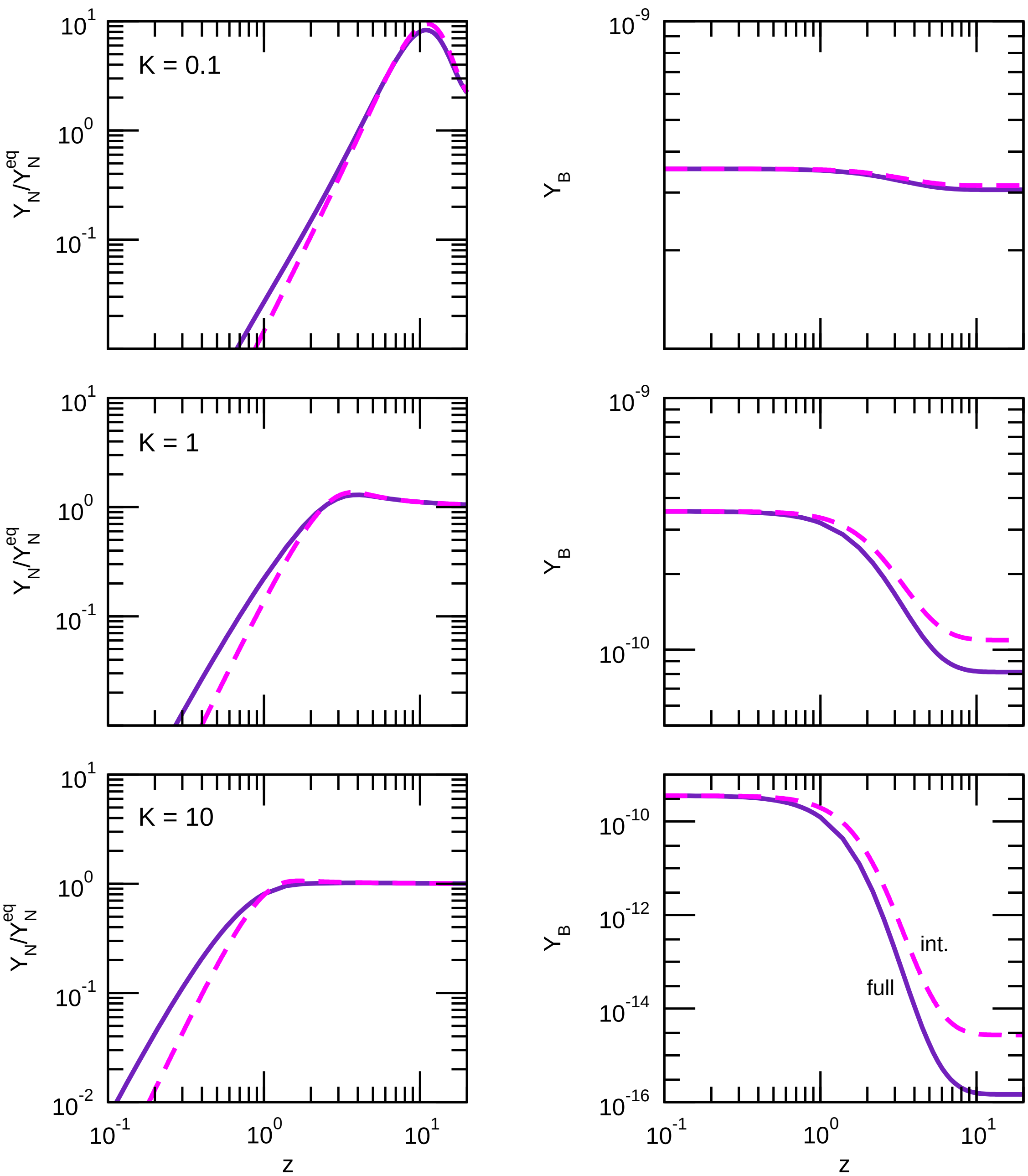

Figure II: Comparison between the integration of the full Boltzmann equations (Eqs. (2.11) and (2.13)) in N_2 leptogenesis (blue solid line) and the approximate integrated equations (Eq. (2.15),Eq. (2.16)) (pink dashed line) for $K_1 = 0.1$, 1, 10, and $Y_{\mathcal{L}} = 10^{-9}$. In the plots on the left we show the heavy neutrino (N_1) abundance as a function of z, and in the ones on the right the surviving Baryon Asymmetry after N_1 washout.

inspired scenarios with flavour effects.

Finally, we consider the addition of flavour effects in this scenario of an initial lepton asymmetry and its washout by N_1 interactions. As shown in Appendix A, the Boltzmann

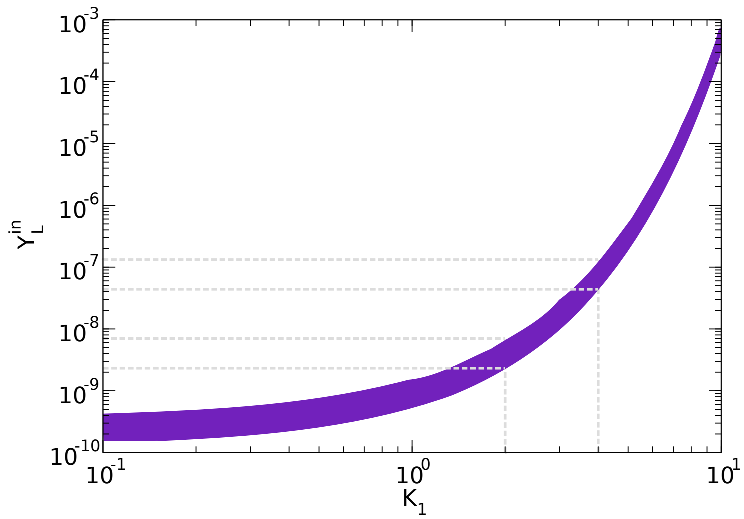

Figure III: The coloured region shows the amount of initial lepton asymmetry that needs to be produced by N_2 in order to obtain the right amount of BAU after N_1 washout, as a function of the washout parameter K_1.

equation for the N_1 abundance is identical to the single flavour case, while now we have to consider different evolution equations for the asymmetries in the different flavours:

$$\frac{\partial f_{N_i}}{\partial z_i} = \frac{K_i z_i^2}{y_i \bar{E}_i} \int_{\frac{\bar{E}_i - y_i}{2}}^{\frac{\bar{E}_i + y_i}{2}} dy_H \left[f_H^{eq} f_L^{eq} (1 - f_{N_i}) - f_{N_i} (1 - f_L^{eq})(1 + f_H^{eq}) \right] , \tag{2.19}$$

$$\begin{aligned} \frac{\partial f_{\mathcal{L}_\alpha}}{\partial z_i} = \frac{z_i^2}{y_L^2} \int_{y_L + \frac{z_i^2}{4 y_L}}^{\infty} d\bar{E}_i & \left\{ K_i \epsilon_i^\alpha \left(f_{N_i} - f_{N_i}^{eq} \right) \left[(1 - f_L^{eq})(1 + f_H^{eq}) - f_H^{eq} f_L^{eq} \right] - \right. \\ & \left. - \frac{K_i^\alpha}{2} f_{\mathcal{L}_\alpha} (f_H^{eq} + f_{N_i}) \right\} \; + \; O(Y_{\alpha i}^2 Y_{\beta i}^2 \times f_{\mathcal{L}_\beta}) , \end{aligned} \tag{2.20}$$

where $K_i^\alpha \equiv \Gamma(N_i \to L_\alpha H, \bar{L}_\alpha \bar{H})/H(T = M_i)$. In this expression we can see that, up to subleading corrections in the Yukawa couplings (barring special cases where $Y_{\beta i}^2 f_{\mathcal{L}_\beta} \gtrsim f_{\mathcal{L}_\alpha}$), the evolution equations for the asymmetries in different flavours decouple and the situation is analogous to the single flavour case. The only difference in this equation is the presence of different CP asymmetries in different flavours and the fact that the washout is produced only by the inverse decays in the relevant flavour [16]. Therefore, we can extend the results obtained above in the single flavour approximation for instance to the three flavour case considering independently the asymmetries in the e, μ and τ channels with independent washout K_1^e, K_1^μ and K_1^τ. Figs. II and III are also valid for the asymmetries in the different flavours with the corresponding washout parameter. Although our results are model independent and can be used in a generic model, they basically agree with the results obtained in Ref [17] in an $SO(10)$ scenario with integrated equations as they have always small washout in the τ-flavour in the relevant parameter space.

3. Soft leptogenesis

In this section, we briefly review the soft leptogenesis scenario, and we present the full Boltzmann equations relevant for soft leptogenesis in the context of the supersymmetric type I seesaw model. We solve them exactly and compare with the approximations used in the literature.

For a hierarchical spectrum of right-handed neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [20], of order $M > 2.4(0.4) \times 10^9$ GeV for vanishing (thermal) initial neutrino densities [20, 21], although flavour effects [11] and/or extended scenarios [22] may affect this limit ‡. The stability of the hierarchy between this new scale and the electroweak one is natural in low-energy supersymmetry, but in the supersymmetric seesaw scenario there is some conflict between the gravitino bound on the reheating temperature and the thermal production of right-handed neutrinos [24]. This is so because in a high temperature plasma, gravitinos are copiously produced, and their late decay could modify the light-nuclei abundances, contrary to observation. This sets an upper bound on the reheating temperature after inflation, $T_{RH} < 10^{6-8}$ GeV, which may be too low for the right-handed neutrinos to be thermally produced [24].

Once supersymmetry (SUSY) has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and the basic mechanism are the same as in the non-supersymmetric case. However, as shown in Refs. [4–6], supersymmetry-breaking terms can play an important role in the lepton asymmetry generated in sneutrino decays because they induce effects which are essentially different from the neutrino ones. Soft supersymmetry-breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation. As a consequence, the mixing between the two sneutrino states generates a CP asymmetry in the decay, which can be sizable for a certain range of parameters. In particular, the asymmetry is large for a right-handed neutrino mass scale relatively low, in the range $10^5 - 10^8$ GeV, below the reheating temperature limits, what solves the cosmological gravitino problem. This is the so called "soft leptogenesis" scenario, which we are going to consider now.

The superpotential of the supersymmetric seesaw model contains the following relevant terms:

$$W = \frac{1}{2} M_{ij} N_i N_j + Y_{\alpha i} N_i (L_\alpha H) \ , \tag{3.1}$$

where the parentheses indicate SU(2) contractions and $\alpha, i = 1, 2, 3$ are flavour indices. L_α, N_i, H are the chiral superfields corresponding to the left-handed lepton doublets, the right-handed (RH) neutrinos and the up-Higgs doublet, respectively, and $Y_{\alpha i}$ denote the neutrino Yukawa couplings (notice that we are using the convention $\alpha \equiv L,\ j \equiv R$). The

‡This bound applies when the lepton asymmetry is generated in the decay of the lightest right-handed neutrino. The possibility to evade the bound producing the asymmetry from the second lightest right-handed neutrino has been considered in [23], and flavour effects have been analysed for this case in [16].

soft supersymmetry-breaking terms involving the heavy sneutrinos $\widetilde{N}_i$ are

$$\mathcal{L}_{soft} = -\widetilde{m}^2_{ij}\widetilde{N}^*_i\widetilde{N}_j - \left[A_{\alpha i}Y_{\alpha i}\widetilde{N}_i(\widetilde{L}_\alpha H) + \frac{1}{2}B_{ij}M_{ij}\widetilde{N}_i\widetilde{N}_j + h.c.\right] \tag{3.2}$$

Contrary to the traditional leptogenesis scenario, where at least two generations of RH neutrinos are required to generate a CP asymmetry in neutrino/sneutrino decays, in this mechanism for leptogenesis, a single generation of heavy RH neutrinos is sufficient to generate a CP asymmetry in sneutrino decays. Therefore, from now on we consider a simplified one-generation model, which refers to the lightest of the three heavy sneutrinos, that we denote as 1. For simplicity we also assume proportionality of soft-trilinear terms, and drop the flavour index for the coefficent A. The sneutrino interaction Lagrangian is then:

$$\begin{aligned}\mathcal{L} =& -(\widetilde{m}^2 + |M|^2)\widetilde{N}^*\widetilde{N} - \frac{1}{2}\left(BM\widetilde{N}\widetilde{N} + h.c.\right) \\ &- \left[Y_{\alpha 1}\widetilde{N}(L_\alpha h) + MY_{\alpha 1}\widetilde{N}^*(\widetilde{L}_\alpha H) + AY_{\alpha 1}\widetilde{N}(\widetilde{L}_\alpha H) + h.c.\right]\end{aligned} \tag{3.3}$$

where h is the fermionic partner of the Higgs doublet H. Under these conditions, a physical CP-violating phase is still present in the neutrino sector,

$$\Phi = \arg(AB)\ , \tag{3.4}$$

which we choose to assign to A. The right-handed neutrino has a mass M, while the sneutrino and antisneutrino states mix in the mass matrix, with mass eigenvectors

$$\begin{aligned}\widetilde{N}_+ &= \frac{1}{\sqrt{2}}\left(e^{i\Phi/2}\widetilde{N} + e^{-i\Phi/2}\widetilde{N}^*\right) \\ \widetilde{N}_- &= \frac{-i}{\sqrt{2}}\left(e^{i\Phi/2}\widetilde{N} - e^{-i\Phi/2}\widetilde{N}^*\right)\end{aligned} \tag{3.5}$$

and mass eigenvalues

$$M^2_\pm = M^2 + \widetilde{m}^2 \pm |BM|\ . \tag{3.6}$$

We define the fermionic and scalar CP asymmetries in the decay of each $\widetilde{N}_i$ ($i = \pm$) as:

$$\epsilon_{s_i} = \frac{|\hat{A}(\widetilde{N}_i \to \widetilde{L}H)|^2 - |\hat{A}(\widetilde{N}_i \to \widetilde{L}^\dagger H^\dagger)|^2}{|\hat{A}(\widetilde{N}_i \to \widetilde{L}H)|^2 + |\hat{A}(\widetilde{N}_i \to \widetilde{L}^\dagger H^\dagger)|^2} = \frac{|\hat{A}(\widetilde{N}_i \to \widetilde{L}H)|^2 - |\hat{A}(\widetilde{N}_i \to \widetilde{L}^\dagger H^\dagger)|^2}{|A^s_i|^2} \tag{3.7}$$

$$\epsilon_{f_i} = \frac{|\hat{A}(\widetilde{N}_i \to Lh)|^2 - |\hat{A}(\widetilde{N}_i \to \bar{L}\bar{h})|^2}{|\hat{A}(\widetilde{N}_i \to Lh)|^2 + |\hat{A}(\widetilde{N}_i \to \bar{L}\bar{h})|^2} = \frac{|\hat{A}(\widetilde{N}_i \to Lh)|^2 - |\hat{A}(\widetilde{N}_i \to \bar{L}\bar{h})|^2}{|A^f_i|^2}\ , \tag{3.8}$$

where we have implicitly summed over flavours, and

$$\begin{aligned}|A^s_i|^2 &= 2\sum_\alpha |Y_{\alpha 1}M|^2\ , \\ |A^f_i|^2 &= 2\sum_\alpha |Y_{\alpha 1}M|^2\frac{M_i^2}{M^2}\left(1 - x_L - x_h\right)\ ,\end{aligned} \tag{3.9}$$

with $M_i = M_+, M_-$ and

$$x_a \equiv \frac{m_a(T)^2}{M^2} \ . \tag{3.10}$$

Notice that, in this section, we keep the thermal masses of the sneutrino decay products, since they break supersymmetry and contribute to obtain a non-vanishing CP asymmetry. They are given by [13]:

$$m_H^2(T) = 2m_h^2(T) = \left(\frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 + \frac{3}{4}Y_t^2\right) T^2 \ , \tag{3.11}$$

$$m_{\widetilde{L}}^2(T) = 2m_L^2(T) = \left(\frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2\right) T^2 \ , \tag{3.12}$$

where g_2 and g_Y are the gauge couplings and Y_t is the top Yukawa. Then, the total sneutrino decay width is [§],

$$\Gamma_i = \frac{1}{16\pi M_i}(\lambda(1, x_{\widetilde{L}}, x_H)|A_i^s|^2 + \lambda(1, x_L, x_h)|A_i^f|^2) \, , \tag{3.13}$$

where

$$\lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x} \tag{3.14}$$

The mixing between the sneutrino states can generate a sizable CP asymmetry in their decay, due to the resonant enhancement of the self-energy contribution. The CP asymmetry can be computed following the effective field-theory approach described in [25], which takes into account the CP violation due to mixing of nearly degenerate states by using resumed propagators for unstable (mass eigenstate) particles. Neglecting supersymmetry breaking in vertices and keeping only the lowest order contribution in the soft terms, it is given by [4, 6]:

$$\epsilon_{s+} = \epsilon_{s-} = -\epsilon_{f+} = -\epsilon_{f-} \equiv \epsilon = \frac{4\Gamma B}{\Gamma^2 + 4B^2}\frac{\mathrm{Im}A}{M} \ . \tag{3.15}$$

Here, we have neglected thermal corrections to the CP asymmetry from the loops, i.e., we have computed the imaginary part of the one-loop graphs using Cutkosky cutting rules at $T = 0$. These corrections are the same for scalar and fermionic decay channels, (only bosonic loops contribute to the wave-function renormalization common to both decays), so they are not expected to introduce significant changes to our results.

In order to generate enough asymmetry, we need $B \simeq \Gamma$, thus the lepton number violating soft bilinear coupling B, responsible of the sneutrino mass splitting, has to be unconventionally small. Moreover, as one can see in the above equation, in soft leptogenesis induced by CP violation in mixing, an exact cancellation occurs between the asymmetry produced in the fermionic and bosonic channels at $T = 0$. Therefore, thermal effects play a fundamental role in this mechanism: final-state Fermi blocking and Bose stimulating factors, together with the effective masses of the particle excitations in the plasma, break supersymmetry and remove this degeneracy.

Several comments are in order:

[§]Neglecting soft SUSY-breaking corrections and thermal masses $|A_i^f|^2 = |A_i^s|^2$.

The effects of flavour in soft leptogenesis have been studied in [26], and we will not consider them here, since the single-flavour approximation is enough to illustrate our results.

It has been recently pointed out that for resonant scenarios the use of quantum BE may be relevant [27]. For **standard resonant leptogenesis** [7], they induce a T dependence in the CP asymmetry which can enhance the produced baryon number. However, in Ref. [28] it has been shown that in **soft leptogenesis**, due to the thermal nature of the mechanism already at the classical level, the introduction of quantum effects does not lead to such enhancement and therefore it is enough to consider only the classical BE in this work.

The authors of Ref. [5] identified new sources for soft leptogenesis, induced by CP violation in right-sneutrino decay and in the interference of mixing and decay. These contributions are relevant both because they can be sizable for natural values of the B parameter and because, unlike the CP violation in mixing, they do not require thermal effects, as they do not vanish at $T = 0$. However, this calculation has been recently revisited in Ref. [29], where it has been found that for all soft SUSY breaking sources of CP violation considered, at $T = 0$ the exact cancellation between the asymetries produced in the fermionic and bosonic channels holds. Therefore it seems that the full Boltzmann equations are always required to calculate the final lepton asymmetry generated in soft leptogenesis.

3.1 Full Bolzmann equations

We assume that the sneutrinos are in a thermal bath with a thermalization time Γ^{-1} shorter than the oscillation time, $(\Delta M)^{-1}$, therefore coherence is lost and we can write the Boltzmann equations for the mass eigenstates Eq. (3.5). As in the previous section, we work in a simplified scenario including only sneutrino decays, inverse decays and resonant scattering. Within this approximation, we can neglect RH neutrino interactions, since in soft leptogenesis only sneutrino interactions generate the lepton asymmetry.

We assume that, because of the fast gauge interactions, the Higgs and higgsino fields are in thermal equilibrium and the leptons and sleptons are in kinetic equilibrium, so that the corresponding distributions are:

$$f_H^{eq} = (e^{E_H/T} - 1)^{-1} \quad , \qquad f_h^{eq} = (e^{E_h/T} + 1)^{-1} \tag{3.16}$$

$$f_L = \frac{1}{\exp[(E_L - \mu_f)/T] + 1} \quad , \qquad f_{\bar{L}} = \frac{1}{\exp[(E_L + \mu_f)/T] + 1} \; , \tag{3.17}$$

$$f_{\widetilde{L}} = \frac{1}{\exp[(E_{\widetilde{L}} - \mu_s)/T] - 1} \quad , \qquad f_{\widetilde{L}^\dagger} = \frac{1}{\exp[(E_{\widetilde{L}} + \mu_s)/T] - 1} \; . \tag{3.18}$$

We have introduced a chemical potential for the leptons, μ_f, and sleptons, μ_s. We are interested in the evolution of the sneutrino density distributions $f_{\widetilde{N}_i}$ and the fermionic and scalar asymmetries, $f_{\mathcal{L}} = f_L - f_{\bar{L}}$, which is given by Eq. (2.7) to first order in μ_f, and

$$f_{\widetilde{\mathcal{L}}} = f_{\widetilde{L}} - f_{\widetilde{L}^\dagger} = \frac{2\, e^{E_L/T}}{(e^{E_L/T} - 1)^2} \frac{\mu_s}{T} + \mathcal{O}((\frac{\mu_s}{T})^3) \; . \tag{3.19}$$

We approximate $f_L + f_{\bar{L}} \simeq 2 f_L^{eq}$ and $f_{\widetilde{L}} + f_{\widetilde{L}^\dagger} \simeq 2 f_{\widetilde{L}}^{eq}$, where:

$$f_L^{eq} = (e^{E_L/T} + 1)^{-1} \,, \qquad f_{\widetilde{L}}^{eq} = (e^{E_{\widetilde{L}}/T} - 1)^{-1} \,. \tag{3.20}$$

In the limit $m_a(T) \ll T$, the chemical potentials are related to the corresponding asymmetries by ¶

$$Y_{\mathcal{L}} = \mu_f \frac{T^2}{3s} + \mathcal{O}((\frac{\mu_f}{T})^3) \,, \tag{3.21}$$

$$Y_{\widetilde{\mathcal{L}}} = \mu_s \frac{2T^2}{3s} + \mathcal{O}((\frac{\mu_s}{T})^3) \,. \tag{3.22}$$

Note that the masses and widths of the two sneutrino states are equal as long as we neglect supersymmetry breaking effects, then $f_{\widetilde{N}_+} = f_{\widetilde{N}_-} \equiv f_{\widetilde{N}}$, and we can write a unique BE for $f_{\widetilde{N}}$; thus, in the following, we do not write the subindex i. The total fermionic (or scalar) asymmetry is then twice the asymmetry generated by one of the two sneutrinos.

The relevant Boltzmann equations are derived in appendix B, and can be written in terms of the dimensionless variables $z = M/T$, $\bar{E}_a = E_a/T$ and $y_a = p_a/T$ ($a = \widetilde{N}, h, L, H, \tilde{L}$) as:

$$\begin{aligned}
\frac{\partial f_{\widetilde{N}}}{\partial z} &= \frac{z^2}{2 y_N \bar{E}_N} \left\{ K_f \int_{\bar{E}_h^m}^{\bar{E}_h^M} d\bar{E}_h \left[f_h^{eq} f_L^{eq} (1 + f_{\widetilde{N}}) - f_{\widetilde{N}} (1 - f_L^{eq})(1 - f_h^{eq}) \right] + \right. \\
&\quad \left. + K_s \int_{\bar{E}_H^m}^{\bar{E}_H^M} d\bar{E}_H \left[f_H^{eq} f_{\widetilde{L}}^{eq} (1 + f_{\widetilde{N}}) - f_{\widetilde{N}} (1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) \right] \right\} \,, && (3.23) \\
\frac{\partial f_{\mathcal{L}}}{\partial z} &= \frac{K_f z^2}{y_L \bar{E}_L} \int_{\bar{E}_N^f}^{\infty} d\bar{E}_N \left\{ -\epsilon (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 - f_L^{eq})(1 - f_h^{eq}) + f_h^{eq} f_L^{eq} \right] - \right. \\
&\quad \left. - \frac{1}{2} f_{\mathcal{L}} (f_{\widetilde{N}} + f_h^{eq}) \right\} + \frac{1}{Hz} S_g \,, && (3.24) \\
\frac{\partial f_{\widetilde{\mathcal{L}}}}{\partial z} &= \frac{K_s z^2}{y_L \bar{E}_L} \int_{\bar{E}_N^s}^{\infty} d\bar{E}_N \left\{ \epsilon (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) + f_H^{eq} f_{\widetilde{L}}^{eq} \right] + \right. \\
&\quad \left. + \frac{1}{2} f_{\widetilde{\mathcal{L}}} (f_{\widetilde{N}_i} - f_H^{eq}) \right\} + \frac{1}{Hz} \widetilde{S}_g \,, && (3.25)
\end{aligned}$$

where

$$f_{\widetilde{N}}^{eq} = (e^{E_N/T} - 1)^{-1} \,, \tag{3.26}$$

and we have used the notation

$$K_f = (1 - x_L - x_h) \lambda(1, x_L, x_h) K \,, \qquad K_s = \lambda(1, x_L, x_h) K \,, \tag{3.27}$$

¶ Taking into account the (s)lepton thermal masses $m_L(T) = 0.3T$ and $m_{\widetilde{L}}(T) = 0.4T$, to first order in μ_f, μ_s we obtain

$$Y_{\mathcal{L}} = \mu_f \, 0.329 \, \frac{T^2}{s} \,, Y_{\widetilde{\mathcal{L}}} = \mu_s \, 0.541 \, \frac{T^2}{s} \,,$$

which are the actual values used in our calculation, instead of the 1/3 and 2/3 of Eqs. (3.21) and (3.21), respectively. We see that the lepton thermal mass does not change significantly the lepton asymmetry, but the slepton thermal mass leads to a 20% reduction of the slepton asymmetry, proportional to 0.54 instead of 0.67.

with $K = \Gamma^0/H(T = M)$ defined in terms of the sneutrino decay width without thermal masses,

$$\Gamma^0 = \frac{M}{4\pi} \sum_\alpha |Y_{\alpha i}|^2 \, . \tag{3.28}$$

Since the thermal masses are different for the final states hL and $H\widetilde{L}$, so are the integration limits, given by:

$$\bar{E}_h^{M,m} = \frac{1}{2} \left\{ E_i(1 - x_L + x_h) \pm y_i \, \lambda(1, x_L, x_h) \right\} \ , \tag{3.29}$$

$$\bar{E}_i^f = \frac{\bar{E}_L + \frac{z_i^2}{4y_L}(1 + x_L - x_h)\lambda(1, x_L, x_h)}{\frac{(1+x_L-x_h)}{2} + \frac{\bar{E}_L}{2y_L} \, \lambda(1, x_L, x_h)} \ , \tag{3.30}$$

and analogously for $\bar{E}_H^{M,m}$ and $\bar{E}_i^s$, just replacing L, h with $\widetilde{L}, H$ in the above equations.

The terms S_g and $\widetilde{S}_g$ in Eqs. (3.24) and (3.25) represent the fast gaugino interactions, defined in Eq. (B.11). These interactions are in equilibrium and mediate processes that transform leptons into scalar leptons and viceversa ($L + L \leftrightarrow \widetilde{L} + \widetilde{L}$). Thus we shall impose that $\mu_f = \mu_s$.

As in the previous section, decays, inverse decays, and on-shell scattering processes must be considered in order to obtain the appropiate out-of-equilibrium condition (see appendix B). Using the approximated distributions (2.7) and (3.19) for $f_{\mathcal{L}}$ and $f_{\widetilde{\mathcal{L}}}$, respectively, and integrating over the dimensionless (s)lepton momentum, y_L, we obtain the following Boltzmann equations for the chemical potentials, defined in Eqs. (3.21) and (3.22):

$$\begin{aligned} \frac{1}{T}\frac{d\mu_f}{dz} &= \frac{3K_f z^2}{\pi^2} \int_z^\infty d\bar{E}_N \int_{\bar{E}_L^m}^{\bar{E}_L^M} d\bar{E}_L \left\{ -\frac{\mu_f}{T} \frac{e^{\bar{E}_L}}{(e^{\bar{E}_L}+1)^2} (f_{\widetilde{N}} + f_h^{eq}) - \right. \\ & \left. - \, \epsilon \, (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 - f_L^{eq})(1 - f_h^{eq}) + f_L^{eq} f_h^{eq} \right] \right\} - (\mu_f - \mu_s)\gamma_g \, , \end{aligned} \tag{3.31}$$

$$\begin{aligned} \frac{1}{T}\frac{d\mu_s}{dz} &= \frac{3K_s z^2}{2\pi^2} \int_z^\infty d\bar{E}_N \int_{\bar{E}_L^m}^{\bar{E}_L^M} d\bar{E}_L \left\{ \frac{\mu_s}{T} \frac{e^{\bar{E}_L}}{(e^{\bar{E}_L}-1)^2} (f_{\widetilde{N}} - f_H^{eq}) + \right. \\ & \left. + \, \epsilon \, (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) + f_{\widetilde{L}}^{eq} f_H^{eq} \right] \right\} + (\mu_f - \mu_s)\widetilde{\gamma}_g \, . \end{aligned} \tag{3.32}$$

where we have summed over the two sneutrino states and the integration limits are:

$$\bar{E}_L^{M,m} = \frac{1}{2} \left\{ \bar{E}_N(1 + x_L - x_h) \pm y_N \, \lambda(1, x_L, x_h) \right\} \ , \tag{3.33}$$

and analogously for $\bar{E}_{\widetilde{L}}^{M,m}$, just replacing L, h with $\widetilde{L}, H$ in the above equation. The γ_g, $\widetilde{\gamma}_g$ terms in Eqs. (3.31) and (3.32) represent gaugino mediated processes $L + L \leftrightarrow \widetilde{L} + \widetilde{L}$.

Schematically, we can write the equations for the chemical potentials as:

$$\frac{d\mu_f}{dz} = \epsilon A + C\mu_f - (\mu_f - \mu_s)\gamma_g T \ , \tag{3.34}$$

$$\frac{d\mu_s}{dz} = \epsilon B + D\mu_s + (\mu_f - \mu_s)\widetilde{\gamma}_g T \ , \tag{3.35}$$

while the lepton and slepton number asymmetries are related to the chemical potentials by

$$Y_{\mathcal{L}} = \alpha_f \mu_f \qquad Y_{\widetilde{\mathcal{L}}} = \alpha_s \mu_s \ . \tag{3.36}$$

Therefore, the evolution equation for the total lepton asymmetry, $Y_{\mathcal{L}_T} = Y_{\mathcal{L}} + Y_{\widetilde{\mathcal{L}}}$, is

$$\frac{dY_{\mathcal{L}_T}}{dz} = \epsilon\,(\alpha_f A + \alpha_s B) + \alpha_f C \mu_f + \alpha_s D \mu_s \tag{3.37}$$

Now, in order to take into account that at shorter time intervals the fast gaugino interactions make $\mu_s = \mu_f$, we replace this equality in the right-hand side of the above equation, obtaining

$$\frac{dY_{\mathcal{L}_T}}{dz} = \epsilon\,(\alpha_f A + \alpha_s B) + \frac{\alpha_f C + \alpha_s D}{\alpha_f + \alpha_s} Y_{\mathcal{L}_T} \tag{3.38}$$

We have solved numerically Eqs. (3.23) and (3.38). Following previous approaches [6], [26] we neglect the thermal masses in the evolution equation for the sneutrino distribution, Eq. (3.23), so the decay channels are always open. If one keeps the thermal masses, there is a range of T for which the decays $\widetilde{N} \to hL, H\widetilde{L}$ are kinematically forbidden [13]. However we have checked that for the relevant values of z, the effect of the thermal masses in the sneutrino distribution is negligible.

It is important, though, to keep the thermal masses in the evolution equation for $Y_{\mathcal{L}_T}$, both, because they contribute significantly to the asymmetry and because the Bose–Einstein distribution is divergent at low values of z for massless scalars, leading to an unphysical enhancement of the slepton number asymmetry. In this case, one should also consider the CP asymmetry produced in the decays $H \to \widetilde{N}_i \widetilde{L}$ and $h \to \widetilde{N}_i L$, allowed at higher temperatures. In [13], this contribution has been found to be negligible in standard leptogenesis, so we assume that this is also the case in soft leptogenesis.

3.2 Approximated Boltzmann Equations

The standard integrated equations, Eq. (2.15) and Eq. (2.16), can not be used in the case of soft leptogenesis, since the CP asymmetry produced in fermionic decays is exactly canceled by the one produced in the scalar channel if Maxwell–Boltzmann statistics is assumed. However one can find some approximate solutions in the literature which try to estimate the baryon asymmetry generated in soft leptogenesis.

One possibility, which was used in [6] and [26], is to neglect all the thermal corrections except for the ones that are crucial to get a non vanishing CP asymmetry, which are evaluated in the approximation of decay at rest of the heavy sneutrinos. Then, the Boltzmann equations are integrated in the standard way, i.e., assuming kinetic equilibrium and Maxwell–Boltzmann statistics for all the particles in the plasma, obtaining:

$$\frac{dY_{\widetilde{N}}}{dz} = -Kz\,(Y_{\widetilde{N}} - Y^{eq}_{\widetilde{N}})\frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)}\,, \tag{3.39}$$

$$\frac{dY_{\mathcal{L}_T}}{dz} = 2\,\epsilon(T)\,Kz(Y_{\widetilde{N}} - Y^{eq}_{\widetilde{N}})\frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)} - \frac{Kz^3}{4}\mathcal{K}_1(z) Y_{\mathcal{L}_T}\,, \tag{3.40}$$

where $Y_{\mathcal{L}_T} = Y_{\mathcal{L}} + Y_{\tilde{\mathcal{L}}}$, and we have summed over the two sneutrino states.

The effective, temperature-dependent CP asymmetry, $\epsilon(T)$, includes the statistical factors and thermal corrections which are different for fermions and scalars, and it is defined as:

$$\epsilon(T) = \epsilon \, \frac{c_B - c_F}{c_B + c_F} \,, \tag{3.41}$$

where

$$c_B = \lambda(1, x_{\widetilde{L}}, x_H)[1 + f_B(\bar{E}_{\widetilde{L}})][1 + f_B(\bar{E}_H)] \,, \tag{3.42}$$

$$c_F = (1 - x_L - x_h)\lambda(1, x_L, x_h)[1 - f_F(\bar{E}_L)][1 - f_F(\bar{E}_h)] \,, \tag{3.43}$$

being f_B and f_F the Bose–Einstein and Fermi–Dirac distributions, respectively, and

$$\bar{E}_{\widetilde{L},H} = \frac{z}{2}(1 + x_{\widetilde{L},H} - x_{H,\widetilde{L}}) \,, \qquad \bar{E}_{L,h} = \frac{z}{2}(1 + x_{L,h} - x_{h,L}) \tag{3.44}$$

We expect this approximation to be accurate in the strong washout regime, $K \gg 1$, since in this case the kinetic equilibrium for the heavy sneutrino is a good approximation and, moreover, the final lepton asymmetry is independent of the initial conditions, and fixed only by the late time $z > 1$ (low temperature) evolution, when the thermal motion of the heavy sneutrino can be neglected. We discuss the details in Sec. 3.3.

We have also explored a different approach, based on Ref. [9]: we again assume kinetic equilibrium for all the particles in the plasma, as well as Maxwell–Boltzmann statistics for the heavy sneutrino, but we keep the phase space and statistical factors which are crucial in soft leptogenesis. Furthermore, we approximate the Fermi and Bose distributions by the Maxwell–Boltzmann one:

$$f_{F(B)} = \frac{1}{e^{E/T} \pm 1} \simeq e^{-E/T} \,, \tag{3.45}$$

since this is enough to have a non vanishing total CP asymmetry in sneutrino decay, and the neglected terms are further suppressed by extra $\mathcal{O}(e^{-E/T})$ factors.

Taking into account the (s)lepton thermal masses, given in Eq. (3.12), the relation between the (s)lepton asymmetry $Y_{\mathcal{L}}$ and the corresponding chemical potential is now

$$Y_{\mathcal{L}} = \mu_f \frac{2m_L^2}{s\pi^2} \mathcal{K}_2(m_L/T) + \mathcal{O}((\frac{\mu_f}{T})^3) \,, \tag{3.46}$$

and the same expression holds for $Y_{\widetilde{\mathcal{L}}}$, just changing $\mu_f \to \mu_s$ and $L \to \widetilde{L}$.

Within these approximations, the Boltzmann equation for the heavy-sneutrino abundance is again the standard one, Eq. (3.39), while the evolution equations for the (s)lepton asimmetries become:

$$\frac{dY_{\mathcal{L}}}{dz} = \frac{K_f z^2}{2\pi^2} \int_z^\infty d\bar{E}_N \int_{\bar{E}_L^m}^{\bar{E}_L^M} d\bar{E}_L \left\{ -\frac{\pi^2}{2} \frac{Y_{\mathcal{L}} T^2}{m_L^2 \mathcal{K}_2(m_L/T)} f_L^{eq} f_h^{eq} \right.$$
$$\left. - \,\epsilon\, (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 - f_L^{eq})(1 - f_h^{eq}) + f_L^{eq} f_h^{eq}\right] / s \right\} \,, \tag{3.47}$$

$$\frac{dY_{\widetilde{\mathcal{L}}}}{dz} = \frac{K_s z^2}{2\pi^2} \int_z^\infty d\bar{E}_N \int_{\bar{E}_L^m}^{\bar{E}_L^M} d\bar{E}_L \left\{ -\frac{\pi^2}{2} \frac{Y_{\widetilde{\mathcal{L}}} T^2}{m_{\widetilde{L}}^2 \mathcal{K}_2(m_{\widetilde{L}}/T)} f_{\widetilde{L}}^{eq} f_H^{eq} \right. \,,$$
$$\left. + \,\epsilon\, (f_{\widetilde{N}} - f_{\widetilde{N}}^{eq}) \left[(1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) + f_{\widetilde{L}}^{eq} f_H^{eq}\right] / s \right\} \,. \tag{3.48}$$

The integration limits $\bar{E}^{m,M}_{L,\widetilde{L}}$ are given in Eq. (3.33). Recall that fast gauge interactions imply that $\mu_f = \mu_s$.

The integration over the (s)lepton energy can be performed analytically, and by writting the result as a series expansion in the heavy-sneutrino momentum, y_N, it is also possible to compute the integral over $\bar{E}_N$, order by order in y_N. The calculation is lengthy but straightforward, and it is described in appendix C. The integrated Boltzmann equation for the total lepton asymmetry $Y_{\mathcal{L}_T}$ can be written as:

$$\frac{dY_{\mathcal{L}_T}}{dz} = 2\,\epsilon\,K(Y_{\widetilde{N}} - Y^{eq}_{\widetilde{N}})\frac{F_1(z)}{\mathcal{K}_2(z)} - \frac{Kz^3}{4}\mathcal{K}_1(z)F_2(z)Y_{\mathcal{L}_T}\,. \tag{3.49}$$

$F_1(z)$ is a complicated function of the thermal masses that can be found in appendix C. We present here the limit of vanishing thermal masses, for the purpose of comparison with Ref. [9]:

$$\begin{aligned} F_1^{(0)}(z) &= 4\sum_{n=0}^{\infty}\frac{z^n}{3^{n+1}2^{2n-1}n!}\mathcal{K}_{n+1}(3z/2) \\ &\simeq \frac{8}{3}\mathcal{K}_1(3z/2) + \frac{2z}{9}\mathcal{K}_2(3z/2) + \frac{z}{36}\mathcal{K}_3(3z/2) + \ldots\,, \end{aligned} \tag{3.50}$$

where the dots stand for negligible contributions. The approximation used in [9] seems to correspond to just keeping the first term in Eq. (3.50), although these authors obtain that the argument of the Bessel function $\mathcal{K}_1$ is $\sqrt{2}z$ instead of $3z/2$. We find that to keep only the first term in $F_1(z)$ is not enough to get an accurate result.

The function $F_2(z)$ is given by

$$F_2(z) = 2T^2\,\frac{[\lambda(1,x_L,x_h)]^2(1-x_L-x_h) + [\lambda(1,x_{\widetilde{L}},x_H)]^2}{m_L^2\mathcal{K}_2(m_L/T) + m_{\widetilde{L}}^2\mathcal{K}_2(m_{\widetilde{L}}/T)}\,. \tag{3.51}$$

It is easy to see that it reduces to the standard washout term in the limit of zero thermal masses, since $\mathcal{K}_2(m/T) \simeq 2T^2/m^2$ for $m/T \ll 1$.

We expect that this approximation works better than the effective CP asymmetry $\epsilon(T)$, since here the thermal motion of the sneutrino is fully accounted for. In fact, we have checked that our approximated equation (3.49) with the first three terms of $F_1(z)$ gives the same results as the numerical integration, when one assumes kinetic equilibrium for the heavy sneutrino and Maxwell–Boltzmann statistics for all the particles.

3.3 Results and discussion

Our main results in the comparison between the three approaches to the Boltzmann equations in soft leptogenesis are summarized in Fig. IV for different values of K. In the left panel, we plot the sneutrino number density normalized to the equilibrium density, from the full (solid blue) and integrated (dashed pink) Boltzmann equations. In the right panel, we show the baryon asymmetry obtained by integrating the full set of Boltzmann equations, Eqs. (3.23) and (3.38), (solid blue), our approximated equations, Eq. (3.49) (dashed

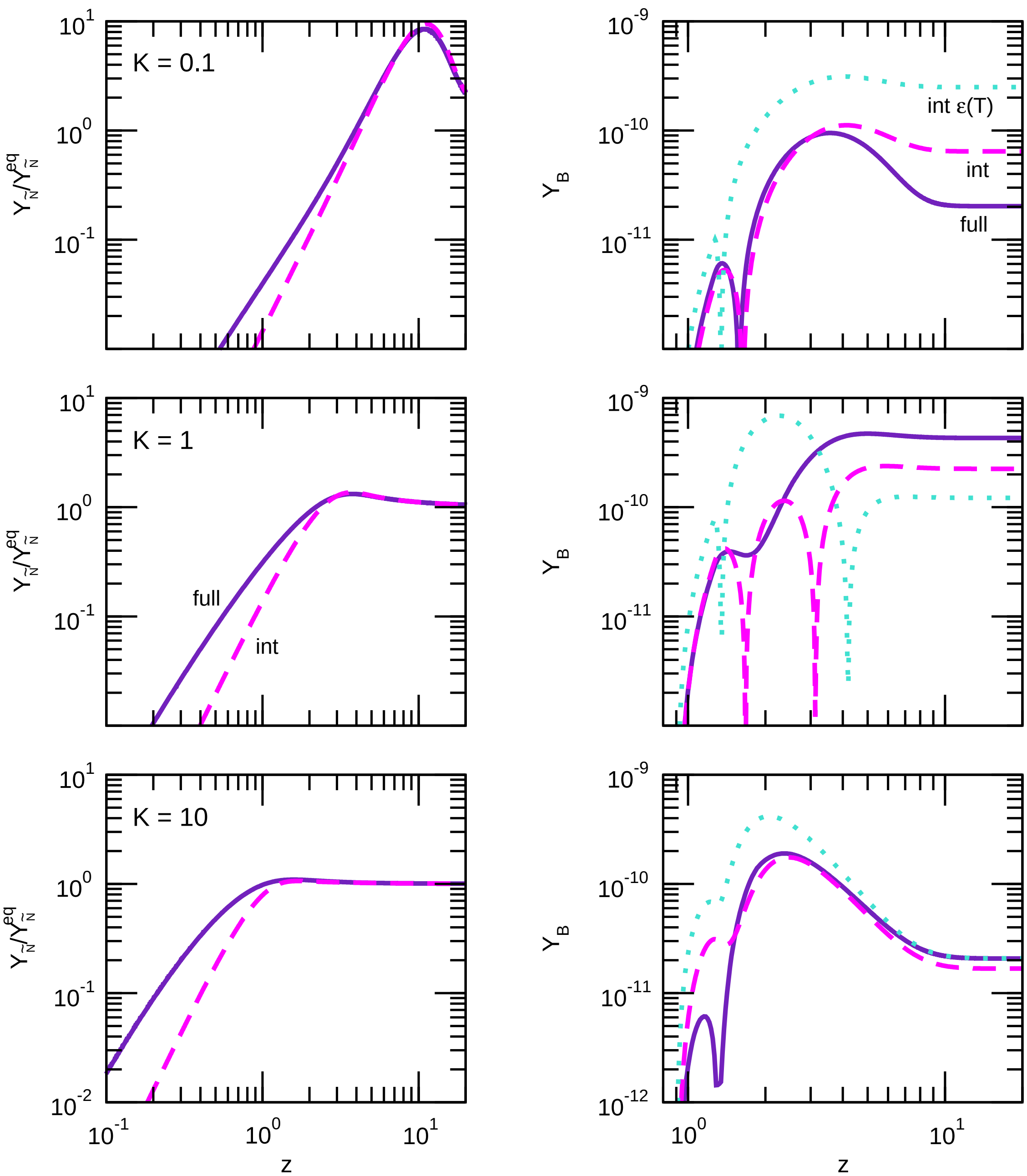

Figure IV: Integration of the full Boltzmann equations in soft leptogenesis and the approximate equations, for $K = 0.1$, 1, 10, and $\epsilon = 4.4 \cdot 10^{-6}$. Left panel: sneutrino distribution as a function of z, obtained from the integration of Eq. (3.23) and the approximate Eq. (3.39). Right panel: baryon asymmetry obtained from the integration of Eqs. (3.31) and (3.32) and the generated in the two approximate equations Eq. (3.49) and Eq. (3.40).

pink), and the effective $\epsilon(T)$ approximation, Eq. (3.40), (dotted light blue). Recall that in

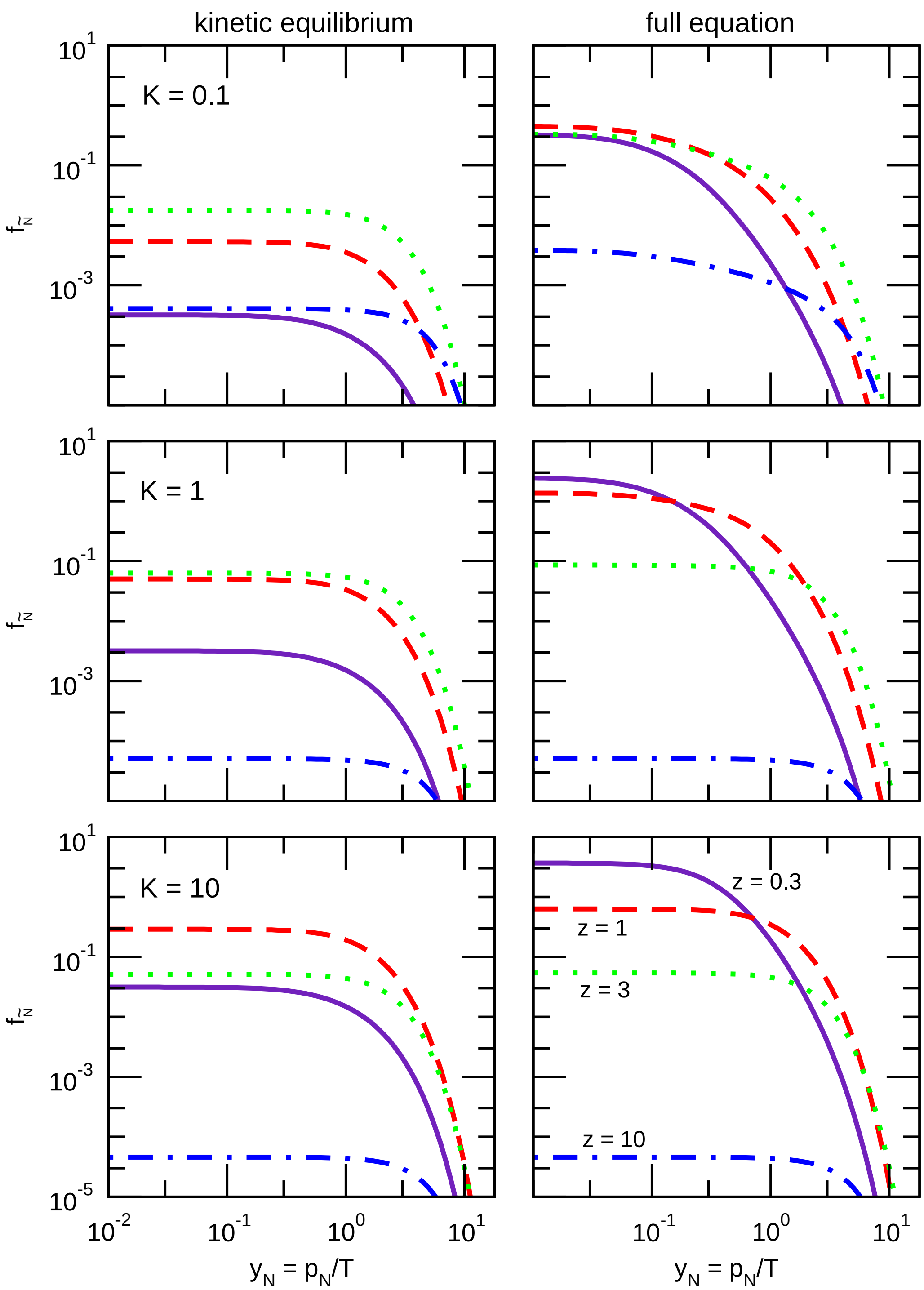

Figure V: Sneutrino momentum distribution assuming kinetic equilibrium (left) and solving the full Boltzmann equation (right), for different values of z, and $K = 0.1$, 1, 10.

the MSSM, the final baryon asymmetry induced by sphaleron transitions is

$$Y_B = \frac{8}{23} Y_{\mathcal{L}_T} \; . \tag{3.52}$$

We find that at high temperatures (small z), the equilibration rate of the sneutrino number density $n_{\tilde{N}}$ is higher when the full Boltzmann equations are used, similarly to what we have in Figure II in the case of the heavy neutrino. The main reason for this effect is that when $z < 1$, the low momentum states of the sneutrino are populated very efficiently, while the population of high momentum modes is small and comparable to the kinetic equilibrium distribution. This can be seen in Fig. V, where we plot the sneutrino phase-space distribution, $f_{\tilde{N}}$, assuming kinetic equilibrium (left panel) and using the full Boltzmann equations (right panel), for different values of z. From these plots, it is clear that, even for $z > 1$ in the weak washout, the density of low momentum states is significantly larger than the density obtained assuming kinetic equilibrium.

The behaviour of the baryon asymmetry, shown in the right panel of Fig. IV, results from the competition of different effects. In soft leptogenesis, due to the inclusion of the thermal masses, the CP asymmetry vanishes for $z \lesssim 0.8$, because both, fermionic and bosonic sneutrino decay channels are kinematically forbiden. For $z \lesssim 1.2$ the fermionic channel of sneutrino (inverse) decay creates an asymmetry, which tends to flip sign when the bosonic channel is open ($z \gtrsim 1.2$), since this one dominates. On the other hand (as it happens also in the standard leptogenesis case), the asymmetry generated during the $\widetilde{N}$ production has opposite sign (we call it "wrong-sign" asymmetry) to the one produced at later times, in $\widetilde{N}$ decay ("right-sign" asymmetry). In the different panels of Fig. IV one can see the competition of these two effects, depending on the strength of the Yukawa sneutrino interactions.

In the case $K = 0.1$, we observe (in agreement with [6]) the flipping of the lepton asymmetry sign when the bosonic channel is open, both in the full (solid blue) and in the approximated calculations. Then, since the Yukawa interactions are weak and the decay occurs at late time, the generation of the right-sign asymmetry is not enough to overcome the wrong-sign one, created during the sneutrino production. However there is an important difference between assuming or not kinetic equilibrium for the heavy sneutrino. As can be seen in the first row of Fig. V, the sneutrino momentum distribution obtained with the full equations and for all values of z is always larger than the kinetic-equilibrium distribution. From Eqs. (3.24) and (3.25), we can see that the source of the asymmetry is always proportional to $f_{\tilde{N}} - f_{\tilde{N}}^{eq}$, where $f_{\tilde{N}}^{eq}$ is the Bose–Einstein distribution. In Figure VI, we can see that up to $z \lesssim 1$, $f_{\tilde{N}} - f_{\tilde{N}}^{eq} \simeq -f_{\tilde{N}}^{eq}$ while for $z \gtrsim 1$, $f_{\tilde{N}} - f_{\tilde{N}}^{eq} \geq 0$ for most values of the momenta. As a consecuence, the right-sign asymmetry for $z \gtrsim 1$ is larger than in the kinetic-equilibrium case, partially compensating the wrong-sign asymmetry generated in the production although it is still not big enough to flip the sign again. Notice that this is not the usual washout, which is very small for $K = 0.1$. If one assumes kinetic equilibrium, the population of the momentum states is more uniform, and the right-sign asymmetry generated is smaller, so one gets a larger final result. However for $K < 1$ one has to be cautious, since scattering processes, that we have not included, can be relevant (see for instance [30]).

For $K = 1$, we see in the second row of Fig. IV that assuming kinetic-equilibrium distributions we find two sign-flips: one due to the opening of the bosonic decay channel and the second, at later time, because for larger K, the washout of the initial wrong-

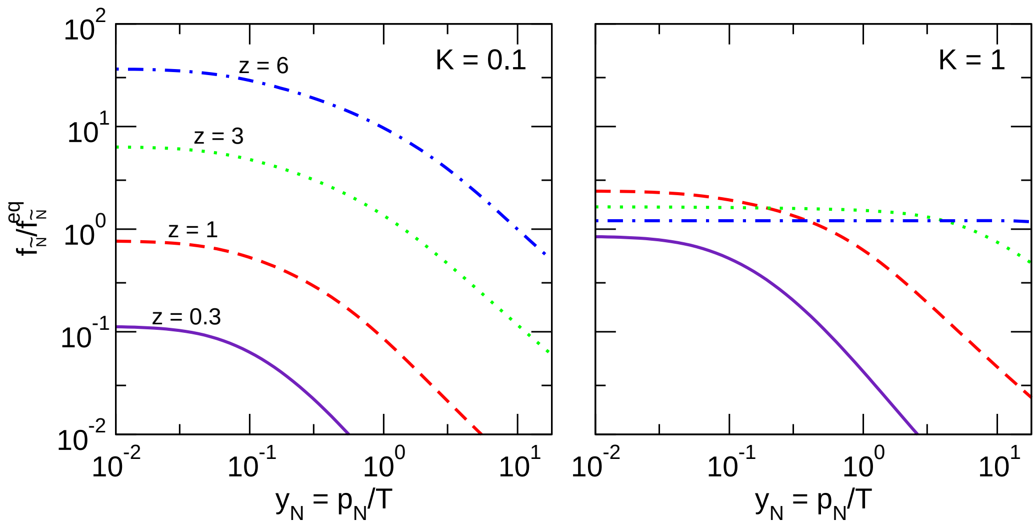

Figure VI: Sneutrino momentum distribution normalized to the equilibrium one, the Bose–Einstein distribution. We show the ratio $f_{\tilde{N}}/f_{\tilde{N}}^{eq}$ for different values of z and $K = 0.1$ (left), $K = 1$ (right).

sign asymmetry is more efficient, and an asymmetry of the right-sign is finally created. However, using the full Boltzmann equation for the sneutrino, these two effects partially compensate each other. In fact, the bosonic channel is open at $z \gtrsim 1.2$ and, as we see in Figure VI, $f_{\tilde{N}} - f_{\tilde{N}}^{eq} \geq 0$ precisely for $1 \lesssim z \lesssim 3$. Therefore, both changes of sign occur at the same time and there is no sign-flip at all. The final baryon asymmetry using the full Boltzmann equations is in this case two (four) times bigger than our (the effective $\epsilon(T)$) approximation. For these values of K, the thermal motion of the sneutrino is important, so our analytic approximation gives a better agreement, specially at high T.

Finally, for $K = 10$, we find no sign-flip of the baryon asymmetry in the full Boltzmann equation calculation, nor in the approximated ones, since the right-sign asymmetry generated in the sneutrino decay (which occurs at earlier times) compensates the sign-flip due to the opening of the bosonic decay channel. As we anticipated, in this strong washout scenario both approximations are very good, since, in this regime, kinetic equilibrium is rapidly reached (Fig. V, third row), and the thermal motion of the sneutrino is negligible for the relevant values of z, so the effective $\epsilon(T)$ approximation gives also accurate results.

4. Conclusions

The integrated Boltzmann equations are generally used to estimate the produced baryon asymmetry through the leptogenesis mechanism; however, these equations are not always a good approximation. In [3], it was shown that in the strong wash-out regime the results may differ by $15-30\%$. In our work, we have extended this study to two different scenarios where the use of the full Boltzmann equations may be relevant: the washout of a preexisting asymmetry (for instance generated by a heavier right handed neutrino, N_2) and soft leptogenesis.

We consider a simplified picture which includes only the heavy right-handed neutrino (N_i) decays, inverse decays, and resonant scattering. This approximation is appropriate for $T < 10^{12}$ GeV, when the off-shell $2 \leftrightarrow 2$ scattering mediated by N_i has only small effects.

A lepton asymmetry produced during the next-to-lightest right-handed neutrino (N_2) decay, in general is washed-out later by the lightest singlet neutrino (N_1) interactions. Then one wonders if a sufficient asymmetry can survive to explain the observed baryon asymmetry. Our results show that the wash-out obtained using the full Boltzmann equations is stronger than the one obtained in the usual Maxwell–Boltzmann and kinetic equilibrium approximation. This result remains valid when flavour effects and different flavour asymmetries are taken into account.

The second scenario that we have explored is soft leptogenesis, where the lepton asymmetry is produced by sneutrino interactions, and the source of lepton number and CP violation are the soft SUSY breaking parameters. The lepton asymmetry produced within this mechanism is a pure thermal effect, because at $T = 0$ the asymmetry in leptons cancels the one in sleptons, and only when $T \neq 0$, the different statistics of bosons and fermions lead to a non-zero asymmetry. In this context, the use of the full Boltzmann equations is mandatory, since in the usual Maxwell–Boltzmann approximation the produced lepton asymmetry is exactly zero. However, there are various simplified approaches available in the literature, always assuming kinetic equilibrium for the heavy sneutrino. A possibility is to use an effective, temperature dependent CP asymmetry, $\epsilon(T)$, which captures the main thermal effects but neglects the thermal motion of the heavy sneutrino [6]. Another approach is to expand the Bose–Einstein and Fermi–Dirac distributions about the Maxwell–Boltzmann one, keeping only the leading order terms [9]. We have found an improved approximation, Eq. (3.49), which describes accurately the temperature dependence of the CP asymmetry, including the thermal motion of the sneutrino. We have compared our results from the full Boltzmann equations with these approximate solutions.

We find that, in the strong washout regime ($K \gg 1$), the results obtained using the full set of Boltzmann equations and those obtained with the approximate equations are in very good agreement, because in both cases the heavy-species distribution is very close to equilibrium in the relevant range of z. However, in the weak washout regime ($K \lesssim 1$), there are important effects. Using the full kinetic equations, we find that the heavy-species low-momentum modes are more efficiently populated, leading to a lepton asymmetry that can be either enhanced (as in the case $K = 1$) or suppressed ($K = 0.1$) with respect to the values obtained using the approximate equations. In fact, the difference bewteen the full kinetic equations and the different approximations can be up to one order of magnitude.

Note added

After the present paper was submitted, a new analysis of standard leptogensis with the full Boltzmann equations appeared [31]. The authors of this work extend previous studies by including, in addition to decays and inverse decays, scattering processes of the right-handed neutrino with the top quark.

Acknowledgments

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A. Full BEs in standard leptogenesis

In this appendix, we describe in detail the derivation of the full Boltzmann equations (BEs) relevant for standard leptogenesis, including the subtraction of the on-shell two-body scattering amplitude which leads to the required Sakharov condition that no lepton asymmetry can be generated in thermal equilibrium.

The Boltzmann equation for the phase-space distribution of the particle species a can be written as:

$$\frac{\partial f_a}{\partial t} - Hp\frac{\partial f_a}{\partial p} = -\frac{1}{2E}C[f_a]\,, \tag{A.1}$$

where the collision integral is given by:

$$\begin{aligned} C[f_a] = &\sum_{aX\leftrightarrow Y} \int d\Pi_X d\Pi_Y (2\pi)^4 \delta^4(p_a + p_X - p_Y) \\ &\times \left[f_a f_X (1 \pm f_Y)|A(aX \to Y)|^2 - f_Y(1 \pm f_a)(1 \pm f_X)|A(Y \to aX)|^2\right] . \end{aligned} \tag{A.2}$$

The sum runs over all allowed processes $aX \leftrightarrow Y$, where X and Y are multiparticle states, and we have used the abbreviations:

$$d\Pi_{X,Y} = \prod_{b\in X,Y} \frac{d^3p_b}{(2\pi)^3 2E_b}\,, \tag{A.3}$$

$$f_{X,Y} = \prod_{b\in X,Y} f_b\,, \qquad (1 \pm f_{X,Y}) = \prod_{b\in X,Y} (1 \pm f_b)\,, \tag{A.4}$$

where $+$ is for bosons and $-$ for fermions.

Then, for the heavy neutrino we have:

$$\begin{aligned} \frac{\partial f_{N_i}}{\partial t} - p_{N_i} H \frac{\partial f_{N_i}}{\partial p_{N_i}} = &\frac{1}{2E_{N_i}} \int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \\ &\times \Big\{ f_H f_L (1 - f_{N_i}) \left|A(LH \to N_i)\right|^2 - f_{N_i}(1 - f_L)(1 + f_H) \left|A(N_i \to LH)\right|^2 \\ &+ f_{\bar H} f_{\bar L}(1 - f_{N_i}) \left|A(\bar L \bar H \to N_i)\right|^2 - f_{N_i}(1 - f_{\bar L})(1 + f_{\bar H}) \left|A(N_i \to \bar L \bar H)\right|^2 \Big\} \,, \end{aligned} \tag{A.5}$$

where $d\vec{p}_X \equiv d^3p_X/2E_X(2\pi)^3$. Using CPT invariance, we can write the neutrino decay amplitudes in terms of the CP conserving total decay amplitude $\left|A^i_D\right|^2$ and the CP asymmetry

$\epsilon_i \ll 1$:

$$|A(N_i \to LH)|^2 = \left|A(\bar{L}\bar{H} \to N_i)\right|^2 = \frac{1+\epsilon_i}{2}\left|A_D^i\right|^2 , \tag{A.6}$$
$$\left|A(N_i \to \bar{L}\bar{H})\right|^2 = |A(LH \to N_i)|^2 = \frac{1-\epsilon_i}{2}\left|A_D^i\right|^2 .$$

We work in the approximations of thermal equilibrium for the Higgs field and kinematic equilibrium for the SM leptons, as described in Sec. 2, Eqs. (2.5) and (2.6). Then, substituting the above decay and inverse decay amplitudes and neglecting terms of order $\mathcal{O}(\epsilon f_{\mathcal{L}})$ (where $f_{\mathcal{L}} = f_L - f_{\bar{L}}$) and of order $\mathcal{O}((\mu/T)^2)$ (where μ is the lepton chemical potential, see Eq. (2.8)), we get

$$\frac{\partial f_{N_i}}{\partial t} - p_{N_i} H \frac{\partial f_{N_i}}{\partial p_{N_i}} = \frac{1}{2E_{N_i}} \int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \left|A_D^i\right|^2$$
$$\times \left[f_H^{eq} f_L^{eq}(1-f_{N_i}) - f_{N_i}(1-f_L^{eq})(1+f_H^{eq})\right] . \tag{A.7}$$

We can perform part of the phase space integration and finally obtain:

$$\frac{\partial f_{N_i}}{\partial t} - p_{N_i} H \frac{\partial f_{N_i}}{\partial p_{N_i}} = \frac{M_i \Gamma_i}{E_{N_i} p_{N_i}} \int_{\frac{E_{N_i}-p_{N_i}}{2}}^{\frac{E_{N_i}+p_{N_i}}{2}} dp_H \left[f_H^{eq} f_L^{eq}(1-f_{N_i}) - f_{N_i}(1-f_L^{eq})(1+f_H^{eq})\right] . \tag{A.8}$$

Although we do not write the arguments of the distribution functions for simplicity, energy conservation implies that $f_L^{eq} = f_L^{eq}(E_{N_i} - E_H)$ in Eq. (A.8).

Regarding the evolution of the lepton asymmetry, besides the right-handed neutrino decays and inverse decays we need to consider the $\Delta L = 2$ scattering terms, in order to obtain the out-of-equilibrium condition, i.e., that no lepton asymmetry is generated in thermal equilibrium. So we have:

$$\frac{\partial f_{\mathcal{L}}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}}}{\partial p_L} = D_i - \bar{D}_i - 2S , \tag{A.9}$$

where:

$$D_i = \frac{1}{2E_L} \int d\vec{p}_{N_i} d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \tag{A.10}$$
$$\times \left\{ f_{N_i}(1-f_L)(1+f_H^{eq}) \left|A(N_i \to LH)\right|^2 - f_H^{eq} f_L (1-f_{N_i}) \left|A(LH \to N_i)\right|^2 \right\} ,$$

$$\bar{D}_i = \frac{1}{2E_L} \int d\vec{p}_{N_i} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_{\bar{L}} - p_{\bar{H}}) \tag{A.11}$$
$$\times \left\{ f_{N_i}(1-f_{\bar{L}})(1+f_H^{eq}) \left|A(N_i \to \bar{L}\bar{H})\right|^2 - f_H^{eq} f_{\bar{L}} (1-f_{N_i}) \left|A(\bar{L}\bar{H} \to N_i)\right|^2 \right\} ,$$

and,

$$S = \frac{1}{2E_L} \int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}})$$
$$\times \left\{ f_L f_H^{eq}(1-f_{\bar{L}})(1+f_H^{eq}) |M_{sub}(LH \to \bar{L}\bar{H})|^2 - \right.$$
$$\left. - \; f_{\bar{L}} f_H^{eq}(1-f_L)(1+f_H^{eq}) |M_{sub}(\bar{L}\bar{H} \to LH)|^2 \right\} , \tag{A.12}$$

The scattering term is defined in terms of the subtracted amplitudes, since the on-shell contribution is already taken into account through the decays and inverse decays in the $D_i - terms$. So, for example:

$$\left|M_{sub}(LH \to \bar{L}\bar{H})\right|^2 = \left|M(LH \to \bar{L}\bar{H})\right|^2 - \left|M_{os}(LH \to \bar{L}\bar{H})\right|^2 \ , \tag{A.13}$$

with

$$\begin{aligned} \left|M_{os}(LH \to \bar{L}\bar{H})\right|^2 &= |A(LH \to N_i)|^2 \, \frac{\pi\delta(s - M_i^2)}{M_i \Gamma_i^{th}} \left|A(N_i \to \bar{L}\bar{H})\right|^2 \\ &\simeq \frac{1 - 2\epsilon_i}{4} \ |A_D^i|^2 \frac{\pi\delta(s - M_i^2)}{M_i \Gamma_i^{th}} |A_D^i|^2 \, , \end{aligned} \tag{A.14}$$

and analogously:

$$\left|M_{os}(\bar{L}\bar{H} \to LH)\right|^2 \simeq \frac{1 + 2\epsilon_i}{4} \ |A_D^i|^2 \frac{\pi\delta(s - M_i^2)}{M_i \Gamma_i^{th}} |A_D^i|^2 \, . \tag{A.15}$$

We define also the off-shell part, which is CP-conserving at leading order, as,

$$|M_{off}(LH \to \bar{L}\bar{H})|^2 = |M(LH \to \bar{L}\bar{H})|^2 - |A_D^i|^2 \ \ \frac{\pi\delta(s - M_i^2)}{4 M_i \Gamma_i} \ \ |A_D^i|^2 \, . \tag{A.16}$$

The width that cutoffs the resonance is Γ_i^{th}, the damping rate at finite temperature:

$$\Gamma_i^{th} = \frac{1}{2M_i} \int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \left[(1 - f_L^{eq})(1 + f_H^{eq}) + f_L^{eq} f_H^{eq}\right] \left|A_D^i\right|^2 \, . \tag{A.17}$$

Using the amplitudes Eq. (A.6) and the approximations in Eqs. (2.7) and (2.8), we can simplify the integrand of the $D_i - terms$ as follows (to first order in ϵ_i and $f_{\mathcal{L}}$):

$$\begin{aligned} &\frac{\left|A_D^i\right|^2}{2} \{ f_{N_i}(1 - f_L)(1 + f_H^{eq})(1 + \epsilon_i) - f_H^{eq} f_L (1 - f_{N_i})(1 - \epsilon_i) \\ &\qquad - f_{N_i}(1 - f_{\bar{L}})(1 + f_H^{eq})(1 - \epsilon_i) + f_H^{eq} f_{\bar{L}} (1 - f_{N_i})(1 + \epsilon_i) \} \\ &= \frac{\left|A_D^i\right|^2}{2} \left\{ 2\,\epsilon_i \left[(1 - f_{N_i}) f_H^{eq} f_L^{eq} + f_{N_i}(1 - f_L^{eq})(1 + f_H^{eq})\right] - f_{\mathcal{L}}(f_H^{eq} + f_{N_i}) \right\} \, . \end{aligned} \tag{A.18}$$

Notice that if we only included the $1 \leftrightarrow 2$ processes, a lepton asymmetry would be generated even in thermal equilibrium, when $f_{N_i} = f_{N_i}^{eq}$ and $f_{\mathcal{L}} = 0$, because the first term on the right-hand side of Eq. (A.18) does not vanish [32]. As anticipated, to remedy this problem we have to include also the subtracted contribution of the $2 \leftrightarrow 2$ scattering. In fact, it is enough to include only the on-shell contribution to the $LH \to \bar{L}\bar{H}$, as the off-shell terms will contribute only at higher order in the Yukawas and the asymmetry. The on-shell scattering is [13]:

$$\begin{aligned} S_{os}(LH \to \bar{L}\bar{H}) &= \frac{1}{2E_L} \int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}}) \\ &\quad \times \ f_L f_H^{eq} (1 - f_{\bar{L}})(1 + f_H^{eq}) \, |A(LH \to N_i)|^2 \, \frac{\pi\delta(s - M_i^2)}{M_i \Gamma_i^{th}} \left|A(N_i \to \bar{L}\bar{H})\right|^2 \, , \end{aligned} \tag{A.19}$$

Since the resonant part of the scattering term is $S_{os} = S_{os}(LH \to \bar{L}\bar{H}) - S_{os}(\bar{L}\bar{H} \to LH)$, substituting the decay and inverse decay amplitudes from Eq. (A.6) it is easy to see that it is order ϵ_i. Thus, within our linear approximation we can consistently use the equilibrium distributions also for the leptons when calculating this term. We then rewrite the product of equilibrium densities as:

$$(1 - f_L^{eq})(1 + f_H^{eq}) = f_{N_i}^{eq} e^{E_{N_i}/T} \left[(1 - f_L^{eq})(1 + f_H^{eq}) + f_L^{eq} f_H^{eq}\right] \; , \tag{A.20}$$

and use the identity:

$$1 = \int d^4 p_{N_i} \delta^{(4)}(p_{N_i} - p_L - p_H) \; , \tag{A.21}$$

to obtain $S_{os}(LH \to \bar{L}\bar{H})$ at the required order in ϵ_i:

$$\begin{aligned} S_{os}(LH \to \bar{L}\bar{H}) = &\frac{1}{2E_L} \int d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) f_L^{eq} f_H^{eq} \left(\frac{1-\epsilon_i}{2}\right)^2 \left|A_D^i\right|^2 \\ &\times \int \frac{d^4 p_{N_i}}{(2\pi)^4} \frac{2\pi\delta(p_{N_i}^2 - M_i^2)}{2M_i \Gamma_i^{th}} f_{N_i}^{eq} e^{E_{N_i}/T} \\ &\times \int d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_{\bar{L}} - p_{\bar{H}}) \left|A_D^i\right|^2 \left[(1 - f_L^{eq})(1 + f_H^{eq}) + f_L^{eq} f_H^{eq}\right] . \end{aligned} \tag{A.22}$$

Notice that the integrals over the final-state particles reproduce the thermal width of N_i, given by Eq. (A.17) [13]. Since $\int \frac{d^4 p_{N_i}}{(2\pi)^4} 2\pi\delta(p_{N_i}^2 - M_i^2) = \int \frac{d^3 p_{N_i}}{(2\pi)^3 2E_{N_i}}$, we can rewrite Eq. (A.22) as:

$$S_{os}(LH \to \bar{L}\bar{H}) = \frac{1}{2E_L} \int d\vec{p}_H d\vec{p}_{N_i} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) f_{N_i}^{eq} e^{E_{N_i}/T} f_L^{eq} f_H^{eq} \left(\frac{1-\epsilon_i}{2}\right)^2 \left|A_D^i\right|^2 . \tag{A.23}$$

To this order, the $S_{os}(\bar{L}\bar{H} \to LH)$ term is the same, just changing the $(1-\epsilon_i)^2$ by $(1+\epsilon_i)^2$, so altogether we find that:

$$S_{os} = -\frac{\epsilon_i}{2E_L} \int d\vec{p}_H d\vec{p}_{N_i} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) f_{N_i}^{eq} e^{E_{N_i}/T} f_L^{eq} f_H^{eq} \left|A_D^i\right|^2 . \tag{A.24}$$

Using the equilibrium relation:

$$f_{N_i}^{eq} e^{E_{N_i}/T} f_L^{eq} f_H^{eq} = f_{N_i}^{eq} (1 - f_L^{eq})(1 + f_H^{eq}) = (1 - f_{N_i}^{eq}) f_L^{eq} f_H^{eq} \; , \tag{A.25}$$

we can write the Boltzmann equation for the lepton number asymmetry as:

$$\begin{aligned} \frac{\partial f_{\mathcal{L}}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}}}{\partial p_L} &= D_i - \bar{D}_i + 2S_{os} = \frac{1}{2E_L} \int d\vec{p}_H d\vec{p}_{N_i} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \qquad \text{(A.26)} \\ &\times \left|A_D^i\right|^2 \left\{\epsilon_i \left(f_{N_i} - f_{N_i}^{eq}\right) \left[(1 - f_L^{eq})(1 + f_H^{eq}) - f_L^{eq} f_H^{eq}\right] - \frac{1}{2} f_{\mathcal{L}} (f_H^{eq} + f_{N_i})\right\} \\ &= \frac{M_i \Gamma_i}{E_L p_L} \int_{p_L + \frac{M_i^2}{4p_L}}^{\infty} dE_{N_i} \left\{\epsilon_i \left(f_{N_i} - f_{N_i}^{eq}\right) \left[(1 - f_L^{eq})(1 + f_H^{eq}) - f_H^{eq} f_L^{eq}\right]\right. \\ &\qquad \left. - \frac{1}{2} f_{\mathcal{L}} (f_H^{eq} + f_{N_i})\right\} \; , \qquad \text{(A.27)} \end{aligned}$$

where we have integrated over part of the phase space and we have used that the heavy-neutrino width at zero temperature is given by Eq. (2.2). This equation does not generate any lepton asymmetry in thermal equilibrium, since the right-hand side of the equation explicitly vanishes in this case. Notice that in this final equation for the evolution of the lepton-asymmetry distribution, (A.26), we have only kept the on-shell contribution to the $\Delta L = 2$ scattering mediated by N_i, because the off-shell part is higher order in the heavy-neutrino Yukawa couplings, and therefore subdominant unless these are large [33].

In the presence of different active lepton flavours in the plasma at $T = M_i$, the Boltzmann equations change. The main difference is that now the right-handed neutrinos decay differently to the different active flavours and generate different phase space distributions and different asymmetries in them. Taking this into account, it is easy to see that the equation for the abundance of the heavy neutrino, Eq. (A.5), now becomes:

$$\begin{aligned}
&\frac{\partial f_{N_i}}{\partial t} - p_{N_i} H \frac{\partial f_{N_i}}{\partial p_{N_i}} = \frac{1}{2E_{N_i}} \int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \qquad \text{(A.28)} \\
&\times \sum_\alpha \Big\{ f_H f_{L_\alpha}(1 - f_{N_i}) \left|A(L_\alpha H \to N_i)\right|^2 - f_{N_i}(1 - f_{L_\alpha})(1 + f_H) \left|A(N_i \to L_\alpha H)\right|^2 \\
&+ f_{\bar{H}} f_{\bar{L}_\alpha}(1 - f_{N_i}) \left|A(\bar{L}_\alpha \bar{H} \to N_i)\right|^2 - f_{N_i}(1 - f_{\bar{L}_\alpha})(1 + f_{\bar{H}}) \left|A(N_i \to \bar{L}_\alpha \bar{H})\right|^2 \Big\} \quad ,
\end{aligned}$$

where

$$\begin{aligned}
\left|A(N_i \to L_\alpha H)\right|^2 = \left|A(\bar{L}_\alpha \bar{H} \to N_i)\right|^2 &= \frac{|A^i_{D,\alpha}|^2 + \epsilon^\alpha_i \left|A^i_D\right|^2}{2} \,, \qquad \text{(A.29)} \\
\left|A(N_i \to \bar{L}_\alpha \bar{H})\right|^2 = \left|A(L_\alpha H \to N_i)\right|^2 &= \frac{|A^i_{D,\alpha}|^2 - \epsilon^\alpha_i \left|A^i_D\right|^2}{2} \,,
\end{aligned}$$

being $A^i_{D,\alpha}$ the CP conserving decay amplitude to the α flavour and ϵ^α_i the flavoured CP asymmetries. In the same approximations as before, i.e. up to terms of order $O(\epsilon f_{\mathcal{L}})$, we obtain for the heavy neutrino abundance the same equation as in the unflavoured case, Eq. (A.7).

In the evolution equations for the lepton asymmetry, we have to consider asymmetries for the different active flavours. The equations are now:

$$\frac{\partial f_{\mathcal{L}_\alpha}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}_\alpha}}{\partial p_L} = D^\alpha_i - \bar{D}^\alpha_i - S^\alpha + S^{\bar{\alpha}} \,, \qquad \text{(A.30)}$$

where D^α_i, $\bar{D}^\alpha_i$ are identical to Eqs. (A.10) and (A.11), taking into account only the relevant flavour in the distribution functions and decay amplitudes, i.e. $f_L \to f_{L_\alpha}$, $|A(N_i \to LH)|^2 \to |A(N_i \to L_\alpha H)|^2$, $\left|A(N_i \to \bar{L}\bar{H})\right|^2 \to \left|A(N_i \to \bar{L}_\alpha \bar{H})\right|^2$, etc.

The scattering terms, in this case, are slightly complicated as the initial and final leptons can have now different flavours and we have additional scatterings changing flavour but not lepton number.

$$S^{\alpha} = \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}}) \times \Bigg\{ f_H^{eq}(1+f_H^{eq}) \tag{A.31}$$
$$\Bigg[f_{L_\alpha} \Bigg(\sum_\beta (1-f_{\bar{L}_\beta})|M_{sub}(L_\alpha H \to \bar{L}_\beta \bar{H})|^2 + \sum_{\beta\neq\alpha}(1-f_{L_\beta})|M_{sub}(L_\alpha H \to L_\beta H)|^2 \Bigg)$$
$$- \ (1-f_{L_\alpha}) \Bigg(\sum_\beta f_{\bar{L}_\beta}|M_{sub}(\bar{L}_\beta \bar{H} \to L_\alpha H)|^2 + \sum_{\beta\neq\alpha} f_{L_\beta}|M_{sub}(L_\beta H \to L_\alpha H)|^2 \Bigg) \Bigg] \Bigg\},$$

and

$$S^{\bar{\alpha}} = \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}}) \times \Bigg\{ f_H^{eq}(1+f_H^{eq}) \tag{A.32}$$
$$\Bigg[f_{\bar{L}_\alpha} \Bigg(\sum_{\beta\neq\alpha} (1-f_{\bar{L}_\beta})|M_{sub}(\bar{L}_\alpha \bar{H} \to \bar{L}_\beta \bar{H})|^2 + \sum_{\beta}(1-f_{L_\beta})|M_{sub}(\bar{L}_\alpha \bar{H} \to L_\beta H)|^2 \Bigg)$$
$$- \ (1-f_{\bar{L}_\alpha}) \Bigg(\sum_{\beta\neq\alpha} f_{\bar{L}_\beta}|M_{sub}(\bar{L}_\beta \bar{H} \to \bar{L}_\alpha \bar{H})|^2 + \sum_{\beta} f_{L_\beta}|M_{sub}(L_\beta H \to \bar{L}_\alpha \bar{H})|^2 \Bigg) \Bigg] \Bigg\}.$$

Using the subtracted amplitudes as in the unflavoured case, the on-shell part of the subtracted amplitudes, Eq. (A.19), now is,

$$S_{os}(L_\alpha H \to \bar{L}_\beta \bar{H}) = \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}}) \tag{A.33}$$
$$\times f_{L_\alpha} f_H^{eq}(1-f_{\bar{L}_\beta})(1+f_H^{eq})\,|A(L_\alpha H \to N_i)|^2 \frac{\pi\delta(s-M_i^2)}{M_i\Gamma_i^{th}} \left|A(N_i \to \bar{L}_\beta \bar{H})\right|^2 .$$

Continuing analogously to the unflavoured case, we obtain the Boltzmann equations for the flavour asymmetries at first order in ϵ and $f_{\mathcal{L}}$:

$$\frac{\partial f_{\mathcal{L}_\alpha}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}_\alpha}}{\partial p_L} = \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{N_i} (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \tag{A.34}$$
$$\times \left\{ \left|A_D^i\right|^2 \epsilon_i^\alpha \,(f_{N_i} - f_{N_i}^{eq}) \left[(1-f_L^{eq})(1+f_H^{eq}) - f_L^{eq} f_H^{eq}\right] - |A_{D,\alpha}^i|^2 \frac{1}{2} f_{\mathcal{L}_\alpha}(f_H^{eq} + f_{N_i}) \right\}$$
$$+ \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}}) f_H^{eq}(1+f_H^{eq})$$
$$\times \sum_\beta \left\{ f_{\mathcal{L}_\beta}|M_{\alpha\beta}|^2_{off} - f_{\mathcal{L}_\beta}|M_{\bar{\alpha}\beta}|^2_{off} - f_{\mathcal{L}_\alpha}|M_{\alpha\beta}|^2_{off} - f_{\mathcal{L}_\alpha}|M_{\bar{\alpha}\beta}|^2_{off} \right\} ,$$

where $|M_{\alpha\bar{\beta}}|^2_{off}$ is defined analogously to Eq. (A.16). We have used CPT invariance, which implies that $|M(\bar{L}_\beta \bar{H} \to \bar{L}_\alpha \bar{H})|^2 = |M(L_\alpha H \to L_\beta H)|^2$, etc., to write all the amplitudes with the flavour α as initial state, and also that the off-shell amplitudes are CP conserving at leading order in the couplings, so that $|M_{\alpha\bar{\beta}}|^2_{off} = |M_{\bar{\alpha}\beta}|^2_{off}$. Finally, we have summed over all flavours β in the last line of Eq. (A.34), since the extra contributions for $\beta = \alpha$ cancel between the first and the third terms.

Notice that in Eq. (A.34), we have kept also the off-shell amplitudes, even taking into account that they are higher order in the Yukawas. In fact, we expect $|M_{\alpha\bar{\beta}}|^2_{off}$ to be of order $Y^2_{\alpha i}Y^2_{\beta i}$ that is indeed smaller than $Y^2_{\alpha i}$ unless $Y_{\beta i} \simeq 1$. However notice that some of these substracted amplitudes contribute proportionally to $f_{\mathcal{L}_\beta}$ and contributions $|Y_{\alpha i}Y_{\beta i}|^2 f_{\mathcal{L}_\beta}$ could be important with respect to the terms $|Y_{\alpha i}|^2 f_{\mathcal{L}_\alpha}$ if $f_{\mathcal{L}\alpha} \ll f_{\mathcal{L}\beta}$ and/or $|Y_{\alpha i}| \ll |Y_{\beta i}|$. Nevertheless, it is not clear a priori when these effects may play a relevant role.

B. Full BEs in soft leptogenesis

Here, we describe in detail the derivation of the full BEs relevant for soft leptogenesis.

Using CPT invariance and the definitions for the CP asymmetries, the decay and inverse decay sneutrino amplitudes can be written as:

$$\begin{aligned}
|\hat{A}(\widetilde{N}_i \to \widetilde{L}H)|^2 &= |\hat{A}(\widetilde{L}^\dagger H^\dagger \to \widetilde{N}_i)|^2 \simeq \frac{1+\epsilon_{s_i}}{2}|A_i^s|^2 \,, \\
|\hat{A}(\widetilde{N}_i \to \widetilde{L}^\dagger H^\dagger)|^2 &= |\hat{A}(\widetilde{L}H \to \widetilde{N}_i)|^2 \simeq \frac{1-\epsilon_{s_i}}{2}|A_i^s|^2 \,, \\
|\hat{A}(\widetilde{N}_i \to Lh)|^2 &= |\hat{A}(\bar{L}\bar{h} \to \widetilde{N}_i)|^2 \simeq \frac{1+\epsilon_{f_i}}{2}|A_i^f|^2 \,, \\
|\hat{A}(\widetilde{N}_i \to \bar{L}\bar{h})|^2 &= |\hat{A}(Lh \to \widetilde{N}_i)|^2 \simeq \frac{1-\epsilon_{f_i}}{2}|A_i^f|^2 \,,
\end{aligned} \tag{B.1}$$

where $|A_i^s|^2$ ($|A_i^f|^2$) is the CP conserving tree-level sneutrino decay amplitude to scalars (fermions).

The evolution equation for the sneutrino distribution with the same approximations as in Appendix A (i.e., to first order in $\epsilon_{f_i}, \epsilon_{s_i}, \mu_f, \mu_s$) is given by

$$\begin{aligned}
\frac{\partial f_{\widetilde{N}_i}}{\partial t} - p_{\widetilde{N}_i} H \frac{\partial f_{\widetilde{N}_i}}{\partial p_{\widetilde{N}_i}} &= \frac{1}{2E_{\widetilde{N}_i}} \int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^4 (p_{\widetilde{N}_i} - p_L - p_H) \\
&\times \left\{ |A_i^f|^2 \left[f_h^{eq} f_L^{eq} (1 + f_{\widetilde{N}_i}) - f_{\widetilde{N}_i} (1 - f_L^{eq})(1 - f_h^{eq}) \right] + \right. \\
&\left. + |A_i^s|^2 \left[f_H^{eq} f_{\widetilde{L}}^{eq} (1 + f_{\widetilde{N}_i}) - f_{\widetilde{N}_i} (1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) \right] \right\} .
\end{aligned} \tag{B.2}$$

For the lepton and slepton asymmetries we have:

$$\frac{\partial f_{\mathcal{L}}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}}}{\partial p_L} = \sum_i \left(D_i - \bar{D}_i \right) - 2S - S_{L\widetilde{L}^\dagger} + \bar{S}_{L\widetilde{L}^\dagger} - S_{L\widetilde{L}} + \bar{S}_{L\widetilde{L}} + S_g \,, \tag{B.3}$$

$$\frac{\partial f_{\widetilde{\mathcal{L}}}}{\partial t} - p_L H \frac{\partial f_{\widetilde{\mathcal{L}}}}{\partial p_L} = \sum_i \left(\widetilde{D}_i - \widetilde{D}_i^\dagger \right) - 2\widetilde{S} - S_{L\widetilde{L}^\dagger} + \bar{S}_{L\widetilde{L}^\dagger} + S_{L\widetilde{L}} - \bar{S}_{L\widetilde{L}} + \widetilde{S}_g \,, \tag{B.4}$$

where

$$\begin{aligned}
D_i = \frac{1}{2E_L} &\int d\vec{p}_{N_i} d\vec{p}_h (2\pi)^4 \delta^{(4)} (p_{N_i} - p_L - p_h) \\
&\times \left\{ f_{\widetilde{N}_i} (1 - f_L)(1 - f_h^{eq}) |\hat{A}(\widetilde{N}_i \to Lh)|^2 - f_h^{eq} f_L (1 + f_{\widetilde{N}_i}) |\hat{A}(Lh \to \widetilde{N}_i)|^2 \right\} ,
\end{aligned} \tag{B.5}$$

$$\widetilde{D}_i = \frac{1}{2E_L}\int d\vec{p}_{N_i} d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{N_i} - p_L - p_H) \tag{B.6}$$
$$\times \left\{ f_{\widetilde{N}_i}(1+f_{\widetilde{L}})(1+f_H^{eq})|\hat{A}(\widetilde{N}_i \to \widetilde{L}H)|^2 - f_H^{eq} f_{\widetilde{L}}(1+f_{\widetilde{N}_i})|\hat{A}(\widetilde{L}H \to \widetilde{N}_i)|^2 \right\} ,$$

and analogous expressions for $\bar{D}_i$ and $\widetilde{D}_i^\dagger$, just changing particles by antiparticles.

As in the previous section, the scattering terms

$$S = \frac{1}{2E_L}\int d\vec{p}_h d\vec{p}_{\bar{L}} d\vec{p}_{\bar{h}} (2\pi)^4 \delta^{(4)}(p_L + p_h - p_{\bar{L}} - p_{\bar{h}})$$
$$\times \left\{ f_L f_h^{eq}(1-f_{\bar{L}})(1-f_h^{eq})|M_{sub}(Lh \to \bar{L}\bar{h})|^2 - \right.$$
$$\left. - \; f_{\bar{L}} f_h^{eq}(1-f_L)(1-f_h^{eq})|M_{sub}(\bar{L}\bar{h} \to Lh)|^2 \right\} , \tag{B.7}$$

$$\widetilde{S} = \frac{1}{2E_L}\int d\vec{p}_H d\vec{p}_{\bar{L}} d\vec{p}_{\bar{H}} (2\pi)^4 \delta^{(4)}(p_L + p_H - p_{\bar{L}} - p_{\bar{H}})$$
$$\times \left\{ f_{\widetilde{L}} f_H^{eq}(1+f_{\widetilde{L}^\dagger})(1+f_H^{eq})|M_{sub}(\widetilde{L}H \to \widetilde{L}^\dagger H^\dagger)|^2 - \right.$$
$$\left. - \; f_{\widetilde{L}^\dagger} f_H^{eq}(1+f_{\widetilde{L}})(1+f_H^{eq})|M_{sub}(\widetilde{L}^\dagger H^\dagger \to \widetilde{L}H)|^2 \right\} , \tag{B.8}$$

etc., are defined in terms of the subtracted amplitudes (see eqs. (A.13),(A.14)). The thermal width of sneutrinos is $\Gamma^{i,th} = \Gamma_f^{i,th} + \Gamma_s^{i,th}$, with

$$\Gamma_f^{i,th} = \frac{1}{2M_i}\int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{\widetilde{N}_i} - p_L - p_H)\left[(1-f_L^{eq})(1-f_h^{eq}) + f_L^{eq} f_h^{eq}\right]|A_i^f|^2 , \tag{B.9}$$

$$\Gamma_s^{i,th} = \frac{1}{2M_i}\int d\vec{p}_L d\vec{p}_H (2\pi)^4 \delta^{(4)}(p_{\widetilde{N}_i} - p_L - p_H)\left[(1+f_{\widetilde{L}}^{eq})(1+f_H^{eq}) + f_{\widetilde{L}}^{eq} f_H^{eq}\right]|A_i^s|^2 . \tag{B.10}$$

Finally, the scattering terms S_g, $\widetilde{S}_g$ correspond to the fast MSSM gaugino interactions $LL \leftrightarrow \widetilde{L}\widetilde{L}$ and are given by

$$S_g = \frac{1}{2E_L}\int d\vec{p}_1 d\vec{p}_2 d\vec{p}_3 (2\pi)^4 \delta^{(4)}(p_L + p_1 - p_2 - p_3)$$
$$\times \left\{ f_{\widetilde{L}}(p_2) f_{\widetilde{L}}(p_3)(1-f_L(p_L))(1-f_L(p_1))|M(\widetilde{L}\widetilde{L} \leftrightarrow LL)|^2 - \right.$$
$$\left. - \; f_L(p_L)) f_L(p_1)(1+f_{\widetilde{L}}(p_2))(1+f_{\widetilde{L}}(p_3))|M(LL \leftrightarrow \widetilde{L}\widetilde{L})|^2 \right\}$$
$$- \{\text{particles} \to \text{antiparticles}\} , \tag{B.11}$$

and analogously for $\widetilde{S}_g$. Since these interactions are in equilibrium, we will not include them in our set of Boltzmann equations, but we shall impose that the chemical potentials for leptons and sleptons are equal, $\mu_s = \mu_f$.

The derivation of the out-of-equilibrium condition is somehow lengthy, but completely analogous to the standard case described in appendix A, so we do not give many details here (see for example [10]). The basic point is that at $\mathcal{O}(\epsilon)$, we can approximate the (s)lepton and anti-(s)lepton distributions by the equilibrium ones, and then use the following relations between them

$$(1-f_h^{eq})(1-f_L^{eq}) = e^{E_{N_i}} f_{\widetilde{N}_i}^{eq}\left[(1-f_h^{eq})(1-f_L^{eq}) + f_h^{eq} f_L^{eq}\right] , \tag{B.12}$$

$$(1+f_H^{eq})(1+f_{\widetilde{L}}^{eq}) = e^{E_{N_i}} f_{\widetilde{N}_i}^{eq}\left[(1+f_H^{eq})(1+f_{\widetilde{L}}^{eq}) + f_H^{eq} f_{\widetilde{L}}^{eq}\right] , \tag{B.13}$$

to reproduce the sneutrino thermal width, following the same procedure as in appendix A. Finally, we obtain:

$$\begin{aligned}
\frac{\partial f_{\mathcal{L}}}{\partial t} - p_L H \frac{\partial f_{\mathcal{L}}}{\partial p_L} &= \sum_i \frac{1}{2E_L} \int d\vec{p}_{\widetilde{N}_i} d\vec{p}_H (2\pi)^4 \delta(p_{\widetilde{N}_i} - p_L - p_H) |A_i^f|^2 \\
&\times \left\{ \epsilon_{f_i} (f_{\widetilde{N}_i} - f_{\widetilde{N}_i}^{eq}) \left[(1 - f_L^{eq})(1 - f_h^{eq}) + f_h^{eq} f_L^{eq} \right] - \right. \\
&\left. - \frac{1}{2} f_{\mathcal{L}} (f_{\widetilde{N}_i} + f_h^{eq}) \right\} , \qquad \text{(B.14)} \\
\frac{\partial f_{\widetilde{\mathcal{L}}}}{\partial t} - p_L H \frac{\partial f_{\widetilde{\mathcal{L}}}}{\partial p_L} &= \sum_i \frac{1}{2E_L} \int d\vec{p}_{\widetilde{N}_i} d\vec{p}_H (2\pi)^4 \delta(p_{\widetilde{N}_i} - p_L - p_H) |A_i^s|^2 \\
&\times \left\{ \epsilon_{s_i} (f_{\widetilde{N}_i} - f_{\widetilde{N}_i}^{eq}) \left[(1 + f_{\widetilde{L}}^{eq})(1 + f_H^{eq}) + f_H^{eq} f_{\widetilde{L}}^{eq} \right] + \right. \\
&\left. + \frac{1}{2} f_{\widetilde{\mathcal{L}}} (f_{\widetilde{N}_i} - f_H^{eq}) \right\} , \qquad \text{(B.15)} \\
\mu_f &= \mu_s \qquad \text{(B.16)}
\end{aligned}$$

C. Analytic approximation in kinetic equilibrium

Our starting point are the Boltzmann equations for the (s)lepton asymmetries (3.47) and (3.48), with the constraint $\mu_f = \mu_s \equiv \mu$ imposed by fast gaugino interactions. In this approximation, we keep the phase space and statistical factors, crucial in soft leptogenesis, but we approximate the Fermi and Bose distributions by the Maxwell-Boltzmann one, since this is enough to obtain a non-vanishing CP asymmetry and allows to perform analitically the energy integrals. After integrating over the (s)lepton energy, we get

$$\begin{aligned}
\frac{dY_{\mathcal{L}}}{dz} &= \frac{K_f z^2}{2\pi^2 s} \int_z^\infty d\bar{E}_N e^{-\bar{E}_N} \left\{ -\epsilon \left(\frac{Y_{\widetilde{N}}}{Y_{\widetilde{N}}^{eq}} - 1 \right) \Big[\lambda(1, x_L, x_h) y_N (1 + 2e^{-\bar{E}_N}) \right. \\
&- \left(e^{-\bar{E}_N(1-x_L+x_h)/2} + e^{-\bar{E}_N(1+x_L-x_h)/2} \right) \left(e^{y_N \lambda(1,x_L,x_h)/2} - e^{-y_N \lambda(1,x_L,x_h)/2} \right) \Big] \\
&- \left. \lambda(1, x_L, x_h) y_N \mu T^2 \right\} \qquad \text{(C.1)} \\
\frac{dY_{\widetilde{\mathcal{L}}}}{dz} &= \frac{K_s z^2}{2\pi^2 s} \int_z^\infty d\bar{E}_N e^{-\bar{E}_N} \left\{ \epsilon \left(\frac{Y_{\widetilde{N}}}{Y_{\widetilde{N}}^{eq}} - 1 \right) \Big[\lambda(1, x_{\widetilde{L}}, x_H) y_N (1 + 2e^{-\bar{E}_N}) \right. \\
&+ \left(e^{-\bar{E}_N(1-x_{\widetilde{L}}+x_H)/2} + e^{-\bar{E}_N(1+x_{\widetilde{L}}-x_H)/2} \right) \left(e^{y_N \lambda(1,x_{\widetilde{L}},x_H)/2} - e^{-y_N \lambda(1,x_{\widetilde{L}},x_H)/2} \right) \Big] \\
&- \left. \lambda(1, x_{\widetilde{L}}, x_H) y_N \mu T^2 \right\} \qquad \text{(C.2)}
\end{aligned}$$

The washout term in the above equation is the standard one, for a non-vanishing (s)lepton mass, so, after integrating over $\bar{E}_N$ gives the Bessel function $\mathcal{K}_1(z)$. Notice that the chemical potential μ is related to the total lepton asymmetry $Y_{\mathcal{L}_T} = Y_{\mathcal{L}} + Y_{\widetilde{\mathcal{L}}}$ by

$$\mu = \frac{\pi^2}{2} \frac{s}{m_L^2 \mathcal{K}_2(m_L/T) + m_{\widetilde{L}}^2 \mathcal{K}_2(m_{\widetilde{L}}/T)} Y_{\mathcal{L}_T} \, . \qquad \text{(C.3)}$$

Using that

$$e^{y_N \lambda/2} - e^{-y_N \lambda/2} = 2 \sinh \left(\frac{y_N \lambda}{2} \right) = 2 \sum_{n=0}^\infty \frac{(y_N \lambda/2)^{2n+1}}{(2n+1)!}, \qquad \text{(C.4)}$$

we can perform the integral over $\bar{E}_N$, order by order in y_N. The resulting integrated Boltzmann equation for $Y_{\mathcal{L}_T}$ can be written as:

$$\frac{dY_{\mathcal{L}_T}}{dz} = 2\,\epsilon\, K(Y_{\widetilde{N}} - Y^{eq}_{\widetilde{N}})\frac{F_1(z)}{\mathcal{K}_2(z)} - \frac{Kz^3}{4}\mathcal{K}_1(z)F_2(z)Y_{\mathcal{L}_T}\,. \tag{C.5}$$

with $F_1(z) = F_1^s(z) + F_1^f(z)$ and

$$\begin{aligned} F_1^s(z) &= \lambda(1, x_{\widetilde{L}}, x_H)\left\{\lambda(1, x_{\widetilde{L}}, x_H)[\mathcal{K}_1(z) + \mathcal{K}_1(2z)]z \right.\\ &\quad \left. + f_1^s[z(3 - x_{\widetilde{L}} + x_H)/2] + f_1^s[z(3 + x_{\widetilde{L}} - x_H)/2]\right\}\,, \qquad (C.6)\\ F_1^f(z) &= (1 - x_L + x_h)\lambda(1, x_L, x_h)\left\{-\lambda(1, x_L, x_h)[\mathcal{K}_1(z) + \mathcal{K}_1(2z)]z \right.\\ &\quad \left. + f_1^f[z(3 - x_L + x_h)/2] + f_1^f[z(3 + x_L - x_h)/2]\right\}\,. \qquad (C.7) \end{aligned}$$

The function f_1^s is given by

$$\begin{aligned} f_1^s(za) &= \sum_{n=0}^{\infty} \frac{1}{2^{3n} n!}\left(\frac{z}{a}\right)^{n+1} [\lambda(1, x_{\widetilde{L}}, x_H)]^{2n+1}\mathcal{K}_{n+1}(za)\\ &\simeq \frac{z}{a}\lambda(1, x_{\widetilde{L}}, x_H)\mathcal{K}_1(za) + \frac{1}{8}\left(\frac{z}{a}\right)^2 [\lambda(1, x_{\widetilde{L}}, x_H)]^3\mathcal{K}_2(za)\\ &\quad + \frac{1}{2^7}\left(\frac{z}{a}\right)^3 [\lambda(1, x_{\widetilde{L}}, x_H)]^5\mathcal{K}_3(za) + \ldots\,, \end{aligned} \tag{C.8}$$

and f_1^f has the same structure, changing $\widetilde{L} \to L$ and $H \to h$.

When thermal masses are neglected, $F_1^s = F_1^f$, and the two different arguments of $f_1^{s,f}$ became the same $(3z/2)$, leading to a global factor of 4 and the simplified function $F^{(0)}(z)$ of Eq. (3.50).

Finally, the function $F_2(z)$ is defined as

$$F_2(z) = 2T^2\,\frac{[\lambda(1, x_L, x_h)]^2(1 - x_L - x_h) + [\lambda(1, x_{\widetilde{L}}, x_H)]^2}{m_L^2\mathcal{K}_2(m_L/T) + m_{\widetilde{L}}^2\mathcal{K}_2(m_{\widetilde{L}}/T)}\,, \tag{C.9}$$

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