Preprint typeset in JHEP style - HYPER VERSION

IFT-UAM/CSIC-10-28
FTUAM-10-07
ULB-TH/10-15

# Dark Coupling and Gauge Invariance

**M.B. Gavela,**

*Departamento de Física Teórica, Universidad Autónoma de Madrid*
*IFT-UAM/CSIC, 28049 Cantoblanco, Madrid, Spain*

**L. Lopez Honorez,**

*Departamento de Física Teórica, Universidad Autónoma de Madrid*
*IFT-UAM/CSIC, 28049 Cantoblanco, Madrid, Spain and Service de Physique*
*Théorique, Université Libre de Bruxelles, Belgium*

**O. Mena,**

*Instituto de Física Corpuscular, IFIC, CSIC and Universidad de Valencia, Spain*

**S. Rigolin,**

*Dipartimento di Fisica, Universitá di Padova and*
*INFN Padova, Via Marzolo 8, I-35131, Padova, Italy*

Abstract: We study a coupled dark energy–dark matter model in which the energy-momentum exchange is proportional to the Hubble expansion rate. The inclusion of its perturbation is required by gauge invariance. We derive the linear perturbation equations for the gauge invariant energy density contrast and velocity of the coupled fluids, and we determine the initial conditions. The latter turn out to be adiabatic for dark energy, when assuming adiabatic initial conditions for all the standard fluids. We perform a full Monte Carlo Markov Chain likelihood analysis of the model, using WMAP 7-year data.

## 1. Introduction

The true substance of dark energy and dark matter is unknown although it should account for about 95% of the matter–energy content of our universe today [1]. While the couplings of dark fluids to photons and normal matter are severely constrained [2], nothing prevents dark matter–dark energy interactions [3–21]. At the level of the background evolution equations, it is customary to parametrize the coupling between the two dark sectors [22] as:

$$\dot{\bar{\rho}}_{dm} + 3\mathcal{H}\bar{\rho}_{dm} = a\overline{Q}_{dm}\,, \tag{1.1}$$

$$\dot{\bar{\rho}}_{de} + 3\mathcal{H}\bar{\rho}_{de}(1+w) = a\overline{Q}_{de}\,, \tag{1.2}$$

where $\bar{\rho}_{dm}$, $\bar{\rho}_{de}$ denote the dark matter and dark energy energy densities, respectively, and $\overline{Q}_{dm} = -\overline{Q}_{de}$ encodes the coupling between those two dark sectors and drives the energy exchange between them. The dot indicates derivative with respect to the conformal time $d\tau = dt/a$, with $\mathcal{H} = \dot{a}/a \equiv a\overline{H}$ denoting the background expansion rate, while $w \equiv w_{de} = \bar{p}_{de}/\bar{\rho}_{de}$ stands for the background dark energy equation of state and pressureless dark matter is assumed: $w_{dm} = \bar{p}_{dm}/\bar{\rho}_{dm} = 0$. From now on, barred quantities are to be considered as the background quantities.

The initial conditions for the several components populating the early universe have been explored to a large extent. They were first analyzed for all cosmic fluids but dark energy (see *e.g.* Ref. [23] and references therein), with the result that adiabatic initial conditions were one possibility. It was also noticed that the choice of gauge could be a delicate issue: a safe alternative proposed was to use a gauge invariant formalism [22,24–26]. The initial conditions for the case of dynamical dark energy as an uncoupled quintessence field have been also derived [27–33], including a gauge invariant treatment [33]: they turned out to be adiabatic if those for the traditional fluids were adiabatic. Furthermore, the formalism in Ref. [33] has been recently applied to the case of a coupled dark energy-dark matter systems which mimic uncoupled models at early times, both at the background and perturbation levels [18] for the viable parameter space: as expected, adiabatic initial conditions for dark energy naturally resulted then. Here we consider a different class of dark couplings, not negligible at early times. It is also illustrated that the gauge invariant formalism is particularly illuminating for the determination of the correct perturbation equations, for a general coupled theory.

The structure of the paper is as follows. In Section 2, the notation is set and the gauge invariant equations -at linear order in perturbation theory- for a coupled fluid are derived. In particular, we study in Section 2.2 the case of a (covariant) dark matter–dark energy interaction proportional to the Hubble rate. In Section 3, following the method proposed in Ref. [33], we derive the corresponding initial conditions for dark energy. Then in Section 4, we constrain the type of coupled models analyzed, using several data sets. Section 5 contains the conclusions.

## 2. Gauge invariant perturbation equations

Following Ref. [22], the FRW metric, up to first order in perturbation theory, can be written as:

$$g_{\mu\nu}dx^\mu dx^\nu = a^2\left[-(1+2A)d\tau^2 - B_i d\tau dx^i + (\gamma_{ij}+2H_{ij})dx^i dx^j\right] , \qquad (2.1)$$

where $\gamma_{ij}$ is the 3D flat metric with positive signature. The perturbations $A$, $B_i$ and $H_{ij}$ are functions of time and space and are in general gauge-dependent, *i.e.* not invariant under an infinitesimal coordinate transformation:

$$(x^0, x^i) \to (\hat{x}^0, \hat{x}^i) = (x^0 - T, x^i - L^i) . \qquad (2.2)$$

Particularizing to the case of scalar metric perturbations, two gauge invariant quantities[1] can be defined [24], the most popular being the so-called Bardeen potentials $\Phi_B$ and $\Psi_B$.

In describing the evolution of a given fluid "$a$", other gauge dependent quantities are introduced, such as the perturbed 4–velocity and the energy–momentum tensor, which can be expressed by:

$$u^\mu_a = \frac{1}{a}(1-A, v^i_a) , \qquad (2.3)$$

$$T^{\mu\nu}_a = \bar{\rho}_a(1+\delta_a)u^\mu_a u^\nu_a + \tau^{\mu\nu}_a , \qquad (2.4)$$

where $v^i_a$ is the peculiar velocity perturbation of the fluid, $\delta_a$ the density perturbation and $\tau^{\mu\nu}_a$ the stress tensor, whose components in first order perturbation theory read

$$\tau_{a0}^{\ 0} = 0 , \qquad \tau_{a0}^{\ i} = \bar{p}_a\, v^i_a , \qquad \tau_{aj}^{\ i} = \bar{p}_a\left[\left(1+\pi^L_a\right)\gamma^i_j + (\pi^T_a)^i_j\right] . \qquad (2.5)$$

In what follows, we deal with the Fourier transformations of the scalar part of the metric and fluid perturbations. See Appendix A for details. In the equations above, $\pi^L_a$ and $\pi^T_a$ denote the isotropic and anisotropic scalar pressure perturbations, respectively, while $v_a$ is the scalar part of the peculiar velocity. Associated gauge invariant quantities can be defined, paralleling the two gauge invariant variables for scalar metric perturbations. Following the notation in Ref. [33], a possible gauge invariant formulation for $\delta_a$, $v_a$ and the stress–tensor components $\pi^L_a$ and $\pi^T_a$ is:

$$\Delta_a = \delta_a - \frac{\dot{\bar{\rho}}_a}{\bar{\rho}_a}\frac{\mathcal{R}}{\mathcal{H}} , \qquad V_a = v_a - \frac{\dot{H}_T}{k} \qquad (2.6)$$

$$\Gamma_a = \pi^L_a - \frac{c^2_{Aa}}{w_a}\delta_a , \qquad \Pi_a = \pi^T_a . \qquad (2.7)$$

The coefficient $c^2_{Aa}$ entering in the entropy perturbation $\Gamma_a$ is the adiabatic sound speed of the fluid $c^2_{Aa} = \dot{\bar{p}}_a/\dot{\bar{\rho}}_a$ and $w_a$ is the equation of state of the fluid.

We focus next on the derivation of the gauge invariant equations for the matter density contrast $\Delta_a$ and the fluid velocity $V_a$, for a generic coupled fluid.

---

[1] The transformation properties of the metric perturbations defined in Eq. (2.1) and the explicit definition of the Bardeen potentials is reminded in Appendix A.

### 2.1 Coupled fluids in general

Consider the full (background plus perturbations) continuity equation for fluid "$a$":

$$\nabla_\mu T^{\mu\nu}_a = Q^\nu_a \qquad , \qquad \sum_a Q^\nu_a = 0\,, \tag{2.8}$$

where $T^{\mu\nu}_a$ denotes the corresponding energy-momentum tensor and the vector $Q^\nu_a$ governs the energy-momentum transfer. The constraint on the right accounts for total energy–momentum conservation. Following Ref. [22], $Q^\nu_a$ can be written as:

$$Q^\mu_a = Q_a u^\mu_a + j^\mu_a\,, \qquad \text{with} \qquad j^\mu_a u^a_\mu = 0\,, \tag{2.9}$$

$$Q_a = \overline{Q}_a\left(1 + \frac{\delta Q_a}{\overline{Q}_a}\right) \equiv \overline{Q}_a\left(1 + \varepsilon_a\right)\,, \tag{2.10}$$

where $j^\mu_a$ and $\varepsilon_a$ are perturbation parameters. In particular, the background contributions reduce to the coupled dark energy-dark matter case in Eqs. (1.1) and (1.2), for $Q^\nu_{de} = -Q^\nu_{dm}$. Defining for simplicity $j^i_a = \bar{\rho}_a f^i_a/a$, the total coupling reads

$$Q^\mu_a = \frac{1}{a}\left(\overline{Q}_a\left[1-(A-\varepsilon_a)\right], \overline{Q}_a v^i_a + \bar{\rho}_a f^i_a\right)\,. \tag{2.11}$$

Let's denote by $f_a$ the Fourier transform of the scalar part of $f^i_a$. One can show that $f_a$ is invariant under gauge transformations, while $\varepsilon_a$ transforms as

$$\widehat{\varepsilon}_a = \varepsilon_a - \frac{\dot{\overline{Q}}_a}{\overline{Q}_a}T\,, \tag{2.12}$$

where the "hat" denotes gauge transformed quantities. This suggest a possible choice of gauge invariant variables for the coupling perturbation parameters, given by

$$E_a = \varepsilon_a - \frac{\dot{\overline{Q}}_a}{\overline{Q}_a}\frac{\mathcal{R}}{\mathcal{H}}\,, \tag{2.13}$$

$$F_a = f_a\,. \tag{2.14}$$

The gauge invariant choice in Eq. (2.13) is analogous to that for $\Delta_a$ in Eq. (2.6).

With the help of these variables, the scalar perturbation equations for the matter density contrast $\Delta_a$ and the peculiar velocity $V_a$, for a generic coupled fluid, read:

$$\dot{\Delta}_a = -3\mathcal{H}\left[\left(c^2_{Aa} - w_a\right)\Delta_a + w_a\Gamma_a\right] - k(1+w_a)V_a + 3\mathcal{H}\bar{q}_a\left[\mathcal{A} + E_a - \Delta_a\right]\,, \tag{2.15}$$

$$\begin{aligned}\dot{V}_a = & -\mathcal{H}\left(1-3c^2_{Aa}\right)V_a + \frac{k}{1+w_a}\left[c^2_{Aa}\Delta_a + w_a\left(\Gamma_a - \frac{2}{3}\Pi_a\right)\right] + k\left(\Psi_B - 3c^2_{Aa}\Phi_B\right) \\ & -3\mathcal{H}\bar{q}_a\frac{c^2_{Aa}}{1+w_a}\left(V_a - k\frac{\Phi_B}{\mathcal{H}}\right) + \frac{aF_a}{1+w_a}\,,\end{aligned} \tag{2.16}$$

where $\mathcal{A}$ is a metric gauge invariant quantity [22], whose expression is given in Eq. (A.19). The quantity $\bar{q}_a$ accounts for the energy transfer $\overline{Q}_a$ in Eqs. (2.15) and (2.16), rescaled as follows

$$\bar{q}_a \equiv \frac{a\overline{Q}_a}{3\mathcal{H}\bar{\rho}_a}\,. \tag{2.17}$$

For vanishing $\bar{q}_a$ and $F_a$, Eqs. (2.15) and (2.16) reduce to those in Ref. [33].

### 2.2 Coupling proportional to $H$

Coupled models with a dark matter-dark energy coupling proportional to the Hubble expansion rate have been studied at the level of linear perturbations in several recent works, see for example Refs. [14–17, 34]. Perturbations in the expansion rate were neglected, though. To analyze the issue, the results of the previous section will be particularized to the following coupling:

$$Q^{\nu}_{dm} = \xi H \rho_{de}\, u^{\nu}_{dm} = -Q^{\nu}_{de}\,. \tag{2.18}$$

Here $Q^{\nu}_{dm}$ is chosen parallel to the dark matter four velocity $u^{\nu}_{dm}$ to avoid momentum transfer in the dark matter rest frame [14]. The evolution equation for the dark matter velocity remains then equal to that of baryons, avoiding the violation of the weak equivalence principle. Moreover, the authors of Ref. [14] pointed out that such a coupled models could suffer from non-adiabatic instabilities if the coupling $Q_{dm}$ is chosen proportional to the dark matter energy density. In Ref. [15–17], it was shown though that such instabilities could be avoided in a minimal way choosing a coupling $Q_{dm}$ proportional to the dark energy density[2].

It is important to notice that, in order to deal with a consistent model, $H$ in Eq. (2.18) must denote the total expansion rate (background plus perturbations), $H = \overline{H} + \delta H$, while in all previous studies only the background quantity was considered. The inclusion of $\delta H$ is mandatory to preserve gauge invariance, as we proceed to illustrate. For the model in Eq. (2.18) one obtains:

$$\overline{Q}_{dm} = \xi \overline{H} \bar{\rho}_{de}\,, \tag{2.19}$$

$$\varepsilon_{dm} = \frac{\delta H}{H} + \frac{\delta\rho_{de}}{\bar{\rho}_{de}} \equiv \mathcal{K} + \delta_{de}\,. \tag{2.20}$$

The $\mathcal{K}$ term (see Eq. (A.9)), represents the expansion rate perturbation, overlooked in all the references mentioned above. Indeed, $\mathcal{K}$ depends on the time slicing, so that the coupling perturbation $\varepsilon_a$ gauge transforms as:

$$\widehat{\varepsilon}_a - \varepsilon_a \equiv \frac{\dot{\overline{Q}}_a}{\overline{Q}_a} T = \frac{\dot{\overline{H}}}{\overline{H}} T + \frac{\dot{\bar{\rho}}_{de}}{\bar{\rho}_{de}} T \;=\; \left(\widehat{\mathcal{K}} - \mathcal{K}\right) + \left(\widehat{\delta}_{de} - \delta_{de}\right). \tag{2.21}$$

---

[2] Would an interaction proportional to the dark matter density be studied instead, it would be necessary to consider a time dependent dark energy equation of state, in order to avoid early time instabilities, thus introducing at least one extra free parameter, see Ref. [18].

To our knowledge this result was not explicitly discussed elsewhere. We will see in Sec. 4 that the extra contribution resulting from $\delta H$ has little quantitative impact on the physical constraints obtained from data, while being essential for gauge invariance.

Before proceeding further let us comment on the covariance of the coupling of Eq. (2.18). First of all, the dark energy density can be rewritten as $\rho_{de} = T^{\mu\nu}_{de} u^{de}_{\mu} u^{de}_{\nu}$. Moreover, we can express the Hubble expansion rate in terms of the covariant derivative of the four velocity defined in Eq. (2.3). Indeed, it is straightforward to verify that the background quantity associated to $u^{\mu}_{a;\mu}$ is directly proportional to the expansion rate $\overline{H}$. Following [22] one has:

$$\Theta_a = u^{\mu}_{a;\mu} = 3\overline{H}(1+\mathcal{K}_a)\,. \tag{2.22}$$

Under gauge transformations, the perturbation $\mathcal{K}_a$ (associated to the $a$–fluid) transforms like:

$$\widehat{\mathcal{K}_a} - \mathcal{K}_a = \frac{\dot{\overline{H}}}{\overline{H}}\, T \tag{2.23}$$

which is exactly what is needed to preserve the gauge invariance of the coupled model under study, see Eq. (2.21). In the following, we will use for definiteness the total matter expansion rate $\Theta_T = u^{\mu}_{T;\mu} = 3\overline{H}(1+\mathcal{K})$, denoting with $\mathcal{K}$ the perturbation associated to the total fluid. Finally the coupling of Eq. (2.18) can be written in a covariant way as:

$$Q^{\nu}_{dm} = \xi \frac{\Theta_T}{3}\, T^{\alpha\beta}_{de} u^{de}_{\alpha} u^{de}_{\beta}\, u^{\nu}_{dm} = -Q^{\nu}_{de}\,. \tag{2.24}$$

We can now particularize Eqs. (2.15) and (2.16) to our coupling. Expressing $\mathcal{K}$ in terms of gauge invariant quantities one obtains:

$$E_a = \Delta_{de} + \left(\frac{x^2}{3} - \frac{3}{2}(1+w_T)\right)\widetilde{V}_T + 2\Phi_B \tag{2.25}$$

where $w_T$ and $V_T$ is the equation of state and velocity of the total fluid. The density and velocity perturbation equations then read:

$$
\begin{aligned}
\frac{\dot{\Delta}_{dm}}{\mathcal{H}} &= -x^2\, \widetilde{V}_{dm} + \xi \frac{\bar{\rho}_{de}}{\bar{\rho}_{dm}} \left[ (\Delta_{de} - \Delta_{dm}) + \frac{x^2}{3} \widetilde{V}_T \right] \ , & (2.26)\\
\frac{\dot{\widetilde{V}}_{dm}}{\mathcal{H}} &= -\left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right) \widetilde{V}_{dm} - \left(\Phi_B + \Omega_\nu \widetilde{\Pi}_\nu\right) \ , & (2.27)\\
\frac{\dot{\Delta}_{de}}{\mathcal{H}} &= -3(c_S^2 - w)\Delta_{de} - (1+w)\, x^2\, \widetilde{V}_{de} + 9\,(1+w)\left(c_S^2 - c_A^2\right)\left(\Phi_B - \widetilde{V}_{de}\right) \\
&\qquad -\xi \left[ \frac{x^2}{3} \widetilde{V}_T - 3\left(c_S^2 - c_A^2\right)\left(\Phi_B - \widetilde{V}_{de}\right)\right] \ , & (2.28)\\
\frac{\dot{\widetilde{V}}_{de}}{\mathcal{H}} &= -\left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - 3c_S^2\right) \widetilde{V}_{de} - \left(1 + 3c_S^2\right)\Phi_B - \Omega_\nu \widetilde{\Pi}_\nu + \frac{c_S^2}{1+w}\Delta_{de} + \\
&\qquad + \frac{\xi}{1+w}\left[\left(1 + c_S^2\right)\widetilde{V}_{de} - \widetilde{V}_{dm} - c_S^2 \Phi_B\right] \ , & (2.29)
\end{aligned}
$$

the rescaled quantities $\widetilde{V} = V/x$ and $\widetilde{\Pi} = \Pi/x^2$ were used, with $x = k/\mathcal{H}$. In deriving these equations, the dark energy entropy perturbation $\Gamma_{de}$ has been rewritten in terms of $\Delta_{de}, V_{de}$ and $\Phi_B$, see Eq. (A.26). $c_A^2$ and $c_S^2$ are the dark energy adiabatic sound speed and the rest frame sound speed, respectively. In the following we work in the framework of constant $w$, $c_A^2 = w$ and $c_S^2 = 1$.

## 3. Initial conditions

In Ref. [33], whose gauge invariant formalism we follow, the solution of the system of differential equations for the perturbations is reduced to that of a simple eigenvalues/eigenvectors problem:

$$
U' \equiv \frac{dU}{d\ln x} = A(x)\, U \, . \qquad (3.1)
$$

Here $A(x)$ encodes the evolution equations for all the universe components, and

$$
U^T \equiv \{\Delta_{dm}, \tilde{V}_{dm}, \Delta_\gamma, \tilde{V}_\gamma, \Delta_b, \Delta_\nu, \tilde{V}_\nu, \tilde{\Pi}_\nu, \Delta_{de}, \tilde{V}_{de}\} \qquad (3.2)
$$

is an array of gauge invariant perturbations, where the subscripts $\gamma$, $b$ and $\nu$ stand for photons, baryons and neutrinos, respectively. No anisotropic stress for dark energy and negligible anisotropic stress for photons (due to large Thompson damping) are assumed.

The evolution equations for baryons, photons, and neutrinos are unaltered by the presence of the dark coupling and we obviate them below. In contrast, the dark matter and dark energy perturbation equations for the case under study are significantly modified. The exact form of the correspondent $A(x)$ matrix can be easily derived from Eqs. (2.26)–(2.29).

To obtain the initial conditions for cosmological perturbations, it is necessary to study the evolution of the several cosmic components at a very early stage, when the universe was radiation dominated and $\mathcal{H} = 1/\tau$. One is interested in the time dependence of all perturbations on super–horizon scales, *i.e* for $x = k\tau \ll 1$.

### 3.1 The $A_0$ matrix

At early times $x \ll 1$, the $A(x)$ matrix can be approached by a constant matrix $A_0$, if no divergence appears when taking the $\lim_{x\to 0} A(x)$. The assumption that the universe is radiation dominated at early times implies $w_T = 1/3$, $\bar{\rho}_T = \bar{\rho}_{rad}$ and

$$\Omega_\nu = \bar{\rho}_\nu/\bar{\rho}_{rad} = R_\nu, \quad \Omega_\gamma = 1 - R_\nu \quad \text{and} \quad \frac{\Omega_{de}}{\Omega_{dm}} = \frac{\bar{\rho}_{de}}{\bar{\rho}_{dm}} \propto x^{-(3w+\xi)} \,. \tag{3.3}$$

$w < -1/3$ is assumed as well, in order to obtain cosmic acceleration, which implies that $(3w + \xi)$ can be always taken negative for $\xi < 0$.

Using Eqs. (2.26)–(2.29) and taking the $x \to 0$ limit, the following entries in the $A_0$ matrix associated to $\Delta_{de}$ and $\tilde{V}_{de}$ result:

$$\begin{pmatrix} 0 & 0 & \frac{R_\gamma}{4}(\alpha+\beta\xi) & R_\gamma(\alpha+\beta\xi) & 0 & \frac{R_\nu}{4}(\alpha+\beta\xi) & R_\nu(\alpha+\beta\xi) & 0 & -\beta & -(\alpha+\beta\xi) \\ 0 & -\xi_r & -R_\gamma\,(1+\xi_r/4) & -4R_\gamma\,(1+\xi_r/4) & 0 & -R_\nu\,(1+\xi_r/4) & -4R_\nu\,(1+\xi_r/4) & -R_\nu & \frac{1}{1+w} & 1+2\xi_r \end{pmatrix}$$

where $\alpha = 9(1-w^2)$, $\beta = 3(1-w)$, $\xi_r = \xi/(1+w)$ and $R_\gamma = 1 - R_\nu$. The other lines in the $A_0$ matrix remain equal to the uncoupled case ones. Indeed the extra term in the $\Delta_{dm}$ equation proportional to $\xi\bar{\rho}_{de}/\bar{\rho}_{dm}$ can be safely neglected in the $x \to 0$ approximation. We thus recover the standard non interacting dark matter perturbation equation in the early universe. Notice that this was also the case of the viable coupled model discussed in Ref. [18].

### 3.2 Adiabatic initial conditions

Let $U_i$ be an eigenvector of $A_0$ with eigenvalue $\lambda_i$. The solution to the system in Eq. (3.1) can then be expressed as a linear combination of $x^{\lambda_i}U_i$ terms. Those corresponding to the largest eigenvalues will dominate the time evolution. We checked that for the model under study, Eq. (2.24), the dominant modes are associated to $\lambda_i = 0$ values and they suffice to specify the initial conditions. The subdominant modes decay in time as they correspond to negative real eigenvalues[3]. The dominant eigenvalue of the evolution matrix, $\lambda_i = 0$, is fourfold degenerate (as was the case for a universe without dynamical dark energy) and the corresponding four eigenvectors serve as a convenient basis to specify the initial conditions.

---

[3] Also, see Ref. [33] for the case of quintessence and Ref. [18] for coupled dark sectors with a different coupling.

Let us assume adiabatic initial conditions for all species but dark energy, as strongly constrained by WMAP data [1,35]. For each pair of components $a_1$ and $a_2$, the relative entropy perturbation, $S_{a_1a_2}$, vanishes:

$$S_{a_1a_2} = \frac{\Delta^0_{a_1}}{\dot{\bar\rho}_{a_1}/\bar\rho_{a_1}} - \frac{\Delta^0_{a_2}}{\dot{\bar\rho}_{a_2}/\bar\rho_{a_2}} = 0 \ . \tag{3.4}$$

For baryons, neutrinos, photons and dark matter this implies:

$$\Delta^0_{dm} = \Delta^0_b = \frac{3}{4}\Delta^0_\gamma = \frac{3}{4}\Delta^0_\nu \ , \tag{3.5}$$

from which one obtains:

$$\widetilde{V}^0_\gamma = \widetilde{V}^0_b = \widetilde{V}^0_\nu = \widetilde{V}^0_{dm} = -\frac{5}{4}\,\mathcal{P}\Delta^0_\gamma \qquad \text{and} \qquad \widetilde{\Pi}^0_\nu = -\mathcal{P}\Delta^0_\gamma \ , \tag{3.6}$$

with $\mathcal{P} = 1/(15+4R_\nu)$. Those are the standard adiabatic initial conditions for velocity perturbations and anisotropic stress. Solving the eigenvalue problem for our $A_0$ matrix, it follows that dark energy also obeys adiabatic initial conditions given by

$$\Delta^0_{de} = \frac{3}{4}\left(1 + w + \frac{\xi}{3}\right)\Delta^0_\gamma \, , \tag{3.7}$$

$$\widetilde{V}^0_{de} = -\frac{5\mathcal{P}}{4}\Delta^0_\gamma \, . \tag{3.8}$$

Consequently, adiabatic initial conditions for the matter and radiation components automatically imply adiabatic initial conditions for dark energy, alike to the case for tracking scalar quintessence [33] or those obtained for dark energy-dark matter couplings which do not depend explicitly on the Hubble rate[4].

As a final comment, notice that the previous results do not depend on the fact that we are using the expansion of the total fluid $\Theta_T$ (and its perturbation $\mathcal{K}$) to define the dark coupling in Eq. (2.24). In fact one could have used the expansion rate of any single specie, $\Theta_a$. In that case, Eq. (2.25) should be replaced by:

$$E_a = \Delta_{de} + \frac{x^2}{3}\widetilde{V}_a - \frac{3}{2}(1 + w_T)\widetilde{V}_T + 2\Phi_B \tag{3.9}$$

which implies that all the contributions of the dark coupling going as $x^2\widetilde{V}_T$ in Eqs. (2.26) and (2.28) should be replaced with $x^2\widetilde{V}_a$. This does not modify the expression of the matrix $A_0$ of Sec. 3.1, that encodes the evolution equations at early times, as in the limit $x \to 0$ all the $x^2$–terms can be neglected. As a consequence our results of Eqs. (3.7), and (3.8) would not be affected, and our conclusion on adiabatic initial conditions remains unchanged.

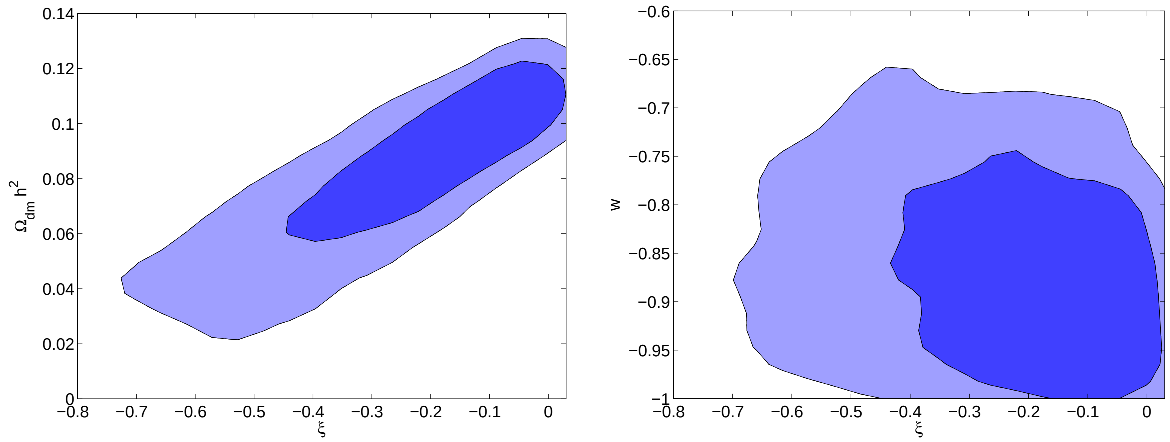

**Figure 1:** *Left (right) panel: 1σ and 2σ marginalized contours in the* $\xi$–$\Omega_{dm}h^2$ ($\xi$–$w$) *plane. The contours show the current constraints from WMAP7, HST, SN,* $H(z)$ *and LSS data taking into account the expansion rate perturbation* $\mathcal{K}$.

## 4. Data constraints

In this section we briefly revisit the constraints on the dark coupling $\xi$ presented in Ref. [16], adding to the analysis the contribution from the expansion rate perturbation $\mathcal{K}$ and imposing adiabatic initial conditions for all fluids. We have therefore modified the Boltzmann CAMB code [36] to incorporate the dark coupling $\xi$ and the $\mathcal{K}$ terms.

In the synchronous gauge, $\mathcal{K} = \theta_T/(3\mathcal{H}) + \dot{h}/(6\mathcal{H})$ and the perturbation equations reduce to:

$$\begin{aligned}
\dot{\delta}_{dm} &= -(kv_{dm} + \frac{1}{2}\dot{h}) + \xi\mathcal{H}\frac{\rho_{de}}{\rho_{dm}}\left(\delta_{de} - \delta_{dm}\right) + \xi\frac{\rho_{de}}{\rho_{dm}}\left(\frac{kv_T}{3} + \frac{\dot{h}}{6}\right) , && (4.1)\\
\dot{v}_{dm} &= -\mathcal{H}v_{dm} , && (4.2)\\
\dot{\delta}_{de} &= -(1+w)(kv_{de} + \frac{1}{2}\dot{h}) - 3\mathcal{H}\left(1-w\right)\left[\delta_{de} + \mathcal{H}\left(3(1+w)+\xi\right)\frac{v_{de}}{k}\right] && (4.3)\\
&\qquad -\xi\left(\frac{kv_T}{3} + \frac{\dot{h}}{6}\right) , \\
\dot{v}_{de} &= 2\mathcal{H}\left(1 + \frac{\xi}{1+w}\right)v_{de} + \frac{k}{1+w}\delta_{de} - \xi\mathcal{H}\frac{v_{dm}}{1+w} , && (4.4)
\end{aligned}$$

where $v_T$ is defined in Eq. (A.10).

We have extracted the cosmological parameters by means of the publicly available Markov Chain Monte Carlo package `cosmomc` [37]. The cosmological model is described by ten free parameters

$$\{\omega_b, \omega_{dm}, \theta_{CMB}, \tau, \Omega_k, f_\nu, w, \xi, n_s, A_s\} \ ,$$

---

[4] See Ref. [18] for $Q_a = \pm\Gamma\rho_{dm}$, where $\Gamma$ is a constant.

where $\omega_b = \Omega_b h^2$ and $\omega_{dm} = \Omega_{dm} h^2$ are the current baryon and dark matter densities respectively, $\theta_{CMB}$ is proportional to the ratio of the sound horizon to the angular diameter distance, $\tau$ is the reionization optical depth, $\Omega_k$ is the spatial curvature, $f_\nu = \Omega_\nu / \Omega_{dm}$ refers to the neutrino fraction, $n_s$ is the scalar spectral index and $A_s$ the amplitude of the primordial spectrum.

The analysis is restricted to negative couplings and also $w > -1$ (to ensure the avoidance of phantom behaviour), exactly as it we did previously in Ref. [16]. The basic data set we exploit here includes a prior on the Hubble parameter of $72 \pm 8$ km/s/Mpc from the Hubble key project (HST) [38], the constraints coming from the latest compilation of supernovae (SN) [39], the matter power spectrum (large scale structure data or LSS data) from the spectroscopic survey of Luminous Red Galaxies from the Sloan Digital Sky Survey survey [40], the $H(z)$ data from galaxy ages [41] and the WMAP7 data [1,35].

CMB constraints the amount of dark matter at redshift $\sim 1000$. In the presence of a negative dark coupling, the energy flows from dark matter to dark energy, thus dark matter energy density is smaller today as it can be seen in Fig. 1 (left panel). This effect is compensated for large scale structures by a larger growth of dark matter perturbation (see *e.g.* [42]). Figure 1, left (right) panel illustrates the $1\sigma$ and $2\sigma$ marginalized contours obtained in the $\xi$–$\Omega_{dm} h^2$ ($\xi$–$w$) plane. We verified that the results do not differ significantly if including WMAP5 data (as we had done in Ref. [16]) instead of WMAP7 data.

Overall, the results show that the addition to the analysis of the perturbation expansion rate $\mathcal{K}$ leaves basically unaffected the quantitative constraints on the cosmological parameters previously obtained in Ref. [16]. Indeed, all the additional terms introduced to make perturbations gauge invariant give negligible contributions at observable scales.

## 5. Conclusions

Interacting dark energy-dark matter cosmologies in which the coupling term is proportional to the Hubble expansion rate are revisited. While in previous works the perturbation in the Hubble expansion rate was neglected, it is illustrated here how the inclusion of such a term is mandatory to satisfy the gauge invariance of the theory. It also serves as a guide to define a covariant formulation of the dark sector interaction. In this work, the latter has been chosen to be expressed in terms of the expansion rate associated to the total fluid. This choice is however not unique, we could have used the expansion rate of any other fluid. For the case under study, we compute the linear perturbation evolution using a gauge invariant formalism. After imposing adiabatic initial conditions on the matter and radiation fluids, we find that the initial conditions for the coupled dark energy fluid are also adiabatic. This result is independent of the choice in the covariant formulation of the expansion rate. The new terms arising from the expansion rate perturbation have negligible quantitative impact on the constraints on cosmological parameters previously obtained in the literature. A new analysis has been performed using the latest WMAP7 data.

## Acknowledgments

B. G. and L. L. H are supported by CICYT through the project FPA2009-09017 and by CAM through the project HEPHACOS, P-ESP-00346. L. L. H. ackowledges the partial support of the F.N.R.S. and the I.I.S.N.. O. M. work is supported by the MICINN Ramón y Cajal contract, AYA2008-03531 and CSD2007-00060. S. R. acknowledges the partial support of an Excellence Grant of Fondazione Cariparo and of the European Program “Unification in the LHC era” under the contract PITN-GA-2009-237920 (UNILHC). All the authors acknowledge partial support by the PAU (Physics of the accelerating universe) Consolider Ingenio 2010.

## A. Gauge invariant formalism

The conventions we use are mostly from Ref. [22] with a few exceptions. For perturbations in flat space time, the perturbation variables can be expanded by harmonic functions $Y^{(S)}(x,k)$ satisfying to $(\nabla_x + k^2)Y^{(S)} = 0$. In the following we focus on scalar perturbations for which we define:

$$Y_i^{(S)} = -\frac{1}{k} Y_{|i}^{(S)} \ , \tag{A.1}$$

$$Y_{ij}^{(S)} = \frac{1}{k^2} Y_{|ij}^{(S)} + \frac{1}{3}\gamma_{ij} Y^{(S)} \ . \tag{A.2}$$

### A.1 Metric perturbations

For the metric defined in Eq. (2.1), expanding in the Fourier basis the independent perturbations, we denote:

$$\begin{aligned} A &\to \widetilde{A} Y^{(S)} \ , \\ B_i &\to \widetilde{B}_L Y_i^{(S)} \ , \\ H_{ij} &\to \widetilde{H}_L \gamma_{ij} + \widetilde{H}_T Y_{ij}^{(S)} \ , \end{aligned}$$

where $\widetilde{H}_{ij}\gamma^{ij} = 0$. From now on, for sake of simplicity we will drop the tilde symbols. Remember that all these quantities are represented by the correspondent Fourier expansion and depend only on time and on the 3-momentum $k$, while the position dependence is left only in the basis $Y$ elements.

Gauge transformations are associated to infinitesimal coordinate transformations under which: $(x^0, x^i) \to (\hat{x}^0, \hat{x}^i) = (x^0 - T, x^i - L^i)$. It can be shown that the metric perturbation transforms as:

$$\widehat{A} - A = \mathcal{H}T + \dot{T} \ , \tag{A.3}$$

$$\widehat{B} - B = -kT - \dot{L} \ , \tag{A.4}$$

$$\widehat{H}_L - H_L = H_L + kL/3 + \mathcal{H}T \ , \tag{A.5}$$

$$\widehat{H}_T - H_T = H_T - kL \ . \tag{A.6}$$

Before going to the gauge invariant variable definition, let us define some useful metric quantities and their transformations:

$$\sigma_g = \frac{1}{k}\left(\dot{H}_T - kB\right) \ , \tag{A.7}$$

$$\mathcal{R} = H_L + \frac{1}{3}H_T \ , \tag{A.8}$$

$$\mathcal{K} = \frac{1}{\mathcal{H}}\left[-\mathcal{H}A + \frac{k}{3}v_T + \dot{H}_L\right] \ , \tag{A.9}$$

where $v_T$ is the center of mass velocity for the total fluid, satisfying

$$(1+w_T)v_T = \sum_a (1+w_a)\Omega_a v_a \, . \tag{A.10}$$

In the text is also sometimes used the following quantity:

$$\mathcal{K}_a = \frac{1}{\mathcal{H}}\left[-\mathcal{H}A + \frac{k}{3}v_a + \dot{H}_L\right] \, . \tag{A.11}$$

The physical meaning of the quantities above is the following: $\sigma_g$ represents the shear perturbation, $\mathcal{R}$ is the curvature perturbation and $\mathcal{K}$ ($\mathcal{K}_a$) is the expansion rate perturbation of the total ($a$) fluid. These quantities are not gauge invariant but transform as:

$$\widehat{\sigma}_g - \sigma_g = kT \ , \tag{A.12}$$

$$\widehat{\mathcal{R}} - \mathcal{R} = \mathcal{R} + \mathcal{H}T \ , \tag{A.13}$$

$$\widehat{\mathcal{K}} - \mathcal{K} = \frac{1}{\mathcal{H}}\left(\dot{\mathcal{H}} - \mathcal{H}^2\right)T = \frac{\dot{\overline{H}}}{\overline{H}}T \ , \tag{A.14}$$

where $\overline{H} = \mathcal{H}/a$ is the usual Hubble parameter defined in the proper time. From the definition of Eq. (A.14) we see explicitly that we can identify $\mathcal{K}$ as the perturbation of $H$.

We now define gauge invariant quantities associated to the metric and fluid perturbations. Bardeen metric gauge invariants are defined [24] as:

$$\Psi_B = A - \frac{\mathcal{H}}{k}\sigma_g - \frac{1}{k}\dot{\sigma}_g \ , \tag{A.15}$$

$$\Phi_B = H_L + \frac{1}{3}H_T - \frac{\mathcal{H}}{k}\sigma_g \ . \tag{A.16}$$

One can also build the following gauge invariant observable related to the expansion rate perturbation:

$$\mathcal{C} = \mathcal{K} - \frac{1}{k}\frac{\dot{\bar{H}}}{\bar{H}}\sigma_g = \mathcal{K} - \frac{3}{2}(1+w_T)\frac{\sigma_g}{k\mathcal{H}} \ , \tag{A.17}$$

$$\mathcal{C}_a = \mathcal{K}_a - \frac{1}{k}\frac{\dot{\bar{H}}}{\bar{H}}\sigma_g = \mathcal{K}_a - \frac{3}{2}(1+w_T)\frac{\sigma_g}{k\mathcal{H}} \ . \tag{A.18}$$

It is also useful to define the following gauge–invariant quantity:

$$\mathcal{A} = \Psi_B - \frac{\dot{\Phi}_B}{\mathcal{H}} - \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\Phi_B = \frac{3}{2}\left(1+w_T\right)\left(\widetilde{V}_T - \Phi_B\right) \ , \tag{A.19}$$

with $\widetilde{V}_T$ the (reduced) gauge invariant velocity of the total fluid defined by:

$$(1+w_T)\widetilde{V}_T = \sum_a (1+w_a)\Omega_a\widetilde{V}_a \ . \tag{A.20}$$

### A.2 Useful equations

The perturbation equations for the metric can be derived from Einstein equations:

$$\Phi_B + \Psi_B = -3\frac{\mathcal{H}^2}{k^2}\frac{p_T\Pi_T}{\rho_T} = -\frac{\mathcal{H}^2}{k^2}\Omega_\nu\Pi_\nu = -\Omega_\nu\widetilde{\Pi}_\nu \qquad \left(\widetilde{\Pi} = \frac{\Pi}{x^2}\right) \ , \tag{A.21}$$

$$\Psi_B - \frac{\dot{\Phi}_B}{\mathcal{H}} = \frac{3}{2}\frac{\mathcal{H}}{k}\left(1+w_T\right)V_T = \frac{3}{2}\sum_a\left(1+w_a\right)\Omega_a\widetilde{V}_a \qquad \left(\widetilde{V} = \frac{V}{x}\right) \ , \tag{A.22}$$

$$\Phi_B = \frac{\Delta_T + 3(1+w_T)\widetilde{V}_T}{3(1+w_T) + \frac{2}{3}x^2} = \frac{\sum_a\left(\Delta_a + 3\left(1+w_a\right)\widetilde{V}_a\right)\Omega_a}{\sum_a 3\left(1+w_a\right)\Omega_a + \frac{2}{3}x^2} \ , \tag{A.23}$$

where we have defined $x = k/\mathcal{H}$. From the previous equation one can obtain the following relation for the expansion rate perturbations:

$$\mathcal{C} = \left[\frac{x^2}{3} - \frac{3}{2}\left(1+w_T\right)\right]\widetilde{V}_T \ , \tag{A.24}$$

$$\mathcal{C}_a = \frac{x^2}{3}V_a - \frac{3}{2}\left(1+w_T\right)\widetilde{V}_T \ . \tag{A.25}$$

For the sake of completeness we also provide the relation between the entropy perturbation $\Gamma_a$, defined in Eq. (2.7), and the sound speed in the rest frame of the fluid $c_{Sa}^2$ which is given by:

$$w_a\Gamma_a = \left(c_{Sa}^2 - c_{Aa}^2\right)\left[\Delta_a - \frac{\dot{\bar{\rho}}_a}{\bar{\rho}_a}\left(\frac{\Phi_B}{\mathcal{H}} - \frac{V_a}{k}\right)\right] \ . \tag{A.26}$$

# References

[1] E. Komatsu et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. 2010.

[2] Sean M. Carroll. Quintessence and the rest of the world. *Phys. Rev. Lett.*, 81:3067–3070, 1998.

[3] T. Damour, G. W. Gibbons, and C. Gundlach. Dark matter, time-varying g, and a dilaton field. *Phys. Rev. Lett.*, 64(2):123–126, Jan 1990.

[4] Thibault Damour and Carsten Gundlach. Nucleosynthesis constraints on an extended Jordan-Brans- Dicke theory. *Phys. Rev.*, D43:3873–3877, 1991.

[5] Christof Wetterich. The Cosmon model for an asymptotically vanishing time dependent cosmological 'constant'. *Astron. Astrophys.*, 301:321–328, 1995.

[6] Luca Amendola. Coupled quintessence. *Phys. Rev.*, D62:043511, 2000.

[7] Winfried Zimdahl and Diego Pavon. Interacting quintessence. *Phys. Lett.*, B521:133–138, 2001.

[8] Glennys R. Farrar and P. James E. Peebles. Interacting Dark Matter and Dark Energy. *Astrophys. J.*, 604:1–11, 2004.

[9] Subinoy Das, Pier Stefano Corasaniti, and Justin Khoury. Super-acceleration as signature of dark sector interaction. *Phys. Rev.*, D73:083509, 2006.

[10] Hong-Sheng Zhang and Zong-Hong Zhu. Interacting Chaplygin gas. *Phys. Rev.*, D73:043518, 2006.

[11] Sergio del Campo, Ramon Herrera, German Olivares, and Diego Pavon. Interacting models of soft coincidence. *Phys. Rev.*, D74:023501, 2006.

[12] Rachel Bean, Eanna E. Flanagan, and Mark Trodden. The Adiabatic Instability on Cosmology's Dark Side. *New J. Phys.*, 10:033006, 2008.

[13] German Olivares, Fernado Atrio-Barandela, and Diego Pavon. Dynamics of Interacting Quintessence Models: Observational Constraints. *Phys. Rev.*, D77:063513, 2008.

[14] Jussi Valiviita, Elisabetta Majerotto, and Roy Maartens. Instability in interacting dark energy and dark matter fluids. *JCAP*, 0807:020, 2008.

[15] Jian-Hua He, Bin Wang, and Elcio Abdalla. Stability of the curvature perturbation in dark sectors' mutual interacting models. *Phys. Lett.*, B671:139–145, 2009.

[16] M. B. Gavela, D. Hernandez, L. Lopez Honorez, O. Mena, and S. Rigolin. Dark coupling. *JCAP*, 0907:034, 2009.

[17] Brendan M. Jackson, Andy Taylor, and Arjun Berera. On the large-scale instability in interacting dark energy and dark matter fluids. *Phys. Rev.*, D79:043526, 2009.

[18] Elisabetta Majerotto, Jussi Valiviita, and Roy Maartens. Adiabatic initial conditions for perturbations in interacting dark energy models. 2009.

[19] Jussi Valiviita, Roy Maartens, and Elisabetta Majerotto. Observational constraints on an interacting dark energy model. 2009.

[20] Kazuya Koyama, Roy Maartens, and Yong-Seon Song. Velocities as a probe of dark sector interactions. 2009.

[21] Christian G. Boehmer, Gabriela Caldera-Cabral, Nyein Chan, Ruth Lazkoz, and Roy Maartens. Quintessence with quadratic coupling to dark matter. *Phys. Rev.*, D81:083003, 2010.

[22] Hideo Kodama and Misao Sasaki. Cosmological Perturbation Theory. *Prog. Theor. Phys. Suppl.*, 78:1–166, 1984.

[23] Chung-Pei Ma and Edmund Bertschinger. Cosmological perturbation theory in the synchronous and conformal Newtonian gauges. *Astrophys. J.*, 455:7–25, 1995.

[24] James M. Bardeen. Gauge Invariant Cosmological Perturbations. *Phys. Rev.*, D22:1882–1905, 1980.

[25] Viatcheslav F. Mukhanov, H. A. Feldman, and Robert H. Brandenberger. Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions. *Phys. Rept.*, 215:203–333, 1992.

[26] Ruth Durrer. The theory of CMB anisotropies. *J. Phys. Stud.*, 5:177–215, 2001.

[27] Pedro T. P. Viana and Andrew R. Liddle. Perturbation evolution in cosmologies with a decaying cosmological constant. *Phys. Rev.*, D57:674–684, 1998.

[28] Rahul Dave, R. R. Caldwell, and Paul J. Steinhardt. Sensitivity of the cosmic microwave background anisotropy to initial conditions in quintessence cosmology. *Phys. Rev.*, D66:023516, 2002.

[29] Michael Malquarti and Andrew R. Liddle. Evolution of large-scale perturbations in quintessence models. *Phys. Rev.*, D66:123506, 2002.

[30] L. R. W. Abramo and F. Finelli. Attractors and isocurvature perturbations in quintessence models. *Phys. Rev.*, D64:083513, 2001.

[31] Masahiro Kawasaki, Takeo Moroi, and Tomo Takahashi. Isocurvature fluctuations in tracker quintessence models. *Phys. Lett.*, B533:294–301, 2002.

[32] Francesca Perrotta and Carlo Baccigalupi. Early time perturbations behaviour in scalar field cosmologies. *Phys. Rev.*, D59:123508, 1999.

[33] Michael Doran, Christian M. Muller, Gregor Schafer, and Christof Wetterich. Gauge-invariant initial conditions and early time perturbations in quintessence universes. *Phys. Rev.*, D68:063505, 2003.

[34] Gabriela Caldera-Cabral, Roy Maartens, and L. Arturo Urena-Lopez. Dynamics of interacting dark energy. *Phys. Rev.*, D79:063518, 2009.

[35] D. Larson et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters. 2010.

[36] Antony Lewis, Anthony Challinor, and Anthony Lasenby. Efficient computation of CMB anisotropies in closed FRW models. *Astrophys. J.*, 538:473–476, 2000.

[37] Antony Lewis and Sarah Bridle. Cosmological parameters from CMB and other data: a Monte- Carlo approach. *Phys. Rev.*, D66:103511, 2002.

[38] W. L. Freedman et al. Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant. *Astrophys. J.*, 553:47–72, 2001.

[39] M. Kowalski et al. Improved Cosmological Constraints from New, Old and Combined Supernova Datasets. *Astrophys. J.*, 686:749–778, 2008.

[40] Max Tegmark et al. Cosmological Constraints from the SDSS Luminous Red Galaxies. *Phys. Rev.*, D74:123507, 2006.

[41] Joan Simon, Licia Verde, and Raul Jimenez. Constraints on the redshift dependence of the dark energy potential. *Phys. Rev.*, D71:123001, 2005.

[42] Gabriela Caldera-Cabral, Roy Maartens, and Bjoern Malte Schaefer. The Growth of Structure in Interacting Dark Energy Models. *JCAP*, 0907:027, 2009.