

Matrix elements of $\Delta I = 3/2$ $K \rightarrow \pi\pi$ decays*

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We present a numerical computation of matrix elements of $\Delta I = 3/2$ $K \rightarrow \pi\pi$ decays by using Wilson fermions. In order to extrapolate to the physical point we work at unphysical kinematics and we resort to Chiral Perturbation Theory at the next-to-leading order. In particular we explain the case of the electroweak penguins $O_{7,8}$ which can contribute significantly in the theoretical prediction of ϵ'/ϵ . The study is done at $\beta = 6.0$ on a $24^3 \times 64$ lattice.

1. INTRODUCTION

The study of non-leptonic kaon decays is a challenging but important task, which is underlined by the recent accurate measurement of a non-zero ϵ'/ϵ parameter [1] (the first confirmed observation of direct CP violation) and the long-standing puzzle of the $\Delta I = 1/2$ rule.

For example, the electroweak penguins (EWPs)

$$O_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q e_q(\bar{q}_\beta q_\beta)_R,$$

$$O_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q e_q(\bar{q}_\beta q_\alpha)_R,$$

although suppressed by the factor $\alpha_{\text{em}}/\alpha_s$ when compared to the QCD penguins, are enhanced by the $\Delta I = 1/2$ rule and tends to cancel significantly the contribution of the QCD penguins in the theoretical prediction of ϵ'/ϵ .

Here we present the first lattice determination of the $\Delta I = 3/2$ $K \rightarrow \pi\pi$ matrix elements (MEs) of the EWPs obtained directly with a two-pion final state. The strategy, explained in [2,3], consists in computing $\mathcal{M}_{\text{kin}}^{(7,8)} \equiv_{I=2} \langle \pi\pi | O_{7,8} | K^0 \rangle$ at some unphysical kinematics on the lattice (called SPQR kinematics) and then using next-to-leading order (NLO) Chiral Perturbation Theory (χ PT) to obtain the MEs at the physical point.

2. OPERATOR MATCHING

We now discuss the problem of the matching of lattice-regularised operators to the continuum renormalization scheme in which the Wilson coefficients are calculated. Even if chirality is preserved by the regularization, $O_7^{3/2}$ and $O_8^{3/2}$ (the $\Delta I = 3/2$ parts of $O_{7,8}$) mix between themselves. In a Wilson-like regularization there is in general also mixing with operators of different naive chiralities. In the $\Delta I = 3/2$ case, $SU(2)$ isospin symmetry forbids mixing with lower dimensional operators while $\mathcal{CP}\mathcal{S}$ symmetry implies that mixing with other dimension six operators with different chiral properties occurs only in the parity conserving sector [5]. The mixing is thus:

$$\begin{pmatrix} O_7^{3/2}(\mu) \\ O_8^{3/2}(\mu) \end{pmatrix} = \hat{Z}(\mu a) \begin{pmatrix} \bar{O}_7^{3/2}(a) \\ \bar{O}_8^{3/2}(a) \end{pmatrix} \quad (1)$$

where μ is the renormalization scale. With an obvious notation we define $\mathcal{Z}_{77}(\mu a)$, $\mathcal{Z}_{78}(\mu a)$, $\mathcal{Z}_{87}(\mu a)$, $\mathcal{Z}_{88}(\mu a)$ to be the MEs of $\hat{Z}(\mu a)$.

To compute $\hat{Z}(\mu a)$ we use the (mass independent) RI-MOM renormalization scheme which can be easily implemented in a non-perturbative way, also in the case of four fermion operators [5]. Spurious effects due to the Goldstone boson contamination have been removed by using the method explained in [6,2].

Once computed $\hat{Z}(\mu a)$ non-perturbatively, we

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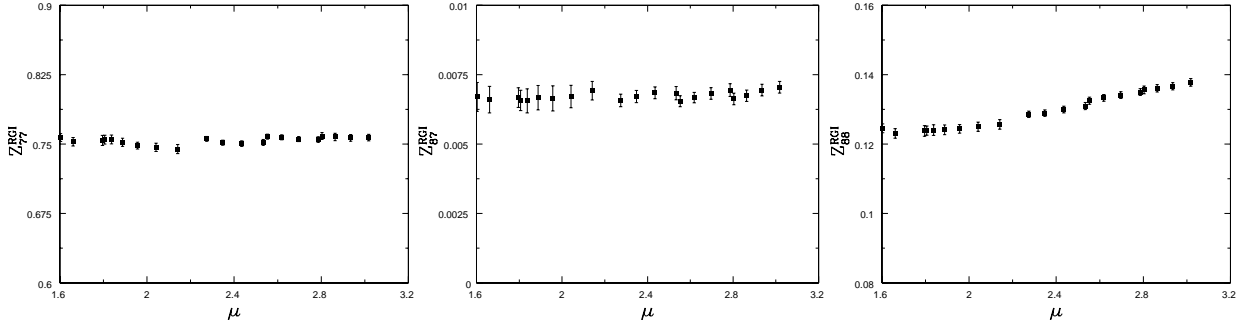


Figure 1. Z_{77}^{RGI} , Z_{87}^{RGI} , Z_{88}^{RGI} from the non-perturbatively determined $\hat{Z}(\mu)$ as defined in Eq. 2.

have to check to what extent it follows the renormalization group behaviour predicted by perturbation theory at the NLO. We thus compute

$$\begin{aligned} \hat{Z}^{RGI}(a) &= (\hat{w}^{-1})^T(\mu) \hat{Z}(\mu) \quad \text{with} \\ (\hat{w}^{-1})^T(\mu) &= [\alpha_s(\mu)]^{-\frac{\hat{\gamma}^{(0)}}{2\beta_0}} \left[1 + \frac{\alpha_s(\mu)}{4\pi} \hat{J}^T \right] \quad (2) \end{aligned}$$

where \hat{J} has been computed in [7]. \hat{Z}^{RGI} should not depend on the scale μ (see Fig. 1).

The observation that no clear $O(a^2p^2)$ lattice artifacts are visible in Z_{77} , Z_{87} together with the very large anomalous dimension of Z_{78} , Z_{88} (6 and 16 at LO) suggest that higher orders in PT could contribute significantly in particular to the RG evolution of Z_{88} .

The central value for the renormalization constants at a given scale μ is extracted in the following way: we compute \hat{Z}^{RGI} by fitting the plateaux to a constant in the interval $2.2 \text{ GeV} \leq \mu \leq 2.6 \text{ GeV}$. We then run \hat{Z}^{RGI} down to the desired scale μ . The systematic error is estimated by computing the variation of the value obtained at this scale from the evolution of the original values (those used to build the combination of Eq. 2 and Fig. 1) at the different scales μ_0 .

3. USE OF THE CHIRAL EXPANSION

The NLO contribution in χ PT, both in the quenched approximation and in the full-QCD case, has been computed for the SPQR kinematics in [4]. One can thus think of extracting the low energy constants (LECs) of the LO and NLO (in the case of the EWPs $\mathcal{O}(p^0)$ and $\mathcal{O}(p^2)$ respectively) from lattice data and to use them in the NLO formula for the physical kinematics. Both forms can be derived from the formula in the most

Table 1

Results in GeV^3 for the MEs at the physical point (the operators are renormalized in $\overline{\text{MS}}\text{-NDR}$ at $\mu = 2 \text{ GeV}$) obtained by using only points with $M_K, E_\pi < \Lambda$. In the quenched case the values $\alpha = 0.1$ and $m_0 = 0.5 \text{ GeV}$ have been used.

fit	Λ	$\langle \mathcal{O}_7 \rangle_{I=2}^{\text{phys}}$	$\frac{\chi}{\text{d.o.f}}$	$\langle \mathcal{O}_8 \rangle_{I=2}^{\text{phys}}$	$\frac{\chi}{\text{d.o.f}}$
poly.	1.2	0.120(11)	0.023	0.714(62)	0.031
poly.	0.8	0.125(13)	0.012	0.715(75)	0.037
χ PT	1.2	0.143(13)	0.817	0.836(73)	0.815
χ PT	0.8	0.123(13)	0.482	0.700(72)	0.626
q χ PT	1.2	0.254(25)	13.2	1.49(16)	10.8
q χ PT	0.8	0.212(24)	5.95	1.19(14)	4.91

general kinematics (energy momentum injection via the weak operator allowed but with all the particles on-shell) which reads

$$\begin{aligned} (\mathcal{M}_{\text{gen}}^{(7,8)})^{\chi\text{PT}} &= A^\chi (1 + [\chi \log]_{\text{gen}}) + B_1^\chi M_K^2 + \\ &B_3^\chi M_\pi^2 + B_2^\chi (p_{\pi^+} + p_{\pi^0}) \cdot p_K + B_4^\chi (p_{\pi^+} \cdot p_{\pi^0}) \quad (3) \end{aligned}$$

where we have symmetrized over p_{π^+}, p_{π^0} since the two pion state is a pure $I = 2$ state.

The SPQR kinematics corresponds to put the K and one π at rest while the other π is either at rest or has the minimum momentum allowed on the lattice. The previous formula can thus be re-expressed in terms of M_K , M_π and E_π , where E_π is the energy of the (possibly) moving pion.

Besides the fit with the complete form predicted by χ PT (Eq. 3), we also try a fit with the polynomial part only (the coefficients of which are now “effective” LECs) which we call $(\mathcal{M}_{\text{gen}}^{(7,8)})^{\text{poly}}$. Tab. 1 reports the results of the fits. It is clear by looking at the $\frac{\chi^2}{\text{d.o.f}}$ that our data are not in the kinematical range where the chiral logarithms are visible. This is visualized in Fig. 2.a-b. A possible explanation for this behaviour is that,

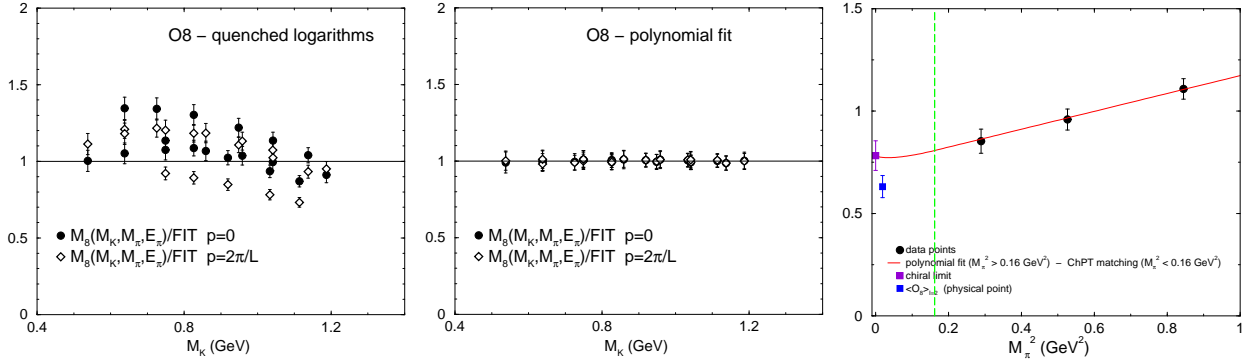


Figure 2. (a)-(b) Quality of the χ PT and polynomial fits for O_8 . (c) χ PT-polynomial matching for O_8 at $M_\pi = E_\pi = 0.4$ GeV, $M_K = 0.41$ GeV (only data with $M_\pi = M_K$ are plotted).

since logarithms come from loops of Goldstone bosons (GBs), they become relevant at a scale of the order of the mass of the GBs. Since the contribution of the counterterms is instead controlled by the physics at the scale of the cut-off (i.e. $\mathcal{O}(1$ GeV)), it is dominant in the kinematical region accessible on the lattice (in our simulation $M_\pi, M_K, E_\pi \in [0.54, 1.2]$ GeV).

On the other hand, one expects that predictions of χ PT (including the logs.) are valid when all the energy scales are sufficiently small (let's say below the mass of the Kaon). If we assume that there exists a region in which the two descriptions of the form factors are reasonably close to each other, we can perform the matching between these two forms [8]. This amounts to fit the data with $(\mathcal{M}_{\text{gen}}^{(7,8)})^{\text{poly}}$ and then to impose the equality of $(\mathcal{M}_{\text{gen}}^{(7,8)})^{\text{poly}}$ with $(\mathcal{M}_{\text{gen}}^{(7,8)})^{\chi\text{PT}}$ and of their first derivatives with respect to the kinematical variables M_K^2 , M_π^2 , $p_{\pi^+} \cdot p_K$, $p_{\pi^+} \cdot p_{\pi^0}$ at a matching point (chosen in the SPQR kinematics).

Results for different matching points are shown in Tab. 2. Lattice data have not yet been corrected for all of the finite volume corrections ex-

plained in [2] and so our results are still preliminary. Nevertheless it's interesting to study the systematics of the χ PT-polynomial matching and the contribution of the NLO at the physical point. Since quenched logarithms are very different from the full QCD ones (and assuming that quenching does not change drastically the kinematical behaviour of the form factors in the range accessible to the lattice) we think that the most sensible way of extrapolating to the physical point is to perform the matching with full χ PT. If we choose the matching point $(M_\pi, M_K) = (0.4, 0.5)$ GeV we get:

$$\langle O_8 \rangle = 0.664(57) \begin{pmatrix} +40 \\ -38 \end{pmatrix} \begin{pmatrix} +50 \\ -40 \end{pmatrix} \quad \langle O_7 \rangle = 0.111(10) \begin{pmatrix} +6 \\ -4 \end{pmatrix} \begin{pmatrix} +9 \\ -6 \end{pmatrix}$$

where the first error is statistical, the second is the systematics of non-perturbative renormalization and the third is the systematics due to the variation of the matching point. At the physical point, the NLO gives a negative contribution of order $15 \div 22\%$ with respects to the LO.

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M_π, M_K	$\langle O_8 \rangle_{I=2}^{\text{phys}}$		$\langle O_7 \rangle_{I=2}^{\text{phys}}$	
poly.	0.714(62)		0.120(11)	
match.	χ PT	$\text{q}\chi$ PT	χ PT	$\text{q}\chi$ PT
0.3,0.5	0.697(60)	0.738(64)	0.117(11)	0.125(12)
0.4,0.5	0.664(57)	0.758(66)	0.111(10)	0.128(12)
0.3,0.31	0.638(55)	0.711(61)	0.106(10)	0.120(11)
0.5,0.51	0.635(55)	0.825(72)	0.106(10)	0.141(13)

Table 2

Results (same units and scheme of Tab. 1) for the matching procedure at the point $(M_\pi = E_\pi, M_K)$.