Non-perturbative renormalization constants and light quark masses * †

SPQcdR Collaboration

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We present the results of an extensive non-perturbative calculation of the renormalization constants of bilinear quark operators for the non-perturbatively $\mathcal{O}(a)$-improved Wilson action. The results are obtained at four values of the lattice coupling, by using the RI/MOM and the Ward identities methods. A new non-perturbative renormalization technique, which is based on the study of the lattice correlation functions at short distance in x-space, is also numerically investigated. We then use our non-perturbative determination of the quark mass renormalization constants to compute the values of the strange and the average up/down quark masses. After performing an extrapolation to the continuum limit, we obtain $m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = (106 \pm 2 \pm 8)$ MeV and $m_\ell^{\overline{\rm MS}}(2\ {\rm GeV}) = (4.4 \pm 0.1 \pm 0.4)$ MeV.

Although the calculation of renormalization constants (RCs) can be performed in perturbation theory, the implementation of non-perturbative renormalization (NPR) techniques has become an essential ingredient of lattice QCD calculations. These techniques allow to control the systematic error associated with the determination of the RCs at the level of accuracy reached at present by the statistical and other systematic uncertainties in lattice calculations.

In this talk we present the results of an extensive NP calculation [1] of the RCs Z_V, Z_A, Z_S, Z_P and Z_T of the bilinear quark operators, for the non-perturbatively $\mathcal{O}(a)$-improved Wilson action [2]. These results are obtained at four values of the lattice coupling, in the range $6.0 \leq \beta \leq 6.45$, by using the RI/MOM method [3] and, for the scale independent RCs, the Ward identity (WI) approach [4]. A NP RI/MOM determination of the RCs of the $\Delta F = 2$ four-fermion operators has been also presented by J. Reyes at this Conference [5].

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The quark mass renormalization constants computed in this study have been used to calculate the strange and the average up/down quark masses [6]. That calculation, which is completely $\mathcal{O}(a)$-improved, uses NPR and involves an extrapolation to the continuum limit, represents at present one of the most accurate determination of the light quark masses within the quenched approximation.

Finally, we present the results of a preliminary numerical investigation of a NPR technique based on the study of the lattice correlation functions at short distance in x-space (XS) [7].

1. RENORMALIZATION CONSTANTS

The lattice parameters used in this study are summarized in table 1. The values of the inverse lattice spacing given in the table have been determined from the study of the static quark anti-quark potential [8], fixing in input $a^{-1}(\beta = 6) = 2.0(1)$ GeV.

In order to study finite size effects, two independent simulations on different lattice sizes have been performed at $\beta = 6.0$. The smallest size corresponds approximately to the same physical volume used at the larger values of β. From this

Table 1
Summary of the lattice parameters used in this work.

β	a^{-1}(GeV)	$L^3 \times T$	# κ_ℓ	# Confs
6.0	2.00(10)	$16^3 \times 52$	4	500
6.0	2.00(10)	$24^3 \times 64$	3	340
6.2	2.75(14)	$24^3 \times 64$	4	200
6.4	3.63(18)	$32^3 \times 70$	4	150
6.45	3.87(19)	$32^3 \times 70$	4	100

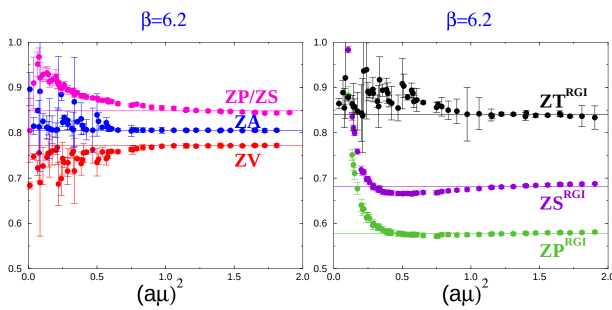

Figure 1. *The scale independent RCs Z_V, Z_A and the ratio Z_P/Z_S (left) and the RGI combinations Z_P^{RGI}, Z_S^{RGI} and Z_T^{RGI} (right) as a function of the renormalization scale, at $\beta = 6.2$. The solid lines represent the results of a constant fit to the data.*

analysis, we find that finite volume effects in the calculation are limited in the range between 1% (Z_V, Z_A) and 3% (Z_S, Z_P, Z_T).

The RI/MOM determination of the RCs is based on the study of the lattice correlation functions at large p^2 and in the chiral limit. The renormalization condition, in the RI/MOM scheme, ensures that leading $\mathcal{O}(a)$-discretization effects vanish in these limits [1]. Therefore, the improvement of the external quark fields and of the composite operators is not necessary for this determination. The on-shell improvement of the vector and axial-vector current operators is required, instead, when the RCs are computed from the study of the lattice WIs. In this case, we use the coefficients c_V and c_A determined non-perturbatively in refs. [2,9].

The results for the scale independent RCs Z_V and Z_A, and for the ratio Z_P/Z_S, obtained with the RI/MOM method from the simulation at $\beta = 6.2$ are shown, as an example, in fig.1 (left) as a function of the renormalization scale. The good quality of the plateau indicates that $\mathcal{O}(a^2)$ discretization effects are well under con-

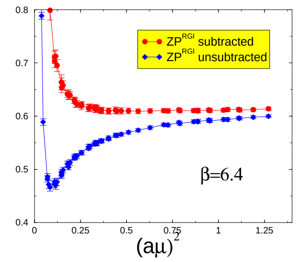

Figure 2. Z_P^{RGI} *as obtained with and without the subtraction of the Goldstone pole, at $\beta = 6.4$.*

trol. In fig.1 (right) we also show the renormalization group invariant (RGI) combinations $Z_{\mathcal{O}}^{RGI} = Z_{\mathcal{O}}(\mu)/C_{\mathcal{O}}(\mu)$, for $\mathcal{O} = S, P, T$, where the evolution function $C_{\mathcal{O}}(\mu)$ is introduced to cancel the scale dependence of the RCs. In the RI/MOM scheme, these functions are known at the NLO in perturbation theory for Z_T and at the N^{3}LO for Z_S and Z_P [10]. From the quality of the plateau, discretization effects appear to be very small, even at quite large values of $(a\mu)^2$. Our estimates of the RCs have been obtained by fitting to a constant the results shown in fig.1 (and similarly for the other βs). The uncertainty due to the residual scale dependence observed in the plots has been used to evaluate the systematic errors.

The RI/MOM approach relies on the fact that NP contributions to the Green functions vanish asymptotically at large p^2. Special care, however, must be taken in the study of the pseudoscalar Green function Γ_P since in this case, due to the coupling to the Goldstone boson, the leading power suppressed contribution is divergent in the chiral limit [3,11]. In order to improve the convergence of Γ_P to its asymptotic behaviour at large p^2, we have subtracted the Goldstone pole contribution by constructing the combinations [12]

$$\Gamma_P^{\rm SUB}(m_1, m_2) = \frac{m_1 \Gamma_P(m_1) - m_2 \Gamma_P(m_2)}{m_1 - m_2}\,, \quad (1)$$

where m_1 and m_2 are different quark masses. The effect of this subtraction on Z_P^{RGI} is shown in fig.2. Though both the unsubtracted and the subtracted determinations of Z_P^{RGI} converge to the same value at large p^2, a clear plateau is never reached in the first case, in the region of momenta

Table 2
Values of the RCs obtained with the RI/MOM *method. The scheme and scale dependent constants, Z_S, Z_P and Z_T are given in the* RI/MOM *scheme at the scale $\mu = 1/a$.*

β	6.0	6.2	6.4	6.45	β	6.0	6.2	6.4	6.45
Z_V	0.766(2)	0.775(3)	0.795(2)	0.797(3)	Z_S	0.668(13)	0.677(9)	0.696(6)	0.707(8)
Z_A	0.804(2)	0.809(3)	0.824(2)	0.825(4)	Z_P	0.535(12)	0.564(4)	0.600(5)	0.612(9)
Z_P/Z_S	0.804(6)	0.831(3)	0.862(3)	0.867(3)	Z_T	0.833(2)	0.847(2)	0.867(6)	0.867(5)

Table 3
Values of the RCs obtained with the WI method.

β	6.0	6.2	6.4
Z_V	0.774(4)	0.789(2)	0.804(2)
Z_A	0.841(17)	0.811(5)	0.843(10)
Z_P/Z_S	0.845(20)	0.863(5)	0.911(10)

explored in this study.

Discretization effects can also be investigated by comparing the results for the RCs obtained at different values of the lattice spacing. Though the RCs depend on the lattice spacing (the UV cutoff in the lattice regularization), their dependence on the renormalization scale is the same for all a, and it is only fixed by the anomalous dimension of the relevant operators. Therefore, the scale dependence should cancel in the ratio

$$Z(a,\mu)/Z(a',\mu) = R(a,a')\,, \qquad (2)$$

up to discretization effects. In fig.3 we show as an example the results for the RCs Z_V and Z_P obtained at four values of β as a function of the renormalization scale. Each RC, $Z(a',\mu)$, has been rescaled by the factor $R(a,a')$ of eq. (2), where we have chosen as a reference scale the value of the lattice spacing at $\beta = 6.4$. In fig.3 we observe that the results obtained at the different values of β all lie on the same universal curve, thus confirming that discretization effects are reasonably small, within the statistical errors. The figure also shows that the renormalization scale dependence is in very good agreement with the one predicted by the anomalous dimensions of the relevant operators.

Our results for the RCs of the bilinear quark operators determined with the RI/MOM and the WI methods are collected in tables 2 and 3 respectively. These results are shown in fig.4, where they are also compared with the predictions of 1-loop boosted perturbation theory (BPT). We

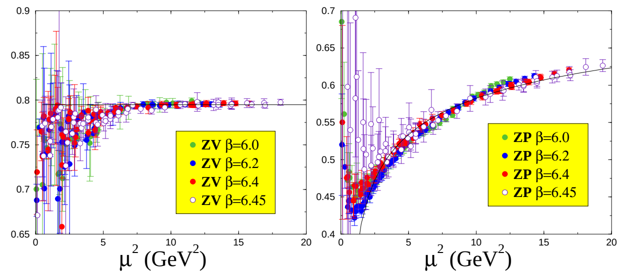

Figure 3. *Z_V and Z_P at the four values of the lattice coupling as a function of the renormalization scale. The results have been rescaled by the factor $R(a,a')$ defined in eq. (2). The solid lines represent the scale dependence predicted by the corresponding anomalous dimensions.*

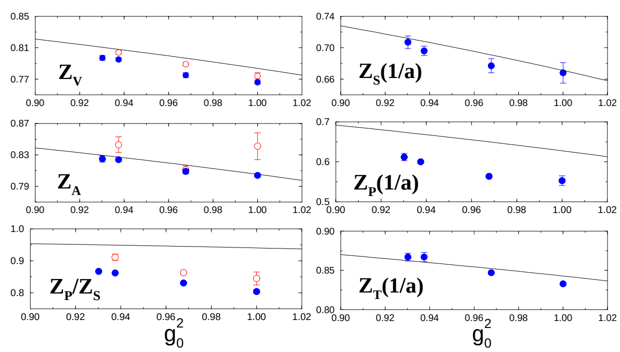

Figure 4. *Values of the RCs obtained from the* RI/MOM *method (filled circles), the WI method (empty circles) and 1-loop BPT (solid lines).*

observe a good agreement between the RI/MOM and the WI determinations for the cases in which the latter method can be used, namely Z_V, Z_A and Z_P/Z_S. There is also a general good agreement between NP results and BPT. A notable exception is the RC of the pseudoscalar density Z_P, whose NP determinations differ from the perturbative prediction by approximately 10-15%.

2. LIGHT QUARK MASSES

An important application of the NPR is the lattice determination of the quark masses. We have computed the strange and the average up/down quark masses by using the standard vector and axial-vector WI methods, for which the relevant RCs are $1/Z_S$ and Z_A/Z_P respectively. We have used the same sets of gauge configurations and quark propagators used for the calculation of the RCs. The physical values of the light quark masses have been fixed by using the experimental pion and kaon masses. Particular attention, in the calculation, has been dedicated to control and reduce the systematic uncertainties, within the quenched approximation. The final results have been obtained by performing an extrapolation to the continuum limit. All details of this calculation are discussed in ref. [6]. Here we only quote our final results, which read:

$$m_s^{\overline{\rm MS}}(2~{\rm GeV}) = (106 \pm 2 \pm 8)~{\rm MeV}$$
$$m_\ell^{\overline{\rm MS}}(2~{\rm GeV}) = (4.4 \pm 0.1 \pm 0.4)~{\rm MeV}\,. \quad (3)$$

3. THE X-SPACE METHOD

In the XS method [7] one imposes the following renormalization condition

$$Z_O(x_0/a)\,\langle O(x)O(0)\rangle|_{x^2=x_0^2} = \langle O(x_0)O(0)\rangle_{\rm cont} \quad (4)$$

where x_0 is a short distance scale. The symbol $\langle\ldots\rangle_{\rm cont}$ denotes the Green function renormalized in a continuum scheme (the simplest choice is to fix this function to its value in the free theory).

With respect to the RI/MOM method, the XS approach presents an important theoretical advantage: the correlation functions in eq. (4) are i) gauge invariant and ii) not affected by the presence of contact terms for $x_0 \neq 0$. This allows to neglect, in the renormalization procedure, the mixing with non gauge invariant operators and/or operators vanishing by the equation of motion.

As a preliminary study of the XS method we show in fig.5 the results for the ratios $\langle V(x)V(0)\rangle/\langle A(x)A(0)\rangle$ and $\langle P(x)P(0)\rangle/\langle S(x)S(0)\rangle$, as a function of the distance x^2 (points corresponding to the same value of x^2 have been averaged together). These quantities provide directly the scheme and scale independent ratios $(Z_A/Z_V)^2$ and $(Z_S/Z_P)^2$. For comparison, the RI/MOM estimates of this RCs are also shown in the figure. We note that, though the results obtained with the XS method are consistent with the expectations, they are affected by large systematic uncertainties. Understanding these uncertainties requires further investigations.

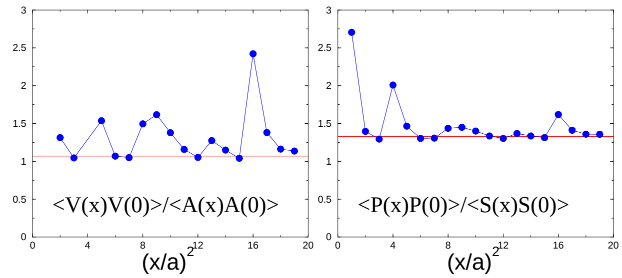

Figure 5. *Ratios of 2-point correlation functions as a function of the distance* $(x/a)^2$*. For comparison, the* RI/MOM *estimates of* $(Z_A/Z_V)^2$ *and* $(Z_S/Z_P)^2$ *are also shown with solid lines.*

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