

CERN-TH/96-163
ROME prep. 96/1144
SHEP 96-14
FTUV 96/21 - IFIC 96/23

A High Statistics Lattice Calculation of The B-meson Binding Energy.

V. Giménez¹, G. Martinelli² and C. T. Sachrajda^{3,*}

¹ Dep. de Física Teórica and IFIC, Univ. de Valencia,
Dr. Moliner 50, E-46100, Burjassot, Valencia, Spain.

² Dip. di Fisica, Univ. “La Sapienza” and
INFN, Sezione di Roma, P.le A. Moro, I-00185 Rome, Italy.

³ Theory Division, CERN, 1211 Geneva 23, Switzerland.

Abstract

We present a high statistics lattice calculation of the B-meson binding energy $\bar{\Lambda}$ of the heavy-quark inside the pseudoscalar B-meson. Our numerical results have been obtained from several independent numerical simulations at $\beta = 6.0, 6.2$ and 6.4 , and using, for the meson correlators, the results obtained by the APE group at the same values of β . Our best estimate, obtained by combining results at different values of β , is $\bar{\Lambda} = 180_{-20}^{+30}$ MeV. For the \overline{MS} running mass, we obtain $\overline{m}_b(\overline{m}_b) = 4.15 \pm 0.05 \pm 0.20$ GeV, in reasonable agreement with previous determinations. The systematic error is the truncation of the perturbative series in the matching condition of the relevant operator of the Heavy Quark Effective Theory.

CERN-TH/96-163
July 1996

* On leave of absence from Dept. of Physics, University of Southampton, Southampton SO17 1BJ,UK.

1 Introduction

In a previous paper [1] we have presented the first results for the binding and kinetic energies of the b -quark in a B -meson, obtained by using the lattice version of the Heavy Quark Effective Theory (HQET) [2]. These are the non-perturbative parameters $\bar{\Lambda}$ and λ_1 , which control the $O(1/m_b)$ and $O(1/m_b^2)$ corrections to the spectroscopy and inclusive decays of B -mesons, and which also appear in the theoretical predictions for several exclusive processes ¹. m_b , m_B and m_{B^*} denote the masses of the b -quark and of the B and B^* mesons respectively. In this letter we report on a detailed study of $\bar{\Lambda}$, which improves on that in ref. [1] in two ways:

- i) We have collected a much larger set of gauge field configurations than was used in ref. [1], with a consequent reduction of the statistical error in the determination of $\bar{\Lambda}$. Moreover, the more accurate knowledge of the heavy quark propagator has allowed for an improved study of its infrared behaviour (i.e. its behaviour at large time separations). This will be discussed in detail in the following.
- ii) We also present new results at a value of the lattice spacing which is smaller than those considered in ref. [1], corresponding to $\beta = 6.4$. Combining the results at three different values of β ($\beta = 6.0, 6.2$ and 6.4) it is possible to study the dependence of $\bar{\Lambda}$ on the lattice spacing. Within the precision of our computations, we find that the results are independent of the lattice spacing.

The presence of renormalon singularities in the pole mass [3, 4] implies that the most intuitive definition of the parameter $\bar{\Lambda}$, i.e. $\bar{\Lambda} \equiv m_B - m_b^{\text{pole}}$, where m_b^{pole} denotes the pole mass of the b -quark, does not correspond to a physical quantity. Thus if $\bar{\Lambda}$ is to be introduced into phenomenological applications (which is not necessary, see below), alternative definitions have to be given. For example, it is possible to define $\bar{\Lambda}$ from the experimentally measured value of some physical quantity, for which the theoretical prediction depends on $\bar{\Lambda}$. Since $\bar{\Lambda}$ does not contribute directly to the leading behaviour of any physical quantity, always appearing as an $O(1/m_b)$ correction, in order to extract $\bar{\Lambda}$, it is first necessary to subtract the perturbation series for the leading term (for a detailed explanation see ref. [5]). This series has a renormalon ambiguity, implying that its sum is not uniquely defined, but the subtraction is performed up to some finite order of perturbation theory. The physical quantity being used, and the order of the perturbation series which has been subtracted, constitute the definition of $\bar{\Lambda}$ ².

Lattice simulations provide the opportunity to compute $\bar{\Lambda}$, defined in some suitable way, non-perturbatively. In ref. [6] a definition of $\bar{\Lambda}$ was proposed which is free of renormalon ambiguities, and from which the linear ultraviolet divergence (i.e. the divergence of $O(a^{-1})$, where a is the lattice spacing) has been subtracted non-perturbatively. The

¹In addition there is the parameter λ_2 , which is the matrix element of the chromomagnetic operator. This parameter gives the leading term in the hyperfine splitting ($m_{B^*} - m_B$), and so can be estimated directly from the experimental data.

² Notice that such definitions of $\bar{\Lambda}$ retain some dependence on the mass of the heavy quark.

necessity for non-perturbative subtractions of power divergences has been stressed in ref. [7]. This definition of $\bar{\Lambda}$, which will be explained in section 2, is obtained after imposing a non-perturbative renormalisation condition on the heavy quark propagator in the Landau gauge. Even though, in practice, the value of $\bar{\Lambda}$ defined in this way is obtained using lattice simulations, it is in fact independent of the method of regularisation.

Although it may be convenient to try to introduce a parameter $\bar{\Lambda}$, which is of $O(\Lambda_{\text{QCD}})$ and is free of renormalon ambiguities, it is not necessary to do so. One can either relate two or more physical quantities directly to the required precision³, or determine physical quantities from non-perturbative computations, such as lattice simulations [5]. Below, as an example of the computation of a physical quantity, we present the results for the b -quark mass (\bar{m}_b) renormalized in the \overline{MS} scheme at the scale $\mu = \bar{m}_b$. The main uncertainty in this quantity comes from the truncation at $O(\alpha_s)$ of the perturbative factor necessary to match the quark mass computed in the HQET to \bar{m}_b . We briefly discuss how to reduce this error which we currently estimate to be about 200 MeV [5].

Any definition of $\bar{\Lambda}$ which is of $O(\Lambda_{\text{QCD}})$ and is independent of any renormalization scale, necessarily involves Green functions defined at large distances. Although we have introduced such a definition in ref. [6], which is free of renormalon singularities, we cannot prove that non-perturbative confining effects do not invalidate the proposed procedure. An important aspect of this paper is to study the infrared behaviour of the heavy quark propagator in order to search for such effects. They are not visible within the precision of our lattice calculations, up to distances of about 1.5 fm, which is the largest distance which we can reach.

The plan of the paper is the following. In sec. 2 we recall the relevant formulae for our definition of $\bar{\Lambda}$ [6]; in sec. 3 we describe the numerical calculation of $\bar{\Lambda}$ and \bar{m}_b and discuss the main results of this study; in the conclusions we present the outlook for future developments and applications of the method discussed in this paper.

2 Non-perturbative definition of $\bar{\Lambda}$

In this section we define the non-perturbative renormalisation prescription which we will use to calculate the “physical” value of $\bar{\Lambda}$ [6]. A detailed discussion of our procedure has been presented in our previous papers [1, 6], and so we only exhibit here those formulae which are necessary to the understanding of the numerical results.

Consider the following Lagrangian of the lattice formulation of the HQET:

$$\mathcal{L}_{\text{eff}} = \frac{1}{1 + a\delta m} \left(\bar{h}(x) D_4 h(x) + \delta m \bar{h}(x) h(x) \right), \quad (1)$$

³This is done implicitly when $\bar{\Lambda}$ is defined using one physical quantity, and used in the prediction for a second one.

where the factor $1/(1 + a \delta m)$ has been introduced to ensure the correct normalization of the heavy quark field h , and the covariant derivative is defined by

$$D_4 f(t) = 1/a \left(f(t) - U_4^\dagger(t-a)f(t-a) \right) . \quad (2)$$

Lattice computations are frequently performed by choosing the bare-mass (δm) to be zero. We propose instead to choose a value of the bare mass by imposing a renormalisation condition on the heavy quark propagator. With our prescription, the heavy-quark propagator can be made finite, up to the standard logarithmic divergences associated with the renormalization of the heavy-quark field.

The mass counter-term can be chosen in many different ways. One possibility is to define it by studying the behaviour of heavy-quark propagator $S(\vec{x}, t)$ in some gauge (we will use the Landau gauge) at large values of the time:

$$- \delta \overline{m} \equiv \frac{\ln(1 + a \delta m)}{a} = \lim_{t \rightarrow \infty} \frac{1}{a} \ln \left[\frac{\text{Tr}(S(\vec{x}, t+a))}{\text{Tr}(S(\vec{x}, t))} \right] , \quad (3)$$

where the traces are over the colour quantum numbers, and we have assumed that the large time limit of the ratio in eq. (3) exists. Our numerical results, to be discussed in detail in the next section, support the validity of this assumption and the use of eq. (3) is our preferred definition of $\delta \overline{m}$. In words, (3) is the condition that the subtracted heavy quark propagator has no exponential fall (or growth) at large times. If, instead, the bare mass δm is chosen to be zero, loop corrections generate a mass of $O(1/a)$, which leads to an exponential behaviour in time.

A possible alternative definition is given by

$$- \delta \overline{m}(t^*) \equiv \frac{1}{a} \ln \left[\frac{\text{Tr}(S(\vec{x}, t^*+a))}{\text{Tr}(S(\vec{x}, t^*))} \right] , \quad (4)$$

for some chosen time t^* . The corresponding definition of $\overline{\Lambda}$, see eq. (7) below, will clearly depend on the choice of t^* : t^* parametrizes the renormalisation prescription dependence and can be considered as the renormalisation point in coordinate space. For physical matrix elements the dependence on t^* is eliminated in the matching of the QCD action and operators with those of the HQET. The use of the propagator at small times, $t^* \Lambda_{\text{QCD}} \ll 1$, to define $\delta \overline{m}(t^*)$, and hence $\overline{\Lambda}(t^*)$, does not require any assumption about the large time behaviour of the heavy quark propagator and in section 3 we will also present the results for $\overline{\Lambda}(t^*)$ obtained in this way. However $\overline{\Lambda}(t^*)$ contains terms of $O(1/t^*)$, and is therefore not a parameter of $O(\Lambda_{\text{QCD}})$.

The divergent parts of the counterterms, either $\delta \overline{m}$ or $\delta \overline{m}(t^*)$, are gauge invariant, in spite of the fact that they are calculated from the heavy quark propagator in a fixed gauge. The argument goes as follows. The linear divergence is eliminated from any correlation function, i.e. for any external state, by subtracting from the action a term

proportional to the gauge-invariant operator $\bar{h}h$. Since in this way one eliminates all divergences both for quark and hadron external states, the coefficient of the mixing has to be gauge-invariant. This must be true also for the finite non-perturbative contributions which arise from the coefficient of the linear divergence. These contributions are generated by terms which are exponentially small in the inverse coupling constant, see ref. [6, 7]. As for the finite parts, the gauge dependence, when it is present, is eliminated from physical quantities up to the order in perturbation theory at which the calculation is performed, see e.g. the case of the running mass discussed below.

In order to obtain the renormalized $\bar{\Lambda}$, in addition to the mass counter-term $\delta\bar{m}$, we need to compute, the “bare” binding energy \mathcal{E} . In the lattice HQET \mathcal{E} , which diverges linearly in $1/a$, is computed from the two-point correlation function

$$C(t) = \sum_{\vec{x}} \langle 0 | J(\vec{x}, t) J^\dagger(\vec{0}, 0) | 0 \rangle, \quad (5)$$

where J (J^\dagger), is any interpolating operator which can annihilate (create) a B -meson ⁴. For a sufficiently large Euclidean time t ,

$$C(t) \rightarrow Z^2 \exp(-\mathcal{E}t) \quad (6)$$

where Z is a constant.

We now define the renormalised binding energy by

$$\bar{\Lambda} \equiv \mathcal{E} - \delta\bar{m}, \quad (7)$$

and a “subtracted” pole mass by

$$m_b \equiv m_B - \mathcal{E} + \delta\bar{m}. \quad (8)$$

$\bar{\Lambda}$ and m_b defined in this way are free of both renormalon ambiguities and linear divergences. Similar formulae hold in the case when $\delta\bar{m}(t^*)$ is used instead to define $\bar{\Lambda}(t^*)$ and $m_b(t^*)$.

3 Numerical implementation of the renormalisation procedure

As explained in the previous section, the determination of $\bar{\Lambda}$ requires the computation of the heavy quark propagator in a fixed gauge (in order to obtain the subtracted operator), as well as the evaluation of the two-point correlation function (5). We have obtained our results using three independent numerical simulations, whose main parameters are given in table 1.

⁴In this paper we only consider pseudoscalar B-mesons, but an analogous discussion holds for other beauty hadrons, such as the Λ_b .

simulation	volume	β	Number of configurations
<u>set A</u>	$24^3 \times 40$	6.0	500
<u>set B</u>	$24^3 \times 40$	6.2	300
<u>set C</u>	$24^3 \times 40$	6.4	300

Table 1: *Parameters of the numerical simulations, the results of which have been used for the present study.*

Our best value of the subtracted binding energy, $\bar{\Lambda} = \mathcal{E} - \delta\bar{m}$, has been determined by combining the values of $\delta\bar{m}$ obtained using set A, set B and set C with the calculation of \mathcal{E} performed by the APE collaboration at $\beta = 6.0, 6.2$ [8] and 6.4 which will be published elsewhere [9]. \mathcal{E} was determined using the SW-Clover fermion action [10] for the light quarks in the quenched approximation. The calculations were performed at several values of the masses of the light quarks, so that extrapolations to the chiral limit are possible. $\delta\bar{m}$ ($\delta\bar{m}(t^*)$) has been determined by using either the standard definition of the lattice heavy-quark propagator or its improved (up to and including $O(a)$ terms) version, the explicit expression of which can be found in ref. [1]. Only the results obtained by using the improved heavy-quark propagator are reported in the following.

All the errors have been computed with the jackknife method by decimating 10 configurations at a time for set A, and 6 configurations at a time for set B and set C. The error on \mathcal{E} was computed with the jackknife method also and we refer the reader to refs. [8, 9] for details.

3.1 Determination of the residual mass $\delta\bar{m}$

To determine the residual mass, we have to compute the effective mass

$$a \delta\bar{m}(t) = - \ln \left(\frac{S_H(t+a)}{S_H(t)} \right) \quad (9)$$

of the heavy quark propagator, $S_H(t)$, defined as

$$S_H(t) = \frac{1}{3V} \sum_{\vec{x}} \langle \text{Tr} [S(t, \vec{x})] \rangle, \quad (10)$$

where the trace is over the colour indices and V denotes the spatial volume of our lattice. It is this averaged propagator $S_H(t)$, which has been used in the computations below; the averaging over all spatial points at time t reduces the statistical errors enormously.

In fig. 1 we present the values of $\delta\bar{m}(t)$ for the improved propagator as a function of t/a . The figure shows the very high accuracy which can be reached in the determination of the effective mass, i.e. in the determination of the logarithmic derivative (3) of the

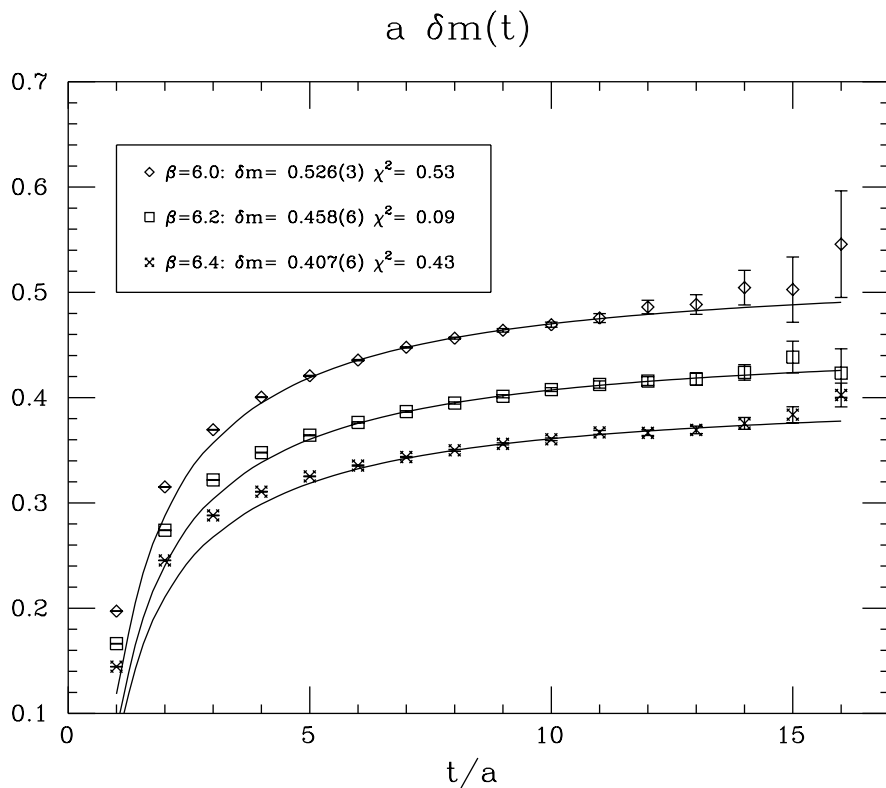


Figure 1: *Effective mass of the improved heavy-quark propagator $S_H(t)$, obtained at $\beta = 6.0$ using 500 configurations, and at $\beta = 6.2$ and 6.4 using 300 configurations, as a function of the time. The curves represent fits of the numerical results to the expression given in eq. (12).*

heavy quark propagator. This will allow us to make a careful study of the infrared behaviour of the quark propagator.

As shown in ref. [1], the effective mass is indistinguishable in the improved and unimproved cases, for $t/a > 4-5$. Thus, in order to minimize lattice artefacts, we have only used the results obtained for $t/a \geq 6$.

Inspired by the results of one-loop perturbation theory [6], we made a fit to $\delta\bar{m}(t)$ using the expression

$$a \delta\bar{m}(t) = a \delta\bar{m} + \gamma \frac{a}{t}. \quad (11)$$

In order to mimic higher order effects, we have also used different expressions to fit $\delta\bar{m}(t)$, e.g.

$$a \delta\bar{m}(t) = a \delta\bar{m} + \gamma' \ln\left(\frac{t+a}{t}\right) \quad (12)$$

or

$$a \delta\bar{m}(t) = a \delta\bar{m} - \gamma'' \ln\left(\frac{\alpha_s[\mathcal{K}/(t+a)]}{\alpha_s[\mathcal{K}/t]}\right) \rightarrow a \delta\bar{m} + \gamma'' \ln\left(\frac{\ln[(t+a)] + \mathcal{C}}{\ln[t] + \mathcal{C}}\right), \quad (13)$$

and changed the interval of the fits in order to check the stability of the determination of $\delta\bar{m}$. Finally, in order to monitor possible confining effects in the heavy-quark propagator, we have also tried a fit of the form

$$a \delta\bar{m}(t) = a \delta\bar{m} + \tilde{\gamma} \ln\left(\frac{t+a}{t}\right) + \rho t, \quad (14)$$

where the last term in eq. (14) will be discussed later on. In eqs. (11)–(14), $\delta\bar{m}$, $\gamma, \dots, \gamma'', \mathcal{C}$ and ρ are free parameters of the fit.

From the different results obtained by varying the fitting functions (11)–(13) and the time intervals, we quote

$$a \delta\bar{m} = 0.526 \pm 0.003 \pm 0.006 \text{ at } \beta = 6.0 \quad (15)$$

$$a \delta\bar{m} = 0.458 \pm 0.005 \pm 0.004 \text{ at } \beta = 6.2 \quad (16)$$

$$a \delta\bar{m} = 0.407 \pm 0.006 \pm 0.006 \text{ at } \beta = 6.4 \quad (17)$$

where in all cases the first error is statistical, and the second is an estimate of the systematic uncertainty, based on the spread of results obtained using different time intervals and fitting functions. Notice the improvement of the above figures, when compared to our old results $a \delta\bar{m} = 0.521 \pm 0.006 \pm 0.010$ at $\beta = 6.0$ and $a \delta\bar{m} = 0.445 \pm 0.008 \pm 0.010$ at $\beta = 6.2$. The improvement is due to the increased statistics, which also allowed us to study the heavy quark propagator at larger time distances, thus reducing also the systematic error.

In order to obtain $\bar{\Lambda}$ we have used:

- $\delta\bar{m}$ from eqs. (15)–(17);
- the SW-Clover determination of \mathcal{E} of the APE collaboration, $a\mathcal{E} = 0.610 \pm 0.010$ at $\beta = 6.0$, $a\mathcal{E} = 0.520 \pm 0.010$ at $\beta = 6.2$ [8] and $a\mathcal{E} = 0.460 \pm 0.007$ at $\beta = 6.4$ [9];

- $a^{-1}(\beta = 6.0) = 2.0 \pm 0.2$ GeV, $a^{-1}(\beta = 6.2) = 2.9 \pm 0.3$ GeV and $a^{-1}(\beta = 6.4) = 3.8 \pm 0.3$ GeV. A significant contribution to the final error in $\bar{\Lambda}$ comes from the calibration of the lattice spacing in quenched simulations which typically has an uncertainty of $O(10\%)$, depending on the physical quantity which is used to set the scale. We take the results given above as a fair representation of the spread of values of a^{-1} .

We then find

$$\bar{\Lambda} = \mathcal{E} - \delta\bar{m} = 170 \pm 30 \text{ MeV at } \beta = 6.0 \quad (18)$$

$$\bar{\Lambda} = \mathcal{E} - \delta\bar{m} = 180 \pm 40 \text{ MeV at } \beta = 6.2 \quad (19)$$

$$\bar{\Lambda} = \mathcal{E} - \delta\bar{m} = 200 \pm 40 \text{ MeV at } \beta = 6.4 \quad (20)$$

where the statistical errors have been combined in quadrature with those due to the uncertainty in the lattice spacing.

Within the uncertainties, the results in eqs. (18)–(20) are compatible with the expected independence of $\bar{\Lambda}$ of the lattice spacing. Given the intrinsic uncertainty in the value of the lattice spacing in quenched simulations it is difficult however, to check this more precisely. Certainly our results exclude large $O(a)$ effects. Assuming that $\bar{\Lambda}$ is indeed constant, we combined together the results of eqs. (18)–(20) to obtain

$$\bar{\Lambda} = (180_{-20}^{+30}) \text{ MeV} , \quad (21)$$

where in the final error we have included also the discretization error which has been conservatively estimated to be of the order of +20 MeV in ref. [1]. The estimate of the discretization error was obtained by comparing the value of \mathcal{E} obtained by using the standard Wilson action and the SW-Clover action at the same values of β .

It would be interesting to compare this value of $\bar{\Lambda}$ with other determinations, obtained by using different methods, e.g. the recent result obtained from the semileptonic width in ref. [11]. The relation between our definition and other ones, provided they are implemented consistently and are free of renormalon ambiguities, can be formally derived in perturbation theory. Even in the case where the perturbative series which connects two different definitions of $\bar{\Lambda}$ is precisely determined, this would not reduce the error in the prediction for physical quantities, such as \bar{m}_b . Indeed, as discussed below, for \bar{m}_b the error of about 200 MeV is dominated by the perturbative series which matches QCD to the effective theory, and not by the error in $\bar{\Lambda}$.

The determination of the mass counter-term $\delta\bar{m}(t^*)$ at fixed $t = t^*$, requires no fitting, and the results obtained using set A–set C are presented in table 2. Only the results obtained in a range of t^* corresponding to values of π/t^* between 1.0 and 2.5 GeV are reported in the table. The results for $\bar{\Lambda}(t^*)$ obtained using the same values of \mathcal{E} and of a^{-1} as before, are also reported in the same table. They show that the values of the binding energy at the same value of the renormalisation scale π/t^* (but at different values of the lattice spacing) are very consistent, implying that lattice artefacts are rather small.

β	t^*/a	$a\delta\bar{m}(t^*)$	π/t^* (GeV)	$\bar{\Lambda}(t^*)$ (MeV)
6.0	3	0.3694(7)	2.1 ± 0.2	$480 \pm 20 \pm 50$
	4	0.4004(8)	1.6 ± 0.2	$420 \pm 20 \pm 40$
	5	0.4207(8)	1.3 ± 0.1	$380 \pm 20 \pm 40$
	6	0.4356(8)	1.0 ± 0.1	$350 \pm 20 \pm 40$
6.2	4	0.3478(6)	2.3 ± 0.2	$500 \pm 20 \pm 50$
	5	0.3643(6)	1.8 ± 0.2	$450 \pm 20 \pm 50$
	6	0.3765(10)	1.5 ± 0.2	$420 \pm 20 \pm 40$
	7	0.3868(12)	1.3 ± 0.1	$390 \pm 20 \pm 40$
	8	0.3949(15)	1.1 ± 0.1	$360 \pm 20 \pm 40$
6.4	5	0.3252(4)	2.4 ± 0.2	$510 \pm 30 \pm 40$
	6	0.3355(4)	2.0 ± 0.2	$470 \pm 30 \pm 40$
	7	0.3435(5)	1.7 ± 0.1	$440 \pm 30 \pm 30$
	8	0.3499(7)	1.5 ± 0.1	$420 \pm 30 \pm 30$
	9	0.3561(7)	1.3 ± 0.1	$390 \pm 30 \pm 30$
	10	0.3604(13)	1.2 ± 0.1	$380 \pm 30 \pm 30$
	11	0.3669(18)	1.1 ± 0.1	$350 \pm 30 \pm 30$

Table 2: Results for $\bar{\Lambda}(t^*) = \mathcal{E} - \delta\bar{m}(t^*)$ for different normalization times t^* , using the results from set A–set C. The first error on $\bar{\Lambda}(t^*)$ is obtained by combining the errors on \mathcal{E} and $\delta\bar{m}(t^*)$ in quadrature; the second error (and the error on π/t^*) comes from the calibration of the lattice spacing.

We conclude this subsection with a brief discussion of the infrared behaviour of the heavy quark propagator. Due to confinement effects, it is not clear that the logarithmic derivative (9) has a finite limit as $t \rightarrow \infty$. Indeed, if (9) goes to a constant at large time distances, then the propagator in the effective theory behaves like an on-shell propagator of a particle of mass $\delta\overline{m}$, $S_H(t) \sim \exp(-\delta\overline{m}t)$. This contradicts the naïve expectation that the Fourier transform of the propagator of a confined object should not have poles, as was found, for example, in lattice computations of the gluon propagator [12]. In the case of the heavy-quark propagator, it is also possible that the “effective” mass $\delta\overline{m}(t)$ does not tend to a constant at large time-distances, and that the propagator decreases faster than an exponential. In order to monitor this, we have allowed for a linear term in t in the effective mass and have fitted (9) to (14). We found that the slope ρ is compatible with zero ⁵ for any fitting interval, and at all the values of β considered in this study. Moreover the value $\delta\overline{m}$ obtained by fitting $\delta\overline{m}(t)$ to eq. (12) or to eq. (14) are equal within the errors. This implies that no sizeable variation of the effective mass, due to confinement effects, is observed down to rather low scales (as small as $\pi/16a \sim 390$ MeV, or distances up to 1.6 fm, at $\beta = 6.0$), which belong to the non-perturbative region.

3.2 The \overline{MS} mass of the b -quark

Although it is convenient and conventional to introduce parameters such as $\overline{\Lambda}$ when studying the $O(1/m_b)$ corrections to quantities in heavy quark physics, it is not necessary. One can instead derive physical quantities from results obtained using lattice simulations. In this subsection we present the results for the \overline{MS} mass of the b -quark, from simulations in the Heavy Quark Effective Theory [1]. The evaluation of a short distance mass does not require any assumptions about the long-distance behaviour of the quark propagator, indeed it can be obtained from computations of the meson propagator only. In ref. [1] it was shown that, at one-loop order,

$$\overline{m}_b = \left(M_B - \mathcal{E} + \alpha_s(a) \frac{X}{a} \right) \left(1 - \frac{4\alpha_s(\overline{m}_b)}{3\pi} \right), \quad (22)$$

where X is the coefficient of the one-loop mass term, obtained in the lattice HQET with zero bare mass. The second factor on the right-hand side of eq. (22) is that which relates the pole mass to \overline{m}_b . The perturbative series of linearly divergent terms (of which only the one-loop term is explicitly exhibited in eq. (22)) removes the linear divergence form \mathcal{E} , but has a renormalon ambiguity of $O(\Lambda_{\text{QCD}})$. This ambiguity is cancelled by that in the second factor, i.e the series relating \overline{m}_b to the pole mass. Thus \overline{m}_b has neither a linear divergence nor a renormalon ambiguity. In evaluating \overline{m}_b we take the experimental value of M_B (5.278 GeV) and the value of \mathcal{E} as measured in the lattice simulations of the APE collaboration. $\alpha_s(a)$ is taken in the range 0.13 and 0.18.

⁵ For example $\rho = 0.002 \pm 0.002$ at $\beta = 6.0$ by fitting in the interval $6 \leq t/a \leq 16$.

We also evaluate \overline{m}_b using

$$\overline{m}_b = m_b(t^*) \times \overline{C}_m(t^*), \quad (23)$$

where

$$m_b(t^*) \equiv M_B - \bar{\Lambda}(t^*) = M_B - \mathcal{E} + \delta\overline{m}(t^*), \quad (24)$$

and

$$\overline{C}_m(t^*) \equiv 1 - \frac{4\alpha_s(\overline{m}_b)}{3\pi} - \frac{1}{\overline{m}_b} \left(\delta\overline{m}(t^*) - \alpha_s(a) \frac{X}{a} \right). \quad (25)$$

Eq. (23) is equivalent to (22) up to one-loop order and neglecting terms of $O(1/m_b)$ in the quark mass. Numerical differences in the results obtained in these two ways are a partial measure of the systematic uncertainties. The values of $a\delta\overline{m}(t^*)$ used in the analysis are taken from eqs. (15)–(17) and table 2.

To obtain a distribution of results, we varied \mathcal{E} , Λ_{QCD} and $a\delta\overline{m}(t^*)$ according to a gaussian distribution; a^{-1} was varied with flat distribution within its error, while $\alpha_s(a)$ was written in terms of the leading quenched expression of the running coupling constant, evaluated at the scale π/a , with Λ_{QCD} distributed according to a flat distribution of width σ and such that $\alpha_s = 0.13$ for $\Lambda_{\text{QCD}} - \sigma$ and $\alpha_s = 0.18$ for $\Lambda_{\text{QCD}} + \sigma$. The resulting distribution is a pseudo-gaussian [1]. From the width of the distribution we estimate the average value and error on \overline{m}_b . The results obtained using eqs. (23) and (22) are given in table 3.

\overline{MS} running b -quark mass, $\overline{m}_b(\overline{m}_b)$					
β	Eq. (24) $t^* = 3 - 6$	Eq. (24) $t^* = \infty$	Eq. (22)	$\overline{m}_b(\overline{m}_b)$ (GeV)	
6.0	4.20(7)	4.18(7)	4.22(7)	4.20(7)	
6.2	4.13(9)	4.10(9)	4.15(8)	4.13(9)	
6.4	4.07(9)	4.04(9)	4.09(8)	4.07(9)	
AVERAGE				4.15 \pm 0.05 \pm 0.05	

Table 3: The \overline{MS} running mass of the b quark. The first error is statistical and the second systematic.

From the results in the table, we estimate $\overline{m}_b = 4.15 \pm 0.05 \pm 0.05$ GeV, where the second error is the systematic error, evaluated by comparing the results obtained using the different formulae given in eqs. (23) and (22). This error, which is of about 50 MeV, can be interpreted as being due to higher order corrections in α_s . It can be compared with the estimate of the effects of higher order corrections, derived from the the bubble-resummation approximation, which is about 200 MeV [5]. We believe that an estimate of 200 MeV is a more realistic one ⁶ and for this reason we quote as our final result

$$\overline{m}_b = 4.15 \pm 0.05 \pm 0.20 \text{ GeV}. \quad (26)$$

⁶The ignorance of higher order corrections also implies an error of this order in any approach.

4 Conclusions

In this study we have presented a high statistics calculation of the binding energy of the b -quark in a B -meson. The improved statistics, with respect to ref. [1] has allowed a better control of the infrared behaviour of the heavy quark propagator. There is no point in further increasing the statistics in the calculation of $\delta\overline{m}$, since the precision on $\overline{\Lambda}$ is mainly limited by the calibration of the lattice spacing in quenched simulations and by the precision in the determination of the bare binding energy \mathcal{E} . It is difficult to imagine a reduction of the uncertainty in the calibration of the lattice spacing in the quenched case, while the error on the bare binding energy is likely to be reduced in the near future.

The main error in the evaluation of the \overline{MS} b -quark mass (~ 200 MeV) comes from the unknown higher order terms in the perturbative matching of the HQET to the full theory [5]. This error can be reduced by computing perturbative corrections at two-loop order. This requires the calculation of the heavy quark propagator in the continuum and of the propagator in the lattice effective theory at $O(\alpha_s^2)$. The continuum calculation has already been performed [13]. We believe that the best approach to the two-loop (and higher order) calculations in the lattice effective theory is the one based on the Langevin equation being developed by the authors of ref. [14]. Based on the summation of light-quark bubble graphs [5, 15], we estimate that knowledge of the matching factor at two-loops would reduce the theoretical uncertainty to about 100 MeV.

Acknowledgments

We acknowledge the partial support by the EC contract CHRX-CT92-0051. V.G. acknowledges the partial support by CICYT under grant number AEN-96/1718. G.M. acknowledges the partial support by M.U.R.S.T. C.T.S. acknowledges the Particle Physics and Astronomy Research Council for its support through the award of a Senior Fellowship.

References

- [1] M. Crisafulli, V. Gimenez, G. Martinelli and C.T. Sachrajda, Nucl. Phys. B 457 (1995) 594.
- [2] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511.
- [3] I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vaishtein, Phys. Rev. D50 (1994) 2234
- [4] M. Beneke and V. M. Braun, Nucl. Phys. B426 (1994) 301
- [5] G. Martinelli and C.T. Sachrajda, CERN-TH/96-117 (hep-ph/9605336).

- [6] G. Martinelli and C.T. Sachrajda, Phys. Lett. B354 (1995) 423.
- [7] L. Maiani, G. Martinelli and C. T. Sachrajda, Nucl. Phys. B368 (1992) 281.
- [8] The APE collaboration, C.R. Allton et al., Nucl. Phys. B413 (1994) 461; Phys. Lett. B326 (1994) 295.
- [9] The APE collaboration, C.R. Allton et al., Nucl. Phys. B (Proc. Suppl.) 42 (1995) 385 and in preparation.
- [10] B.Sheikholeslami and R. Wohlert, Nucl. Phys. B259 (1985) 572; G. Heatlie et al., Nucl. Phys. B352 (1991) 266; Nucl. Phys. B (Proc. Suppl.) 17 (1990) 607.
- [11] M. Gremm, A. Kapustin, Z. Ligeti and M.M. Wise, CALT-68-2043, hep-ph/9603314.
- [12] P. Marenzoni, G. Martinelli, N. Stella and M. Testa, Phys. Lett. B 318 (1993) 511; C. Bernard, C. Parrinello and A. Soni, Nucl. Phys. B (Proc.Suppl.) 30 (1993) 535 and Phys. Rev. D49 (1994) 1585; P. Marenzoni, G. Martinelli and N. Stella, Nucl. Phys. B455 (1995) 339.
- [13] N. Gray, D.J. Broadhurst, W Grafe and K. Schilcher, Z. Phys. C48 (1990) 673.
- [14] F. Di Renzo, G. Marchesini and E. Onofri, Nucl. Phys. B457 (1995) 202.
- [15] P. Ball, M. Beneke and V.M. Braun, Nucl. Phys. B461 (1995) 563