

# Combined Relativistic and static analysis for all $\Delta B = 2$ operators

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We analyse matrix elements of  $\Delta B = 2$  operators by combining QCD results with the ones obtained in the static limit of HQET. The matching of all the QCD operators to HQET is made at NLO order. To do that we have to include the anomalous dimension matrix up to two loops, both in QCD and HQET, and the one loop matching for all the  $\Delta B = 2$  operators. The matrix elements of these operators are relevant for the prediction of the  $B - \bar{B}$  mixing,  $B_s$  meson width difference and supersymmetric effects in  $\Delta B = 2$  transitions.

## 1. INTRODUCTION

Matrix elements of  $\Delta B = 2$  operators crucially enter theoretical determinations of important phenomenological quantities. Due to the high mass of the  $b$  quark and to the present computing power, it is impossible to simulate  $b$  quarks directly on the lattice. Two approaches have been employed to overcome this problem:

- Perform relativistic simulations with several masses around the  $c$ -quark mass and extrapolate the results to physical  $b$ -quark mass.
- By means of an effective theory, the heavy degrees of freedoms can be integrated out and only the light modes are simulated. So far, HQET and NRQCD have been employed. In HQET an expansion in  $\Lambda^{QCD}/m_b$  is made whereas in NRQCD the limit  $v \ll 1$  is taking, being  $v$  the heavy quark velocity.

In this work we have combined, for the first time, full QCD and HQET simulations to compute the matrix elements of 4-quark  $\Delta B = 2$  operators. To do so appropriately, the computation of the matching coefficients of the operators between QCD and HQET at next-to-leading order (NLO) accuracy is needed.

\*Talk given by Juan Reyes.

We have considered the following basis of 4-quark  $\Delta B = 2$  operators:

$$\begin{aligned}
 O_1 &= \bar{b}^i \gamma_\mu (1 - \gamma_5) q^i \bar{b}^j \gamma_\mu (1 - \gamma_5) q^j, \\
 O_2 &= \bar{b}^i (1 - \gamma_5) q^i \bar{b}^j (1 - \gamma_5) q^j, \\
 O_3 &= \bar{b}^i (1 - \gamma_5) q^j \bar{b}^j (1 - \gamma_5) q^i, \\
 O_4 &= \bar{b}^i (1 - \gamma_5) q^i \bar{b}^j (1 + \gamma_5) q^j, \\
 O_5 &= \bar{b}^i (1 - \gamma_5) q^j \bar{b}^j (1 + \gamma_5) q^i,
 \end{aligned} \tag{1}$$

with  $i, j$  colour indices, and  $q$  stands for either  $d$ - or  $s$ - light quark flavour. The first of the above operators enters the Standard Model (SM) description of the  $B^0 - \bar{B}^0$  mixing amplitude [1], whereas  $O_2$  and  $O_3$  are relevant for the SM estimates of the relative width difference in the neutral  $B$ -meson system,  $(\Delta\Gamma/\Gamma)_{B_s}$  [2]. The matrix elements of all operators parametrize supersymmetric effects in  $\Delta B = 2$  transitions [3].

It is usual to express the matrix elements of the operators (1) in terms of the so-called  $B$ -parameters, which are introduced as a measure of the deviation from the vacuum saturation approximation (VSA), namely [3,4],

$$\begin{aligned}
 \langle \bar{B}_q^0 | \hat{O}_1(\mu) | B_q^0 \rangle &= b_1 m_{B_q}^2 f_{B_q}^2 B_1(\mu), \\
 \langle \bar{B}_q^0 | \hat{O}_i(\mu) | B_q^0 \rangle &= b_i \chi m_{B_q}^2 f_{B_q}^2 B_i(\mu), \quad (2) \\
 &2 \leq i \leq 5
 \end{aligned}$$

with  $\vec{b} = \{8/3, -5/3, 1/3, 2, 2/3\}$  and  $\chi \equiv m_{B_q}^2 / (m_b(\mu) + m_q(\mu))^2$ . The hat symbol denotes operators renormalized in some renormalization

scheme at the scale  $\mu$ . To determine the values of the B-parameters  $B_{1-5}(\mu)$ , we have combined the results of a (quenched) QCD numerical simulation on the lattice with 3 values of the heavy quark mass, in the range of heavy-light pseudoscalar masses  $m_P \in (1.7, 2.4)$  GeV, with HQET results in order to constrain the extrapolation towards the physical point,  $m_{B_{s/d}}$ . This extrapolation is guided by the Heavy Quark Scaling Laws (HQSL) which are the properly ones of HQET. Therefore, to use the HQSL we have to match the B-parameters obtained in QCD onto the HQET ones. We refer the reader to ref. [5] where this issue is explained in great detail.

## 2. SIMULATION DETAILS

We enumerate, briefly, the main elements of our lattice simulations:

- For full QCD: the details can be found in refs. [6–8]. The simulation is performed in a lattice of the size  $24^3 \times 48$ , at  $\beta = 6.2$  corresponding to  $a^{-1} = 2.7$  GeV, with the non-perturbatively improved Wilson action. The number of gauge configurations is 200, with 3 values of the heavy and 3 values of the light quark masses, corresponding to the Wilson hopping parameters:  $\kappa_q \in \{0.1344, 0, 1349, 0.1352\}$ , and  $\kappa_Q \in \{0.125, 0, 122, 0.119\}$ .
- For HQET: the details are in ref. [9]. The simulation is performed in a lattice of the size  $24^3 \times 40$ , at  $\beta = 6.0$  corresponding to  $a^{-1} = 2.0$  GeV, with the tree level Clover improved Wilson action. The number of gauge configurations is 600, with 3 values of the light quark masses, corresponding to the Wilson hopping parameters  $\kappa_q \in \{0.1425, 0.1432, 0.1440\}$ .

## 3. EXTRAPOLATION TO THE B-MESON

Once the B-parameters of the renormalized operators, in QCD and HQET, are obtained from lattice simulations in some scheme and at some scale, the matching between the two theories

reads:

$$\begin{aligned} W_{QCD}^T[m_P, \mu]^{-1} \cdot \vec{B}(m_P, \mu) = \\ C(m_P) \cdot W_{HQET}^T[m_P, \mu]^{-1} \cdot \vec{\tilde{B}}(\mu) \\ + \mathcal{O}\left(\frac{1}{m_P}\right) + \dots \end{aligned} \quad (3)$$

where  $W_{QCD}^T[\mu_2, \mu_1]^{-1}$  is the matrix encoding the full QCD evolution from a scale  $\mu_1$  to  $\mu_2$  of all five  $\Delta B = 2$  B-parameters, likewise for  $W_{HQET}^T[\mu_2, \mu_1]^{-1}$  in HQET. The matrix  $C$  is the matching matrix of the B-parameters between QCD and HQET. These matrices are specified in [5].  $\vec{B}(m_P, \mu)$  is a five-component vector which collects all five  $\Delta B = 2$  B-parameters renormalized at the scale  $\mu$  simulated with a heavy quark mass corresponding to a pseudoscalar meson mass of  $m_P$ .  $\vec{\tilde{B}}(\mu)$  is the corresponding vector in HQET.

In order to account for the logarithm dependence in eq. (3) we put the HQET evolution matrix in the l.h.s. and construct the quantity  $\vec{\Phi}(m_P, \mu)$ :

$$\begin{aligned} \vec{\Phi}(m_P, \mu) = \\ W_{HQET}^T[m_P, \mu] \cdot C^{-1}(m_P) \cdot W_{QCD}^T[m_P, \mu]^{-1} \\ \cdot \vec{B}(m_P, \mu) \end{aligned} \quad (4)$$

which can be fit either freely as

$$\vec{\Phi}(m_P, \mu) = \vec{a}_0(\mu) + \frac{\vec{a}_1(\mu)}{m_P}, \quad (5)$$

where  $\vec{a}_0(\mu)$  and  $\vec{a}_1(\mu)$  are the fit parameters, or by constraining it by the static HQET results, *i.e.*

$$\vec{\Phi}(m_P, \mu) = \vec{a}'_0(\mu) + \frac{\vec{a}'_1(\mu)}{m_P} + \frac{\vec{a}'_2(\mu)}{m_P^2}, \quad (6)$$

where the coefficient  $\vec{a}'_0(\mu)$  is completely constrained by the static value,  $\vec{\tilde{B}}(\mu)$ , so that one can probe the term  $\mathcal{O}(1/m_P^2)$ . In figure 1 we show the two extrapolation of eqs. 5 and 6 for the first three B-parameters. In [5] the analogous plots for  $\Phi_4$  and  $\Phi_5$  can be found. As a result, we obtain the HQET values of the B-parameters, *i.e.*  $\vec{\Phi}(m_{B_{s/d}}, \mu)$ , which are then to be matched back onto their QCD counterparts. The final results in the constrained case are presented in table

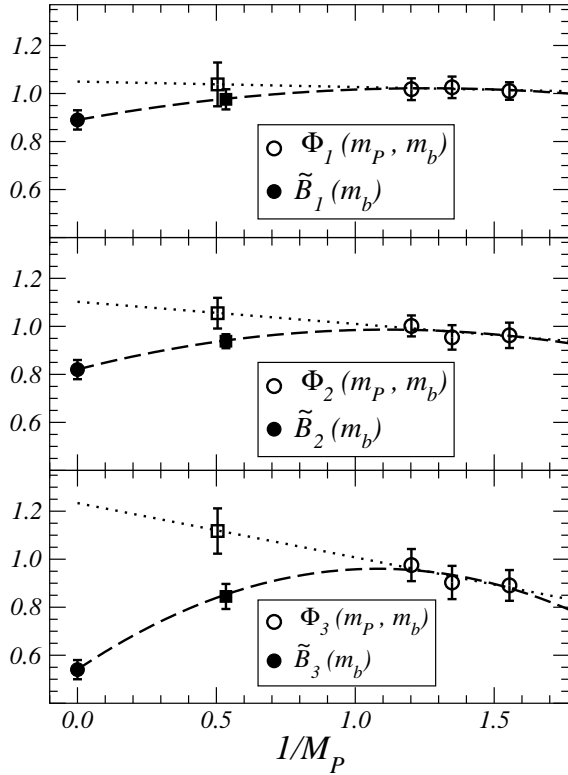


Figure 1. Extrapolation to the physical  $B_d$  meson mass (squared symbols) in the inverse heavy meson mass. The dotted line corresponds to the unconstrained linear extrapolation for each of the components  $\Phi_i(m_P, m_b)$  from our data (empty circles) to  $\Phi_i(m_{B_d}, m_b)$  (empty square). The result of the constrained extrapolation (filled squares) by the static HQET B-parameters (filled circles) is marked by the dashed line.

1, where the first error is the statistical one and the second the systematic error, due to the uncertainty in the lattice renormalization constants, both in QCD and HQET (see ref. [5] for more details).

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## REFERENCES

Table 1

$B$ -parameters defined in eq. (2), renormalized at  $\mu = m_b = 4.6$  GeV and in two renormalization schemes: RI/MOM and  $\overline{\text{MS}}$  of ref. [10].

	Scheme	
	RI/MOM	$\overline{\text{MS}}$ (NDR) [10]
$B_1^{(d)}(m_b)$	0.87(4) $\left(\begin{smallmatrix} +5 \\ -4 \end{smallmatrix}\right)$	0.87(4) $\left(\begin{smallmatrix} +5 \\ -4 \end{smallmatrix}\right)$
$B_2^{(d)}(m_b)$	0.82(3)(4)	0.79(2)(4)
$B_3^{(d)}(m_b)$	1.02(6)(9)	0.92(6)(8)
$B_4^{(d)}(m_b)$	1.16(3) $\left(\begin{smallmatrix} +5 \\ -7 \end{smallmatrix}\right)$	1.15(3) $\left(\begin{smallmatrix} +5 \\ -7 \end{smallmatrix}\right)$
$B_5^{(d)}(m_b)$	1.91(4) $\left(\begin{smallmatrix} +22 \\ -7 \end{smallmatrix}\right)$	1.72(4) $\left(\begin{smallmatrix} +20 \\ -6 \end{smallmatrix}\right)$
$B_1^{(s)}(m_b)$	0.86(2) $\left(\begin{smallmatrix} +5 \\ -4 \end{smallmatrix}\right)$	0.87(2) $\left(\begin{smallmatrix} +5 \\ -4 \end{smallmatrix}\right)$
$B_2^{(s)}(m_b)$	0.83(2)(4)	0.80(1)(4)
$B_3^{(s)}(m_b)$	1.03(4)(9)	0.93(3)(8)
$B_4^{(s)}(m_b)$	1.17(2) $\left(\begin{smallmatrix} +5 \\ -7 \end{smallmatrix}\right)$	1.16(2) $\left(\begin{smallmatrix} +5 \\ -7 \end{smallmatrix}\right)$
$B_5^{(s)}(m_b)$	1.94(3) $\left(\begin{smallmatrix} +23 \\ -7 \end{smallmatrix}\right)$	1.75(3) $\left(\begin{smallmatrix} +21 \\ -6 \end{smallmatrix}\right)$

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