

# Width difference in the $B_s^0 - \bar{B}_s^0$ system from lattice HQET

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We present recent results for the prediction of the  $B_s^0 - \bar{B}_s^0$  lifetime difference from lattice Heavy Quark Effective Theory simulations. In order to get a next-to-leading order result we have calculated the matching between QCD and HQET and the two loop anomalous dimension in the HQET for all the  $\Delta B = 2$  operators, in particular for the operators which enter in the width difference. We obtain for the  $B_s^0 - \bar{B}_s^0$  lifetime difference,  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = (5.1 \pm 1.9 \pm 1.7) \times 10^{-2}$ .

## 1. Introduction.

The width difference ( $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$ ) in the  $B_s^0 - \bar{B}_s^0$  system is expected to be the largest among bottom hadrons. The Standard Model prediction for  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$  relies on an operator product expansion, where the short distance scale is provided by the  $b$  quark mass [1]. The theoretical estimations for  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$  are in the range  $5 \div 15\%$  and hadronic matrix elements of four quark operators are crucial inputs. We present a next-to-leading order (NLO) computation of the dominant matrix elements which contribute to  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$ .

## 2. Standard Model formula for the width difference in the $B_s^0 - \bar{B}_s^0$ system.

The theoretical expression for the width difference reads [1]

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = \frac{G_F^2 m_b^2}{12\pi} f_{B_s}^2 M_{B_s} |V_{cs} V_{cb}|^2 \tau_{B_s} \cdot \left( G(z) \frac{8}{3} B(m_b) + G_S(z) \frac{5}{3} B_S(m_b) \mathcal{X} + \delta_{1/m_b} \right), \quad (1)$$

where

$$\mathcal{X} = \frac{M_{B_s}^2}{(m_b + m_s)^2}, \quad (2)$$

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$z = m_c^2/m_b^2$ ,  $G(z)$  and  $G_S(z)$  are NLO Wilson coefficients calculated in [2],  $\delta_{1/m_b}$  are  $O(1/m_b)$  corrections which depend on matrix elements of higher dimension operators (the explicit expression for  $\delta_{1/m_b}$  can be found in [1]).

$B$  and  $B_S$  are bag parameters defined as vacuum insertion deviations,

$$\langle \bar{B}_s | O_L(\mu) | B_s \rangle \equiv \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B(\mu), \quad (3)$$

$$\langle \bar{B}_s | O_S(\mu) | B_s \rangle \equiv -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \mathcal{X} B_S(\mu) \quad (4)$$

where

$$O_L = \bar{b}^i \gamma^\mu (1 - \gamma_5) q^i \bar{b}^j \gamma_\mu (1 - \gamma_5) q^j$$

$$O_S = \bar{b}^i (1 - \gamma_5) s^i \bar{b}^j (1 - \gamma_5) s^j$$

$i, j$  are color indices.

One of the most important sources of uncertainty in (1) is the  $B_s$  decay constant,  $f_{B_s}$ . In order to reduce it we follow ref. [3] and write  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$  in the following way,

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = \frac{m_{B_s}}{m_{B_d}} \frac{4\pi}{3} \frac{m_b^2}{m_W^2} \left| \frac{V_{cb} V_{cs}}{V_{td} V_{tb}} \right|^2 \frac{\tau_{B_s} \Delta m_{B_d}}{\eta_B(m_b) S_0(x_t)} \xi^2 \cdot \left( G(z) + G_S(z) \mathcal{R}(m_b) + \tilde{\delta}_{1/m} \right), \quad (5)$$

Notice that all the non-perturbative contribution, at leading order in  $1/m_b$ , comes from the  $\mathcal{R}$  parameter,

$$\mathcal{R}(m_b) \equiv \frac{\langle \bar{B}_s | O_S(m_b) | B_s \rangle}{\langle \bar{B}_s | O_L(m_b) | B_s \rangle} = -\frac{5}{8} \frac{B_S(m_b)}{B(m_b)} \mathcal{X}, \quad (6)$$

The parameter  $\xi = \frac{f_{B_s} \sqrt{\bar{B}_{B_s}}}{f_{B_d} \sqrt{\bar{B}_{B_d}}}$ , is accurately computed from lattice simulations [4],  $x_t = m_t^2/m_W^2$  and,  $\eta_B(m_b)$  and  $S_0(x_t)$  are well known theoretically [5,6]. Finally  $\Delta m_{B_d}$  and  $\tau_{B_s}$  are accurately measured experimentally.

### 3. NLO matching from continuum QCD to lattice HQET.

Our aim is to compute the two B-parameters  $B$  and  $B_S$  from lattice simulations. The b quark, however, is too heavy to be simulated dynamically in accessible simulations since the typical inverse lattice spacings, are in the range  $2 \div 3$  Gev. There are two ways to overcome this problem: performing simulations around the charm mass and extrapolate the results to the physical b quark mass or using an effective field theory (HQET, NRQCD) in which the heavy degrees of freedom have been integrated out. In our work we follow the second method using HQET. Our task is to extract matrix elements of operators in QCD from lattice HQET simulations. To obtain a NLO result the following four steps, described in detail in [7], should be computed:

#### 1. The continuum QCD–HQET matching.

The QCD operators are expressed as linear combinations of HQET ones in the continuum at a given high scale, say,  $\mu = m_b$ .

#### 2. Running down to $\mu = a^{-1}$ in the HQET.

The HQET operators obtained in step 1 at the high scale  $\mu = m_b$  are evolved down to a lower scale  $\mu = a^{-1}$ , appropriate for lattice simulations, using the HQET NLO renormalization group equations, which implies the computation of the two loop anomalous dimension of the relevant operators.

#### 3. Continuum–lattice HQET matching.

Having obtained the continuum HQET operators at the scale  $\mu = a^{-1}$ , they are expressed as a linear combination of lattice HQET operators at this scale.

#### 4. Lattice computation of the matrix elements.

The matrix elements of the lattice HQET

operators in Step 3, are measured by Monte Carlo numerical simulations on the lattice.

We stress that the first three steps depend on the renormalization scheme used in the HQET. When the two loop anomalous dimensions are included in the renormalization group evolution (RGE) this scheme dependence must cancel up to  $O(\alpha_s)$ . So far, the complete chain of matching equations have been calculated for  $O_L$  only. However, the two last steps are known for all operators [4,8,9]. We have extended the calculation to all the  $\Delta B = 2$  operators computing the continuum QCD – HQET matching and the two loop anomalous dimensions [10,11]. We have explicitly verified the cancellation of the dependence of our expressions on the intermediate HQET scheme [10]. This is a strong check of our calculation of the matching.

### 4. B-parameters from numerical simulations on the lattice.

The lattice B-parameters are extracted from the large time behavior of the ratio between three- and two-point correlation functions (see for example [4]):

$$R_{O_i} \equiv \frac{C_{O_i}(-t_1, t_2)}{C(-t_1)C(t_2)} \xrightarrow{t_1, t_2 \rightarrow \infty} \frac{\langle \bar{B}_{P_S} | O_i(a) | B_{P_S} \rangle}{|\langle 0 | A_0(a) | B_{P_S} \rangle|^2}$$

where

$$C_{O_i}(t_1, t_2) \equiv \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | A_0(\vec{x}_1, t_1) O_i(\vec{0}, 0) A_0^\dagger(\vec{x}_2, t_2) | 0 \rangle$$

$$C(t) \equiv \sum_{\vec{x}} \langle 0 | A_0(\vec{x}, t) A_0^\dagger(\vec{0}, 0) | 0 \rangle$$

and  $A^\mu$  is the HQET axial current.

The results presented here are obtained from a quenched simulation performed by APE collaboration in a  $24^3 \times 40$  lattice with 600 gauge configuration at  $\beta = 6.0$  with  $a^{-1} = 2$  Gev. The details of the simulation can be found in [4] and references therein.

An important remark we want to make is that, as firstly pointed out in [8], all lattice B-parameters obtained in the static limit are very close to their vacuum insertion values. This result has been obtained in static simulations by

APE coll. [4], UKQCD coll. [12], Christensen et al. [13] and also by JLQCD coll. in NRQCD when their results are extrapolated to the infinite mass limit [14]. Notice that although these simulations are performed with different actions (static-clover, static-Wilson, NRQCD-clover) at different lattice spacings and by different groups, the conclusion is the same in all cases: the lattice B-parameters in the static limit are compatible with the vacuum insertion value within the statistical errors. This surprising property of the HQET deserves further investigation which is in progress.

### 5. $1/m_b$ dependence of the B-parameters.

Having performed the calculation described in the previous section one obtains the B-parameters in QCD to leading order in  $1/m_b$ . By using vacuum insertion approximation (VIA) for the subleading operators [15] and the fact that the bare lattice B-parameters are very close to the VIA value, we have shown [10] that all  $\Delta B = 2$  B-parameters, defined as vacuum insertion deviations, have small  $O(\bar{\Lambda}/m_b)$  corrections ( $\bar{\Lambda} \equiv M_{B_s} - m_b$ ),

$$B_i(m_b) = \bar{B}_i(m_b) + \underbrace{O(0.3 \frac{\bar{\Lambda}}{m_b})}_{\sim 0.05} + O(\frac{1}{m_b^2}), \quad (7)$$

where  $B_i(m_b)$  and  $\bar{B}_i(m_b)$  are the full and the static QCD B-parameters respectively. The factor 0.3 is an estimation of the deviation from VIA in the calculation of the subleading operators and  $O(\alpha_s)$  corrections.

### 6. Results.

Before presenting our results, let us stress that  $m_s$  and  $m_b$  parameters in previous sections are the corresponding pole masses. The latter coincides with the expansion parameter of the HQET because we have set the residual mass to zero. We calculate the  $b$  quark pole mass from the running  $\overline{\text{MS}}$  mass, which can be accurately determined. Since our computation is performed at NLO, we use the perturbative relation between the pole and the running mass at the same order. From

Table 1

Values of the B-parameters in the NDR- $\overline{\text{MS}}$  scheme (see text).

B-parameters	
$B(m_b)$	0.83(5)(4)(5)
$B_S(m_b)$	0.96(8)(5)(5)
$B^{LR}(m_b)$	0.94(5)(4)(5)
$B_S^{LR}(m_b)$	1.03(3)(5)(5)

the world average running mass [16],  $\overline{m}_b(\overline{m}_b) = 4.23 \pm 0.07$  GeV one obtains  $m_b = 4.6 \pm 0.1$  GeV. Notice that the contribution of  $m_s$  is very small because it always appears divided by  $m_b$ .

In table 1 we present the values for all  $\Delta B = 2$  B-parameters. We have included, for the sake of completeness, the other  $\Delta B = 2$  operators in the HQET,

$$\frac{\langle \bar{B}_s | O^{LR}(\mu) | B_s \rangle}{-2f_{B_s}^2 M_{B_s}^2} \left(1 + \frac{2}{3}\mathcal{X}\right)^{-1} \equiv B^{LR}(\mu), \quad (8)$$

$$\frac{\langle \bar{B}_s | O_S^{LR}(\mu) | B_s \rangle}{\frac{1}{3}f_{B_s}^2 M_{B_s}^2} (1 + 6\mathcal{X})^{-1} \equiv B_S^{LR}(\mu), \quad (9)$$

and

$$O^{LR} = \bar{b}^i \gamma^\mu (1 - \gamma_5) q^i \bar{b}^j \gamma_\mu (1 + \gamma_5) q^j, \quad (10)$$

$$O_S^{LR} = \bar{b}^i (1 - \gamma_5) q^i \bar{b}^j (1 + \gamma_5) q^j. \quad (11)$$

The first error in table 1 comes from lattice simulations and includes statistical and systematic errors. The second one is an estimate of the error due to the uncertainties in the values of the lattice coupling constant and to higher order contributions to the matching. The third is an estimate of  $1/m_b$  corrections to the static result (see eq.(7)). The analysis of the perturbative matching is explained in detail in our previous work [9] where we gave  $B(m_b) = 0.81 \pm 0.05 \pm 0.04$ . The tiny difference is due to the fact that there we used, in the perturbative evolution, a number of flavours  $n_f = 4$  instead of  $n_f = 0$  as in the present paper. The B-parameters in table 1 are QCD scheme dependent, they all are in the NDR- $\overline{\text{MS}}$  scheme. In order to make contact with the NLO computation of [2], we have subtracted the evanescent operators in the renormalization of  $O$  and  $O_S$  as in [2]. For  $B^{LR}$  and  $B_S^{LR}$  we use the prescription of [17].

Table 2  
Values of  $B$ ,  $B_S$  and  $\mathcal{R}$  from different groups.

Group	$B(m_b)$	$B_S(m_b)$	$\mathcal{R}(m_b)$
[14]	0.85(3)(11)	0.95(2)(12)	-0.91(5)(17)
[3]	0.91(3) $^{+0.00}_{-0.06}$	1.02(2) $^{+0.02}_{-0.04}$	-0.93(3) $^{+0.00}_{-0.06}$
This work	0.83(5)(6)	0.96(8)(7)	-0.95(7)(9)

For the parameter  $\mathcal{R}$  (see eq.(6)), which contains all the non-perturbative contribution to the width difference, at leading order in  $1/m_b$ , we obtain

$$\mathcal{R}(m_b) = -0.95(7)(9) \quad (12)$$

where as usual the first error comes from lattice simulations, and the second is systematic due to the uncertainty in the perturbative matching and to the contribution of higher orders in  $1/m_b$ .

### 6.1. Comparison with other recent results

In table 2 we present our result for the B-parameters relevant for  $\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}$  compared with other recent determinations. In [3] the  $B_S$  is defined as in eq.(4) but in terms of the running mass instead of the pole mass. Therefore, we have multiplied their value by  $(m_b/\overline{m}_b(\overline{m}_b))^2$  in order to compare with our result. On the other hand, in the definition used in [14] does not appear the factor  $\mathcal{X}$  (see eq. (2)), therefore we have divided their result by  $\mathcal{X}$ . Notice that in spite of using different methods to obtain the B-parameters, there is a good agreement between the three computations, in particular in the value of the ratio  $\mathcal{R}$ .

From equation (5) we get our prediction,

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = (5.1 \pm 1.9 \pm 1.7) \times 10^{-2}. \quad (13)$$

The first error is systematic obtained from the spread of values of all input parameters in eq.(5). The second one comes from the uncertainty in the value of  $\tilde{\delta}_{1/m_b}$ . Since in the estimate of  $\tilde{\delta}_{1/m_b}$  the operator matrix elements were computed using VIA and the radiative corrections were not included, we assume an error of 30% [3]. As can be seen, this parameter is still affected by a large uncertainty, so that a precise determination of the width difference requires the computation of the

subleading matrix elements using lattice QCD. This simulation is missing to date.

Our result is to be compared with the present experimental status [18]

$$\left(\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}\right)^{\text{exp.}} = \left(17_{-10}^{+09}\right) \times 10^{-2} \quad (14)$$

As can be seen the central values are rather different but still compatible within the large errors. More work is needed on the theoretical and experimental sides to reduce the uncertainty in the width difference in the  $B_s$  system.

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