K^0–$\bar{K}^0$ Mixing with Wilson Fermions without Subtractions

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Abstract

By using suitable Ward identities, we show that it is possible to compute K^0–$\bar{K}^0$ mixing without subtracting the terms generated by explicit chiral symmetry breaking present in Wilson-like lattice actions. The accuracy in the determination of the amplitudes is of $\mathcal{O}(a)$, which is the best one attainable in the absence of improvement.

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1 Introduction

A key ingredient in the study of CP violation in the Standard Model is the theoretical prediction of the K^0–$\bar{K}^0$ mixing amplitude. This involves the computation of the $\Delta S = 2$ matrix element

$$\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle \equiv \frac{8}{3} f_K^2 m_K^2 B_K \qquad (1)$$

of the operator

$$O^{\Delta S=2} \equiv O_1 = (\bar{s}^A \gamma_\mu (1-\gamma_5) d^A)(\bar{s}^B \gamma_\mu (1-\gamma_5) d^B), \qquad (2)$$

where s and d stand for strange and down quarks and A, B are colour indices.

In general, important information on the physics beyond the Standard Model, such as various SUSY extensions (MSSM, NMSSM,...), can be obtained by studying $\Delta F = 2$ transitions (see [1] and references therein for a discussion). For neutral kaons, such processes require, besides O_1, also the knowledge of the matrix elements of the operators (we adopt here the notation of ref. [1])

$$\begin{aligned}
O_2 &= (\bar{s}^A (1-\gamma_5) d^A)(\bar{s}^B (1-\gamma_5) d^B) \\
O_3 &= (\bar{s}^A (1-\gamma_5) d^B)(\bar{s}^B (1-\gamma_5) d^A) \\
O_4 &= (\bar{s}^A (1-\gamma_5) d^A)(\bar{s}^B (1+\gamma_5) d^B) \\
O_5 &= (\bar{s}^A (1-\gamma_5) d^B)(\bar{s}^B (1+\gamma_5) d^A)
\end{aligned} \qquad (3)$$

On the lattice, the matrix elements of the four-fermion operators above are typically extracted from the large-time asymptotic behaviour of three-point correlation functions of the form $\langle K_P^0(x_1) O^{\Delta S=2}(0) K_P^0(x_2) \rangle$, where K_P^0 are pseudoscalar sources with suitable quark flavour, $K_P^0(x) = \bar{d}^A(x) \gamma_5 s^A(x)$. Expressed in terms of traces of quark propagators, these correlation functions correspond to the so-called "eight"-shaped quark diagrams given in fig. 1.

In order to obtain the physical amplitudes, it is necessary to compute the matrix elements of the renormalized operators corresponding to those defined in eqs. (2) and (3). With Wilson-like fermions, the renormalization procedure is complicated by the presence of explicit chiral-symmetry breaking (χSB) in the lattice fermion action: because of the Wilson term, dimension-six operators belonging to different chiral representations can mix with each other.

The testing ground for the restoration of chiral symmetry has been the chiral behaviour of $\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle$, which, if properly renormalized, vanishes when the K-meson becomes massless [2]. Although several attempts with Wilson fermions [3]–[8] have given reasonable measurements of B_K, it remains true that the control of the renormalization of the relevant operator is rather problematic [1].

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[1] B_K has also been obtained with staggered fermions [9]–[12] mainly in the quenched approximation (see ref. [13] for a review). The (surviving) chiral symmetry in the staggered fermion formalism ensures the vanishing of the relevant matrix element in the chiral limit.
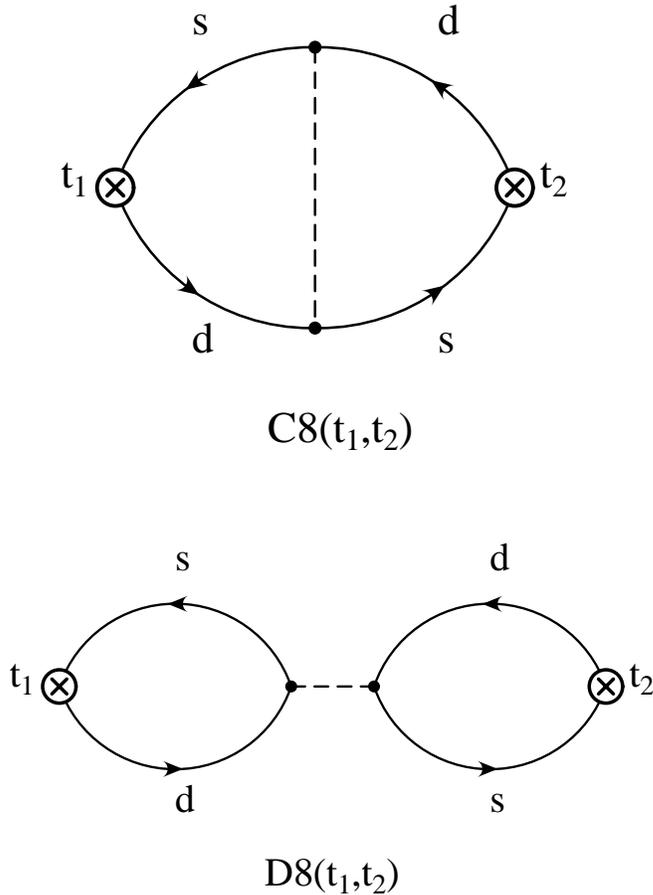
C8(t$_1$,t$_2$)

D8(t$_1$,t$_2$)

Figure 1: *Connected and disconnected eight diagrams. The dashed line represents the four-fermion operator.*

The root of the problem is the operator subtraction outlined above. $O^{\Delta S=2}$ mixes with other operators O_i of the same dimension but with "wrong naïve chirality". Thus, the (μ-dependent) K^0–$\bar{K}^0$ matrix element of the renormalized operator $\hat{O}^{\Delta S=2}$ is given in terms of the (a-dependent) bare matrix elements by:

$$\langle \bar{K}^0|\hat{O}^{\Delta S=2}(\mu)|K^0\rangle = \lim_{a\to 0}\langle \bar{K}^0|Z_0^{\Delta S=2}(a\mu, g_0^2)\left[O^{\Delta S=2}(a) + \sum_i \Delta_i(g_0^2)O_i(a)\right]|K^0\rangle \tag{4}$$

The overall renormalization constant $Z_0^{\Delta S=2}(a\mu, g_0^2)$ is logarithmically divergent and its determination does not affect the chiral behaviour of the matrix element. The constants $\Delta_i(g_0^2)$'s are finite mixing coefficients which depend on the lattice bare coupling (also expressed as $\beta = 6/g_0^2$) only [14]. They have been either computed in perturbation theory (PT) [15]–[16], or fixed by using the non-perturbative (NP) method of refs. [17]–[21], or determined using the relevant Ward identities (WI) on quark states (QWI) [22] [2].

When PT is used for the calculation of the mixing coefficients Δ_i, the mass dependence of the renormalized-operator matrix element, $\langle \bar{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$, shows large deviations from the expected chiral behaviour. Past experience suggests, instead, that

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[2] Another method, suggested in refs. [23, 24], is the use of gauge-invariant WI on hadronic states. This method has never been implemented though.

the chiral behaviour of $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle$ is satisfactory if a non-perturbative method, either the NP renormalization [19]–[20] or the QWI [22], is used.

In spite of this progress, the determination of the mixing coefficients is a long and painful procedure. The non-perturbative renormalization techniques proposed in the past have their limitations and, for this reason, an accurate and systematic study of the uncertainties in the determination of the mixing coefficients would be necessary before keeping the final error on the matrix element fully under control.

In this paper, *we propose a new method which allows the calculation of $\Delta S = 2$ amplitudes with Wilson fermions without determining the mixing coefficients. The method is based on the lattice Ward identities and it is non-perturbative. It can be applied to any Wilson-like formulation of the action (Wilson action, tree-level improved, NPI and alike). Its accuracy, in the absence of improvement (of the action and of the operators), is of $\mathcal{O}(a)$, which is the best possible accuracy, attainable only with a perfect determination of the mixing coefficients Δ_i in previous approaches.*

The same method can be used to compute $\Delta I = 3/2$ K-π matrix elements, e.g. those relative to the electro-penguin operators, and $\Delta I = 3/2$ parity-conserving amplitudes in hyperon decays. It fails, unfortunately, for $\Delta I = 1/2$ transitions because of the mixing with lower-dimension operators.

An alternative approach, based on the twisted mass QCD formalism [25], also allows the determination of the $\Delta S = 2$ mixing amplitude without lattice subtractions [26].

2   Description of the Method

In this section, we show how it is possible to compute the physical K^0–$\bar{K}^0$ mixing amplitude without knowing, or determining, the mixing coefficients induced by the explicit χSB of the lattice action. We first recall some basic notions about operator renormalization and mixing with Wilson fermions and then explain our proposal.

2.1   Operator Renormalization with Wilson Fermions

Following [21] (see also [5] and [27]), we classify the complete basis of dimension-six, four-fermion operators which mix under renormalization, relying on general symmetry arguments based on the vector-flavour symmetry, which survives on the lattice. To this purpose it is convenient to consider separately the parity-even and parity-odd parts of the operators in eqs. (2) and (3). Thus, for example, the parity-even part of the operator $O_1 = Q_1 - \mathcal{Q}_1$ is given by

$$Q_1 = O_{[VV+AA]} = (\bar{s}^A \gamma_\mu d^A)(\bar{s}^B \gamma_\mu d^B) + (\bar{s}^A \gamma_\mu \gamma_5 d^A)(\bar{s}^B \gamma_\mu \gamma_5 d^B), \qquad (5)$$

whereas the parity odd is defined as

$$\mathcal{Q}_1 = O_{[VA+AV]} = 2(\bar{s}^A \gamma_\mu d^A)(\bar{s}^B \gamma_\mu \gamma_5 d^B). \qquad (6)$$

On the basis of CPS symmetries, it can been shown that the renormalization of the $\mathcal{Q}_i$ operators is not affected by the explicit χSB of the lattice action and proceeds exactly as in the "continuum" theory. The corresponding renormalization matrix $\mathcal{Z}_{ij}$ (which obviously depends on the renormalization-scheme) is a block diagonal matrix [21]:

$$
\begin{pmatrix} \hat{\mathcal{Q}}_1 \\ \hat{\mathcal{Q}}_2 \\ \hat{\mathcal{Q}}_3 \\ \hat{\mathcal{Q}}_4 \\ \hat{\mathcal{Q}}_5 \end{pmatrix} = \begin{pmatrix} \mathcal{Z}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_{22} & \mathcal{Z}_{23} & 0 & 0 \\ 0 & \mathcal{Z}_{32} & \mathcal{Z}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ 0 & 0 & 0 & \mathcal{Z}_{54} & \mathcal{Z}_{55} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}. \tag{7}
$$

Thus the lattice does not induce extra subtractions (Δ_i) for the parity-odd sector ($\mathcal{Q}_k$; $k = 1, \ldots, 5$), since the mixing in eq. (7) is the same as it would appear in the absence of χSB. In particular, $\mathcal{Q}_1$ renormalizes multiplicatively.

The parity-even sector, on the contrary, is not protected by CPS symmetries, and all the five relevant operators get mixed because of the χSB of the lattice action. In this case, it is convenient to separate the operator mixing into two classes: i) the first which consists in correcting the operator mixing induced by the breaking of chiral symmetry; ii) the second is the renormalization which survives in the continuum limit. In the absence of explicit χSB, the mixing structure is the same as the one considered above for the parity-odd counterparts. The corresponding parity-even operators ($\tilde{Q}_k$; $k = 1, \ldots, 5$) would renormalize according to:

$$
\begin{pmatrix} \hat{Q}_1 \\ \hat{Q}_2 \\ \hat{Q}_3 \\ \hat{Q}_4 \\ \hat{Q}_5 \end{pmatrix} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{Q}_5 \end{pmatrix}, \tag{8}
$$

where the $\tilde{Q}_i$ represent the bare operators, which transform as elements of irreducible representations of the chiral group (obviously up to terms of $\mathcal{O}(a)$). In the presence of the Wilson term, the $\tilde{Q}_i$ are defined as

$$
\begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{Q}_5 \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix} + \begin{pmatrix} 0 & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & 0 & 0 & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & 0 & 0 & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 & 0 \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & 0 & 0 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix}. \tag{9}
$$

In other words, first the lattice subtraction is performed, followed by the renormalization of the remaining logarithmic divergencies. The above mixing pattern is abbreviated, in matrix form, as

$$
\begin{aligned} \hat{Q} &= Z\tilde{Q} \\ \tilde{Q} &= [I + \Delta]Q \end{aligned} \tag{10}
$$

where I is the 5×5 unit matrix.

4

2.2 K^0–$\bar{K}^0$ Mixing without Subtractions

For the sake of illustration, we discuss here the determination of the K^0–$\bar{K}^0$ matrix element of the operator $O^{\Delta S=2} = O_1$ only. The extension to the other operators (O_i, $i = 2, \ldots 5$) is straightforward.

Le us consider the Ward identities which can be derived from the τ_3 axial rotation

$$\delta u = \gamma_5 u \,, \quad \delta \bar{u} = \bar{u} \gamma_5 \,,$$
$$\delta d = -\gamma_5 d \,, \quad \delta \bar{d} = -\bar{d} \gamma_5 \,, \tag{11}$$

where u and d are the up and down quarks taken with degenerate masses, $m_u = m_d = m$. For further use, we also introduce the following bilinear operators [3]

$$\Pi^0(x) = \bar{d}(x) \gamma_5 d(x) - \bar{u}(x) \gamma_5 u(x) \,, \quad K_P^0(t) = \sum_{\vec{x}} \bar{d}(\vec{x},t) \gamma_5 s(\vec{x},t) \,,$$
$$K_S^0(t) = \sum_{\vec{x}} \bar{d}(\vec{x},t) s(\vec{x},t) \,, \tag{12}$$

and the corresponding renormalized quantities $\hat{K}_P^0(t) = Z_P K_P^0(t)$ and $\hat{K}_S^0(t) = Z_S K_S^0(t)$. The useful Ward identity in our case is then [14]

$$\langle \delta \left[\hat{K}_P^0(t_1) \hat{O}_1(0) \hat{K}_P^0(t_2) \right] \rangle - \langle \delta [S] \, \hat{K}_P^0(t_1) \hat{O}_1(0) \hat{K}_P^0(t_2) \rangle = 0 \,, \tag{13}$$

where $\delta [\ldots]$ denotes the rotation of the argument of δ and $\delta [S]$ is the rotation of the action under the transformation defined in eq. (11) [4].

In terms of the fields defined in eq. (12), and of the parity-even and parity-odd operators Q_1 and $\mathcal{Q}_1$, the Ward identity reads [14]

$$2 \langle \hat{K}_P^0(t_1) \hat{Q}_1(0) \hat{K}_P^0(t_2) \rangle = 2m \sum_x \langle \Pi^0(x) \hat{K}_P^0(t_1) \hat{\mathcal{Q}}_1(0) \hat{K}_P^0(t_2) \rangle$$
$$- \langle \hat{K}_S^0(t_1) \hat{\mathcal{Q}}_1(0) \hat{K}_P^0(t_2) \rangle - \langle \hat{K}_P^0(t_1) \hat{\mathcal{Q}}_1(0) \hat{K}_S^0(t_2) \rangle + \mathcal{O}(a) \,, \tag{14}$$

where, see eqs. (7)–(9),

$$\hat{Q}_1 = Z_{11} \left(Q_1 + \sum_{i=2,5} \Delta_{1i} Q_i \right) \,, \quad \hat{\mathcal{Q}}_1 = \mathcal{Z}_{11} \mathcal{Q}_1 \,. \tag{15}$$

The term on the l.h.s. of eq. (14), corresponding to the rotation of the operator $\mathcal{Q}_1$, is the quantity from which we may extract the physical K^0–$\bar{K}^0$ mixing amplitude.

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[3] As kaon source, we may as well use the fourth component of the axial current $\bar{d} \gamma_0 \gamma_5 s$ instead of the pseudoscalar density.

[4] In general, different choices of the flavour content of the operators, sources and rotation are possible or may be necessary. For example for the operators O_4 and O_5, the suitable choice is $O_{4,5} = (\bar{s}\Gamma u)(\bar{s}\Gamma d)$ (Γ matrices are not specified) with $K_1 = \bar{d}\gamma_5 s$ and $K_2 = \bar{u}\gamma_5 s$) and the same rotation as in eq. (11).
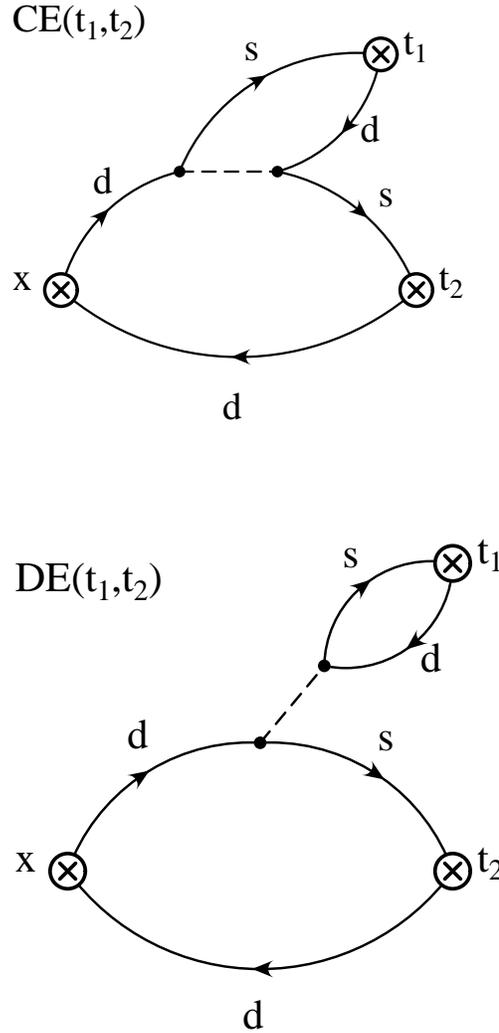
Figure 2: *Connected and disconnected emission diagrams. The dashed line represents the four-fermion operator. We denote as $DE(t_1, t_2)$ and $CE(t_1, t_2)$ the diagrams where the $\bar{K}^0$ meson is emitted in t_1; $DE(t_2, t_1)$ and $CE(t_2, t_1)$ the diagrams where the $\bar{K}^0$ meson is emitted in t_2.*

The first term on the r.h.s. corresponds to the insertion of the rotation of the action. In terms of Feynman diagrams, it can be seen as the "decay" of a neutral (zero four-momentum) pion, π^0, into two $\bar{K}^0$s under the action of $\mathcal{Q}_1$. In the $SU(2)$ isospin symmetric case ($m_u = m_d$), only the emission diagrams shown in fig. 2 must be considered. The last two terms in eq. (14) correspond to the rotation of the pseudoscalar kaon sources. These terms are necessary to saturate the Ward identity.

One could envisage the following method to extract the coefficients Δ_{1i} and the ratio $\Delta_{11} = Z_{11}/\mathcal{Z}_{11}$, thus determining the subtracted operator $\tilde{Q}_1$. Since eq. (14) must be satisfied for any value of t_1 and t_2, the Ward identity corresponds to a system of linear equations in the unknown quantities Δ_i ($i = 11, 12, \ldots, 15$) [24]. At least in principle, one has an independent equation for any assigned value of t_1 and t_2. In practice, the equations become dependent when the Ward identity, for large values of t_1 and t_2, is saturated by the contribution of the lowest lying states, namely the two K-mesons.

This method to extract the Δ_i, which in practice may be very difficult to implement, is however unnecessary and a much easier procedure gives us directly the wanted quantity, namely the correlation function of $\hat{Q}_1$. Let us consider the Ward identity (14) in the limit $t_1 \to \infty$ and $t_2 \to -\infty$ (in practice for large values of the time distances). In this limit, we may safely neglect the last two terms of eq. (14), because they correspond to the propagation of scalar states, which are exponentially suppressed with respect to the kaon contribution [5]. Then, up to exponentially suppressed terms, we have ($\Delta_{11} = Z_{11}/\mathcal{Z}_{11}$ and we have divided all the terms of the Ward identity by a factor of two)

$$
\lim_{t_1 \to \infty, t_2 \to -\infty} \Delta_{11} \langle K_P^0(t_1) \tilde{Q}_1(0) K_P^0(t_2) \rangle \to
$$
$$
\frac{|\langle 0 | K_P^0 | K^0 \rangle|^2}{4 m_K^2} e^{-m_K(t_1 + |t_2|)} \times \mathcal{Z}_{11}^{-1} \langle \bar{K}^0 | \hat{Q}_1 | K^0 \rangle \sim
$$
$$
m \sum_x \langle \Pi^0(x) K_P^0(t_1) \mathcal{Q}_1(0) K_P^0(t_2) \rangle + \mathcal{O}(a) . \tag{16}
$$

Note that the last correlation function, $\langle \Pi^0(x) K_P^0(t_1) \mathcal{Q}_1(0) K_P^0(t_2) \rangle$, is expressed in terms of bare quantities only. Indeed a single constant is sufficient to obtain the physical amplitude, namely $\mathcal{Z}_{11}$ which relate the bare lattice parity-odd operator to the continuum one, renormalized in a specified renormalization scheme. Obviously this constant cannot be determined from the Ward identity, since its values depend on the renormalization condition imposed to the renormalized operator [14, 21, 22].

In terms of the Feynman diagrams defined in figs. 1 and 2, eq. (16) can be written as

$$
2\Big(C8(t_1, t_2) + D8(t_1, t_2) \Big) = \tag{17}
$$
$$
2m \Big(CE(t_1, t_2) + CE(t_2, t_1) + DE(t_1, t_2) + DE(t_2, t_1) \Big) + \mathcal{O}(a) .
$$

In summary, the strategy to obtain the physical matrix element $\langle \bar{K}^0 | \hat{Q}_1 | K^0 \rangle$ is extremely simple:

- one computes the correlation function

$$
G_3(t_1, t_2) = m \sum_x \langle \Pi^0(x) K_P^0(t_1) \mathcal{Q}_1(0) K_P^0(t_2) \rangle
$$

  of the bare parity-odd operator at large time distances $t_{1,2}$; as "mass" m, it is more convenient to use the quark mass defined as $m = Z_A \rho$, where ρ is defined using the axial Ward identity [14, 28, 29]

$$
2\rho = \frac{\langle \partial_\mu A_\mu \rangle}{\langle P \rangle} .
$$

---

[5] This point can be explicitly checked by computing $\langle \hat{K}_S^0(t_1) \hat{\mathcal{Q}}_1(0) \hat{K}_P^0(t_2) \rangle$ in the same numerical simulation as the other correlation functions appearing in eq. (14).

In the above equation A_μ and P are the bare (eventually improved) axial current and pseudoscalar densities and the matrix elements are usually taken between the vacuum and a pion (in our case the π^0) at rest. Z_A is the axial-current renormalization factor.

- then one divides $G_3(t_1, t_2)$ by the factor

$$\mathcal{F}(t_1, t_2) = \frac{Z_5}{4m_K^2} e^{-m_K(t_1 + |t_2|)} \,,$$

where $Z_5 = |\langle 0 | K_P^0 | K^0 \rangle|^2$, obtaining the quantity $\mathcal{R} = \mathcal{Z}_{11}^{-1} \langle \bar{K}^0 | \hat{Q}_1 | K^0 \rangle$. $\mathcal{F}$ can be readily computed from the study of the kaon two-point correlator $\langle K_P^0(t) K_P^{0\dagger}(0) \rangle$. Alternatively, one may construct the ratio

$$R(t_1, t_2) = Z_5 \frac{G_3(t_1, t_2)}{\langle K_P^0(t_1) K_P^{0\dagger}(0) \rangle \langle K_P^0(t_2) K_P^{0\dagger}(0) \rangle} \to \mathcal{R} \,,$$

at large time distances.

- finally, the physical amplitude is given by

$$\langle \bar{K}^0 | \hat{Q}_1 | K^0 \rangle = \mathcal{Z}_{11} \times \mathcal{R} \,.$$

The constant $\mathcal{Z}_{11}$ cannot be determined from the Ward identity and has to be fixed either using perturbation theory, or non-perturbatively on quark states [17]–[21] or with the Schrödinger functional method [29].

The application of the method discussed above to the operators of eq. (3), appearing in extensions of the Standard Model, is so easy that it does not require further discussion. The same approach can be used to compute the $\Delta I = 3/2$ K-π matrix elements, $\langle \pi | O_i^{\Delta S=1} | K \rangle$. A further interesting application is related to hyperon decays. In hyperon decays both the parity-odd and parity-even terms contribute. For the parity-odd case, as explained in 2.1, there are no subtractions induced by the chiral symmetry of the lattice action; for the parity-even case, as done before, we add to the relevant correlation function a soft-pion (zero four momentum) field, i.e. a pseudoscalar density summed over x, multiplied by the factor $2m$ and replace the parity-even operator with the corresponding parity-odd one. Unfortunately, this method works only for $\Delta I = 3/2$ transitions since, in the $\Delta I = 1/2$ case, the presence of power divergences, due to mixing with lower dimensional operators, makes the strategy proposed in this paper almost impossible to implement in practice.

3   Conclusion

With Wilson-like fermions, the calculation of the K^0–$\bar{K}^0$ amplitude is complicated by the necessity of accurately determining many mixing coefficients (either perturbatively

or non-perturbatively), which arise from the explicit chiral symmetry breaking in the lattice action. In this paper, we have shown that, by using suitable Ward identities, it is possible to compute the physical amplitudes without any subtraction. The error on the final answer is of $\mathcal{O}(a)$, which corresponds to the best accuracy attainable with a perfect determination of the mixing coefficients in previous approaches. The extension of this method to the improved case, i.e. to obtain matrix elements with an accuracy of $\mathcal{O}(a^2)$, does not seem possible at present, but it is worth being investigated. Our method can also be applied to $\Delta I = 3/2$ hyperon decays. A pioneering numerical calculation of the K^0–$\bar{K}^0$ matrix elements using the strategy proposed in this paper is already underway.

Acknowledgements

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