New Results From Lattice QCD: Non–Perturbative Renormalization and Quark Masses.

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For the first time, we compute non–perturbatively, i.e. without lattice perturbation theory, the renormalization constants of two–fermion operators in the quenched approximation at $\beta = 6.0$, 6.2 and 6.4 using the Wilson and the tree–level improved SW–Clover actions. We apply these renormalization constants to fully non–perturbatively estimate quark masses in the $\overline{MS}$ scheme from lattice simulations of both the hadron spectrum and the Axial Ward Identity in the quenched approximation. Some very preliminary unquenched Wilson results obtained from the gluon configurations generated by the TχL Collaboration at $\beta = 5.6$ and $N_f = 2$ are also discussed.

1. Introduction.

The values of the quark masses are very important for phenomenology. Quark masses are free parameters of the QCD Lagrangian which cannot be determined within QCD from theoretical considerations only. Since quarks are confined inside hadrons, they are not observed as physical particles. Therefore, **quark masses cannot be measured directly and their values depend on how they are defined from observations, i.e. they are scheme and scale dependent quantities**.

The QCD Lagragian has a chiral symmetry for vanishing quark masses that is spontaneously broken by dynamics and explicitly broken by quark masses. The masses of the Goldstone bosons π, K and η can be computed in a systematic expansion in the quark masses, the so–called chiral perturbation theory (χPT), plus certain unknown non–perturbative parameters, A, B, C, so that to lowest order one has:

$$
\begin{aligned}
m_{PS}^2 &= \mathrm{C}\,(m_1 + m_2) \\
m_V &= \mathrm{A} + \mathrm{B}\,(m_1 + m_2)
\end{aligned}
\tag{1}
$$

where m_{PS} (m_V) is the mass of a pseudoscalar (vector) meson in terms of the quark masses $m_{1,2}$. As a consequence, χPT can only estimate ratios

of quarks masses but **not their absolute values**. Therefore, to determine the absolute normalization of quark masses, we have to use methods which go beyond χPT, such as QCD Sum Rules or Lattice QCD. In this work, we will use Lattice QCD to compute the meson masses and then, by utilizing the χPT results, extract the quark masses. As an alternative procedure, we will also simulate the Axial Ward Identity on the lattice to calculate quark masses. We do not compute the bottom quark mass which can be extracted from the Lattice HQET [1].

2. The lattice definition of quark masses.

In this work, we will consider the Wilson and the tree–level improved SW–Clover lattice actions in Euclidean space. They are parameterized in terms of (β, κ) which are the only tunable input parameters related to the bare lattice coupling constant, g_0, and the bare lattice quark mass, m_0, via $\beta = 6/g_0^2$ and $\kappa = 1/(2\,m_0 a + 8)$ respectively, with a the lattice spacing. As is well known, these actions contain a term proportional to a, the so–called Wilson term, which is needed to avoid the fermion doubling problem, but explictly breaks the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry. On the lattice, the chiral properties of QCD are lost so that the chiral limit does not correspond to vanishing bare lattice quark masses and operators of different chirality can mix among themselves.

In spite of this, it is possible to find some lattice currents which are partially conserved and obey the Current Algebra in the limit $a \to 0$ [2].

Vector Ward Identities (VWI) are obtained by applying infinitesimal vector flavour transformations to the quark fields of the lattice actions and exploting the invariance of the path integral under local changes of the fermionic variables. Using them, it is easy to show that there is a lattice vector current, $\tilde{V}_\mu$, which is both conserved in the limit of degenerate quarks, i.e. $Z_{\tilde{V}} = 1$, and improved, i.e. its matrix elements have errors of order $O(g_0^2 a)$ only. For the Wilson action, the lattice VWI between on–shell hadronic states α and β takes the form

$$\langle\alpha|\partial_\mu \tilde{V}_\mu|\beta\rangle = \frac{1}{2}\left(\frac{1}{\kappa_2} - \frac{1}{\kappa_1}\right)\langle\alpha|S|\beta\rangle \qquad (2)$$

where $\kappa_{1,2}$ are the hopping parameter of the quarks of the hadron, $S = \bar{q}_2 q_1$ is the bare scalar density and ∂_μ is the forward lattice derivative. By comparing with the continuum QCD VWI, one obtains the relation between the renormalized lattice quark mass and the hopping parameter

$$m_R a = Z_S^{-1} m a \equiv Z_S^{-1}\frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) \qquad (3)$$

where κ_c is the critical value of the hopping parameter corresponding to a vanishing quark mass, and Z_S is the renormalization constant of the scalar density. For the SW–Clover action, a similar relation can be derived (see [3]). In order to determine κ, one could compute the ratio of the vector and scalar densities on the lattice but, unfortunately, the scalar density matrix elements turn out to be extremely noisy, preventing any reliable analysis. Alternatively, one can calculate κ by fixing the mass of an hadron containing a quark of a given flavour to its experimental value using eq.(1). This is the *Spectroscopy method.*

Axial Ward Identities (AWI) are obtained analogously by applying axial–vector transformations. In this case, however, there is no partially conserved lattice axial current either for the Wilson or SW–Clover action. Therefore, any lattice definition of the axial current get renormalized, even in the limit of vanishing bare quark masses. For degenerate quarks, close to the chiral limit ($\kappa = \kappa_c$) and neglecting terms of order $O(a)$, one obtains the AWI [2]

$$Z_A \langle\alpha|\partial_\mu A_\mu|\beta\rangle = \left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)\frac{Z_P}{Z_S}\langle\alpha|P|\beta\rangle \qquad (4)$$

where $P = \bar{q}_2\gamma_5 q_1$ is the bare pseudoscalar density, $A_\mu = \bar{q}_2\gamma_\mu\gamma_5 q_1$ the bare axial current and Z_P and Z_A are their renormalization constants. From (4), one can define the renormalized quark mass as

$$m_R a = \frac{Z_A}{Z_P}\frac{\rho a}{2} \equiv \frac{Z_A}{Z_P}\frac{a}{2}\frac{\langle 0|\partial_0 A_0|\pi(\vec{p}=0)\rangle}{\langle 0|P|\pi(\vec{p}=0)\rangle} \qquad (5)$$

where $|\pi(\vec{p} = 0)\rangle$ is a pseudoscalar meson state containing a quark of hopping parameter κ. For the SW–Clover improved case, the only difference is that one has to consider the ratio of improved operators and take the symmetric derivative. This is the *Axial Ward Identity method* which does not use χPT.

From the VWI and the AWI some interesting properties of the renormalization constants can be obtained:

1. Z_V, where $V_\mu = \bar{q}_2\gamma_\mu q_1$, and Z_A are finite functions of g_0^2 with Z_V, $Z_A \neq 1$.

2. Z_P and Z_S are logarithmically divergent in μa but Z_S/Z_P is a finite function of g_0^2 only with $Z_S/Z_P \neq 1$ for the Wilson and SW–Clover lattice actions.

3. Evaluation of the renormalization constants.

Matching between lattice and continuum is necessary to convert the lattice results to the continuum renormalization scheme one has chosen to analyze the experimental data. The matching factors Z's are called lattice renormalization constants. At scales $a^{-1} = 2 - 4$ GeV of our simulations, one expects small non–perturbative effects on the Z's so that lattice (LPT) and continuum (CPT) perturbation theory may be used to calculate them. In almost all cases, however, the Z's are known only to one–loop and, further, is well known that LPT corrections are large due to **tadpole** diagrams. It is claimed that the so–called *Boosted perturbation theory* (BPT) can re-

sum tadpole diagrams by replacing the bare lattice coupling constant with a non–perturbatively renormalized one, $\alpha_P = g_0^2/(4\pi)P$, where P is the plaquette expectation value, so that the observables calculated with this coupling are closer to their non–perturbative counterparts. Unfortunately, data show that even after the implementation of the BPT, $O(\alpha_s^2)$ corrections can be large, of the order of $10-50\%$, depending on the quantity at hand. Alternatively, one can use the non–perturbative renormalization technique proposed by Martinelli *et al* [5]. The idea is to impose renormalization conditions directly on quark and gluon Green functions in a fixed gauge with given off–shell external states with large virtualities μ^2, mimicking the continuum renormalization procedure but performing all computations non–perturbatively on the lattice. This procedure defines the Regularization Independent scheme (RI), also called MOM. For example, the renormalized scalar density in the RI scheme is $\hat{P}^{\rm RI}(\mu) = Z_P(\mu,a)\,P(a)$ where Z_P is fixed by imposing the renormalization condition

$$Z_P(\mu,a)\,\langle p|P(a)|p\rangle\,|_{p^2=-\mu^2} = \langle p|P(a)|p\rangle_o \qquad (6)$$

where $\langle p|P(a)|p\rangle_o$ is the tree level matrix element. Since (6) can be evaluated non–perturbatively through lattice simulations, Z_P is extracted without LPT. Notice that Z_P is affected by possible $O(a)$ effects and depends on the external states and on the gauge. This dependence, however, will cancel in the final results at a given order when combined with the continuum $\overline{MS}$–RI matching calculation. For this method, which defines the same renormalized operators in all regularization schemes, to work the following window in μ must exist

$$\Lambda_{QCD} << \mu << a^{-1} \qquad (7)$$

where the first term is required to avoid large higher–order corrections in CPT while the second one is necessary to avoid $O(a)$ effects. The evolution of a bilinear operator renormalized in the RI scheme $\hat{O}_\Gamma^{RI}(\mu)$ $(\Gamma = V, A, P$ or $S)$ can be expressed in the form

$$\hat{O}_\Gamma^{RI}(\mu) = \frac{c_\Gamma^{RI}(\mu)}{c_\Gamma^{RI}(\mu_0)}\,\hat{O}_\Gamma^{RI}(\mu_0) \qquad (8)$$

where c_Γ^{RI} are the solutions of the RGE which are known up to N²LO [6]. Therefore, we can define a renormalization group invariant quantity

$$Z_\Gamma^{RGI}(a) \equiv \frac{Z_\Gamma^{RI}(\mu a)}{c_\Gamma^{RI}(\mu)} \qquad (9)$$

which, up to higher order terms in CPT, should be independent of μ, in the region in which CPT is valid $\mu \geq 2$ GeV, of the renormalization scheme, of the external states and gauge invariant. In fig.1 and 2, we show our preliminary results for the Z's for the unquenched Wilson action (see ref.[7]). We can clearly see a window in μ where the Z's are scale independent. The corresponding Z's for the Wilson and SW–Clover actions in the quenched approximation are similar, but somewhat less stable, and can be found in ref.[6]. We have used them to perform the first measurement of quark masses without LPT.

4. The continuum $\overline{MS}$ quark masses.

Since the $\overline{MS}$–RI matching factor depends on typical scales of order $\mu \approx a^{-1} \approx 2-4$ GeV, it can be calculated in continuum perturbation theory only. We consider two cases:

Perturbative calculation of the Z's, i.e. one uses both BPT and CPT at NLO

$$m^{\overline{MS}}(\mu) = U_m^{\overline{MS}}(\mu, \pi/a) \times \left\{ \begin{array}{ll} 1/Z_S^{\overline{MS}} & m \\ Z_A^{\overline{MS}}/Z_P^{\overline{MS}} & \frac{\rho}{2} \end{array} \right.$$

where $U_m^{\overline{MS}}$ is the evolutor operator for the quark mass at NLO in the continuum. The renormalization constants $1/Z_S^{\overline{MS}}$ and $Z_A^{\overline{MS}}/Z_S^{\overline{MS}}$ are known to $O(\alpha_s)$ only (see ref.[6] for details).

Non–Perturbative calculation of the Z's, i.e. one uses CPT but the Z's are computed non–perturbatively as explained in section 3. In this case, a N²LO matching can be performed

$$m^{\overline{MS}}(\mu) = U_m^{\overline{MS}}(\mu, \mu') \left[1 + \frac{\alpha_s(\mu')}{4\pi}c^{(1)} \right.$$
$$+ \left. \left(\frac{\alpha_s(\mu')}{4\pi}\right)^2 c^{(2)} \right] \times \left\{ \begin{array}{ll} 1/Z_S^{RI}(\mu'a) & m \\ Z_A^{RI}/Z_P^{RI}(\mu'a) & \frac{\rho}{2} \end{array} \right.$$

where $c^{(1)}$ and $c^{(2)}$ are $\overline{MS}$–RI matching constants (see ref.[6] for details).

Figure 1. Z_V and Z_A for the unquenched Wilson action as a function of μ at $\kappa_{sea} = 0.1580$.
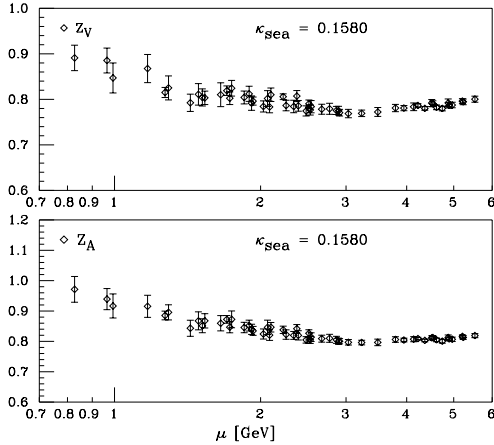

Figure 2. Z_P and Z_S for the unquenched Wilson action as a function of μ at $\kappa_{sea} = 0.1580$.

5. Results.

We have analyzed about 1000 quenched configurations for the Wilson action and about 2000 quenched configurations for the SW–Clover action in a variety of physical volumes (ranging from $16^3 \times 32$ to $24^3 \times 64$) and lattice spacings corresponding to $\beta = 6.0$, 6.2 and 6.4. We refer the reader to the ref.[3] for technical details. For the spectroscopy method, we find a resonable agreement between the quark masses obtained with perturbative (PT) evaluated renormalization constants and non–perturbative (NP) ones and are compatible with previous determinations. This not the case, however, for the AWI method where the PT results are lower than the NP ones by more than two standard deviations. Only when we use non–perturbative renormalization constants, the spectroscopy and AWI methods give consistent values of the quark masses, as it should be. We think that the reason is that lattice perturbation theory fails to determine Z_P even if the Boosted recipes are implemented. As for the charm quark mass, our results are less stables than for the light and strange quarks. We think that the reason is that there is a rather large $O(ma)$ contamination in our data for the charm quark mass. Finally, we want to stress that we cannot extrapolate to the continuum limit, even though we have simulated at three values of β,
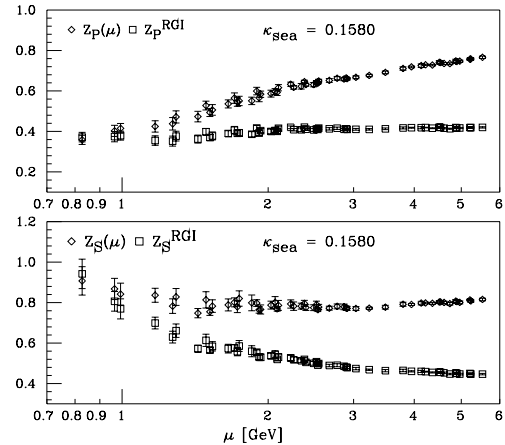
due to the small physical volume at $\beta = 6.4$. Since our results at $\beta = 6.0$ and $\beta = 6.2$ are definitely stables within errors, i.e. we do not see any dependence on a in our results, we average the NP AWI results and obtain our best estimates:

$$
\begin{aligned}
\bar{m}^{\overline{MS}}(2\,\text{GeV}) &= 5.7 \pm 0.1 \pm 0.8 \ \text{MeV} \\
m_s^{\overline{MS}}(2\,\text{GeV}) &= 130 \pm 2 \pm 18 \ \text{MeV} \\
m_c^{\overline{MS}}(2\,\text{GeV}) &= 1662 \pm 30 \pm 230 \ \text{MeV} \quad (10)
\end{aligned}
$$

where the first error is statistical and the second is an estimate of the systematic error evaluated from the spread in the values of the quark masses extracted from different mesons, lattices and chiral extrapolation methods.

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