Quark masses and the chiral condensate with a non-perturbative renormalization procedure

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We determine the quark masses and the chiral condensate in the $\overline{MS}$ scheme at NNLO from Lattice QCD in the quenched approximation at $\beta = 6.0$, $\beta = 6.2$ and $\beta = 6.4$ using both the Wilson and the tree-level improved SW-Clover fermion action. We extract these quantities using the Vector and the Axial Ward Identities and non-perturbative values of the renormalization constants. We compare the results obtained with the two methods and we study the $O(a)$ dependence of the quark masses for both actions.

1. INTRODUCTION

To describe the light hadron spectrum one considers massless QCD a good approximation to this world and the vacuum not symmetric under chiral transformations. The chiral condensate is the order parameter which governs the spontaneous chiral symmetry breaking. The pseudoscalar mesons are identified with the goldstone bosons and their physical masses are attributed to the mass term in the QCD Hamiltonian. Since free quarks are not physical states, quark masses cannot be measured directly in the experiments and can be determined from the meson spectrum using non-perturbative techniques. On the lattice one can compute the quark masses and the chiral condensate from first principles and it is the only procedure that can be systematically improved. The methods and the symbols we have used and all the results we have obtained are fully described in [1,2].

2. QUARK MASSES

The usual on-shell mass definition cannot be used for quark masses and their values depend on the theoretical definition adopted. In the following we will give our final results for the running quark masses defined in the $\overline{MS}$ scheme. Quark masses can be defined from the Vector Ward Identity (VWI). Neglecting terms of $O(a)$, the VWI between on-shell hadronic states can be written as [3]

$$\langle\alpha|\partial^\mu \tilde{V}_\mu|\beta\rangle = \frac{1}{2}\Big(\frac{1}{k_2} - \frac{1}{k_1}\Big)\langle\alpha|S|\beta\rangle \ . \qquad (1)$$

Eq. (1) fixes the relation between the lattice bare quark mass in lattice units and the hopping parameter, i.e. for the Wilson action $m = 1/2(1/k - 1/k_c)$. Quark masses can also be extracted from the Axial Ward Identity (AWI). Neglecting terms of $O(a)$, the AWI can be written as [3]

$$Z_A\langle\alpha|\partial^\mu A^a_\mu|\beta\rangle = 2(m_0 - \overline{m})\langle\alpha|P^a|\beta\rangle \ , \qquad (2)$$

where $\overline{m}$ is defined in [3]. The light and strange quark masses are determined by fixing to their experimental values the masses of the π and K mesons. The standard perturbative approach [1,4] uses lattice and the continuum perturbation theory to connect the "bare" lattice quark mass to the renormalized $\overline{MS}$ one. The scale $1/a$, where a is the lattice spacing, of our simulations is $a^{-1} \simeq 2-4$ GeV. At these scales we expect small non-perturbative effects. However the

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Table 1
Summary of the parameters of the runs analyzed in this work.

	Matrix Elements						
	C60a	C60b	W60	C62	W62	C64	W64
β	6.0	6.0	6.0	6.2	6.2	6.4	6.4
Action	SW	SW	Wil	SW	Wil	SW	Wil
# Confs	490	600	320	250	250	400	400
Volume	$18^3 \times 64$	$24^3 \times 40$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$
	Renormalization Constants						
	C60Z	W60Z	C62Z	W62Z	C64Z	W64Z	
β	6.0	6.0	6.2	6.2	6.4	6.4	
Action	SW	Wil	SW	Wil	SW	Wil	
# Confs	100	100	180	100	60	60	
Volume	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$24^3 \times 32$	$24^3 \times 32$	

Table 2
Quark Masses an the Chiral Condensate from the VWI in MeV. $\overline{MS}$ values are at $\mu = 2$ GeV.

Run	$m_l^{\overline{MS}}$	$m_s^{\overline{MS}}$	$-\frac{1}{N_f}\langle\bar{\psi}\psi\rangle_1$
W60	5.8(2)(1)	130(3)(2)	$(247 \pm 2 \pm 1)^3$
W62	5.4(2)(1)	124(4)(2)	$(250 \pm 3 \pm 1)^3$
W64	4.9(2)(1)	112(5)(2)	$(258 \pm 4 \pm 1)^3$

"tadpole" diagrams, which are present in lattice perturbation theory, can give rise to large perturbative corrections and hence to large uncertainties in the 1-loop matching procedure at values of $\beta = 6/g_L^2 = 6.0 - 6.4$. A non-perturbative renormalization (NP) technique eliminate these uncertainties [5,6]. Z_A, Z_P and Z_S can be calculated by imposing the renormalization conditions, proposed in [5], on the quark states of momentum $p^2 = \mu^2$ and in the Landau gauge [2]. This procedure works if μ satisfies the condition $\Lambda_{QCD} \ll \mu \ll 1/a$ to avoid chiral symmetry breaking effects, large higher-order perturbative corrections and discretization errors. In figure 1 we show the scalar renormalization constant divided by its Renormalization Group (RG) evolution (Z_S^{RGI}). The data show that discretization errors are within statistical errors in the range $0.5 < \mu a < 2$ where the NP renormalization is applied to compute the quark masses. The NP method allows a fully non perturbative definition of the renormalized quark masses in the RI scheme

$$m^{RI}(\mu) = \frac{1}{Z_S^{RI}(\mu a)} m a^{-1}$$

$$m^{RI}(\mu) = \frac{Z_A^{RI}}{Z_P^{RI}(\mu a)} \rho a^{-1} \qquad (3)$$

where for large time separations

$$\rho(a) = \frac{1}{2}\sinh(M_{PS})\frac{\langle A_0(\tau)P(0)\rangle}{\langle P(\tau)P(0)\rangle} \ . \qquad (4)$$

The $\overline{MS}$ definition of the renormalized quark masses is intrinsically perturbative and can be related to the RI one through continuum perturbation theory only:

$$m^{\overline{MS}}(\mu) = U_m^{\overline{MS}}(\mu,\mu')\frac{Z_m^{\overline{MS}}(\mu')}{Z_m^{RI}(\mu')} m^{RI}(\mu') \ , \qquad (5)$$

where $U_m^{\overline{MS}}(\mu,\mu')$ is the RG evolution of the quark mass. $Z_m^{\overline{MS}}/Z_m^{RI}$ is the matching factor computed in perturbation theory at scales $\mu \simeq 2-4$ GeV large enough to avoid non-perturbative effects and/or higher order corrections.

3. THE CHIRAL CONDENSATE

The chiral condensate can be defined using the AWI arising from the variation of the non-singlet pseudoscalar density. The integrated AWI becomes [2,3]

$$\frac{1}{N_f}\langle\bar{\psi}\psi\rangle = \lim_{m_0 \to m_C} 2(m_0 - \overline{m})\int d^4x \langle P(x)P(0)\rangle \ .$$

Table 3
Quark Masses and the Chiral Condensate from the AWI in MeV. $\overline{MS}$ values are at $\mu = 2$ GeV.

Run	$m_l^{\overline{MS}}$	$m_s^{\overline{MS}}$	$-\frac{1}{N_f}\langle\bar\psi\psi\rangle_2$
C60a	6.0(2)(9)	136(4)(20)	$(242 \pm 3 \pm 12)^3$
C60b	5.7(2)(8)	132(4)(19)	$(244 \pm 2 \pm 12)^3$
W60	5.7(2)(8)	127(4)(17)	$(248 \pm 2 \pm 11)^3$
C62	5.8(5)(6)	131(7)(14)	$(245 \pm 4 \pm 9)^3$
W62	5.4(2)(5)	122(4)(12)	$(251 \pm 3 \pm 8)^3$
C64	4.4(3)(3)	104(5)(6)	$(265 \pm 4 \pm 5)^3$
W64	4.7(2)(3)	108(5)(8)	$(262 \pm 4 \pm 6)^3$

In the chiral limit one can show that the above expression is equivalent to

$$\frac{1}{N_f}\langle\bar\psi\psi\rangle = -\lim_{m_0\to m_C}\frac{f_P^2 M_P^2}{4(m_0-\overline{m})}\ , \qquad (6)$$

which is the familiar Gell-Mann–Oakes–Renner (GMOR) relation. We write the relation (6) for the renormalized condensate as

$$\frac{1}{N_f}\langle\bar\psi\psi\rangle_1 = -\frac{1}{2}a^{-1}f_\chi^2 Z_S C^{HS}$$
$$\frac{1}{N_f}\langle\bar\psi\psi\rangle_2 = -\frac{1}{2}a^{-1}f_\chi^2 \frac{Z_P}{Z_A} C^{AWI}\ , \qquad (7)$$

where $f_\chi = 0.1282$ GeV is the "experimental" value in physical units [2] and

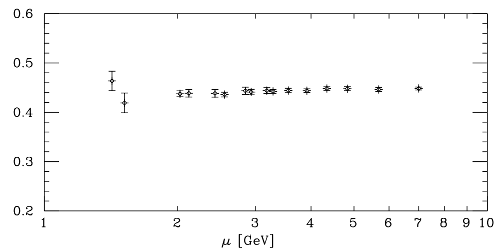
$$M_P^2 = C^{HS}\left(\frac{1}{\kappa}-\frac{1}{\kappa_C}\right)$$
$$2a\rho = \frac{1}{C^{AWI}}M_P^2\ . \qquad (8)$$

The main advantages of the above formulas are to avoid the error amplification of the standard method because we are left with only one power of the UV cutoff a^{-1} and to determine the slope C^{HS} (C^{AWI}) without extrapolation to the chiral limit. Note that since we work at κ values typical of the strange quark mass, we are implicitly assuming that the slope will not change in the chiral region.

4. RESULTS

We have computed the renormalization constants of bilinear quark operators with the NP method proposed in [5]. The μ-dependence of Z_A and Z_S are in excellent agreement with the RG predictions. Z_P is in good agreement but the chiral symmetry breaking effects deserve a more accurate analysis. The parameters and the action used in each simulation are listed in Table 1 and the main results we have obtained are reported in Tables 2 and 3. The data at $\beta = 6.4$ have to be consider for an exploratory study only, since the physical volume and the time extension of the lattice are too small to be considered reliable. The $\overline{MS}$ values of the quark masses and the chiral condensate reported in Tables 2 and 3 show a very good agreement between the values extracted from the VWI and from the AWI using the non perturbative determinations of the renormalization constants, while the same comparison using the perturbative values of Z_A, Z_P and Z_S fails giving inconsistent results for the two methods [1]. This pattern is also found by [7,8] for different fermion actions. In the β range studied, there is *no* statistical evidence for an "a" dependence of the quark masses and the chiral condensate. We believe that the best estimates for the light and strange quark masses and the chiral condensate are $m_l^{\overline{MS}}(2\ GeV) = (5.7\pm5\pm8\pm8)\ MeV$, $m_s^{\overline{MS}}(2\ GeV) = (130 \pm 8 \pm 15 \pm 15)\ MeV$ and $-\frac{1}{N_f}\langle\bar\psi\psi\rangle^{\overline{MS}}(2\ GeV) = (245 \pm 4 \pm 9 \pm 7\ MeV)^3$, where the first error is statistical, the second is due to the non-perturbative renormalization and the third is an estimate of the overall systematic errors on the bare quantities [1].

Figure 1. Z_S^{RGI} vs μ for the run W64.



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