Neutrino masses and mixing in A_4 models with three Higgs doublets

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We study neutrino masses and mixing in the context of flavor models with A_4 symmetry, three scalar doublets in the triplet representation, and three lepton families. We show that there is no representation assignment that yields a dimension-5 mass operator consistent with experiment. We then consider a type-I seesaw with three heavy right-handed neutrinos, explaining in detail why it fails, and allowing us to show that agreement with the present neutrino oscillation data can be recovered with the inclusion of dimension-3 heavy neutrino mass terms that break softly the A_4 symmetry.

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I. INTRODUCTION

One can attempt to explain the structure of neutrino masses and mixing by building full theories subject to discrete symmetries (for recent reviews, see e.g. Refs. [1–4]). As an alternative, one may consider effective operators in the low-energy limit and their remnant symmetries, without recourse to the full theory [5]. In either case, it is important to ensure that the vacuum structure chosen for the scalar sector of the theory does indeed correspond to a global (and not merely a local) minimum. Recently [6], the global minima of theories with three Higgs doublets, Φ_k ($k = 1, 2, 3$), in triplet representations of A_4 [7] or S_4 have been identified. It turns out that the allowed alignments for the vacuum expectation values (VEVs) in A_4 are [6]

$$\begin{aligned} &v\,(1,0,0),\\ &v\,(1,1,1),\\ &v\,(\pm1,\eta,\eta^*) \text{ with } \eta = e^{i\pi/3},\\ &v\,(1,e^{i\alpha},0) \text{ with any phase } \alpha. \end{aligned} \tag{1}$$

Permutations of these VEVs are still global minima; other solutions of the stationarity conditions are not. The quark sector of such theories was explored by us in Ref. [8]. It was shown that, at tree level, there is no consistent theory with only three families of standard quark fields that can explain the fact that the quark masses and the Cabibbo-Kobayashi-Maskawa CP-violating phase are nonvanishing.

In this article, we extend the analysis of Ref. [8] to the neutrino sector, assuming that neutrino masses are generated through a low-energy mass operator in an effective theory. We consider models with three Higgs doublets Φ_k in a triplet representation of A_4, so that the only possible VEV structures are those in Eq. (1).

In Sec. II we recall the features that apply to the charged-lepton Yukawa matrices in A_4 [8]. Then, in Sec. III, we turn to the neutrino sector. In Sec. III A, we address the question of whether it is possible to get a consistent picture in an effective theory with the following particle content: three scalar doublets in a triplet representation of A_4, three left-handed lepton doublets L_L and three right-handed charged leptons ℓ_R in any representation of A_4. We conclude that, although an A_4-invariant dimension-5 effective operator $(L_L\Phi)(L_L\Phi)$ can always be built, it is not possible to assign suitable A_4 representations to the fields in order to obtain viable (nondegenerate and nonvanishing) mass spectra, with the VEV alignments given in Eq. (1) and neutrino masses arising solely from the effective operator. As a result, in such a minimal A_4 framework, none of the standard seesaw mechanisms, usually invoked to give small masses to neutrinos, is consistent with the experimental data. With the aim of identifying what features need to be corrected when building a complete viable model, in Sec. III B we discuss in detail type-I (type-III) seesaw models [1] that contain a minimal particle content, namely, three generations of left-handed lepton doublets and right-handed charged lepton singlets, and three right-handed neutrino singlets ν_R (fermion triplets Σ_R). Since, at the Lagrangian level, the A_4 flavor group structure is the same for both seesaw cases, so are the conclusions. Clearly, the above minimal setup is not sufficient to build a consistent A_4 flavor model that leads to nonzero nondegenerate charged lepton and neutrino masses, and to the correct leptonic mixing. The detailed discussion presented in Sec. III B is used in Sec. IV to uncover examples where a soft breaking of the A_4 symmetry is enough to provide consistency with experiment. Our conclusions are briefly summarized in Sec. V.

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[1] We do not consider here the type-II seesaw scenario since it involves triplet scalars, thus making, in general, the identification of the global minima of the scalar potential extremely hard.

II. A_4 AND CHARGED LEPTONS

The A_4 group has four irreducible representations: three singlets $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, and one triplet $\mathbf{3}$. Here we adopt the basis used in Ref. [8] for the generators of the group, and place the three Higgs doublets in the triplet representation, $\Phi \sim \mathbf{3}$. The conclusions reached in Table II of Ref. [8] for the down-type quarks also hold for charged leptons. In particular, requiring nonvanishing nondegenerate charged lepton masses forces the VEV alignments to be $v(1,1,1)$ or $v(\pm1,\eta,\eta^*)$, restricting the possible representation assignments to the cases listed in Table I.

L_L	ℓ_R
$\mathbf{3}$	$\mathbf{3}$
$\mathbf{3}$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$
$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	$\mathbf{3}$

TABLE I. Possible representations of the left-handed lepton doublet (L_L) and right-handed charged-lepton singlets (ℓ_R), which lead to nonvanishing nondegenerate charged lepton masses, when the three Higgs doublets are in a triplet representation $\mathbf{3}$. The case $L_L \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$ leads to nonrealistic charged-lepton masses.

Let us consider in detail the case $L_L \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$. We may parametrize the charged lepton mass matrix as

$$m_\ell = \begin{pmatrix} 0 & b\,e^{i\beta}\,v_3 & a\,e^{i\alpha}\,v_2 \\ a\,e^{i\alpha}\,v_3 & 0 & b\,e^{i\beta}\,v_1 \\ b\,e^{i\beta}\,v_2 & a\,e^{i\alpha}\,v_1 & 0 \end{pmatrix}. \tag{2}$$

Taking the VEV alignment $(1,1,1)$ or $(\pm1,\eta,\eta^*)$,[2] the eigenvalues of $m_\ell m_\ell^\dagger$ are

$$\begin{aligned} &a^2+b^2+2ab\cos(\alpha-\beta), \\ &a^2+b^2-2ab\cos(\alpha-\beta\pm\pi/3), \end{aligned} \tag{3}$$

which coincide with the squared masses of the charged leptons. Although the eigenvalues depend on three parameters, one can see that they cannot be used to fit the hierarchical structure of the charged lepton masses. Indeed, it is easy to show that

$$\begin{aligned} 3(a^2+b^2) &= m_e^2+m_\mu^2+m_\tau^2\,, \\ 3ab &= \sqrt{m_e^4+m_\mu^4+m_\tau^4-m_e^2m_\mu^2-m_e^2m_\tau^2-m_\mu^2m_\tau^2}\,. \end{aligned} \tag{4}$$

Substituting for the experimental masses [9], we find that $(a-b)^2 \simeq -1.05\ \mathrm{GeV}^2$. Thus, in this case, the charged lepton masses cannot be fitted.

[2] Henceforth, we assume without loss of generality that $v=1$.

III. A_4 AND MAJORANA NEUTRINOS

If neutrinos were Dirac particles, with masses arising from Yukawa couplings with Φ_k, our conclusions would be the same as those reached in Ref. [8] for the up-type quarks. Clearly, in that case, the existence of one massless neutrino or the lack of leptonic CP violation would not contradict current experiments [10]. In this work, however, we focus on Majorana neutrinos whose masses are given by some seesaw mechanism or else are generated effectively by integrating out unspecified heavy degrees of freedom.

A. Effective dimension-5 operator for neutrino masses

There is only one dimension-5 operator made out of the SM fields which respects the SM gauge symmetry [11],

$$\mathcal{L}_{\rm eff} = \left(\bar{L}_L\tilde{\Phi}\right)\,K\,\left(\tilde{\Phi}^T L_L^c\right) + \text{H.c.}, \tag{5}$$

where K is a complex symmetric matrix, and $\tilde{\Phi} = i\sigma_2\Phi$. Since we are only interested in the group structure, we will only refer to $L_L L_L$ and $\tilde{\Phi}\tilde{\Phi}$.

If $L_L \sim \mathbf{3}$, we can form the symmetric bilinears

$$\begin{aligned} (L_L\otimes L_L)_{\mathbf{1}} &= L_1^2+L_2^2+L_3^2, &(6)\\ (L_L\otimes L_L)_{\mathbf{1}'} &= L_1^2+\omega^2L_2^2+\omega L_3^2, &(7)\\ (L_L\otimes L_L)_{\mathbf{1}''} &= L_1^2+\omega L_2^2+\omega^2 L_3^2, &(8)\\ (L_L\otimes L_L)_{\mathbf{3}s} &= 2(L_2L_3, L_3L_1, L_1L_2), &(9) \end{aligned}$$

where $\omega = e^{2i\pi/3}$. Similar combinations hold for $\tilde{\Phi}\tilde{\Phi}$. Forming all combinations that can lead to a singlet, we find

$$\begin{aligned} \mathcal{L}_{\rm eff} = {} & \lambda_1(L_1^2+L_2^2+L_3^2)(\tilde{\Phi}_1^2+\tilde{\Phi}_2^2+\tilde{\Phi}_3^2) \\ & +\lambda_2(L_1^2+\omega^2L_2^2+\omega L_3^2)(\tilde{\Phi}_1^2+\omega\tilde{\Phi}_2^2+\omega^2\tilde{\Phi}_3^2) \\ & +\lambda_3(L_1^2+\omega L_2^2+\omega^2 L_3^2)(\tilde{\Phi}_1^2+\omega^2\tilde{\Phi}_2^2+\omega\tilde{\Phi}_3^2) \\ & +\lambda_4(L_2L_3\tilde{\Phi}_2\tilde{\Phi}_3+L_3L_1\tilde{\Phi}_3\tilde{\Phi}_1+L_1L_2\tilde{\Phi}_1\tilde{\Phi}_2), \end{aligned} \tag{10}$$

where $\lambda_1\dots\lambda_4$ are complex parameters.

After spontaneous symmetry breaking, the fields get VEVs $\langle\Phi_k\rangle = v_k$, and the elements of the symmetric matrix K become

$$\begin{aligned} k_{11} &= \lambda_1(v_1^{*2}+v_2^{*2}+v_3^{*2})+\lambda_2(v_1^{*2}+\omega^2v_2^{*2}+\omega v_3^{*2}) \\ &\quad \lambda_3(v_1^{*2}+\omega v_2^{*2}+\omega^2v_3^{*2}), \\ k_{22} &= \lambda_1(v_1^{*2}+v_2^{*2}+v_3^{*2})+\lambda_2\omega(v_1^{*2}+\omega^2v_2^{*2}+\omega v_3^{*2}) \\ &\quad \lambda_3\omega^2(v_1^{*2}+\omega v_2^{*2}+\omega^2v_3^{*2}), \\ k_{33} &= \lambda_1(v_1^{*2}+v_2^{*2}+v_3^{*2})+\lambda_2\omega^2(v_1^{*2}+\omega^2v_2^{*2}+\omega v_3^{*2}) \\ &\quad \lambda_3\omega(v_1^{*2}+\omega v_2^{*2}+\omega^2v_3^{*2}) \\ k_{12} &= \frac{1}{2}v_1^*v_2^*\lambda_4, \quad k_{13} = \frac{1}{2}v_1^*v_3^*\lambda_4, \quad k_{23} = \frac{1}{2}v_2^*v_3^*\lambda_4. \end{aligned} \tag{11}$$

Only the VEV alignments $(1,1,1)$ and $(\pm1,\eta,\eta^*)$ lead to nonvanishing charged lepton masses. With $(1,1,1)$, we find

$$K = \frac{1}{2}\begin{pmatrix} 6\lambda_1 & \lambda_4 & \lambda_4 \\ \lambda_4 & 6\lambda_1 & \lambda_4 \\ \lambda_4 & \lambda_4 & 6\lambda_1 \end{pmatrix}, \tag{12}$$

meaning that $KK^\dagger$ has a doubly degenerate eigenvalue $9|\lambda_1|^2 + |\lambda_4/2|^2 - 3\mathrm{Re}(\lambda_1\lambda_4^*)$ and a third eigenvalue $9|\lambda_1|^2 + |\lambda_4|^2 + 6\mathrm{Re}(\lambda_1\lambda_4^*)$. With $(\pm1, \eta, \eta^*)$ the matrix K is slightly different, but the eigenvalues of $KK^\dagger$ are the same with $\lambda_1 \to \lambda_3$. So, the effective dimension-5 term with A_4 symmetry, $\Phi \sim \mathbf{3}$, and $L_L \sim \mathbf{3}$ is ruled out.

Finally, we turn to the possibility that L_L is a singlet of A_4. We know from Table I that the only choice compatible with the charged lepton masses is $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim \mathbf{3}$. The $L_L L_L$ group structures obtainable when $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ are contained in the right-hand side of Eq. (A5). Entries with $\mathbf{1}$ couple to the $\tilde\Phi\tilde\Phi$ combination $(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}}$. This can be read from Eq. (6) by changing $L_L \to \tilde\Phi$, which after spontaneous symmetry breaking leads to $(v_1^{*2} + v_2^{*2} + v_3^{*2})$. Similarly, entries with $\mathbf{1}'$ in Eq. (A5) couple to the $(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}''}$ combination $(v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2})$. Finally, entries with $\mathbf{1}''$ in Eq. (A5) couple to the $(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}'}$ combination $(v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2})$.

For the VEV $(1, 1, 1)$, the above combinations give $v_1^{*2} + v_2^{*2} + v_3^{*2} = 3$, $v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2} = 0$, and $v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2} = 0$. In this case, the matrix K is of the form

$$K = 3\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 \\ 0 & \lambda_2 & 0 \end{pmatrix}. \tag{13}$$

The matrix $KK^\dagger$ has a doubly degenerate eigenvalue $3|\lambda_2|^2$, and a third eigenvalue $3|\lambda_1|^2$. Similarly, for the VEV $(\pm1, \eta, \eta^*)$, the relevant VEV combinations are 0, 3, and 0, respectively, with

$$K = 3\begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}. \tag{14}$$

Again, $KK^\dagger$ has a doubly degenerate eigenvalue $3|\lambda_2|^2$ and a third eigenvalue $3|\lambda_1|^2$. As a result, all cases are ruled out.

B. Type-I and type-III seesaw

We consider first the type-I [12] seesaw mechanism with $n_R = 3$ right-handed neutrino fields. The relevant Lagrangian is

$$-\mathcal{L}_{\mathrm{I}} = \bar L_L \sum_{k=1}^{3} Y_{\ell,k}\Phi_k \ell_R + \bar L_L \sum_{k=1}^{3} Y_{\nu,k}^* \tilde\Phi_k \nu_R + \frac{1}{2}\bar\nu_R M_R \nu_R^c + \mathrm{H.c.}, \tag{15}$$

where $L_L = (\ell_L, \nu_L)^T$, ℓ_R, and ν_R are vectors in the three-dimensional generation spaces of left-handed doublets, right-handed charged lepton singlets, and right-handed neutrino singlets, respectively. For each scalar doublet Φ_k there is a charged lepton Yukawa matrix $Y_{\ell,k}$, and a neutrino Yukawa matrix $Y_{\nu,k}$. After the spontaneous symmetry breaking we obtain the mass terms

$$-\mathcal{L}_{\mathrm{I}} = \bar\ell_L m_\ell \ell_R + \bar\nu_L m_D \nu_R + \frac{1}{2}\bar\nu_R M_R \nu_R^c + \mathrm{H.c.}, \tag{16}$$

where

$$m_\ell = \sum_{k=1}^{3} Y_{\ell,k} v_k, \quad m_D = \sum_{k=1}^{3} Y_{\nu,k}^* v_k^*, \tag{17}$$

and M_R is a symmetric matrix. To correctly reproduce the light neutrino masses, the eigenvalues of M_R should be much larger than $(|v_1|^2 + |v_2|^2 + |v_3|^2)^{1/2}$. Integrating out the heavy right-handed Majorana fields, the low-energy effective Lagrangian becomes

$$-\mathcal{L}_{\mathrm{eff}} = \bar\ell_L m_\ell \ell_R + \frac{1}{2}\nu_L^T C m_\nu \nu_L + \mathrm{H.c.}, \tag{18}$$

and the light neutrinos acquire an effective mass

$$m_\nu = -m_D M_R^{-1} m_D^T. \tag{19}$$

In the basis where the charged lepton mass matrix is diagonal,

$$m_\ell = \mathrm{diag}\,(m_e, m_\mu, m_\tau), \tag{20}$$

the neutrino mass matrix m_ν is diagonalized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13] leptonic mixing matrix U as

$$U^T m_\nu U = \mathrm{diag}\,(m_1, m_2, m_3), \tag{21}$$

where m_i are the light neutrino masses.

We will try to assign the lepton fields to A_4 representations, subject to the following constraints:

1. The matrix M_R corresponding to the heavy Majorana fields cannot have a zero eigenvalue;

2. The charged lepton masses cannot vanish or be degenerate.

The first condition forces the right-handed neutrino fields to be in one of the following three representations. One can have $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$ and

$$M_R = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \tag{22}$$

where $\times$ represents an independent complex entry. Alternatively, one can have $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ and

$$M_R = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}. \tag{23}$$

Finally, if $\nu_R \sim \mathbf{3}$, then

$$M_R = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (24)$$

where M is an arbitrary complex number. This corresponds to degenerate heavy neutrinos. Other combinations are ruled out by our first requirement.

Before proceeding, let us look back at Eq. (19). Because $\det M_R \neq 0$, the existence (or absence) of massless light neutrinos depends on the nature of m_D. If $\det m_D = 0$ ($\det m_D \neq 0$), then $\det m_\nu = 0$ ($\det m_\nu \neq 0$). As a result, we will consider the constraints coming from charged leptons in each of these two cases, separately. Notice that, regardless of the ν_R representation, no case with both L_L and ℓ_R in singlet representations is possible because $\Phi \sim \mathbf{3}$, leading to $m_\ell = 0$. Similarly, the cases where both L_L and ν_R are in singlet representations are excluded because they lead to $m_D = 0$ and, through Eq. (19), to $m_\nu = 0$. Finally, as shown in Sec. II, cases when both L_L and ℓ_R are in the triplet representation of A_4 are not possible. The remaining cases are listed in Table II.

Case	L_L	ν_R	ℓ_R	Neutrino masses
i)	$\mathbf{3}$	$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	2 degenerate
ii)	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{3}$	2 degenerate
iii)	$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	2 degenerate
iv)	$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	2 massless

TABLE II. Possible representations of the left-handed lepton doublet (L_L), the right-handed neutrino singlets (ν_R), and right-handed charged lepton singlets (ℓ_R), when the three Higgs doublets are in a triplet representation $\mathbf{3}$.

1. Nonvanishing neutrino masses

We start by looking at the cases in which $\nu_R \sim \mathbf{3}$. From Table II one concludes that there are two possibilities which may lead to nonvanishing neutrino and charged lepton masses: i) $L_L \sim \mathbf{3}$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ and ii) $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$.

Let us consider in detail case i). Then, Eq. (24) holds and we may parametrize

$$m_\ell = \begin{pmatrix} a\, e^{i\alpha}\, v_1 & b\, e^{i\beta}\, v_1 & c\, e^{i\gamma}\, v_1 \\ a\, e^{i\alpha}\, v_2 & \omega\, b\, e^{i\beta}\, v_2 & \omega^2\, c\, e^{i\gamma}\, v_2 \\ a\, e^{i\alpha}\, v_3 & \omega^2\, b\, e^{i\beta}\, v_3 & \omega\, c\, e^{i\gamma}\, v_3 \end{pmatrix}, \qquad (25)$$

and

$$m_D = \begin{pmatrix} 0 & f\, e^{i\epsilon}\, v_3^* & d\, e^{i\delta}\, v_2^* \\ d\, e^{i\delta}\, v_3^* & 0 & f\, e^{i\epsilon}\, v_1^* \\ f\, e^{i\epsilon}\, v_2^* & d\, e^{i\delta}\, v_1^* & 0 \end{pmatrix}. \qquad (26)$$

Taking the VEV alignment $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$, the eigenvalues of $m_\ell m_\ell^\dagger$ are $3a^2$, $3b^2$, and $3c^2$, which can be properly chosen to fit the experimental values of the charged lepton masses.

As for the light neutrino mass matrix, we obtain for the VEV alignment $(1,1,1)$

$$m_\nu = -M^{-1} m_D m_D^T = -M^{-1} \begin{pmatrix} x & y & y \\ y & x & y \\ y & y & x \end{pmatrix}, \qquad (27)$$

where $x = d^2\, e^{2i\delta} + f^2\, e^{2i\epsilon}$ and $y = d\, f\, e^{i(\delta+\epsilon)}$. The eigenvalues of $m_\nu m_\nu^\dagger$ are

$$M^{-2} \left[d^2 + f^2 + 2df \cos(\delta - \epsilon)\right]^2, \qquad (28)$$
$$M^{-2} \left\{\left[d^2 + f^2 - df \cos(\delta - \epsilon)\right]^2 - 3d^2 f^2 \sin^2(\delta - \epsilon)\right\},$$

with the latter twice degenerate. This in turn implies that two light neutrinos are degenerate in mass, in contradiction with experiment. This feature remains for the VEV alignment $(\pm 1, \eta, \eta^*)$, although the expressions for the eigenvalues become more involved in that case.

Let us now analyze case ii). In this case, the mass matrices become

$$m_\ell = \begin{pmatrix} a\, e^{i\alpha}\, v_1 & a\, e^{i\alpha}\, v_2 & a\, e^{i\alpha}\, v_3 \\ b\, e^{i\beta}\, v_1 & \omega\, b\, e^{i\beta}\, v_2 & \omega^2\, b\, e^{i\beta}\, v_3 \\ c\, e^{i\gamma}\, v_1 & \omega^2\, c\, e^{i\gamma}\, v_2 & \omega\, c\, e^{i\gamma}\, v_3 \end{pmatrix}, \qquad (29)$$

$$m_D = \begin{pmatrix} d\, e^{i\delta}\, v_1^* & d\, e^{i\delta}\, v_2^* & d\, e^{i\delta}\, v_3^* \\ f\, e^{i\epsilon}\, v_1^* & \omega\, f\, e^{i\epsilon}\, v_2^* & \omega^2\, f\, e^{i\epsilon}\, v_3^* \\ g\, e^{i\xi}\, v_1^* & \omega^2\, g\, e^{i\xi}\, v_2^* & \omega\, g\, e^{i\xi}\, v_3^* \end{pmatrix}. \qquad (30)$$

For both VEVs, $(1,1,1)$ and $(\pm 1, \eta, \eta^*)$, $m_\ell m_\ell^\dagger = 3\,\mathrm{diag}\,(a^2, b^2, c^2)$, so we can easily accommodate the charged lepton masses. Furthermore, for the VEV $(1,1,1)$, we obtain

$$m_\nu = -M^{-1} m_D m_D^T = -M^{-1} \begin{pmatrix} z & 0 & 0 \\ 0 & 0 & t \\ 0 & t & 0 \end{pmatrix}, \qquad (31)$$

where $z = 3d^2\, e^{2i\delta}$ and $t = 3fg\, e^{i(\epsilon+\xi)}$. Thus, $m_\nu m_\nu^\dagger = 9\,\mathrm{diag}\,(d^4, f^2 g^2, f^2 g^2)$, and we get two degenerate light neutrinos. The matrices for the VEV $(\pm 1, \eta, \eta^*)$ are slightly different, but the conclusions are the same. As a result, cases i) and ii) are ruled out by experiment.

The analysis of the remaining case iii), for which $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $L_L \sim \mathbf{3}$ and $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, is easily carried out. Indeed, the charged lepton sector coincides with that of case i). Furthermore, from Eq. (25), the matrix m_D can be inferred:

$$m_D = \begin{pmatrix} d\, e^{i\delta}\, v_1^* & f\, e^{i\epsilon}\, v_1^* & g\, e^{i\xi}\, v_1^* \\ d\, e^{i\delta}\, v_2^* & \omega\, f\, e^{i\epsilon}\, v_2^* & \omega^2\, g\, e^{i\xi}\, v_2^* \\ d\, e^{i\delta}\, v_3^* & \omega^2\, f\, e^{i\epsilon}\, v_3^* & \omega\, g\, e^{i\xi}\, v_3^* \end{pmatrix}. \qquad (32)$$

Writing

$$M_R = \begin{pmatrix} r_1\, e^{i\sigma_1} & 0 & 0 \\ 0 & 0 & r_2\, e^{i\sigma_2} \\ 0 & r_2\, e^{i\sigma_2} & 0 \end{pmatrix}, \qquad (33)$$

we can use Eq. (19) to determine m_ν. The expression is long, but the eigenvalues of $m_\nu\, m_\nu^\dagger$ are simply given by $(3d^2/r_1)^2$ and $(3fg/r_2)^2$, with the latter twice degenerate. Thus, we also get two degenerate light neutrinos, so this case is also excluded.

2. Vanishing neutrino masses

We now turn to the possibility that m_D, and thus m_ν, have determinants equal to zero, with at most one massless light neutrino. The only case consistent with realistic charged lepton masses is iv) $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$, $\ell_R \sim (\mathbf{1}, \mathbf{1'}, \mathbf{1''})$, provided that the VEV alignment is $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$.

Let us show that this possibility is also inconsistent with experiment. In this case, m_ℓ has the same matrix structure as case i), while the Dirac neutrino mass matrix has the form

$$m_D = \begin{pmatrix} d\, e^{i\delta}\, v_1^* & f\, e^{i\epsilon}\, v_1^* & g\, e^{i\xi}\, v_1^* \\ d\, e^{i\delta}\, v_2^* & f\, e^{i\epsilon}\, v_2^* & g\, e^{i\xi}\, v_2^* \\ d\, e^{i\delta}\, v_3^* & f\, e^{i\epsilon}\, v_3^* & g\, e^{i\xi}\, v_3^* \end{pmatrix}. \qquad (34)$$

Taking the VEV $(1,1,1)$, we find that

$$m_D = V_L\ D_D\ V_R^\dagger\,, \qquad (35)$$

where $D_D = \sqrt{3}\, e^{i\xi} R_3\, \mathrm{diag}(0,0,1)$,

$$V_L = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \qquad (36)$$

$$V_R = \begin{pmatrix} -\frac{g}{R_2}\, e^{-i(\delta-\xi)} & \frac{df}{R_2 R_3}\, e^{i(\epsilon-\xi)} & \frac{d}{R_3}\, e^{-i(\delta-\xi)} \\ 0 & -\frac{R_2}{R_3}\, e^{i(\delta-\xi)} & \frac{f}{R_3}\, e^{-i(\epsilon-\xi)} \\ \frac{d}{R_2} & \frac{fg}{R_2 R_3}\, e^{i(\delta+\epsilon-2\xi)} & \frac{g}{R_3} \end{pmatrix}, \qquad (37)$$

$R_2 = (d^2+g^2)^{1/2}$ and $R_3 = (d^2+f^2+g^2)^{1/2}$. Here, V_L and V_R are the unitary matrices that diagonalize the Hermitian matrices $m_D m_D^\dagger$ and $m_D^\dagger m_D$, respectively.

Using Eq. (19), we get

$$m_\nu = -V_L\, D_D\, X\, D_D\, V_L^T = -e^{2i\xi} R_3^2 X_{33} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad (38)$$

where $X = V_R^\dagger\, M_R^{-1}\, V_R^*$.

From Eq. (38), we find the eigenvalues of $m_\nu m_\nu^\dagger$ to be $m_1^2 = m_2^2 = 0$ and $m_3^2 = 9R_3^4\, |X_{33}|^2$. Since there are two massless neutrinos, this case is ruled out. For the VEV alignment $(1, \eta, \eta^*)$, the intermediate steps get more involved, but the eigenvalues of $m_\nu m_\nu^\dagger$ have the same expressions, and therefore this possibility is also ruled out.

Before concluding this section, let us comment on the type-III seesaw mechanism [14]. In the type-III seesaw framework, instead of three right-handed singlet neutrino fields, one adds three Majorana neutrinos, Σ_R, in the triplet representation of the gauge group $SU(2)_L$,

$$\Sigma_{iR} = \begin{pmatrix} \Sigma_i^0/\sqrt{2} & \Sigma_i^+ \\ \Sigma_i^- & -\Sigma_i^0/\sqrt{2} \end{pmatrix}, \quad i = 1,2,3. \qquad (39)$$

The relevant Lagrangian is very similar to Eq. (15) for a type-I seesaw:

$$-\mathcal{L}_{\rm II} = \bar{L}_L \sum_{k=1}^{3} Y_{\ell,k} \Phi_k \ell_R + \bar{L}_L \sum_{k=1}^{3} Y_{\Sigma,k}^* \tilde{\Phi}_k \Sigma_R + \frac{1}{2} \sum_{i,j=1}^{3} (M_\Sigma)_{ij} \mathrm{Tr}\left(\bar{\Sigma}_{iR} \Sigma_{jR}^C\right) + \mathrm{H.c.} \qquad (40)$$

The effective light neutrino mass matrix acquires the same seesaw structure as Eq. (19), with M_R replaced by M_Σ. As a result, the analysis of flavor structures under the A_4 symmetry is the same as before, and all the conclusions hold. In particular, Table II applies, with the obvious replacement $\nu_R \to \Sigma_R$.

IV. SOFTLY BROKEN A_4 SYMMETRY

We now consider the possibility that the effective operator is not invariant under A_4. This situation is well behaved as long as we guarantee that the non-invariance comes, at the UV level, from terms that do not spoil renormalizability. We therefore assume that A_4 is broken softly by dimension-3 terms contributing to the right-handed neutrino mass matrix M_R [15].

We start again from the cases listed in Table II and analyzed in Sec. III B. Let us consider first case iv) of Table II, where $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$, and M_R is, according to Eq. (22), the most general 3×3 symmetric complex matrix. Including soft-breaking terms does not alter this feature. Since Eq. (38) leads to two massless eigenvalues in $m_\nu m_\nu^\dagger$, we conclude that case iv) is not viable, even after the inclusion of soft-breaking terms.

Next, we show that the remaining cases listed in Table II lead to viable fits of the current experimental neutrino data after the inclusion of soft-breaking terms in M_R.

We start with case i). The charged lepton and Dirac neutrino mass matrices are given by Eqs. (25) and (26), respectively. For simplicity, we study the VEV alignment $(1,1,1)$—the results for $(\pm 1, \eta, \eta^*)$ will be equivalent. We can always change the basis of L_L, corresponding to multiplying the matrices m_ℓ and m_D on the left by the same unitary matrix. In this case, the matrices $m_\ell m_\ell^\dagger$ and $m_D m_D^\dagger$ are diagonalized by the same matrix,

$$V_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{41}$$

Indeed, multiplying the mass matrices on the left by $V_\omega^\dagger$, we obtain in the new basis

$$\begin{aligned} m_\ell &= \sqrt{3}\ \mathrm{diag}\left(a\, e^{i\,\alpha}, b\, e^{i\,\beta}, c\, e^{i\,\gamma}\right), \\ m_D &= D_D\, V_R^\dagger, \end{aligned} \tag{42}$$

where

$$V_R^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \end{pmatrix}, \tag{43}$$

$$D_D = \mathrm{diag}\,(de^{i\delta} + fe^{i\epsilon}, de^{i\delta} + \omega^2 fe^{i\epsilon}, de^{i\delta} + \omega fe^{i\epsilon}).$$

The PMNS mixing matrix and the light neutrino masses are then obtained from the diagonalization of m_ν [cf. Eqs. (19) and (21)].

We have randomly generated matrices m_D and M_R satisfying the current experimental data, via a procedure described in detail in Appendix B. In particular, we define in Eq. (B7) a figure of merit σ probing how much the matrix M_R differs from its form in the exact A_4 limit, given in the present case by Eq. (24). Values of $\sigma \sim 1$ correspond to soft-breaking terms of the order of the A_4-symmetric terms. Smaller values of σ correspond to cases where the terms that break the symmetry are perturbative; that is, they are about 1 order of magnitude smaller than the terms which preserve A_4.

Our aim is to show that there exist viable solutions where the deviations from Eq. (24) are perturbative. This becomes clear from Fig. 1, which shows σ as a function of m_1 (in eV). We notice several features: a) one can produce fits with values of m_1 in almost the whole range attempted (from 0 to 0.2 eV), except for very small values of m_1; b) solutions with $\sigma < 0.2$ can be found, meaning that the soft-breaking terms are perturbative; c) smaller values of σ tend to prefer values for m_1 around 0.05 eV.

In our simulation, smaller values of σ have no correlation with the measured quantities—namely, the neutrino mixing angles θ_{12}, θ_{13}, θ_{23}, and the mass-squared differences $\Delta m^2_{21} = m^2_2 - m^2_1$, or $\Delta m^2_{31} = m^2_3 - m^2_1$. Although $\sigma < 0.2$ implies values of the Dirac phase δ_D close to $\pm\pi/2$, all values of δ_D are possible if one allows $\sigma < 1$. In contrast, even a loose cut of $\sigma < 1$, forces the Majorana phases $\alpha_M \sim 0$ and $\beta_M \sim \pi$ (recall that the phases are defined mod 2π). This is illustrated in Fig. 2.

Next, we consider case ii), where $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$, so that Eqs. (29) and (30) hold for the charged lepton mass matrix m_ℓ and the Dirac neutrino mass matrix m_D, respectively. We choose the VEV alignment $(1,1,1)$. This case is interesting because m_ℓ is diagonalized exclusively through a unitary transformation on the right-handed fields ℓ_R, implying that the PMNS matrix arises exclusively from the diagonalization of m_ν. The eigenvalues of $m_\ell\, m_\ell^\dagger$ from Eq. (29) are $3a^2$, $3b^2$, and $3c^2$. Thus, we take $a = m_e/\sqrt{3}$, $b = m_\mu/\sqrt{3}$, and $c = m_\tau/\sqrt{3}$, and consider $\alpha = \beta = \gamma = 0$.

We will now assume that A_4 is softly broken in the right-handed neutrino sector, such that the form of M_R in Eq. (24) is altered. As before, we follow the fit procedure described in Appendix B. Figure 3 shows σ as a function of $s^2_{23} = \sin^2\theta_{23}$. We notice that all values of s^2_{23} are possible, but that smaller values of σ show some preference for smaller values of s^2_{23}. There is no such correlation with θ_{12}, θ_{13}, m_1, Δm^2_{21}, or Δm^2_{31}. The

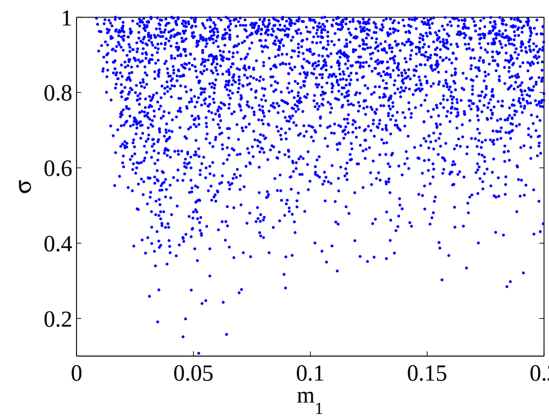

FIG. 1. Simulation points in case i), showing how the figure of merit σ varies with m_1 (in eV). The quantity σ measures the deviation of M_R, in the softly broken case, from its form in the A_4-symmetric case.

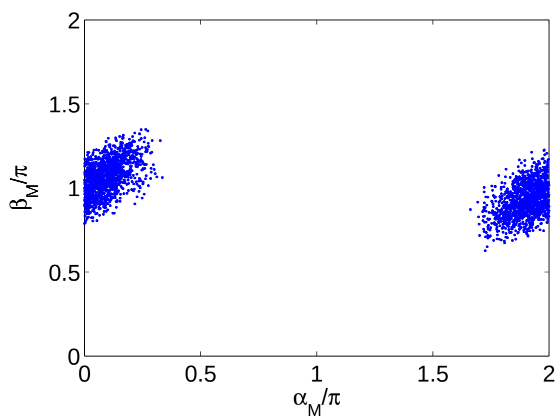

FIG. 2. Simulation in the (α_M, β_M) plane for case i). Points verifying the condition $\sigma < 1$ are represented.

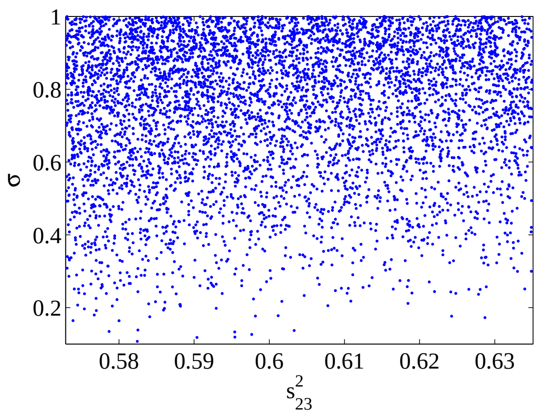

FIG. 3. Simulation points for case ii), showing how σ varies with s_{23}^2.

CP-violating phases δ_D, α_M, and β_M exhibit the same behavior as in case i).

It is interesting to look at a numerical example in detail. We consider the symmetric matrix $M_R = MY_R$, with $M = 10^{14}$ GeV and

$$\begin{aligned}(Y_R)_{11} &= 0.6622 + 7.7769\,i,\\ (Y_R)_{12} &= -0.4304 + 0.5404\,i,\\ (Y_R)_{13} &= -0.0490 - 0.4532\,i,\\ (Y_R)_{22} &= 5.0726 + 5.9385\,i,\\ (Y_R)_{23} &= 0.1819 + 0.1479\,i,\\ (Y_R)_{33} &= 6.5927 + 4.2281\,i.\end{aligned} \tag{44}$$

For use in the matrix m_D of Eq. (30), we take

$$\begin{aligned} d\,e^{i\delta} &= 0.7663 + 0.2958\,i,\\ f\,e^{i\epsilon} &= 0.7516 - 0.0537\,i,\\ g\,e^{i\xi} &= -0.7477 - 0.5013\,i,\end{aligned} \tag{45}$$

and $v_1 = v_2 = v_3 = 246$ GeV. Upon diagonalization of the neutrino mass matrix m_ν given by Eq. (19), we find a nearly degenerate normal neutrino mass spectrum with $m_1 \simeq 0.1545$ eV, $\Delta m^2_{21} = 7.46 \times 10^{-5}\,\text{eV}^2$ and $\Delta m^2_{31} = 2.55 \times 10^{-3}\,\text{eV}^2$, which are well within the 1σ ranges given in Ref. [16]. Next, we equate the diagonalizing matrix U [see Eq. (21)] with the PMNS neutrino mixing matrix written in the standard PDG form [9]. This leads to $\sin^2\theta_{12} = 0.336$, $\sin^2\theta_{23} = 0.613$, and $\sin^2\theta_{13} = 0.0247$. According to Ref. [16], the first value is close to its 1σ upper bound, while the other two coincide with the best-fit values. Finally, the as-yet-unmeasured CP-violating phases turn out to be $\delta = 0.53\pi$, $\alpha_M = 0.28\pi$, and $\beta_M = 1.08\pi$.

In the above numerical example, the magnitudes of the entries in M_R are

$$|M_R| = 7.83 \times 10^{14}\,\text{GeV} \times \begin{pmatrix} 0.997 & 0.088 & 0.058 \\ 0.088 & 0.997 & 0.030 \\ 0.058 & 0.030 & 1.000 \end{pmatrix}. \tag{46}$$

This shows that the current neutrino oscillation data can be easily fitted in flavor models of the type discussed in this article, if the A_4 symmetry is softly broken in the heavy Majorana neutrino mass matrix M_R by coefficients that deviate perturbatively (of the order of 10% or less) from the leading A_4-symmetric terms. Notice also that in the above example a normal (quasidegenerate) neutrino mass spectrum is obtained. Numerical examples that lead to an inverted neutrino mass hierarchy, and simultaneously to low-energy neutrino parameters in agreement with the present 1σ ranges, can be equally constructed.

Finally, we turn to case iii), where $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. The charged lepton mass matrix is given in Eq. (25), while the Dirac neutrino mass matrix m_D is given by Eq. (32). As before, we concentrate on the VEV $(1, 1, 1)$. Using Eq. (41), we multiply the matrices m_ℓ and m_D by $V_\omega^\dagger$, obtaining, in the new basis,

$$\begin{aligned} m_\ell &= \sqrt{3}\ \text{diag}\left(a\,e^{i\,\alpha}, b\,e^{i\,\beta}, c\,e^{i\,\gamma}\right),\\ m_D &= \sqrt{3}\ \text{diag}\left(d\,e^{i\,\delta}, f\,e^{i\,\epsilon}, g\,e^{i\,\xi}\right).\end{aligned} \tag{47}$$

The numerical results for this case exhibit many similarities with case ii). In particular, there are no fundamental constraints on the observables, except for $\delta_D \sim \pm\pi/2$, $\alpha_M \sim 0$, and $\beta_M \sim \pi$.

V. CONCLUSIONS

In this paper we have studied the possibility of generating the neutrino masses and mixing in the context of models with three scalar doublets in the triplet representation of the A_4 group and three lepton families. We have shown that none of the possible VEV alignments that correspond to a global minimum of the scalar potential yields phenomenologically viable charged lepton and neutrino mass matrices. In particular, there is no representation assignment that leads to a dimension-5 neutrino mass operator consistent with the present oscillation data. This in turn implies that, in this minimal A_4 construction, the canonical type-I (type-III) seesaw mechanism is not consistent with experiment. Notice that, from the point of view of the low-energy effective operator, this conclusion holds for any number of right-handed singlet (triplet) neutrinos, since the dimension-5 operator is the same regardless of the number of heavy fields. Furthermore, since A_4 is a subgroup of S_4, our conclusions also remain valid in flavor models based on the latter group.

In the context of a type-I seesaw mechanism with three heavy right-handed neutrinos, we have analyzed in detail what happens when the A_4-symmetric Lagrangian is enlarged by adding soft breaking, through dimension-3 right-handed neutrino mass terms. We find three cases where this framework can be implemented in good agreement with neutrino oscillation data. We have also pointed out that this is possible perturbatively, i.e., by keeping the soft-breaking terms much smaller than the A_4-symmetric terms of the right-handed neutrino mass matrix.

At this point, it is worthwhile to comment on other possibilities considered in the literature. Several studies propose various renormalizable extensions of the A_4 three-Higgs-doublet model, including always additional fields as well as new symmetries. For example, Ma and Rajasekaran [17] discuss a very simple extension, where an additional scalar doublet η is added to case i). This scalar will be the one responsible for the Dirac neutrino mass term. Therefore, an additional Z_2 is added in order to forbid Φ and η to interchange sectors. This model still needs some soft-breaking terms to split the degeneracy in the light neutrino spectrum. There are also models using two scalar A_4 triplets, one for each sector. In most models these are flavon fields, which lead to a non-renormalizable theory [18, 19]. Renormalizable models with two scalar triplets of A_4 will lead to a six-Higgs-doublet model, with a large increase in the number of parameters. In Ref. [20] a case iii) is studied, but with a Higgs triplet for each sector, and with an additional Z_2 symmetry. However, the vacuum alignments utilized — (v_1, v_2, v_3) and $v(1, 1, -2)$ — are not shown to be the absolute minimum. To our knowledge, our work is the first to fully study all possibilities consistent with one Higgs A_4 triplet, for which the global minima of the scalar potential have been recently identified.

The analysis performed in this paper dealt with the most general case of three-Higgs-doublet models with the scalars in a triplet of A_4. This study can serve as a starting point for more elaborate models. Nevertheless, we emphasize that the scalar potential in such new extensions has to be fully analyzed, in order to ensure that the vacua utilized can indeed be global minima. In general, this is not a trivial task.

ACKNOWLEDGMENTS

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Appendix A: Couplings of $L_L L_L$ when L_L is in a singlet representation of A_4

Let us consider the group structure of the combination $L_L L_L$, when L_L is an A_4 singlet. We concentrate on the group constraints, ignoring any spinor or $SU(2)_L$ characteristics. We also note that, in all cases of interest to us, the $L_L L_L$ coupling matrix *must be symmetric*. Disregarding irrelevant permutations, we must consider the cases

$$\begin{aligned} &(\mathbf{1},\mathbf{1},\mathbf{1}), \quad (\mathbf{1},\mathbf{1},\mathbf{1}'),\\ &(\mathbf{1},\mathbf{1}',\mathbf{1}'), \quad (\mathbf{1},\mathbf{1},\mathbf{1}''),\\ &(\mathbf{1},\mathbf{1}'',\mathbf{1}''), \quad (\mathbf{1},\mathbf{1}',\mathbf{1}''),\\ &(\mathbf{1}',\mathbf{1}',\mathbf{1}'), \quad (\mathbf{1}',\mathbf{1}',\mathbf{1}''),\\ &(\mathbf{1}',\mathbf{1}'',\mathbf{1}''), \quad (\mathbf{1}'',\mathbf{1}'',\mathbf{1}''). \end{aligned} \tag{A1}$$

We can combine the couplings of $L_L L_L$ with some other group structure in a 3×3 matrix. To explain our notation, we will use the example of $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1})$. We construct the matrix of all field products

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}, \tag{A2}$$

where a matrix element $\mathbf{1}$ means that there is at that matrix position an arbitrary complex entry, if we are coupling $L_L L_L$ to some other group structure transforming like $\mathbf{1}$ of A_4, and zero otherwise. For example, the scalar combination $(\Phi\Phi) \sim \mathbf{1}$ has couplings to all bilinears of $L_L L_L$ when $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1})$, while a $(\Phi\Phi) \sim \mathbf{1}'$ will couple to none.

As a further example, consider $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1}')$. The corresponding matrix is

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}'' \end{pmatrix}. \tag{A3}$$

This means that a $(\Phi\Phi) \sim \mathbf{1}$ will only introduce couplings in the upper-left 2×2 submatrix, and a $(\Phi\Phi) \sim \mathbf{1}''$ will only have couplings to $(L_i L_3 + L_3 L_i)$ with $i = 1, 2$, while $(\Phi\Phi) \sim \mathbf{1}'$ would only couple to $L_3 L_3$. For simplicity, we write $L_L = (L_1, L_2, L_3)$. We also recall that $\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$.

The remaining possibilities in Eq. (A1) lead to

$$\begin{pmatrix} \mathbf{1} & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1}'' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1}'' \\ \mathbf{1} & \mathbf{1} & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}' \end{pmatrix}, \tag{A4}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}'' & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1} & \mathbf{1}' & \mathbf{1}'' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1}'' & \mathbf{1} & \mathbf{1}' \end{pmatrix}, \tag{A5}$$

$$\begin{pmatrix} \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1}'' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1}' \end{pmatrix}, \tag{A6}$$

$$\begin{pmatrix} \mathbf{1}'' & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1} & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \tag{A7}$$

respectively.

Appendix B: The fit procedure

In this appendix we present the fit procedure adopted in Sec. IV for the cases in which A_4 is softly broken. We define the Hermitian matrices

$$H_\ell = m_\ell m_\ell^\dagger = V_{\ell L}\,\mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)\,V_{\ell L}^\dagger,$$
$$H_\nu = m_\nu m_\nu^\dagger = U_\nu^*\,\mathrm{diag}(m_1^2, m_2^2, m_3^2)\,U_\nu^T. \quad \text{(B1)}$$

With this notation, the PMNS matrix U entering the charged current interactions as

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\bar{\ell}_L^{\mathrm{mass}}\,U\,\gamma^\mu \nu_L^{\mathrm{mass}}\,W_\mu^- + \mathrm{H.c.}, \quad \text{(B2)}$$

is given by $U = V_{\ell L}^\dagger\,U_\nu$.

With a suitable phase choice, we write

$$U = V\,K, \quad \text{(B3)}$$

where the parametrization

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_D} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} \quad \text{(B4)}$$

follows the Particle Data Group notation [9], and

$$K = \mathrm{diag}(1, e^{i\alpha_M/2}, e^{i\beta_M/2}) \quad \text{(B5)}$$

contains the Majorana phases. Recent constraints on the mixing angles and phases of the PMNS matrix can be found in Refs. [16, 21]. The Majorana phases α_M and β_M are unconstrained, while interpretations differ about constraints on the Dirac phase δ_D. We leave δ_D free, and follow Ref. [16] for the ranges of the other parameters.

In the cases of interest to us, it is easy to diagonalize m_ℓ. In the basis where m_ℓ is diagonal, Eqs. (20) and (21) hold. Using the type-I seesaw relation (19) and Eq. (21), we find

$$M_R = -m_D^T\,U\,\mathrm{diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})\,U^T\,m_D. \quad \text{(B6)}$$

We perform our fits in the following fashion:

1. We use random values for s_{12}, s_{13}, s_{23}, Δm_{21}^2, and Δm_{31}^2, within the 1σ intervals found in Ref. [16], corresponding to the normal hierarchy.
2. We generate random values for δ_D, α_M, β_M, and for m_1, keeping the latter between 0 and 0.2 eV.
3. We generate random values for the theoretical parameters in the matrix m_D, written in the basis where m_ℓ is diagonal.
4. We obtain M_R from Eq. (B6)—by construction, m_D and M_R are consistent with the experimental observations.
5. We define a figure of merit σ, which measures the difference between the form of M_R obtained from Eq. (B6) and that predicted in Eqs. (23) or (24) when the symmetry is exact.
6. To keep the soft breaking perturbative, only cases where σ is smaller than some reference value are maintained.

The only cases where soft breaking of A_4 allows the type-I seesaw mechanism to fit the experimental data are cases i), ii), and iii) in Table II. In the first two cases, $\nu_R \sim \mathbf{3}$, and M_R has, in the exact A_4 limit, the form in Eq. (24). We wish to keep deviations from this form somewhat small. To be precise, we define

$$\sigma = \sum_{i,j=1}^{3} |\Sigma_{ij}|^2\,, \quad \text{(B7)}$$

with

$$\Sigma = \frac{M_R}{|(M_R)_{11}|} - \mathrm{diag}\,(1,1,1), \quad \text{(B8)}$$

where M_R comes from Eq. (B6) in the fit explained above. We only keep points where $\sigma < 1$.

In case iii), $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, and M_R has the form in Eq. (23) in the exact A_4 limit. For this case we define

$$\sigma = \frac{|(M_R)_{12}|^2 + |(M_R)_{13}|^2 + |(M_R)_{22}|^2 + |(M_R)_{33}|^2}{\min\left(|(M_R)_{11}|^2, |(M_R)_{23}|^2\right)} \quad \text{(B9)}$$

and only keep points where $\sigma < 1$.

[1] G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010).

[2] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu,

H. Okada and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010).
[3] W. Grimus and P. O. Ludl, J. Phys. A **45**, 233001 (2012).
[4] S. F. King and C. Luhn, Rept. Prog. Phys. **76**, 056201 (2013).
[5] See, for example, S.-F. Ge, D. A. Dicus and W. W. Repko, Phys. Rev. Lett. **108**, 041801 (2012); D. Hernandez and A. Y. .Smirnov, Phys. Rev. D **87**, 053005 (2013); B. Hu, Phys. Rev. D. **87**, 033002 (2013); and references therein.
[6] A. Degee, I. P. Ivanov and V. Keus, J. High Energy Phys. **02**, 125 (2013).
[7] Implications of minima (including local) in A_4 models have been studied in R. de Adelhart Toorop, F. Bazzocchi, L. Merlo and A. Paris, J. High Energy Phys. **03**, 035 (2011); **01**, 098(E) (2013); J. High Energy Phys. **03**, 040 (2011).
[8] R. González Felipe, H. Serôdio and J. P. Silva, Phys. Rev. D **87**, 055010 (2013).
[9] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[10] For a recent review, see G. C. Branco, R. González Felipe and F. R. Joaquim, Rev. Mod. Phys. **84**, 515 (2012).
[11] S. Weinberg, Phys. Rev. D **22**, 1694 (1980).
[12] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories* (Tsukuba, Japan, 1979); S.L. Glashow, in *Cargèse Lectures*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 687; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[13] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968).
[14] R. Foot, H. Lew, X.G. He, and G.C. Joshi, Z. Phys. C **44**, 441 (1989).
[15] G. C. Branco, R. González Felipe, M. N. Rebelo, and H. Serôdio, Phys. Rev. D **79**, 093008 (2009).
[16] D. V. Forero, M. Tórtola, and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012).
[17] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001).
[18] G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006).
[19] M.-C. Chen and S. F. King, J. High Energy Phys. **06**, 072 (2009).
[20] P. H. Frampton and S. Matsuzaki, arXiv:0806.4592 [hep-ph].
[21] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Phys. Rev. D **86**, 013012 (2012); M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, J. High Energy Phys. **12**, 123 (2012).