Tetraquarks in a chiral constituent quark model

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Abstract

We analyze the possibility of heavy-light tetraquark bound states by means of a chiral constituent quark model. The study is done in a variational approach. Special attention is paid to the contribution given by the different terms of the interacting potential and also to the role played by the different color channels. We find a stable state for both $qq\bar{c}\bar{c}$ and $qq\bar{b}\bar{b}$ configurations. Possible decay modes of these structures are analyzed.

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I. INTRODUCTION

The potentiality of the quark model for hadron physics in the low-energy regime was manifest when it was used to classify all the known hadron states. Describing hadrons as $q\bar{q}$ or qqq configurations, their quantum numbers were correctly explained. This assignment was based on the comment by Gell-Mann [1] introducing the notion of quark: *'It is assuming that the lowest baryon configuration (qqq) gives just the representations 1, 8 and 10, that have been observed, while the lowest meson configuration ($q\bar{q}$) similarly gives just 1 and 8'*. Since then, it is assumed that these are the two configurations involved in the description of physical hadrons. However, color confinement is also compatible with other multiquark structures like the tetraquark $qq\bar{q}\bar{q}$ introduced by Jaffe [2]. During the last two decades of the past century there appear a number of experimental data that are hardly accommodated in the traditional scheme defined by Gell-Mann. Besides, new experiments are being designed looking for these unusual structures.

The possible existence of tetraquarks has been suggested in two different scenarios. The first one is a system composed of two light quarks (antiquarks) and two heavy antiquarks (quarks). Although such an object is experimentally difficult to produce and also to detect [3] it has been argued that for sufficiently large heavy quark mass the tetraquark system should be bound [4]. The second scenario corresponds to the scalar mesons, $J^{PC} = 0^{++}$, where a huge amount of experimental data is available nowadays. However, in spite of that the situation is not yet conclusive. From the theoretical point of view the scalar sector is very particular, because to obtain a positive parity state from a $q\bar{q}$ pair one needs at least one unit of angular momentum. Apparently this costs an energy around 1 GeV, since similar meson states (1^{++} and 2^{++}) lie above 1.2 GeV. However, a $q^2\bar{q}^2$ state can couple to $J^{PC} = 0^{++}$ without orbital excitation and, as a consequence, it could be a serious candidate to describe the lightest scalar mesons. In this paper we focus our attention on the first subject, the heavy-light tetraquarks, whose existence has been recently examined [5] from experimental data of the SELEX collaboration [6].

The stability of the $qq\bar{Q}\bar{Q}$ system relies on the mass of the heavy quark [4]. The heavier the quark the stronger the short-range Coulomb attraction, in such a way that it could play a decisive role to bind the tetraquark system. The binding energy of a $1/r$ potential is proportional to the mass and it must lead to a bound state in the limit of infinite quark mass. Moreover the $\bar{Q}\bar{Q}$ pair brings a small kinetic energy into the system and then contributes to stabilize it. On the other hand, heavy-light tetraquarks are ideal systems to study the interplay between the different quark interactions because chiral symmetry is spontaneously broken in the light sector but it is explicitly broken in the heavy one.

Heavy-light tetraquarks have been studied in the past in different ways. Carlson *et al.* [7] used a potential derived from the MIT bag model in the Born-Oppenheimer approximation. The calculations were done by means of the Green's function Monte Carlo Method. They found a $J^P = 1^+$ isoscalar $bb\bar{u}\bar{d}$ bound state and they also concluded that the $cc\bar{u}\bar{d}$ is not bound. A different approach was followed by Manohar and Wise [8] who studied systems of two heavy-light ($Q\bar{q}$) mesons interacting by a potential determined at long distances by a one-pion exchange computed using Chiral Perturbation Theory. They found that for $Q = b$ this long range potential may be sufficiently attractive to produce a weakly bound two-meson state, although states where $Q = c$ are not bound. In the framework of the nonrelativistic quark potential models, Silvestre-Brac and Semay [9] have studied possible tetraquark structures using different parametrizations of the Bhaduri potential [10]. They found several bound state candidates in the $bb\bar{q}\bar{q}$ and $bc\bar{q}\bar{q}$ configurations but not for the $cc\bar{q}\bar{q}$ structure. A different conclusion is obtained by Pepin *et al.* [11] using a pseudoscalar meson-exchange interaction coming from the breaking of chiral symmetry instead of the chromomagnetic potential. Their results indicate that such interaction binds the heavy tetraquark systems both for $Q = b$ and $Q = c$. Therefore, the theoretical situation seems to be uncertain depending on whether chromomagnetic or chiral interactions are used. Even in the case of the same kind of interaction (one-pion exchange) the results of Refs. [8] and [11] are different.

Such a model dependence of the possible existence of tetraquarks claims for calculations with interactions constrained in other sectors. For this purpose, the chiral constituent quark model of Ref. [12] is an ideally suited starting point. It is based on the idea that between the chiral symmetry breaking scale and the confinement scale, QCD may be formulated for the light quark sector as an effective theory of constituent quarks interacting through gluons and Goldstone modes associated to the spontaneous breaking of $SU(2)$ chiral symmetry. Its parameters have been determined in the description of non-strange two- and three-baryon systems and the hadron spectra [12–14]. For the present study one needs to include strange and heavy flavors and therefore the model has to be generalized, fixing the new parameters in the meson spectra. This interaction will be used to solve the Schrödinger equation for the tetraquark system using a variational method. For the spatial wave function we assume a linear combination of gaussians and we will consider the two possible color configurations: $\{\bar{3}3\}$ and $\{6\bar{6}\}$.

The paper is organized as follows. In Sec. II we describe the chiral constituent quark model. The method and details of the calculation are shown in Sec. III. Section IV is devoted to the analysis and discussion of the results. Finally, a summary is presented in Sec. V.

II. THE CHIRAL CONSTITUENT QUARK MODEL

Since the origin of the quark model hadrons have been considered to be built by constituent (massive) quarks. Nowadays it is widely recognized that the constituent quark mass, very different from the current quark mass of the QCD lagrangian, appears because of the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. The picture of the QCD vacuum as a dilute medium of instantons [15] explains nicely such a symmetry breaking, which is the most important nonperturbative phenomenon for hadron structure at low energies. Quarks interact with fermionic zero modes of the individual instantons in the medium and therefore the propagator of a light quark gets modified and quarks acquire a momentum dependent mass which drops to zero for momenta higher than the inverse of the average instanton size $\overline{\rho}$. The momentum dependent quark mass acts as a natural cutoff of the theory. In the domain of momenta $k < 1/\overline{\rho}$, a simple lagrangian invariant under the chiral transformation can be derived as [15]

$$L = \overline{\psi}(i\gamma^\mu \partial_\mu - MU^{\gamma_5})\psi \tag{1}$$

where $U^{\gamma_5} = \exp(i\pi^a\lambda^a\gamma_5/f_\pi)$. π^a denotes the pseudoscalar fields $(\vec{\pi}, K_i, \eta_8)$ with i=1,...,4, and M is the constituent quark mass. An expression of the constituent quark mass can be obtained from the theory, but it also can be parametrized as $M(q^2) = m_q F(q^2)$ with

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{\frac{1}{2}} \tag{2}$$

where Λ determines the scale at which chiral symmetry is broken. Even if one does not believe in instantons as the microscopic mechanism of spontaneous chiral symmetry breaking, one has to admit that once a constituent quark mass is generated by some mechanism such quarks inevitably have to interact through Goldstone modes. Whereas the lagrangian $\overline{\psi}(i\gamma^\mu\partial_\mu - M)\psi$ is not invariant under chiral rotations, the lagrangian of Eq. (1) is invariant since the rotation of the quark fields can be compensated renaming the bosons fields. U^{γ_5} can be expanded in terms of boson fields as,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma^5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + ... \tag{3}$$

The first term generates the quark constituent mass and the second one gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which will be simulated by means of the one-sigma exchange potential. Based on the non-relativistic approximation of the above lagrangian, and making use of the physical η instead the octect one (this is the reason why a mixing angle θ_p appears), one can write the following potentials between quarks,

$$V_{PS}(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_{PS}^2}{12m_im_j}\frac{\Lambda_{PS}^2}{\Lambda_{PS}^2 - m_{PS}^2}m_{PS}\left[Y(m_{PS}\,r_{ij}) - \frac{\Lambda_{PS}^3}{m_{PS}^3}Y(\Lambda_{PS}\,r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=1}^{3}(\lambda_i^a\cdot\lambda_j^a)\,, \tag{4}$$

$$V_S(\vec{r}_{ij}) = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_S^2}{\Lambda_S^2 - m_S^2}m_S\left[Y(m_S\,r_{ij}) - \frac{\Lambda_S}{m_S}Y(\Lambda_S\,r_{ij})\right], \tag{5}$$

$$V_K(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2}m_K\left[Y(m_K\,r_{ij}) - \frac{\Lambda_K^3}{m_K^3}Y(\Lambda_K\,r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=4}^{7}(\lambda_i^a\cdot\lambda_j^a)\,, \tag{6}$$

$$V_\eta(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2}m_\eta\left[Y(m_\eta\,r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3}Y(\Lambda_\eta\,r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\left[cos\theta_p(\lambda_i^8\cdot\lambda_j^8) - sin\theta_p\right] \tag{7}$$

where $g_{ch} = m_q/f_\pi$, the $\lambda's$ are the $SU(3)$ flavor Gell-Mann matrices, and $Y(x)$ is the standard Yukawa function. The chiral coupling constant g_{ch} is related to the πNN coupling constant by

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2} \tag{8}$$

Proceeding on this way one assumes that flavor $SU(3)$ is an exact symmetry, only broken by the different mass of the strange quark. m_i is the quark mass, m_{PS}, m_K and m_η are the masses of the $SU(3)$ Goldstone bosons, taken to be their experimental values, and m_S is taken from the PCAC relation $m_S^2 \sim m_{PS}^2 + 4\,m_{u,d}^2$ [16]. In the heavy quark sector, chiral symmetry is explicitly broken and therefore these interactions will not appear.

For higher momentum transfer quarks still interact through gluon exchanges. De Rújula *et al.* [17] proposed that gluon exchange between constituent quarks can be described as an effective interaction according to the lagrangian

$$L_{gqq} = i\sqrt{4\pi}\alpha_s \overline{\psi}\gamma_\mu G^\mu \lambda^c \psi \tag{9}$$

where λ^c are the $SU(3)$ color matrices and G^μ is the gluon field. Using a nonrelativistic reduction one obtains coulomb and contact potentials that can be parametrized as follows [18]:

$$V_{OGE}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s\, \vec{\lambda^c}_i \cdot \vec{\lambda^c}_j \left\{\frac{1}{r_{ij}} - \frac{2\,\pi}{3m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j \delta(\vec{r}_{ij})\right\}. \tag{10}$$

In order to obtain a unified description of light, strange and heavy mesons, a running strong coupling constant has to be used [18]. The standard expression for $\alpha_s(Q^2)$ diverges when $Q \to \Lambda_{QCD}$ and therefore the coupling constant has to be frozen at low energies [19]. We parametrize this behavior by means of an effective scale dependent strong coupling constant similar to the one used in Refs. [20,21]

$$\alpha_s(\mu) = \frac{\alpha_0}{ln\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)}, \tag{11}$$

where μ is the reduced mass of the $q\bar{q}$ system and α_0, μ_0 and Λ_0 are fitted parameters. This equation gives rise to $\alpha_s \sim 0.54$ for the light quark sector, a value consistent with the one used in the study of the nonstrange hadron phenomenology [12–14], and it also has an appropriate high Q^2 behavior, $\alpha_s \sim 0.127$ at the Z_0 mass [22]. In order to avoid an unbound spectrum from below the delta function has to be regularized. Taken into account that for a coulombic system the typical size scales with the reduced mass, we use a flavor-dependent regularization $r_0(\mu) = \hat{r}_0/\mu$,

$$\delta(\vec{r}_{ij}) \;\Rightarrow\; \frac{1}{4\pi r_0^2(\mu)} \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}}. \tag{12}$$

The other nonperturbative property of QCD, which cannot be explained by the instanton liquid model, is confinement. Up to now it still remains a problem to derive this property from QCD in an analytic manner. The only indication we have on the nature of confinement is through lattice QCD studies. These calculations show that $q\bar{q}$ systems are well reproduced at short distances by a linear potential. This potential can be physically interpreted in a picture in which the quark and the antiquark are linked with a one-dimensional color flux tube or string with a string tension $\sigma_{q\overline{q}}$ and hence the $q\overline{q}$ potential is proportional to the distance between the quark and the antiquark. However, pair creation screens the potential at large distances [23]. A screened potential simulating the results of lattice calculations is given by

$$V_{CON}(\vec{r}_{ij}) = \{-a_c\,(1-e^{-\mu_c\,r_{ij}}) + \Delta\}(\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j) \tag{13}$$

where Δ is a global constant to fit the origin of energies. At short distances this potential presents a linear behavior with an effective confinement strength $a = a_c\,\mu_c\,(\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j)$ while it becomes constant at large distances.

Let us resume the different pieces of the interacting potential as a function of the quarks involved:

$$V_{ij} = \begin{cases} (ij) = (qq) \Rightarrow V_{CON} + V_{OGE} + V_{PS} + V_S + V_\eta \\ (ij) = (qs) \Rightarrow V_{CON} + V_{OGE} + V_S + V_K + V_\eta \\ (ij) = (ss) \Rightarrow V_{CON} + V_{OGE} + V_S + V_\eta \\ (ij) = (qQ) \Rightarrow V_{CON} + V_{OGE} \\ (ij) = (QQ) \Rightarrow V_{CON} + V_{OGE} \end{cases} \tag{14}$$

The corresponding $q\bar{q}$ potential is obtained from the qq one as detailed in Ref. [24]. In the case of $V_K(\vec{r}_{ij})$, where G-parity is not well defined, the transformation is given by $\lambda_1^a \cdot \lambda_2^a \to \lambda_1^a \cdot (\lambda_2^a)^T$, which recovers the standard change of sign in the case of the pseudoscalar exchange between two nonstrange quarks. Assuming a nonrelativistic expression for the kinetic energy, the hamiltonian of the system takes the form:

$$H = \sum_i \left(m_i + \frac{\vec{p}_i^{\,2}}{2m_i} \right) + \sum_{i<j} V_{ij} \tag{15}$$

where V_{ij} is given by Eq. (14).

III. TETRAQUARK WAVE FUNCTION

For the description of the $qq\bar{Q}\bar{Q}$ system we introduce the Jacobi coordinates

$$\vec{x} = \vec{r}_1 - \vec{r}_2 \qquad \vec{y} = \vec{r}_3 - \vec{r}_4 \tag{16}$$

$$\vec{z} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} - \frac{m_3\vec{r}_3 + m_4\vec{r}_4}{m_3+m_4} \qquad \vec{R} = \frac{\sum m_i\vec{r}_i}{\sum m_i} \tag{17}$$

where particles 1 and 2 are quarks and 3 and 4 antiquarks. The four-body problem is solved using a wave function that includes all the possible flavor-spin-color channels that contribute to a given configuration. For each channel s, such a wave function will be a tensor product of a color (g_s), flavor (f_s), spin (χ_s) and spatial (R_s) wave functions

$$\mid \phi_s >= g_s(1234) f_s(1234) \chi_s(1234) R_s(1234) \tag{18}$$

Concerning the spatial wave function, the most general one with $L = 0$ may depend on six scalar quantities

$$R_s(1234) = R_s(x^2, y^2, z^2, \vec{x}\cdot\vec{y}, \vec{x}\cdot\vec{z}, \vec{y}\cdot\vec{z}) \tag{19}$$

Most part of tetraquarks studies have been done by means of a completely symmetric spatial wave function, depending only on the quadratic terms, x^2, y^2 and z^2. This is a restricted choice which we will analyze in our calculation. We define our variational spatial wave function as a linear combination of gaussians

$$R_s(1234) = \sum_{i=1}^{n} \beta_s^{(i)} R_s^{(i)} = \sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)}\vec{x}^2 - b_s^{(i)}\vec{y}^2 - c_s^{(i)}\vec{z}^2 - d_s^{(i)}\vec{x}\vec{y} - e_s^{(i)}\vec{x}\vec{z} - f_s^{(i)}\vec{y}\vec{z}} \tag{20}$$

where n is the number of gaussians we use to expand the spatial wave function of each color-spin-flavor component.

With respect to the color wave function, one can couple the two quarks $(1,2)$ and the two antiquarks $(3,4)$ to color singlet in different ways

$$| 1_{13}, 1_{24} > \ , \ | 8_{13}, 8_{24} > \tag{21}$$
$$| 1_{14}, 1_{23} > \ , \ | 8_{14}, 8_{23} > \tag{22}$$
$$| \overline{3}_{12}, 3_{34} > \ , \ | 6_{12}, \overline{6}_{34} > \tag{23}$$

The first two couplings are convenient for asymptotic meson-meson channels (or meson-meson molecules) while the third one is more appropriate for tetraquark bound states. With our choice of the Jacobi coordinates the last color basis results to be more suitable and essentially to treat the Pauli principle in an easy way.

The spin part of the wave function can be written as

$$| S_i >= [(12)_{S_{12}}(34)_{S_{34}}]_S \tag{24}$$

where the spin of the two quarks is coupled to S_{12} and that of the antiquarks to S_{34}. Then we will have the following basis vectors as a function of the total spin S=0,1,2:

$$S = 0 \Rightarrow \begin{cases} | S_1 >= [(12)_0(34)_0]_0 \\ | S_2 >= [(12)_1(34)_1]_0 \end{cases} \tag{25}$$

$$S = 1 \Rightarrow \begin{cases} | S_3 >= [(12)_0(34)_1]_1 \\ | S_4 >= [(12)_1(34)_0]_1 \\ | S_5 >= [(12)_1(34)_1]_1 \end{cases} \tag{26}$$

$$S = 2 \Rightarrow \left\{ | S_6 >= [(12)_1(34)_1]_2 \right. \tag{27}$$

Concerning the flavor part, we consider light quarks those with flavor u and d and heavy ones those with flavor s, c and b. The heavy quarks have isospin zero so they do not contribute to the total isospin. Therefore we can classify the tetraquark wave function by the isospin of the light quarks $I = 0, 1$. Taken into account all degrees of freedom, the Pauli principle must be satisfied for each subsystem of identical quarks (antiquarks). It imposes restrictions on the quantum numbers of the basis states, which is the justification to use the coupling $[(qq)(\bar{Q}\bar{Q})]$.

Using the wave function described above, we search for a variational solution of the hamiltonian of Eq. (15). The spin, color and flavor parts are integrated out and the $\beta_s^{(i)}$ coefficients of the spatial wave function are obtained by solving the system of linear equations

$$\sum_s \sum_{i=1}^{n} \beta_s^{(i)} \left[\langle R_{s'}^{(j)} |\, H \,| R_s^{(i)} \rangle - E \, \langle R_{s'}^{(j)} | R_s^{(i)} \rangle \delta_{s,s'} \right] = 0 \qquad \forall \, j, s' \tag{28}$$

once the eigenvalues $E(a_s^{(i)}, ..., f_s^{(i)})$ are obtained using a minimization procedure. In practice, we use the MINUIT package [25] to obtain the energies. The stable tetraquark states are identified by comparing the obtained eigenvalues with the corresponding meson-meson threshold calculated with the same hamiltonian.

IV. RESULTS

As we have already discussed, the parameters appearing in our hamiltonian corresponding to the light sector are fixed from the NN interaction [12]. However there are others which cannot be determined in this way either because the NN interaction does not depend on them (e.g. the confinement parameters) or because they appear as a consequence of the generalization to strange and heavy flavors. Among the thoughtful criteria for fixing these remaining parameters the most commonly used one is to fit the meson spectra. Being the meson masses the ones that will govern the tetraquark thresholds and therefore will determine if the tetraquark is bound or not, we think that a good fit of meson spectra must be the most important criterium. This criterium is the one proposed long ago by Isgur and collaborators [26] and also by Manohar and Wise [8], and it has been also adopted in many other works, which use the Bhaduri potential whose parameters are fitted to charmonium [9]. Therefore, we have performed fits of the $q\bar{q}$ sector using the tetraquark wave functions in the limit where the two component mesons are isolated, what is equivalent to solve the $q\bar{q}$ system [26]. The complete set of parameters is shown in Table I and the corresponding meson ground state masses are given in Table II.

Tetraquarks will be stable under the strong interaction if their total energy lies below all the possible, and allowed, two-meson thresholds. Therefore we will use the quantity

$$\Delta E = E_T(qq\bar{Q}\bar{Q}) - E_{m_1}(q\bar{Q}) - E_{m_2}(q\bar{Q}) \tag{29}$$

to discriminate the stable tetraquark states.

We focus our attention on the lowest $L = 0$, positive parity, states of the $qq\bar{Q}\bar{Q}$ system with $Q = b, c, s$ and $q = u, d$ for all possible spin-isospin combinations. As a first step we consider a spatial wave function with a single gaussian and we compare the results obtained for the tetraquark energies using the most general wave function [see Eq. (20)] or a completely symmetric spatial wave function. The energy difference obtained are always smaller than 1%. Therefore we will use for the spatial wave function combinations of the quadratic terms, $R(x^2, y^2, z^2)$, except for those spin-isospin channels which are Pauli forbidden if we use such a symmetric spatial wave function. For these channels, which correspond to $(S, I) = (0, 0)$ and $(2, 0)$, we make the calculation with the full wave function.

The variational calculation is done using as spatial wave function a sum of gaussians. We start with one gaussian and we increase its number until convergence in the tetraquark energy is reached. The typical number of gaussians needed is six.

The results are presented in Table III. Our calculation predicts two bound states, one for the $qq\bar{b}\bar{b}$ system and one for the $qq\bar{c}\bar{c}$, in both cases with $(S, I) = (1, 0)$. No bound states are found for the $qq\bar{s}\bar{s}$ system, the $(S, I) = (1, 0)$ channel being again the most attractive one. The other (S, I) channels are strongly repulsive, specially those where the Pauli principle forbids the spatially symmetric wave function, except for the $(S, I) = (1, 1)$ and $(2, 1)$ for the $qq\bar{b}\bar{b}$ system. Comparing with the existing literature we find two different types of calculations: those including only confinement and one-gluon exchange, Refs. [9] and [28] and those considering different models of Goldstone boson exchanges between quarks, Refs. [8] and [11]. Models including only one-gluon exchange predict systematically a bound state in the $(S, I) = (1, 0)$ channel for the $qq\bar{b}\bar{b}$ system and no bound state for the $qq\bar{c}\bar{c}$ system. Among the others one finds different conclusions. In Ref. [8] a weakly bound state is found only in the $qq\bar{b}\bar{b}$ system, whereas Ref. [11] reports bound states for both $qq\bar{b}\bar{b}$ and $qq\bar{c}\bar{c}$.

To clarify the importance of the different pieces of the interacting hamiltonian we have redone our calculations switching-off the pseudoscalar part, Eq.(4), of the Goldstone boson exchanges. The results are shown in Table IV. A first glance to the results tells us that the bound states have disappeared. This part of the interaction plays a relevant role to bind the $(S, I) = (1, 0)$ channel [11] and it also favors the binding in the $(S, I) = (0, 0)$ but not in the other channels. One can easily understand the difference with the results of Ref. [9], quoted in Table IV, due to the larger strong coupling constant used in these works. This is due to an obvious renormalization of the gluon exchange parameters in order to reproduce the meson spectra in two different formalisms (gluon exchange alone and gluon exchange + boson exchanges).

There is an important aspect that should be emphasized at this point. For the predicted bound systems the thresholds are determined by D and B mesons. Such mesons are described just in terms of a confining and a one-gluon exchange interaction [see Eq. (14)]. Therefore, one could use the same restricted interaction to calculate the total energy of the tetraquark, but one would obtain different results (see Table IV). The reason for this discrepancy stems from the fact that in this case there also appear interactions between light quarks that need to be described by means of a more elaborated interaction. As we have previously mentioned this shows the importance of reproducing the full meson spectrum (and not a reduced set of states) to make thoughtful predictions.

The structure of our interaction also allows for a study of the influence of the different color configurations in the tetraquarks binding energy. In Table V we present the results for the probabilities of the two color components $\{\bar{3}3\}$ and $\{6\bar{6}\}$ in the tetraquark wave function. We observe that for the bound states channel the probability of $\{6\bar{6}\}$ is almost negligible, but its influence increases when the heavy quark mass decreases. Similar results have also been reported in the context of a pure one-gluon exchange model [4]. However, for unbound channels this probability tends to increase reaching values of 25% for the $qq\bar{s}\bar{s}$ $(S, I) = (0, 1)$. This would be an indication that the $\{6\bar{6}\}$ channel may be important for the pure light tetraquark sector and therefore should not be neglected a priori.

The dominance of the $\{\bar{3}3\}$ channel has a more clear physical interpretation if we change the color basis from $\{| \,\bar{3}_{12}, 3_{34} >, | \,6_{12}, \bar{6}_{34} >\}$ to $\{| \,1_{13}, 1_{24} >, | \,8_{13}, 8_{24} >\}$. As stated above, the second basis is more appropriate to describe asymptotic channels because the first vector describes two physical mesons, whereas the second one describes two colored meson states. The relation between them is given by

$$
\begin{aligned}
\mid \overline{3}_{12}, 3_{34} >&= \sqrt{\frac{1}{3}} \mid 1_{13}, 1_{24} > - \sqrt{\frac{2}{3}} \mid 8_{13}, 8_{24} > \\
\mid 6_{12}, \overline{6}_{34} >&= \sqrt{\frac{2}{3}} \mid 1_{13}, 1_{24} > + \sqrt{\frac{1}{3}} \mid 8_{13}, 8_{24} >
\end{aligned} \tag{30}
$$

Being the $\mid \overline{3}_{12}, 3_{34} >$ channel the dominant one, the contribution to the tetraquark binding energy of the colored meson channel is more important than the two physical meson states. This fact can be interpreted in the sense that our tetraquarks are a more complicated object than a pure meson-meson resonance.

Let us finally discuss the possible decay channels of these structures. The allowed decay modes depend on the relationship between the tetraquark mass and the sum of the masses of the possible decay products. In the case of the charmed tetraquark the results shown in Table III satisfy $m_D + m_{D^*} \geq m_{(qq)(\bar{c}\bar{c})} \geq 2m_D$, and therefore the strong decay is forbidden, being the most probable decay the electromagnetic process $T[(qq)(\bar{c}\bar{c})] \rightarrow D^+D^0\gamma$, that could be identified detecting a relatively soft photon, $E_\gamma \leq 140$ MeV. In the case of the $qq\bar{b}\bar{b}$ structure we obtain (see Table III) $m_{(qq)(\bar{b}\bar{b})} - 2m_B < 0$, and therefore it can only decay through two consecutive weak processes.

V. SUMMARY

We were faced in this work with the possible existence of tetraquark structures with heavy-light flavors in a variational approach. To be as much predictive as possible we have started from a chiral constituent quark model which is able to describe the NN interaction and also gives a reasonable description of the non-strange baryon spectrum. The generalization to strange and heavy flavors has been asked to produce a nice fit of the meson spectrum. We have proved that for the tetraquark description it is enough to consider a simplified variational wave function depending only on the square of the Jacobi coordinates except for those cases for which this wave function is forbidden by the Pauli principle. We have found that the influence of the $\{6\overline{6}\}$ color channel could be important in the light sector. For the heavy flavors its contribution is almost negligible which suggests a non-trivial color structure for the tetraquark in terms of meson-meson components.

Our calculation predicts the existence of only one bound tetraquark with $(S, I) = (1, 0)$ in both the bottom and the charm sectors. While the first one should decay weakly, the second one would do it electromagnetically.

VI. ACKNOWLEDGMENTS

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TABLES

TABLE I. Quark-model parameters.

a_c (MeV)	430.0	m_S (fm^{-1})	3.42	$m_{u,d}$ (MeV)	313
μ_c (fm^{-1})	0.7	m_{PS} (fm^{-1})	0.7	m_s (MeV)	555
$g_{ch}^2/4\pi$	0.54	m_K (fm^{-1})	2.509	m_c (MeV)	1752
α_0	2.1181	m_η (fm^{-1})	2.772	m_b (MeV)	5100
Λ_0 (fm^{-1})	0.113178	$\Lambda_{PS,S}$ (fm^{-1})	4.2	θ_p (rad)	-0.2618
μ_0 (MeV)	36.9762	$\Lambda_{K,\eta}$ (fm^{-1})	5.2	$\hat{r}_0$ (fm MeV)	28.17
		Δ (MeV)	181.1		

TABLE II. Meson spectra for the model described on the text. Experimental data are taken from Ref. [27]. All masses are given in MeV.

Meson	π	η	ρ	η'	ϕ	K	K^*
Result	139.5	571.6	771.6	955.7	1020.4	496.0	910.5
Experiment	138.04	547.30	771.10	957.78	1019.46	495.0	891.66
Meson	D	D^*	η_c	J/ψ	B	B^*	Υ
Result	1883.2	2008.9	2989.9	3097.0	5280.8	5321.2	9505.0
Experiment	1867.7	2008.9	2979.7	3096.87	5279.2	5325.0	9460.30

TABLE III. $E_T(qq\bar{Q}\bar{Q})$ and ΔE energies in MeV.

(S,I)		(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$[(qq)(\bar{s}\bar{s})]$	E_T	2239	1804	1528	1940	2681	2020
	ΔE	+1247	+812	+126	+538	+860	+199
$[(qq)(\bar{c}\bar{c})]$	E_T	4351	4150	3764	4186	4849	4211
	ΔE	+585	+384	-129	+293	+830	+192
$[(qq)(\bar{b}\bar{b})]$	E_T	10820	10690	10261	10698	11350	10707
	ΔE	+258	+128	-341	+96	+708	+65

TABLE IV. $E_T(qq\bar{Q}\bar{Q})$ and ΔE in MeV neglecting the pseudoscalar part of the Goldstone boson exchanges.

(S,I)		(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$[(qq)(\bar{s}\bar{s})]$	E_T	2630	1999	1996	2139	2772	2186
	ΔE	+1638	+1007	+594	+737	+951	+365
$[(qq)(\bar{c}\bar{c})]$	E_T	4666	4175	4101	4231	4851	4254
	ΔE	+900	+409	+208	+338	+832	+235
	ΔE from Ref. [9]			+19			
$[(qq)(\bar{b}\bar{b})]$	E_T	11136	10735	10612	10743	11352	10752
	ΔE	+574	+173	+10	+141	+710	+110
	ΔE from Ref. [9]			−131	+56		+30

TABLE V. Probability of the different color channels.

(S,I)	(0,1)		(1,0)	
	$P_{\{\bar{3}3\}}$	$P_{\{6\bar{6}\}}$	$P_{\{\bar{3}3\}}$	$P_{\{6\bar{6}\}}$
$[(qq)(\bar{s}\bar{s})]$	0.7469	0.2531	0.9706	0.0294
$[(qq)(\bar{c}\bar{c})]$	0.8351	0.1649	0.9940	0.060
$[(qq)(\bar{b}\bar{b})]$	0.9895	0.0105	0.9993	0.0007