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A $SU(4) \otimes O(3)$ scheme for nonstrange baryons

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Abstract. We show that nonstrange baryon resonances can be classified according to multiplets of $SU(4) \otimes O(3)$. We identify spectral regularities and degeneracies that allow us to predict the high spin spectrum from 2 to 3 GeV.

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1 Introduction

The theoretical study of the high energy part of the baryonic spectrum has been a subject of interest in the last decade, the aim being to get a better understanding of the dynamics (in particular the confinement mechanism of quarks in the baryon and the decay hadronization process), or, at least, the symmetries involved. In particular the idea of a parity multiplet classification scheme at high excitation energies as due to chiral symmetry was suggested some years ago [1] and put in question later on [2]. The lack of precise and complete data prevents, at the current moment, to extract any definitive conclusion.

From the point of view of dynamics quark model refined potentials with linear confinement have provided a reasonably accurate description of the whole nonstrange light baryon spectrum once the coupling to πN formation channels is taken into account [3]. The low probability obtained for many resonances to be formed would explain their no experimental detection providing a solution to the so-called missing state problem (the difference between the number of predicted states above 1 GeV excitation energy -infinite with a linear potential- and the number of known resonances).

Alternatively, the use of a quark-quark screened potential [4,5,6], motivated by recent unquenched QCD lattice calculations showing string breaking in the static potential between two quarks [7,8], allows to obviate the missing state problem (up to the limit of applicability of the model). In Refs. [4] and [6] a correct prediction of the number and ordering of the known N and Δ resonances, up to 2.4 GeV mass or 1.5 GeV excitation energy, is obtained. Above this limit the 3-free quark state is energetically favored pointing out the need to implement the coupling to the continuum. Nonetheless the unambiguous assignment of quantum numbers to experimental states in the region of applicability translates, as we shall show, into a well defined symmetry pattern.

In this article we identify the symmetry pattern as the one corresponding to $SU(4) \otimes O(3)$, $SU(4)$ containing $SU(2)_{\rm spin} \otimes SU(2)_{\rm isospin}$ and $O(3)$ standing for the orbital symmetry, and we analyze spectral regularities and degeneracies according to it. The extension of this pattern to energies above the applicability limit of the model allows us to predict the spectrum in the range $2-3$ GeV where only incomplete and non-precise data exist.

2 $SU(4) \otimes O(3)$ Pattern

In Ref. [6] a quark model including confinement and minimal one gluon exchange (coulomb + hyperfine) interactions has been developed. Screening is imposed by requiring that the interaction potential saturates (i.e., becomes constant) at a certain distance to be fixed phenomenologically. Though one cannot obtain a precise fit to the spectrum with such a simplistic model it is amazing that one can make an unambiguous assignment of quantum numbers to the dominant configuration of any J^P ground and first non-radial states up to $J = 11/2$. This assignment agrees completely with the ones available in the literature (only up to $J = 7/2$) with much more refined theoretical models [9] or purely phenomenological analysis [10].

In Tables 1 and 2 we group experimental resonances according to their dominant configuration (the symmetry pattern obtained has been extended up to 2 GeV excitation energy). To express the spatial part we use the hyperspherical harmonic notation, i.e., the quantum numbers $(K, L, Symmetry)$. The so-called great orbital, K, defines the parity of the state, $P = (-)^K$, and its centrifugal barrier energy, $\frac{\mathcal{L}(\mathcal{L}+1)}{2m\langle\rho^2\rangle}$ ($\mathcal{L} = K + \frac{3}{2}$, ρ: hyperradius). L is the total orbital angular momentum. $Symmetry$ specifies the spatial symmetry ([3] : symmetric, [21] : mixed, [111] : antisymmetric) which combines to the spin, S, and isospin, T, symmetries ($S,T = 3/2$: symmetric; $S,T = 1/2$: mixed) to have a symmetric wave function (the color part is antisymmetric). More precisely, $T = 1/2$ for N and $T = 3/2$ for Δ, hence the spatial-spin wave function must be mixed for N and symmetric for Δ.

A look at the Tables makes manifest the underlying symmetry.

For positive parity each box in the upper part of Table 1 groups four $N(J^P)$ ground states -two isospin and two spin projections- and sixteen $\Delta(J^P)$ ground states -four isospin and four spin projections- corresponding to an orbitally symmetric configuration. It is worth to mention that the N states have also some probability of mixed orbital symmetry with the same values of K and L. This mixing explains the appearance of an additional $N(3/2^+)$, which is the symmetry partner of $\Delta(1/2^+)$, in the second upper box as a consequence of the bigger hyperfine attraction for orbitally symmetric $S = 1/2$ states. Another consequence of the mixing is the presence of corresponding N excitations with reverse probabilities that appear in the boxes of the lower part of Table I. Any of these excitations adds four states -two isospin and two spin projections- to the eight N's and eight Δ's ground states present in each box.

For negative parity, Table 2, the same box pattern repeats with different combinations of N's and Δ's.

This 20 member box picture where all the members of the same box have the same parity given by $P = (-)^L$, is a reflection of an underlying $SU(4) \otimes O(3)$ symmetry providing a $(20, L^P)$ classification scheme, the 20plet structure coming out naturally from the product of irreducible quark representations: $4 \otimes 4 \otimes 4 = 20_S \oplus 20_M \oplus 20_M \oplus \overline{4}$. The only difference in content between a box and the corresponding 20plet refers to mixed N resonances being a linear combination of N members of the 20plets with well defined orbital symmetry.

It is worth to emphasize that $SU(4)$ goes beyond a factorization $SU(2) \otimes SU(2)$ as can be checked through the $N - \Delta$ degeneracies appearing within the same box when the $SU(4)$ breaking spin-spin interaction plays a minor role.

3 Spectral regularities and degeneracies

From the spectral pattern represented by Tables 1 and 2 experimental regularities and degeneracies for $J \geq 5/2$ ground states come out:

i) $E_{N,\Delta}(J+2) - E_{N,\Delta}(J) \approx 400 - 500$ MeV

ii) $N(J^{\pm}) \approx \Delta(J^{\pm})$ for $J = \frac{4n+3}{2}$, $n = 1, 2...$

iii) $N(J^+) \approx N(J^-)$ for $J = \frac{4n+1}{2}$, $n = 1, 2.$

These rules can also be obtained theoretically by refitting the quark model to reproduce precisely the $J \geq 5/2$ states. Rule i) expresses the increasing of the centrifugal barrier between states with the same orbital symmetry and the slowly varying spin-spin contribution for the same S when increasing L for $L \geq 2$. Rule ii) for positive parity reflects the small spin-spin contribution for $S = 3/2$ when $L \geq 2$, and for negative parity reflects the $SU(4) \otimes O(3)$ degeneracy for $N's$ and $\Delta's$ in the same multiplet once the centrifugal barrier suppresses greatly the hyperfine splitting. Rule iii) for N parity doublets comes from the balance between a bigger repulsion (due to bigger K and L) and a bigger hyperfine attraction (due to lower S) for $N(J^+)$ against $N(J^-)$. No parallel degeneracy for Δ's is found since the J^- states should be higher in mass (bigger centrifugal repulsion and spin-spin repulsion as well) than the J^+ ones. These results are in disagreement with the parity multiplet classification scheme proposed in Ref. [1].

For excited states the absence of spin-orbit and tensor forces in our dynamical model suggests a new rule for $J \geq 5/2$:

iv) $(N(J), \Delta(J))^{\bullet} \approx (N(J+1), \Delta(J+1))$

say the first non-radial excitation of $N(J)$ and the ground state of $N(J+1)$ are almost degenerate (the same for Δ). This rule is well satisfied by experimental data.

Taking into account rules i)-iv) and the developed symmetry pattern we can make predictions for, until now, unknown states from 2 to 3 GeV, Table 3. Though some of our predicted states might be masked by experimental uncertainties and others could not be easily detected (small coupling to formation channels) we hope the results in Table 3 may be of some help to guide future experimental searches.

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References

1. D. Jido, T.Hatsuda, and T. Kunihiro, Phys. Rev. Lett. **84** (2000) 3252; D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. **106** (2001) 873; T.D. Cohen and L. Ya. Glozman, Phys. Rev. D **65** (2002) 016006.
2. R.L. Jaffe, D. Pirjol, and A. Scardicchio, Phys. Rev. Lett. **96** (2006) 121601; hep-ph/0602010.
3. S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. **45**, (2000) S241, and references therein.
4. J. Vijande, P. González, H. Garcilazo, and A. Valcarce, Phys. Rev. D **69** (2004) 074019.
5. Z. Zhang, Y.Yu, P. Shen, X. Shen, and Y. Dong, Nucl. Phys. A **561** (1993) 595.
6. P. González, J. Vijande, A.Valcarce, and H. Garcilazo, Eur. Phys. J. A **29** (2006) 235.
7. G.S. Bali, Phys. Rep. **343** (2001) 1.
8. SESAM Collaboration, G.S. Bali, H. Neff, T. Düssel, T. Lippert, and K. Schilling, Phys. Rev. D **71** (2005) 114513.
9. S. Capstick and W. Roberts, Phys. Rev. D **47** (1993) 1994.
10. E. Klempt, Phys. Rev. C **66** (2002) 058201.
11. S. Eidelman *et al.*, Phys. Lett. B **592** (2004) 1.

Table 1. Positive parity N and Δ states (masses in MeV) for different dominant spatial-spin configurations up to $\simeq 3$ GeV. Experimental data are from PDG [11]. Stars have been omitted for four-star resonances. States denoted by a question mark correspond to predicted resonances that do not appear in the PDG (their predicted masses appear in Table 3).

$(K, L, Symmetry)$	$S = 1/2$	$S = 3/2$
$(0, 0, [3])$	$N(1/2^+)(940)$	
		$\Delta(3/2^+)(1232)$
$(2, 2, [3])$	$N(5/2^+)(1680), N(3/2^+)(1720)$	
		$\Delta(7/2^+)(1950)$
$(4, 4, [3])$	$N(9/2^+)(2220)$	
		$\Delta(11/2^+)(2420)$
$(6, 6, [3])$	$N(13/2^+)(**)(2700)$	
		$\Delta(15/2^+)(**)(2950)$
$(2, 0, [21])$	$N(1/2^+)(***)(1710)$	
	$\Delta(1/2^+)(1750)$	
$(2, 2, [21])$	$N(5/2^+)(**)(2000)$	
		$N(7/2^+)(**)(1990)$
	$\Delta(5/2^+)(1905)$	
$(4, 4, [21])$	$N(9/2^+)(2220)$	
		$N(11/2^+)(?)$
	$\Delta(9/2^+)(**)(2300)$	
$(6, 6, [21])$	$N(13/2^+)(2700)$	
		$N(15/2^+)(?)$
	$\Delta(13/2^+)(?)$	

Table 2. Negative parity N and Δ states (masses in MeV) for different dominant spatial-spin configurations up to $\simeq 3$ GeV. Experimental data are from PDG [11]. Stars have been omitted for four-star resonances. States denoted by a question mark correspond to predicted resonances that do not appear in the PDG (their predicted masses appear in Table 3).

$(K, L, Symmetry)$	$S = 1/2$	$S = 3/2$
$(1, 1, [21])$	$N(3/2^-)(1520), N(1/2^-)(1535)$	
		$N(5/2^-)(1675)$
	$\Delta(3/2^-)(1700), \Delta(1/2^-)(1620)$	
$(3, 3, [21])$	$N(7/2^-)(2190)$	
		$N(9/2^-)(2250)$
	$\Delta(7/2^-)(*)(2200)$	
$(5, 5, [21])$	$N(11/2^-)(***)(2600)$	
		$N(13/2^-)(?)$
	$\Delta(11/2^-)(?)$	
$(3, 3, [3])$	$N(7/2^-)(?)$	
		$\Delta(9/2^-)(**)(2400)$
$(5, 5, [3])$	$N(11/2^-)(?)$	
		$\Delta(13/2^-)(**)(2750)$

Table 3. Predicted N and Δ states in the interval $[2.2, 3.0]$ MeV. We denote by a black dot the first non-radial excitation.

	N		Δ	
$J = 7/2$	$N(7/2^+)^\bullet(2220)$	$N(7/2^-)^\bullet(2250)$		$\Delta(7/2^-)^\bullet(2400)$
$J = 9/2$	$N(9/2^+)^\bullet(2450)$	$N(9/2^-)^\bullet(2600)$	$\Delta(9/2^+)^\bullet(2420)$	$\Delta(9/2^-)^\bullet(2650)$
$J = 11/2$	$N(11/2^+)(2450)$			$\Delta(11/2^-)(2650)$
	$N(11/2^+)^\bullet(2700)$	$N(11/2^-)^\bullet(2650)$	$\Delta(11/2^+)^\bullet(2850)$	$\Delta(11/2^-)^\bullet(2750)$
$J = 13/2$		$N(13/2^-)(2650)$	$\Delta(13/2^+)(2850)$	
	$N(13/2^+)^\bullet(2900)$		$\Delta(13/2^+)^\bullet(2950)$	
$J = 15/2$	$N(15/2^+)(2900)$			