

On the existence of exotic and non-exotic multiquark meson states

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Abstract. To obtain an exact solution of a four-body system containing two quarks and two antiquarks interacting through two-body terms is a cumbersome task that has been tackled with more or less success during the last decades. We present an exact method for the study of four-quark systems based on the hyperspherical harmonics formalism that allows us to solve it without resorting to further approximations, like for instance the existence of diquark components. We apply it to systems containing two heavy and two light quarks using different quark-quark potentials. While $QQ\bar{n}\bar{n}$ states may be stable in nature, the stability of $Q\bar{Q}n\bar{n}$ states would imply the existence of quark correlations not taken into account by simple quark dynamical models.

The discoveries on several fronts [1], of unusual charmonium states like $X(3872)$ and $Y(4260)$ and open-charm mesons with unexpected masses like $D_{sJ}^*(2317)$ and $D_0^*(2308)$, have re-invigorated the study of hadronic resonances. Any debate on the possible multiquark structure of meson resonances should be based on our capability to find an exact solution of the four-body problem [2]. Theoretical predictions often differ because of the approximation method used. A powerful tool to solve a few-particle system is an expansion of the trial wave function in terms of hyperspherical harmonics (HH) basis functions. In Ref. [3] a generalization of the HH formalism to study four-quark systems in an exact way was presented. Due to their actual interest and having in mind that systems with unequal masses are more promising [2], we will center our attention on $Q\bar{Q}n\bar{n}$ and $Q\bar{Q}n\bar{n}$ states (n stands for a light quark and Q for a heavy one). We will analyze the possible existence of compact four-quark bound states using two standard quark-quark interactions, a Bhaduri-like potential (BCN) [4] and a constituent quark model considering boson exchanges (CQC) [5]. Both interactions fulfill the requirement of giving a reasonable description of meson and baryon spectroscopy. Assuming non-relativistic quantum mechanics we solve the

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four-body Schrödinger equation. The grand angular momentum K is the main quantum number in our expansion and the calculation is truncated at some K value. Further details of the numerical method can be found in Ref. [3].

In spite of the shortcomings of the methods used to study four-quark systems, in the past, many four-quark bound states have been suggested. To analyze their stability against dissociation, parity and total angular momentum must be preserved. Additionally, C -parity is a good quantum number for $c\bar{c}n\bar{n}$ and the Pauli principle must be fulfilled in the $cc\bar{n}\bar{n}$ case. The corresponding thresholds can be computed by adding the meson masses of the dissociation channel. Four-quark states will be stable under strong interaction, and therefore very narrow, if their total energy lies below all allowed two-meson thresholds. Sometimes, results of four-quark calculations have been directly compared to experimental thresholds. In this case one could misidentify scattering wave functions as bound states. When they are referred to the thresholds within the same model, theoretical predictions do not imply an abundance of multiquark states in the data.

Table 1. Energy (MeV) and probability of the different color components for the $c\bar{c}n\bar{n}$ $J^{PC} = 1^{++}$ both for CQC and BCN models. The last rows indicate the lowest theoretical two-meson thresholds. P_{11} (P_{88}) stands for the probability of singlet-singlet (octet-octet) color components.

K	CQC			BCN		
	E	P_{11}	P_{88}	E	P_{11}	P_{88}
18	3791	0.9962	0.0038	3840	0.9995	0.0005
20	3786	0.9968	0.0032	3822	0.9996	0.0004
22	–	–	–	3808	0.9997	0.0003
$J/\psi \omega _S$	3745	1	0	3874	1	0
$\chi_{cJ} \eta _P$	4281	1	0	3655	1	0

Once the method has been established, we concentrate on the $c\bar{c}n\bar{n}$ systems as a potential structure for the $X(3872)$. To make the physics clear we compare with the $cc\bar{n}\bar{n}$ system. In particular, we focus on the $J^{PC} = 1^{++}$ $c\bar{c}n\bar{n}$ and $J^P = 1^+$ $cc\bar{n}\bar{n}$ quantum numbers to illustrate their similitudes and differences. A complete study of all the quantum numbers have been reported in Ref. [3]. For the $c\bar{c}n\bar{n}$ system, independently of the quark-quark interaction, the system evolves to a well separated two-meson state, see Table . This is clearly seen in the energy, approaching the corresponding two free-meson threshold, but also in the probabilities of the different color components of the wave function. Comparing the theoretical predictions with the experimental threshold, $M_{J/\psi \omega|_S} = 3879.57 \pm 0.13$ MeV, one could be tempted to claim for the existence of a bound state. However, the experimental threshold is not reproduced by the effective Hamiltonians. Thus, in any manner one can claim for the existence of a bound state. Similar conclusions are drawn for all quantum numbers of this system. A completely different behavior is observed in the case of $J^P = 1^+$ $cc\bar{n}\bar{n}$. The energy quickly stabilizes below the lowest theoretical thresholds (3937 MeV for CQC and 3906 for BCN), being the results obtained for $K_{max} = 24$ com-

pletely converged, 3861 MeV for CQC and 3900 for BCN. Besides, the radius is also stable and is smaller than the sum of the radius of the two-meson threshold. We obtain $r_{4q} = 0.37$ fm compared to $r_{M_1} + r_{M_2} = 0.44$ fm.

It is thus important to realize that a bound state should be pursued not only by looking at the energy, but also with a careful analysis of the radius and probabilities. This detailed analysis allows us to distinguish between compact states and meson-meson molecules [6] and it does consider the contribution of all meson-meson channels to a particular set J^{PC} of quantum numbers. Inherent to our discussion is a much richer decay spectrum of compact states due to the presence of octet-octet color components in their wave function.

Let us notice that there is an important difference between the two physical systems studied. While for the $c\bar{c}n\bar{n}$ there are two allowed physical *decay channels*, $(c\bar{c})(n\bar{n})$ and $(c\bar{n})(\bar{c}n)$, for the $cc\bar{n}\bar{n}$ only one physical system contains the possible final states, $(c\bar{n})(c\bar{n})$. This has important consequences if both systems (two- and four-quark states) are to be described within the same two-body Hamiltonian, the $c\bar{c}n\bar{n}$ will hardly present bound states, because the system will reorder itself to become the lightest two-meson state, either $(c\bar{c})(n\bar{n})$ or $(c\bar{n})(\bar{c}n)$. In other words, if the attraction is provided by the interaction between particles i and j , it does also exist in the asymptotic two meson state reflecting this attraction. This may not happen for the $cc\bar{n}\bar{n}$ if the interaction between, for example, the two quarks is strongly attractive. In this case there is no asymptotic two-meson state with such attraction, and therefore the system will bind.

Therefore, our conclusions can be made more general. If we have an N -quark system described by two-body interactions in such a way that there exists a subset of quarks that cannot make up a physical subsystem, then one may expect the existence of N -quark bound states by means of central two-body potentials. If this is not true one will hardly find N -quark bound states [7].

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